mathematics vision project

Transforming Mathematics Education

SECONDARY MATH ONE

An Integrated Approach

MODULE 8 HONORS

Connecting Algebra & Geometry

MATHEMATICSVISIONPROJECT.ORG

The Mathematics Vision Project

Scott Hendrickson, Joleigh Honey, Barbara Kuehl, Travis Lemon, Janet Sutorius

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CONNECTING ALGEBRA AND GEOMETRY

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READY, SET, GO Homework: Connecting Algebra and Geometry 8.7H

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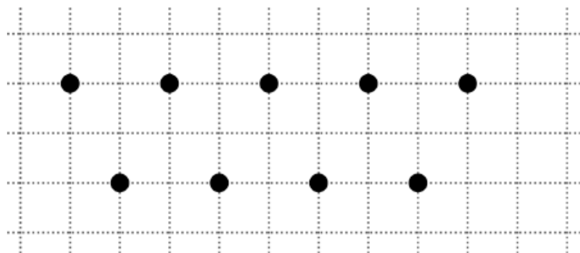
READY, SET, GO Homework: Connecting Algebra and Geometry 8.12H

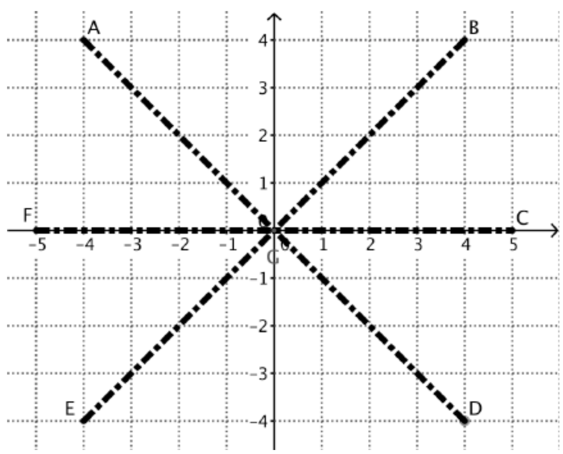
8.1 Go the Distance

A Develop Understanding Task

The performances of the Podunk High School drill team are very popular during half-time at the school's football and basketball games. When the Podunk High School drill team choreographs the dance moves that they will do on the football field, they lay out their positions on a grid like the one below:



In one of their dances, they plan to make patterns holding long, wide ribbons that will span from one dancer in the middle to six other dancers. On the grid, their pattern looks like this:

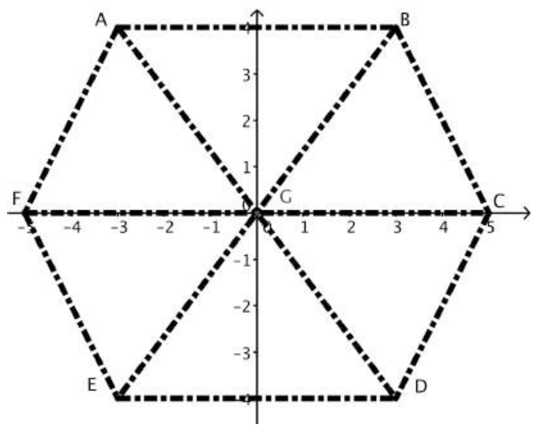


The question the dancers have is how long to make the ribbons. Gabriela (G) is standing in the center and some dancers think that the ribbon from Gabriela (G) to Courtney (C) will be shorter than the one from Gabriela (G) to Brittney (B).

1. How long does each ribbon need to be?

2. Explain how you found the length of each ribbon.

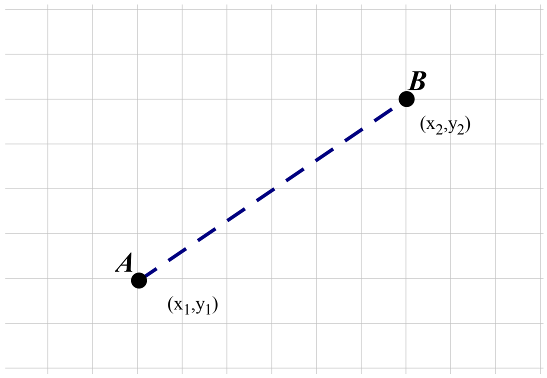
When they have finished with the ribbons in this position, they are considering using them to form a new pattern like this:



3. Will the ribbons they used in the previous pattern be long enough to go between Britney (B) and Courtney (C) in the new pattern? Explain your answer.

Gabriela notices that the calculations she is making for the length of the ribbons reminds her of math class. She says to the group, "Hey, I wonder if there is a process that we could use like what we have been doing to find the distance between any two points on the grid." She decides to think about it like this:

"I'm going to start with two points and draw the line between them that represents the distance that I'm looking for. Since these two points could be anywhere, I named them A (x_1, y_1) and B (x_2, y_2). Hmmmmm. . . . when I figured the length of the ribbons, what did I do next?"



4. Think back on the process you used to find the length of the ribbon and write down your steps here, in terms of (x_1, y_1) and (x_2, y_2).

5. Use the process you came up with in #4 to find the distance between two points located far enough away from each other that using your formula from #4 is more efficient than graphing and counting. For example find the distance between (-11, 25) and (23, -16)

6. Use your process to find the perimeter of the hexagon pattern shown in #3.

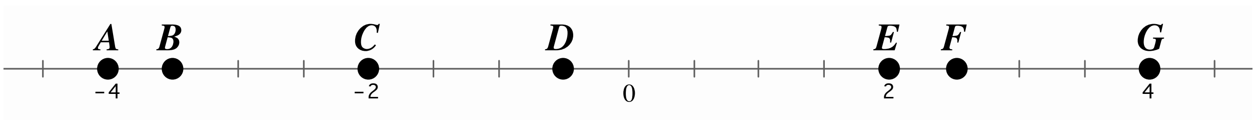
READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Finding the distance between two points

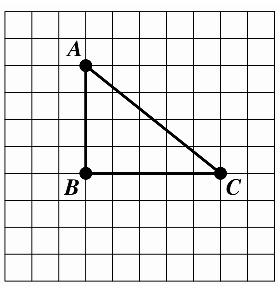
Use the number line to find the distance between the given points. (The notation AB means the distance between the points A and B.)

1. AE 2. CF 3. GB 4. CA 5. BF 6. EG



7. Describe a way to find the distance between two points on a number line without counting the spaces.

8.



a. Find AB.

b. Find BC.

c. Find AC.

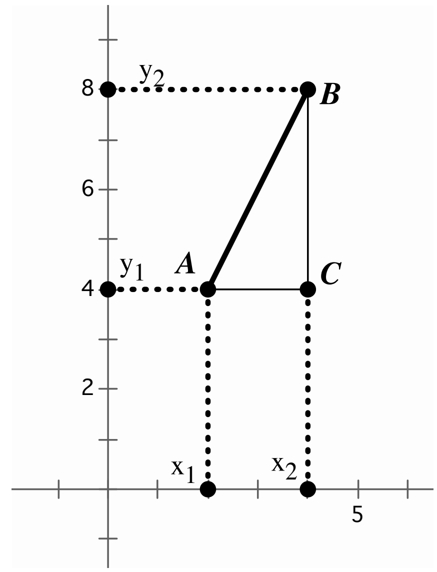
9. Why is it easier to find the distance between point A and point B and point B and point C than it is to find the distance between point A and point C?

10. Explain how to find the distance between point A and point C.

SET

Topic: Slope triangles and the distance formula

Triangle ABC is a slope triangle for the line segment AB where BC is the rise and AC is the run. Notice that the length of segment BC has a corresponding length on the y-axis and the length of AC has a corresponding length on the x-axis. The slope formula is written as $m = \frac{y_2 - y_1}{x_2 - x_1}$ *where m is the slope.*

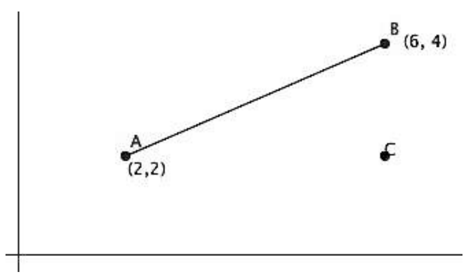


11. a. What does the value $(y_2 - y_1)$ tell you?

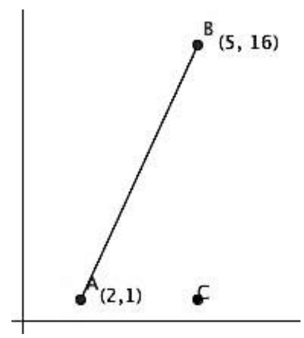
b. What does the value $(x_2 - x_1)$ tell you?

In the previous unit you found the length of a slanted line segment by drawing the slope triangle and then using the Pythagorean theorem on the two sides of the triangle. In this exercise, try to develop a more efficient method of calculating the length of a line segment by using the meaning of $(y_2 - y_1)$ *and* $(x_2 - x_1)$ *combined with the Pythagorean theorem.*

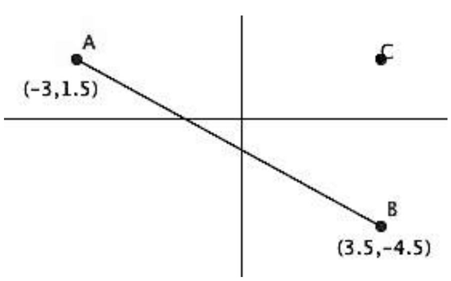
12. Find AB.



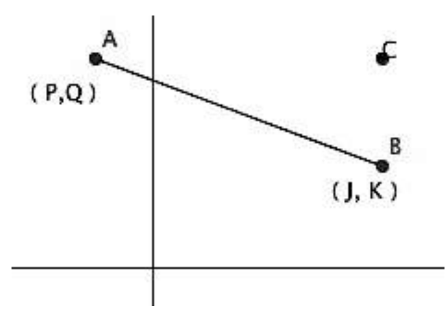
13. Find AB.



14. Find AB.



15. Find AB.

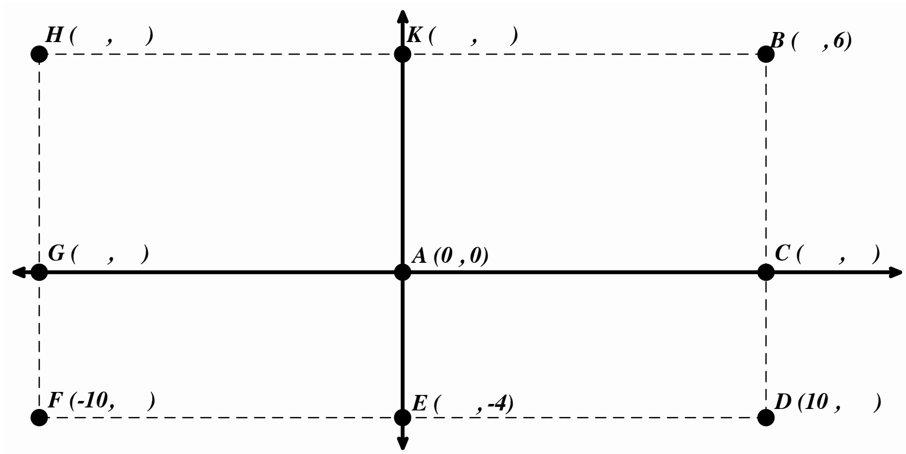


GO

Topic: Rectangular coordinates

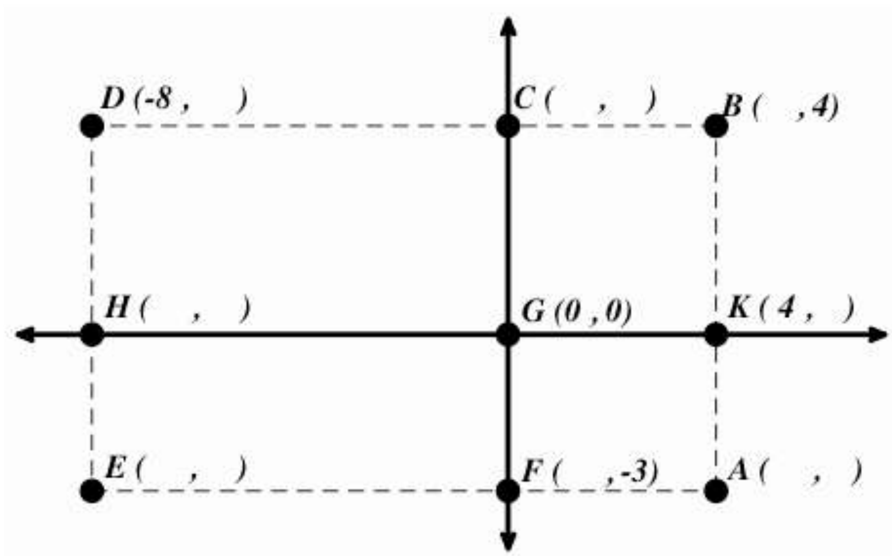
Use the given information to fill in the missing coordinates. Then find the length of the indicated line segment.

16. a) Find HB.



b) Find BD.

17. a) Find DB

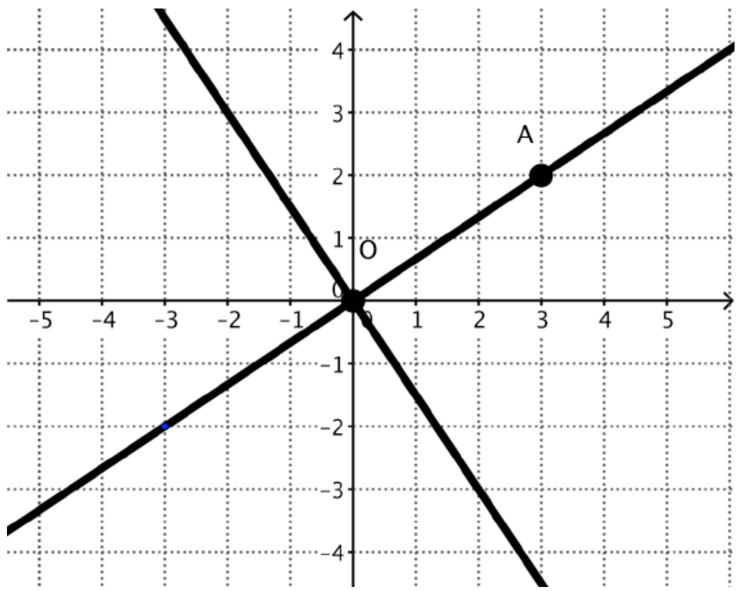


b) Find CF

8.2 Slippery Slopes

A Solidify Understanding Task

While working on "Is It Right?" in the previous module you looked at several examples that lead to the conclusion that the slopes of perpendicular lines are negative reciprocals. Your work here is to formalize this work into a proof. Let's start by thinking about two perpendicular lines that intersect at the origin, like these:



1. Start by drawing a right triangle with the segment $\overline{OA}$ as the hypotenuse. These are often called slope triangles. Based on the slope triangle that you have drawn, what is the slope of $\overleftrightarrow{OA}$?

