TOPIC 3

Coordinate geometry

3.1 Overview

Why learn this?

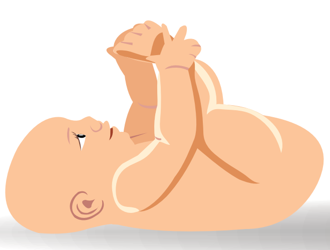
What did you weigh as a baby, and how tall were you? Did you grow at a steady (linear) rate, or were there periods in your life when you grew rapidly? What is the relationship between your height and your weight? We constantly seek to find relationships between variables, and coordinate geometry provides a picture, a visual clue as to what the relationships may be.

What do you know?

assess on

1 **THINK** List what you know about linear graphs and their equations. Use a thinking tool such as a concept map to show your list.
2 **PAIR** Share what you know with a partner and then with a small group.
3 **SHARE** As a class, create a large concept map that shows your class's knowledge of linear graphs and their equations.

Learning sequence



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The story of mathematics: Descartes

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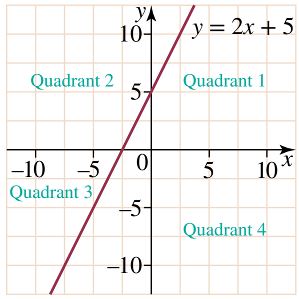
3.2 Sketching linear graphs



- If a series of points (x, y) is plotted using the rule $y = mx + c$, then the points always lie in a straight line whose gradient equals m and whose y-intercept equals c.
- The rule $y = mx + c$ is called the equation of a straight line written in 'gradient–intercept' form.

Plotting linear graphs

- To plot a **linear graph**, complete a table of values to determine the points.

WORKED EXAMPLE 1 TI CASIO

Plot the linear graph defined by the rule $y = 2x - 5$ for the x-values $-3, -2, -1, 0, 1, 2$ and 3.

THINK / **WRITE/DRAW**

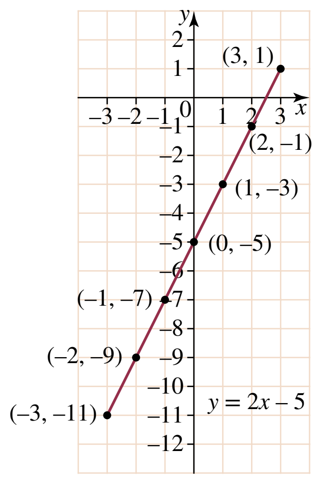
1 Create a table of values using the given x-values.

x	–3	–2	–1	0	1	2	3
y							

2 Find the corresponding y-values by substituting each x-value into the rule.

x	–3	–2	–1	0	1	2	3
y	–11	–9	–7	–5	–3	–1	1

3 Plot the points on a Cartesian plane and rule a straight line through them. Since the x-values have been specified, the line should only be drawn between the x-values of -3 and 3.



4 Label the graph.

Sketching straight lines

- A minimum of two points are necessary to plot a straight line.
- Two methods can be used to plot a straight line:
  – Method 1: The x- and y-intercept method.
  – Method 2: The gradient–intercept method.

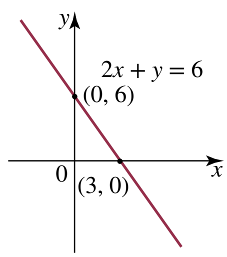
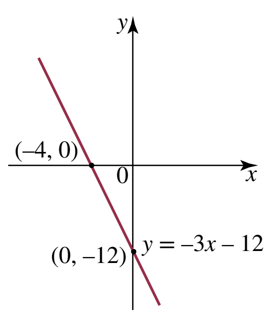
Method 1: Sketching a straight line using the x- and y-intercepts

- As the name implies, this method involves plotting the x- and y-intercepts, then joining them to sketch the straight line.
- The line cuts the y-axis where $x = 0$ and the x-axis where $y = 0$.

