

Continuum Soil Modeling in the Static Analysis of Buried Structures

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ABSTRACT

Soil loading traditionally has been modeled as a hydrostatic pressure, a practice acceptable for many design applications. In the analyses of buried structures with predictive goals, soil compliance and load redistribution in the presence of soil plasticity are important factors to consider in determining the appropriate response of the structure. In the analysis of existing buried waste-storage tanks at the U.S. Department of Energy's Hanford Site, three soil-tank interaction modeling considerations are addressed. First, the soil interacts with the tank as the tank expands and contracts during thermal cycles associated with changes in the heat generated by the waste material as a result of additions and subtractions of the waste. Second, the soil transfers loads from the surface to the tank and provides support by resisting radial displacement of the tank haunch. Third, conventional finite-element mesh development causes artificial stress concentrations in the soil associated with differential settlement.

In predicting the response of the buried high-heat single-shell waste-storage tank 241-C-106 to thermal cycling and significant surcharge loading, a Drucker-Prager plasticity model is used to address soil compliance and surcharge load distribution. Triaxial test data from the Hanford Site are used to derive soil model parameters, which are needed to describe the Drucker-Prager constitutive model.

Finite-element meshes normally are developed to represent the unloaded condition, including the absence of gravity. Because of the significant stiffness and weight differences between the soil and the buried structure, significant differential settlement occurs as the gravity load is introduced. To address the differential settlement, three methods of mesh development and corresponding gravity application are described, in order of increasing complexity. The first method involves the application of a prestress condition. The second method uses built-in vertical "slip planes" where relative displacement is allowed to release artificial stresses at locations of discontinuity. The third method uses a mesh that is developed incrementally in layers to simulate the actual construction sequence.

INTRODUCTION

In the static analysis of a buried structure, the benefits of modeling soil as a continuum are apparent.

Unlike the representation of soil via invariant traction loads or grounded spring elements, the properly formulated continuum model will distribute soil deadweight and surface loads correctly to the structure.

Furthermore, it automatically will capture soil-structure interaction. Continuum soil models also allow the analyst to study the effects of backfilling and compaction.

These benefits come at the expense of increased complexity and cost of the analysis. Nonlinear finite-element analysis is the method of choice, because soil constitutive models are usually nonlinear. The material parameters for the analysis must be calibrated from nonstandard material test data. The finite-element mesh representing the soil must be defined so that there is adequate refinement in regions of large stress gradients, and a sufficient volume of soil must be modeled to preclude spurious boundary effects. Other important modeling considerations are contact conditions between the soil and the structure, and the soil's initial stress state.

This paper discusses briefly a few of the prevalent constitutive models for soil and describes various techniques for developing suitable finite-element models for the static analysis of buried structures. These techniques are demonstrated by describing their use in the axisymmetric analysis of a buried waste tank. This paper concludes by identifying several topics needing additional study.

SURVEY OF SOIL CONSTITUTIVE MODELS

Several constitutive soil models are available in the ABAQUS [1] general-purpose finite-element computer code, and many others are described in the literature [2]. In general soil constitutive models are more complicated than linear elastic models. Until the development of modern computers, practical problems involving soils modeled as continua were not solvable partly because of the complexity of the soil constitutive relations. Finite-element computer codes that can compute soil deformations effectively are now available.

The most widely known soil constitutive model is the Mohr-Coulomb model. The Mohr-Coulomb model is actually a failure criterion. The material is assumed to behave as a linearly elastic solid until failure occurs. In its simplest form, the Mohr-Coulomb criterion states that the absolute value of shear stress in a plane at failure is an affine function of the normal stress in the plane [3]. The two parameters that define the failure line are called the cohesion and the angle of internal friction. The cohesion defines the intercept of the failure line with the shear stress axis. The angle of internal friction determines the slope of the failure line.

Another widely known soil constitutive model is the Drucker-Prager model. The classical Drucker-Prager model [4] postulates a yield function that depends on the hydrostatic pressure and the magnitude of the deviator stress. Until failure, the material behaves as an elastic solid. On yielding, the material becomes perfectly plastic. The Drucker-Prager model has been modified with post-yield strain hardening and a compression cap on the yield surface. This modified Drucker-Prager model is included in the ABAQUS Version 5.2 finite-element computer code.