2. Now, rotate the slope triangle 90° about the origin. What are the coordinates of the image of point A?

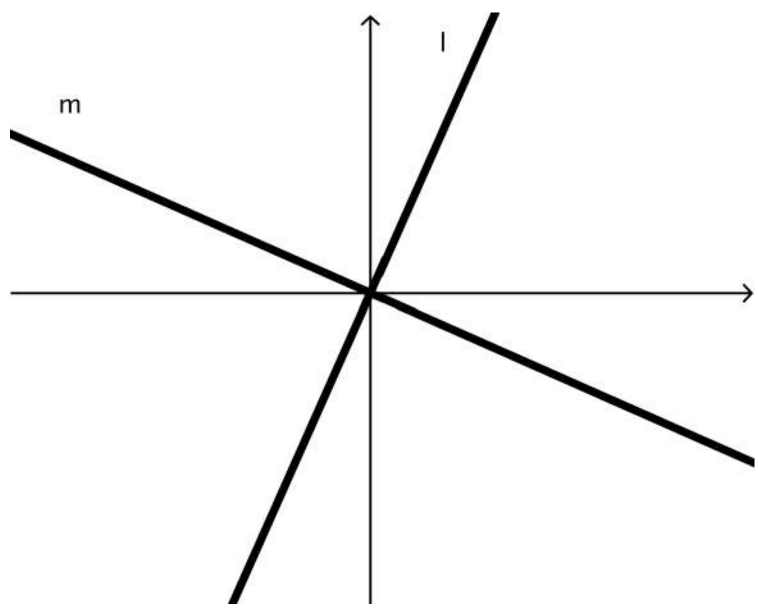
3. Using this new point, A', draw a slope triangle with hypotenuse $\overline{OA'}$. Based on the slope triangle, what is the slope of the line $\overleftrightarrow{OA'}$?

4. What is the relationship between these two slopes? How do you know?

5. Is the relationship changed if the two lines are translated so that the intersection is at (-5, 7)?

   How do you know?

To prove a theorem, we need to demonstrate that the property holds for any pair of perpendicular lines, not just a few specific examples. It is often done by drawing a very similar picture to the examples we have tried, but using variables instead of numbers. Using variables represents the idea that it doesn't matter which numbers we use, the relationship stays the same. Let's try that strategy with the theorem about perpendicular lines having slopes that are negative recipricals.



- Lines *l* and *m* are constructed to be perpendicular.
- Start by labeling a point P on the line *l.*
- Label the coordinates of P.
- Draw the slope triangle from point P.
- Label the lengths of the sides of the slope triangle using variables like *a* and *b* for the run and the rise.

6. What is the slope of line *l*?

Rotate point P 90° about the origin, label it P' and mark it on line m. What are the coordinates of P'?

7. Draw the slope triangle from point P'. What are the lengths of the sides of the slope triangle? How do you know?

8. What is the slope of line *m*?

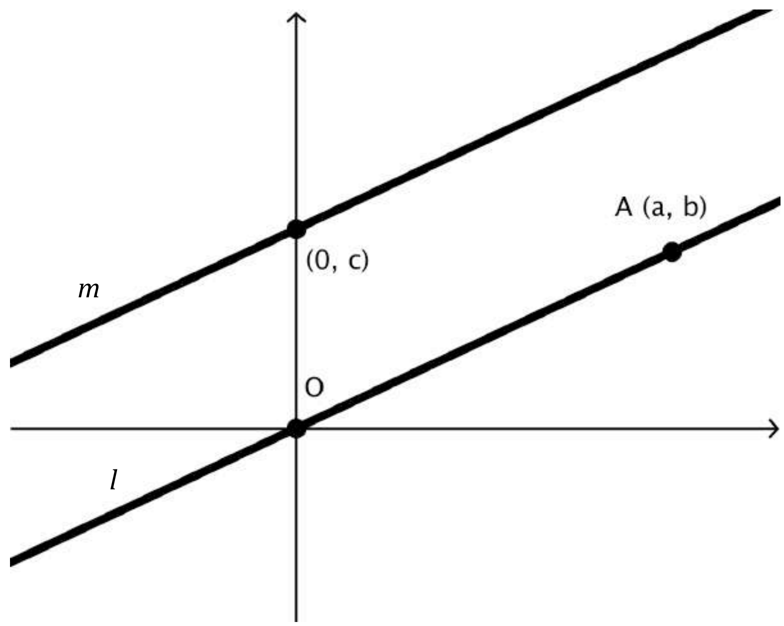
9. What is the relationship between the slopes of line *l* and line *m*? How do you know?

10. Is the relationship between the slopes changed if the intersection between line *l* and line *m* is translated to another location? How do you know?

11. Is the relationship between the slopes changed if lines *l* and *m* are rotated?

12. How do these steps demonstrate that the slopes of perpendicular lines are negative reciprocals for any pair of perpendicular lines?

Think now about parallel lines like the ones below.

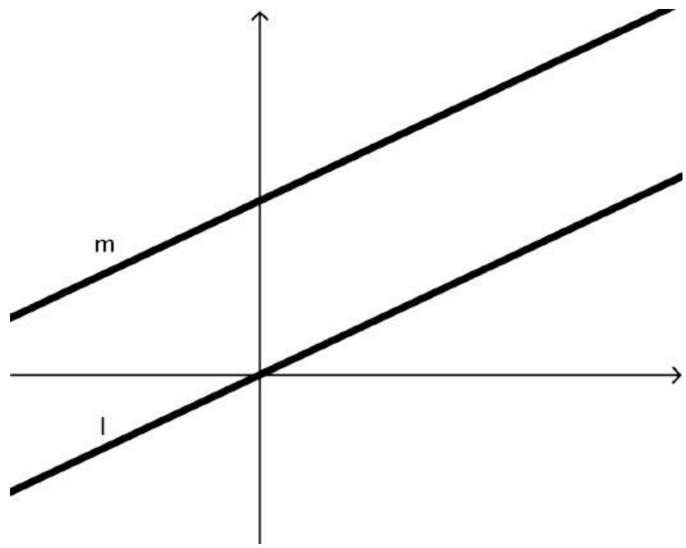


13. Draw the slope triangle from point A to the origin. What is the slope of $\overleftrightarrow{OA}$?

14. What transformation(s) maps the slope triangle with hypotenuse $\overline{OA}$ onto the other line m?

15. What must be true about the slope of line l? Why?

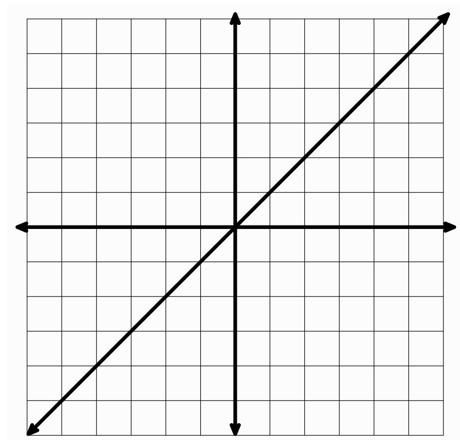
Now you're going to try to use this example to develop a proof, like you did with the perpendicular lines. Here are two lines that have been constructed to be parallel.



16. Show how you know that these two parallel lines have the same slope and explain why this proves that all parallel lines have the same slope.

READY, SET, GO! **Name** **Period** **Date**

READY



Topic: Using translations to graph lines

The equation of the line in the graph is $y = x$.

1. a) On the same grid graph a parallel line that is 3 units above it .

   b) Write the equation for the new line in slope-intercept form.

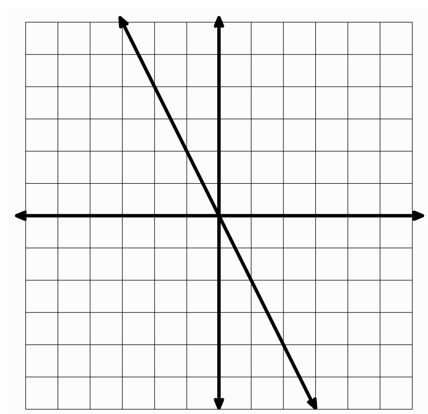
   c) Write the y-intercept of the new line as an ordered pair.

   d) Write the x-intercept of the new line as an ordered pair.

   e) Write the equation of the new line in point-slope form using the *y-intercept.*

   f) Write the equation of the new line in point-slope form using the *x-intercept.*

   g) Explain in what way the equations are the same and in what way they are different.



The graph at the right shows the line $y = -2x$.

2. a) On the same grid, graph a parallel line that is 4 units below it.

   b) Write the equation of the new line in slope-intercept form.

   c) Write the y-intercept of the new line as an ordered pair.

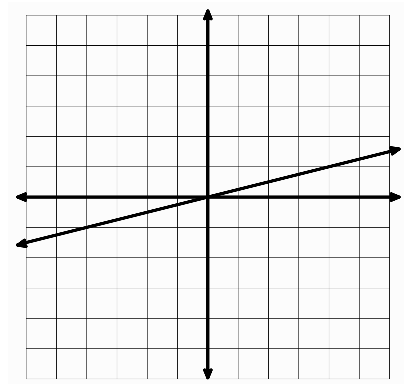
   d) Write the x-intercept of the new line as an ordered pair.

   e) Write the equation of the new line in point-slope form using the *y-intercept.*

   f) Write the equation of the new line in point-slope form using the *x-intercept.*

   g) Explain in what way the equations are the same and in what way they are different.

The graph at the right shows the line $y = \frac{1}{4}x$.



3. a) On the same grid, graph a parallel line that is 2 units below it.

   b) Write the equation of the new line in slope-intercept form.

   c) Write the y-intercept of the new line as an ordered pair.

   d) Write the x-intercept of the new line as an ordered pair.

   e) Write the equation of the new line in point-slope form using the *y-intercept.*

   f) Write the equation of the new line in point-slope form using the *x-intercept.*

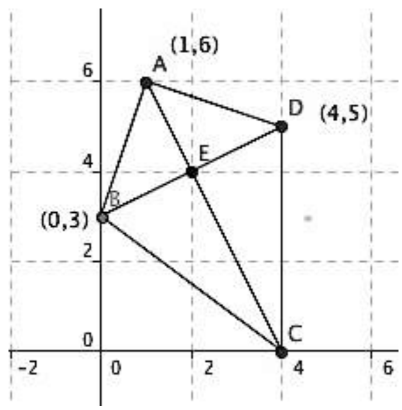
   g) Explain in what way the equations are the same and in what way they are different.

SET

Topic: Verifying and proving geometric relationships

The quadrilateral at the right is called a **kite.**
Complete the mathematical statements about the kite using the given symbols. Prove each statement algebraically. (A symbol may be used more than once.)

$\cong$ $\perp$ $\parallel$ $<$ $>$ $=$



Proof

4. $\overline{BC}$_________$\overline{DC}$ ____

5. $\overline{BD}$_________$\overline{AC}$ ____

6. $\overline{AB}$_________$\overline{BC}$ ____

7. ΔABC_____ ΔADC ____

8. $\overline{BE}$________$\overline{ED}$ ____

9. $\overline{AE}$________$\overline{ED}$ ____

10. $\overline{AC}$________$\overline{BD}$ ____

GO

Topic: Writing equations of lines

Use the given information to write the equation of the line in standard form. $(\boldsymbol{Ax + By = C})$

11. $\boldsymbol{Slope: -\frac{1}{4} \quad point\ (12, 5)}$

12. $\boldsymbol{P(11, -3),\ Q(6, 2)}$

13. $\boldsymbol{x - intercept: -2;\ \ y - intercept: -3}$

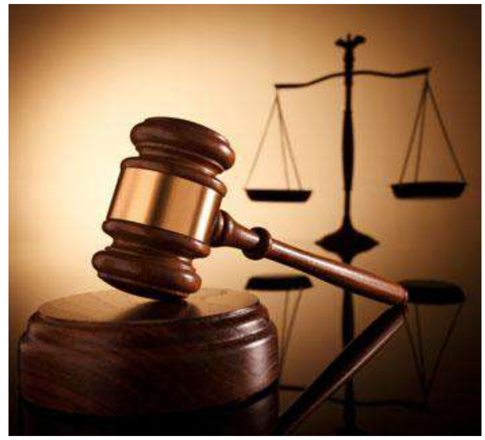
14. $\boldsymbol{All\ x\ values\ are\ (-7).\ Y\ is\ any\ number.}$

15. $\boldsymbol{Slope: \frac{1}{2};\ \ x - intercept: 5}$

16. $\boldsymbol{E\ (-10, 17),\ G\ (13, 17)}$

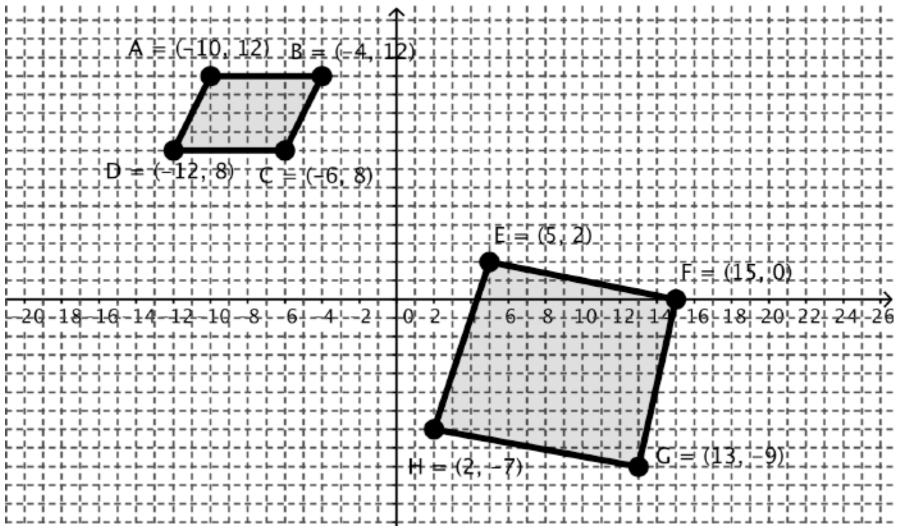
8.3 Prove It!

A Practice Understanding Task



In this task you need to use all the things you know about quadrilaterals, distance, and slope to prove that the shapes are parallelograms, rectangles, rhombi, or squares. Be systematic and be sure that you give all the evidence necessary to verify your claim.

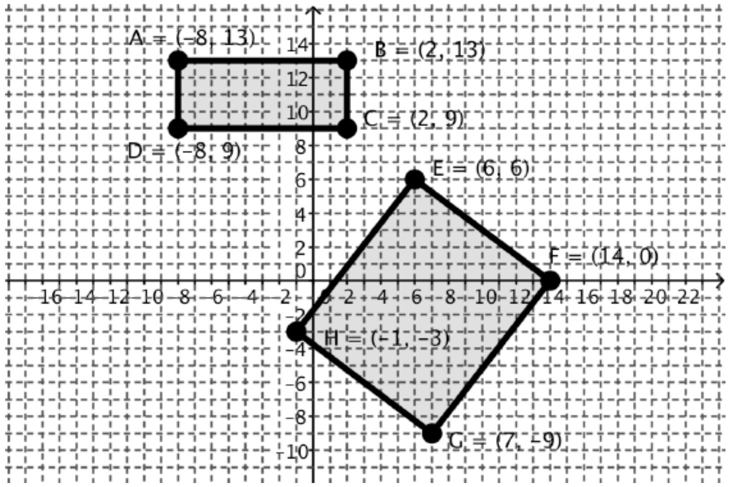
1.



a. Is ABCD a parallelogram? Explain how you know.

b. Is EFGH a parallelogram? Explain how you know.

