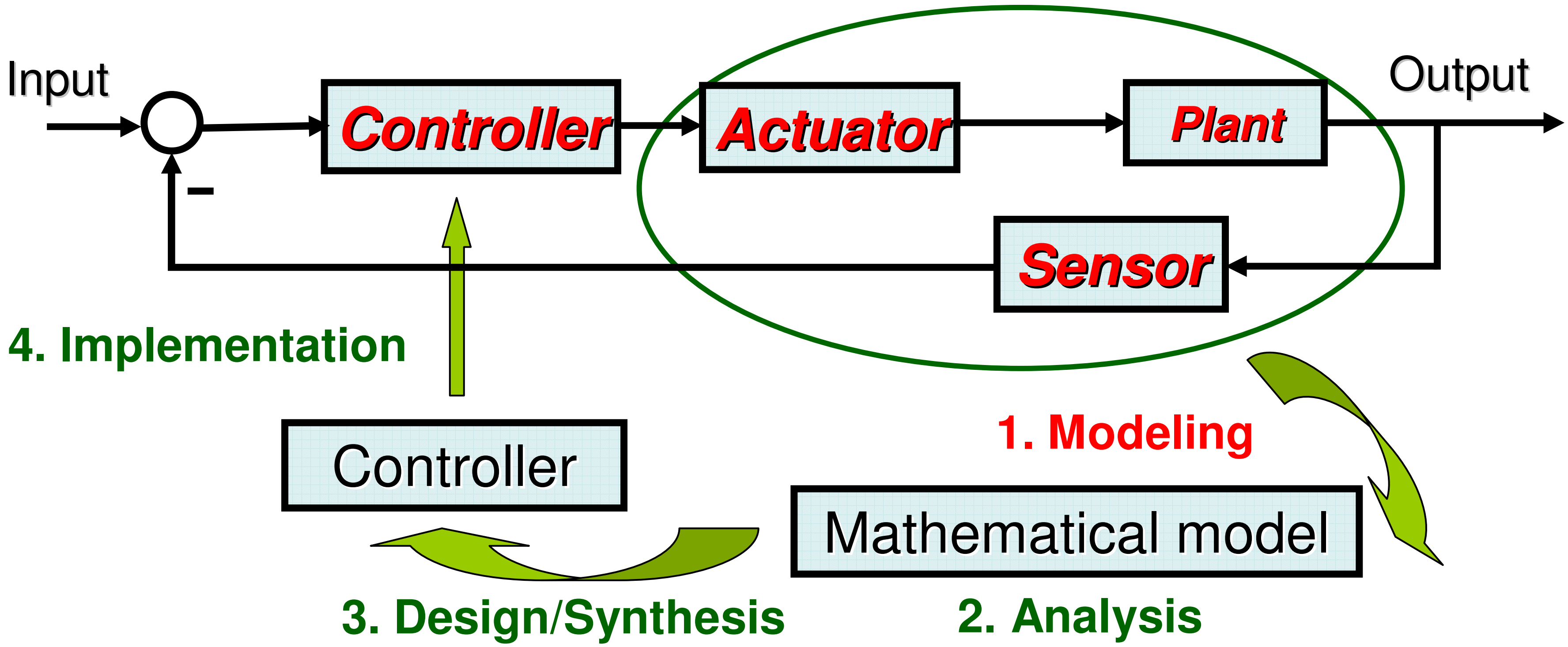
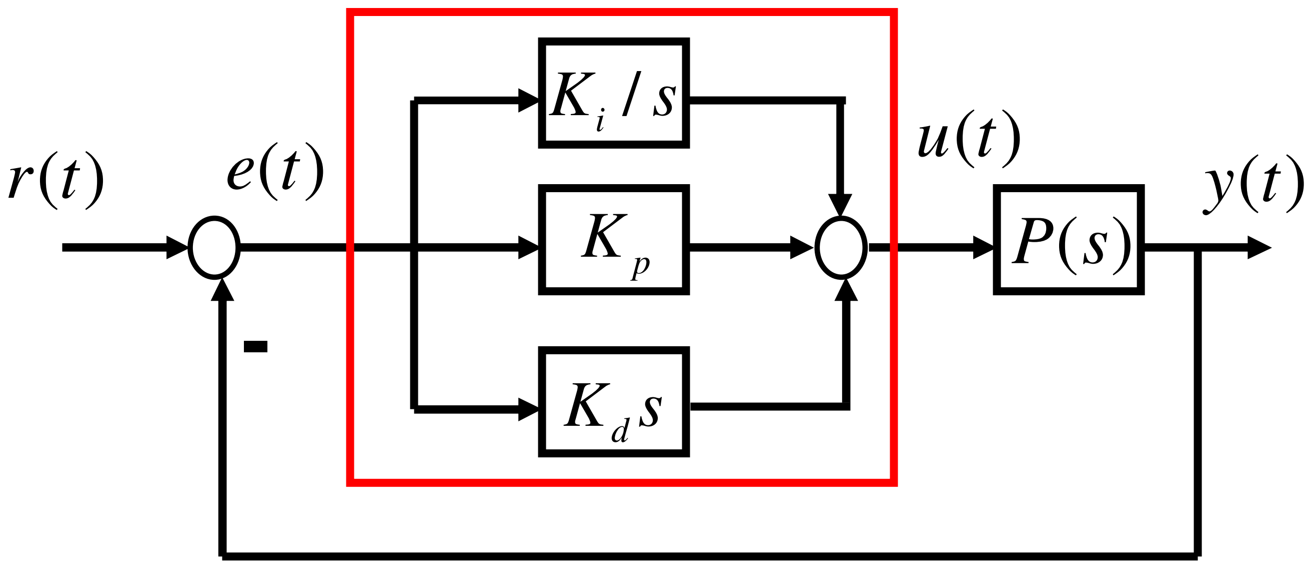
Control Design (CD) – Control Design Process



Control Design based upon
- RL (Root Locus)
- Frequency Response Design
  - Bode Diagram (BD)
  - Nyquist Approach

Control Method to be discussed:
- PID control
- Gain compensation
- Phase lead/lag compensation
- Lead/lag compensation

CD – PID Controller



t-domain: $$u(t) = \underbrace{K_p e(t)}_{\text{Proportional}} + \underbrace{K_i \int_0^t e(\tau) d\tau}_{\text{Integal}} + \underbrace{K_d \frac{de(t)}{dt}}_{\text{Derivative}}$$

s-domain: $$C(s) = K_p + \frac{K_i}{s} + K_d s = K_p \left(1 + \frac{1}{K_I s} + K_D s \right)$$

CD – PID Controller Remarks

- Most popular in process and robotics industries
  - Good performance
  - Functional simplicity (Operators can easily tune.)
- To avoid high frequency noise amplification, derivative term is implemented as

$$K_d s \approx \frac{K_d s}{\tau_d s + 1}$$

with τ_d much smaller than plant time constant.

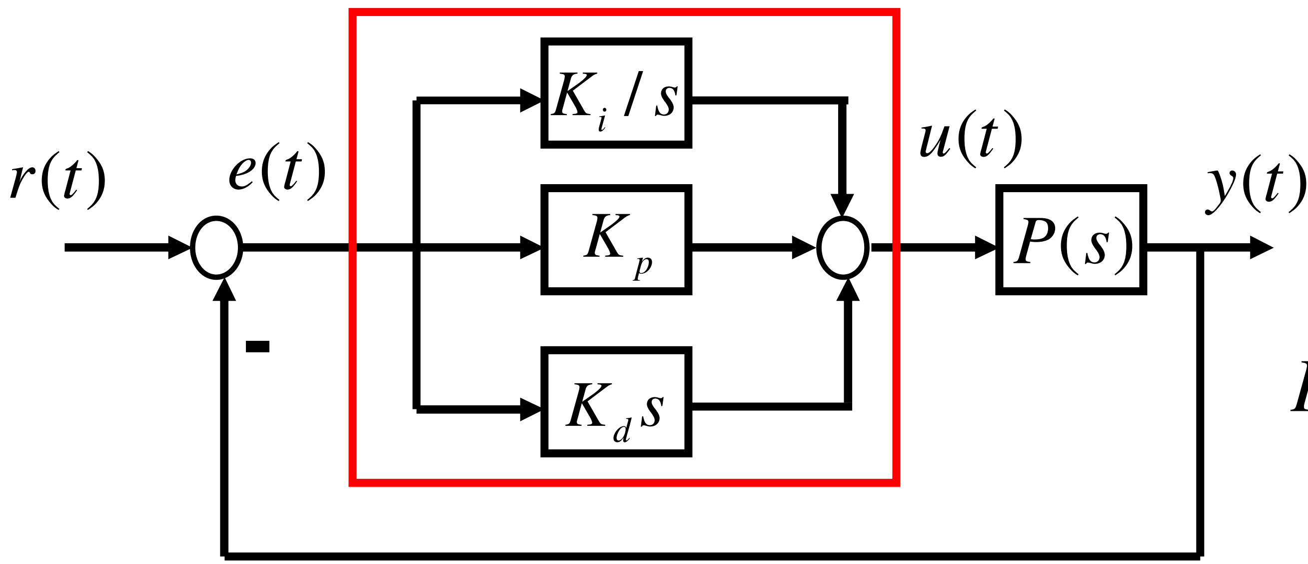
- PI controller

$$C(s) = K_p + \frac{K_i}{s}$$

- PD controller

$$C(s) = K_p + K_d s$$

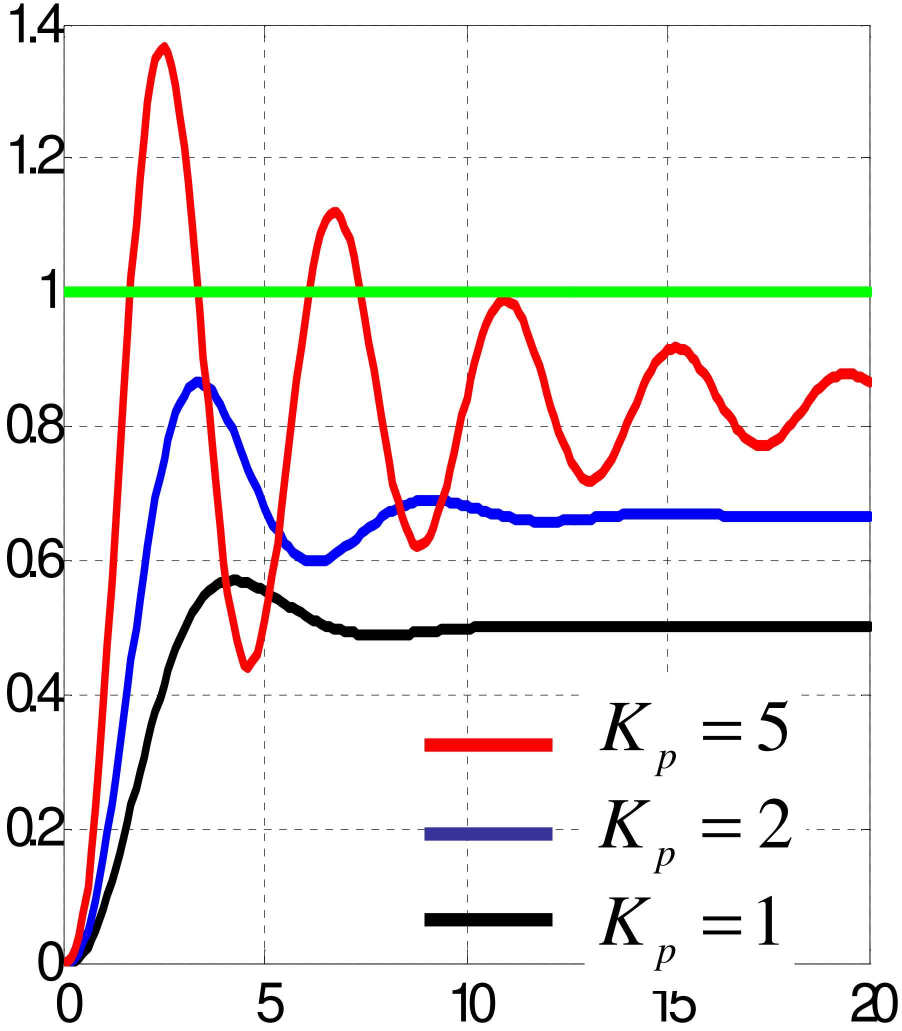
CD – A Simple Example (1)



$$P(s) = \frac{1}{(s+1)^3}$$

- We plot $y(t)$ for step reference $r(t)$ with
  - P controller
  - PI controller
  - PID controller

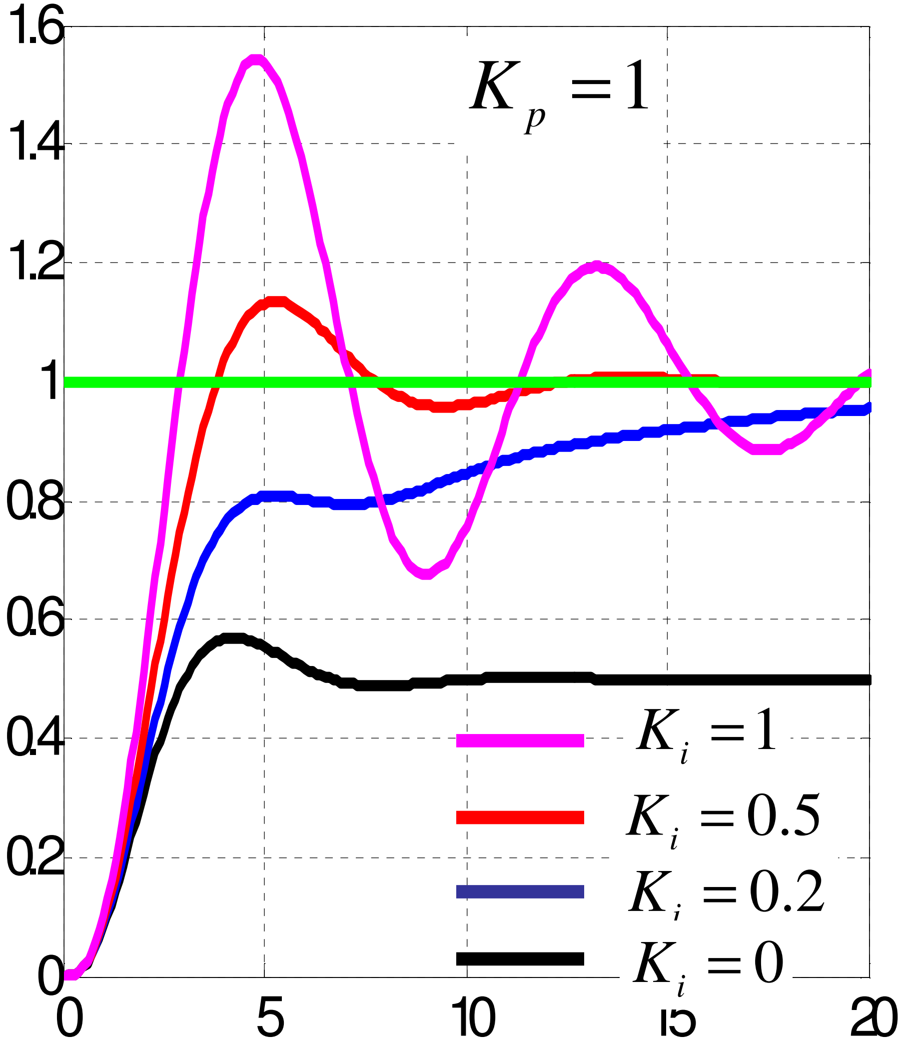
CD – A Simple Example (P Controller (2))



$$C(s) = K_p$$

- Simple
- Steady state error
  - Higher gain gives smaller error
- Stability
  - Higher gain gives faster and more oscillatory response

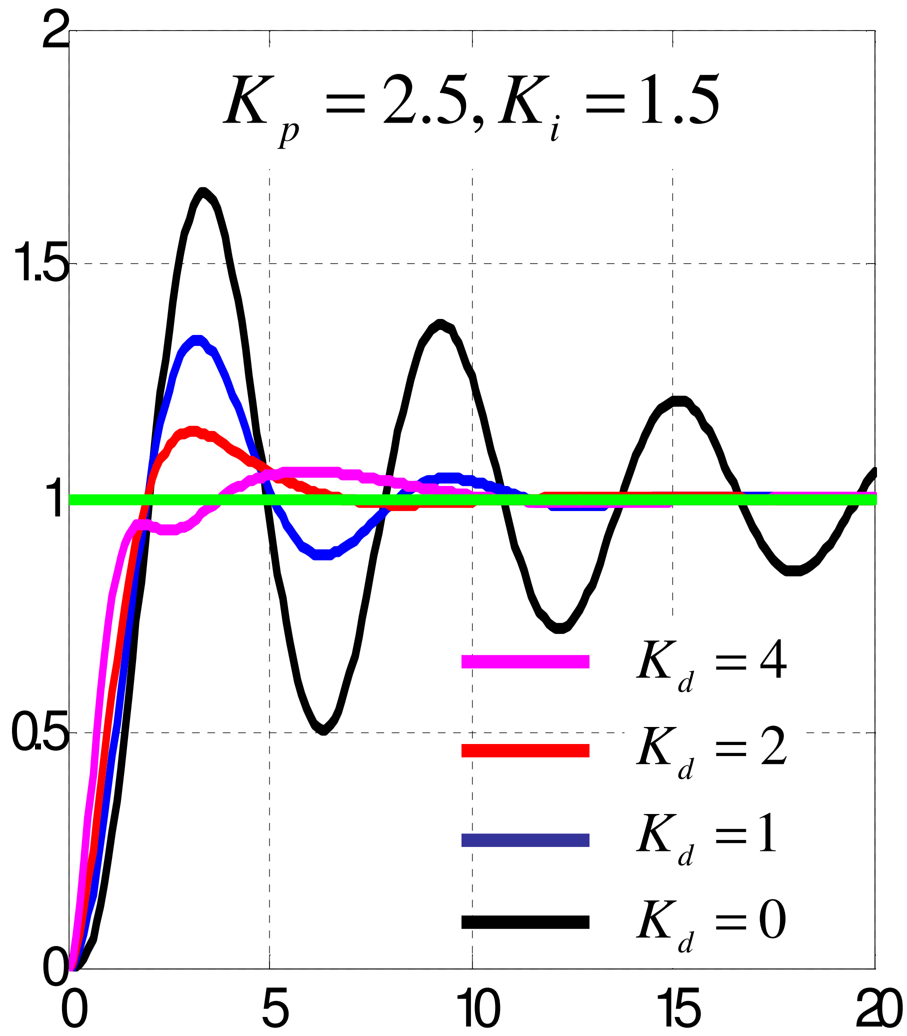
CD – A Simple Example (PI Controller (3))



$$C(s) = K_p + \frac{K_i}{s}$$

- Zero steady state error (provided that CL is stable.)
- Stability
  - Higher gain gives faster and more oscillatory response

CD – A Simple Example (PID Controller (4))



$$C(s) = K_p + \frac{K_i}{s} + K_d s$$

- Zero steady state error (due to integral control)
- Stability
  - Higher gain gives more damped response
- Too high gain worsen performance.

CD – How to Turn PID Parameters

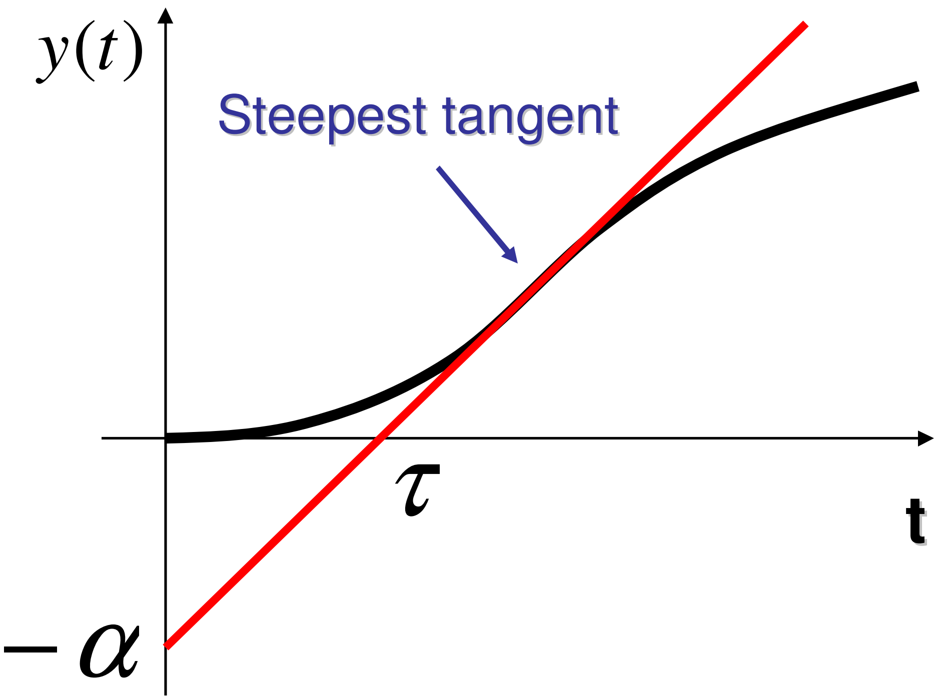
- Model-based
  - Root locus
  - Frequency response approach
  - Useful only when a model is available
  - Necessary if a system has to work at the first trial

- Empirical (without model)
  - Ziegler-Nichols tuning rule (1942)
  - Simple
  - Useful even if a system is too complex to model
  - Useful only when trial-and-error tuning is allowed

CD – Ziegler-Nichols PID Tuning Rules (1)

- Step response method (for only stable systems)

Open-loop step response → **PID parameters**



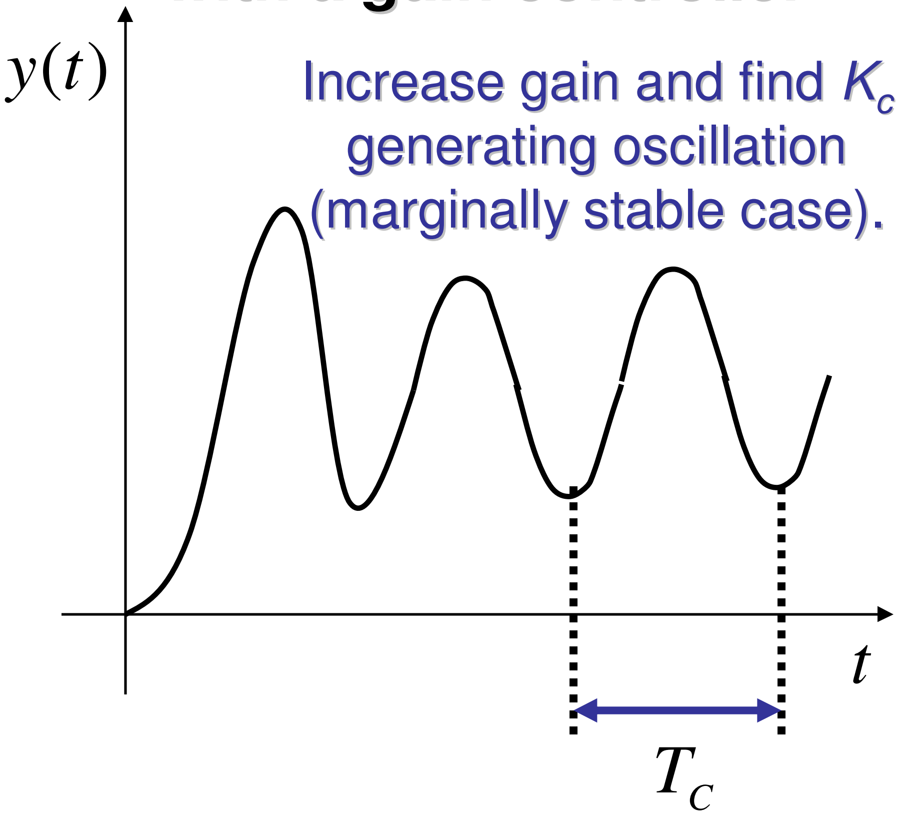
$$C(s) = K_p\left(1 + \frac{1}{T_I s} + T_d s\right)$$

Type	K_P	T_I	T_D
P	$1/\alpha$		
PI	$0.9/\alpha$	3τ	
PID	$1.2/\alpha$	2τ	0.5τ

CD – Ziegler-Nichols PID Tuning Rules (2)

- Ultimate sensitivity method

Closed-loop step response with a gain controller



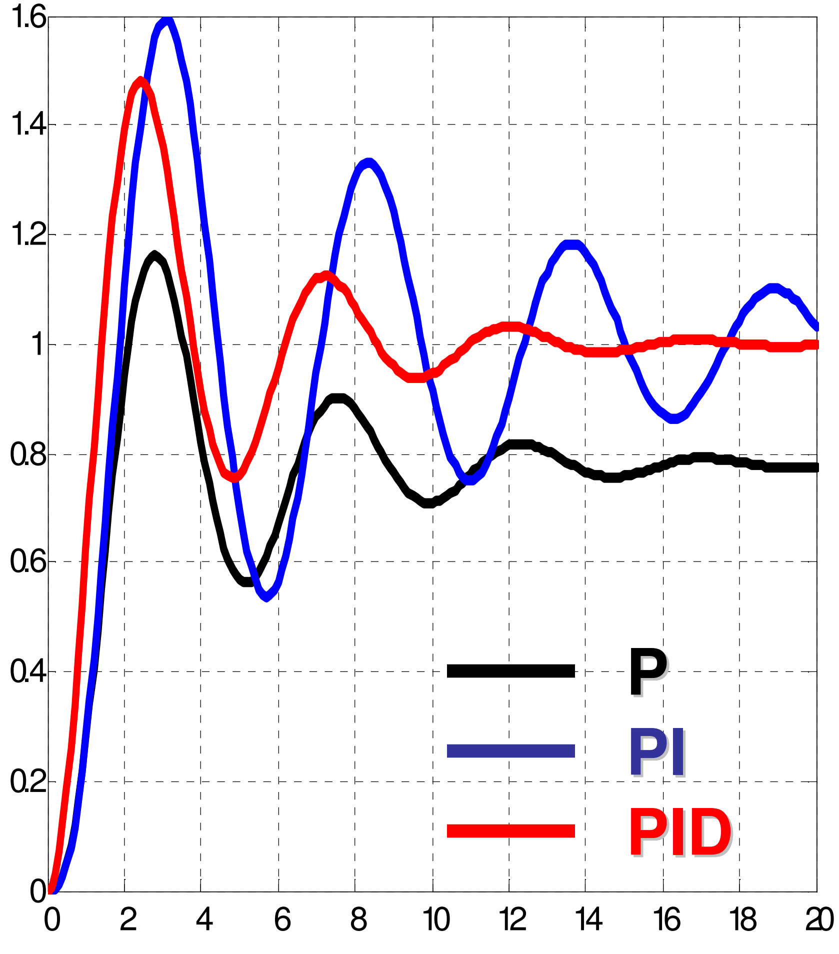
PID parameters

$$C(s) = K_p\left(1 + \frac{1}{T_I s} + T_d s\right)$$

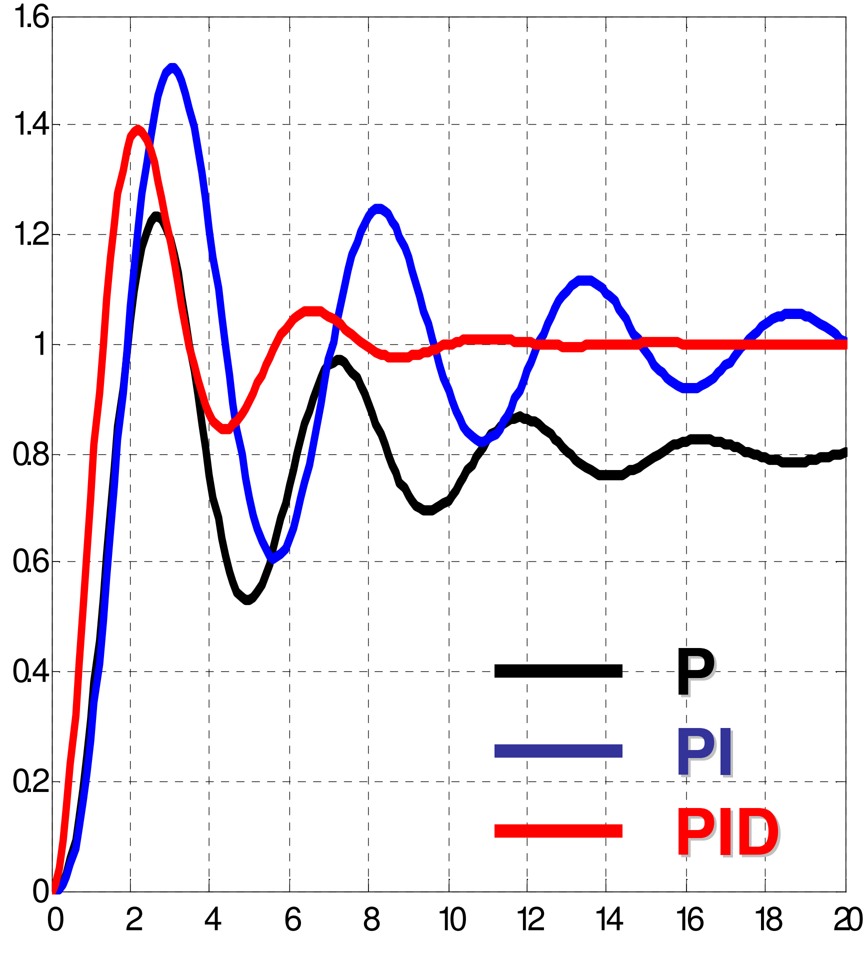
Type	K_P	T_I	T_D
P	$0.5K_C$		
PI	$0.4K_C$	$0.8T_C$	
PID	$0.6K_C$	$0.5T_C$	$0.125T_C$

CD – A Simple Example (Revisited (5))

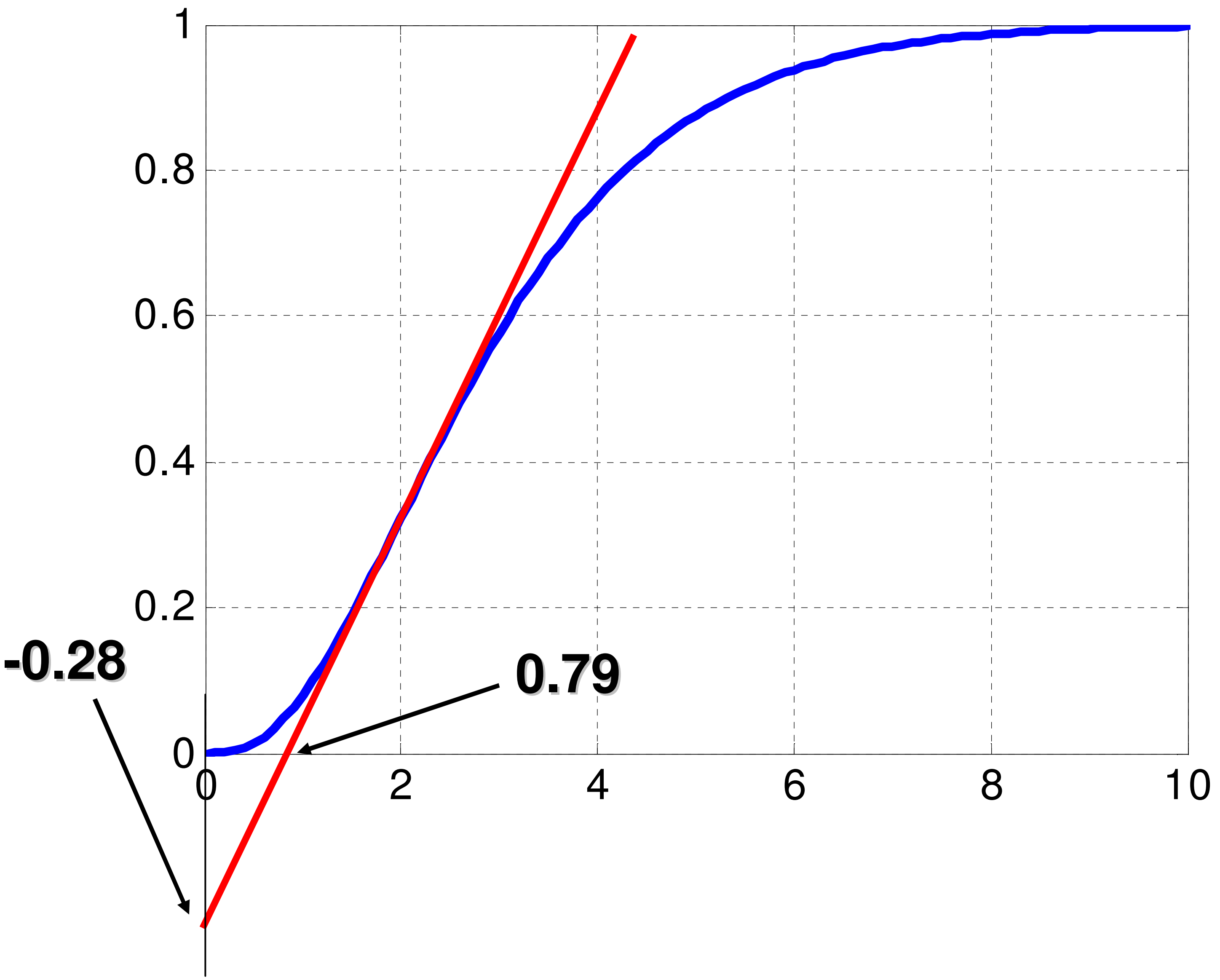
- Step response method



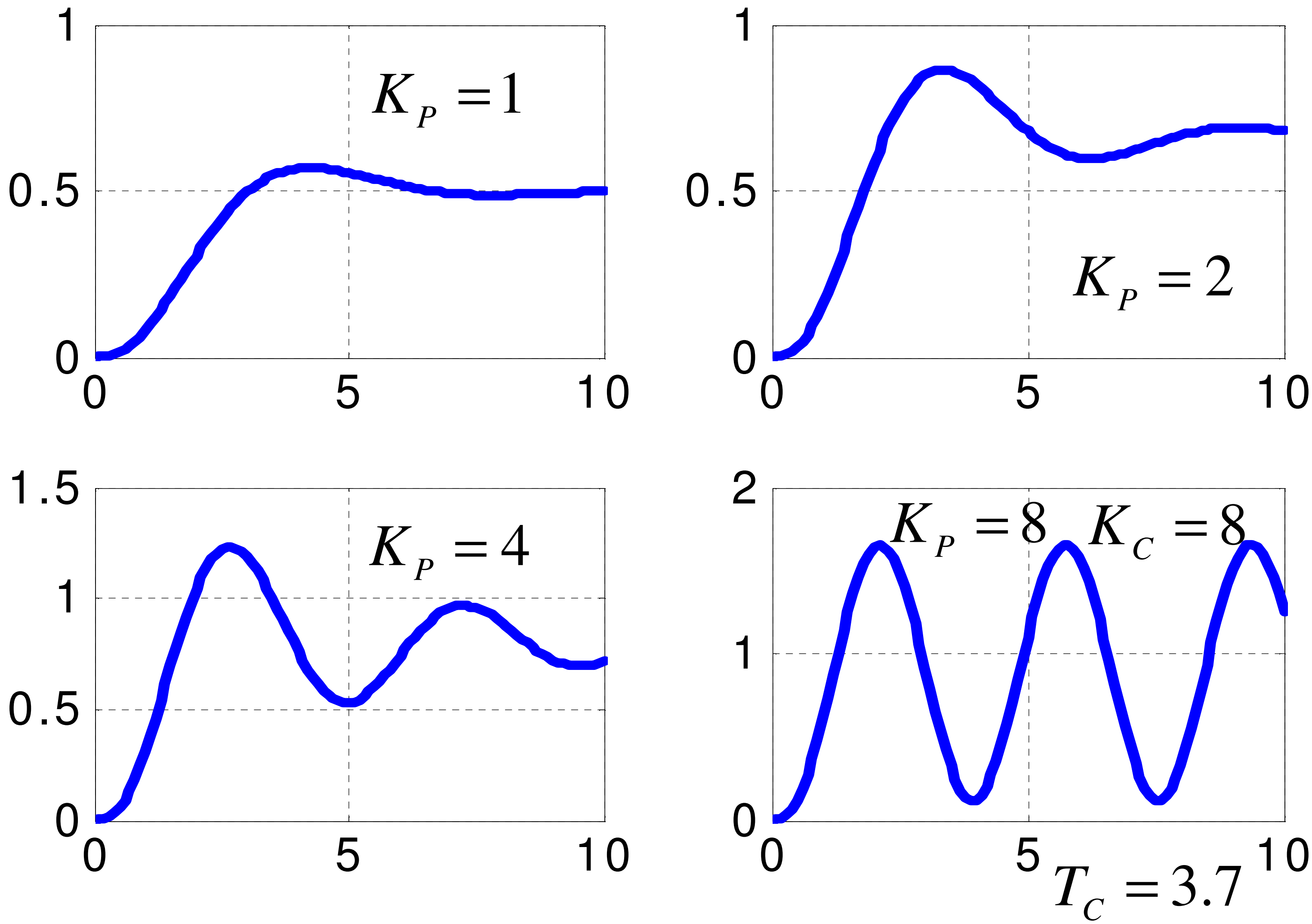
- Ultimate sensitivity



CD – OL Step Response for "Step Response method"