Many other types of soil constitutive models have been proposed. In the 1960's, researchers at Cambridge proposed the critical state theories after observing the behavior of soil samples in a uniform state of stress and strain. The simplest critical state model, often called the Cam-clay model, is a relatively complicated four-parameter model. The critical state is the constant-volume state reached by samples undergoing shearing deformation. The hyperbolic model [5] is yet another soil constitutive model. In the hyperbolic model, the stress-strain relation is a hyperbolic equation. The equation is implicit in stress, i.e., stress terms appear on both sides of the equation.

FINITE-ELEMENT MODELING

Practical problems involving buried structures are solved best via the finite-element method. This section describes some of the finite-element techniques used to analyze the 241-C-106 buried waste tank located at the U.S. Department of Energy's Hanford Site [6]. The techniques were applied with the ABAQUS finite-element program; however, the procedures are useful to analysts using other nonlinear finite-element programs.

SOIL CONSTITUTIVE MODEL

The University of California at Berkeley has done considerable research on soil constitutive modeling [5], [7], [8] that addresses the variation of the soil properties with depth and confining pressure. The goal has been to develop a model suitable for finite-element analysis that models properly the soil structure interaction as well as the lateral loads introduced by soil compaction. The Berkeley research employed the existing soil triaxial-test procedures with confining pressures; this research resulted in the development of the hyperbolic model for stress-strain and bulk moduli [2], [5].

The hyperbolic model (see Figure 1) assumes that stress-strain curves for soils can be approximated as hyperbolas. The local slope of the hyperbolic stress-strain curve is the tangent modulus E_t , which is defined by

$$E_t = (1 - R_f SL)^2 K P_a \left(\frac{\sigma_3}{P_a} \right)^n, \quad (1)$$

where

- R_f = Constant (0.6 to 0.9)
- SL = Ratio of deviatoric stress to deviatoric stress at Mohr-Coulomb failure
- K, n = Constants relating the initial tangent modulus to confining pressure
- P_a = Atmospheric pressure.

The hyperbolic model is really a family of hyperbolic stress-strain curves that shift with confining pressure or stress (σ_3) and the axial compression stress minus the confining pressure ($\sigma_1 - \sigma_3$).

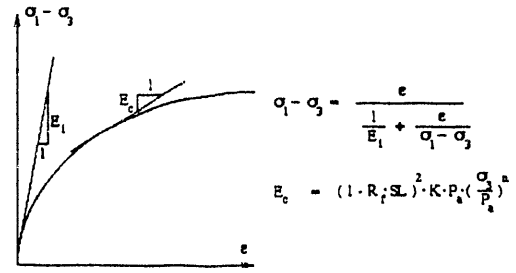
The hyperbolic model uses the tangent modulus to model all situations corresponding to primary loading, where all loading occurs at a stress level equal to or higher than all previous stress levels, triaxial testing of the Hanford soil shows good agreement with the power equation relation for initial tangent modulus. Triaxial testing of the Hanford soil also shows that, when the stress level is less than the previous maximum stress, the soil no longer follows the primary load curve. The soil responds in an unload-reload path that is defined by the unload-reload modulus as follows:

$$E_{ur} = K_{ur} P_a \left(\frac{\sigma_3}{P_a} \right)^n \quad (2)$$

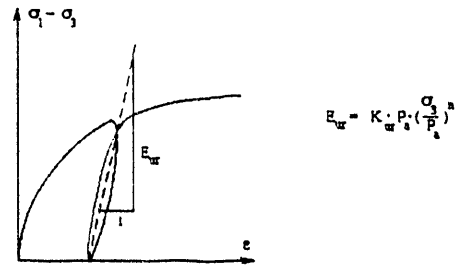
where

K_{ur} is typically 1.2 to 3 times greater than K .

The hyperbolic model was programmed and interfaced to ABAQUS as a user-defined material subroutine. Validation testing of the programming and performance of the hyperbolic model as implemented with the ABAQUS structural analysis program showed very positive results for simple test cases. The simple test cases included numerical simulation of a series of triaxial compression tests with different confining pressures and compaction of rigidly confined volumes of soil.



(a) Hyperbolic Representation of Stress-Strain Curve for Primary Loading



(b) Linear Unloading-Reloading Stress-Strain Relationship

Figure 1. Hyperbolic Soil Model.