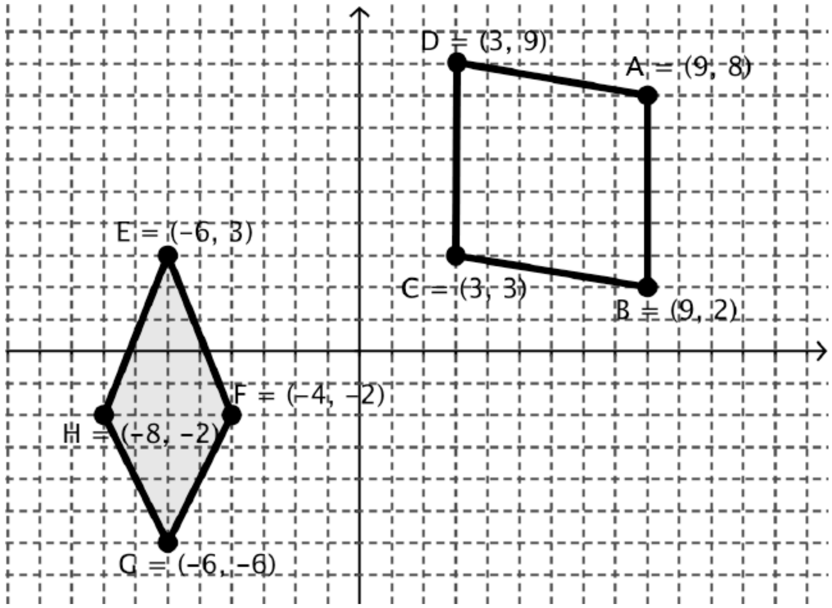
2.



a. Is ABCD a rectangle? Explain how you know.

b. Is EFGH a rectangle? Explain how you know.

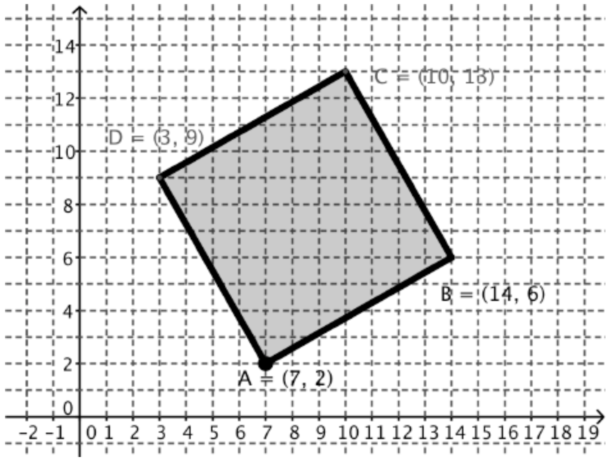
3.



a. Is ABCD a rhombus? Explain how you know.

b. Is EFGH a rhombus? Explain how you know.

4.



a. Is ABCD a square? Explain how you know.

READY, SET, GO! **Name** **Period** **Date**

READY

Topic: Interpreting tables of value as ordered pairs.

Find the value of $f(x)$ for the given domain. Write x and $f(x)$ as an ordered pair.

1. $f(x) = 3x - 2$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

2. $f(x) = x^2$

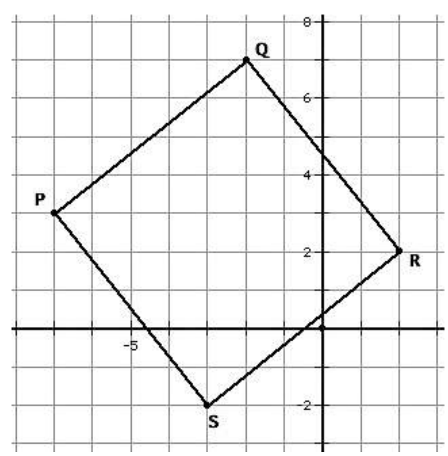
x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

3. $f(x) = 5^x$

x	$f(x)$	$(x, f(x))$
-2		
-1		
0		
1		
2		

SET

Topic: Identifying specific quadrilaterals



4. a) Is the figure at the right a rectangle? Justify your answer.

b) Is the figure at the right a rhombus? Justify your answer.

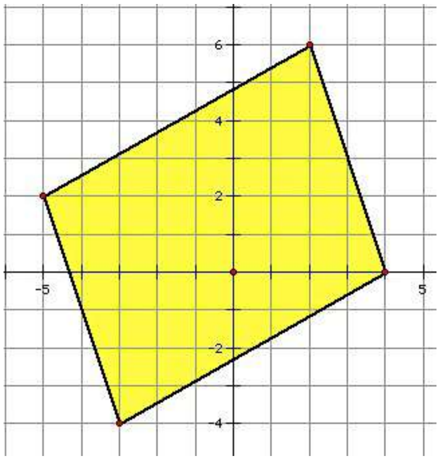
c) Is the figure at the right a square? Justify your answer.

GO

Topic: Calculating perimeters of geometric shapes

Find the perimeter of each figure below. Round answers to the nearest hundredth.

5.



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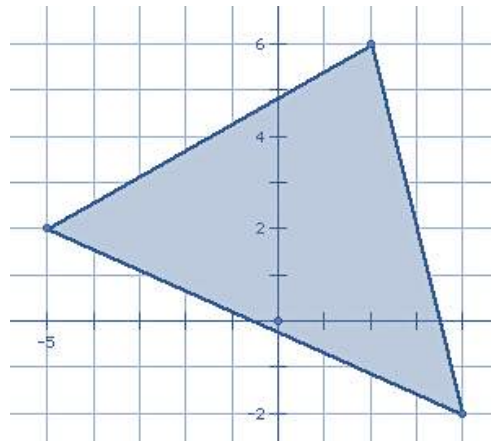

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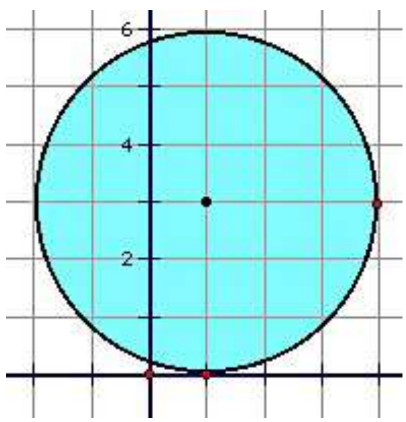

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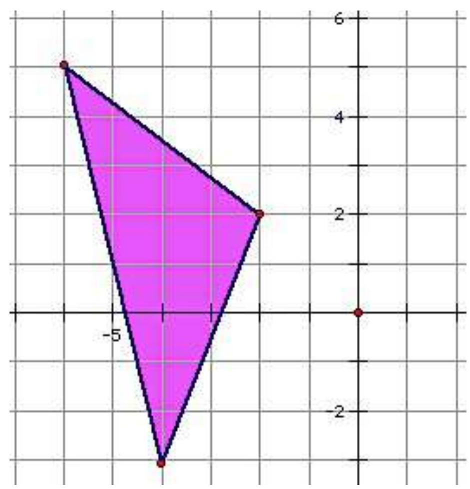

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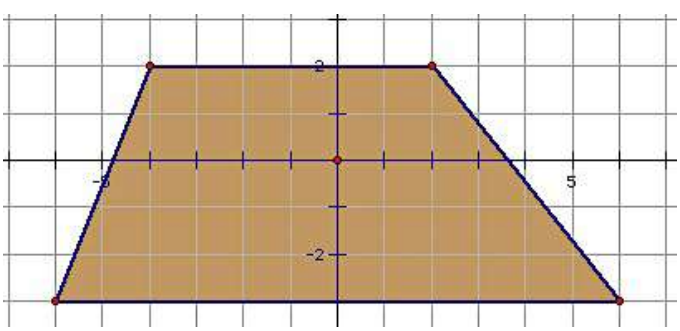

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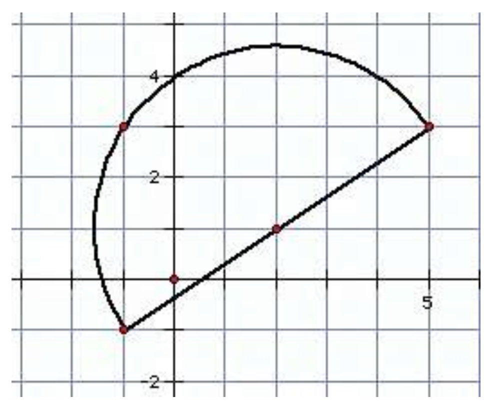


8.4 Training Day

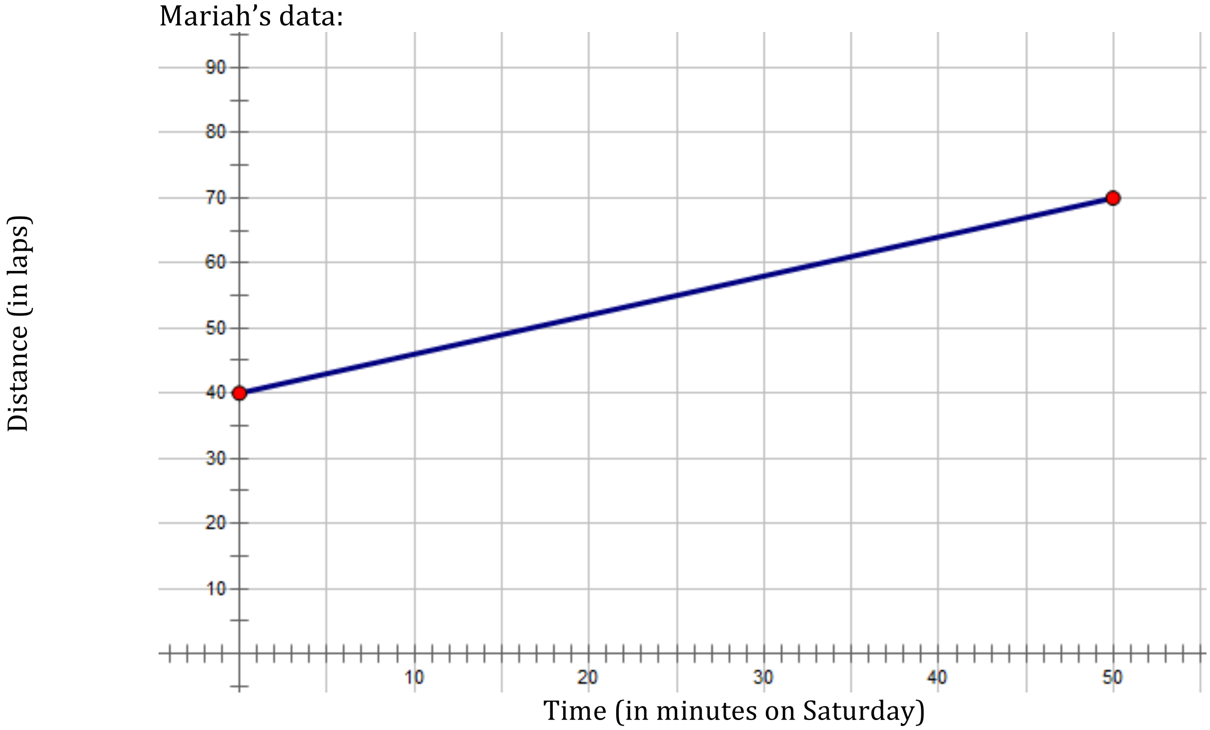
A Develop Understanding Task

Fernando and Mariah are training for six weeks to run in a marathon. To train, they run laps around the track at Eastland High School. Since their schedules do not allow them to run together during the week, they each keep a record of the total number of laps they run throughout the week and then always train together on Saturday morning. The following are representations of how each person kept track of the total number of laps that they ran throughout the week plus the number of laps they ran on Saturday.

Fernando's data:

Time (in minutes on Saturday)	0	10	20	30	40	50
Distance (in laps)	60	66	72	78	84	90

Mariah's data:



1. What observations can be made about the similarities and differences between the two trainers?

2. Write the equation, $m(t)$, that models Mariah's distance.

3. Fernando and Mariah both said they ran the same rate during the week when they were training separately. Explain in words how Fernando's equation is similar to Mariah's. Use the sentence frame:

   The rate of both runners is the same throughout the week, however,

   Fernando____.

4. In mathematics, sometimes one function can be used to build another. Write Mariah's equation, $m(t)$, by starting with Fernando's equation, $f(t)$.

   $f(t) =$

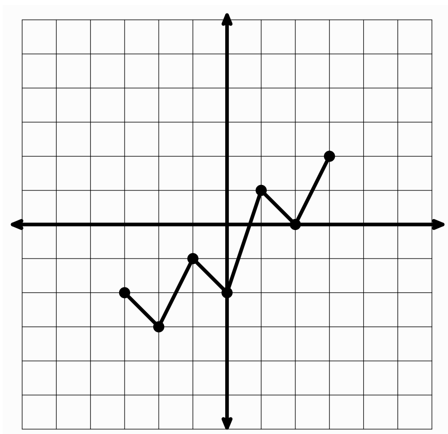
5. Use the mathematical representations given in this task (table and graph) to model the equation you wrote for number 4. Write in words how you would explain this new function to your class.

READY, SET, GO! **Name** **Period** **Date**

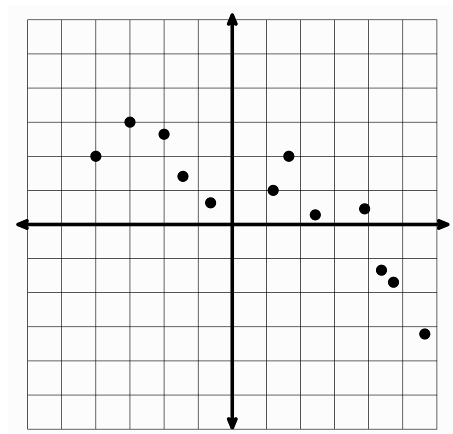
READY

Topic: Vertical transformations on graphs

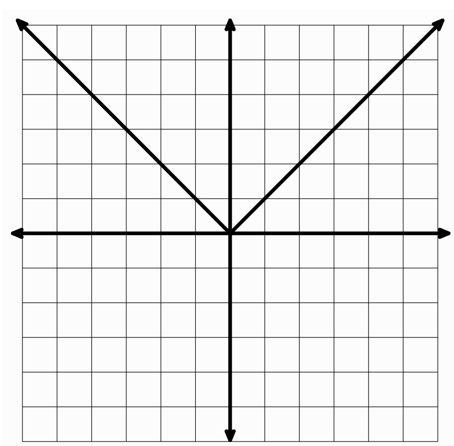
1. Use the graph below to draw a new graph that is translated up 3 units.



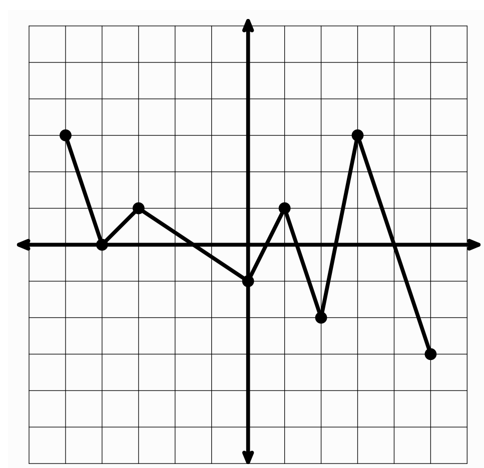
2. Use the graph below to draw a new graph that is translated down 1 unit.