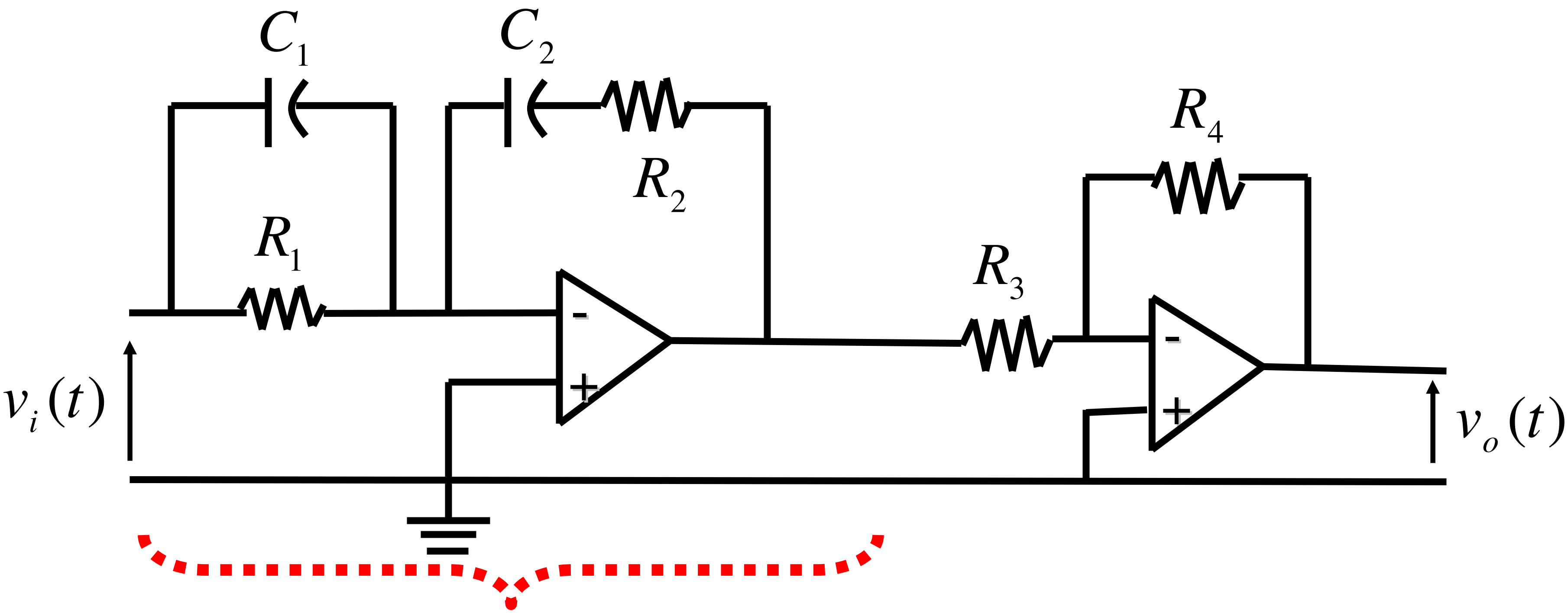


CD – CL Step Responses for "Ultimate Sensitivity method"



CD – PID Controller Realization

- One example: Using OP amp



$$-\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2C_1s+\frac{1}{R_1C_2s}\right]$$

Exercise: Derive this!

CD – PID Control Summary and Exercise

- PID control
  - Most popular controller in industry
  - Model-free methods for design are available.
  - Simple controller structure
  - Simple controller tuning
  - Widely applicable

- Ziegler-Nichols tuning rules provide a starting point for fine tuning, rather than final settings of controller parameters in a single shot.

CD – Nyquist Stability Criterion (Review)

$$\text{CL system is stable} \Leftrightarrow Z := P + N = 0$$

- Z: # of CL poles in open RHP
- P: # of OL poles in open RHP (given)
- N: # of clockwise encirclement around -1
  by Nyquist plot of OL transfer function $L(s)$
  (counted by using Nyquist plot of $L(s)$)

Remark: N = -1: a counter-clockwise encirclement

CD – Nyquist Stability Criterion: A Special Case

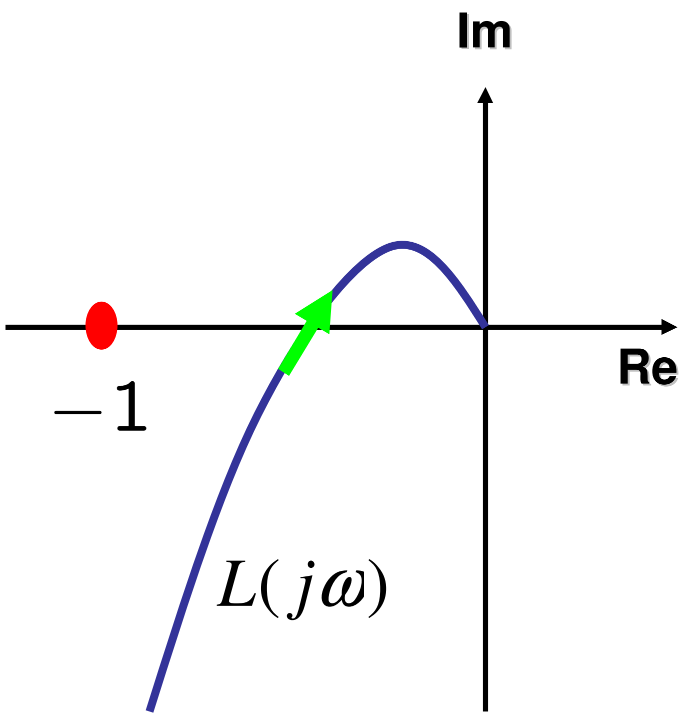
$$\text{CL system is stable} \Leftrightarrow Z := P + N = 0$$

- IF P=0 (i.e., if $L(s)$ has no pole in open RHP or stable)

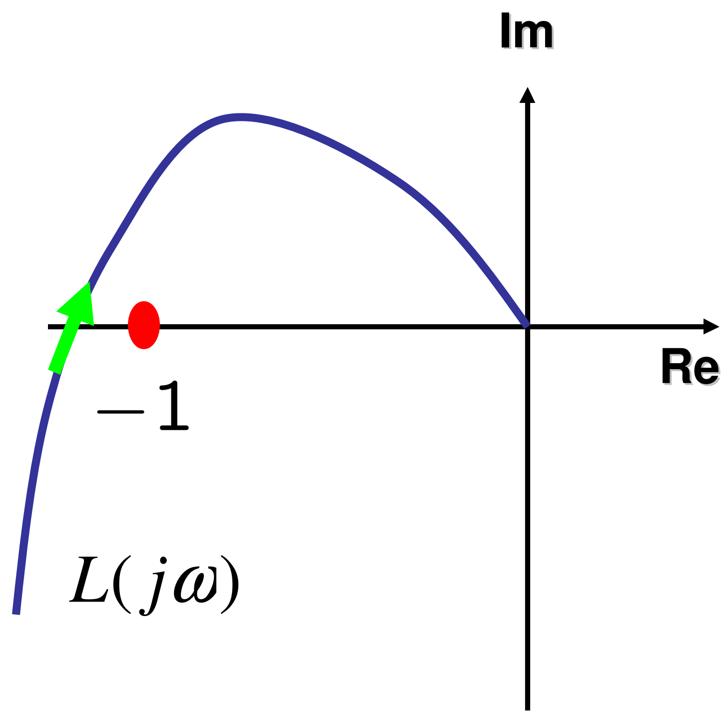
$$\text{CL system is stable} \Leftrightarrow N = 0$$

This fact is very important since open-loop systems in many practical problems have no pole in open RHP!

CD – Examples with P = 0 (stable OL system)



CL stable



CL unstable

CD – Nyquist Stability Remarks

- Nyquist stability criterion gives not only *absolute* but also *relative stability*.
  - Absolute stability: Is the closed-loop system stable or not? (Answer is yes or no.)
  - Relative stability: How "much" is the closed-loop system stable? (Margin of safety)

- Relative stability is important because a math model is never accurate.

- How to measure relative stability?
  - Use a "distance" from the critical point -1.
  - Gain margin (GM) & Phase margin (PM)

CD – Nyquist Gain Margin (GM)

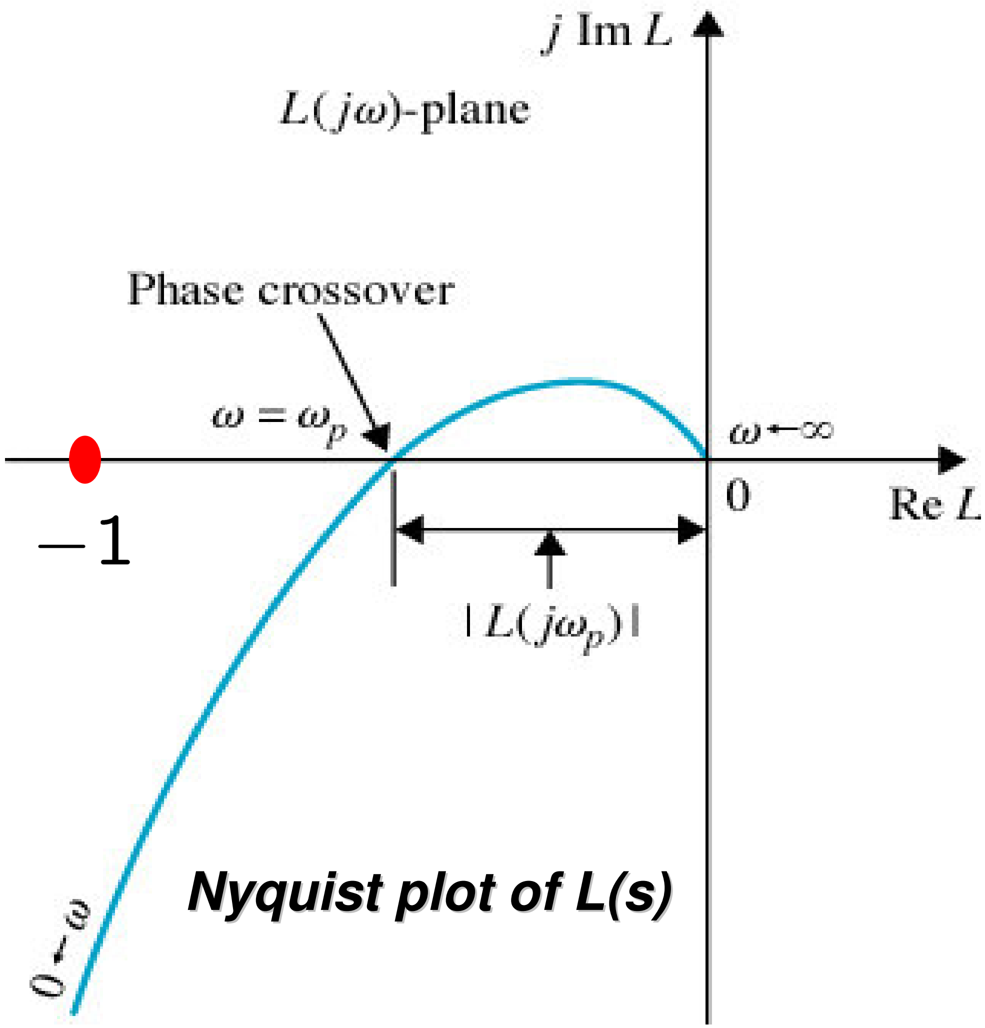
- Phase crossover frequency ωp:

$$\angle L(j\omega_p) = -180$$

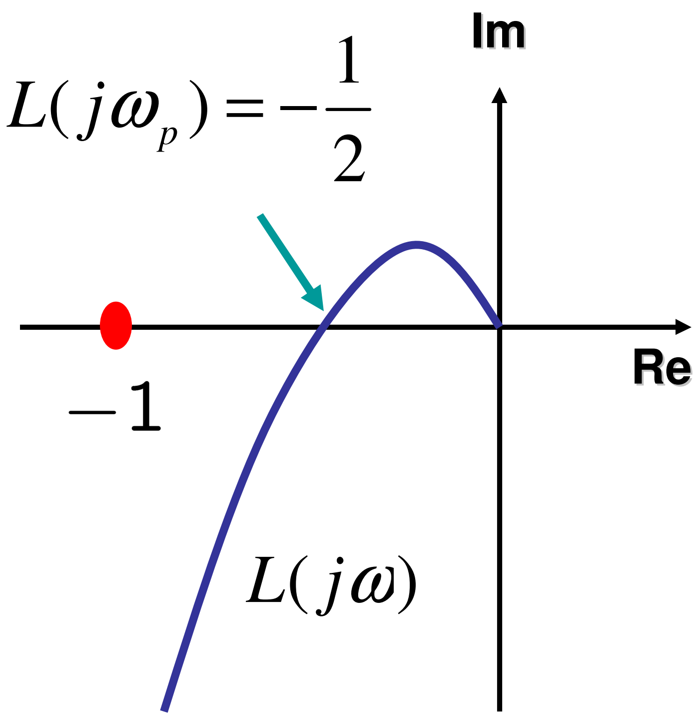
- Gain margin (in dB)

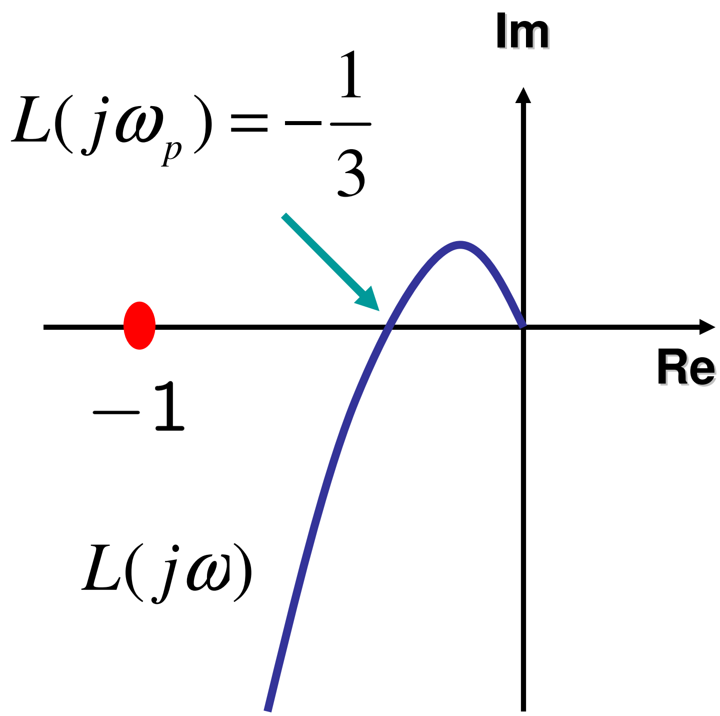
$$GM = 20\log_{20}\frac{1}{\left|L(j\omega_p)\right|}$$

- Indicates how much OL gain can be multiplied without violating CL stability.



CD – GM Examples

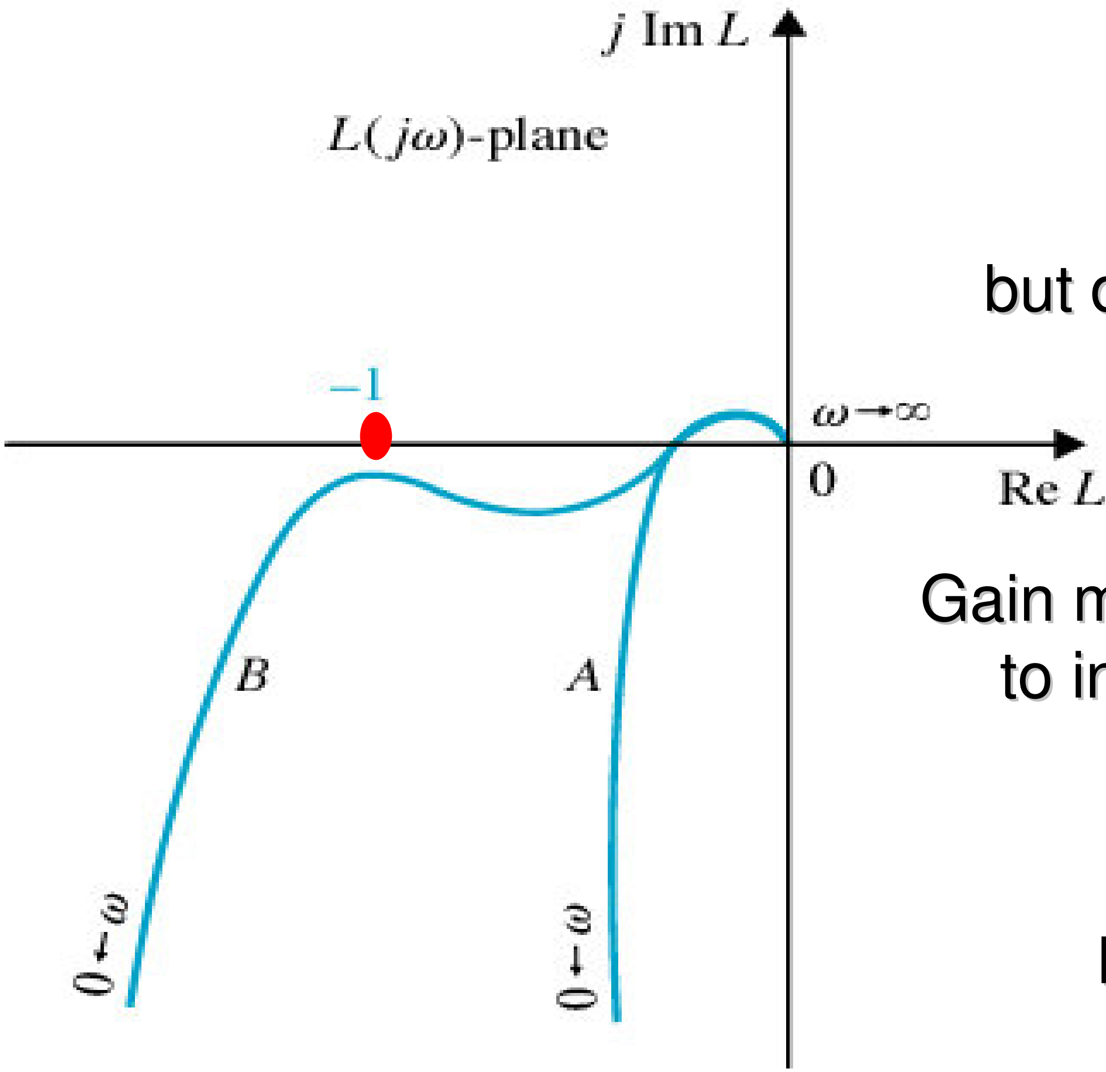




$$GM = 20\log_{10}\frac{1}{\underbrace{|L(j\omega_p)|}_{2}} \approx 6\text{dB}$$

$$GM = 20\log_{10}\frac{1}{\underbrace{|L(j\omega_p)|}_{3}} \approx 9.5\text{dB}$$

CD – Why GM Alone is Inadequate



Same gain margin,
but different relative stability

Gain margin is often inadequate
to indicate relative stability

Phase margin!

CD – Nyquist Phase Margin (PM)

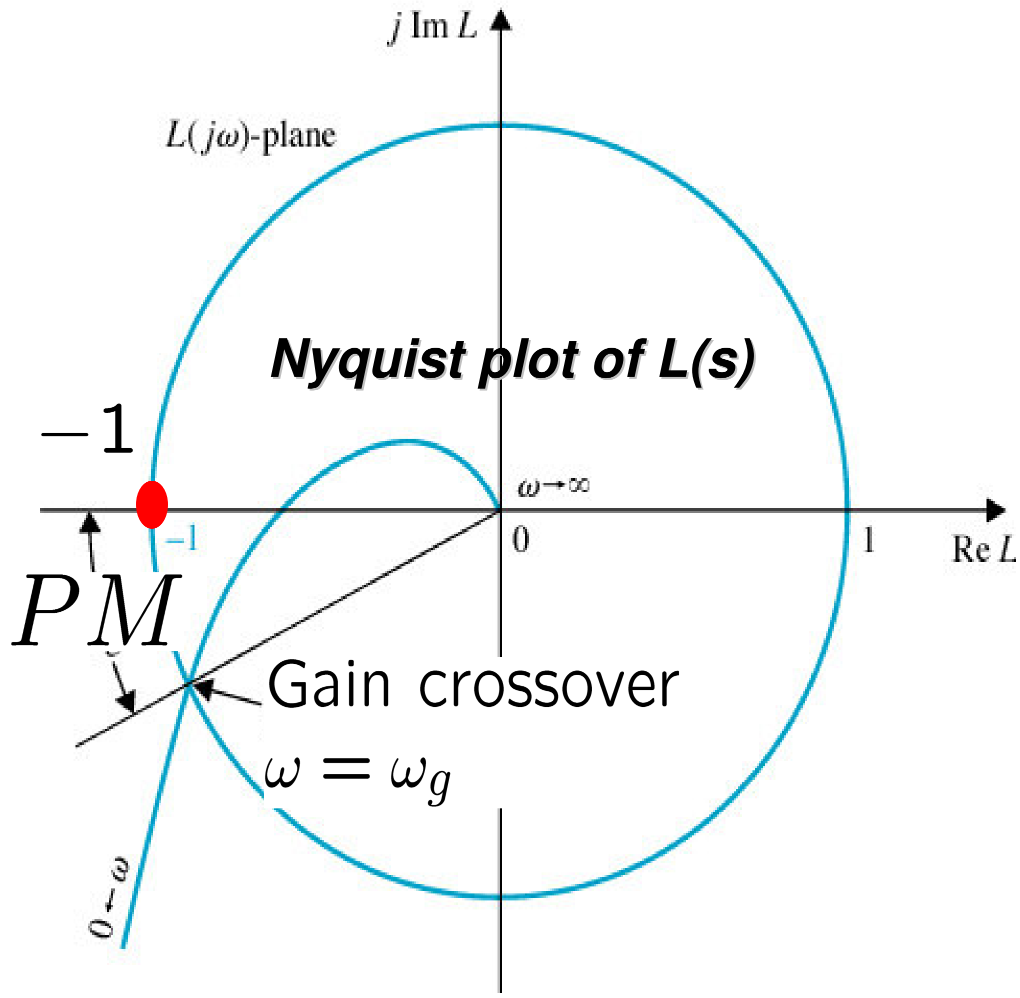
- Gain crossover frequency ωg:

$$\angle L(j\omega_g) = 1$$

- Phase margin

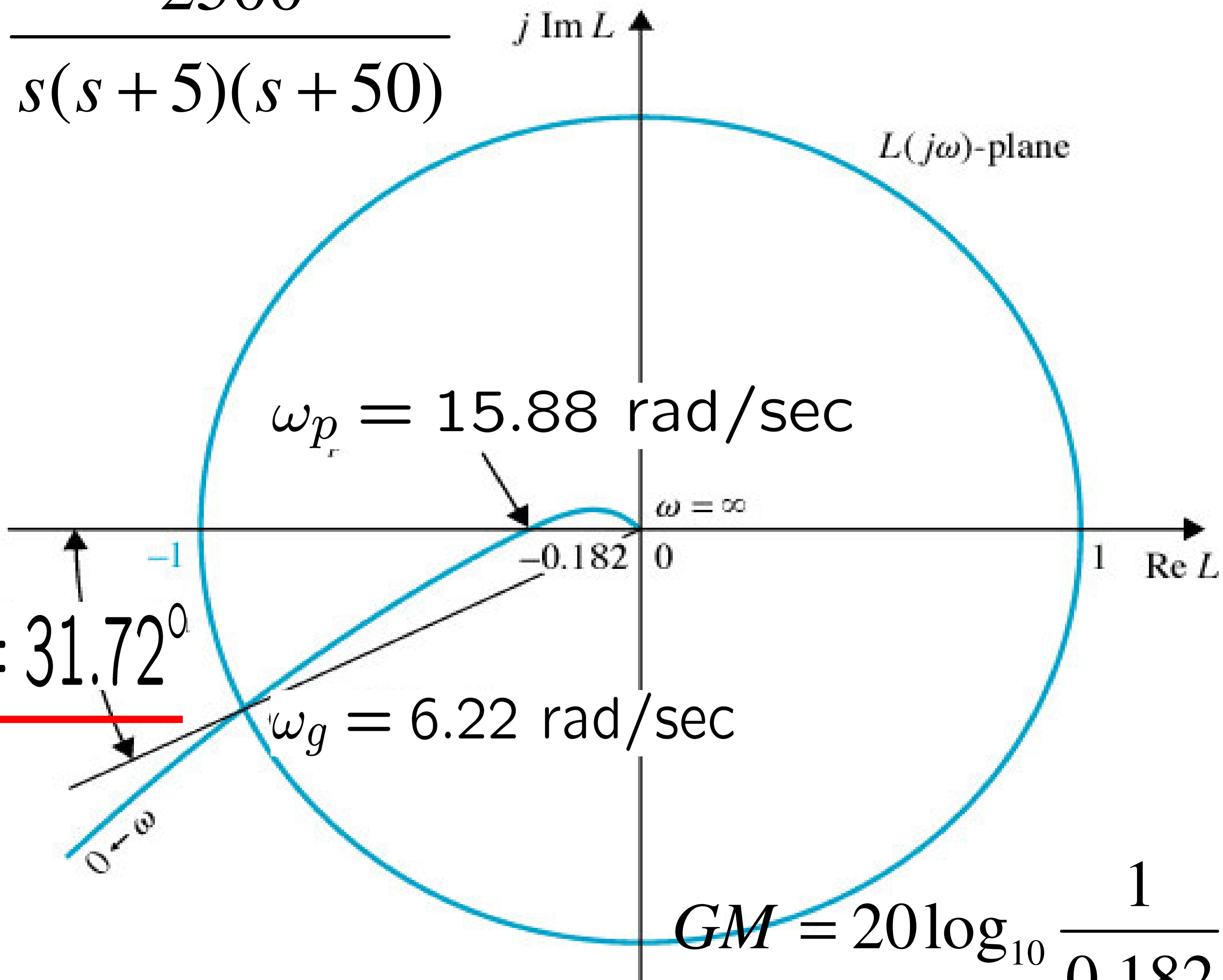
$$PM = \angle L(j\omega_g) - 180^o$$

- Indicates how much OL phase can be added without violating CL stability.



CD – PM Example

$$L(s) = \frac{2500}{s(s+5)(s+50)}$$



$\omega_{p_r} = 15.88$ rad/sec

$\omega_g = 6.22$ rad/sec

$$PM = 31.72^0$$

$$GM = 20\log_{10}\frac{1}{0.182} = 14.80\text{dB}$$

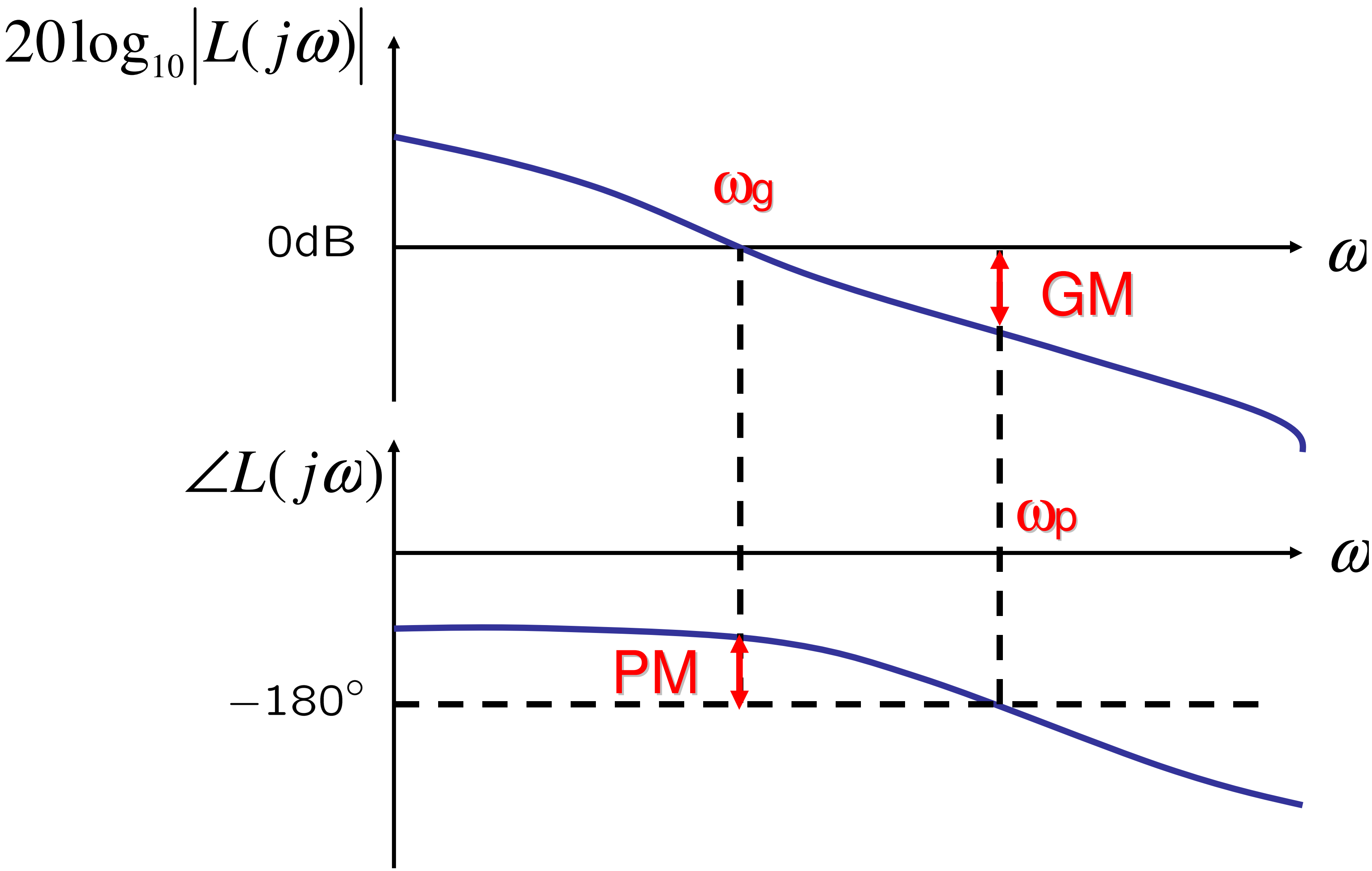
CD – Nyquist Plot Remarks

- Advantages
  – Nyquist plot can be used for study of closed-loop stability, for open loop systems which is unstable and includes time-delay.

- Disadvantage
  – Controller design on Nyquist plot is difficult. (Controller design on Bode plot is much simpler.)

We translate GM and PM on Nyquist plot into those in Bode plot!