The hyperbolic model was tested with the 241-C-106 single-shell tank structural model. The hyperbolic model, as implemented through the ABAQUS user-defined material subroutine, encountered numerical instabilities in regions of structural discontinuities. The ABAQUS program estimates the deflections and strains, then calculates stresses, and iterates until equilibrium force balance is obtained. The hyperbolic model as used directly defines the modulus of stiffness as being dependent on the stress history; the modulus is used by ABAQUS to define the stress field so errors compound themselves with stress fields that can get out of step with the strains. Additional work on the convergence criteria is needed before the Hyperbolic model can be implemented reliably in complex problems.

Although the hyperbolic stress-strain relation was unsuccessful because of its numerical instabilities as implemented in ABAQUS, the power equation confining pressure relations for tangent modulus and unload-reload modulus have proven useful for interpolation and extrapolation of the test data. The power equation relations were used to help define the variation with confining pressure and depth in the soil Drucker-Prager constitutive model.

The Drucker-Prager plasticity model is one of several constitutive models that defines a material's yield surface as a function of confining pressure. Although a modified Drucker-Prager plasticity model in ABAQUS is capable of including effects of non-associated flow, strain hardening, and a capped yield surface, the classical Drucker-Prager model (elastic, perfectly plastic) was used in the analysis of tank 241-C-106 for two reasons. First, nonstandard material test data required to calibrate the material parameters of the modified model were lacking, and second, the classical model is more numerically stable. In the 241-C-106 tank model, the ANACAP-U [9] concrete constitutive subroutine disables the ABAQUS automatic load incrementation as described in [10]. The Drucker-Prager strain-hardening model does not work efficiently without automatic load incrementation.

Available test data indicated that Young's modulus and the uniaxial compressive yield stress of the soil around 241-C-106 varies as a function of depth, or more precisely, as a function of the mean stress (pressure) in the soil. In ABAQUS, many of the constitutive parameters can be made a function of a user-specified field variable. Assigning the field variable at any given location in the soil a value equal to the expected confining pressure can make the soil constitutive parameters pressure-dependent. The field-variable profile (expected confining pressure field) in the soil was fixed throughout time for the 241-C-106 analysis and was calculated as follows:

$$F.V. = \text{mean stress} = -(\sigma_x + \sigma_y + \sigma_z)/3 \quad (3)$$

where

σ_x	= radial stress	= $K_o \gamma h$
σ_y	= vertical stress	= γh
σ_z	= hoop stress	= $K_o \gamma h$
K_o	= Rankine coefficient of lateral earth pressure (at rest)	
	= $\nu/(1-\nu)$, where ν is Poisson's Ratio	
γ	= soil density	
h	= depth of node measured from surface of soil.	

The field-variable approach is an approximation as the confining pressure generally changes as the analysis progresses, and the expected confining pressure is not known a priori. To improve accuracy, the field-variable profile can be updated periodically throughout the analysis to correspond to the changing confining

pressure. Analyses in which the field-variable profile remains constant throughout will generate useful results if the changing pressure field resembles the specified field-variable profile with pressure variances remaining small. Even in cases where nontrivial pressure changes occur locally in the soil over the course of the analysis, load redistribution will tend to diminish the error attributable to the use of a fixed field-variable profile.

The Drucker-Prager model used in the 241-C-106 tank analysis accurately reproduces most of the results of triaxial tests on soil from the Hanford Site. However, the constitutive model compromises some of the soil behavior for the sake of numerical simplicity. As mentioned previously, the 241-C-106 analysis did not use automatic load incrementation because of the concrete constitutive model. Although the strain-hardening Drucker-Prager theory could replicate almost exactly the available triaxial tests, it could not function properly without automatic load incrementation. Consequently, preliminary analyses of the 241-C-106 tank which used the strain-hardening model failed. The strain-hardening model was abandoned in favor of the classical Drucker-Prager model. The Drucker-Prager model used in the final 241-C-106 analyses has a reduced modulus to capture some of the softening caused by yielding. It also has an artificially high yield stress at any given confining pressure so that the maximum compressive stress developed during a triaxial test simulation is representative of measured values.

SOIL DISCRETIZATION

Eight-node biquadratic, reduced integration, axisymmetric solid elements (CAX8R) were used to model the soil surrounding tank 241-C-106. The finite-element mesh of the tank and the surrounding soil are shown in Figure 2. Because quadratic elements were used, the mesh is relatively coarse. However, the degree of soil mesh refinement increases slightly near the tank.