WORKED EXAMPLE 2

Sketch graphs of the following linear equations by finding the x- and y-intercepts.

a $2x + y = 6$ **b** $y = -3x - 12$

THINK	WRITE/DRAW
a 1 Write the equation.	**a** $2x + y = 6$
2 Find the x-intercept by substituting $y = 0$.	x-intercept: when $y = 0$, $2x + 0 = 6$ $2x = 6$ $x = 3$ x-intercept is $(3, 0)$.
3 Find the y-intercept by substituting $x = 0$.	y-intercept: when $x = 0$, $2(0) + y = 6$ $y = 6$ y-intercept is $(0, 6)$.
4 Plot both points and rule the line.	
5 Label the graph.	
b 1 Write the equation.	**b** $y = -3x - 12$
2 Find the x-intercept by substituting $y = 0$. i Add 12 to both sides of the equation. ii Divide both sides of the equation by -3.	x-intercept: when $y = 0$, $-3x - 12 = 0$ $-3x = 12$ $x = -4$ x-intercept is $(-4, 0)$.
3 Find the y-intercept. The equation is in the form $y = mx + c$, so compare this with our equation to find the y-intercept, c.	$c = -12$ y-intercept is $(0, -12)$.
4 Plot both points and rule the line.	
5 Label the graph.	

Method 2: Sketching a straight line using the gradient–intercept method

- This method is often used if the equation is in the form $y = mx + c$, where m represents the gradient (slope) of the straight line, and c represents the y-intercept.
- The steps below outline how to use the gradient–intercept method to sketch a linear graph.

  Step 1: Plot a point at the y-intercept.

  Step 2: Write the gradient in the form $m = \frac{\text{rise}}{\text{run}}$. (To write a whole number as a fraction, place it over a denominator of 1.)

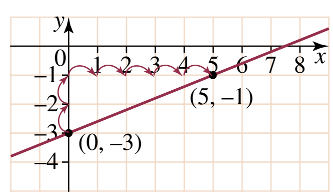
  Step 3: Starting from the y-intercept, move up the number of units suggested by the rise (move down if the gradient is negative).

  Step 4: Move to the right the number of units suggested by the run and plot the second point.

  Step 5: Rule a straight line through the two points.

WORKED EXAMPLE 3 TI CASIO

Sketch the graph of $y = \frac{2}{5}x - 3$ using the gradient–intercept method.

THINK	WRITE/DRAW
1 Write the equation of the line.	$y = \frac{2}{5}x - 3$
2 Identify the value of c (that is, the y-intercept) and plot this point.	$c = -3$, so y-intercept: $(0, -3)$.
3 Write the gradient, m, as a fraction.	$m = \frac{2}{5}$
4 $m = \frac{\text{rise}}{\text{run}}$, note the rise and run.	So, rise = 2; run = 5.
5 Starting from the y-intercept at $(0, -3)$, move 2 units up and 5 units to the right to find the second point $(5, -1)$. We have still not found the x-intercept.	

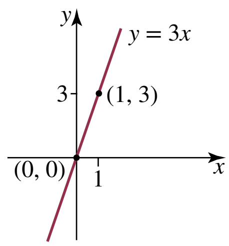
Sketching linear graphs of the form $y = mx$

- Graphs given by $y = mx$ pass through the origin (0, 0), since $c = 0$.
- A second point may be determined using the rule $y = mx$ by substituting a value for x to find y.

WORKED EXAMPLE 4

Sketch the graph of $y = 3x$.

THINK	WRITE/DRAW
1 Write the equation.	$y = 3x$
2 Find the x- and y-intercepts. *Note:* By recognising the form of this linear equation, $y = mx$ you can simply state that the graph passes through the origin, (0, 0).	x-intercept: when $y = 0$, $0 = 3x$ $x = 0$ y-intercept: (0, 0) Both the x- and y-intercepts are at (0, 0).

3 Find another point to plot by finding the y-value when $x = 1$.	When $x = 1$, $y = 3 \times 1$ $= 3$ Another point on the line is (1, 3).
4 Plot the two points (0, 0) and (1, 3) and rule a straight line through them.	
5 Label the graph.	

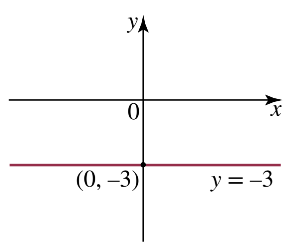
Sketching linear graphs of the form $y = c$ and $x = a$

- The line $y = c$ is parallel to the x-axis, having a gradient of zero and a y-intercept of c.
- The line $x = a$ is parallel to the y-axis and has an undefined (infinite) gradient.

WORKED EXAMPLE 5

Sketch graphs of the following linear equations.