3. Use the graph below to draw a new graph that is translated down 4 units.



4. Use the graph below to draw a new graph that is translated down 3 units.

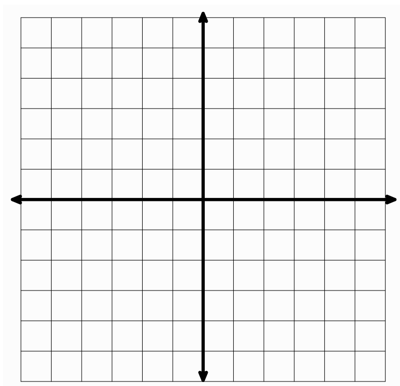


SET

Topic: Graphing transformations and writing the equation of the new graph

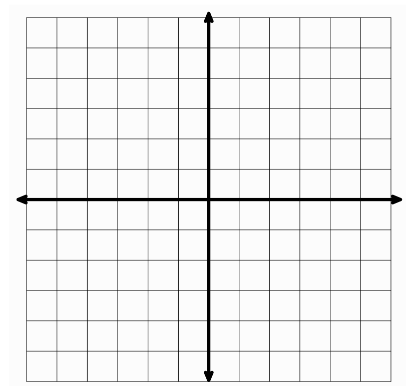
You have been given the equations of $f(x)$ and the transformation $g(x) = f(x) + k$. Graph both $f(x)$ and $g(x)$. Then write the linear equation for $g(x)$ in the space provided.

5. $f(x) = 2x - 4$; $g(x) = f(x) + 3$



$g(x) =$ ______________________________

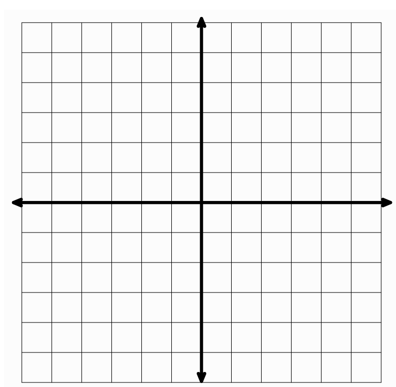
6. $f(x) = 0.5x$; $g(x) = f(x) - 3$



$g(x) =$ ______________________________

Based on the given graph, write the equation of $g(x)$ in the form of $g(x) = f(x) + k$. Then simplify the equation of $g(x)$ into slope-intercept form. The equations of $f(x)$ is given.

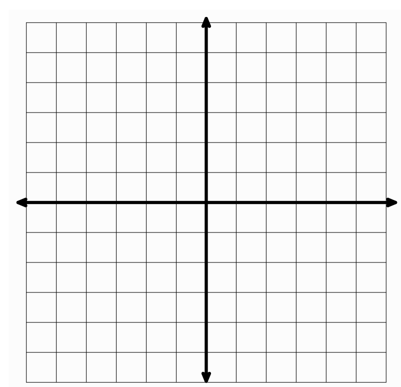
7. $f(x) = \frac{1}{4}x - 3$



a. $g(x) =$ ______________________________
Translation form

b. $g(x) =$ ______________________________
Slope-intercept form

8. $f(x) = -2x + 5$



a. $g(x) =$ ______________________________
Translation form

b. $g(x) =$ ______________________________
Slope-intercept form

GO

Topic: Converting units and making decisions based on data

9. Fernando and Mariah are training for a half marathon. The chart below describes their workouts for the week just before the half marathon. A half marathon is equal to 13.1 miles. If four laps make up one mile, do you think Mariah and Fernando are prepared for the event?

Describe how you think each person will perform in the race. Include who you think will finish first and predict what you think each person's finish time will be. Use the data to inform your conclusions and to justify your answers.

Day of the week	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Fernando: Distance (in laps)	34	45	52	28	49	36
Time per day (in minutes)	60	72	112	63	88	58
Mariah: Distance (in laps)	30	48	55	44	38	22
Time per day (in minutes)	59	75	119	82	70	45

8.5 Training Day Part II

A Solidify Understanding Task

Fernando and Mariah continued training in preparation for the half marathon. For the remaining weeks of training, they each separately kept track of the distance they ran during the week. Since they ran together at the same rate on Saturdays, they took turns keeping track of the distance they ran and the time it took. So they would both keep track of their own information, the other person would use the data to determine their own total distance for the week.

1. **Week 2**: Mariah had completed 15 more laps than Fernando before they trained on Saturday.
   a. Complete the table for Mariah.

Time (in minutes on Saturday)	0	10	20	30	40	50	60
Fernando: Distance (in laps)	50	56	62	68	74	80	86
Mariah: Distance (in laps)							

   b. Write the equation for Mariah as a transformation of Fernando. Equation for Mariah: $m(t) = f(t)$ ________

2. **Week 3:** On Saturday morning before they started running, Fernando saw Mariah's table and stated, "My equation this week will be $f(t) = m(t) + 30$."
   a. What does Fernando's statement mean?

   b. Based on Fernando's translated function, complete the table.

Time (in minutes on Saturday)	0	20	40	60	70
Fernando: Distance (in laps)					
Mariah: Distance (in laps)	45	57	69	81	87

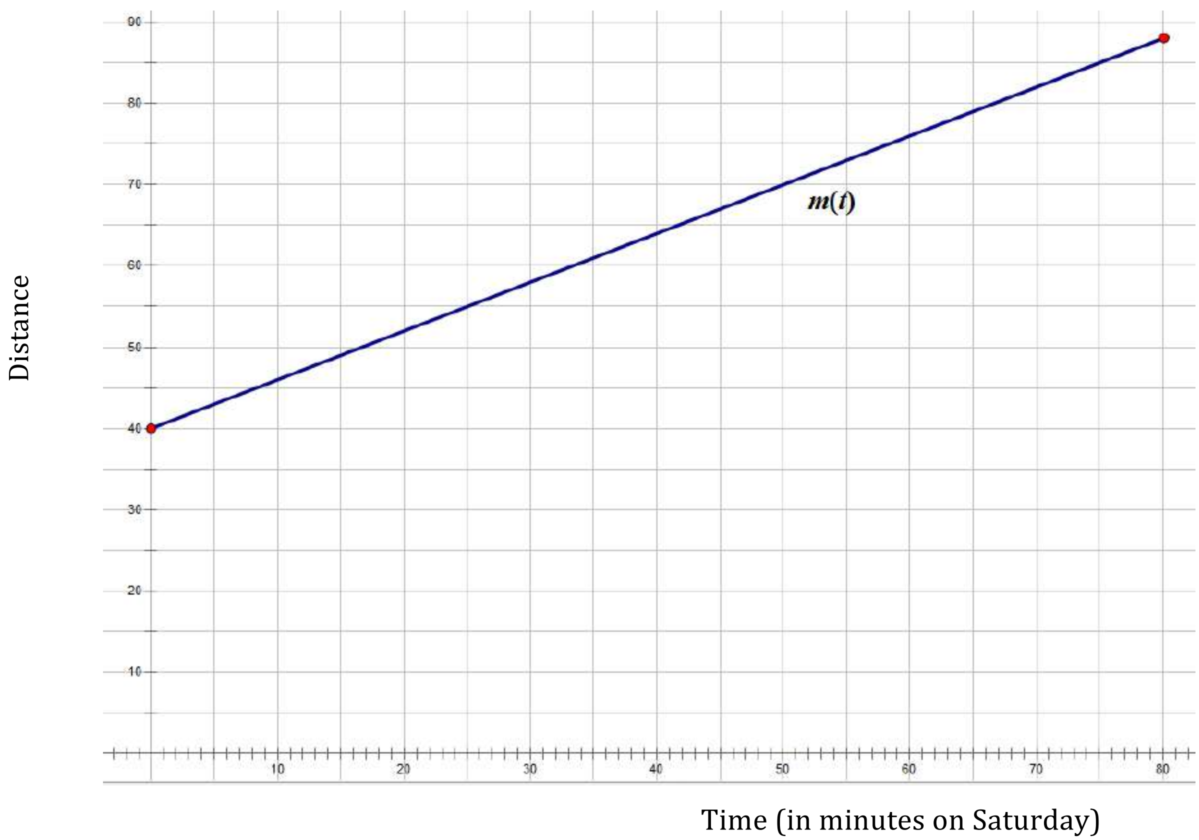
c. Write the equation for both runners in slope-intercept form:

d. Write the equation for Mariah, transformed from Fernando.

e. What relationship do you notice between your answers to parts c and d?

3. Week 4: The marathon is only a couple of weeks away!

a. Use Mariah's graph to sketch *f(t)*. *f(t) = m(t)* – 10



b. Write the equations for both runners in slope-intercept form.

c. What do you notice about the two graphs? Would this always be true if one person ran "*k*" laps more or less each week?

4. **Week 5:** This is the last week of training together. Next Saturday is the big day. When they arrived to train, they noticed they had both run 60 laps during the week.

   a. Write the equation for Mariah on Saturday given that they run at the same rate as the week before.

   b. Write Fernando's equation as a transformation of Mariah's equation.

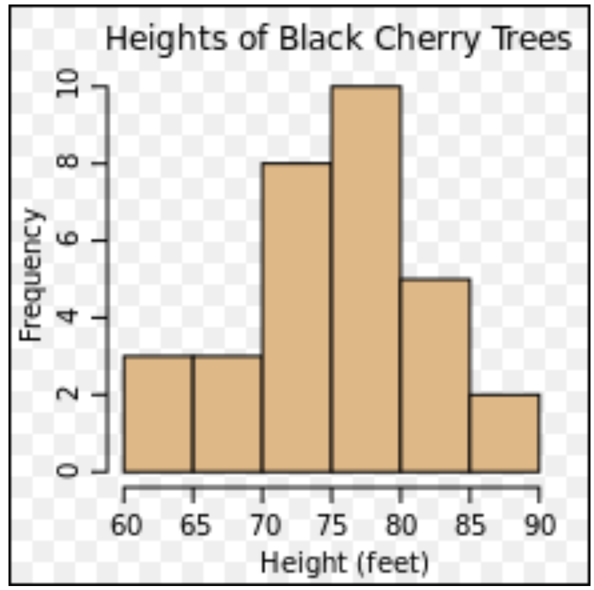
5. What conjectures can you make about the general statement: "$g(x) = f(x)+k$" when it comes to linear functions?

READY, SET, GO! **Name** **Period** **Date**

READY

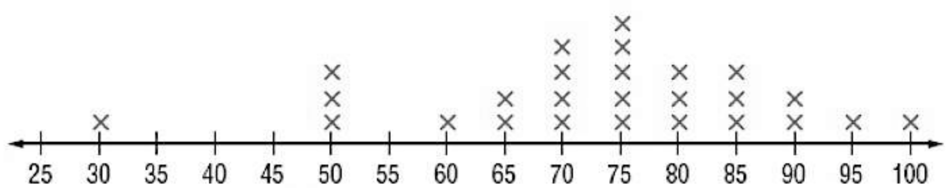
Topic: Describing spread.

1. Describe the spread in the histogram below.

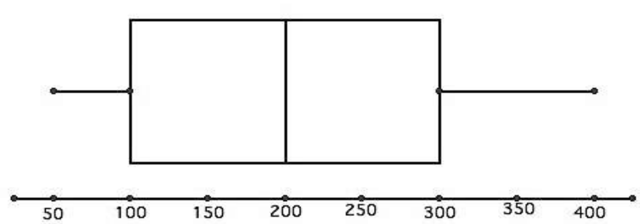


https://commons.wikimedia.org/wiki/File:Black cherry tree

2. Describe the spread in the line plot below.



3. Describe the spread in the box and whisker plot.



SET
Topic: Writing functions in translation form.

You are given information about $f(x)$ and $g(x)$.
Rewrite $g(x)$ in translation form: $g(x) = f(x) + k$

4. $f(x) = 7x + 13$
$g(x) = 7x - 5$

$g(x)$ = ______________________
Translation form

5. $f(x) = 22x - 12$
$g(x) = 22x + 213$

$g(x)$ = ______________________
Translation form

6. $f(x) = -15x + 305$
$g(x) = -15x - 11$

$g(x)$ = ______________________
Translation form

7.

x	f(x)	g(x)
3	11	26
10	46	61
25	121	136
40	196	211

$g(x)$ = ______________________
Translation form

8.

x	f(x)	g(x)
-4	5	-42
-1	-1	-48
5	-13	-60
20	-43	-90

$g(x)$ = ______________________
Translation form

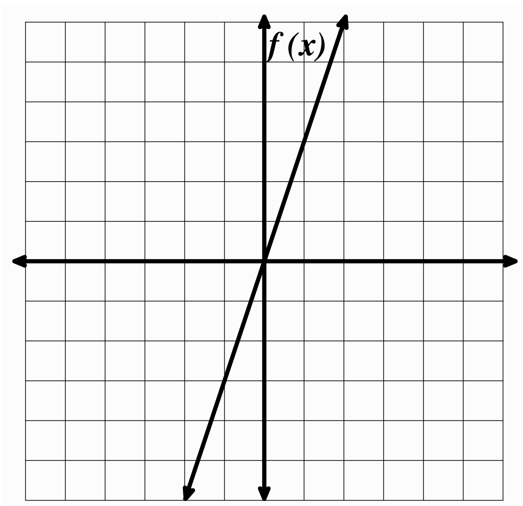
9.

x	f(x)	g(x)
-10	4	-15.5
-3	7.5	-12
22	20	0.5
41	29.5	10

$g(x)$ = ______________________
Translation form

GO
Topic: Vertical and horizontal translations.

10. Use the graph of $f(x) = 3x$ to do the following:



a. Sketch the graph of $g(x) = 3x - 2$ on the same grid.

b. Sketch the graph of $h(x) = 3(x - 2)$

c. Describe how $f(x), g(x),$ *and* $h(x)$ are different and how they are the same.

d. Explain in what way the parentheses affect the graph. Why do you think this is so?

8.6 Shifting Functions

A Practice Understanding Task

Part I: Transformation of an exponential function.

The table below represents the property value of Rebekah's house over a period of four years.

Rebekah's Home

Time (years)	Property Value	Common Ratio
0	150,000	
1	159,000	
2	168,540	
3	178,652	
4	189,372	

Rebekah says the function $P(t) = 150{,}000(1.06)^t$ represents the value of her home.

1. Explain how this function is correct by using the table to show the initial value and the common ratio between terms.

Jeremy lives close to Rebekah and says that his house is always worth \$20,000 more than Rebekah's house. Jeremy created the following table of values to represent the property value of his home.

Jeremy's Home

Time (years)	Property Value	Relationship to Rebekah's table
0	170,000	
1	179,000	
2	188,540	
3	198,652	
4	209,372	

When Rebekah and Jeremy tried to write an exponential function to represent Jeremy's property value, they discovered there was not a common ratio between all of the terms.

2. Use your knowledge of transformations to write the function that could be used to determine the property value of Jeremy's house.

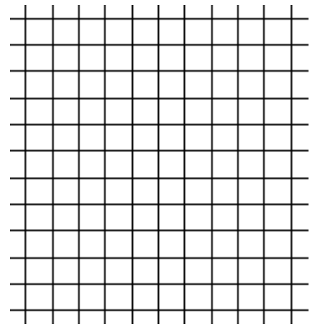
Part 2: Shifty functions.

Given the function *g(x)* and information about *f(x),*

- write the function for *f(x),*
- graph both functions on the set of axes, and
- show a table of values that compares *f(x)* and *g(x)*.