Approximating the soil as a finite continuum requires that the distance to the outer boundaries of the soil be established so that the location of these have little influence on the stress state at the soil-tank interface. Three test cases were evaluated with preliminary models to establish the outside radius of the soil that would not influence the local effects at the inside radius adjacent to the structure (tank). These

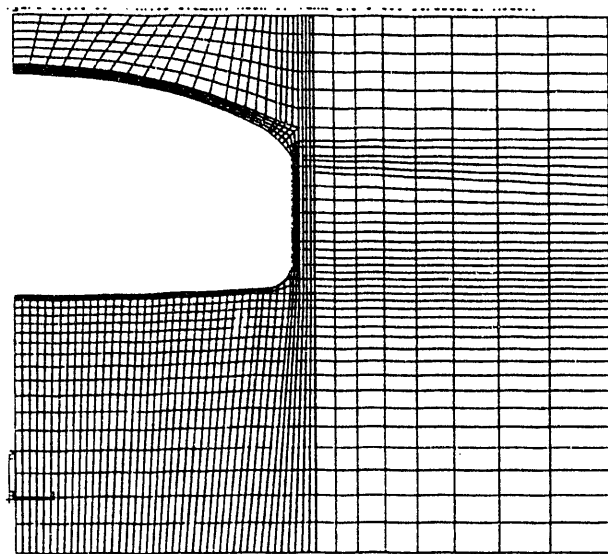


Figure 2. Finite-Element Mesh for 241-C-106 Model.

cases correspond to $R_o = 1.6R_i$, $2R_i$, and $3R_i$ where R_o is the distance from the center of the tank to the outer boundary of the soil and R_i is the outer radius of the tank. The results indicated that an outside soil radius of $2R_i$ is sufficient to define a fixed-lateral-displacement boundary condition, i.e., the computed stresses at the outer boundary approach the theoretical free-field stresses. In keeping with this observation, the radius of the outer soil boundary in the 241-C-106 model was specified as approximately two times the outside radius of the tank wall. The depth of soil underneath the tank was established as one tank radius. A lesser depth would likely suffice as the high confining pressure at depth effectively stiffens the soil to a point where it is insensitive to tank loads.

SOIL-STRUCTURE INTERFACE CONDITIONS

When modeling a buried structure, the analyst must specify contact conditions between the soil and the structure that are capable of realistically simulating soil-structure interaction. Advanced finite-element codes such as ABAQUS have special contact elements that may be used at the soil-structure interface. Unfortunately, these contact elements tend to be costly. Alternately, nonlinear spring elements can be used to maintain the proper contact conditions. The appropriate spring element is defined as very stiff in compression but very compliant in tension, to allow separation of the soil from

the structure. The 241-C-106 structural model uses nonlinear spring elements (SPRING2) to enforce the contact conditions between the tank and the surrounding soil. These springs act in a fixed direction normal to the initial soil-tank interface surface. Thus, tangential forces from friction at the tank/soil interface are neglected.

INITIAL STRESS STATE

In analyzing most structures, it is appropriate to begin with a complete mesh of stress-free, undeformed elements and subsequently apply the specified loads to obtain the desired stress state. Buried structures are an exception in that their response generally is nonlinear and depends on the history of the loading. The analyst must take steps to ensure that the actual stress condition developed during construction, primarily during backfilling operations, is represented adequately in the model before applying subsequent loads. In pursuit of this goal, backfilling around the structure may be simulated by adding stress-free layers of soil elements to a previously loaded and deformed mesh. Alternately, ad hoc modeling techniques may be used to approximately calculate a stress state representative of sequential backfilling. The primary advantage of the ad hoc procedures is the relative ease of generating the model. Three such ad hoc modeling approaches are described below.

In the first approach, gravity is applied to the structure and all the soil in one computational step. Both the structure and soil are stress free before the application of gravity. This method is simple to use, but may produce unrealistic and sometimes large tensile stresses in the soil. Soil deformation tends to be overpredicted when this approach is employed.

A second ad hoc approach is to impose a user-defined stress field onto the undeformed soil mesh. In ABAQUS, geostatic stress states can be imposed onto a mesh using the *GEOSTATIC option. Gravity then is applied in the first computational load step and displacements are computed to obtain force equilibrium. This approach is usually not practical because the quality of the final solution largely depends on the accuracy of the prescribed preliminary stress state.