CD – Bode Diagram Relative Stability



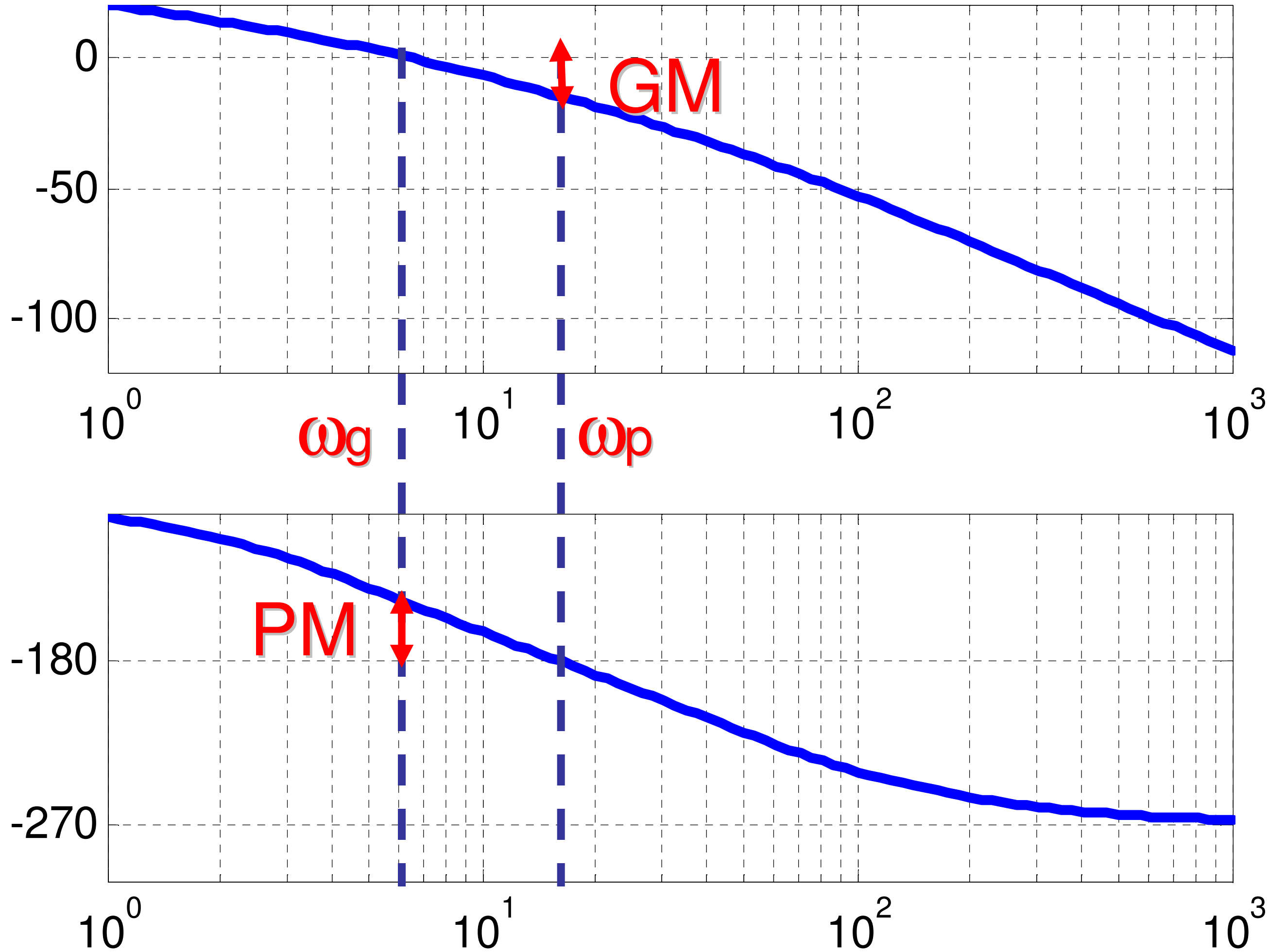
CD – Bode Diagram Remarks

- Advantages
  - Without computer, Bode plot can be sketched easily.
  - GM, PM, crossover frequencies are easily determined on Bode plot.
  - Controller design on Bode plot is simple.

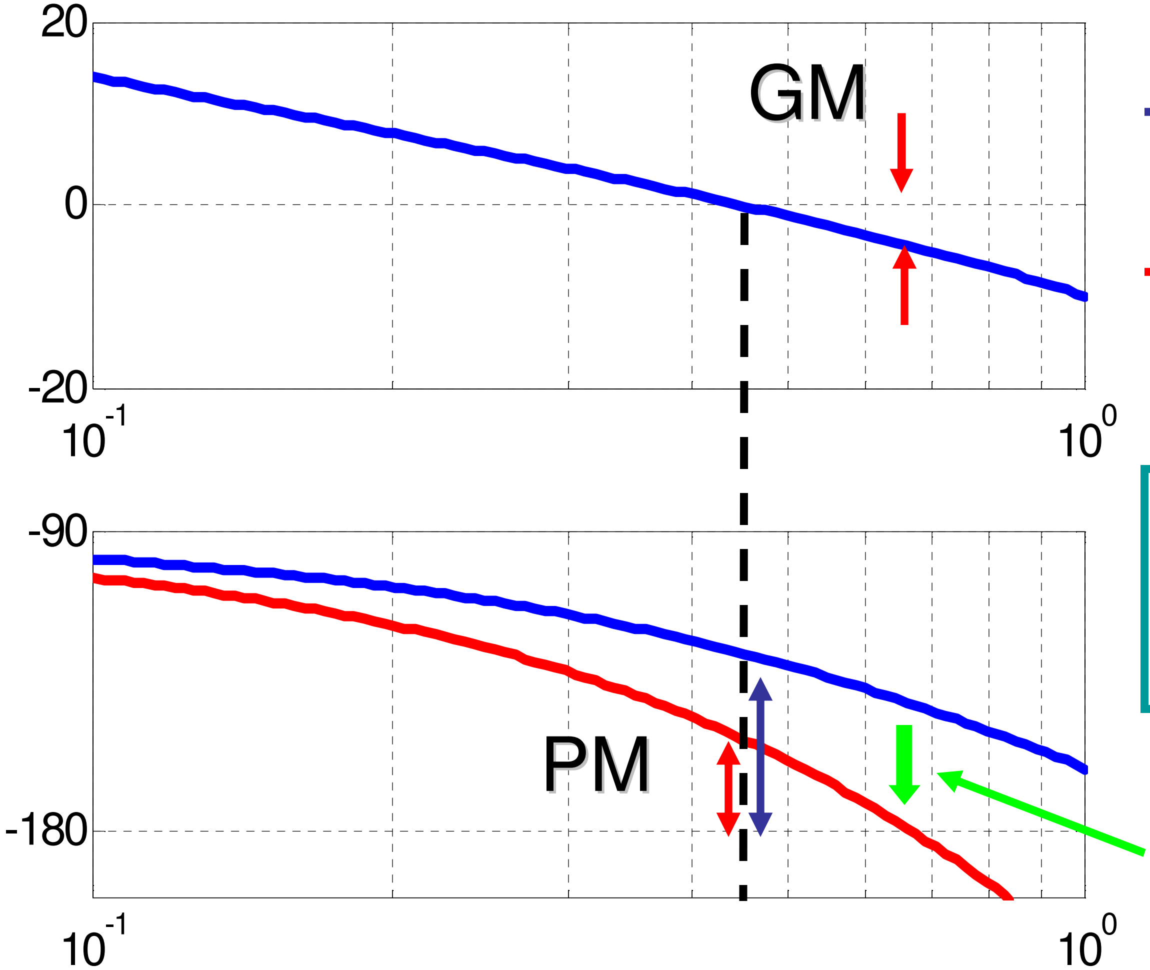
- Disadvantage
  - If OL system is unstable, we cannot use Bode plot for stability analysis.

CD – Bode Diagram Example

$$L(s) = \frac{2500}{s(s+5)(s+50)}$$



CD – Bode Diagram Relative Stability (time Delay)



$$L(s) = \frac{1}{s(s+1)(s+2)}$$

$$L(s) = \frac{e^{-s}}{s(s+1)(s+2)}$$

Time delay reduces relative stability!

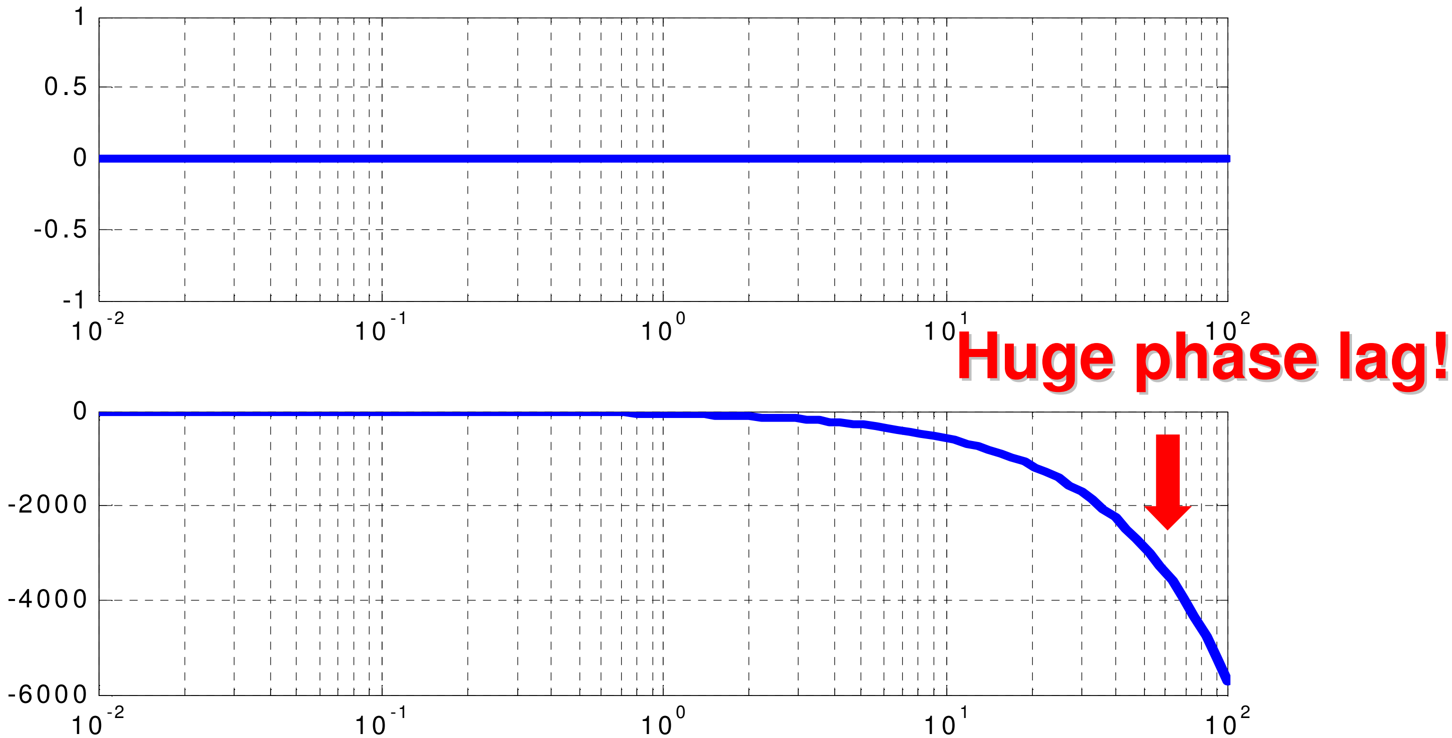
$57.3\omega T_d$ deg phase lag

Delay time

CD – Body Diagram of A Time Delay

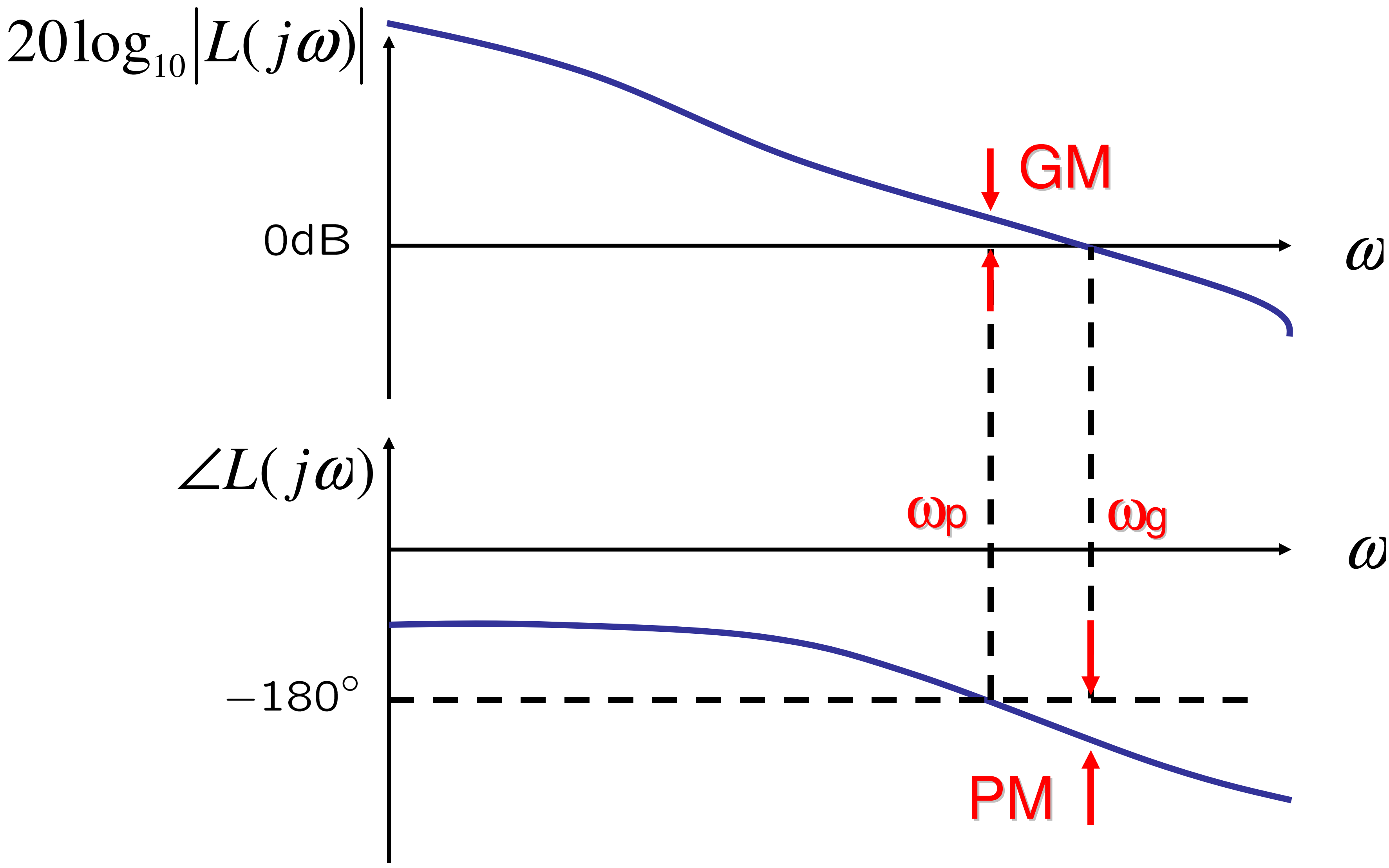
- TF

$$G(s) = e^{-Ts} \Rightarrow \left|G(j\omega)\right| = 1,\ \forall \omega,\ \angle G(j\omega) = -\omega T\ \text{(rad)}$$



As can be explained with Nyquist stability criterion, this phase lag causes instability of the closed-loop system, and hence, the difficulty in control.

CD – Body Diagram Unstable CL Case



CD – Body Diagram Summary and Exercises

- Relative stability: Closeness of Nyquist plot to the critical point -1
  - Gain margin, phase crossover frequency
  - Phase margin, gain crossover frequency

- Relative stability on Bode plot

- We normally emphasize PM in controller design.

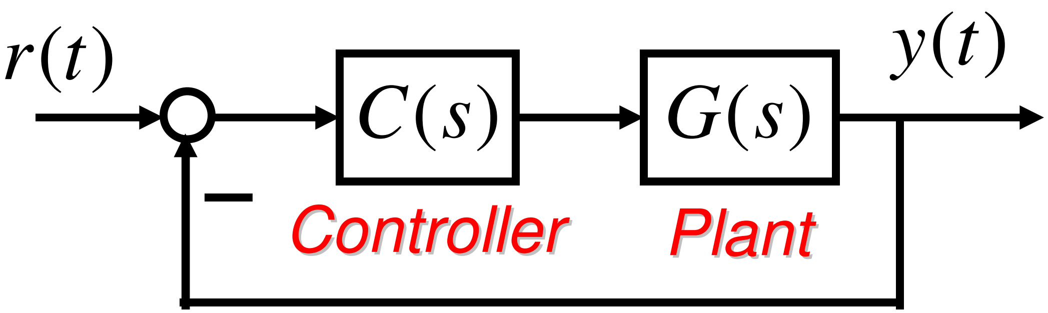
CD – Control Design Comparison

Design specifications in time domain
(Rise time, settling time, overshoot, steady state error, etc.)

Approximate translation

Desired closed-loop pole location in s-domain	Constraints on open-loop frequency response in s-domain
Root locus shaping	***Frequency response shaping (Loop shaping)***

CD – Feedback Control System Design

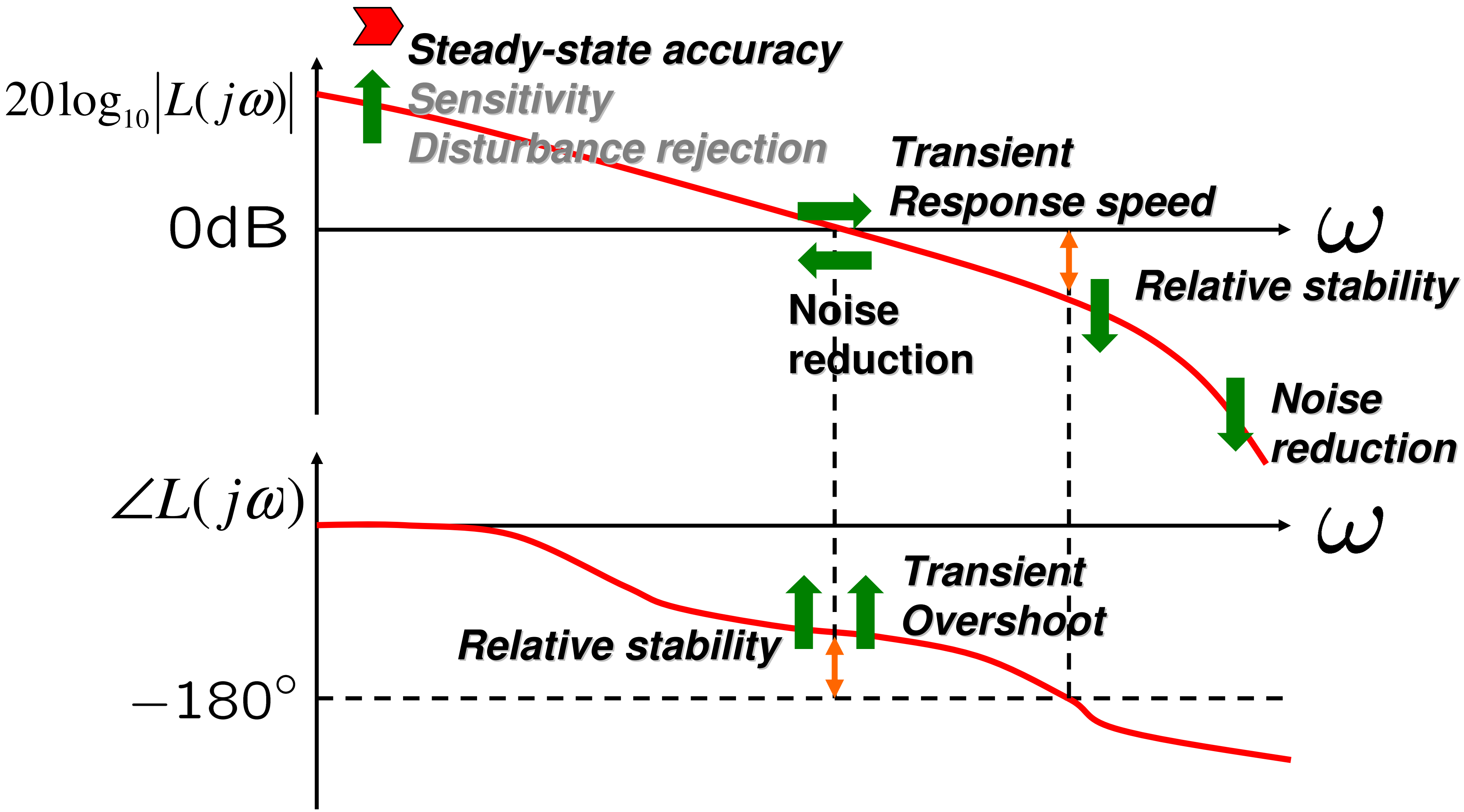


OL: $L(s) := G(s)C(s)$

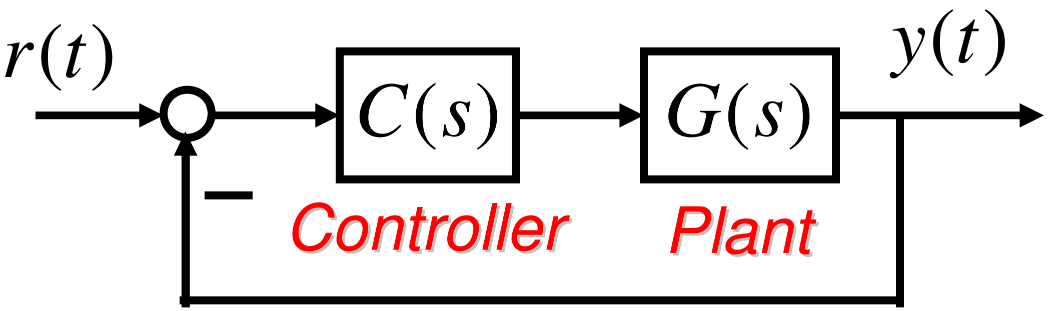
CL: $T(s) := \dfrac{L(s)}{1 + L(s)}$

- Given $G(s)$, design $C(s)$ that satisfies time domain specs, such as stability, transient, and steady-state responses.
- We learn typical qualitative relationships between open-loop Bode plot and time-domain specifications.

CD – Typical Desired OL Body Diagram



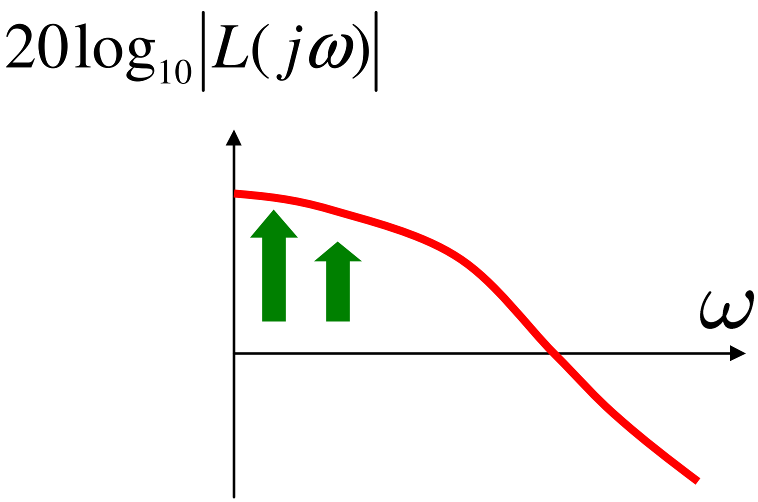
CD – Steady State Accuracy (1)



OL: $L(s) := G(s)C(s)$

CL: $T(s) := \dfrac{L(s)}{1 + L(s)}$

For steady-state accuracy,
L should have high gain at low frequencies.



Large $|L(j\omega)|$

$\Rightarrow$ $Y(j\omega) = \dfrac{L(j\omega)}{1 + L(j\omega)} \approx 1$

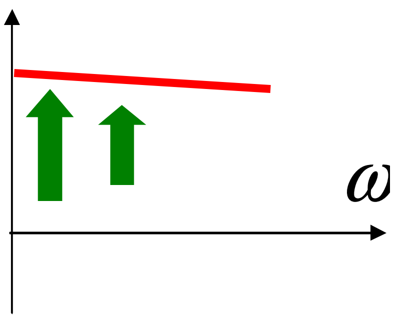
$\Rightarrow$ $y(t)$ tracks $r(t)$ composed of low frequencies very well.

CD – Steady State Accuracy (2)

- Step $r(t)$
  Increase

$$K_p := L(0)$$

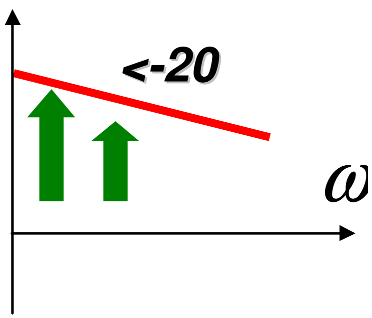
$$20\log_{10}|L(j\omega)|$$



- Ramp $r(t)$
  Increase

$$K_v := \lim_{s\to 0} sL(s)$$

$$20\log_{10}|L(j\omega)|$$

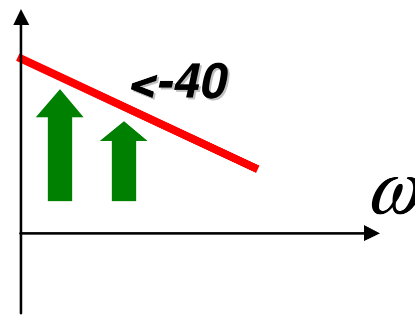


For Kv to be nonzero, L must contain at least one integrator.

- Parabolic $r(t)$
  Increase

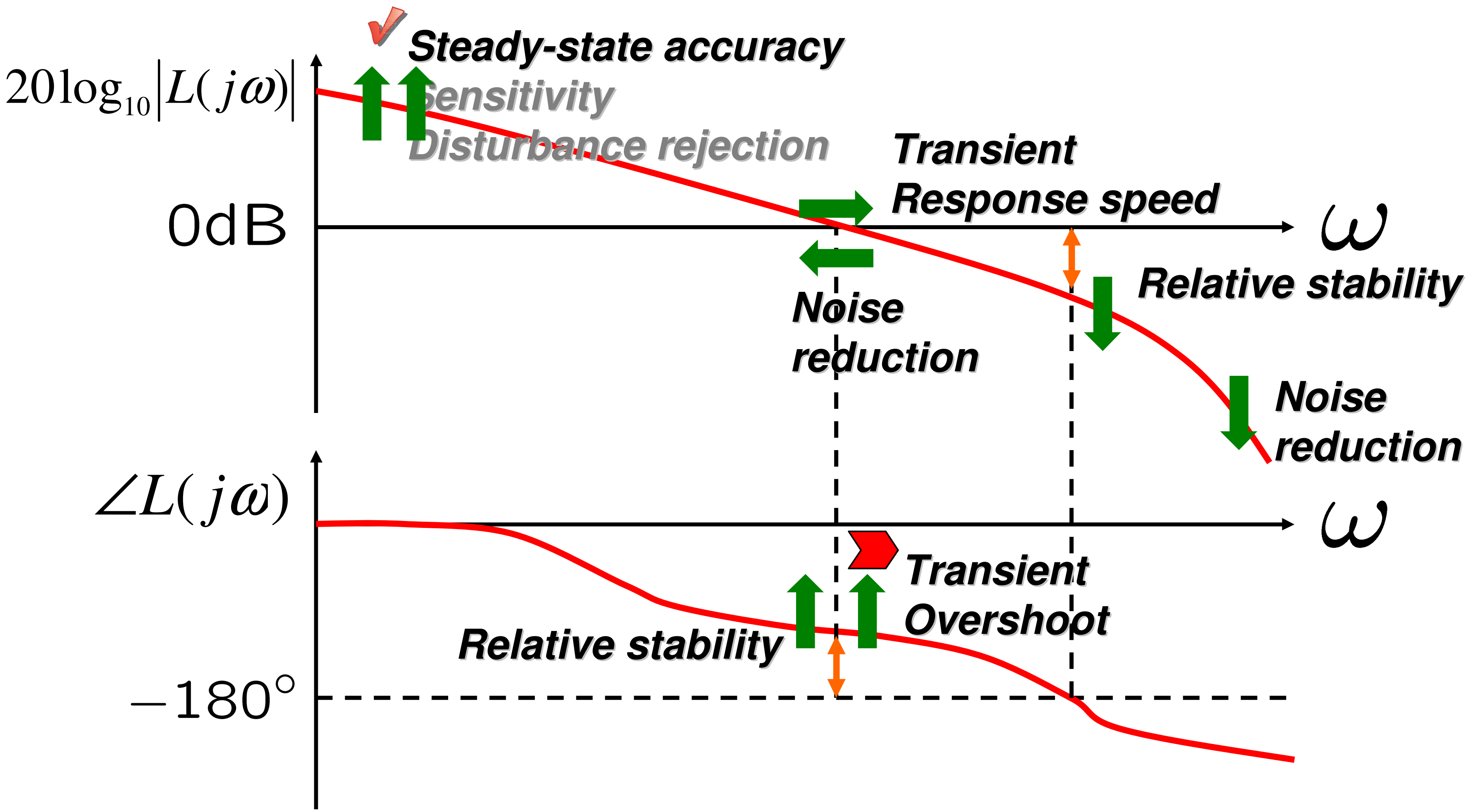
$$K_a := \lim_{s\to 0} s^2 L(s)$$

$$20\log_{10}|L(j\omega)|$$



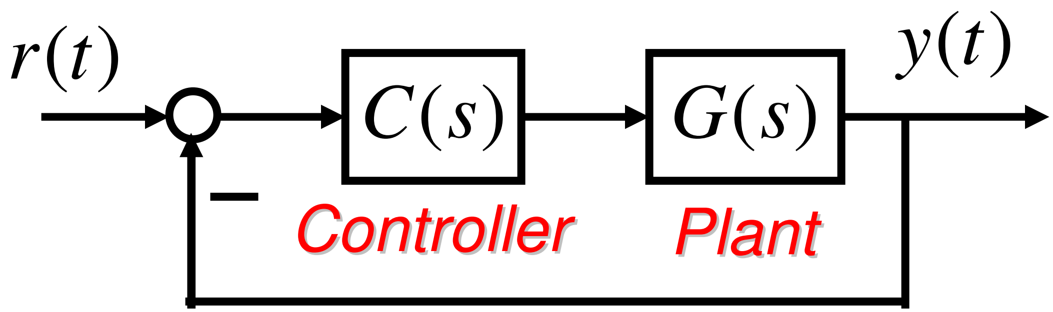
For Ka to be nonzero, L must contain at least two integrators.

CD – Typical Desired OL Body Diagram (Revisited)



CD – A 2nd Order System Example

- For illustration, we use the feedback system:

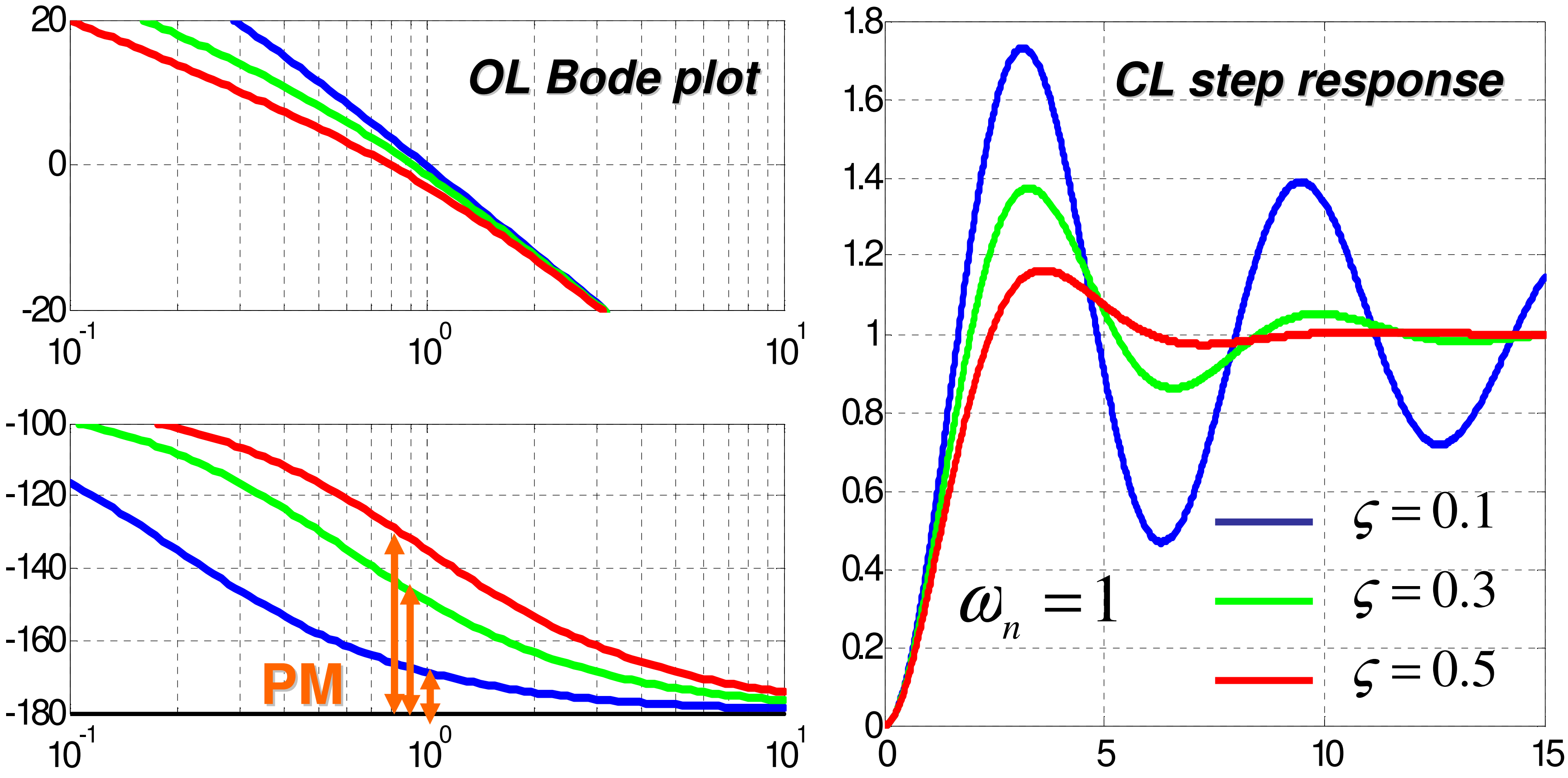


$$L(s) := G(s)C(s) = \frac{\omega_n^2}{s(s+2\varsigma\omega_n)}$$

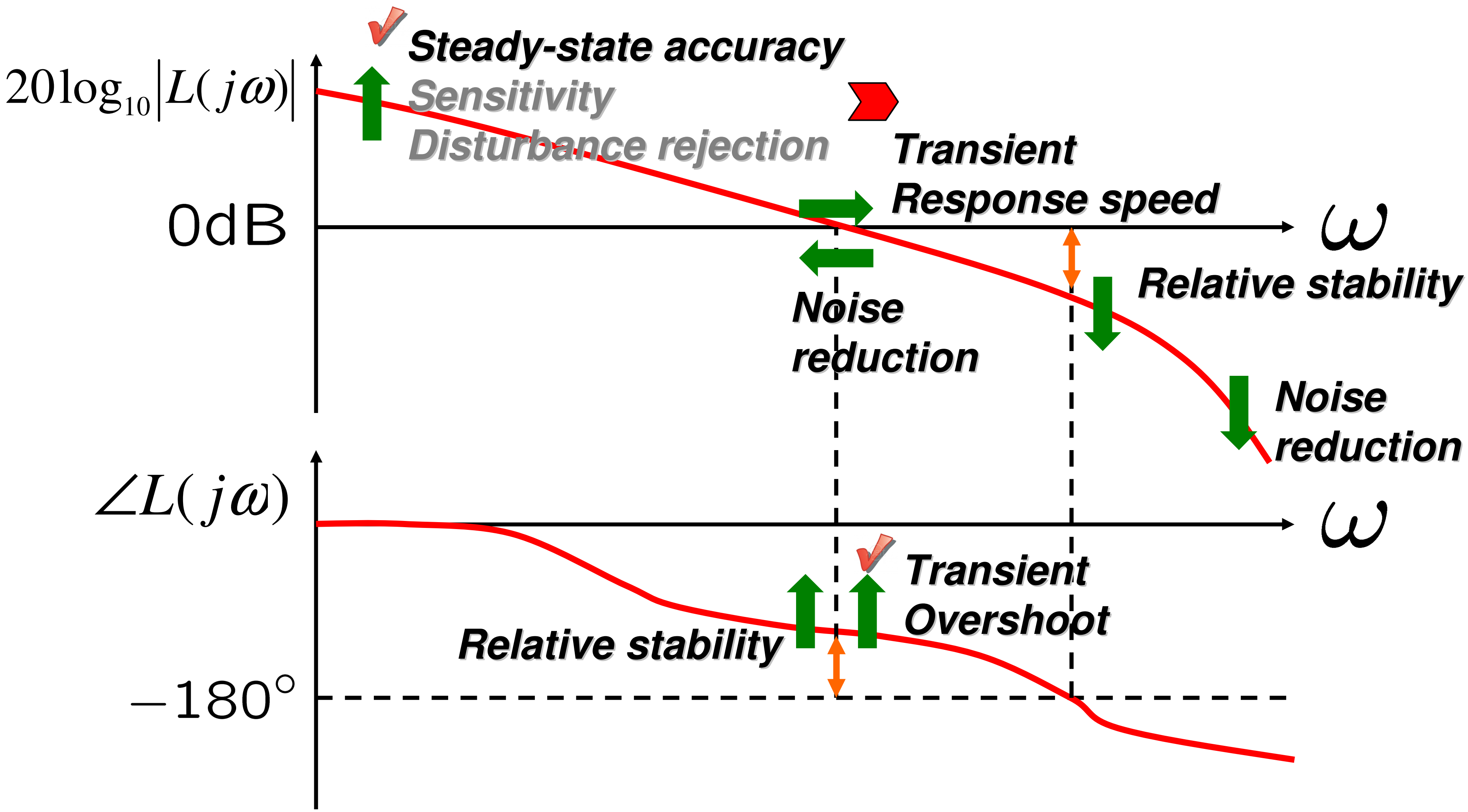
$$T(s) := \frac{L(s)}{1+L(s)} = \frac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

CD – Percent Overshoot

For small percent overshoot,
L should have larger phase margin.