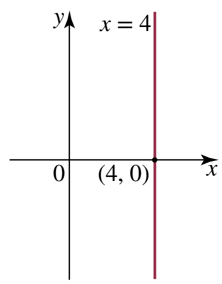
a $y = -3$ **b** $x = 4$

THINK	WRITE/DRAW
a **1** Write the equation.	**a** $y = -3$
2 The y-intercept is -3. As x does not appear in the equation, the line is parallel to the x-axis, such that all points on the line have a y-coordinate equal to -3. That is, this line is the set of points $(x, -3)$ where x is an element of the set of real numbers.	y-intercept $= -3$, $(0, -3)$
3 Sketch a horizontal line through $(0, -3)$.	
4 Label the graph.	

b 1 Write the equation. — **b** $x = 4$

2 The x-intercept is 4. As y does not appear in the equation, the line is parallel to the y-axis, such that all points on the line have an x-coordinate equal to 4. That is, this line is the set of points $(4, y)$ where y is an element of the set of real numbers. — x-intercept = 4, (4, 0)

3 Sketch a vertical line through (4, 0).



4 Label the graph.

assess on

Exercise 3.2 Sketching linear graphs

INDIVIDUAL PATHWAYS

REFLECTION
What types of straight lines have an x-and y-intercept of the same value?

PRACTICE	CONSOLIDATE	MASTER
Questions: 1, 2, 3a–h, 4a–e, 5a–d, 6a–f, 7a–d, 8a–d, 9, 10, 12	Questions: 1, 2, 3f–m, 4a–e, 5a–d, 6a–f, 7c–f, 8a–f, 9–12	Questions: 1, 2, 3h–o, 4d–i, 5c–f, 6e–i, 7d–h, 8c–h, 9–13

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FLUENCY

1 **WE1** Generate a table of values and then plot the linear graphs defined by the following rules for the given range of x-values.

	Rule	x-values
a	$y = 10x + 25$	−5, −4, −3, −2, −1, 0, 1
b	$y = 5x - 12$	−1, 0, 1, 2, 3, 4
c	$y = -0.5x + 10$	−6, −4, −2, 0, 2, 4
d	$y = 100x - 240$	0, 1, 2, 3, 4, 5
e	$y = -5x + 3$	−3, −2, −1, 0, 1, 2
f	$y = 7 - 4x$	−3, −2, −1, 0, 1, 2

2 Plot the linear graphs defined by the following rules for the given range of x-values.

Rule — **x-values**

a $y = -3x + 2$

x	-6	-4	-2	0	2	4	6
y							

b $y = -x + 3$

x	-3	-2	-1	0	1	2	3
y							

c $y = -2x + 3$

x	-6	-4	-2	0	2	4	6
y							

3 **WE2** Sketch graphs of the following linear equations by finding the x- and y-intercepts.

a $5x - 3y = 10$ **b** $5x + 3y = 10$ **c** $-5x + 3y = 10$
d $-5x - 3y = 10$ **e** $2x - 8y = 20$ **f** $4x + 4y = 40$
g $-x + 6y = 120$ **h** $-2x + 8y = -20$ **i** $10x + 30y = -150$
j $5x + 30y = -150$ **k** $-9x + 4y = 36$ **l** $6x - 4y = -24$
m $y = 2x - 10$ **n** $y = -5x + 20$ **o** $y = -\frac{1}{2}x - 4$

4 **WE3** Sketch graphs of the following linear equations using the gradient–intercept method.

a $y = 4x + 1$ **b** $y = 3x - 7$ **c** $y = -2x + 3$
d $y = -5x - 4$ **e** $y = \frac{1}{2}x - 2$ **f** $y = -\frac{2}{7}x + 3$
g $y = 0.6x + 0.5$ **h** $y = 8x$ **i** $y = x - 7$

5 **WE4** Sketch the graphs of the following linear equations.

a $y = 2x$ **b** $y = 5x$ **c** $y = -3x$
d $y = \frac{1}{2}x$ **e** $y = \frac{2}{3}x$ **f** $y = -\frac{5}{2}x$

6 **WE5** Sketch the graphs of the following linear equations.

a $y = 10$ **b** $y = -10$ **c** $x = 10$
d $x = -10$ **e** $y = 100$ **f** $y = 0$
g $x = 0$ **h** $x = -100$ **i** $y = -12$

7 Transpose each of the equations to standard form (that is, $y = mx + c$). State the x- and y-intercept for each.

a $5(y + 2) = 4(x + 3)$ **b** $5(y - 2) = 4(x - 3)$
c $2(y + 3) = 3(x + 2)$ **d** $10(y - 20) = 40(x - 2)$
e $4(y + 2) = -4(x + 2)$ **f** $2(y - 2) = -(x + 5)$
g $-5(y + 1) = 4(x - 4)$ **h** $8(y - 5) = -4(x + 3)$
i $5(y + 2.5) = 2(x - 3.5)$ **j** $2.5(y - 2) = -6.5(x - 1)$