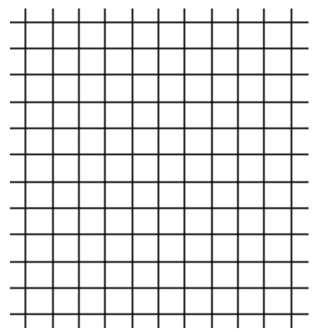
3. $If\ \ g(x) = 3(2)^x\ and\ \ f(x) = \ g(x) - 5, then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



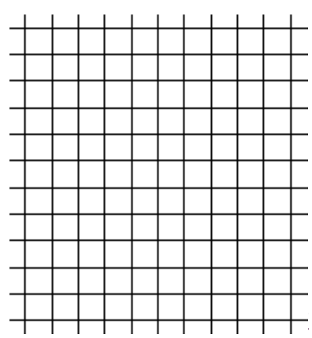
4. $If\ \ g(x) = 4(.5)^x\ and\ \ f(x) = \ g(x) + 3,\ then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



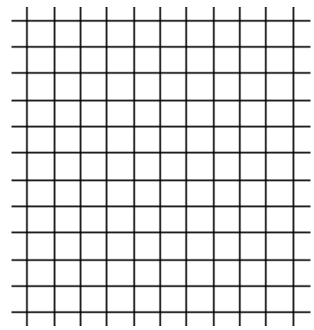
5. $If\ \ g(x) = 4x + 3\ and\ \ f(x) = \ g(x) + 7, then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



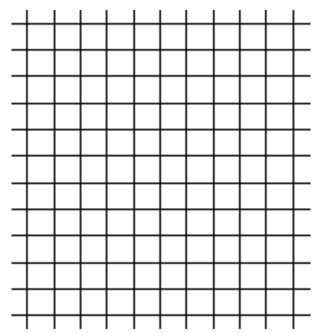
6. $If\ \ g(x) = 2x + 1\ and\ \ f(x) = \ g(x) - 4, then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



7. $If\ \ g(x) = -x\ and\ \ f(x) = \ g(x) + 3, then\ \ f(x) =$ ______________________________

x				
f(x)				
g(x)				



Part III: Communicate your understanding.

8. If *f(x) = g(x) + k,* describe the relationship between *f(x)* and *g(x).* Support your answers with tables and graphs.

READY, SET, GO! Name Period Date

READY

Topic: Finding percentages

Mrs. Gonzalez noticed that her new chorus class had a lot more girls than boys in it. There were 32 girls and 17 boys. (Round answers to the nearest %.)

1. What percent of the class are girls?

2. What percent are boys?

3. 68% of the girls were sopranos.
   a. How many girls sang soprano?

   b. What percent of the entire chorus sang soprano?

4. Only 30% of the boys could sing bass.
   a. How many boys were in the bass section?

   b. What percent of the entire chorus sang bass?

5. Compare the number of girls who sang alto to the number of boys who sang tenor. Which musical section is larger? Justify your answer.

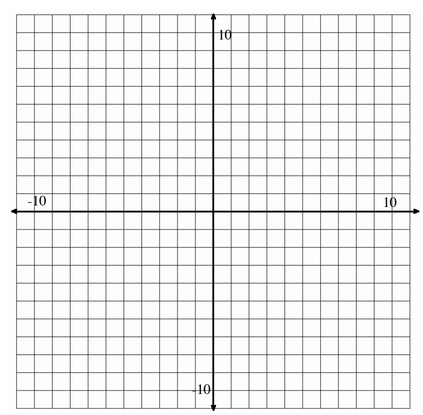
SET

Topic: Graphing exponential equations.

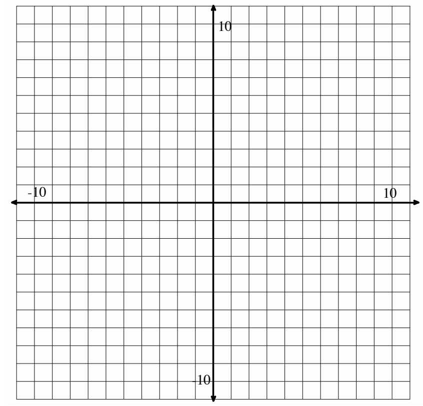
6. Think about the graphs of $y = 2^x$ and $y = 2^x - 4$.

   a. Predict what you think is the same and what is different.

   b. Use your calculator to graph both equations on the same grid. Explain what stayed the same and what changed when you subtracted 4. Identify in what way it changed. (If you don't have a graphing calculator, this can easily be done by hand.)



7. Think about the graphs of $y = 2^x$ and $y = 2^{(x-4)}$.



   a. Predict what you think is the same and what is different.

   b. Use your calculator to graph both equations on the same grid. Explain what stayed the same and what changed. Identify in what way it changed.

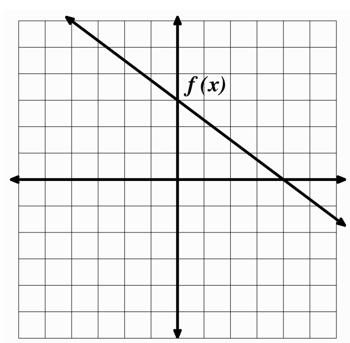
GO

Topic: Vertical translations of linear equations

The graph of $f(x)$ and the translation form equation of $g(x)$ are given. Graph $g(x)$ on the same grid as $f(x)$ and write the slope-intercept equation of $f(x)$ and $g(x)$.

8. $g(x) = f(x) - 5$

a.

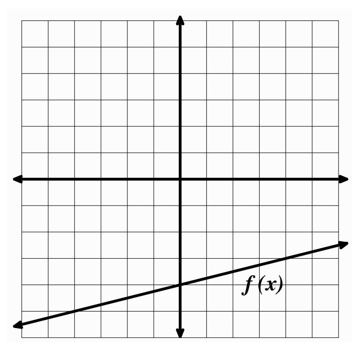


b. $f(x) =$ ____________________

c. $g(x) =$ ____________________
Slope-intercept form

9. $g(x) = f(x) + 4$

a.

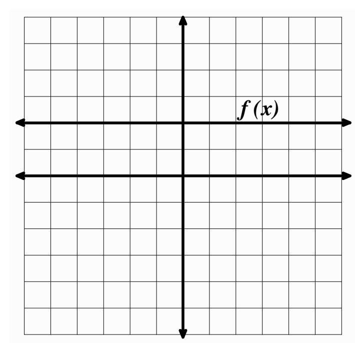


b. $f(x) =$ ____________________

c. $g(x) =$ ____________________
Slope-intercept form

10. $g(x) = f(x) - 6$

a.



b. $f(x) =$ ____________________

c. $g(x) =$ ____________________
Slope-intercept form

8. 7H The Arithmetic of Vectors

A Solidify Understanding Task

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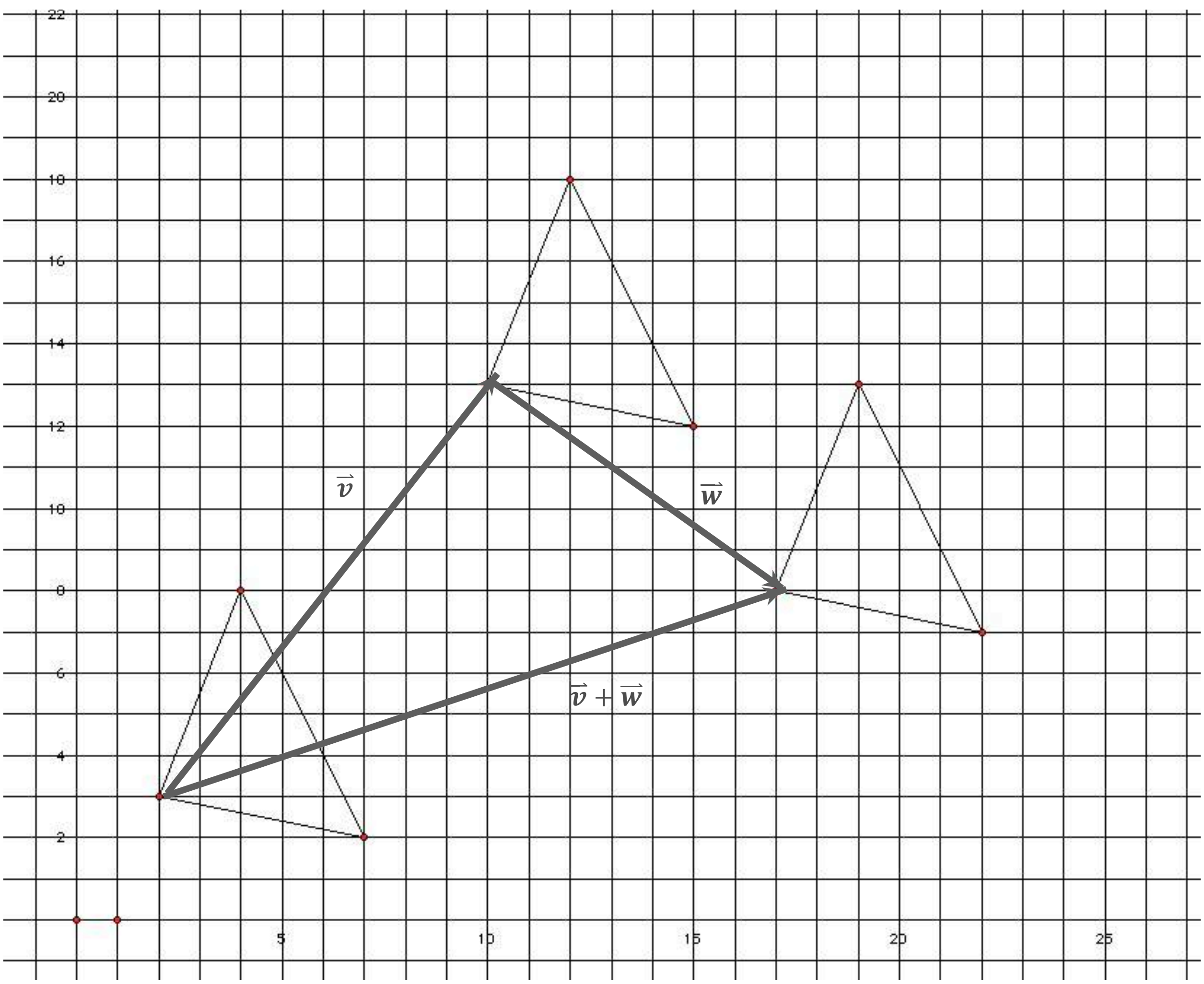
The following diagram shows a triangle that has been translated to a new location, and then translated again. Arrows have been used to indicate the movement of one of the vertex points through each translation. The result of the two translations can also be thought of as a single translation, as shown by the third arrow in the diagram.



Draw arrows to show the movement of the other two vertices through the sequence of translations, and then draw an arrow to represent the resultant single translation. What do you notice about each set of arrows?

A **vector** is a quantity that has both **magnitude** and **direction**. The arrows we drew on the diagram represent translations as vectors—each translation has *magnitude* (the distance moved) and *direction* (the direction in which the object is moved). Arrows, or *directed line segments*, are one way of representing a vector.

Addition of Vectors

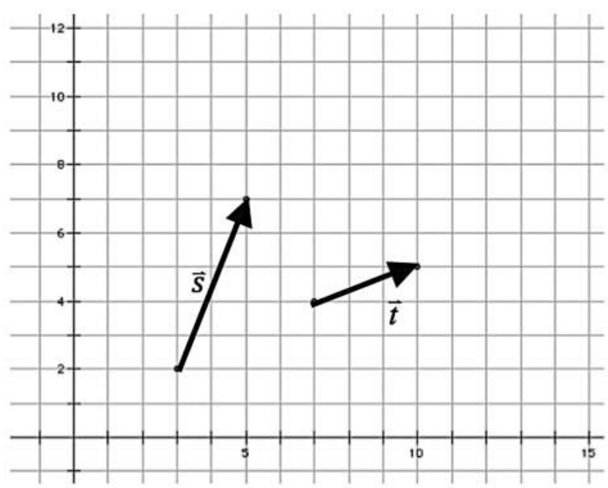
In the example above, two vectors $\vec{v}$ and $\vec{w}$ were combined to form vector $\vec{r}$. This is what is meant by "adding vectors".

1. Study each of the following three methods for adding vectors, then try each method to add vectors $\vec{s}$ and $\vec{t}$ given in the diagrams to find $\vec{q}$, such that $\vec{s} + \vec{t} = \vec{q}$.

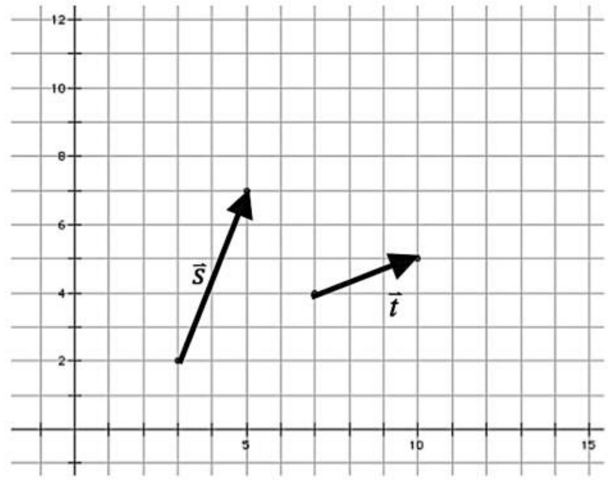
2. Explain why each of these methods gives the same result.

Method 1: *End-to-end*

The diagram given above illustrates the end-to-end strategy of adding two vectors to get a resultant vector that represents the sum of the two vectors. In this case, the resulting vector shows that a single translation could accomplish the same movement as the combined sum of the two individual translations, that is $\vec{v} + \vec{w} = \vec{r}$.



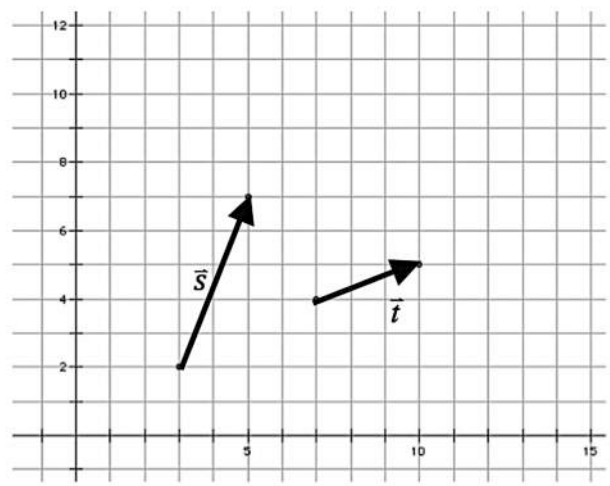
Method 2: *The parallelogram rule*



Since we can relocate the arrow representing a vector, draw both vectors starting at a common point. Often both vectors are relocated so they have their *tail* ends at the origin. These arrows form two sides of a parallelogram. Draw the other two sides. The resulting sum is the vector represented by the arrow drawn from the common starting point (for example, the origin) to the opposite vertex of the parallelogram.

Question to think about: How can you determine where to put the missing vertex point of the parallelogram?

Method 3: *Using horizontal and vertical components*



Each vector consists of a horizontal component and a vertical component. For example, vector $\vec{v}$ can be thought of as a movement of 8 units horizontally and 13 units vertically. This is represented with the notation $\langle 8,13\rangle$. Vector $\vec{w}$ consists of a movement of 7 units horizontally and -5 units vertically, represented by the notation $\langle 7,-5\rangle$.

Question to think about: How can the components of the individual vectors be combined to determine the horizontal and vertical components of the resulting vector $\vec{r}$?

