CONSERVATIVE FORCES, POTENTIAL ENERGY AND CONSERVATION OF ENERGY

Today's Objectives:

Students will be able to:

1. Use the concept of conservative forces and determine the potential energy of such forces.
2. Apply the principle of conservation of energy.

READING QUIZ

1. The potential energy of a spring is ________

   A) always negative. B) always positive.

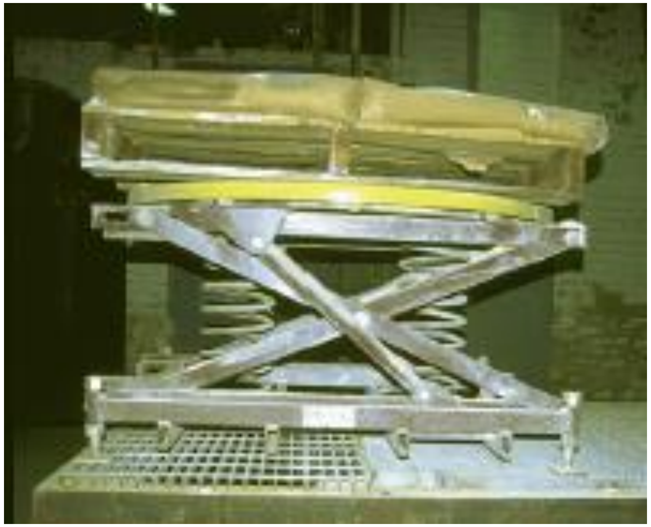
   C) positive or negative. D) equal to ks.

2. When the potential energy of a conservative system increases, the kinetic energy _________

   A) always decreases. B) always increases.

   C) could decrease or increase. D) does not change.

APPLICATIONS



The weight of the sacks resting on this platform causes potential energy to be stored in the supporting springs.

As each sack is removed, the platform will *rise* slightly since some of the potential energy within the springs will be transformed into an increase in gravitational potential energy of the remaining sacks.

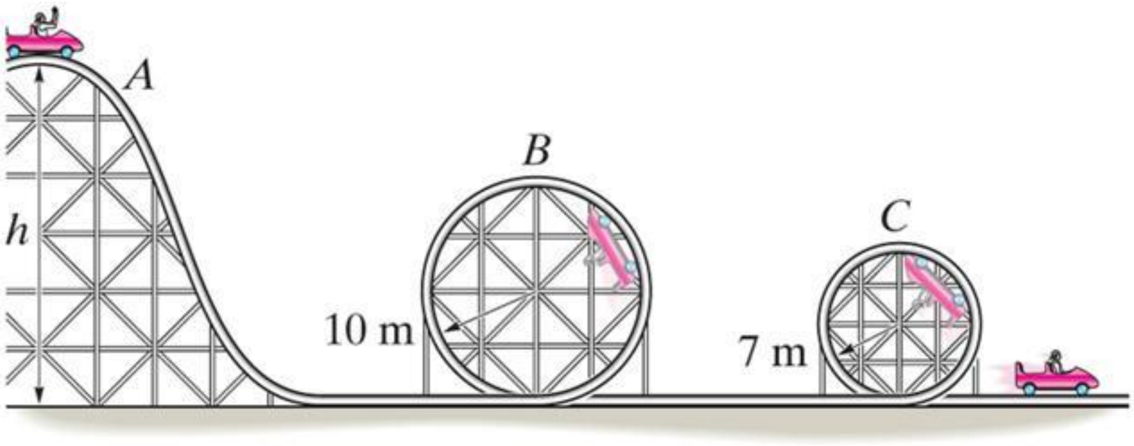
If the sacks weigh 100 lb and the equivalent spring constant is k = 500 lb/ft, what is the energy stored in the springs?

APPLICATIONS (continued)

The young woman pulls the water balloon launcher back, stretching each of the four elastic cords.

If we know the unstretched length and stiffness of each cord, can we estimate the maximum height and the maximum range of the water balloon when it is released from the current position? Would we need to know any other information?