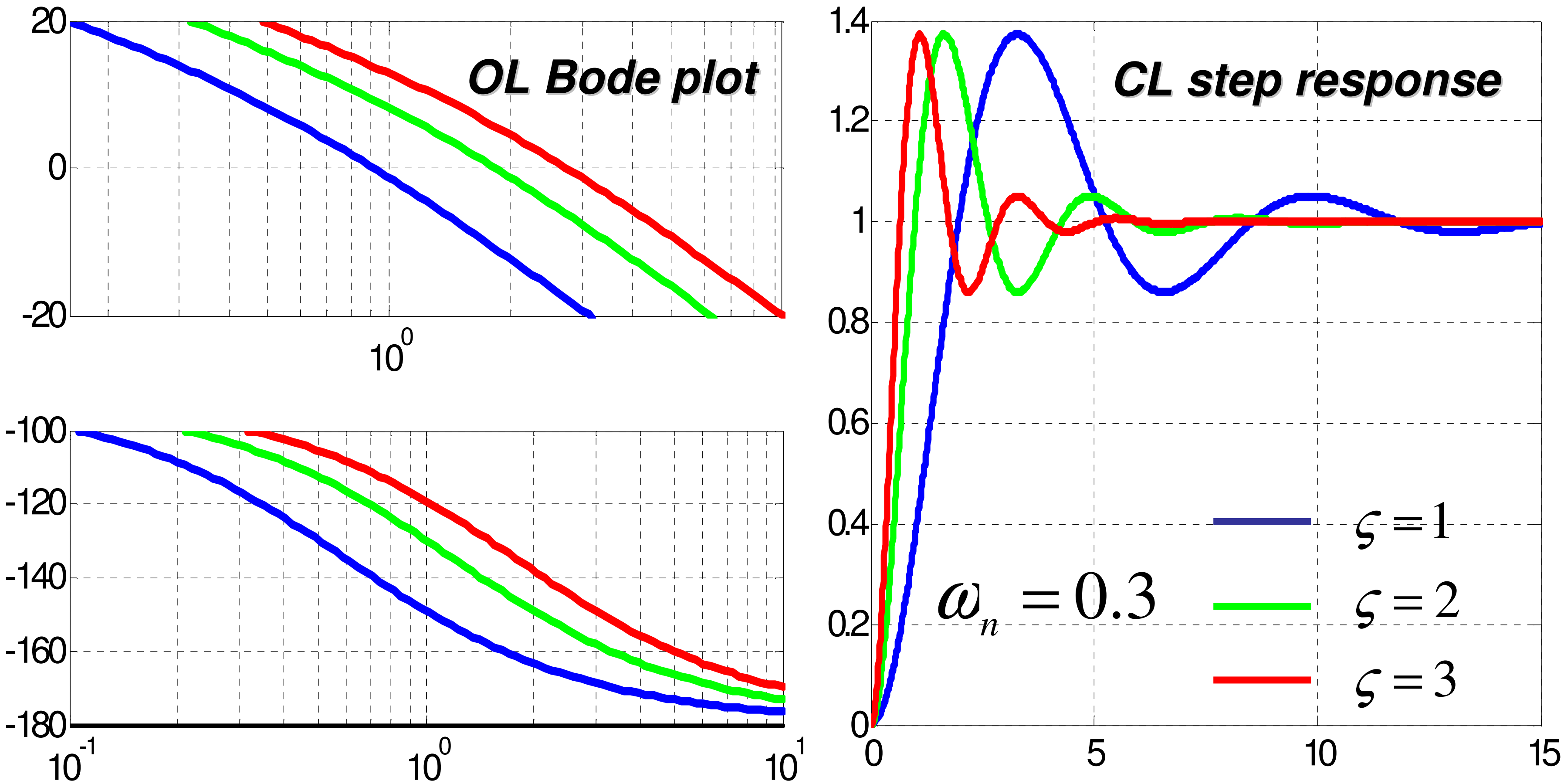


CD – Typical Desired OL Body Diagram (Revisited)

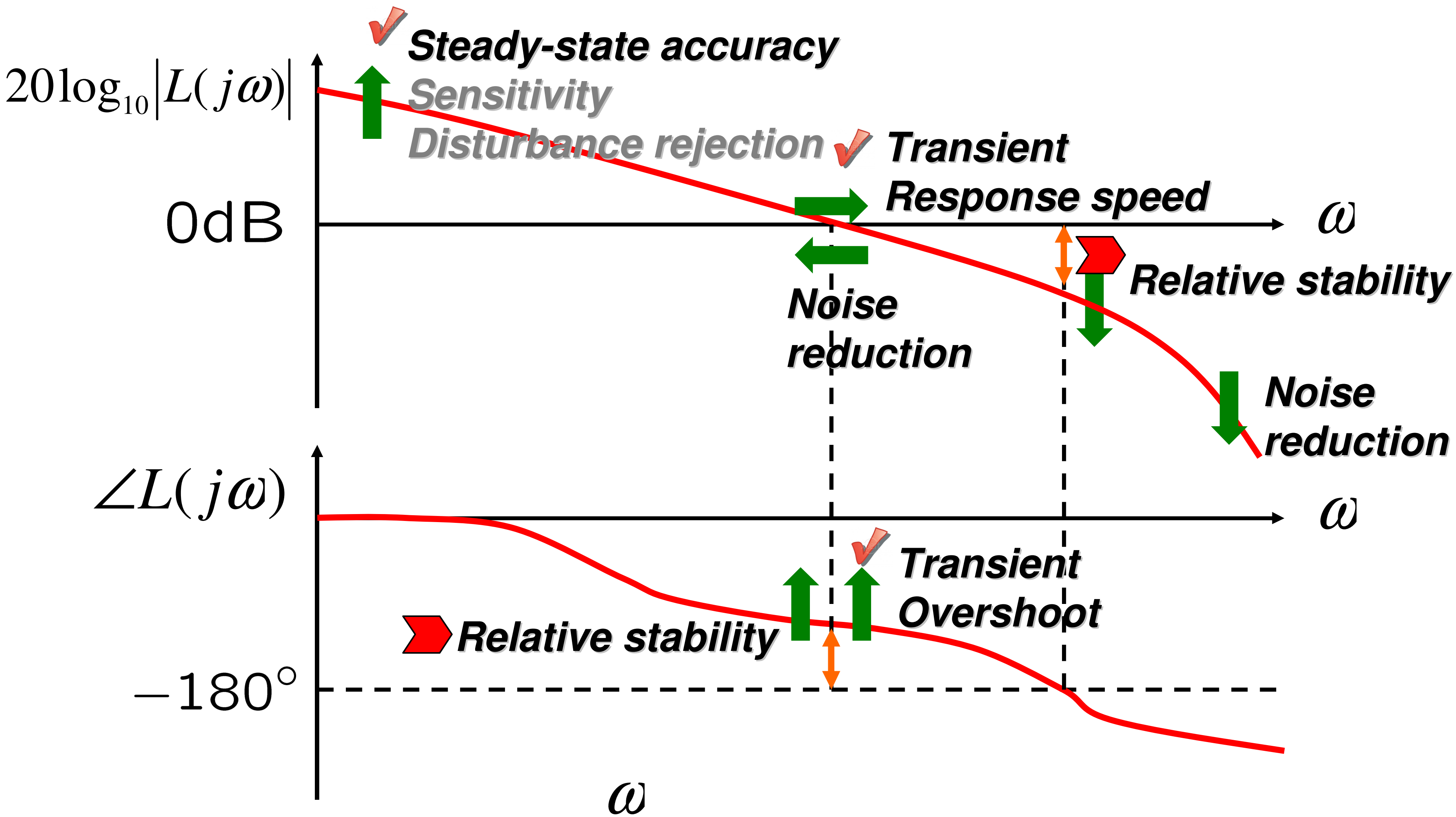


CD – Response Speed

For fast response,
L should have larger gain crossover frequency.



CD – Typical Desired OL Body Diagram (Revisited)



CD – Relative Stability

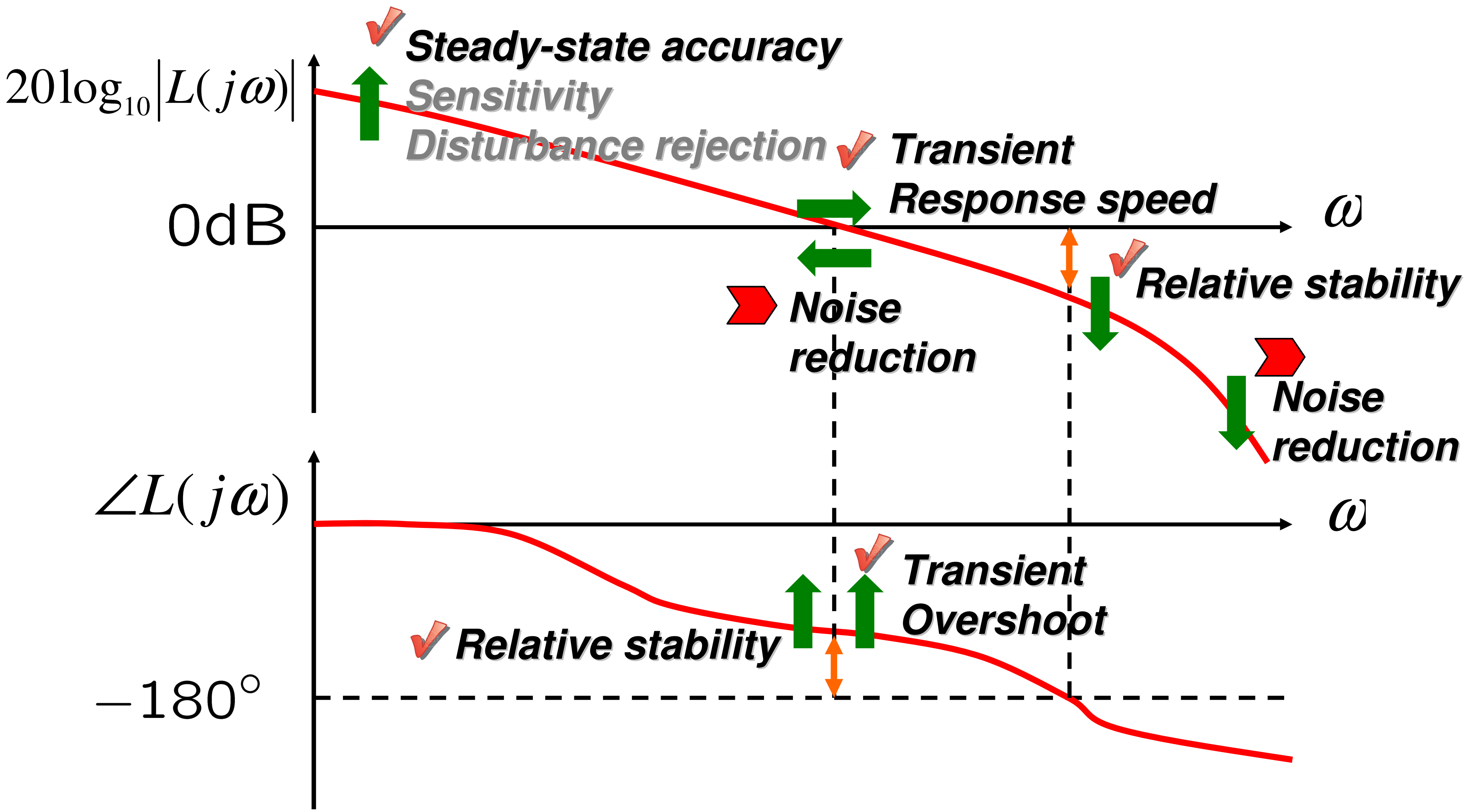
- We require adequate GM and PM for:
  - safety against inaccuracies in modeling
  - reasonable transient response

- It is difficult to give reasonable numbers of GM and PM for general cases, but usually,
  - GM should be at least 6dB
  - PM should be at least 45deg

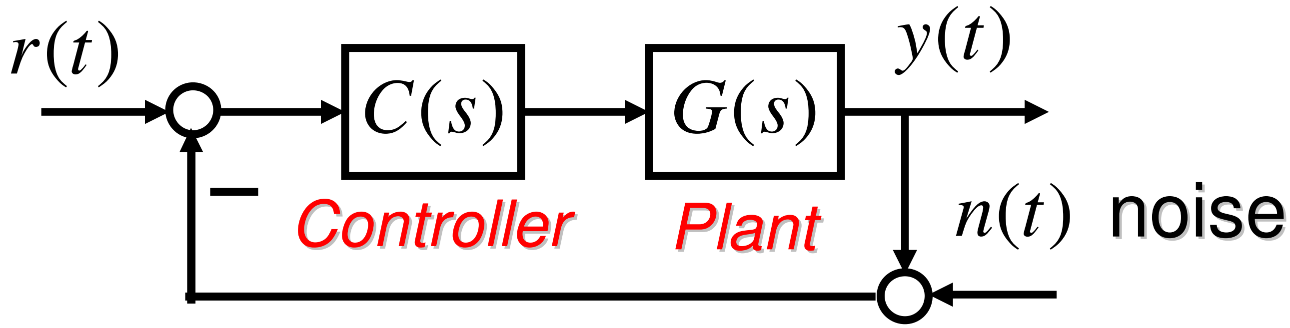
  (These values are not absolute but approximate!)

- In controller design, we are especially interested in PM (which typically gives good GM).

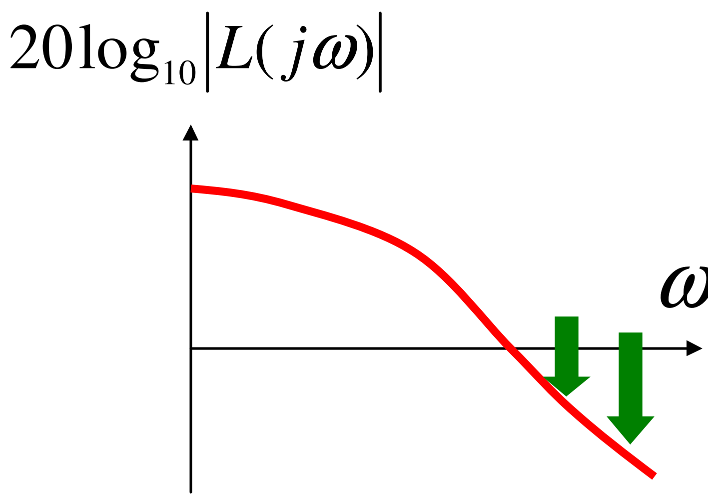
CD – Typical Desired OL Body Diagram (Revisited)



CD – Noise Rejection



For noise rejection,
L should have small gain at high frequencies.

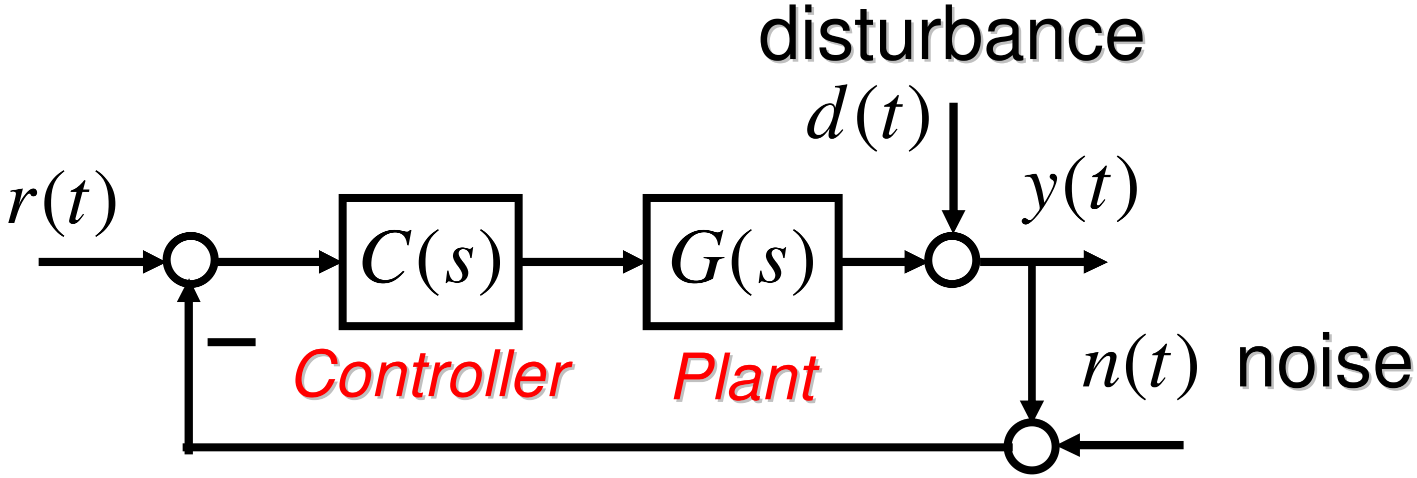


Small $|L(j\omega)|$

$$\Rightarrow \frac{Y}{N}(j\omega) = -\frac{L(j\omega)}{1+L(j\omega)} \approx 0$$

$\Rightarrow$ $y(t)$ is not affected by $n(t)$ composed of high frequencies.

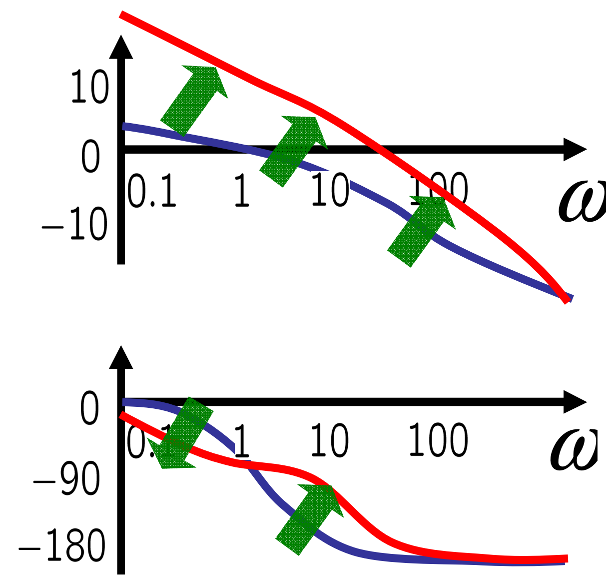
CD – Frequency Shaping (Loop Shaping)



- Reshape Bode plot of G(jω) into a "desired" shape of

$$L(j\omega) := G(j\omega)C(j\omega)$$

by a series connection of appropriate $C(s)$.



CD – Advantages of Body Diagram

- Bode plot of a series connection $G_1(s)G_2(s)$ is the addition of each Bode plot of G_1 and G_2.
  - Gain

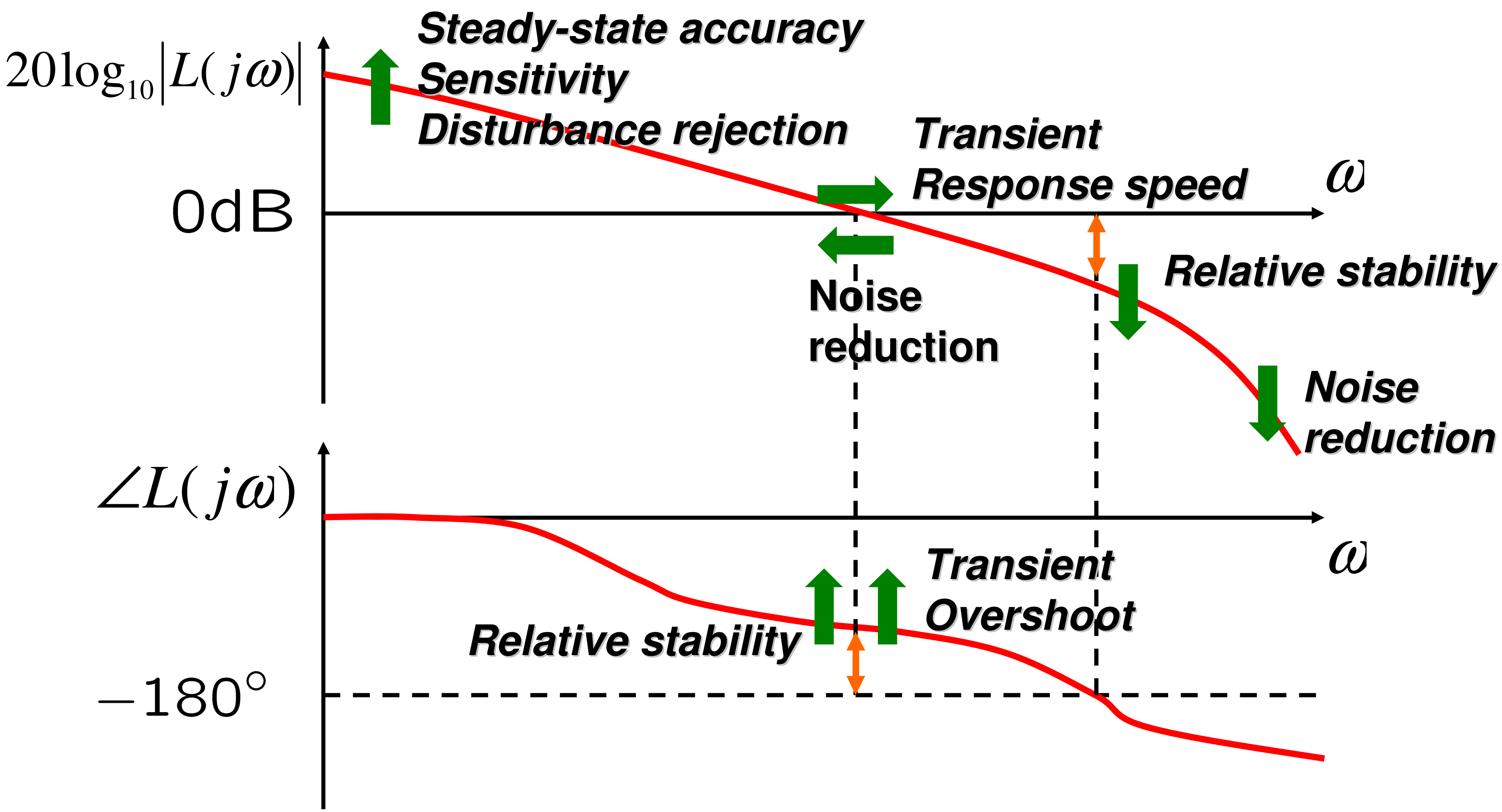
$$20\log_{10}\left|G_1(j\omega)G_2(j\omega)\right| = 20\log_{10}\left|G_1(j\omega)\right| + 20\log_{10}\left|G_2(j\omega)\right|$$

  - Phase

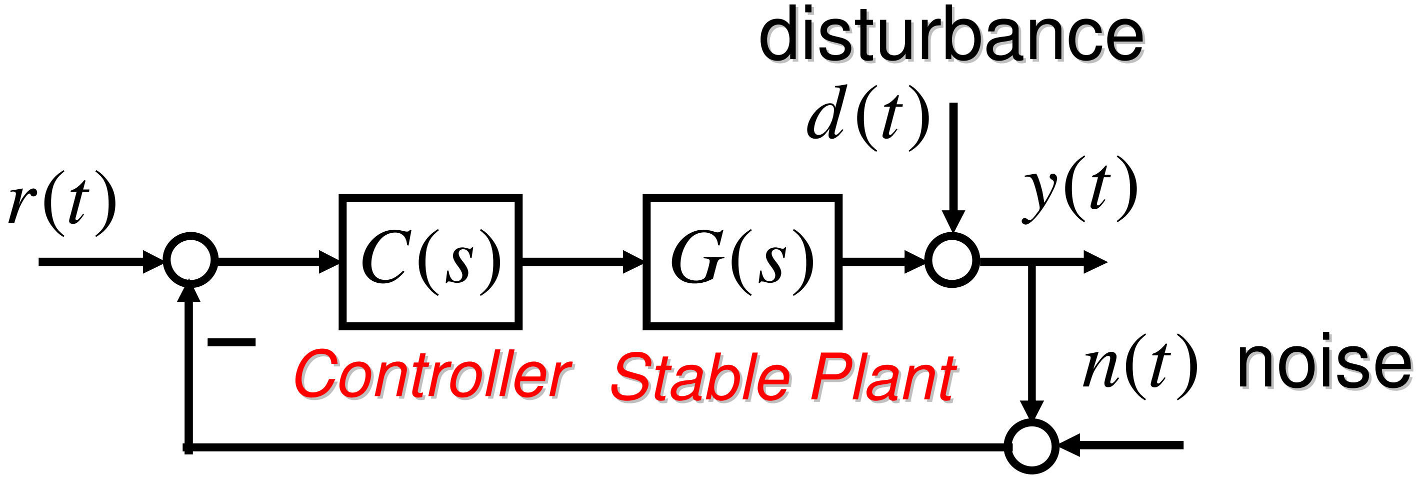
$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_1(j\omega)$$

- We use this property to design $C(s)$ so that $G(s)C(s)$ has a "desired" shape of Bode plot.

CD – Typical Shaping Goal (Review)



CD – Simple Controllers



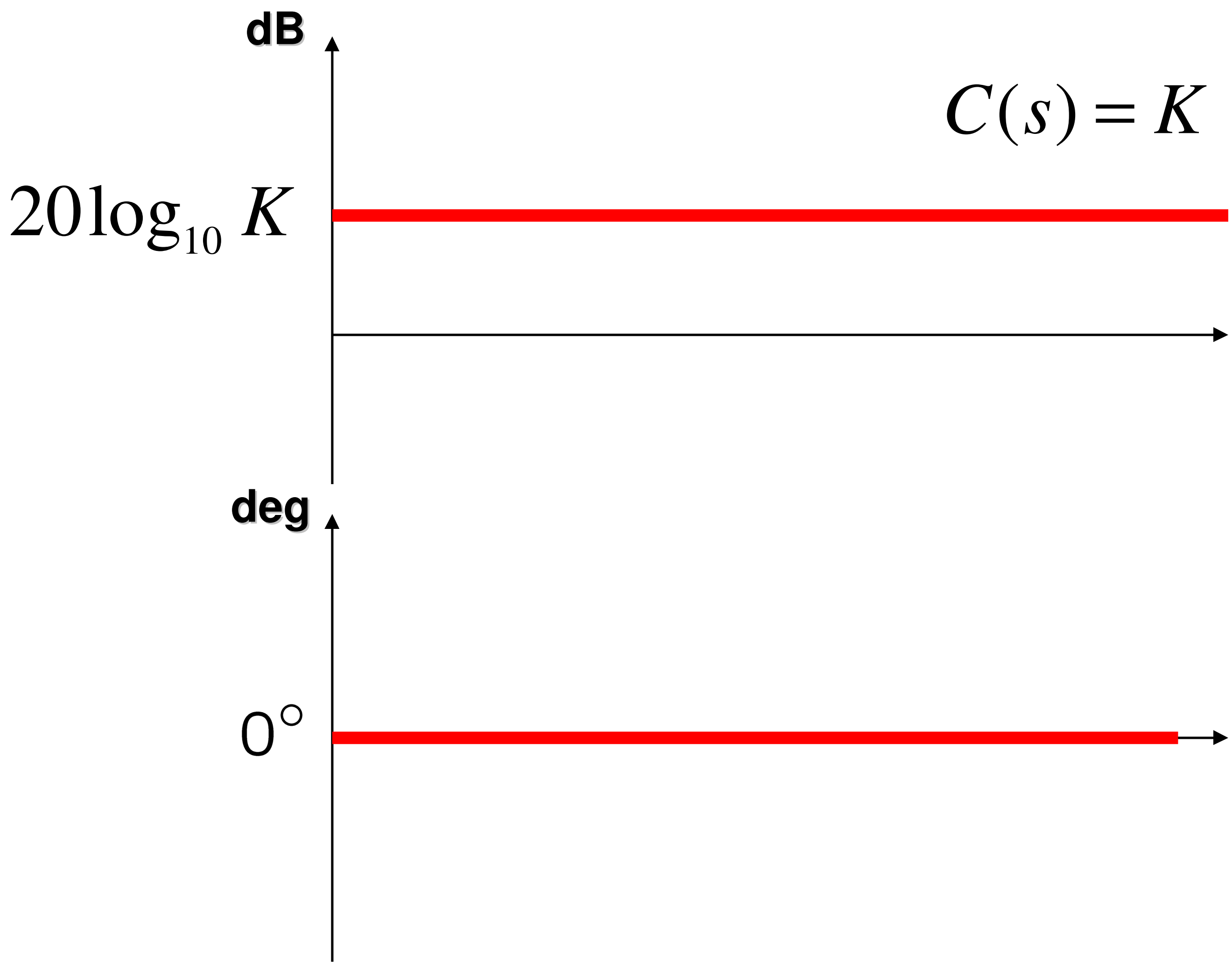
- We use simple controllers for shaping.
  - Gain

$$C(s) = K$$

  - Lead and lag compensators

$$C(s) = \frac{(\text{1st - order poly.})}{(\text{1st - order poly.})} = \frac{\frac{s}{z}+1}{\frac{s}{p}+1}$$

CD – Bode Plot of a Gain

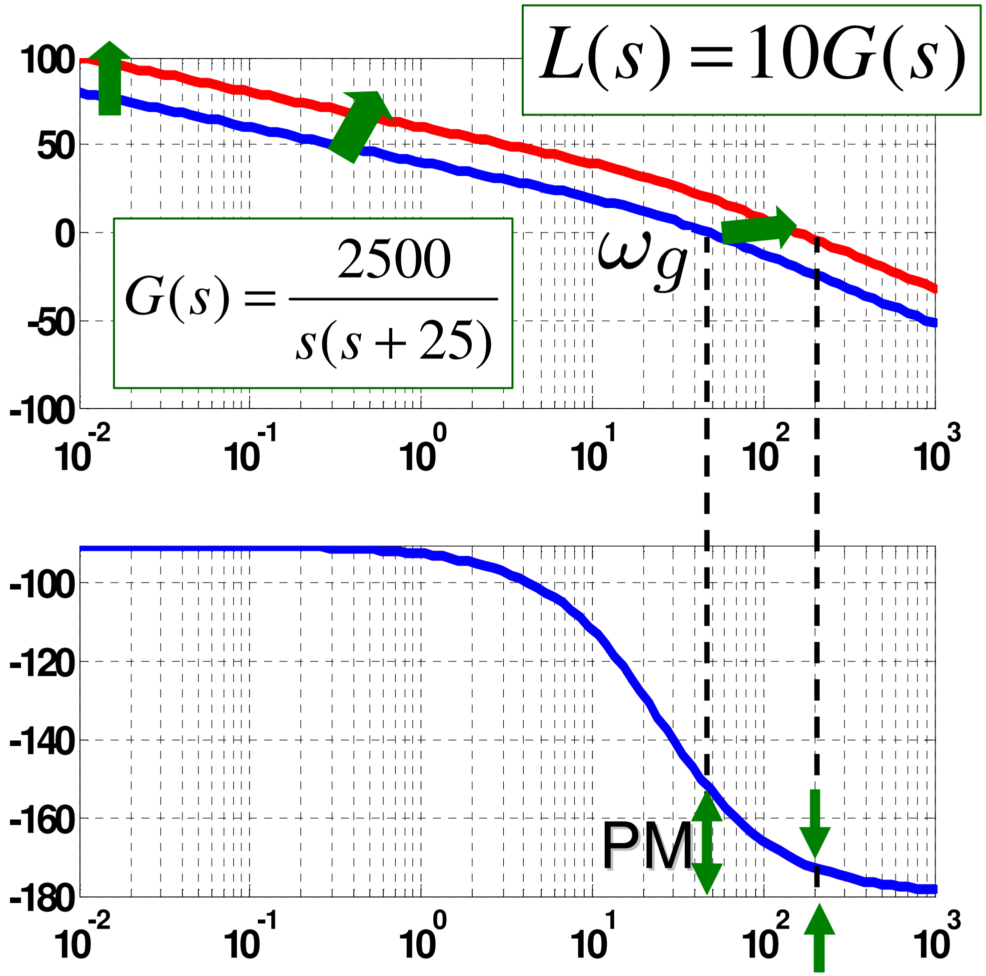


CD – Effect of a Gain C(s) of L(s)

$$C(s) = K(>0)$$

In case of $K > 1$,

- Gain increases uniformly, but phase does not change.
- Typically,
  - (Steady state) $L(0)$ ↑
  - (Speed) ω_g ↑
  - (Stability & overshoot) PM ↓