3. Examine vector $\vec{s}$ given above. While we can relocate the vector, in the diagram the *tail* of the vector is located at (3, 2) and the *head* of the vector is located at (5, 7). Explain how you can determine the horizontal and vertical components of a vector from just the coordinates

of the point at the tail and the point at the head of the vector? That is, how can we find the horizontal and vertical components of movement without counting across and up the grid?

Magnitude of Vectors

The symbol $\|\vec{v}\|$ is used to denote the magnitude of the vector, in this case the length of the vector.

Devise a method for finding the magnitude of a vector and use your method to find the following.

Be prepared to describe your method for finding the magnitude of a vector.

4. $\|\vec{v}\|$

5. $\|\vec{w}\|$

6. $\|\vec{v} + \vec{w}\|$

Scalar Multiples of Vectors

We can stretch a vector by multiplying the vector by a scale factor. For example, $2\vec{v}$ represents the vector that has the same direction as $\vec{v}$, but whose magnitude is twice that of $\vec{v}$.

Draw each of the following vectors on a coordinate graph:

7. $3\vec{s}$

8. $-2\vec{t}$

9. $3\vec{s} + (-2\vec{t})$

10. $3\vec{s} - 2\vec{t}$

Other Applications of Vectors

We have illustrated the concept of a vector using translation vectors in which the magnitude of the vector represents the distance a point gets translated. There are other quantities that have magnitude and direction, but the magnitude of the vector does not always represent length.
For example, a car traveling 55 miles per hour along a straight stretch of highway can be represented by a vector since the speed of the car has magnitude, 55 miles per hour, and the car is traveling in a specific direction. Pushing on an object with 25 pounds of force is another example. A vector can be used to represent this push since the force of the push has magnitude, 25 pounds of force, and the push would be in a specific direction.

11. A swimmer is swimming across a river with a speed of 20 ft/sec and at a 45° angle from the bank of the river. The river is flowing at a speed of 5 ft/sec. Illustrate this situation with a vector diagram and describe the meaning of the vector that represents the sum of the two vectors that represent the motion of the swimmer and the flow of the river.

12. Two teams are participating in a tug-of-war. One team exerts a combined force of 200 pounds in one direction while the other team exerts a combined force of 150 pounds in the other direction. Illustrate this situation with a vector diagram and describe the meaning of the vector that represents the sum of the vectors that represent the efforts of the two teams.

READY, SET, GO! Name Period Date

READY

Topic: Solving equations using properties of arithmetic

1. Here are the steps Zac used to solve the following equation. State or describe the properties of equality or otherwise that he is using in each step.

$2(x + 5) + 7x = 4x + 15$	*the distributive property*
$(2x + 10) + 7x = 4x + 15$	a.
$2x + (10 + 7x) = 4x + 15$	b.
$2x + (7x + 10) = 4x + 15$	c.
$(2x + 7x) + 10 = 4x + 15$	d.
$(2 + 7)x + 10 = 4x + 15$	e.
$9x + 10 = 4x + 15$	f.
$9x + 10 - 10 = 4x + 15 - 10$	g.
$9x + 0 = 4x + 5$	h.
$9x = 4x + 5$	i.

$9x - 4x = 4x + 5 - 4x$	j.
$(9 - 4)x = 4x + 5 - 4x$	k.
$5x = 4x + 5 - 4x$	l.
$5x = 4x - 4x + 5$	m.
$5x = 0 + 5$	n.
$5x = 5$	o.
$\frac{1}{5} \cdot 5x = \frac{1}{5} \cdot 5$	p.
$1x = 1$	q.
$x = 1$	

Solve each of the following equations for *x*, carefully recording each step. Then state or describe the properties of arithmetic (for example, the distributive property, or the associative property of multiplication, etc.) or properties of equality (for example, the addition property of equality) that justify each step.

2. $2(3x + 5) = 4(2x - 1)$

3. $\frac{4}{5}x + 3 = 2x - 1$

SET

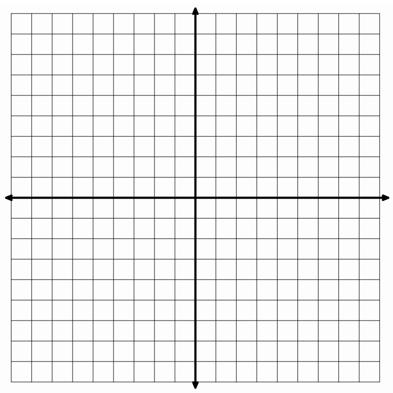
Topic: Adding vectors

Two vectors are described in component form in the following way:

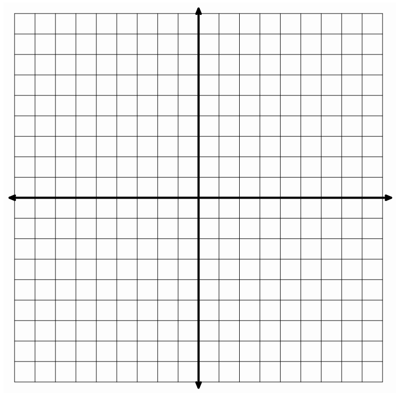
$\vec{\boldsymbol{v}} : \langle -2 , 3 \rangle$ **and** $\vec{\boldsymbol{w}} : \langle 3 , 4 \rangle$

On the grids below, create vector diagrams to show:

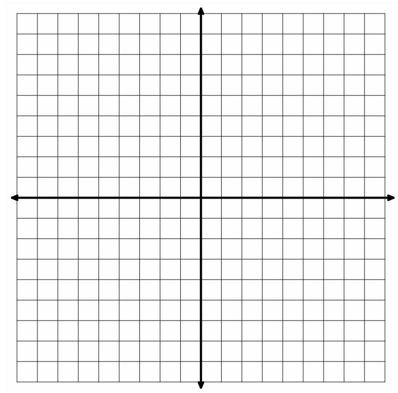
4. $\vec{v} + \vec{w} =$



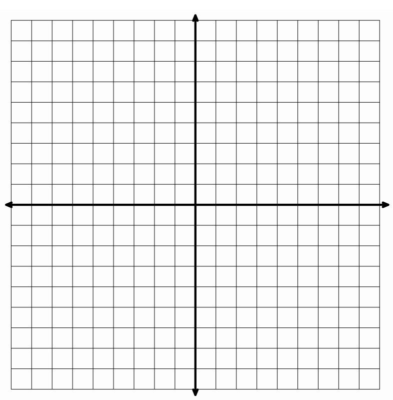
5. $\vec{v} - \vec{w} =$



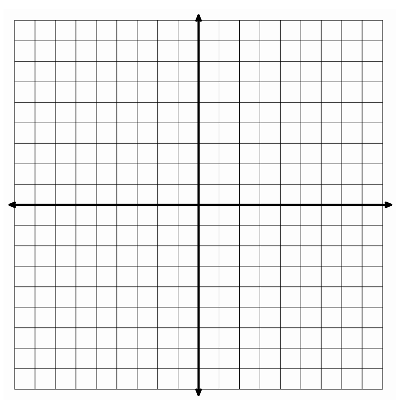
6. $3\vec{v} =$



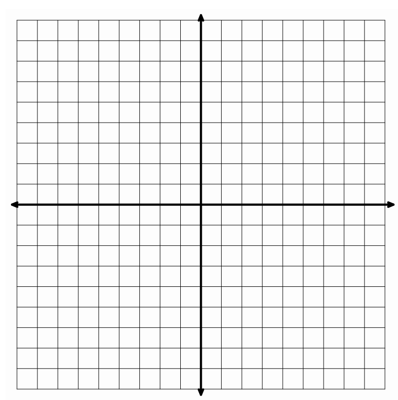
7. $-2\vec{w} =$



8. $3\vec{v} - 2\vec{w} =$



9. Show how to find $\vec{v} + \vec{w}$ using the parallelogram rule.



GO

Topic: The arithmetic of matrices

$$A = \begin{bmatrix} 2 & -3 \\ -1 & 5 \end{bmatrix},\ B = \begin{bmatrix} 2 & 5 \\ -3 & 2 \end{bmatrix},\ \textbf{and}\ C = \begin{bmatrix} 4 & 2 & -1 \\ 5 & 2 & 3 \end{bmatrix}$$

Find the following sums, differences or products, as indicated. If the sum, difference or product is undefined, explain why.

10. $A + B$

11. $A + C$

12. $2A - B$

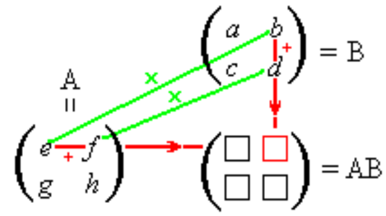
13. $A \cdot B$

14. $B \cdot A$

15. $A \cdot C$

16. $C \cdot A$

8. 8H More Arithmetic of Matrices



http://commons.wikimedia.org/wiki/File:Matriz_A_por_B.png

A Solidify Understanding Task

In this task you will have an opportunity to examine some of the properties of matrix addition and matrix multiplication. We will restrict this work to square 2×2 matrices.

The table below defines and illustrates several properties of addition and multiplication for real numbers and asks you to determine if these same properties hold for matrix addition and matrix multiplication. While the chart asks for a single example for each property, you should experiment with matrices until you are convinced that the property holds or you have found a counter-example to show that the property does not hold. Can you base your justification on more that just trying out several examples?

Property	**Example with Real Numbers**	**Example with Matrices**
Associative Property of Addition $(a + b) + c = a + (b + c)$		
Associative Property of Multiplication $(ab)c = a(bc)$		
Commutative Property of Addition $a + b = b + a$		

Commutative Property of Multiplication $ab = ba$		
Distributive Property of Multiplication Over Addition $a(b + c) = ab + ac$		

In addition to the properties listed in the table above, addition and multiplication of real numbers include properties related to the numbers 0 and 1. For example, the number 0 is referred to as the *additive identity* because $a + 0 = 0 + a = a$, and the number 1 is referred to as the *multiplicative identity* since $a \cdot 1 = 1 \cdot a = a$. Once the additive and multiplicative identities have been identified, we can then define additive inverses a and $-a$ since $a + -a = 0$, and multiplicative inverses a and $1/a$ since $a \cdot \frac{1}{a} = 1$. To decide if these properties hold for matrix operations, we will need to determine if there is a matrix that plays the role of 0 for matrix addition, and if there is a matrix that plays the role of 1 for matrix multiplication.

The Additive Identity Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of 0, or the additive identity matrix, for the following matrix addition. Will this same matrix work as the additive identity for all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

The Multiplicative Identity Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of 1, or the multiplicative identity matrix, for the following matrix multiplication. Will this same matrix work as the multiplicative identity for all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

Now that we have identified the additive identity and multiplicative identity for 2×2 matrices, we can search for the additive inverses and multiplicative inverses of matrices.

Finding an Additive Inverse Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of the additive inverse of the first matrix. Will this same process work for finding the additive inverse of all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Finding a Multiplicative Inverse Matrix

Find values for *a, b, c* and *d* so that the matrix below that contains these variables plays the role of the multiplicative inverse of the first matrix. Will this same process work for finding the multiplicative inverse of all 2×2 matrices?

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

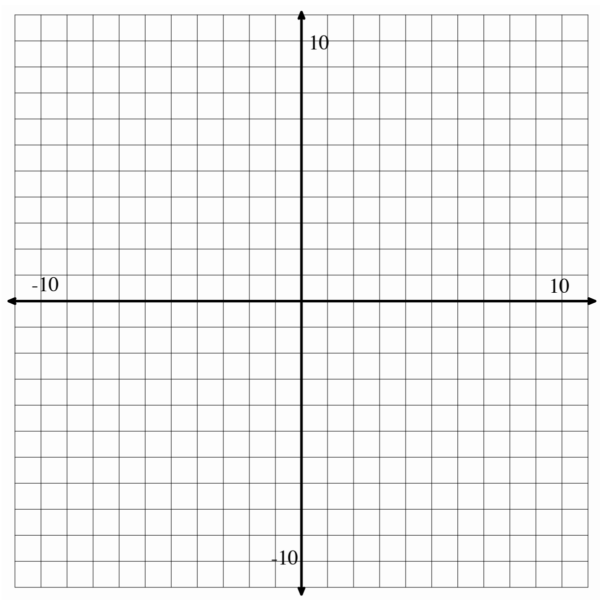
READY, SET, GO! Name Period Date

READY

Topic: Solving systems of linear equations

1. Solve the system of equations $\begin{cases} 5x - 3y = 3 \\ 2x + y = 10 \end{cases}$

a. By graphing



b. By substitution

c. By elimination

SET

Topic: Inverse matrices

2. Given: Matrix $A = \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix}$

a. Find the additive inverse of matrix A

b. Find the multiplicative inverse of matrix A

mathematics vision project

3. Given: Matrix $B = \begin{bmatrix} 4 & 2 \\ 3 & 2 \end{bmatrix}$

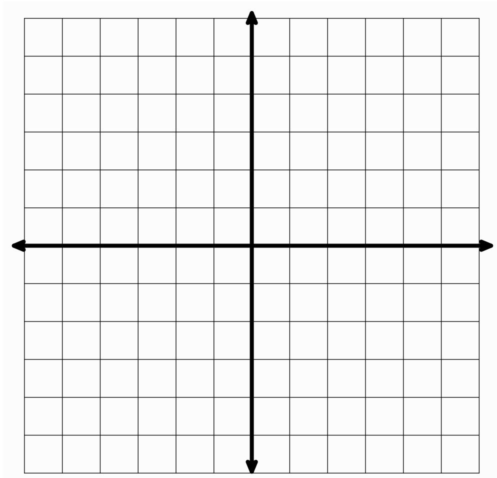
a. Find the additive inverse of matrix B.

b. Find the multiplicative inverse of matrix B

GO

Topic: Parallel lines, perpendicular lines, and length from a coordinate geometry perspective

Given the four points: A (2, 1), B (5, 2), C (4, 5), and D (1, 4)



4. Is $ABCD$ a parallelogram?

Provide convincing evidence for your answer.

5. Is $ABCD$ a rectangle?

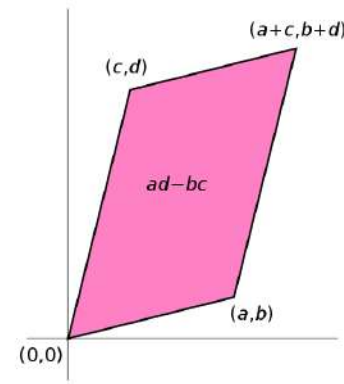
Provide convincing evidence for your answer.

6. Is $ABCD$ a rhombus? Provide convincing evidence for your answer.

7. Is $ABCD$ a square? Provide convincing evidence for your answer.

8. 9H The Determinant of a Matrix

A Solidify Understanding Task



http://en.wikipedia.org/wiki/
File:Area_parallellogram_as_determinant.svg

In the previous task we learned how to find the multiplicative inverse of a matrix. Use that process to find the multiplicative inverse of the following two matrices.

1. $\begin{bmatrix} 5 & 1 \\ 6 & 2 \end{bmatrix}$

2. $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$

3. Were you able to find the multiplicative inverse for both matrices?

There is a number associated with every square matrix called the **determinant**. If the determinant is not equal to zero, then the matrix has a multiplicative inverse.