UNDERSTANDING

8 Find the x- and y-intercepts of the following lines.

a $-y = 8 - 4x$ **b** $6x - y + 3 = 0$ **c** $2y - 10x = 50$

9 Explain why the gradient of a horizontal line is equal to zero and the gradient of a vertical line is undefined.

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Digital docs
SkillSHEET Describing the gradient of a line **doc-5197**
SkillSHEET Plotting a line using a table of values **doc-5198**
SkillSHEET Stating the y-intercept from a graph **doc-5199**
SkillSHEET Solving linear equations that arise when finding x- and y-intercepts **doc-5200**
SkillSHEET Using Pythagoras' theorem **doc-5201**
SkillSHEET Substitution into a linear rule **doc-5202**
SkillSHEET Transposing linear equations to standard form **doc-5203**

REASONING

10 Determine whether $\frac{x}{3} - \frac{y}{2} = \frac{7}{6}$ is the equation of a straight line by rearranging into an appropriate form and hence sketch the graph, showing all relevant features.

11 Your friend loves to download music. She earns \$50 and spends some of it buying music online at \$1.75 per song. She saves the remainder. Her saving is given by the function $f(x) = 50 - 1.75x$.

a What does x represent?

b Sketch the function.

c How many songs can she buy and still save \$25?

PROBLEM SOLVING

12 A straight line has a general equation defined by $y = mx + c$. This line intersects the lines defined by the rules $y = 7$ and $x = 3$. The lines $y = mx + c$ and $y = 7$ have the same y-intercept while $y = mx + c$ and $x = 3$ have the same x-intercept.

a On the one set of axes, sketch all three graphs.

b Determine the y-axis intercept for $y = mx + c$.

c Determine the gradient for $y = mx + c$.

d MC The equation of the line defined by $y = mx + c$ is:

A $x + y = 3$ **B** $7x + 3y = 21$ **C** $3x + 7y = 21$

D $x + y = 7$ **E** $7x + 3y = 7$

13 Water is flowing from a tank at a constant rate. The equation relating the volume of water in the tank, V litres, to the time the water has been flowing from the tank, t minutes, is given by $V = 80 - 4t$, $t \geq 0$.

a How much water is in the tank initially?

b Why is it important that $t \geq 0$?

c At what rate is the water flowing from the tank?

d How long does it take for the tank to empty?

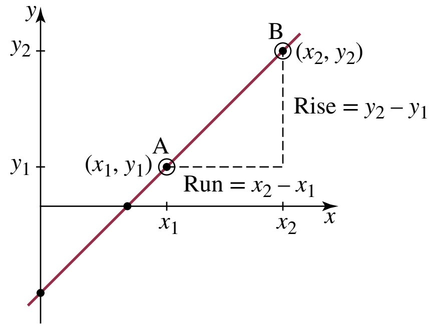
e Sketch the graph of V versus t.

3.3 Determining linear equations

Finding a linear equation given two points

- The gradient of a straight line can be calculated from the coordinates of two points (x_1, y_1) and (x_2, y_2) that lie on the line.

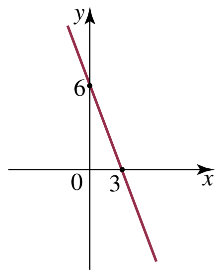
$$\text{Gradient} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



- The equation of the straight line can then be found in the form $y = mx + c$, where c is the y-intercept.

WORKED EXAMPLE 6

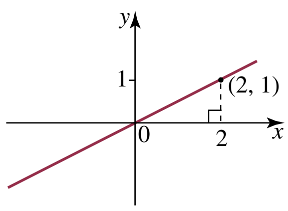
Find the equation of the straight line shown in the graph.



THINK	WRITE
1 There are two points given on the straight line: the x-intercept $(3, 0)$ and the y-intercept $(0, 6)$.	$(3, 0), (0, 6)$
2 Find the gradient of the line by applying the formula $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$, where $(x_1, y_1) = (3, 0)$ and $(x_2, y_2) = (0, 6)$.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{6 - 0}{0 - 3}$ $= \frac{6}{-3}$ $= -2$ The gradient $m = -2$.
3 The graph has a y-intercept of 6, so $c = 6$. Substitute $m = -2$, and $c = 6$ into $y = mx + c$ to find the equation.	$y = mx + c$ $y = -2x + 6$

WORKED EXAMPLE 7

Find the equation of the straight line shown in the graph.