APPLICATIONS (continued)



The roller coaster is released from rest at the top of the hill A. As the coaster moves down the hill, potential energy is transformed into kinetic energy.

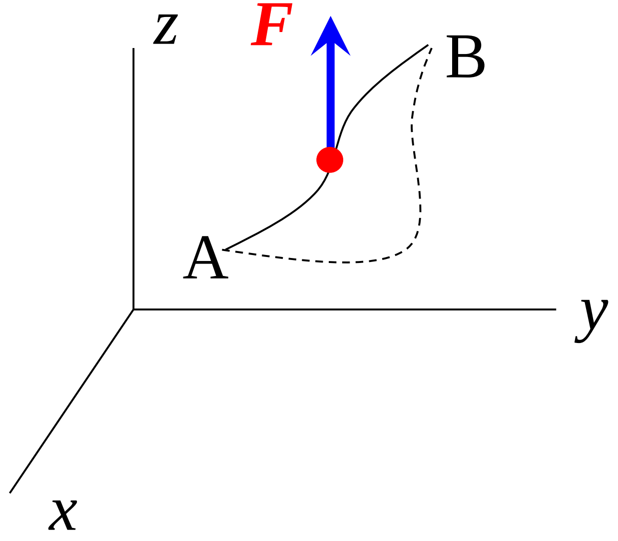
What is the velocity of the coaster when it is at B and C?

Also, how can we determine the minimum height of hill A so that the car travels around both inside loops without leaving the track?

CONSERVATIVE FORCE (Section 14.5)

A force ***F*** is said to be conservative if the work done is independent of the path followed by the force acting on a particle as it moves from A to B. This also means that the work done by the force ***F*** in a closed path (i.e., from A to B and then back to A) is zero.

$$\oint \mathbf{F} \cdot d\mathbf{r} = 0$$



Thus, we say the work is conserved.

The work done by a conservative force depends only on the positions of the particle, and is independent of its velocity or acceleration.

CONSERVATIVE FORCE (continued)

A more rigorous definition of a conservative force makes use of a potential function (V) and partial differential calculus, as explained in the text. However, even without the use of the these more complex mathematical relationships, much can be understood and accomplished.

The “conservative” potential energy of a particle/system is typically written using the potential function V. There are two major components to V commonly encountered in mechanical systems, the potential energy from gravity and the potential energy from springs or other elastic elements.

$$V_{total} = V_{gravity} + V_{springs}$$

POTENTIAL ENERGY

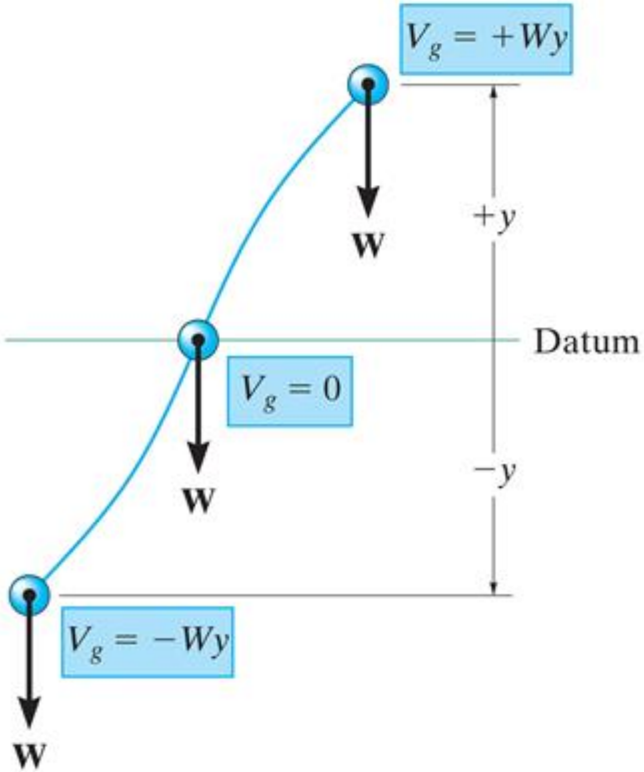
Potential energy is a measure of the amount of work a conservative force will do when a body changes position.