In the third ad hoc approach, strategically-placed vertical "slip planes" in the soil are activated during the application of gravity and deactivated for subsequent loading. These planes allow the two "columns" of soil on either side of a slip plane vertically to displace independently of each other.

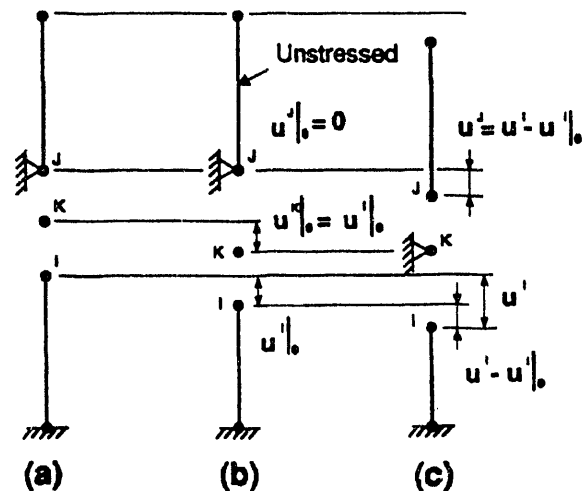
This approach inhibits the formation of spurious localized stresses near geometrical irregularities in the buried structure. These local stresses are bounded to some degree by the soil plasticity model, irrespective of the inclusion of slip planes.

The method of developing the initial stress state that is preferred over the ad hoc approaches is the "construction sequence" approach that simulates the actual construction/backfill sequence by adding the backfill soil a layer at a time. The first layer is added and the system then is allowed to deform under the gravitational body force. An undeformed second layer then is added. Thus, in the analysis, as in reality, the deformation of any soil layer is due only to its own weight and the weight of fill layers above it.

The so-called "dummy node" technique described in [11] may be used to add an undeformed soil layer to a previously deformed mesh. In brief, a row of dummy nodes is defined along the interface between the soil layers. The dummy nodes track the displacement of the top row of the bottom layer as it deforms. The displacements at the dummy nodes are held fixed at the final displacements obtained by the top row of the bottom layer. A constraint equation is prescribed to subtract these displacements from the subsequent displacements of the bottom row of nodes on the top layer. Thus, the top layer is not affected by the initial deformation of the bottom layer.

The approach is illustrated in Figure 3 in terms of one-dimensional elements. The constraint equation $u^I - u^J - u^K = 0$ relates nodal displacements. Figure 3(a) shows the load-free/undeformed initial state of the system. In the next step, illustrated in Figure 3(b), a load is applied to the bottom element while displacement at node *J* is restrained. Displacements at nodes *I* and *K* are identical. In the step shown in Figure 3(c), the displacement boundary condition at node *J* is removed to allow the top element to deform, node *K* is held fixed at its location at the end of the preceding step, and additional load is applied. Displacement of node *J* is equal to the change in displacement of node *I* from the end of the preceding step. For this and subsequent steps, nodes *I* and *J* displace together.

The above approach is applied in the tank analysis to simulate the stress-free addition of soil backfill layers. In reality, backfilling involves the placement of numerous thin layers of soil. The analysis approximates the backfilling sequence by considering four relatively



Note: Nodes I, J, and K are initially coincident. Nodes are shown offset for clarity.

Figure 3. One-Dimensional Illustration of the "Dummy Node" Technique.

thick backfill layers (footing soil layer, first wall soil layer, second wall soil layer, and top soil layer). The backfill layers modeled are shown in Figure 4. The bottom soil layer comprises all the soil beneath the tank floor elevation and is not considered as backfill.

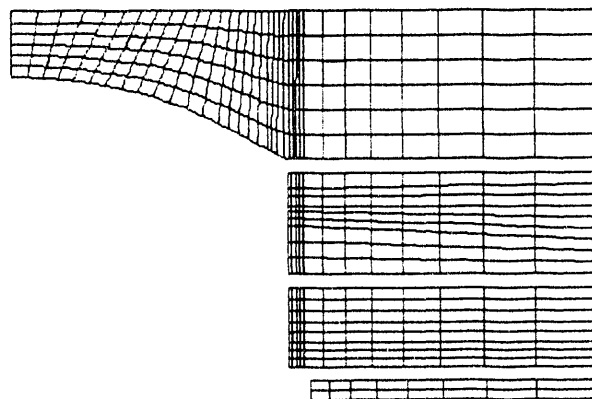


Figure 4. Backfill Layers in the 241-C-106 Model.