THINK	WRITE
1 There are two points given on the straight line: the x- and y-intercept $(0, 0)$ and another point $(2, 1)$.	$(0, 0), (2, 1)$

THINK	WRITE
2 Find the gradient of the line by applying the formula $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$, where $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (2, 1)$.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{1 - 0}{2 - 0}$ $= \frac{1}{2}$ The gradient $m = \frac{1}{2}$.
3 The y-intercept is 0, so $c = 0$. Substitute $m = \frac{1}{2}$ and $c = 0$ into $y = mx + c$ to determine the equation.	$y = mx + c$ $y = \frac{1}{2}x + 0$ $y = \frac{1}{2}x$

WORKED EXAMPLE 8 TI CASIO

Find the equation of the straight line passing through (−2, 5) and (1, −1).

THINK	WRITE
1 Write the general equation of a straight line.	$y = mx + c$
2 Write the formula for calculating the gradient of a line between two points.	$m = \frac{y_2 - y_1}{x_2 - x_1}$
3 Let (x_1, y_1) and (x_2, y_2) be the two points $(-2, 5)$ and $(1, -1)$ respectively. Substitute the values of the pronumerals into the formula to calculate the gradient.	$m = \frac{-1 - 5}{1 - -2}$ $= \frac{-6}{3}$ $= -2$
4 Substitute the value of the gradient into the general rule.	$y = -2x + c$
5 Select either of the two points, say $(1, -1)$, and substitute its coordinates into $y = -2x + c$.	Point $(1, -1)$: $-1 = -2 \times 1 + c$
6 Solve for c; that is, add 2 to both sides of the equation.	$-1 = -2 + c$ $1 = c$
7 State the equation by substituting the value of c into $y = -2x + c$.	The equation of the line is $y = -2x + 1$.

Finding the equation of a straight line using the gradient and one point

- If the gradient of a line is known, only one point is needed to determine the equation of the line.

WORKED EXAMPLE 9

Find the equation of the straight line with gradient of 2 and y-intercept of −5.

THINK	WRITE
1 Write the known information. The other point is the y-intercept, which makes the calculation of c straightforward.	Gradient = 2, y-intercept = −5
2 State the values of m and c.	$m = 2, c = -5$
3 Substitute these values into $y = mx + c$ to find the equation.	$y = mx + c$ $y = 2x - 5$

WORKED EXAMPLE 10 TI CASIO

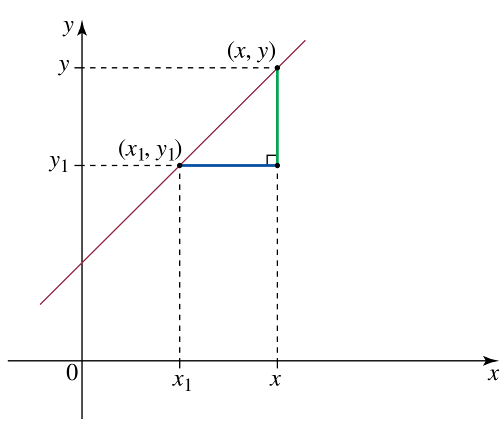
Find the equation of the straight line passing through the point (5, −1) with a gradient of 3.

THINK	WRITE
1 Write the known information.	Gradient = 3, point (5, −1).
2 State the values of m, x and y.	$m = 3,\ (x, y) = (5, -1)$
3 Substitute the values $m = 3, x = 5$ and $y = -1$ into $y = mx + c$ and solve to find c.	$y = mx + c$ $-1 = 3(5) + c$ $-1 = 15 + c$ $-16 = c$
4 Substitute $m = 3$ and $c = -16$ into $y = mx + c$ to determine the equation.	The equation of the line is $y = 3x - 16$.

A simple formula

- The diagram shows a line of gradient m passing through the point (x_1, y_1).
- If (x, y) is any other point on the line, then:

$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{y - y_1}{x - x_1}$$
$$m(x - x_1) = y - y_1$$
$$y - y_1 = m(x - x_1)$$



- The formula $y - y_1 = m(x - x_1)$ can be used to write down the equation of a line, given the gradient and the coordinates of one point.