In general, for any conservative force system, we can define the potential function (V) as a function of position. The work done by conservative forces as the particle moves equals the change in the value of the potential function (e.g., the sum of $V_{gravity}$ and $V_{springs}$).

It is important to become familiar with the two types of potential energy and how to calculate their magnitudes.

POTENTIAL ENERGY DUE TO GRAVITY

The potential function (formula) for a gravitational force, e.g., weight (W = mg), is the force multiplied by its elevation from a datum. The datum can be defined at any convenient location.

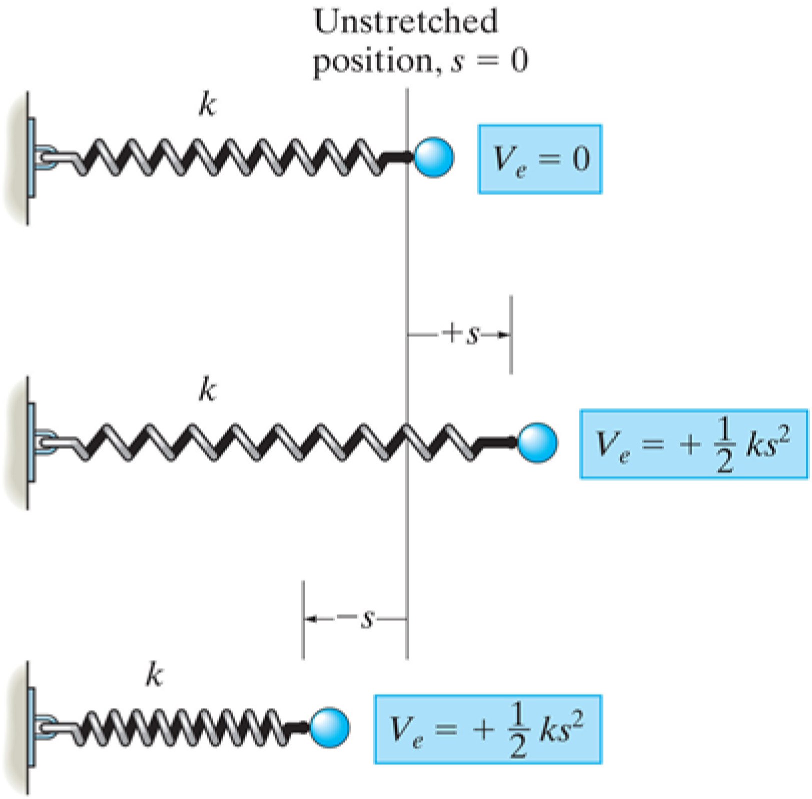


$$V_g = \pm W y$$

V_g is positive if y is above the datum and negative if y is below the datum. Remember, YOU get to set the datum.

ELASTIC POTENTIAL ENERGY

Recall that the force of an elastic spring is $F = ks$. It is important to realize that the potential energy of a spring, while it looks similar, is a different formula.



V_e (where 'e' denotes an elastic spring) has the distance "s" raised to a power (the result of an integration) or

$$V_e = \frac{1}{2} k\, s^2$$

Notice that the potential function V_e always yields positive energy.