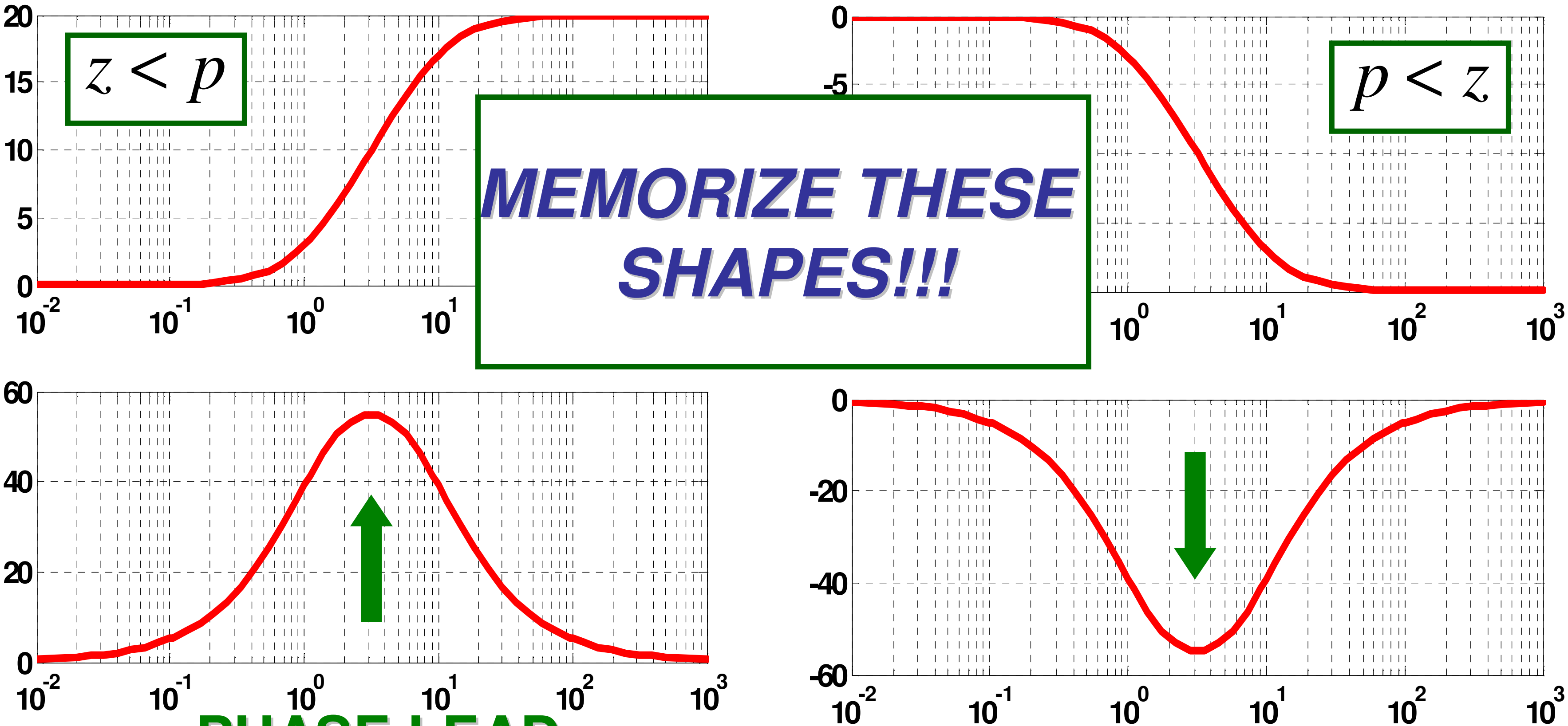


CD – Bode Diagrams of a Lead and Lag C(s)

$$C(s)=\frac{\frac{s}{z}+1}{\frac{s}{p}+1}$$

Lead compensator

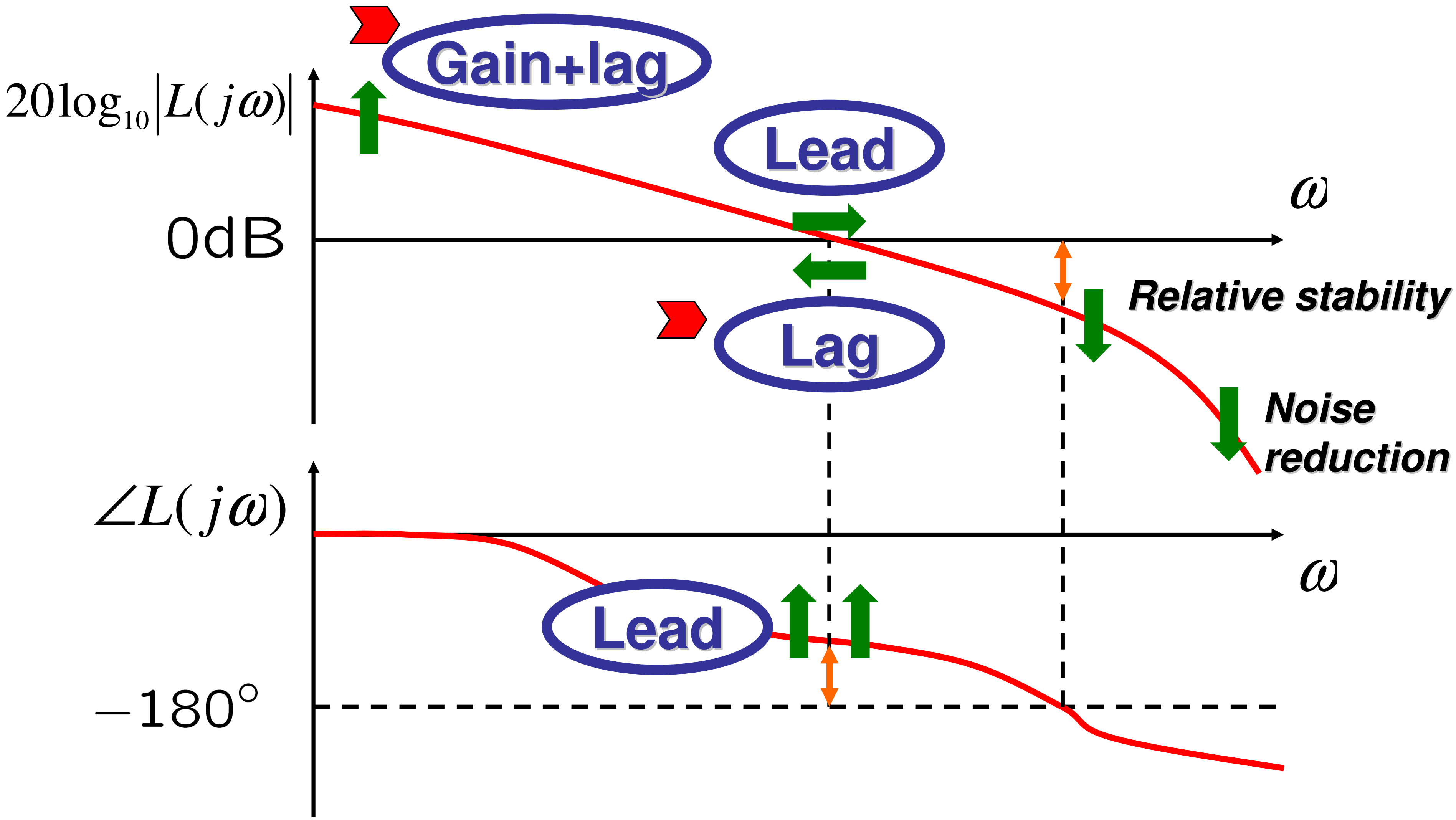
Lag compensator



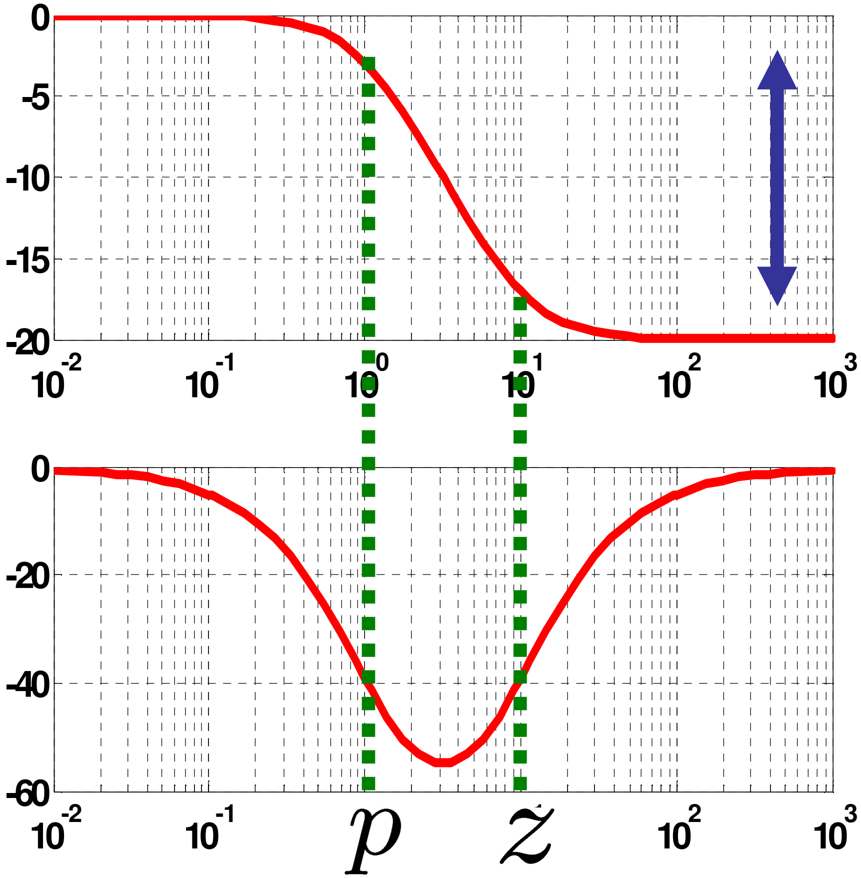
PHASE LEAD

PHASE LAG

CD – Guideline of a Lead and Lag Design



CD – Effect of a Lag C(s) on L(s)



Destabilizing effect

$$20\log_{10}\frac{z}{p}$$

- ***Decreasing*** ω_g

Select z much (at least 1 decade) less than ω_g

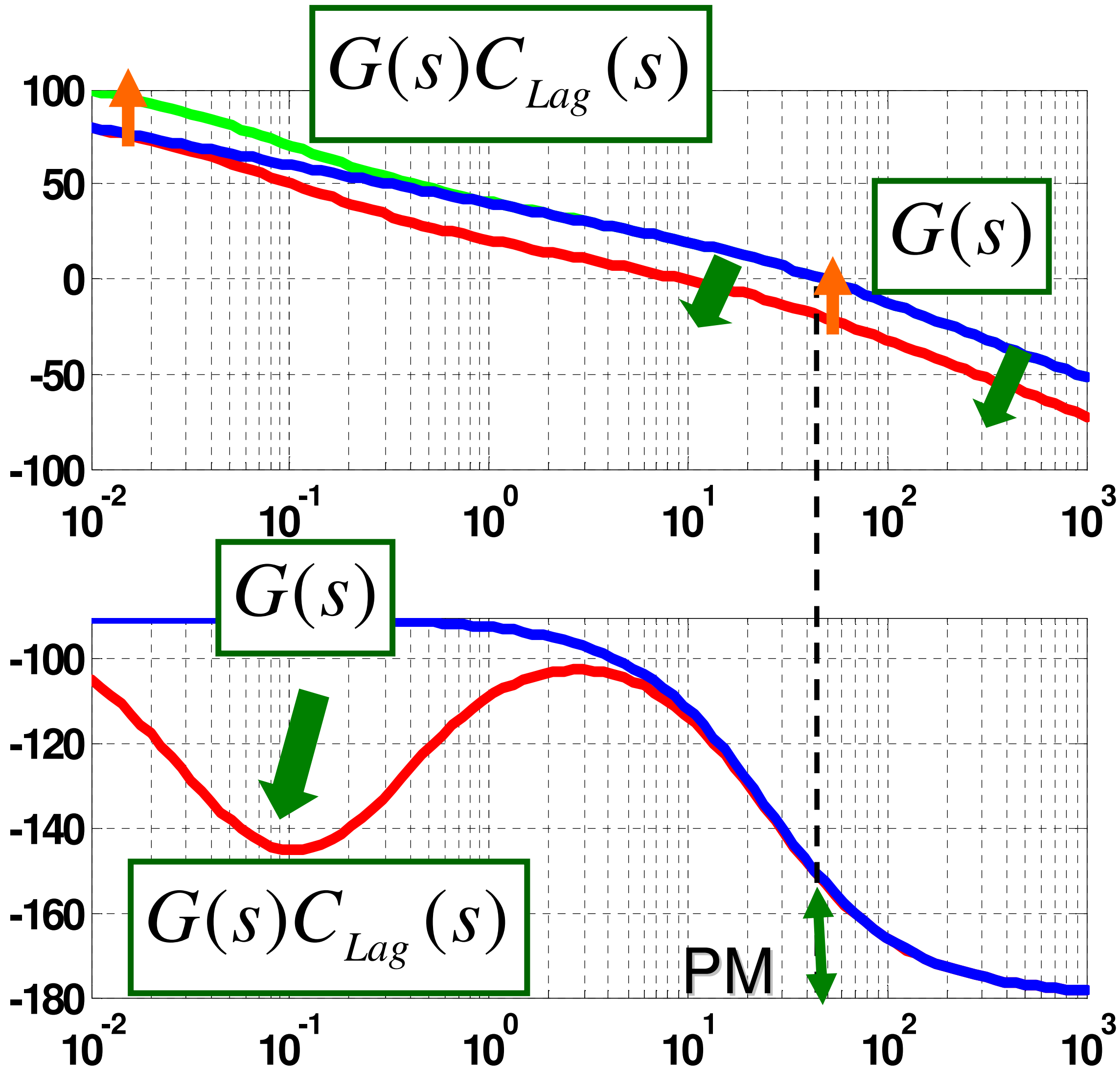
CD – Lag + Gain C(s) Design

$$G(s)=\frac{2500}{s(s+25)}$$

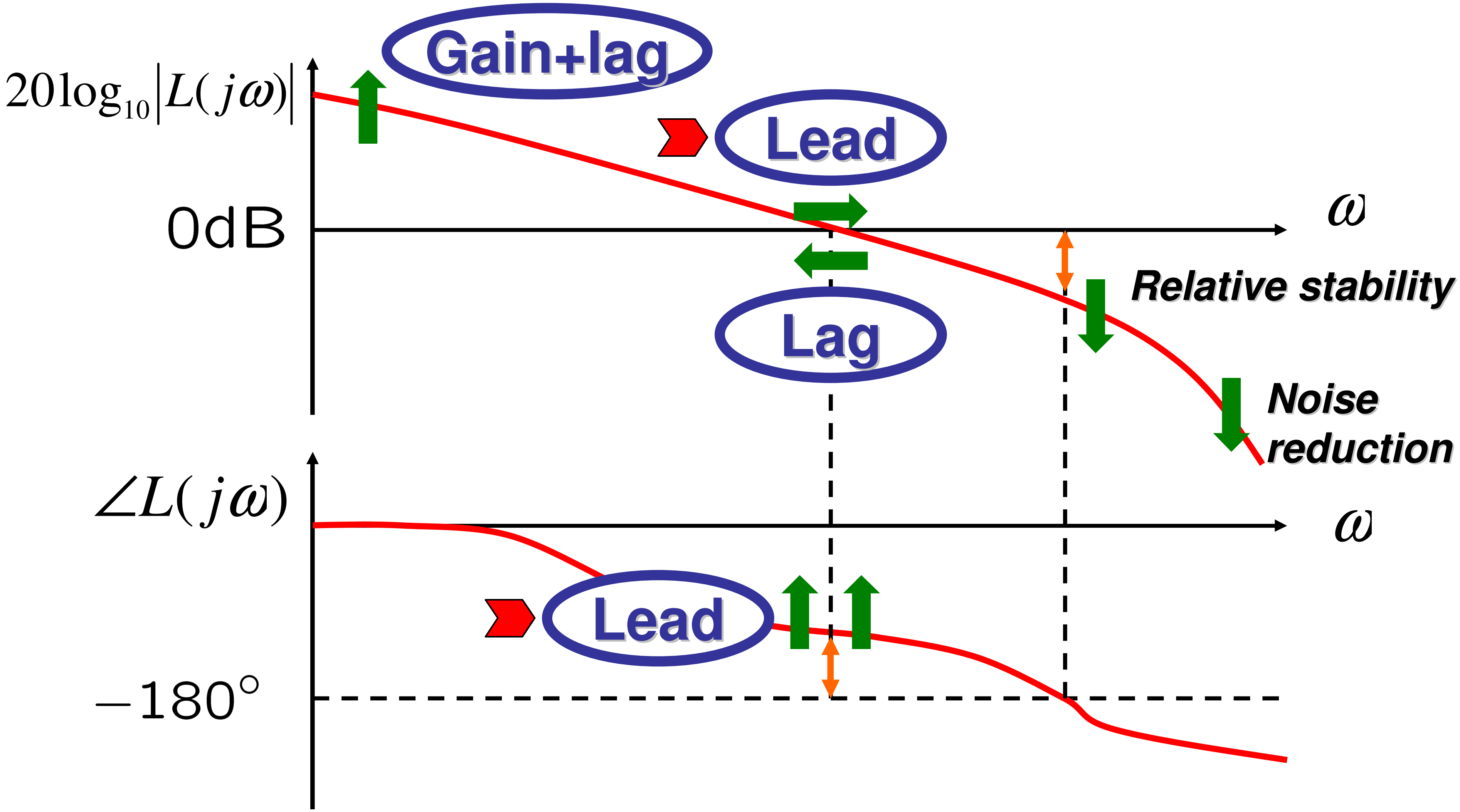
PM: 28 deg at ω_g=47 rad/s

$$C_{Lag}(s)=10\,\frac{1+\frac{s}{1/3}}{1+\frac{s}{1/30}}$$

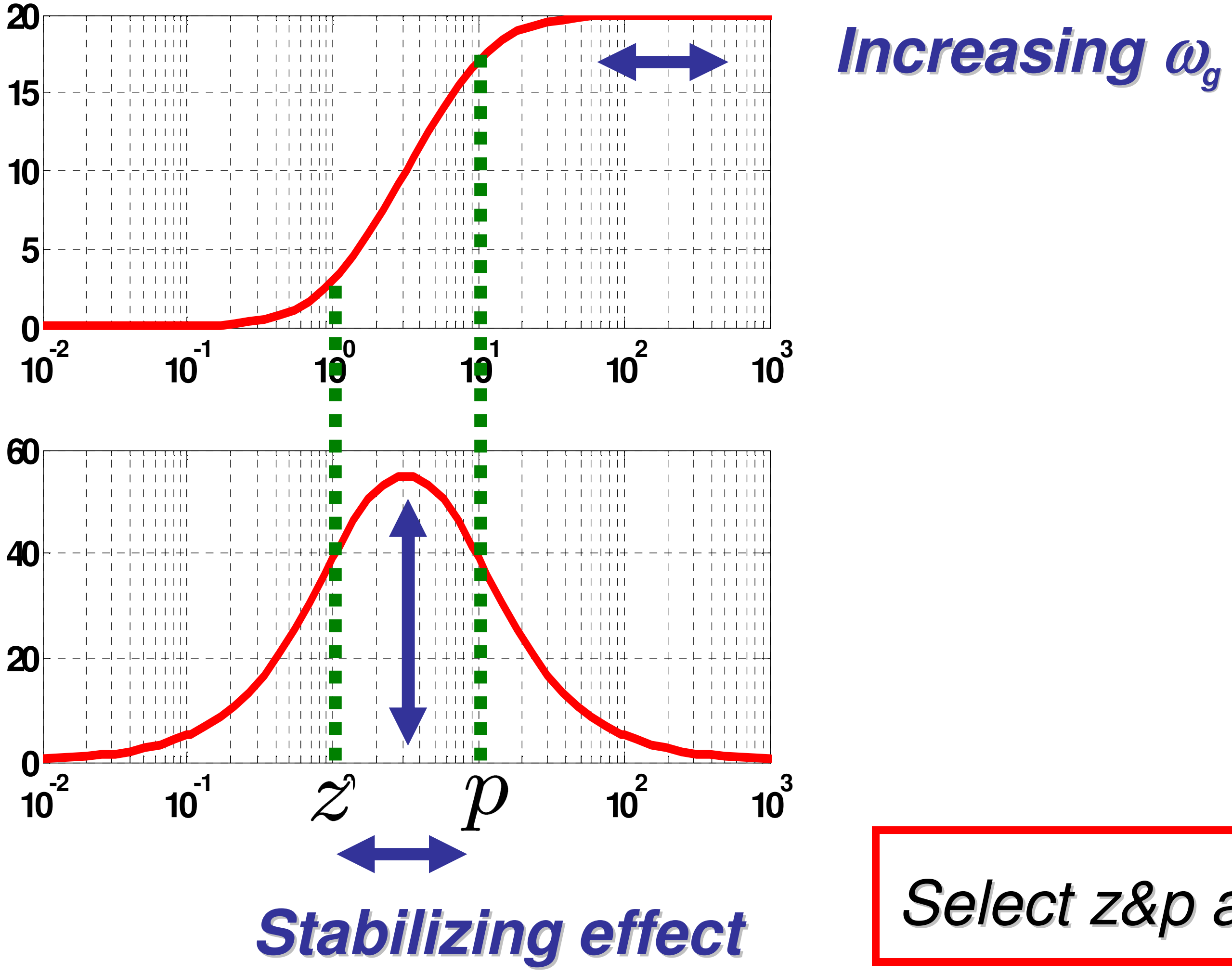
PM: 27 deg at ω_g=47 rad/s



CD – Guideline of a Lead and Lag Design (revisited)



CD – Effect of a Lead C(s) on L(s)



Select z&p around ω_g

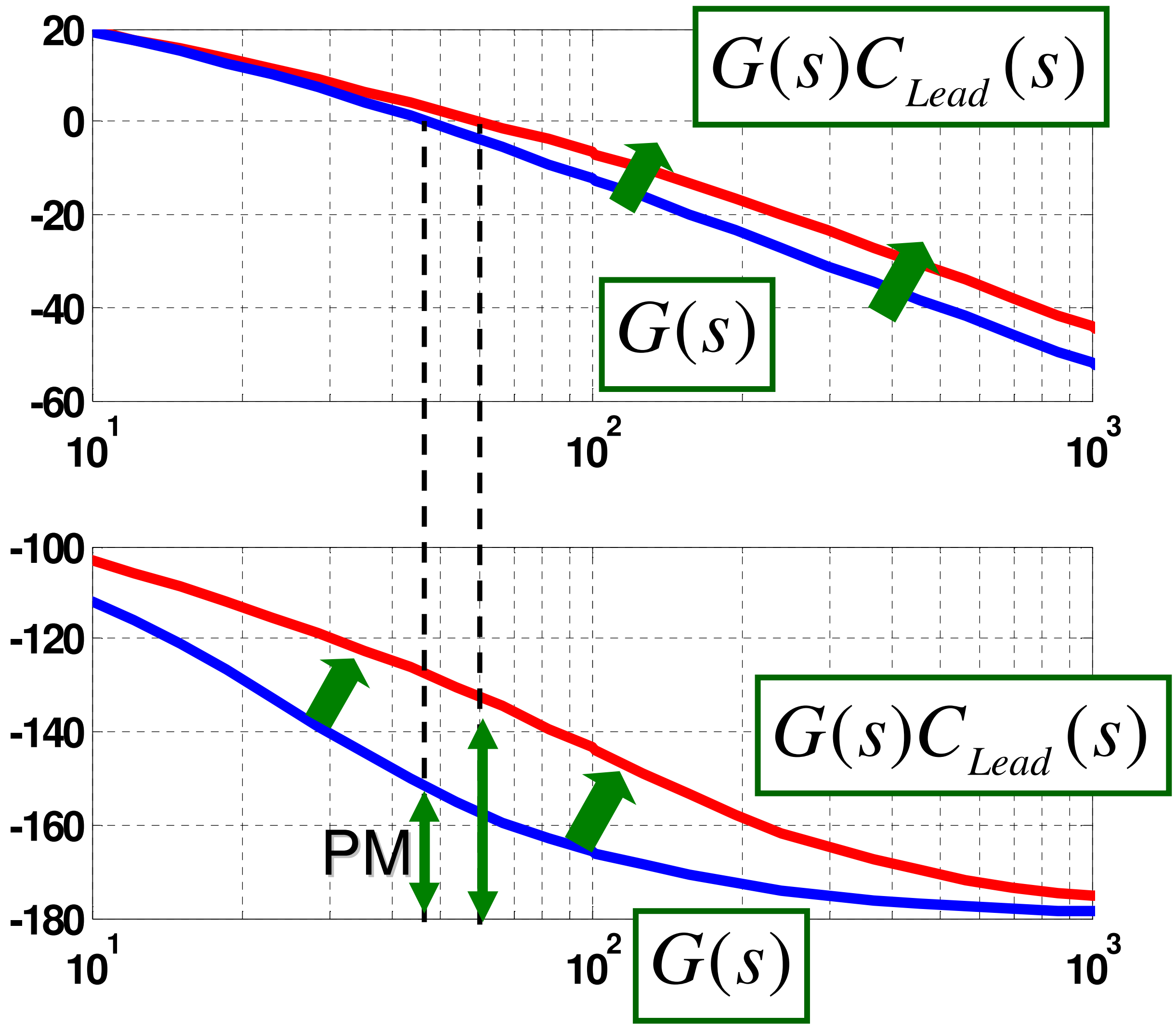
CD – Example of a Lead Design

$$G(s) = \frac{2500}{s(s+25)}$$

PM: 28 deg at ω_g=47 rad/s

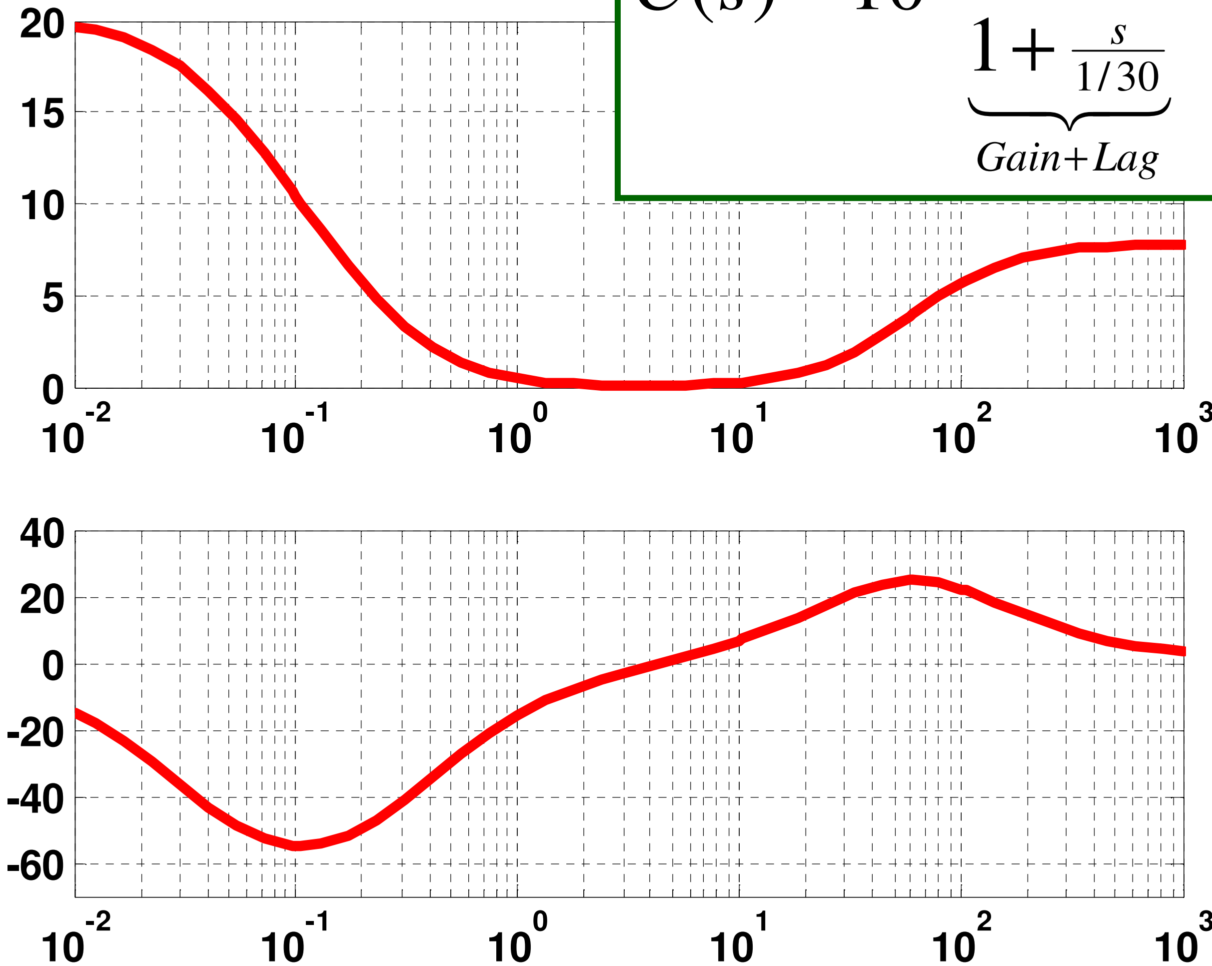
$$C_{Lead}(s) = \frac{1+\frac{s}{38.21}}{1+\frac{s}{94.1}}$$

PM: 47 deg at ω_g=60 rad/s



CD – Lead-Lag Compensator

$$C(s) = 10 \cdot \underbrace{\frac{1+\frac{s}{1/3}}{1+\frac{s}{1/30}}}_{Gain+Lag} \cdot \underbrace{\frac{1+\frac{s}{38.21}}{1+\frac{s}{94.1}}}_{Lead}$$