WORKED EXAMPLE 11

Find the equation of the line with a gradient of −2 which passes through the point (3, −4). Write the equation in general form, that is in the form $ax + by + c = 0$.

THINK	WRITE
1 Use the formula $y - y_1 = m(x - x_1)$. Write the values of x_1, y_1, and m.	$m = -2, x_1 = 3, y_1 = -4$ $y - y_1 = m(x - x_1)$
2 Substitute for x_1, y_1, and m into the equation.	$y - (-4) = -2(x - 3)$ $y + 4 = -2x + 6$
3 Transpose the equation into the form $ax + by + c = 0$.	$y + 4 + 2x - 6 = 0$ $2x + y - 2 = 0$

assess on

Exercise 3.3 Determining linear equations

REFLECTION
What problems might you encounter when calculating the equation of a line whose graph is actually parallel to one of the axes?

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–d, 2, 3, 4, 5a–d, 7	Questions: 1a–f, 2, 3, 4, 5c–g, 7, 9	Questions: 1d–h, 2, 3, 4, 5e–j, 6–10

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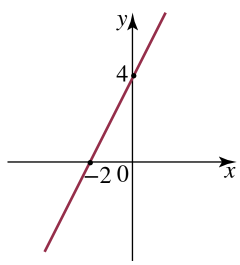
Digital docs
SkillSHEET
Measuring the rise and the run
doc-5196
SkillSHEET
Finding the gradient given two points
doc-5204

FLUENCY

1 WE6 Determine the equation for each of the straight lines shown.

a


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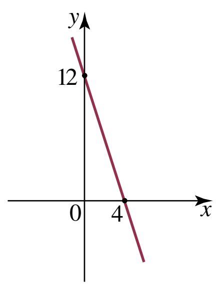
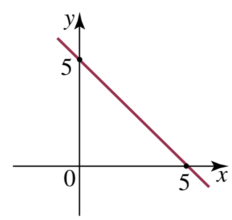
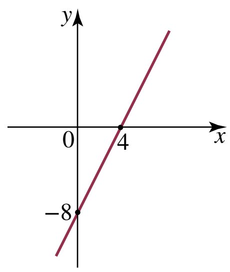
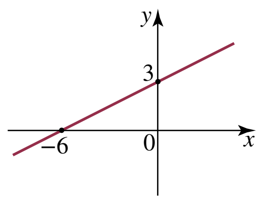
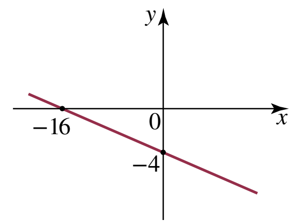


c


d


e


f


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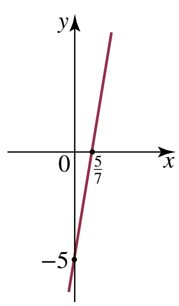

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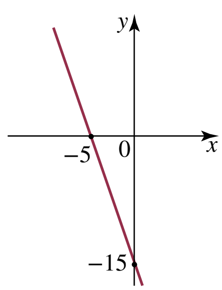
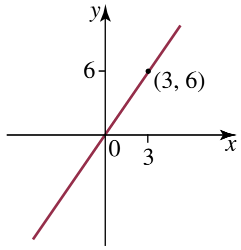


2 WE7 Determine the equation of each of the straight lines shown.

a


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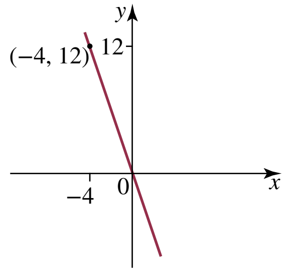

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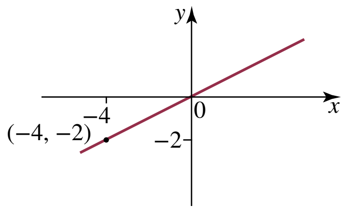

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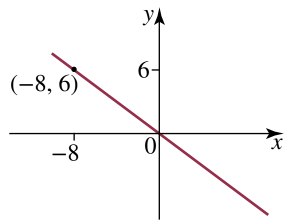


3 WE8 Find the equation of the straight line that passes through each pair of points.

a $(1, 4)$ and $(3, 6)$
b $(0, -1)$ and $(3, 5)$
c $(-1, 4)$ and $(3, 2)$
d $(3, 2)$ and $(-1, 0)$
e $(-4, 6)$ and $(2, -6)$
f $(-3, -5)$ and $(-1, -7)$