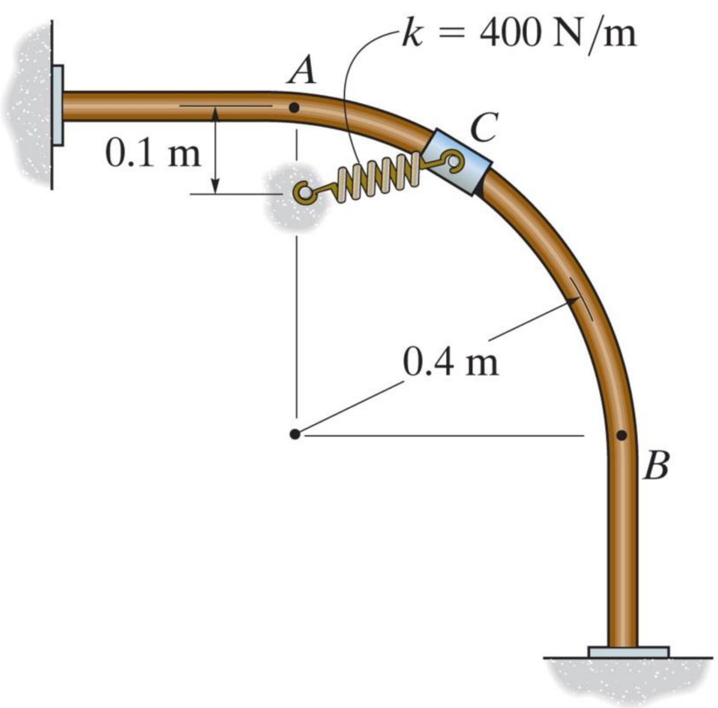
CONSERVATION OF ENERGY (Section 14.6)

When a particle is acted upon by a system of conservative forces, the work done by these forces is conserved and the sum of kinetic energy and potential energy remains constant. In other words, as the particle moves, kinetic energy is converted to potential energy and vice versa. This principle is called the principle of conservation of energy and is expressed as

$$T_1 + V_1 = T_2 + V_2 \quad = \text{ Constant}$$

T_1 stands for the kinetic energy at state 1 and V_1 is the potential energy function for state 1. T_2 and V_2 represent these energy states at state 2. Recall, the kinetic energy is defined as $T = ½ mv^2$.

EXAMPLE



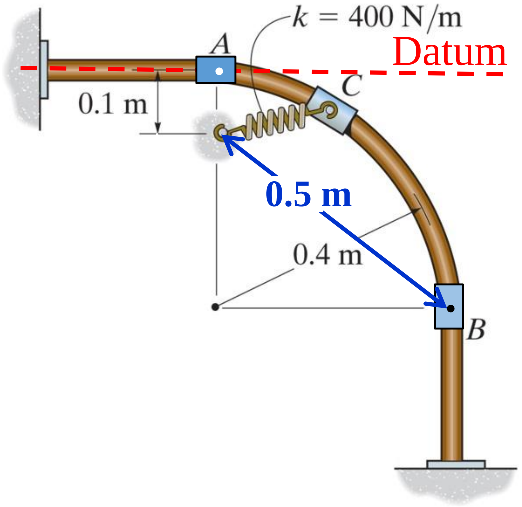
Given: The 4 kg collar, C, has a velocity of 2 m/s at A. The spring constant is 400 N/m. The unstretched length of the spring is 0.2 m.

Find: The velocity of the collar at B.

Plan: Apply the conservation of energy equation between A and B. Set the gravitational potential energy datum at point A or point B (in this example, choose point A—why?).

EXAMPLE (continued)

Solution:



Note that the potential energy at *B* has two parts.

$$V_B = (V_B)_e + (V_B)_g$$

$$V_B = 0.5\,(400)\,(0.5 - 0.2)^2 - 4\,(9.81)\,0.4$$

The kinetic energy at B is

$$T_B = 0.5\,(4)\,v_B^{\,2}$$

Similarly, the potential and kinetic energies at A will be

$$V_A = 0.5\,(400)\,(0.1 - 0.2)^2, \quad T_A = 0.5\,(4)\,2^2$$

The energy conservation equation becomes $T_A + V_A = T_B + V_B$.