For a 2×2 matrix the determinant can be found using the following rule: (note: the vertical lines, rather than the square brackets, are used to indicate that we are finding the determinant of the matrix)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

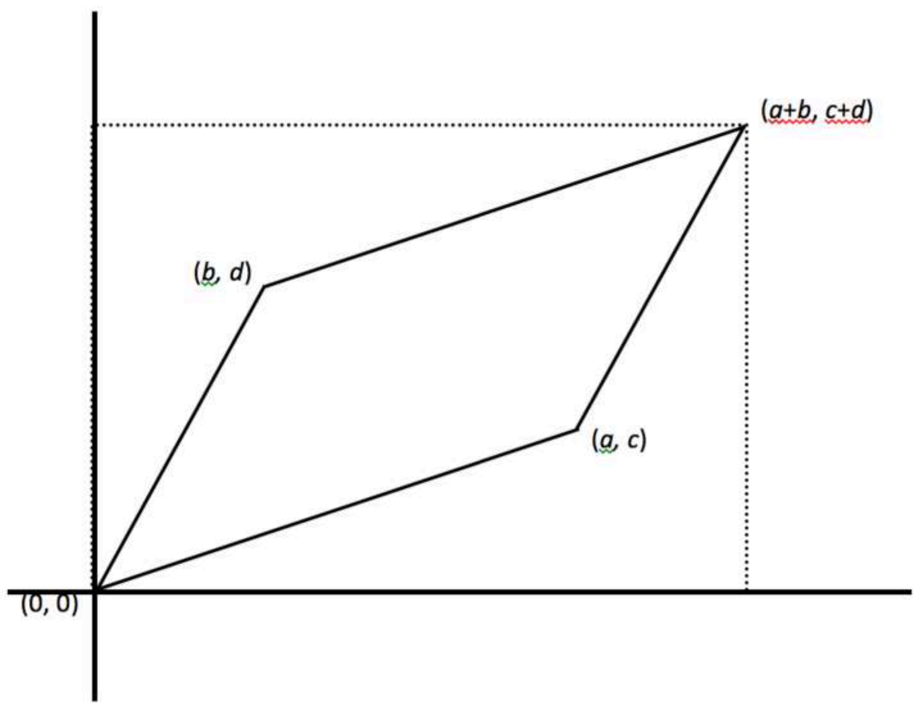
4 Using this rule, find the determinant of the two matrices given in problems 1 and 2 above.

The absolute value of the determinant of a 2×2 matrix can be visualized as the area of a parallelogram, constructed as follows:

- Draw one side of the parallelogram with endpoints at (0, 0) and (a, c).
- Draw a second side of the parallelogram with endpoints at (0, 0) and (b, d).
- Locate the fourth vertex that completes the parallelogram.

(Note that the elements in the columns of the matrix are used to define the endpoints of the vectors that form two sides of the parallelogram.)

5. Use the following diagram to show that the area of the parallelogram is given by $ad - bc$.



6. Draw the parallelograms whose areas represent the determinants of the two matrices listed in questions 1 and 2 above. How does a zero determinant show up in these diagrams?

7. Create a matrix for which the determinant will be negative. Draw the parallelogram associated with the determinant of your matrix and find the area of the parallelogram.

The determinant can be used to provide an alternative method for finding the inverse of 2×2 matrix.

8. Use the process you used previously to find the inverse of a generic 2×2 matrix whose elements are given by the variables *a, b, c* and *d*. For now, we will refer to the elements of the inverse matrix as M_1, M_2, M_3 and M_4 as illustrated in the following matrix equation. Find expressions for M_1, M_2, M_3 and M_4 in terms of the elements of the first matrix, *a, b, c* and *d*.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$M_1 =$

$M_2 =$

$M_3 =$

$M_4 =$

Use your work above to explain this strategy for finding the inverse of a 2×2 matrix: (note: the $^{-1}$ superscript is used to indicate that we are finding the multiplicative inverse of the matrix)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ where $ad - bc$ is the determinant of the matrix

READY, SET, GO! Name Period Date

READY

Topic: Solving systems of equations using row reduction

Given the system of equations $\begin{cases} 5x - 3y = 3 \\ 2x + y = 10 \end{cases}$

1. Zac started solving this problem by writing $\begin{bmatrix} 5 & -3 & 3 \\ 2 & 1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -5 & -17 \\ 2 & 1 & 10 \end{bmatrix}$

Describe what Zac did to get from the matrix on the left to the matrix on the right.

2. Lea started solving this problem by writing $\begin{bmatrix} 5 & -3 & 3 \\ 2 & 1 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 5 & -3 & 3 \\ 1 & \frac{1}{2} & 5 \end{bmatrix}$

Describe what Lea did to get from the matrix on the left to the matrix on the right.

3. Using either Zac's or Lea's first step, continue solving the system using row reduction. Show each matrix along with notation indicating how you got from one matrix to another. Be sure to check your solution.

SET

Topic: The determinant of a 2 X 2 matrix

4. Use the determinant of each 2×2 matrix to decide which matrices have multiplicative inverses, and which do not.

a. $\begin{bmatrix} 8 & -2 \\ 4 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$ c. $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$

5. Find the multiplicative inverse of each of the matrices in 4, provided the inverse matrix exists.

a. b. c.

6. Generally matrix multiplication is not commutative. That is, if A and B are matrices, typically $A \cdot B \neq B \cdot A$. However, multiplication of inverse matrices is commutative. Test this out by showing that the pairs of inverse matrices you found in question 7 give the same result when multiplied in either order.

GO

Topic: Parallel and perpendicular lines

Determine if the following pairs of lines are parallel, perpendicular or neither. Explain how you arrived at your answer.

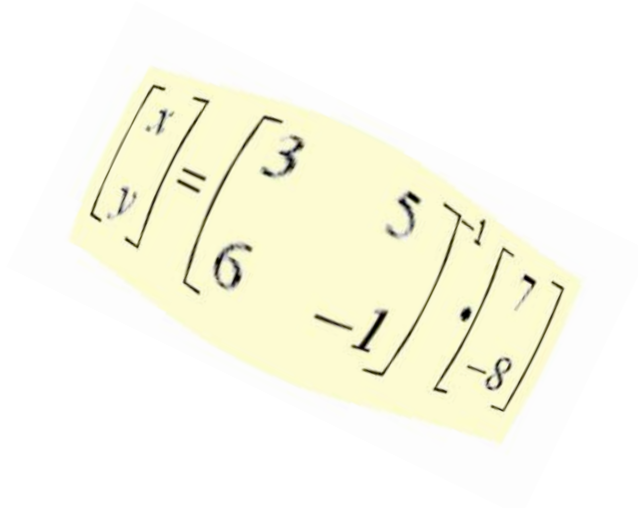
7. $3x + 2y = 7$ and $6x + 4y = 9$

8. $y = \frac{2}{3}x - 5$ and $y = -\frac{2}{3}x + 7$

9. $y = \frac{3}{4}x - 2$ and $4x + 3y = 3$

10. Write the equation of a line that is parallel to $y = \frac{4}{5}x - 2$ and has a y-intercept at (0, 4).

8.10H Solving Systems with Matrices, Revisited



A Solidify Understanding Task

When you solve linear equations, you use many of the properties of operations that were revisited in the task *More Arithmetic of Matrices*.

1. Solve the following equation for x and list the properties of operations that you use during the equation solving process.

$$\frac{2}{3}x = 8$$

The list of properties you used to solve this equation probably included the use of a multiplicative inverse and the multiplicative identity property. If you didn't specifically list those properties, go back and identify where they might show up in the equation solving process for this particular equation.

Systems of linear equations can be represented with matrix equations that can be solved using the same properties that are used to solve the above equation. First, we need to recognize how a matrix equation can represent a system of linear equations.

2. Write the linear system of equations that is represented by the following matrix equation. (Think about the procedure for multiplying matrices you developed in previous tasks.)

$$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

3. Using the relationships you noticed in question 3, write the matrix equation that represents the following system of equations.

$$\begin{cases} 2x + 3y = 14 \\ 3x + 4y = 20 \end{cases}$$

4. The rational numbers $\frac{2}{3}$ and $\frac{3}{2}$ are multiplicative inverses. What is the multiplicative inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$? Note: The inverse matrix is usually denoted by $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1}$.

5. The following table lists the steps you may have used to solve $\frac{2}{3}x = 8$ and asks you to apply those same steps to the matrix equation you wrote in question 4. Complete the table using these same steps.

Original equation	$\frac{2}{3}x = 8$	$\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \end{bmatrix}$
Multiply both sides of the equation by the multiplicative inverse	$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 8$	
The product of multiplicative inverses is the multiplicative identity on the left side of the equation	$1 \cdot x = \frac{3}{2} \cdot 8$	
Perform the indicated multiplication on the right side of the equation	$1 \cdot x = 12$	

Apply the property of the multiplicative identity on the left side of the equation	$x = 12$	

6. What does the last line in the table in question 5 tell you about the system of equations in question 3?

7. Use the process you have just examined to solve the following system of linear equations.

$$\begin{cases} 3x + 5y = -1 \\ 2x + 4y = 4 \end{cases}$$

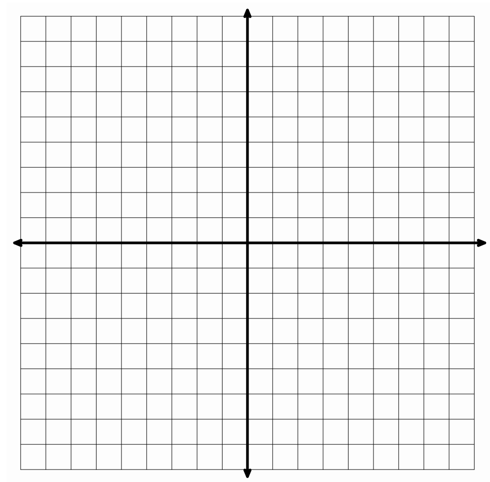
READY, SET, GO! Name Period Date

READY

Topic: Reflections and Rotations

1. The following three points form the vertices of a triangle: (3, 2), (6, 1), (4, 3)

a. Plot these three points on the coordinate grid and then connect them to form a triangle.



b. Reflect the original triangle over the *y*-axis and record the coordinates of the vertices here:

c. Reflect the original triangle over the *x*-axis and record the coordinates of the vertices here:

d. Rotate the original triangle 90° counter-clockwise about the origin and record the coordinates of the vertices here:

e. Rotate the original triangle 180° about the origin and record the coordinates of the vertices here.

SET

Topic: Solving Systems Using Inverse Matrices

Two of the following systems have unique solutions (that is, the lines intersect at a single point).

2. Use the determinant of a 2×2 matrix to decide which systems have unique solutions, and which one does not.

a. $\begin{cases} 8x - 2y = -2 \\ 4x + y = 5 \end{cases}$ b. $\begin{cases} 3x + 2y = 7 \\ 6x + 4y = -5 \end{cases}$ c. $\begin{cases} 4x + 2y = 0 \\ 3x + y = 2 \end{cases}$

3. For each of the systems in #2 which have a unique solution, find the solution to the system by solving a matrix equation using an *inverse matrix*.

a. b. c.

GO

Topic: Properties of Arithmetic

Match each example on the left with the name of a property of arithmetic on the right. Not all answers will be used.

____ 4. $2(x + 3y) = 2x + 6y$

____ 5. $(2x + 3y) + 4y = 2x + (3y + 4y)$

____ 6. $2x + 3y = 3y + 2x$

____ 7. $2(3y) = (2 \cdot 3)y = 6y$

____ 8. $\frac{2}{3} \cdot \frac{3}{2} x = 1x$

____ 9. $x + -x = 0$

____ 10. $xy = yx$

a. multiplicative inverses

b. additive inverses

c. multiplicative identity

d. additive identity

e. commutative property of addition

f. commutative property of multiplication

g. associative property of addition

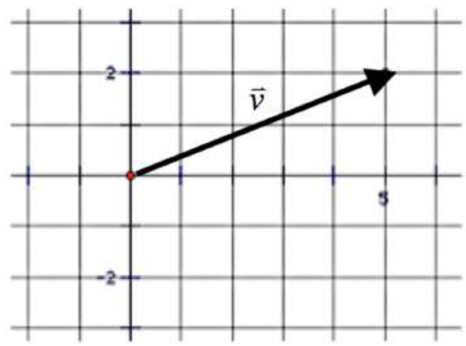
h. associative property of multiplication

i. distributive property of addition over multiplication

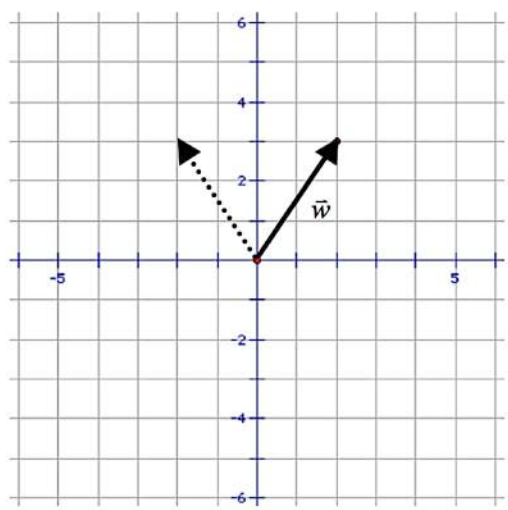
8.11H Transformations with Matrices

A Solidify Understanding Task

Various notations are used to denote vectors: bold-faced type, ***v***; a variable written with a harpoon over it, $\vec{v}$; or listing the horizontal and vertical components of the vector, $\langle v_x, v_y \rangle$. In this task we will represent vectors by listing their horizontal and vertical components in a matrix with a single column, $\begin{bmatrix} v_x \\ v_y \end{bmatrix}$.



1. Represent the vector labeled $\vec{v}$ in the diagram at the right as a matrix with one column.



Matrix multiplication can be used to transform vectors and images in a plane.

Suppose we want to reflect $\vec{w}$ over the y-axis. We can represent $\vec{w}$ with the matrix $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$, and the reflected vector with the matrix $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

2. Find the 2×2 matrix that we can multiply the matrix representing the original vector by in order to obtain the matrix that represents the reflected vector. That is, find *a, b, c* and *d* such that $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

3. Find the matrix that will reflect $\vec{w}$ over the *x*-axis.

4. Find the matrix that will rotate $\vec{w}$ 90° counterclockwise about the origin.

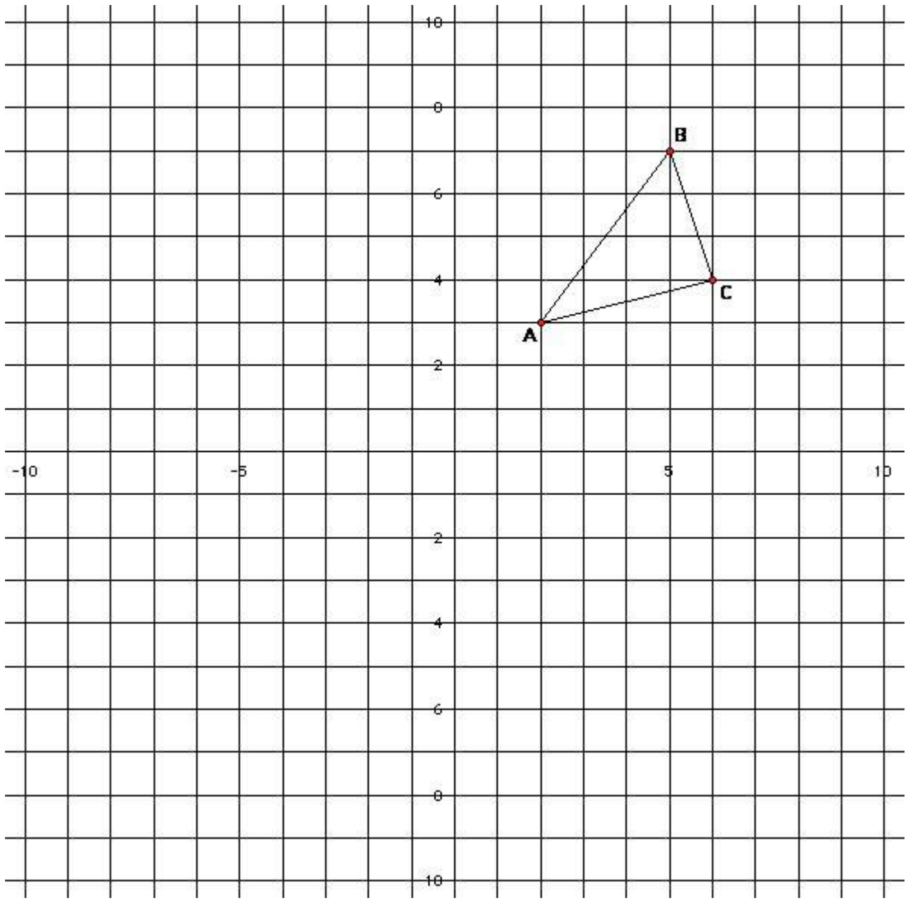
5. Find the matrix that will rotate $\vec{w}$ 180° counterclockwise about the origin.