4 WE9 Find the linear equation given the information in each case below.

a Gradient $= 3$, y-intercept $= 3$
b Gradient $= -3$, y-intercept $= 4$
c Gradient $= -4$, y-intercept $= 2$
d Gradient $= 4$, y-intercept $= 2$
e Gradient $= -1$, y-intercept $= -4$
f Gradient $= 0.5$, y-intercept $= -4$
g Gradient $= 5$, y-intercept $= 2.5$
h Gradient $= -6$, y-intercept $= 3$
i Gradient $= -2.5$, y-intercept $= 1.5$
j Gradient $= 3.5$, y-intercept $= 6.5$

5 WE10, 11 For each of the following, find the equation of the straight line with the given gradient and passing through the given point.

a Gradient $= 5$, point $= (5, 6)$
b Gradient $= -5$, point $= (5, 6)$
c Gradient $= -4$, point $= (-2, 7)$
d Gradient $= 4$, point $= (8, -2)$
e Gradient $= 3$, point $= (10, -5)$
f Gradient $= -3$, point $= (3, -3)$
g Gradient $= -2$, point $= (20, -10)$
h Gradient $= 2$, point $= (2, -0.5)$
i Gradient $= 0.5$, point $= (6, -16)$
j Gradient $= -0.5$, point $= (5, 3)$

UNDERSTANDING

6 a If t represents the time in hours and C represents cost ($), construct a table of values for 0–3 hours for the cost of playing ten-pin bowling at the new alley.

> Save $$$ with Supa-Bowl!!!
> NEW Ten-Pin Bowling Alley
> *Shoe rental just $2 (fixed fee)*
> Rent a lane for ONLY $6/hour!

b Use your table of values to plot a graph of time versus cost. (*Hint:* Ensure your time axis (horizontal axis) extends to 6 hours and your cost axis (vertical axis) extends to $40.)

c i What is the y-intercept?

ii What does the y-intercept represent in terms of the cost?

d Calculate the gradient.

e Write a linear equation to describe the relationship between cost and time.

f Use your linear equation from part **e** to calculate the cost of a 5-hour tournament.

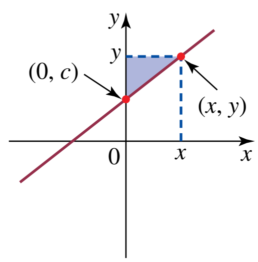
g Use your graph to check your answer to part **f**.

REASONING

7 When using the gradient to draw a line, does it matter if you rise before you run or run before you rise? Explain your answer.

8 a Using the graph below, write a general formula for the gradient m in terms of x, y and c.

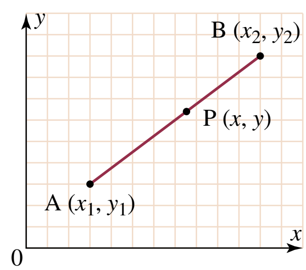
b Transpose your formula to make y the subject. What do you notice?



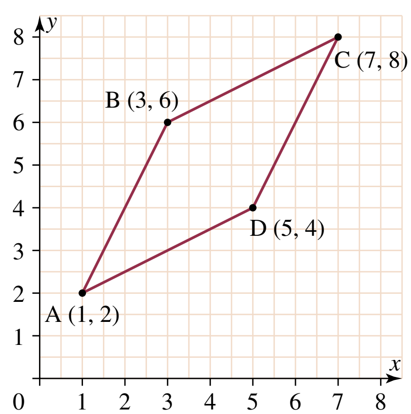
PROBLEM SOLVING

9 The points A (x_1, y_1), B (x_2, y_2) and P (x, y) are co-linear. P is a general point that lies anywhere on the line.
Show that an equation relating these three points is given by

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$



10 Show that the quadrilateral ABCD is a parallelogram.



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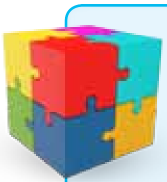
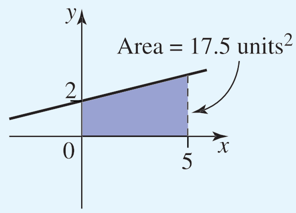


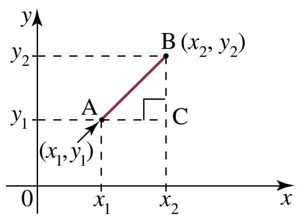
CHALLENGE 3.1

The graph of the straight line crosses the *y*-axis at (0, 2). The shaded section represents an area of 17.5 square units. Use this information to determine the equation of the line.