$$[\,0.5(400)\,(0.5 - 0.2)^2 - 4(9.81)0.4\,] + 0.5\,(4)\,v_B^{\,2}$$

$$= [0.5\,(400)\,(0.1 - 0.2)^2\,] + 0.5\,(4)\,2^2$$

$$\Rightarrow \; v_B = \underline{1.96 \text{ m/s}}$$

CONCEPT QUIZ

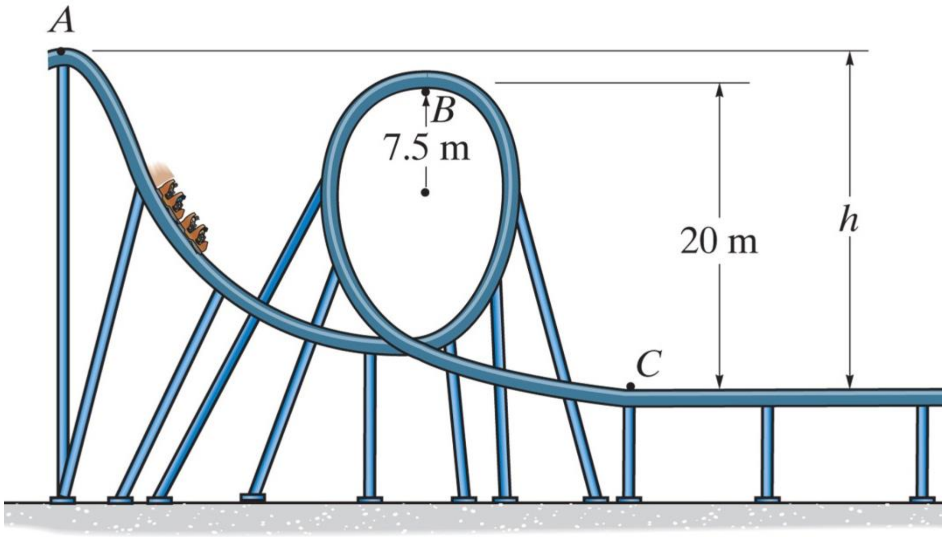
1. If the work done by a conservative force on a particle as it moves between two positions is –10 ft·lb, the change in its potential energy is _______

   A) 0 ft·lb.
   B) -10 ft·lb.
   C) +10 ft·lb.
   D) None of the above.

2. Recall that the work of a spring is $U_{1\text{-}2} = -\frac{1}{2}\, k(s_2^2 - s_1^2)$ and can be either positive or negative. The potential energy of a spring is $V = \frac{1}{2}\, ks^2$. Its value is __________

   A) always negative.
   B) either positive or negative.
   C) always positive.
   D) an imaginary number!

GROUP PROBLEM SOLVING I

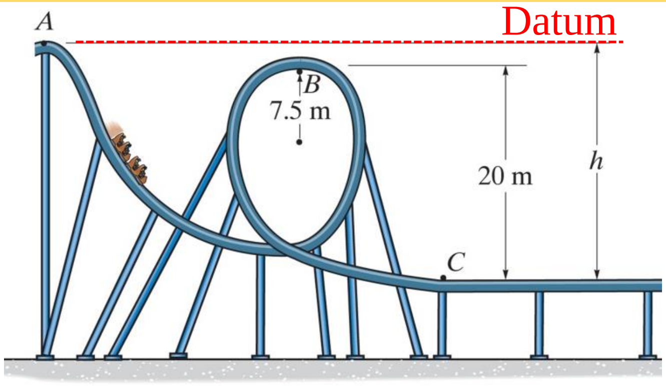


Given: The 800 kg roller coaster car is released from rest at A.

Find: The minimum height, h, of Point A so that the car travels around inside loop at B without leaving the track. Also find the velocity of the car at C for this height, h, of A.

Plan: Note that only kinetic energy and potential energy due to gravity are involved. Determine the velocity at B using the equation of motion and then apply the conservation of energy equation to find minimum height h .

Solution:



1) Placing the datum at A:

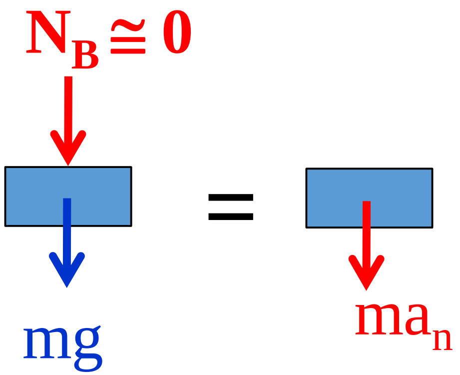
$$T_A + V_A = T_B + V_B$$

$$\Rightarrow 0.5\,(800)\,0^2 + 0$$

$$= 0.5\,(800)\,(v_B)^2 - 800(9.81)\,(h - 20) \qquad (1)$$

2) Find the required velocity of the coaster at B so it doesn't leave the track.