Compaction of each soil layer can be included by applying a compaction load to each layer before the addition of the next layer. Typically, backfilling is performed by placing a thin layer of soil (approximately 1-ft thick), compacting the soil to a specified density, and repeating these steps until the compacted soil surface is at the finish elevation. Unfortunately, backfill compaction cannot be simulated realistically in an axisymmetric analysis because any load applied in such an analysis is a "ring" load and not in character with a compaction load applied in the field.

Seed and Duncan [7] point out that the horizontal stress profile in compacted backfill tends to be more uniform and creates a larger resultant force than a triangular stress profile predicted by Rankine theory. Neglecting compaction of fill layers in the model leads to underestimation of the initial horizontal earth pressure towards the top of the structure; earth pressure near the bottom of the structure is reasonably accurate. This deviation from reality in the model is conservative with respect to determining the ultimate structural capacity of 241-C-106 tank because the horizontal earth pressure near the haunch provides resistance to dome collapse from a vertical dome load.

A model of a generic buried tank was used to compare the ad hoc methods of calculating the initial geostatic stress state to the "construction sequence" method. Geometric irregularities in the generic tank are similar to those of 241-C-106. The nonlinear springs at the soil-tank interface and the soil parameters used in the generic model are identical to those used in the 241-C-106 model.

Figure 5 shows the sequential addition of soil layers as used in the construction sequence simulation. Gaps appear between soil layers as a consequence of using the dummy node method; however, proper interface conditions are maintained via constraint equations. A final deformed mesh of the tank with displacements scaled by a factor of 200 is shown in Figure 6 for each of the ad hoc modeling approaches (test1, test2, test3) and the "construction sequence" modeling approach (test4). The original location of the tank mesh is indicated by dashed lines. Although absolute displacements vary significantly among the approaches, the deformed shapes of the tank are similar. Horizontal and vertical soil-tank interface spring forces are plotted as a function of distance along the outside surface of the tank in Figures 7 and 8, respectively. In the plot of horizontal spring forces, the midside node spring forces are distributed to the corner node springs to provide a

more realistic indication of the horizontal load distribution.

These results indicate that the displacements and interface forces calculated using any of the ad hoc techniques were noticeably different from those calculated using the "construction sequence" approach. To some degree the magnitude of these differences is dependent on the parameters of the generic model. Because of the demonstrated sensitivity of the results to the modeling approach, it is recommended that the most realistic approach, i.e., the construction sequence approach, be used.

CONCLUSION

Soil continuum models may be used effectively in the static analysis of buried structures, as demonstrated by the analysis of the 241-C-106 buried waste tank [6]. The benefits of explicitly modeling the soil (or any similar material in which a structure is buried) are two-fold. First, such modeling provides a means of accurately distributing surface load and soil weight to the structure. Second, it appropriately addresses the soil-structure interaction. Of course, soil continuum models increase the complexity of the analysis because in general, they are nonlinear. Furthermore, they increase the size of the finite-element model because the soil region must be discretized.

Many soil constitutive models are described in the literature. Some, like the Mohr-Coulomb theory or the Drucker-Prager theory, are actually failure or yield criteria joined with a theory of elasticity, usually the linear theory. Others, like the hyperbolic model, are complicated, fully nonlinear constitutive models. Each model has its inherent limitations and advantages. For example, the Mohr-Coulomb failure criterion has the advantage of being rather simple to comprehend. The simplicity, however, limits its ability to capture the behavior of a soil over a wide range of confining pressures. More complex soil constitutive models are available that are capable of capturing more than just the rudimentary aspects of soil behavior; however, calibration of the parameters for the model requires material testing beyond what is generally provided to the analyst.