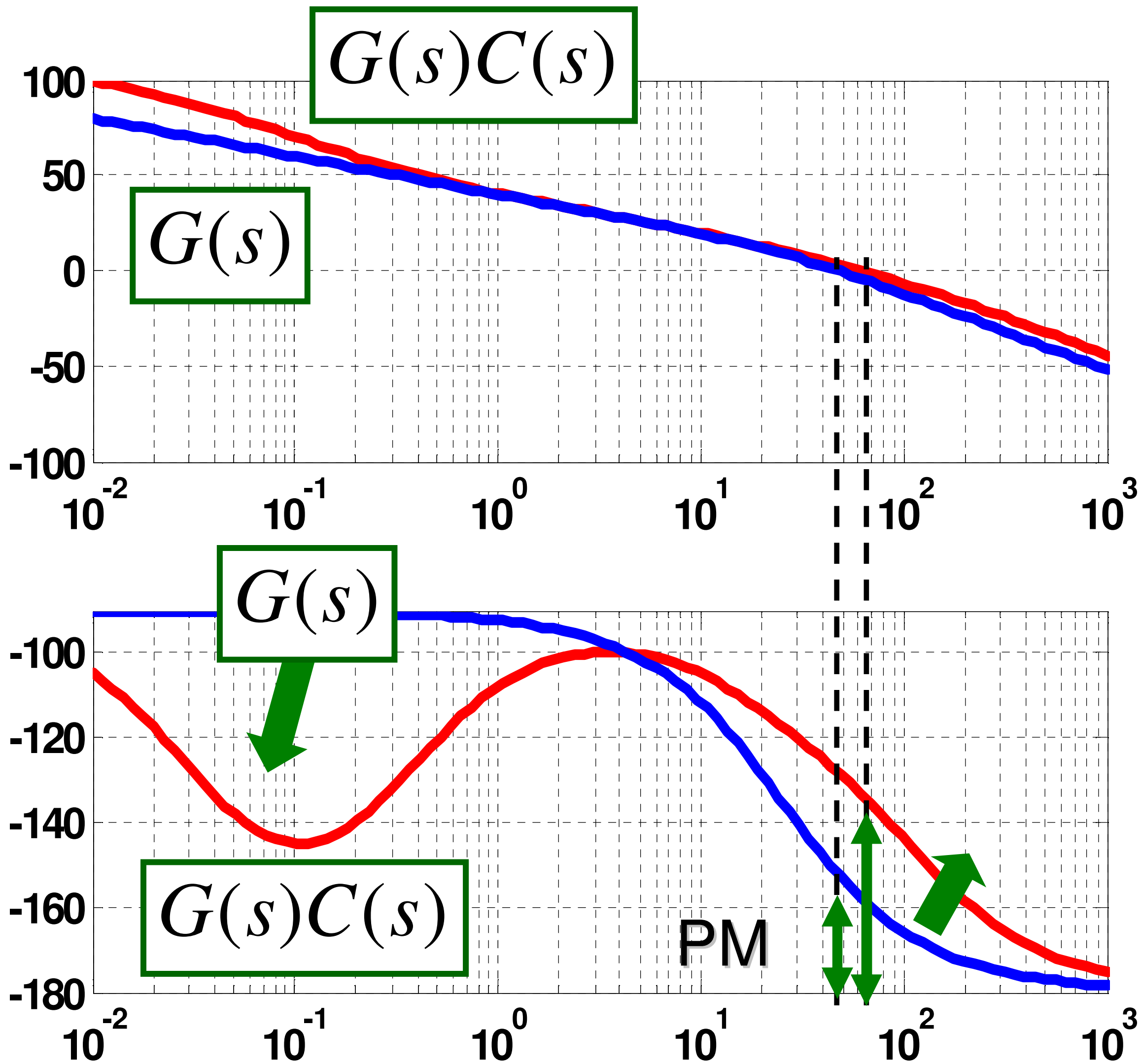
CD – Example of a Lead-Lag Design

$$G(s) = \frac{2500}{s(s+25)}$$

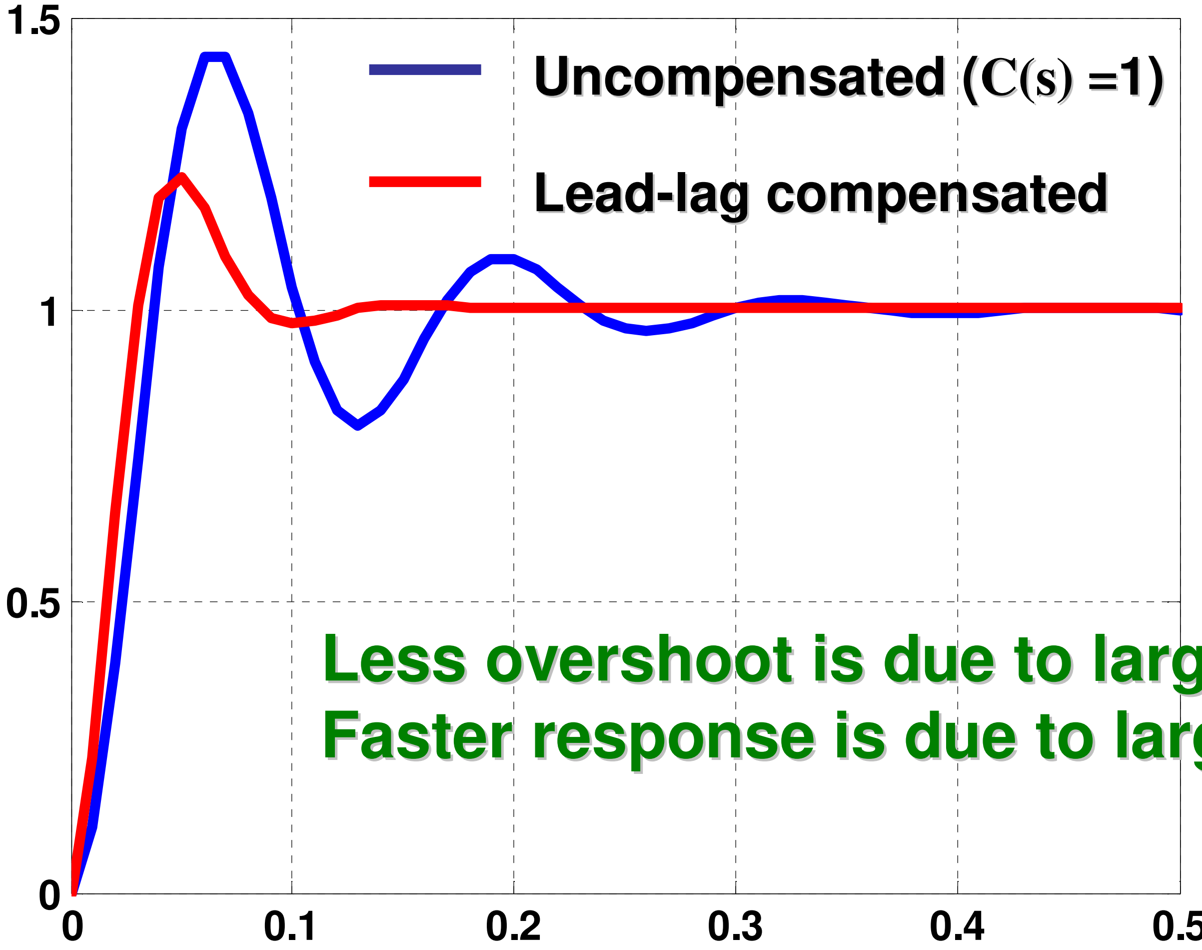
PM: 28 deg at
$\omega_g = 47$ rad/s

$$C(s) = C_{Lead}(s)C_{Lag}(s)$$

PM: 47 deg at
$\omega_g = 60$ rad/s

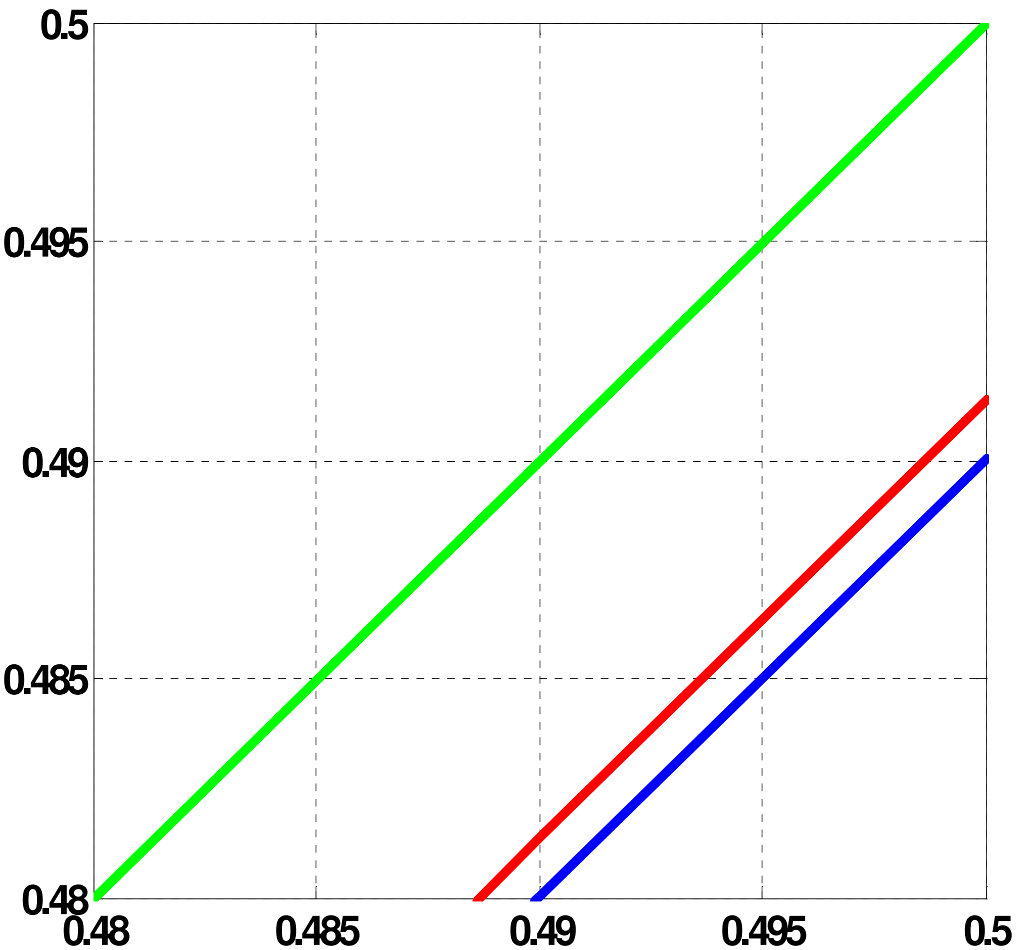


CD – Step Responses



Less overshoot is due to larger PM.
Faster response is due to larger w_g.

CD – Ramp Responses



Ramp reference

Uncompensated (C(s)=1)
K_v **= 100**

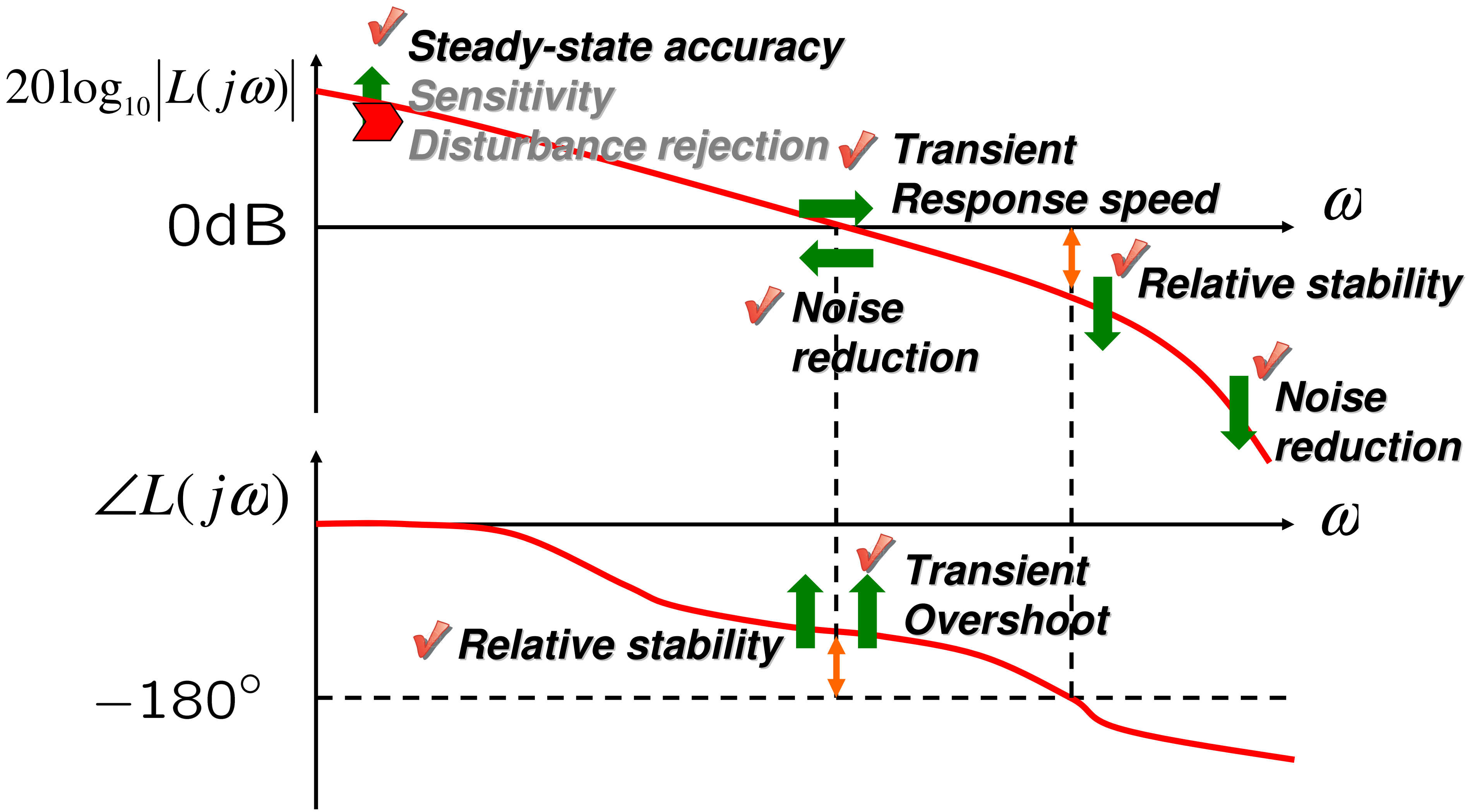
Lead-lag compensated
K_v **= 1000**

Smaller steady-state error is due to larger K_v.

CD – Loop Shaping Summary

- Frequency shaping (Loop shaping) on Bode plot
- Effect of lead, lag, and lead-lag compensators
- Qualitative explanation
- In actual design, one needs to use Matlab.
- Next, more detail about
  - Lag design
  - Lead design

CD – Typical Desired OL Body Diagram



CD – Sensitivity Reduction

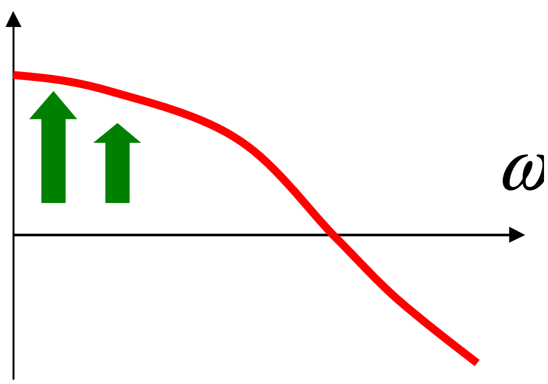
- Sensitivity indicates the influence of plant variations (due to temperature, humidity, age.) on closed-loop performance.
- Sensitivity function

$$S(s) := \frac{\partial T(s)/T(s)}{\partial G(s)/G(s)} = \frac{1}{1+G(s)C(s)} = \frac{1}{1+L(s)}$$

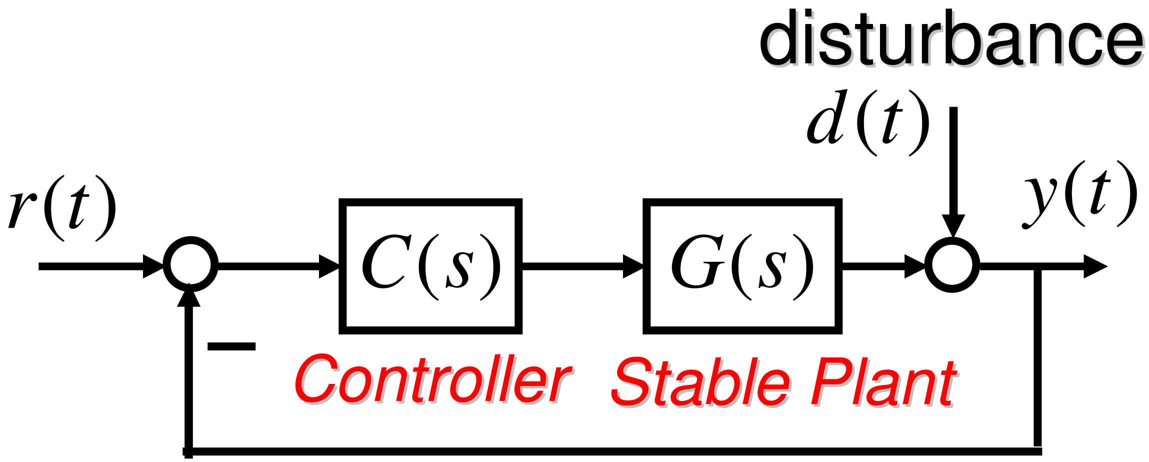
For sensitivity reduction, L should have large gain at low frequencies.

$$\text{Large}\left|L(j\omega)\right| \Rightarrow S(j\omega) = \frac{1}{1+L(j\omega)} \approx 0$$

$20\log_{10}\left|L(j\omega)\right|$

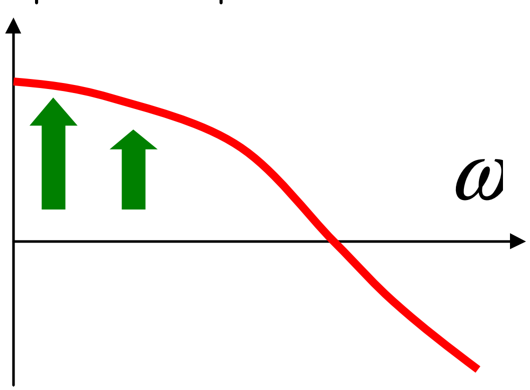


CD – Disturbance Rejection



For disturbance rejection,
L should have large gain at low frequencies.

$20\log_{10}|L(j\omega)|$



Large $|L(j\omega)|$

$$\Rightarrow \quad \frac{Y}{D}(j\omega) = \frac{1}{1+L(j\omega)} \approx 0$$

$\Rightarrow$ $y(t)$ is not affected by $d(t)$ composed of low frequencies.

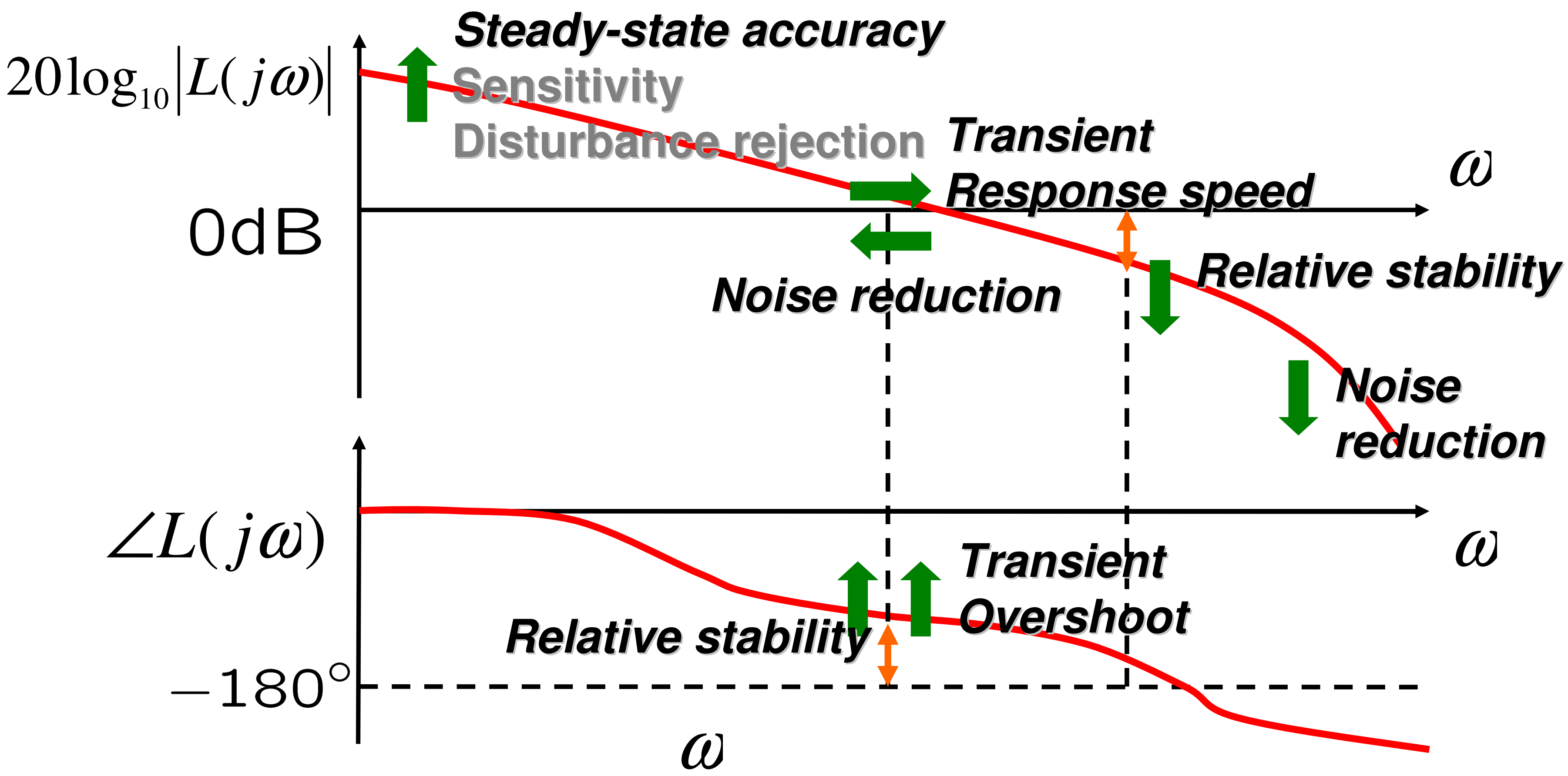
CD – Disturbance

- Unwanted signals

- Examples
  - Wind turbulence in airplane altitude control
  - Wave in ship direction control
  - Sudden temperature change outside the temperature-controlled room
  - Air pressure brake to DC motor
  - Bumpy road in cruise control

- Often, disturbance is neither measurable nor predictable. (Use feedback to compensate it!)

CD – Summary

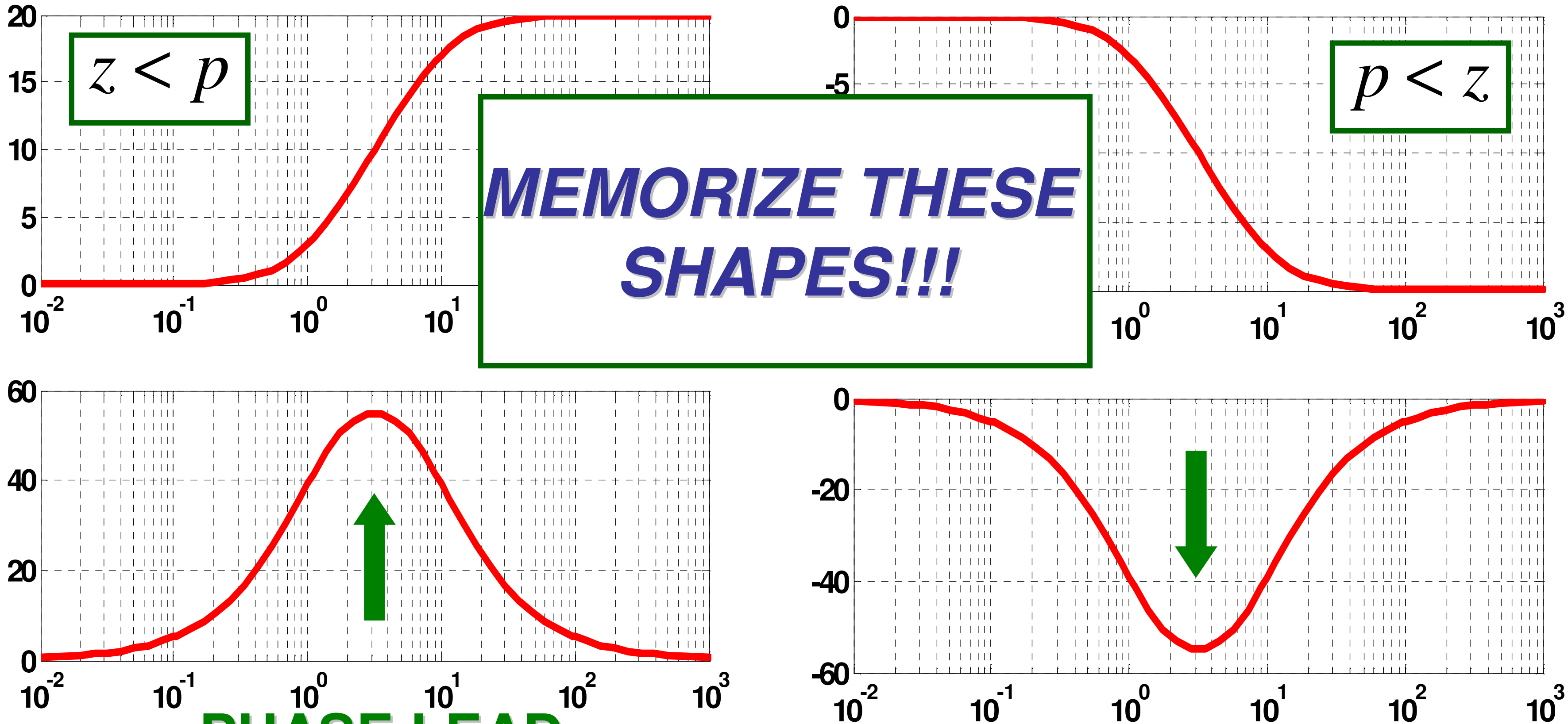


- Next, *frequency shaping (loop shaping) design*

CD – Body Diagram of a Lead/Lag C(s) (Review)

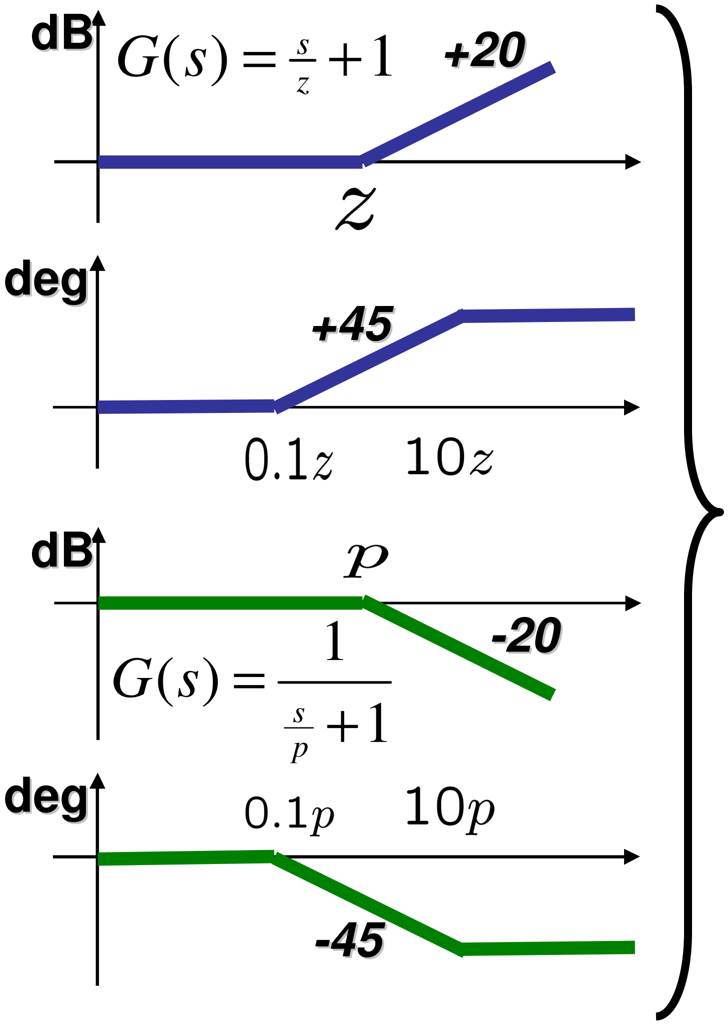
$$C(s) = \frac{\frac{s}{z} + 1}{\frac{s}{p} + 1}$$

Lead compensator | **Lag** compensator

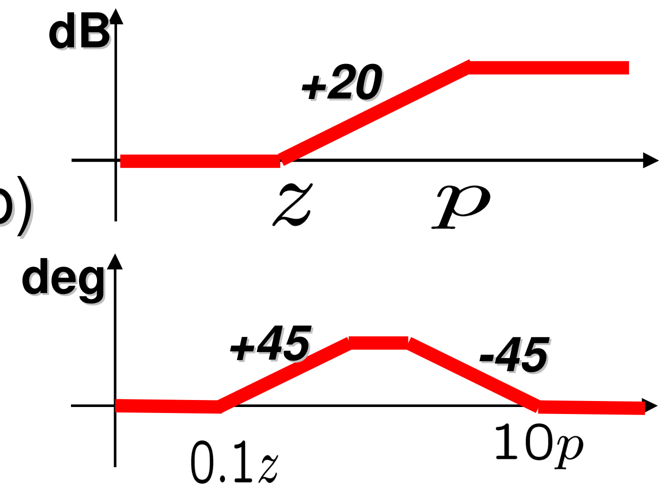


PHASE LEAD | **PHASE LAG**

CD – Straight-Line Approximations

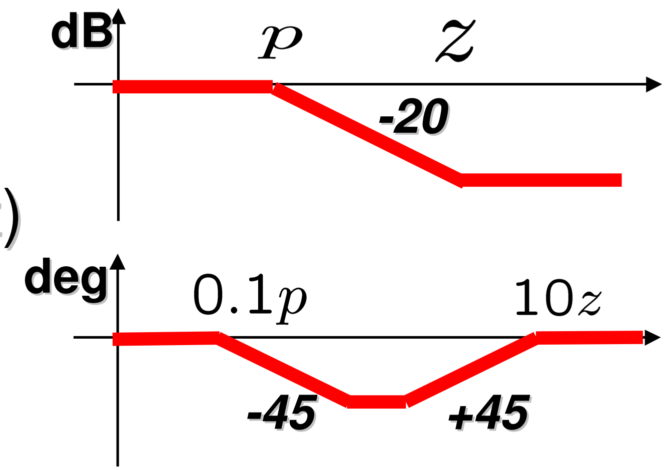


Lead (z < p)

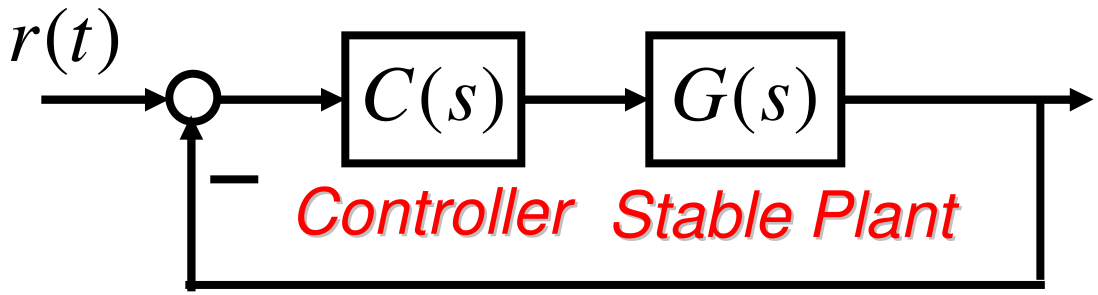


$$G(s) = \frac{\frac{s}{z} + 1}{\frac{s}{p} + 1}$$

Lag (p < z)

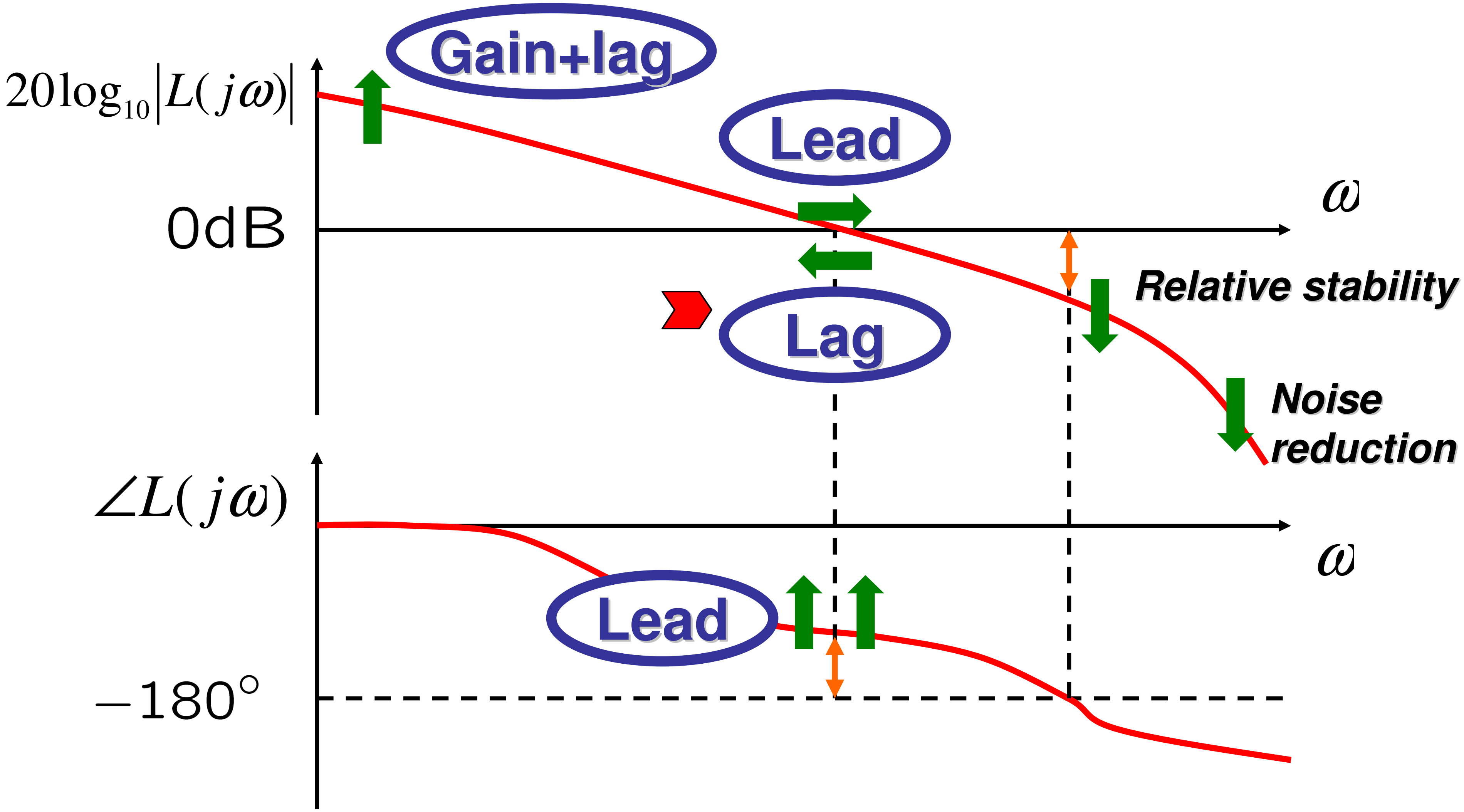


CD – Frequency Shaping (Loop Shaping)



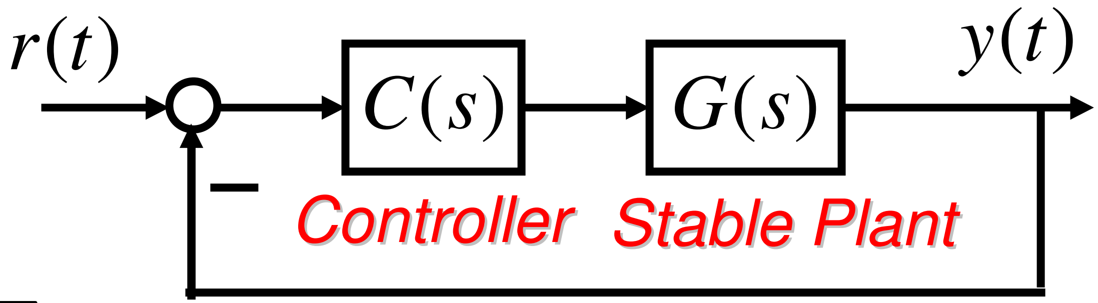
- Design $C(s)$ so that $L(j\omega) := G(j\omega)C(j\omega)$ has a desired shape.
- We study the design of simple compensators:
  – Gain compensator (Today)
  – Lag compensator (Today)
  – Lead compensator (Next lecture)

CD – Guideline of Lead-Lag Design (Review)



CD – An Example (Lead-Lag Design)

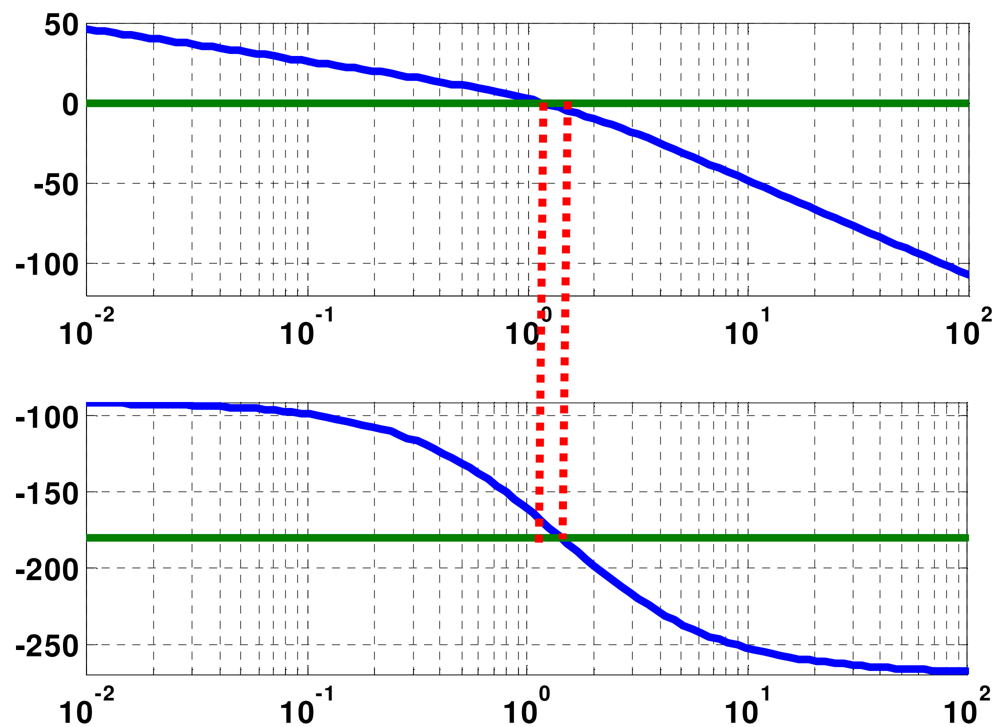
- Consider a system



$$G(s) = \frac{4}{s(s+1)(s+2)}$$

- Analysis for $C(s) = 1$
  - Stable
  - PM at least 12 deg
  - GM at least 3.5 dB

These values are too small for good transient response!