Equation of motion applied at B:



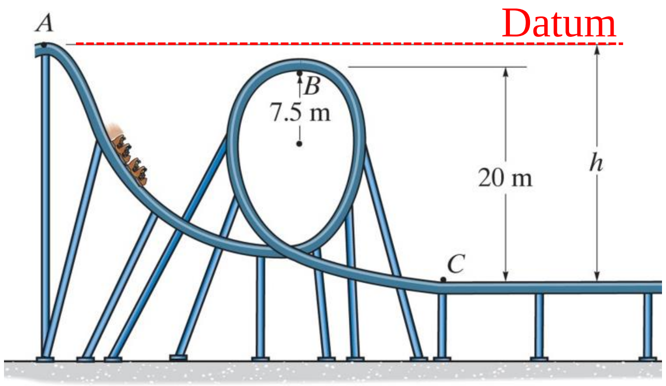
$$\sum F_n = ma_n = m\frac{v^2}{\rho}$$

$$800\,(9.81) = 800\,\frac{(v_B)^2}{7.5}$$

$$\Rightarrow\ v_B = 8.578 \text{ m/s}$$

GROUP PROBLEM SOLVING I (continued)

Now using the energy conservation, eq. (1), the minimum h can be determined.



$$0.5\,(800)\,0^2 + 0 = 0.5\,(800)\,(8.578)^2 - 800(9.81)\,(h - 20)$$

$\Rightarrow$ h = 23.75 m

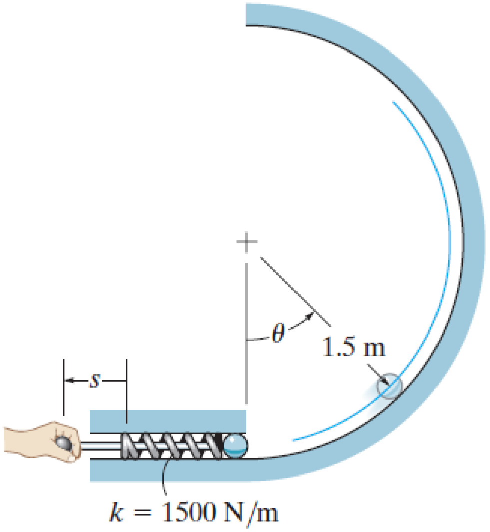
3) Find the velocity at C applying the energy conservation.

$$T_A + V_A = T_C + V_C$$

$$\Rightarrow 0.5\,(800)\,0^2 + 0 = 0.5\,(800)\,(v_C)^2 - 800(9.81)\,(23.75)$$

$\Rightarrow V_C$ = 21.6 m/s

GROUP PROBLEM SOLVING II



Given: The arm is pulled back such that s = 100 mm and released. When s = 0, the spring is unstretched. Assume all surfaces of contact to be smooth. Neglect the mass of the spring and the size of the ball.

Find: The speed of the 0.3-kg ball and the normal reaction of the circular track on the ball when $\theta = 60°$.

Plan: Determine the velocity at $\theta = 60°$ using the conservation of energy equation and then apply the equation of motion to find the normal reaction on the ball.