The development of a finite-element model of a soil region requires special procedures. Nonlinear contact conditions between the soil and the structure must be specified. A realistic initial stress state must either be specified or created. Pressure dependence of

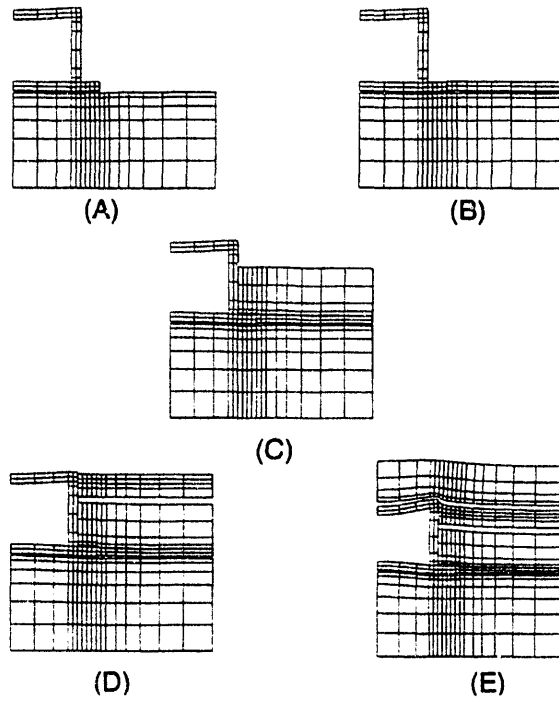


Figure 5. Sequential Addition of Soil Layers in the Generic Buried-Tank Model.

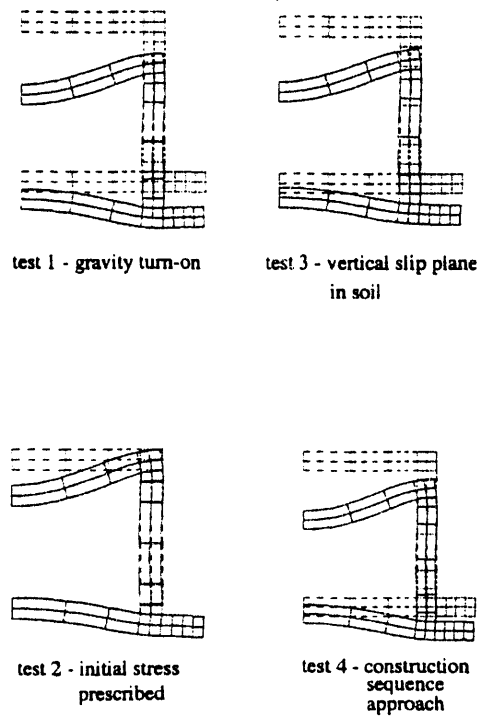


Figure 6. Deformed Shape of the Generic Buried-Tank Model.

Horizontal Spring Forces (Midside node forces lumped to corner nodes)