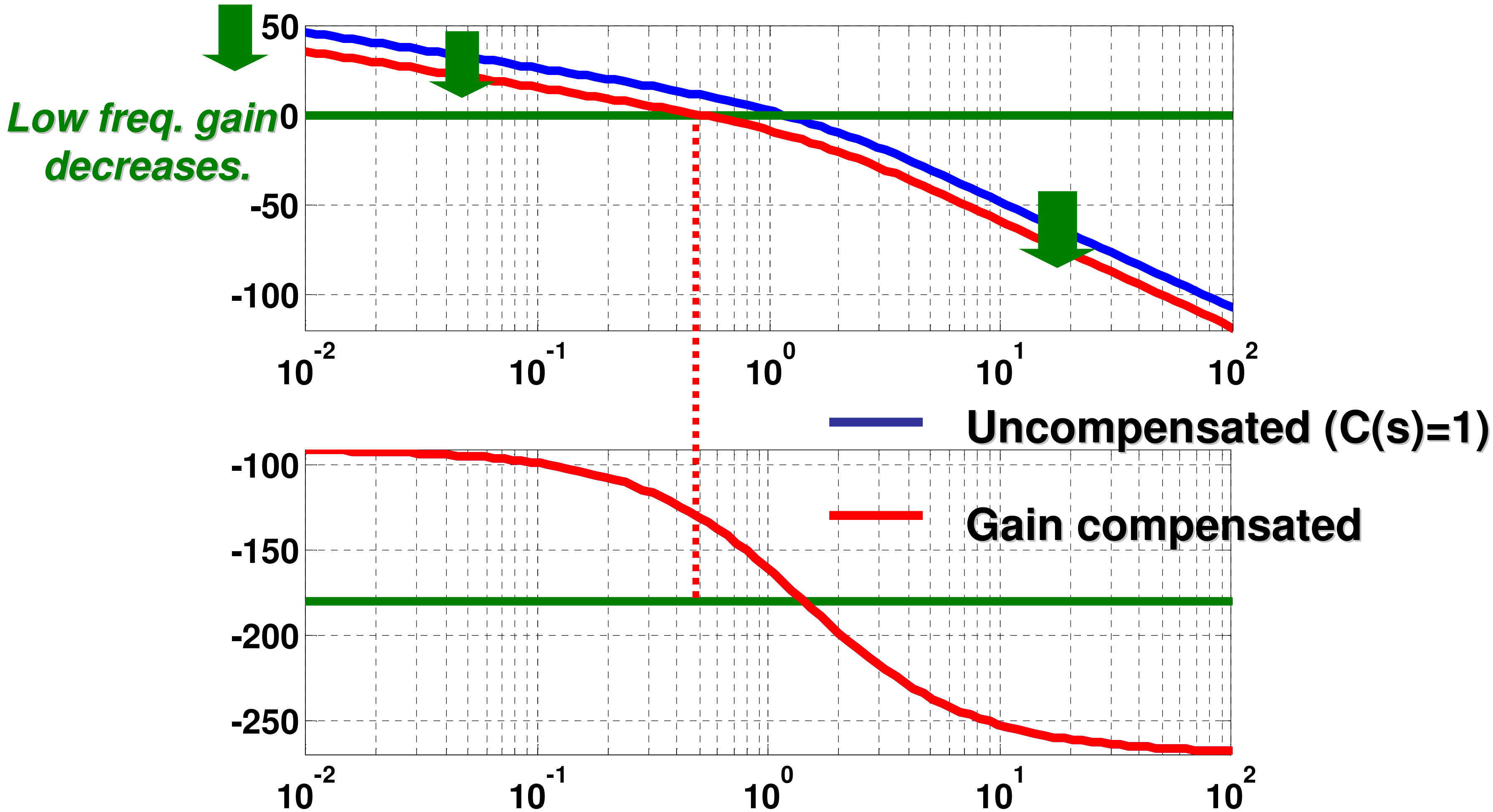


CD – Gain Margin Compensation (Example (2))

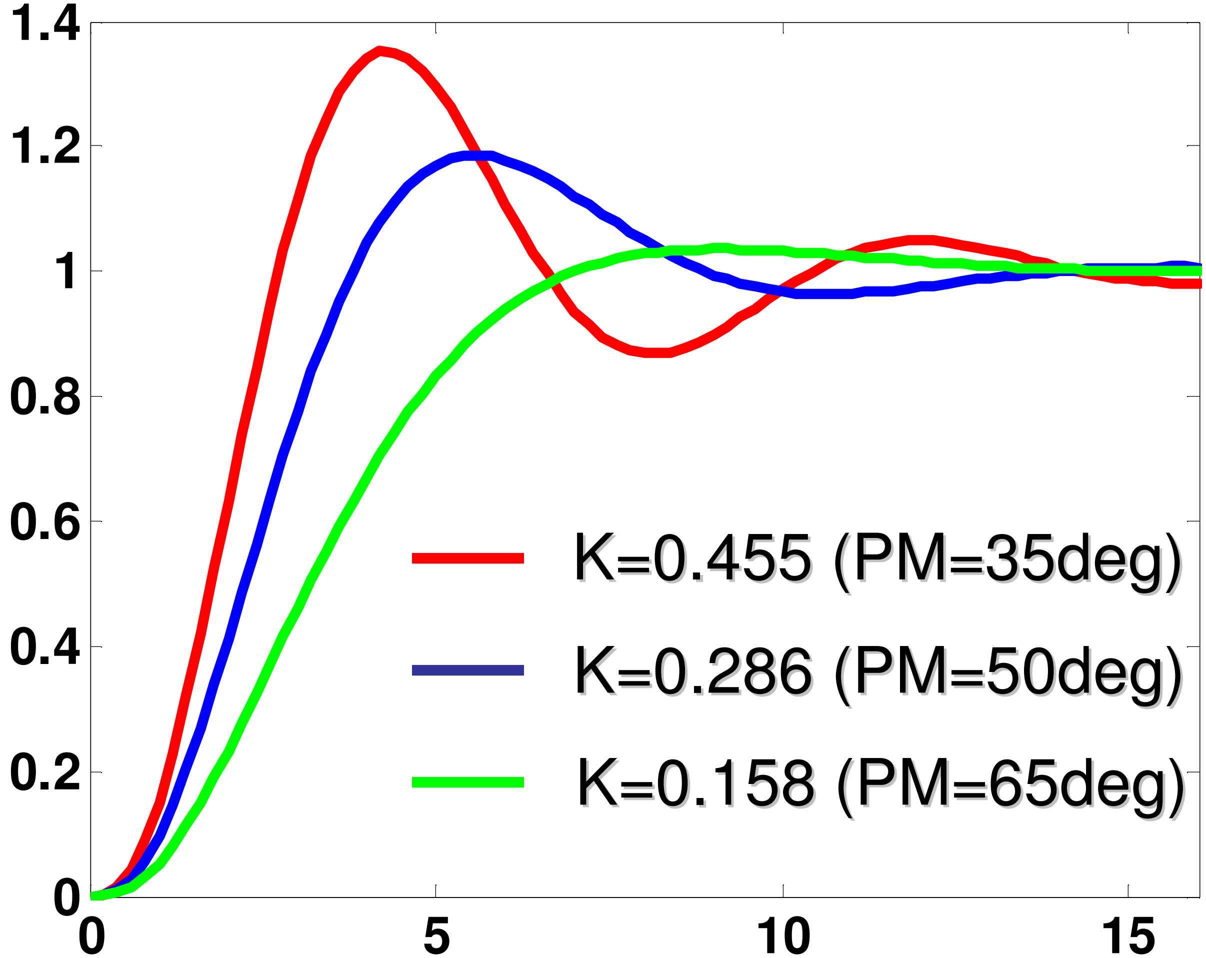
- PM is specified to be 50 deg.
- In this example, to increase PM by gain compensation, we need to lower the gain curve.

K	PM deg	Overshoot %	ω_g rad/sec	Rise time sec
0.455	35	35	0.7	1.7
0.286	50	18	0.5	2.4
0.158	65	3.7	0.3	4.5

CD – Bode Diagram for C(s) = 0.286 (Example (3))

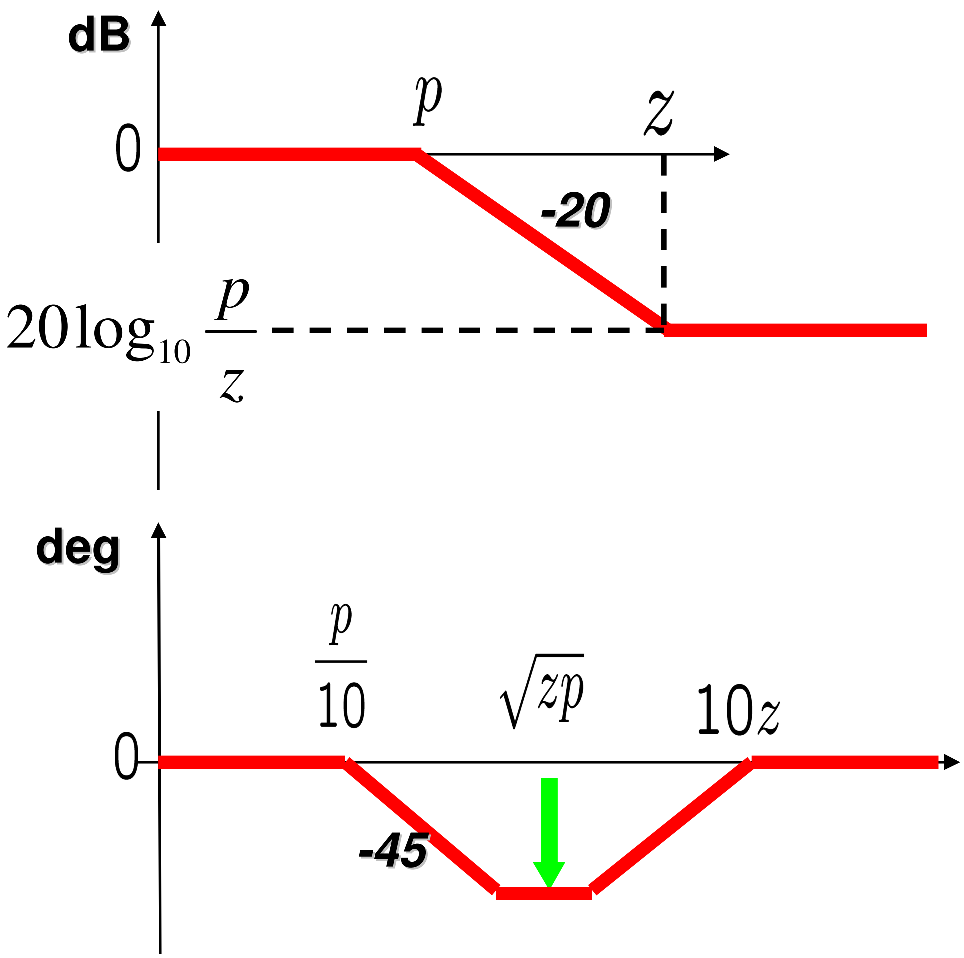


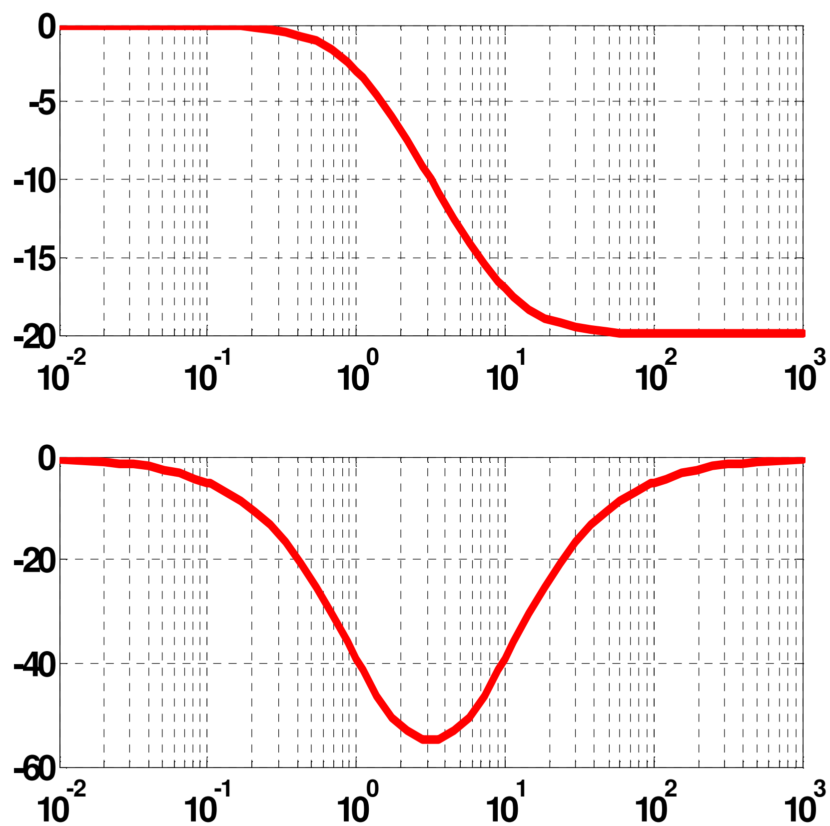
CD – Step Responses (Example (4))



CD – Phase-Lag Compensator (Review)

$$G(s) = \frac{\frac{s}{z}+1}{\frac{s}{p}+1}, 0 < p < z$$

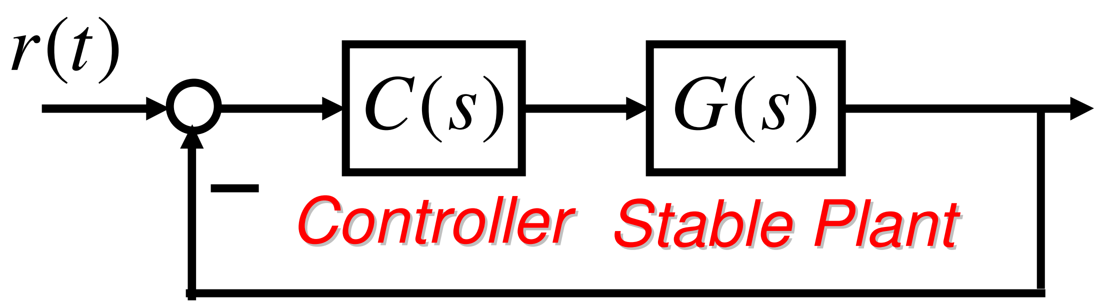




CD – Phase-Lag Compensator C(s) Design

We try to design phase-lag C(s) which gives
- *PM 50deg*
- *Low frequency gain same as the original plant.*

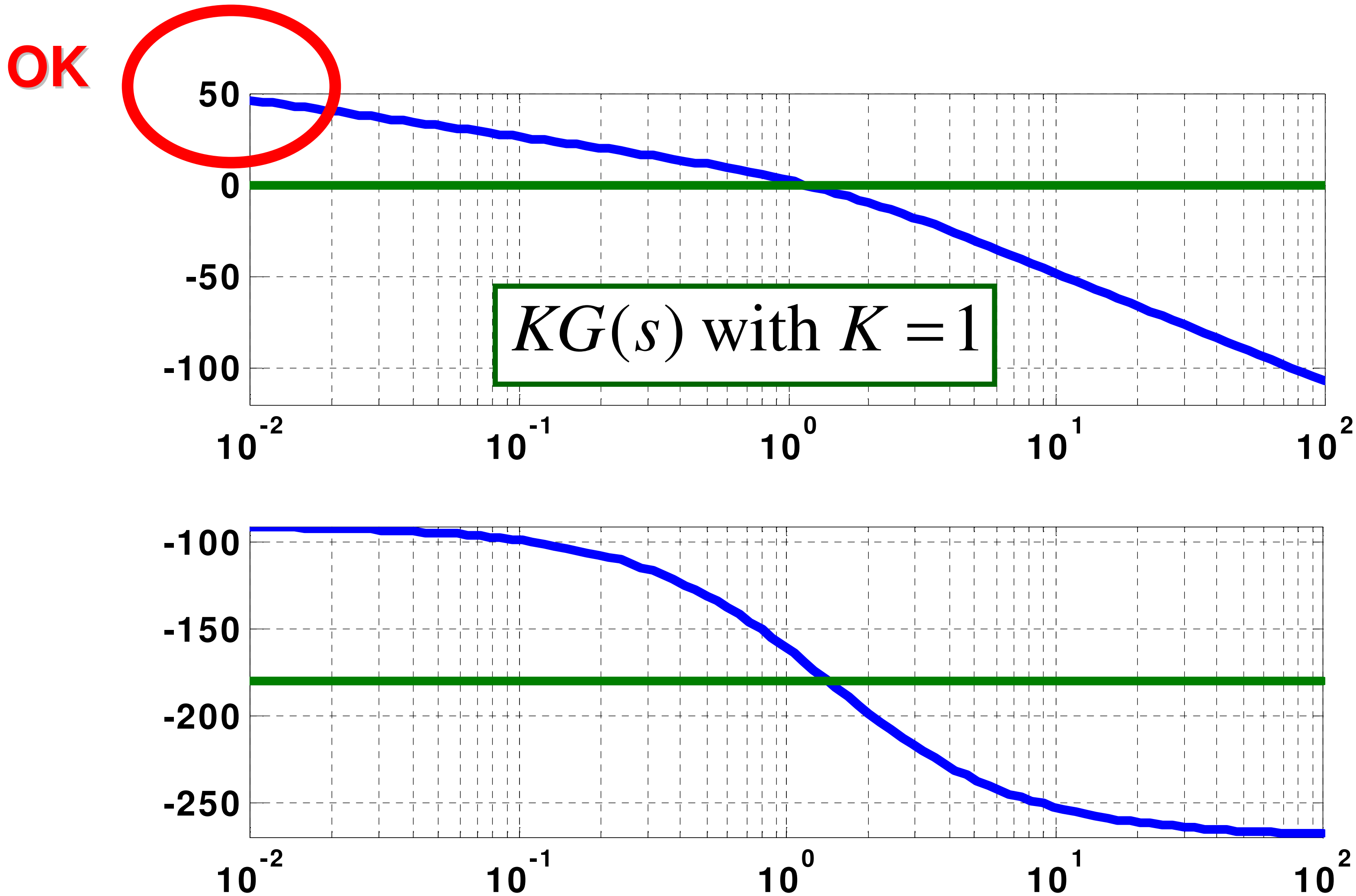
Step 1: To satisfy low frequency requirement, adjust DC gain of OL system by a constant gain K.



- Analysis for $C(s) = 1$
  - Stable
  - PM at least 12 deg
  - GM at least 3.5 dB

$$G(s) = \frac{4}{s(s+1)(s+2)}$$

CD – Phase-Lag Design Step 1 (C(s) = 1)



CD – Phase-Lag Design Step 2 (C(s) = 1)

Step 2: Find the frequency ωg (which will become gain crossover frequency after compensation) where

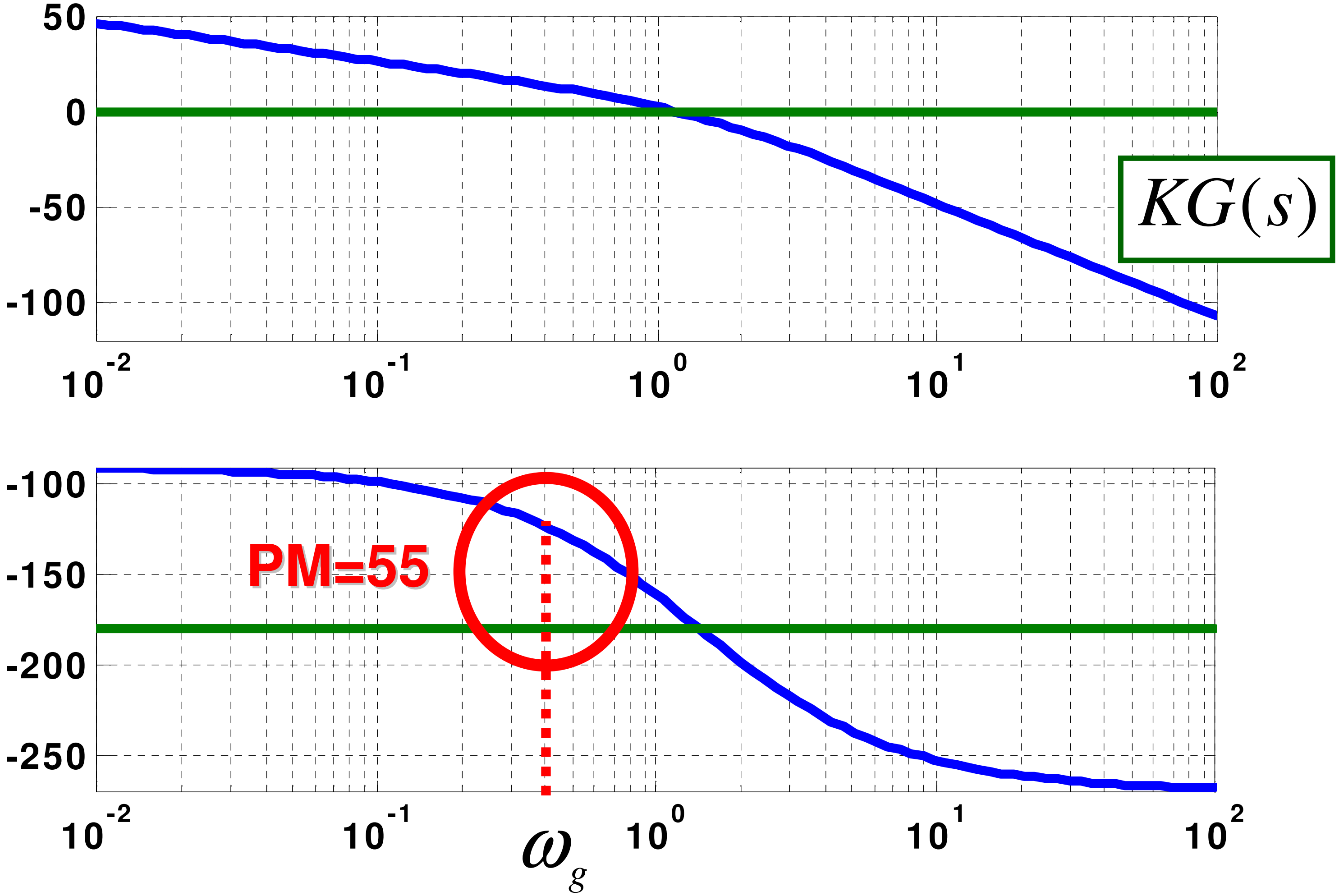
$$\angle G(j\omega_g) = -180^o + \phi_m + 5^o,\ \phi_m :\ \text{required PM}$$

In this example,

$$\angle G(j\omega_g) = -180^o + \underbrace{50^o}_{\phi_m} + 5^o = -125^o \Rightarrow \omega_g = 0.4$$

Note: The reason of +5 deg is explained later.

CD – Phase-Lag Design Step 2 (C(s) = 1)



CD – Phase-Lag Design Step 3 (C(s) = 1)

Step 3:

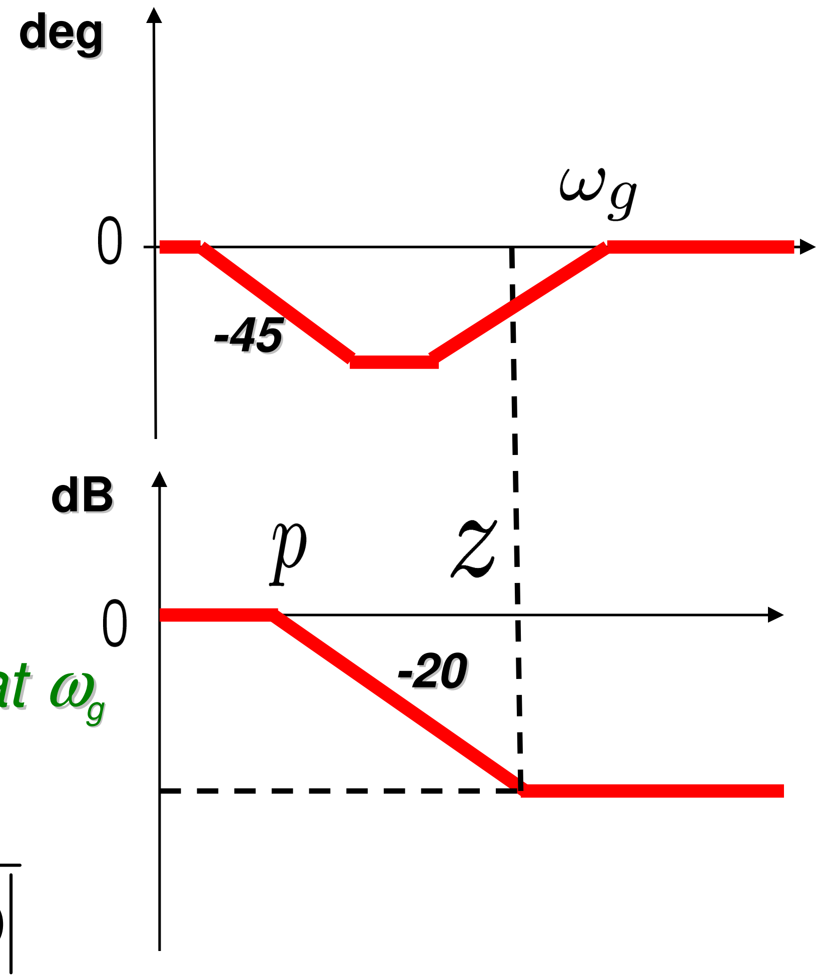
$$z = 0.1\omega_g \; (= 0.04)$$

For small phase lag at ω_g

$$p = \frac{0.1\omega_g}{|KG(j\omega_g)|} \left(= \frac{0.04}{4.55}\right)$$

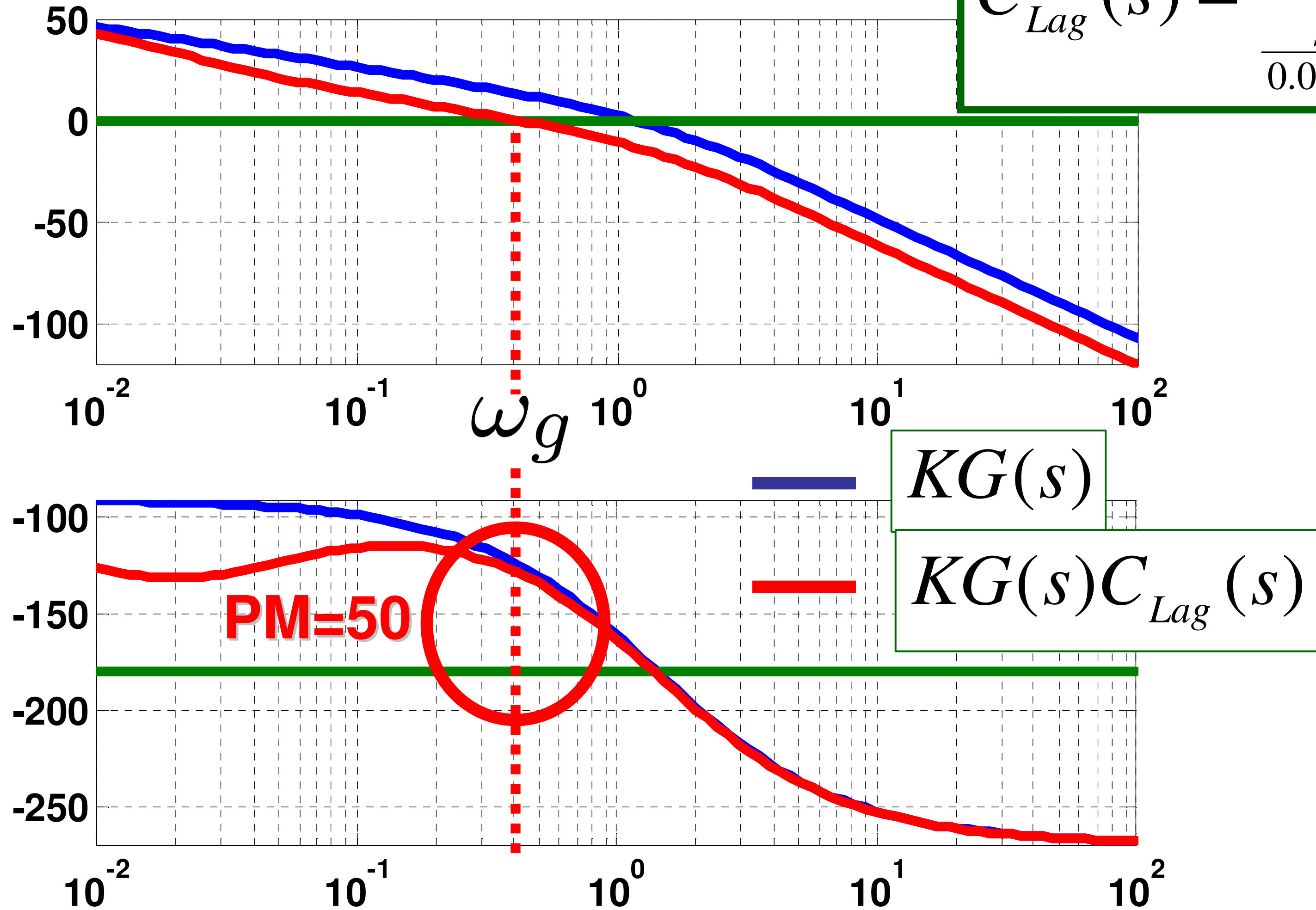
For setting new gain crossover at ω_g

$$20\log_{10}\frac{p}{z} = 20\log_{10}\frac{1}{|KG(j\omega_g)|}$$

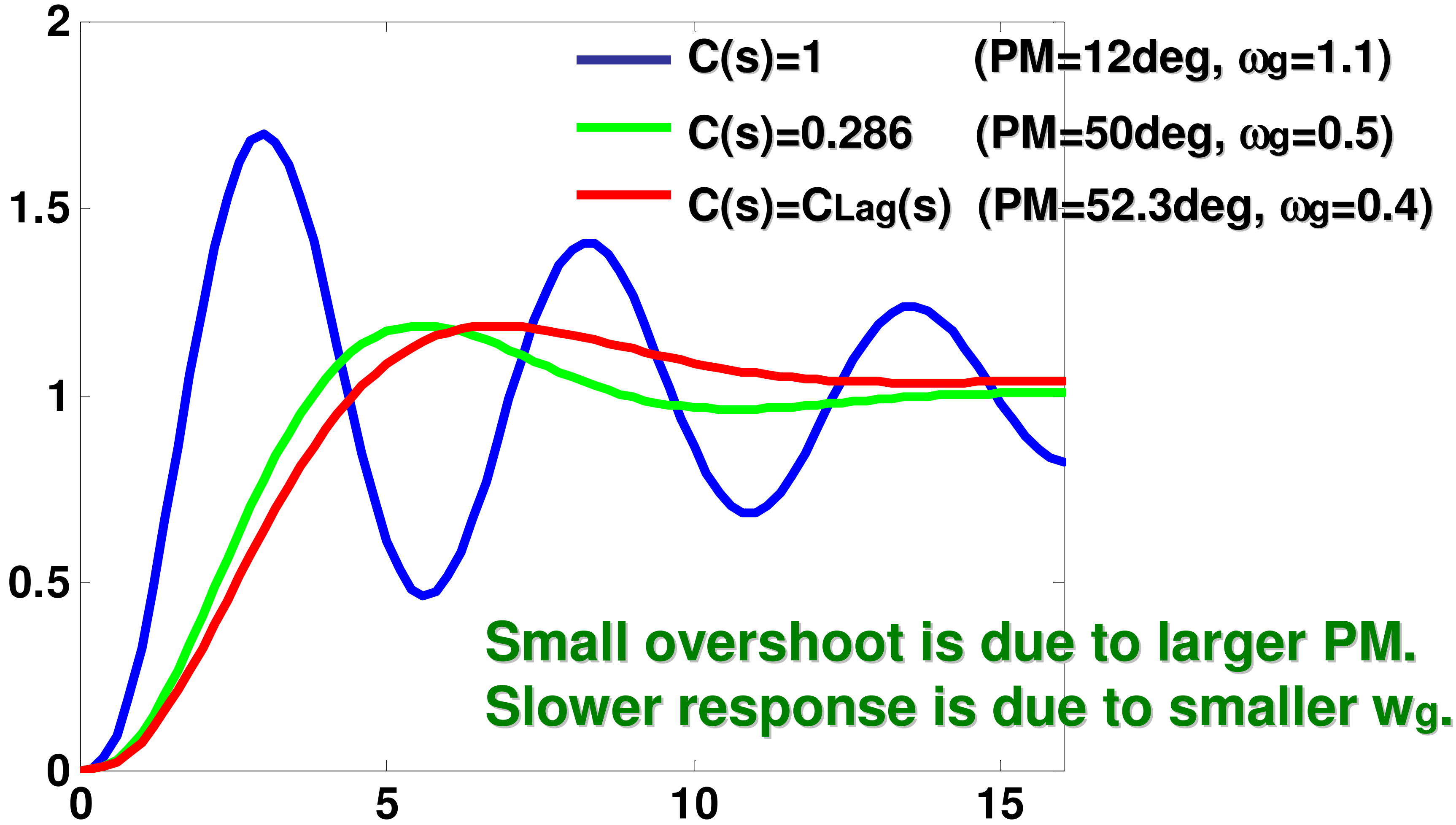


CD – Phase-Lag Design Step 3 ($C(s) = C_{lag}(s)$)

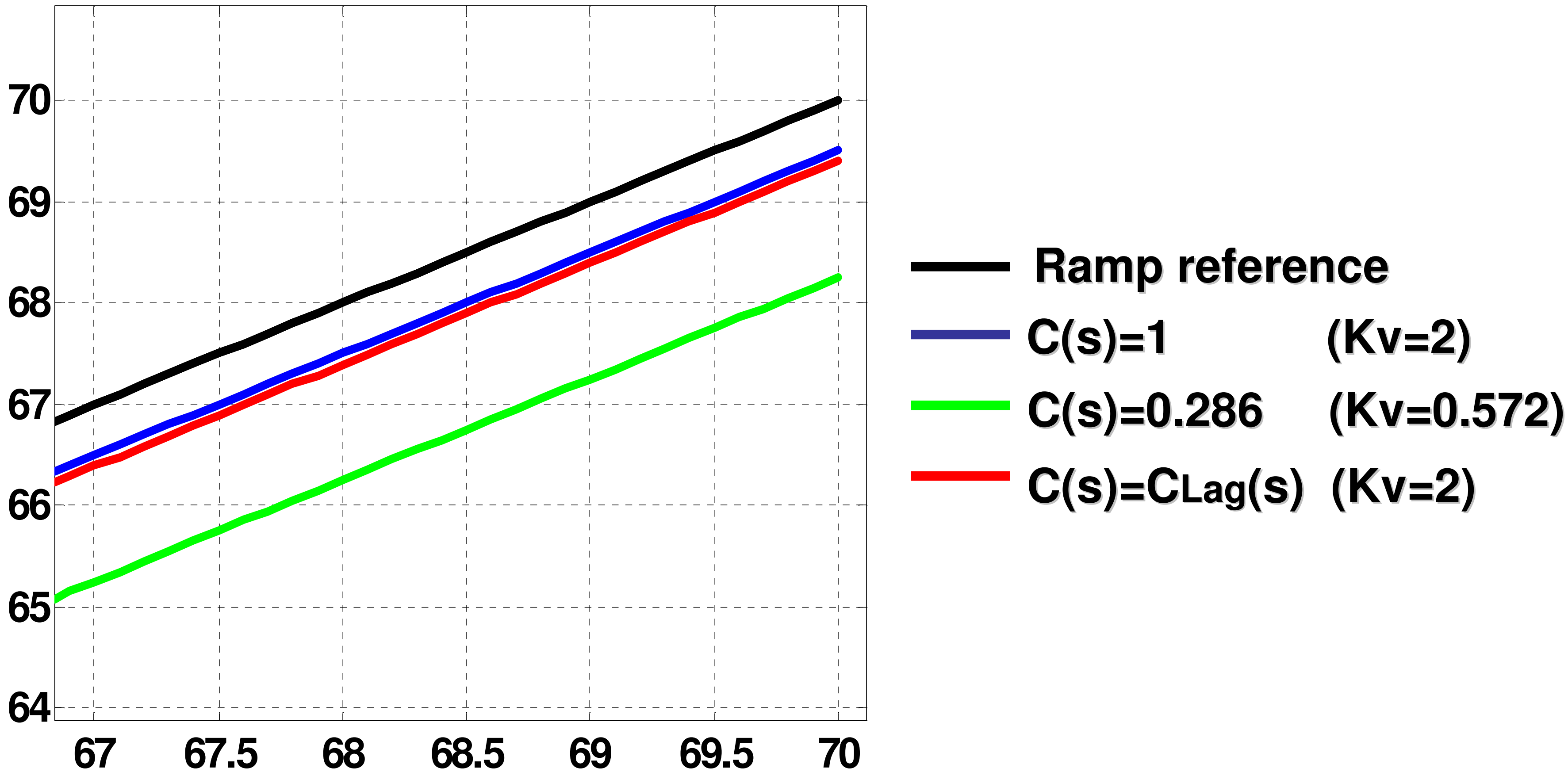
$$C_{Lag}(s) = \frac{\frac{s}{0.04}+1}{\frac{s}{0.0088}+1}$$



CD – Phase-Lag Design Step Responses



CD – Phase-Lag Design Ramp Responses



Smaller steady-state error is due to larger Kv.