GROUP PROBLEM SOLVING II (continued)

Solution:

1) Placing the datum at A:

$$T_A + V_A = T_B + V_B$$

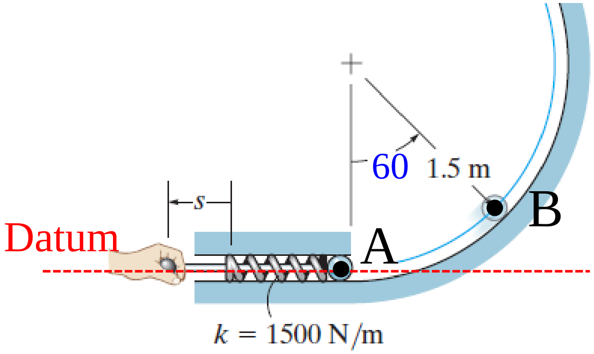
where

$T_A = 0.5\,(0.3)\,0^2$

$V_A = 0 + 0.5\,(1500)\,0.1^2$

$T_B = 0.5\,(0.3)\,0^2$

$V_B = 0.3\,(9.81)\,1.5\,(1 - \cos 60°)$



The conservation of energy equation is

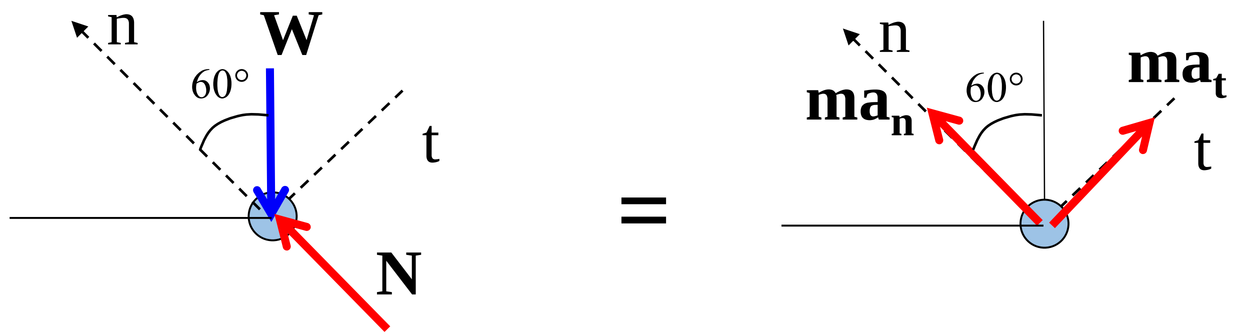
$$0 + 0.5\,(1500)\,0.1^2 = 0.5\,(0.3)\,(v_B)^2 + 0.3\,(9.81)\,1.5\,(1 - \cos 60°)$$

v_B = <u>5.94 m/s</u>

GROUP PROBLEM SOLVING II (continued)

2) Find the normal reaction on the ball when θ = 60°.

Free-body diagram Kinetic diagram



Equation of motion applied at θ = 60° :

$$\sum F_n = ma_n = m\frac{v_B^2}{\rho}$$

$$N - 0.3\,(9.81)\cos 60° = 0.3\,\frac{5.94^2}{1.5}$$

N = <u>8.53 N</u>

PEARSON

ATTENTION QUIZ

1. The principle of conservation of energy is usually ______ to apply than the principle of work & energy.

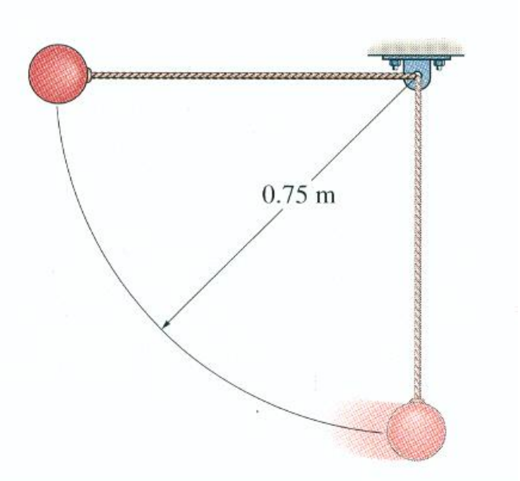
   A) harder B) easier

   C) the same amount of work D) It is a mystery!

2. If the pendulum is released from the horizontal position, the velocity of its bob in the vertical position is _____

   A) 3.8 m/s. B) 6.9 m/s.

   C) 14.7 m/s. D) 21 m/s.



End of the Lecture

Let Learning Continue