6. Find the matrix that will rotate $\vec{w}$ 270° counterclockwise about the origin.

7. Is there another way to obtain a rotation of 270° counterclockwise about the origin other than using the matrix found in question 6? If so, how?

We can represent polygons in the plane by listing the coordinates of its vertices as columns of a matrix.

For example, the triangle below can be represented by the matrix $\begin{bmatrix} 2 & 5 & 6 \\ 3 & 7 & 4 \end{bmatrix}$.



8. Multiply this matrix, which represents the vertices of ΔABC, by the matrix found in question 2. Interpret the product matrix as representing the coordinates of the vertices of another triangle in the plane. Plot these points and sketch the triangle. How is this new triangle related to the original triangle?

9. How might you find the coordinates of the triangle that is formed after ΔABC is rotated $90°$ counterclockwise about the origin using matrix multiplication? Find the coordinates of the rotated triangle.

10. How might you find the coordinates of the triangle that is formed after ΔABC is reflected over the x-axis using matrix multiplication? Find the coordinates of the reflected triangle.

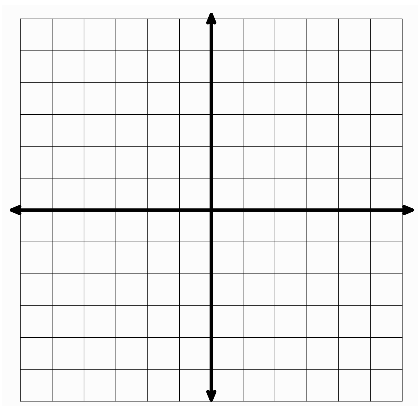
READY, SET, GO! Name Period Date

READY

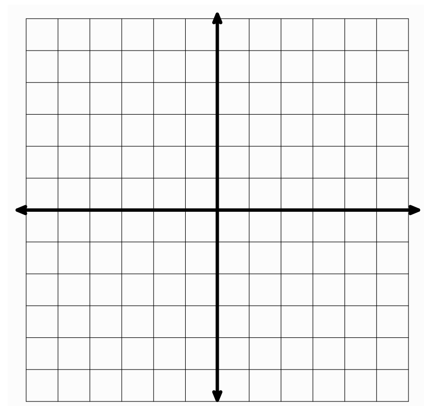
Topic: Adding Vectors

Given vectors $\vec{v}:\langle -2,\ 4\rangle$ and $\vec{w}:\langle 5,-2\rangle$, find the following using the parallelogram rule:

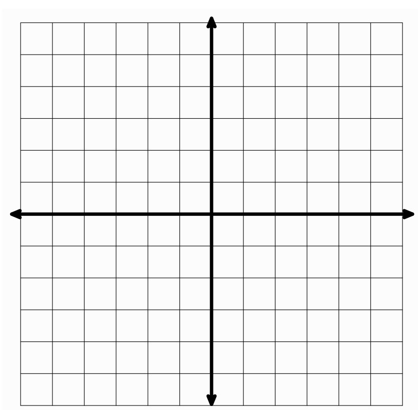
1. $\vec{v}+\vec{w}=$



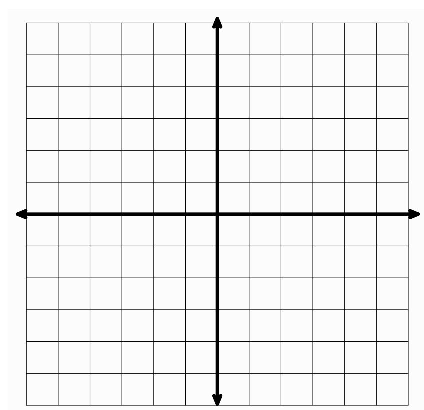
2. $\vec{v}-\vec{w}=$



3. $2\vec{v}+\vec{w}=$



4. $\vec{v}-2\vec{w}=$



SET

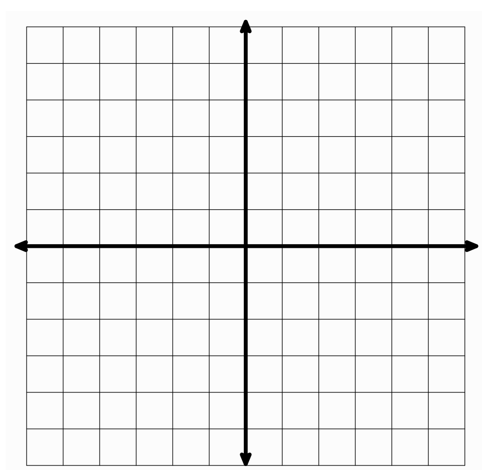
Topic: Matrices and Transformations of the Plane

3. List the coordinates of the four vertices of the parallelogram you drew in question 1 as a matrix. The x-values will be in the left column, and the y-values will be in the right column.

$$\begin{array}{r c} & \begin{matrix} \quad x \quad & \quad y \quad \end{matrix} \\ \begin{matrix} \text{Point 1} \\ \text{Point 2} \\ \text{Point 3} \\ \text{Point 4} \end{matrix} & \begin{bmatrix} \quad & \quad \\ \quad & \quad \\ \quad & \quad \\ \quad & \quad \end{bmatrix} \end{array}$$

4. Multiply the matrix you wrote in question 3 by the following matrix: $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

5. Plot the original parallelogram. Then, using the ordered pairs from your answer in question 4, using the points from the matrix in number 4 on the following coordinate grid. Connect those 4 points. What transformation occurred between your original parallelogram and the new one?



GO

Topic: Transformation of Functions

Function $f(x)$ is defined by the following table below:

x	2	4	6	8	10	12	14	16
$f(x)$	-8	-3	2	7	12	17	22	27
$g(x)$								
$h(x)$								

6. Write an equation for $f(x)$.

7a. Fill in the values for $g(x)$ assuming that $g(x) = f(x) + 3$

b. Write an equation for $g(x)$.

8a. Fill in the values for $h(x)$ assuming that $h(x) = 2f(x)$

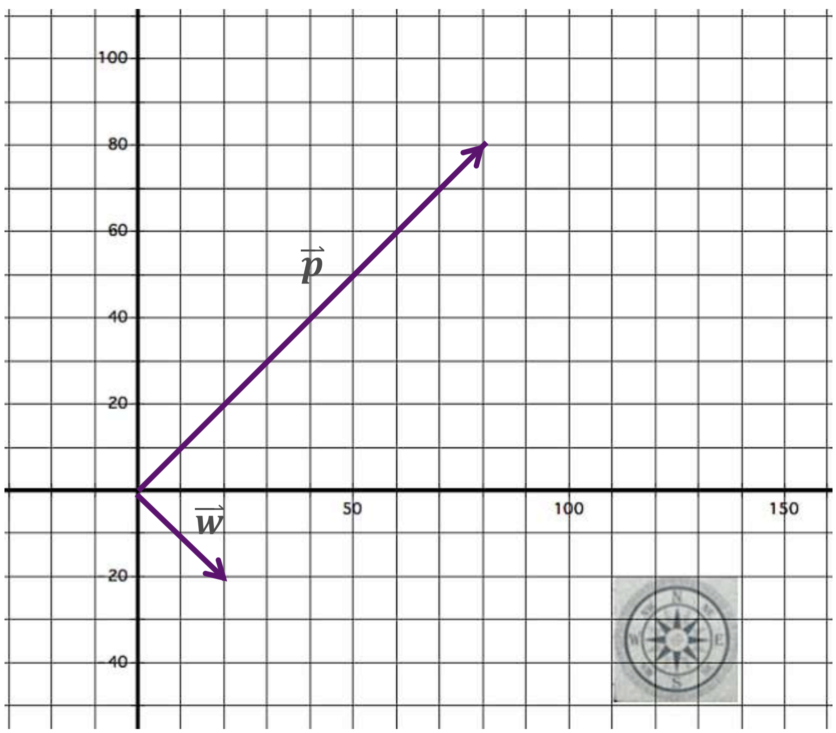
b. Write an equation for $h(x)$.

8.12H Plane Geometry

A Practice Understanding Task

Jon's father is a pilot and he is using vector diagrams to explain some principles of flight to Jon. His father has drawn the following diagram to represent a plane that is being blown off course by a strong wind. The plane is heading northeast as represented by $\vec{p}$ and the wind is blowing towards the southeast as represented by $\vec{w}$.

1. Based on this diagram, what is the plane's speed and what is the wind's speed? (The vector diagram represents the speed of the plane in still air.) Note: Each grid unit is 10 $^{miles}/_{hour}$



2. Use this diagram to find the ground speed of the plane, which will result from a combination of the plane's speed and the wind's speed. Also, indicate on the diagram the direction of motion of the plane relative to the ground.

3. Jon drew a parallelogram to determine the ground speed and direction of the plane. If you have not already done so, draw Jon's parallelogram and explain how it represents the original problem situation as well as the answers to the question asked in problem 2.

4. Write a matrix equation that will reflect the parallelogram you drew in problem 3 over the y-axis. Use the solution to the matrix equation to draw the resulting parallelogram.

5. Prove that the resultant figure of the reflection performed in problem 4 is a parallelogram. That is, explain how you know opposite sides of the resulting quadrilateral are parallel.

6. Find the area of the parallelogram drawn in problem 3. Explain your method for determining the area.

8.12H

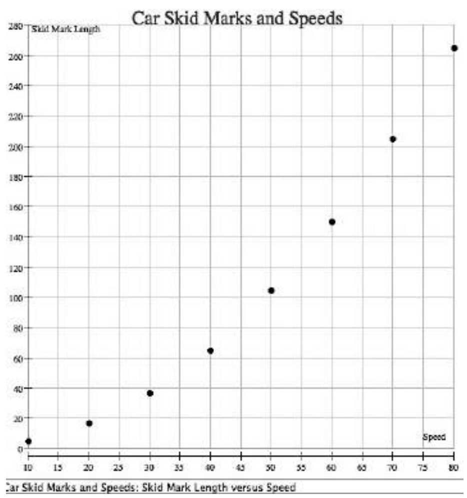
READY, SET, GO! Name Period Date

READY

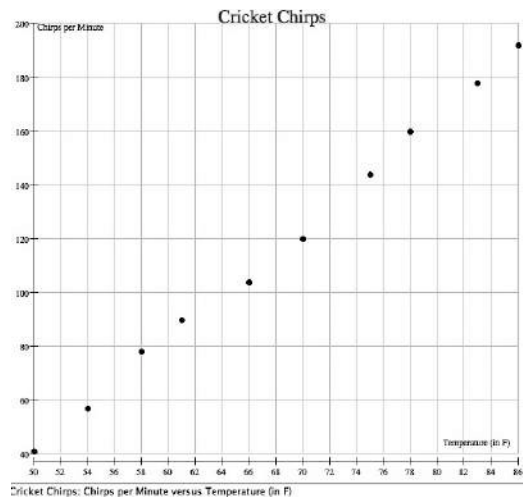
Topic: Scatter Plots and Trend Lines

Examine each of the scatterplots shown below. If possible, make a statement about relationships between the two quantities depicted in the scatterplot.

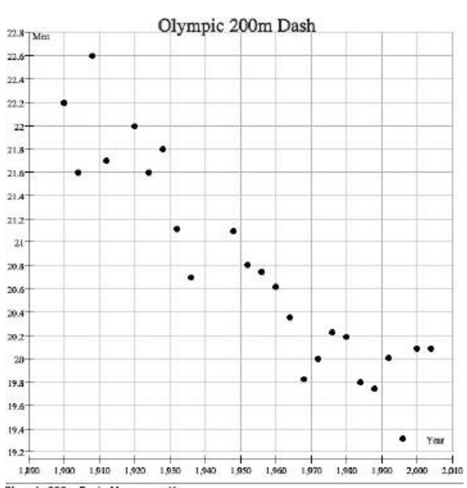
1.



2.



3.



4. For each scatterplot, write the equation of a trend line that you think best fits the data.

a. Trend line #1

b. Trend line #2

c. Trend line #3

SET

Topic: Applications of vectors.

Given: $\vec{u}:\langle -5,1\rangle$, $\vec{v}:\langle 3,5\rangle$, $\vec{w}:\langle 4,-3\rangle$. **Each of these three vectors represents a force pulling on an object—such as in a three-way tug of war—with force exerted in each direction being measured in pounds.**

5. Find the magnitude of each vector. That is, how many pounds of force are being exerted on the object by each tug? (Round to the nearest hundredth)

a. $\|\vec{u}\| =$

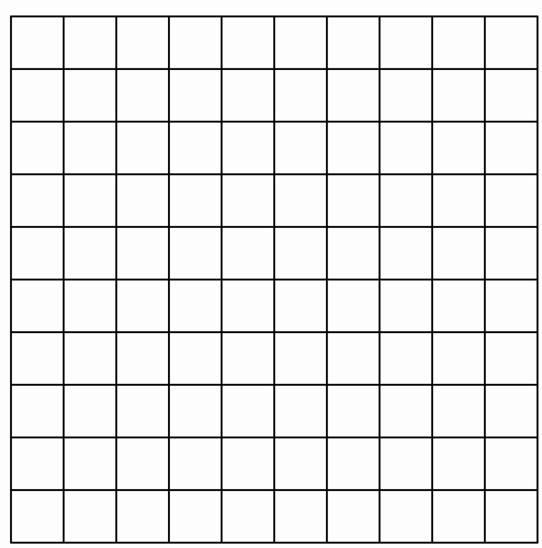
b. $\|\vec{v}\| =$

c. $\|\vec{w}\| =$

6. Find the magnitude of the sum of the three forces on the object.

$\|\vec{u} + \vec{v} + \vec{w}\| =$

7. Draw a vector diagram showing the resultant direction and magnitude of the motion resulting from this three-way tug of war.



GO

Topic: Solving Systems

8. Solve the given system in each of the following ways.

Given: $\begin{cases} 4x - 4y = 7 \\ 6x - 8y = 9 \end{cases}$

a. By substitution

b. By elimination

c. Using matrix row reduction

d. Using an inverse matrix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