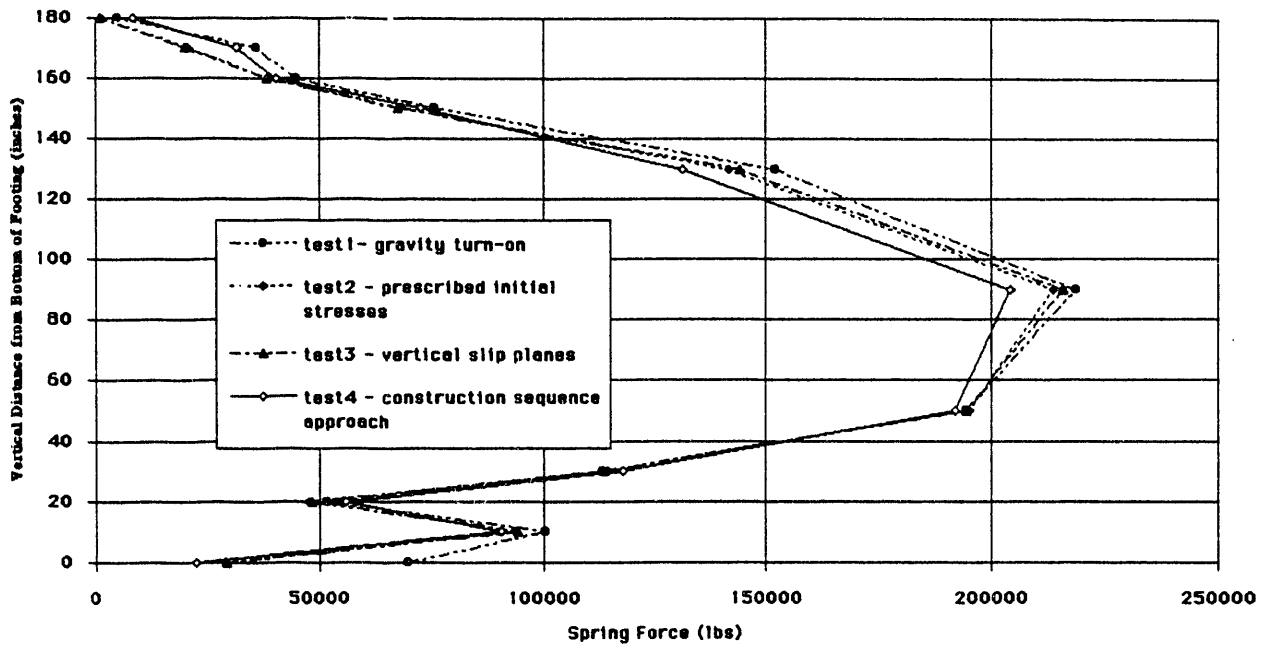


Figure 7. Horizontal Soil-Tank Interface Forces.

Vertical Spring Forces

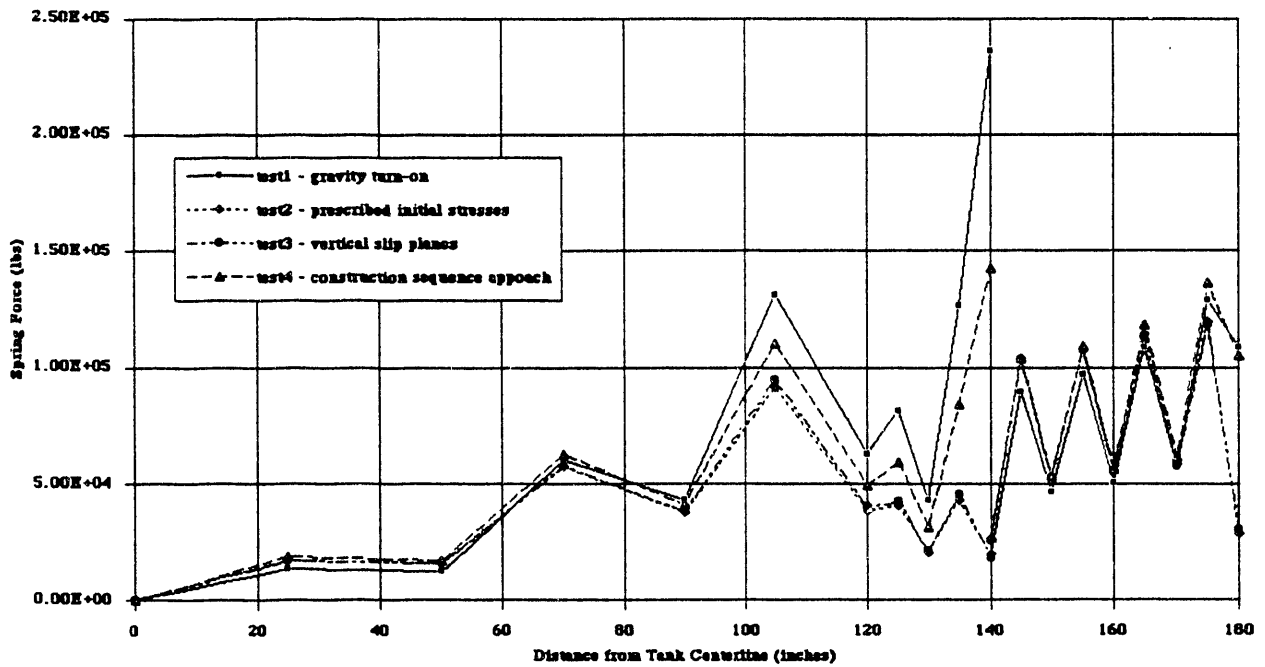


Figure 8. Vertical Soil-Tank Interface Forces.

the soil constitutive relation may have to be refined with a field variable.

Several topics in the field of soil continuum modeling need additional study. Soil compaction is one such area. The dummy node method can be used to model soil compaction; however, the correct magnitude and manner of load application to simulate compaction accurately and practically are not readily apparent, particularly for an axisymmetric analysis. Another subject that warrants additional study is soil-structure interface friction.

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