CD – Phase-Lag Design Summary

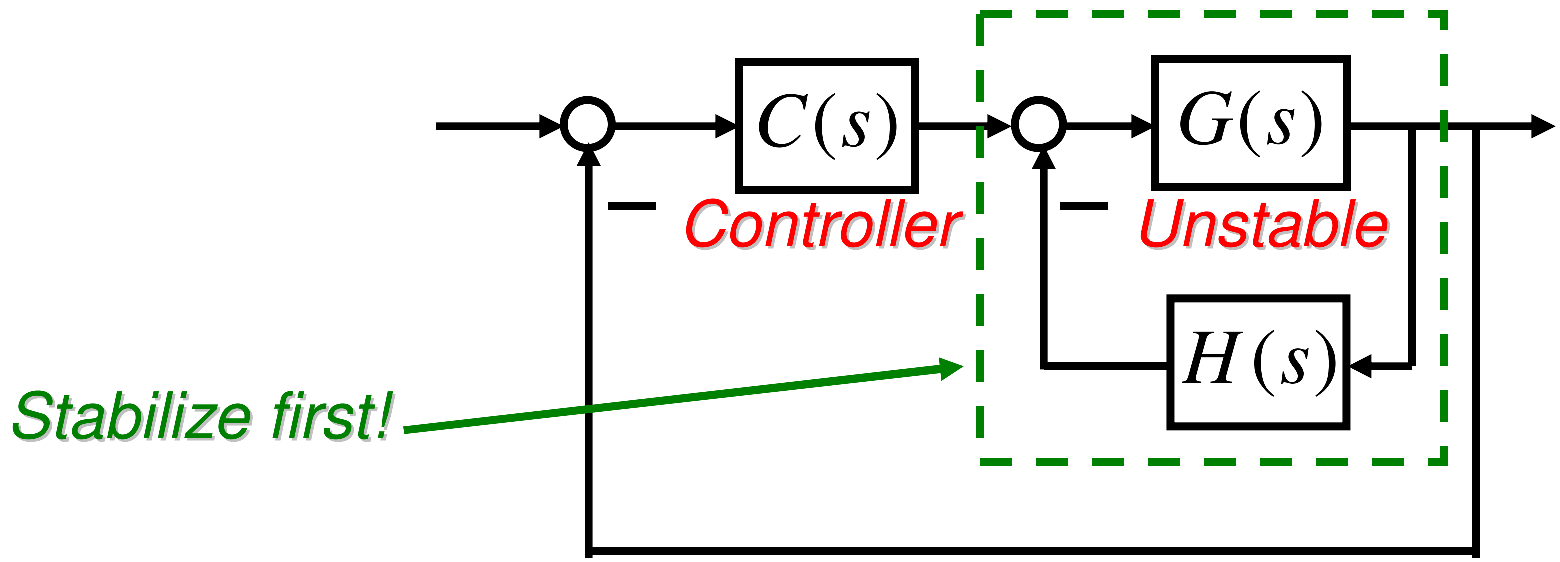
- Gain controller design in Bode plot
  - Gain changes uniformly over frequencies.
  - Phase does not change.

- Lag compensator design in Bode plot
  - Lag compensator can be used for
    - Improving PM by maintaining low freq. gain, or
    - Improving low freq. gain by maintaining PM

- Low freq. gain determines steady state error, disturbance rejection, while PM does overshoot.
- Next, lead compensator design

CD – If G(s) has OL RHP Poles

- What is problematic?
  - Nyquist stability criterion says that, for closed-loop stability, Nyquist plot of open-loop system must encircle -1 point.
  - It is hard to translate this condition into Bode plot.
- To use FR technique…



CD – Phase-Lead Design Procedure

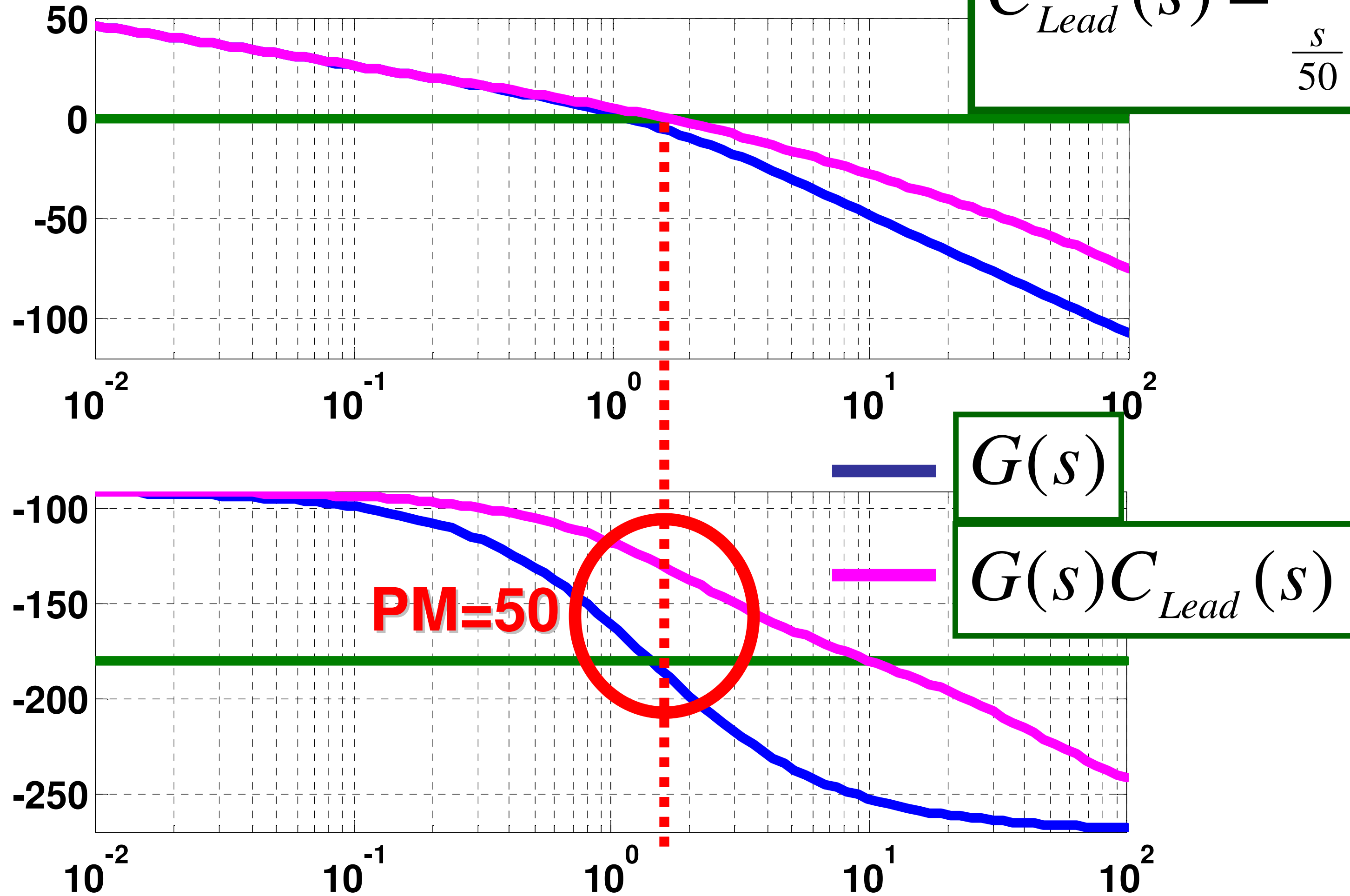
1. Select z near uncompensated ω_g.
   In the example, $\omega_g = 1.14$. So, select, for example, $z = 1$.

2. Select $p > z$ by trial-and-error.

3. Check PM and settling time. If not satisfactory, move the pole p. If moving pole does not give the desired results, try to move the zero z.

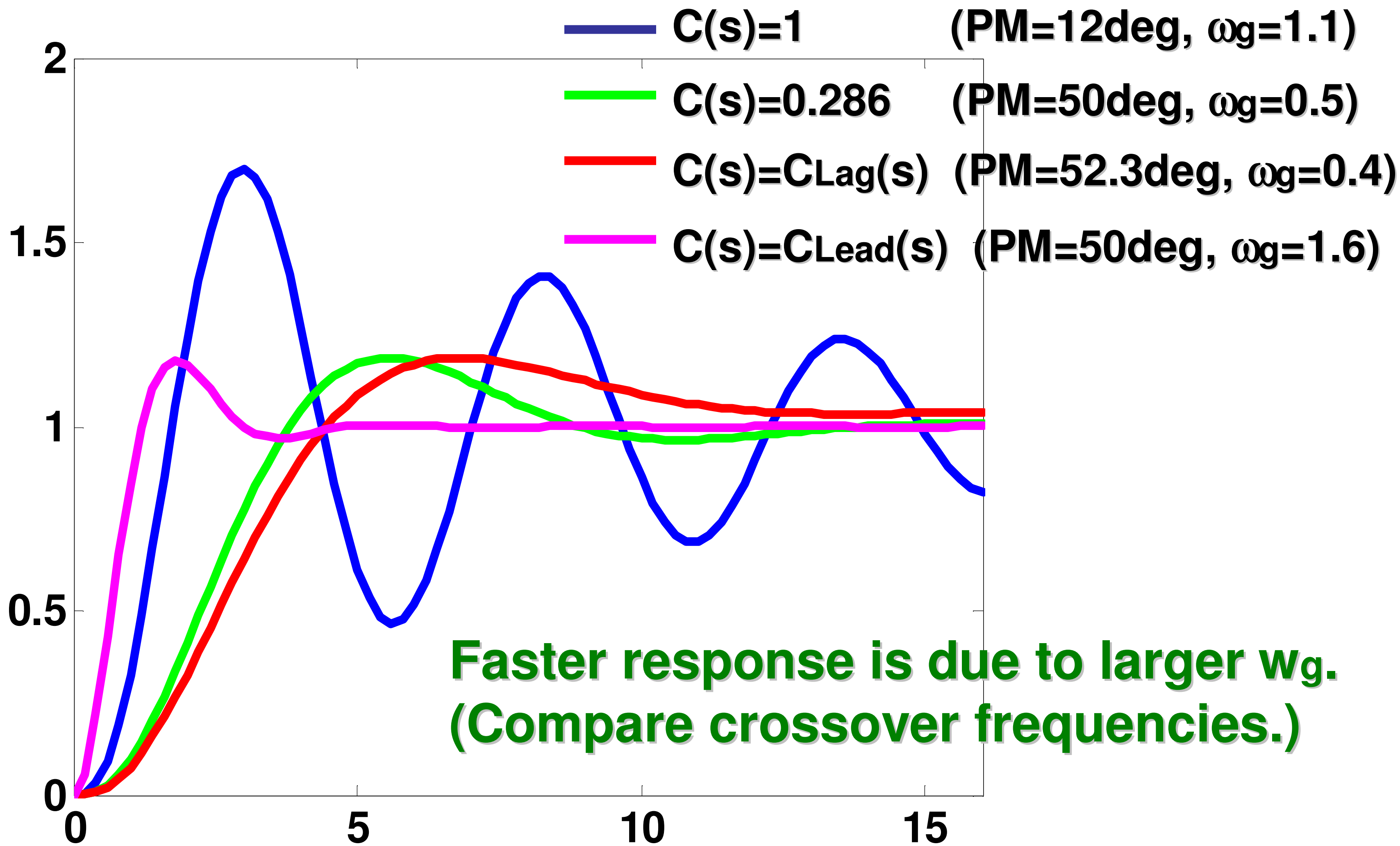
CD – Phase-Lead Design Example

$$C_{Lead}(s) = \frac{s+1}{\frac{s}{50}+1}$$



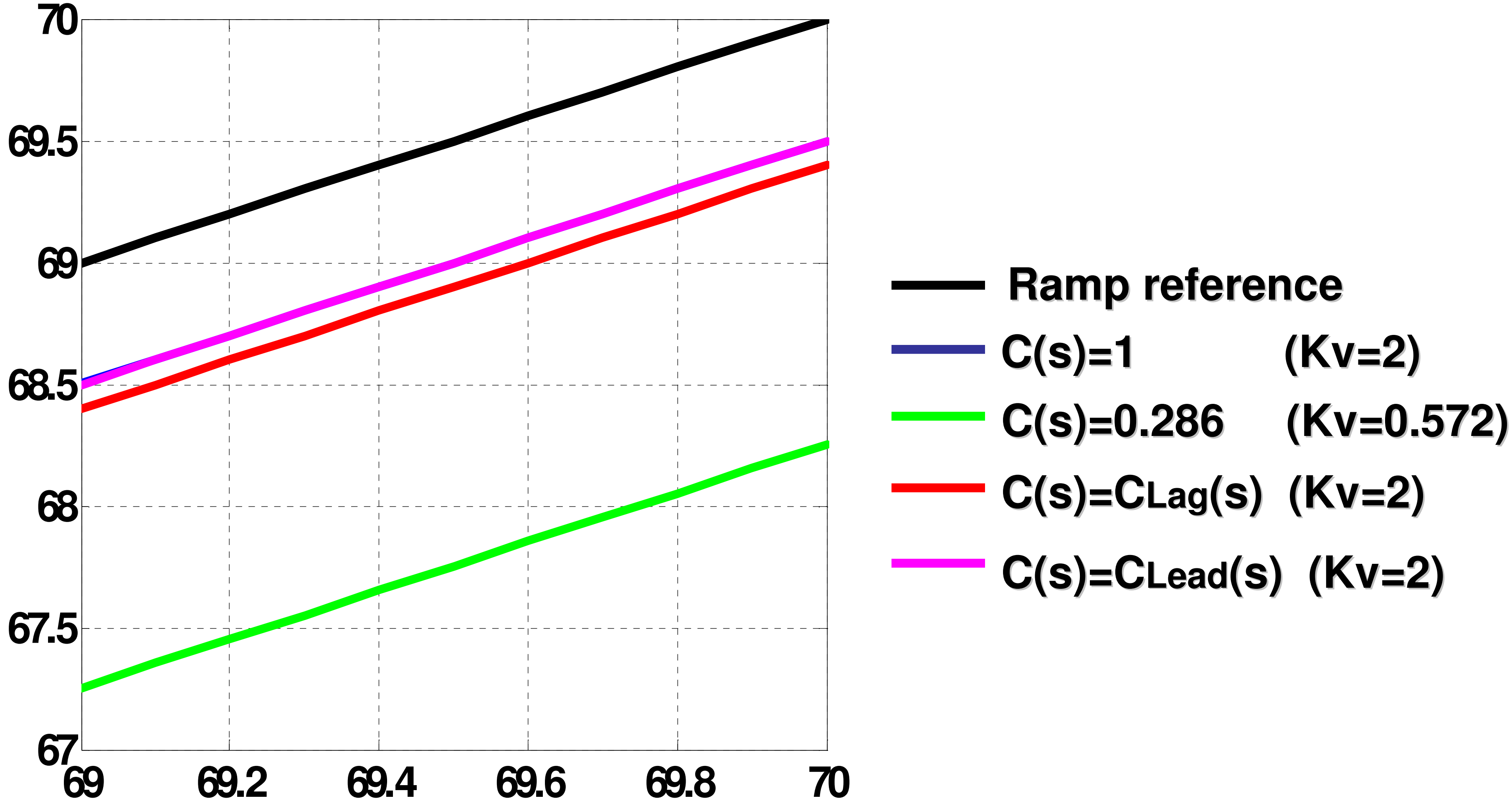
$$\omega_g = 1.57$$

CD – Phase-Lead Design Step Responses

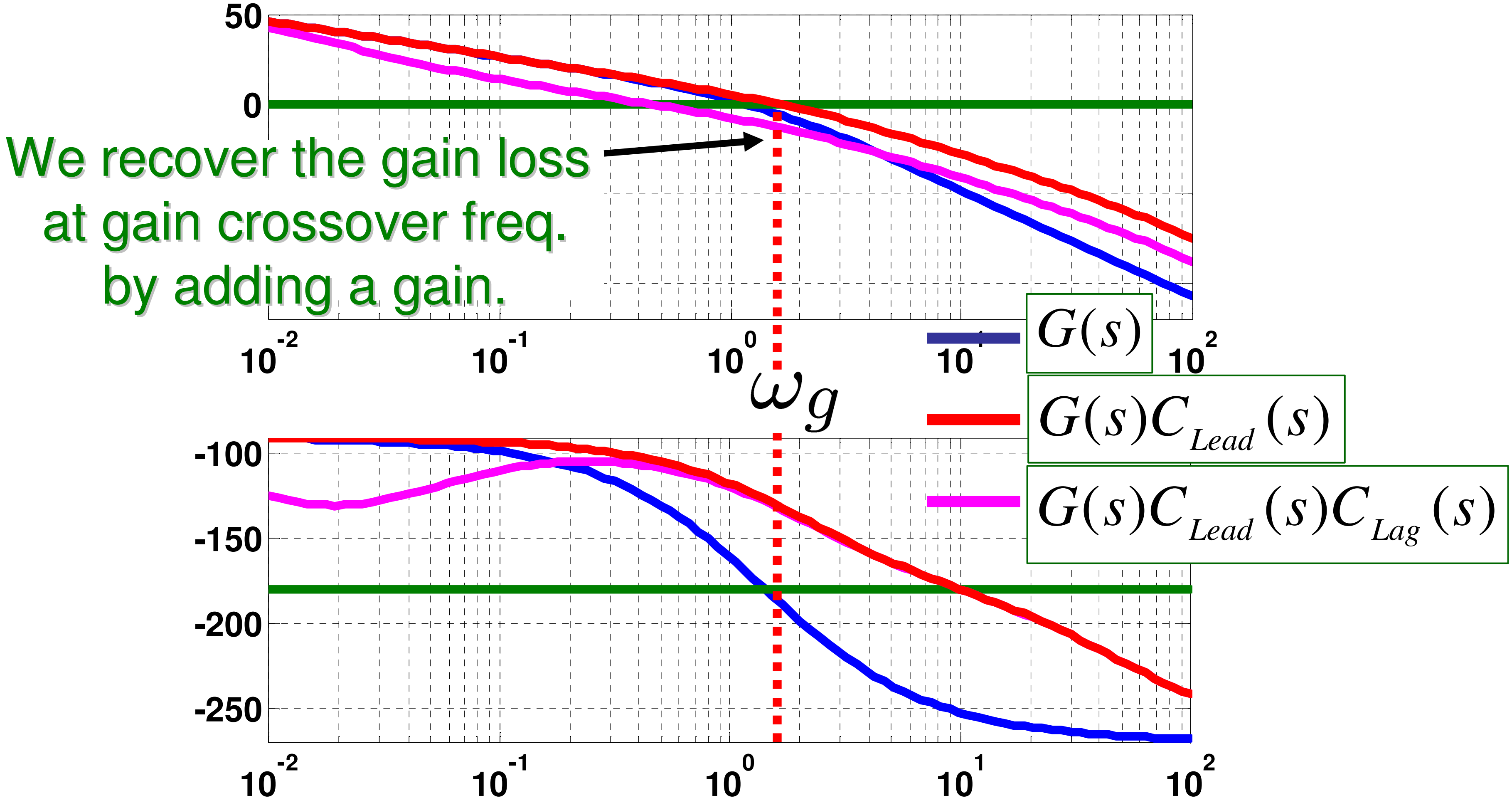


Faster response is due to larger wg.
(Compare crossover frequencies.)

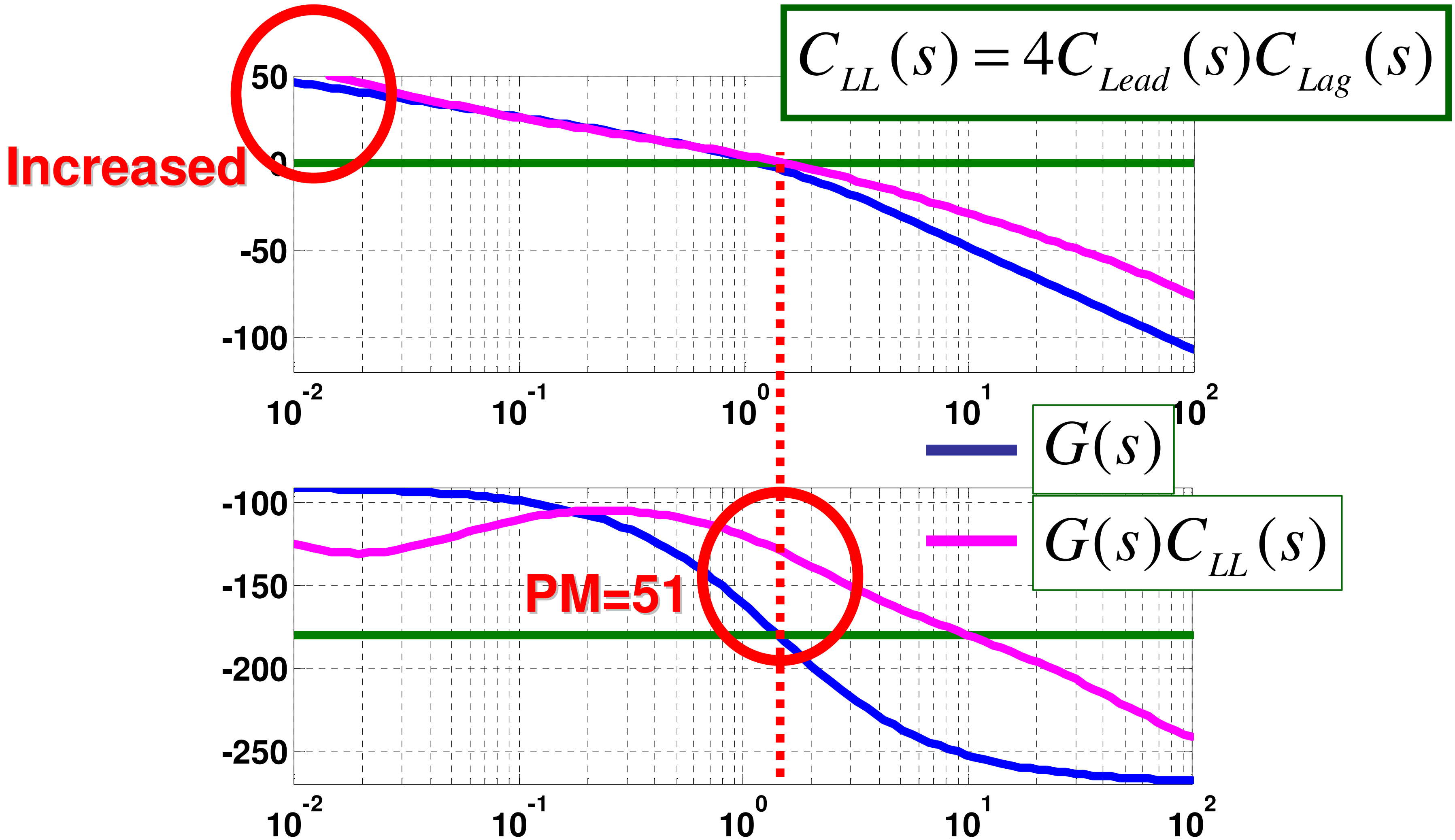
CD – Phase-Lead Design Ramp Responses



CD – Phase Lead-Lag Design Example (1)

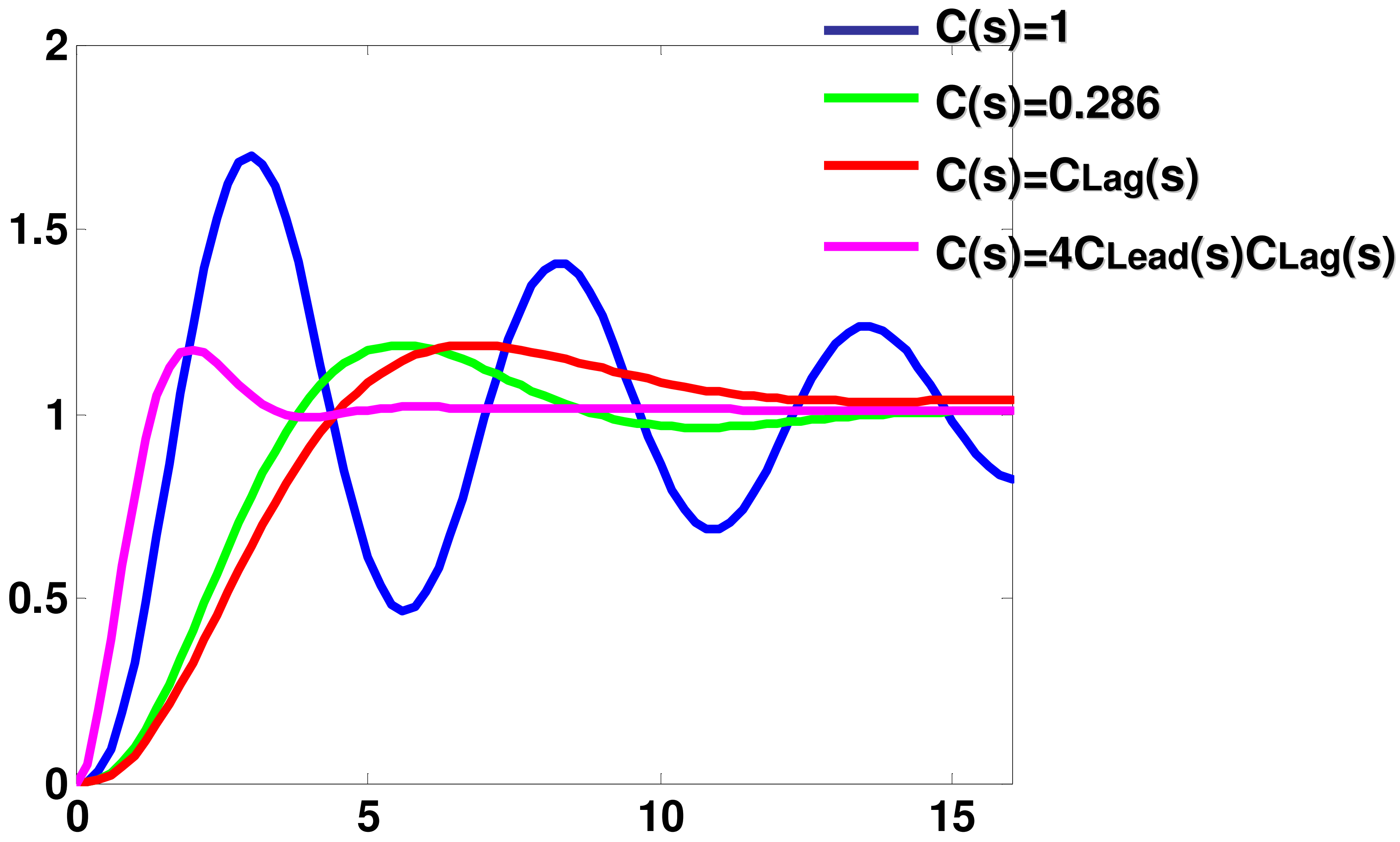


CD – Phase Lead-Lag Design Example (2)

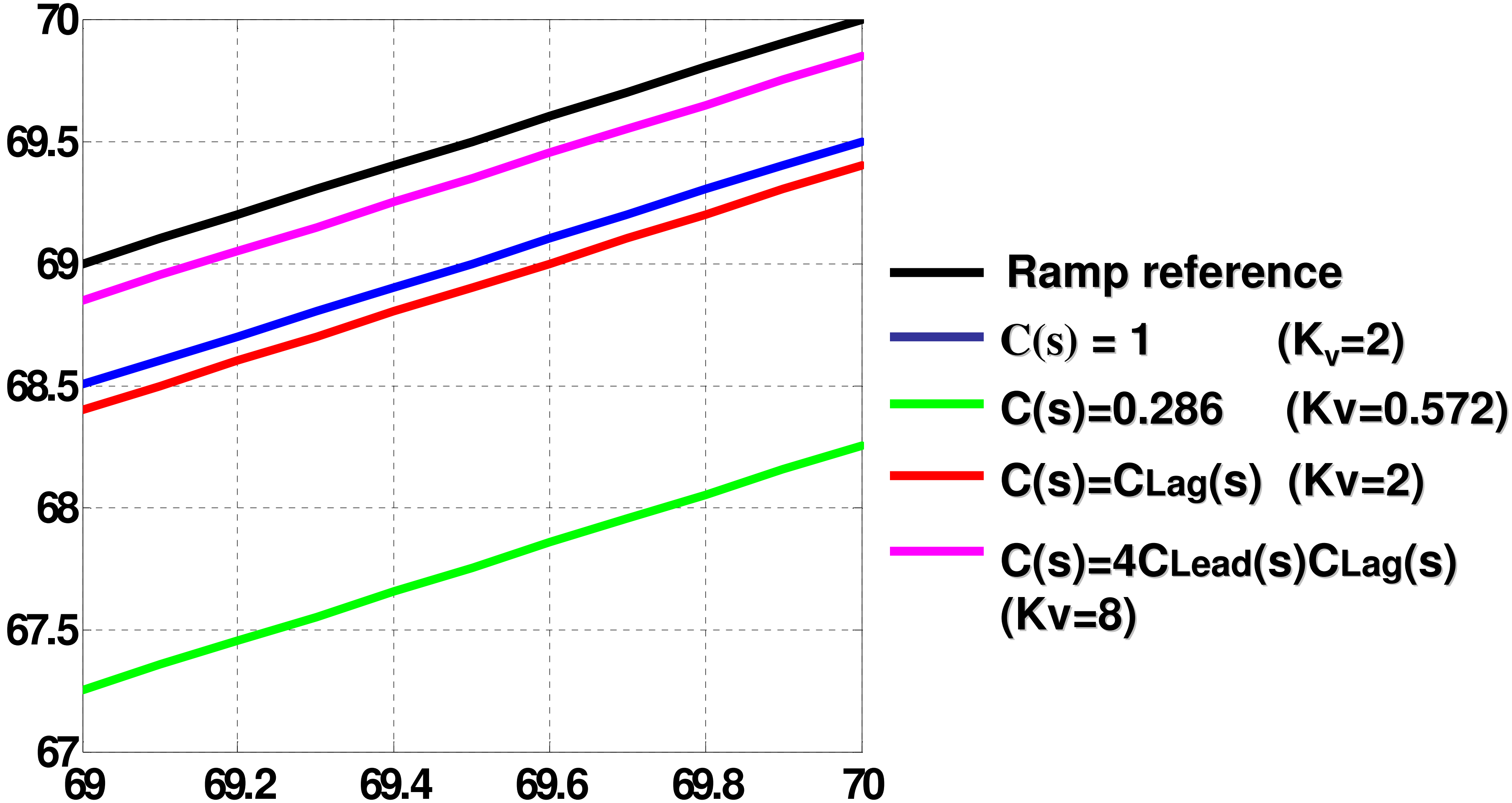


$\omega_g = 1.43$

CD – Phase Lead-Lag Design Step Responses



CD – Phase Lead-Lag Ramp Responses



CD – Lead-Lag Compensator Summary

- Lead compensator can be used for improving
    - Gain crossover frequency
    - Phase margin

  by maintaining low frequency gain,

- Lead-lag compensator can improve
    - Transient (ω_g for speed, PM for overshoot)
    - Steady state (low frequency gain for error constant)

- Next, case studies
    - Antenna azimuth position control
    - Hard disk drive control