3.4 The distance between two points

- The distance between two points can be calculated using Pythagoras' theorem.
- Consider two points A (x_1, y_1) and B (x_2, y_2) on the Cartesian plane as shown below.



- If point C is placed as shown, ABC is a right-angled triangle and AB is the hypotenuse.

$$AC = x_2 - x_1$$
$$BC = y_2 - y_1$$

By Pythagoras' theorem:

$$AB^2 = AC^2 + BC^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Hence $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

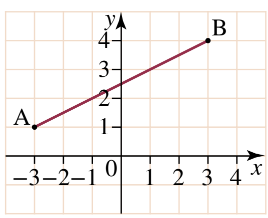
The distance between two points A (x_1, y_1) and B (x_2, y_2) is:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- This distance formula can be used to calculate the distance between any two points on the Cartesian plane.
- The distance formula has many geometric applications.

WORKED EXAMPLE 12

Find the distance between the points A and B in the figure below. Answer correct to two decimal places.



THINK	WRITE
1 From the graph, locate points A and B.	A(−3, 1) and B(3, 4)
2 Let A have coordinates (x_1, y_1).	Let $(x_1, y_1) = (-3, 1)$

THINK	WRITE
3 Let B have coordinates (x_2, y_2).	Let $(x_2, y_2) = (3, 4)$
4 Find the length AB by applying the formula for calculating the distance between two points.	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{[3 - (-3)]^2 + (4 - 1)^2}$ $= \sqrt{(6)^2 + (3)^2}$ $= \sqrt{36 + 9}$ $= \sqrt{45}$ $= 3\sqrt{5}$ $= 6.71$ (correct to 2 decimal places)

Note: If the coordinates were named in the reverse order, the formula would still give the same answer. Check this for yourself using $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-3, 1)$.

WORKED EXAMPLE 13 TI CASIO

Find the distance between the points P (−1, 5) and Q (3, −2).

THINK	WRITE
1 Let P have coordinates (x_1, y_1).	Let $(x_1, y_1) = (-1, 5)$
2 Let Q have coordinates (x_2, y_2).	Let $(x_2, y_2) = (3, -2)$
3 Find the length PQ by applying the formula for the distance between two points.	$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{[3 - (-1)]^2 + (-2 - 5)^2}$ $= \sqrt{(4)^2 + (-7)^2}$ $= \sqrt{16 + 49}$ $= \sqrt{65}$ $= 8.06$ (correct to 2 decimal places)

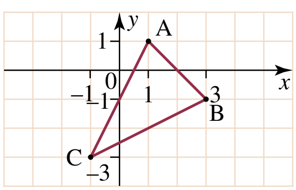
WORKED EXAMPLE 14

Prove that the points A (1, 1), B (3, −1) and C (−1, −3) are the vertices of an isosceles triangle.

THINK

1 Plot the points and draw the triangle.
Note: For triangle ABC to be isosceles, two sides must have the same magnitude.

WRITE/DRAW



2 AC and BC seem to be equal. Find the length AC.
A (1, 1) = (x_2, y_2)
C (−1, −3) = (x_1, y_1)

$AC = \sqrt{[1 - (-1)]^2 + [1 - (-3)]^2}$
$= \sqrt{(2)^2 + (4)^2}$
$= \sqrt{20}$
$= 2\sqrt{5}$

3 Find the length BC. B $(3, -1) = (x_2, y_2)$ C $(-1, -3) = (x_1, y_1)$	$BC = \sqrt{[3-(-1)]^2 + [-1-(-3)]^2}$ $= \sqrt{(4)^2 + (2)^2}$ $= \sqrt{20}$ $= 2\sqrt{5}$
4 Find the length AB. A $(1, 1) = (x_1, y_1)$ B $(3, -1) = (x_2, y_2)$	$AB = \sqrt{[3-(1)]^2 + [-1-(1)]^2}$ $= \sqrt{(2)^2 + (-2)^2}$ $= \sqrt{4+4}$ $= \sqrt{8}$ $= 2\sqrt{2}$
5 State your proof.	Since AC = BC ≠ AB, triangle ABC is an isosceles triangle.

Exercise 3.4 The distance between two points

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1, 2a–d, 5, 8, 9	Questions: 1, 2c–f, 5, 7, 9, 11	Questions: 1, 2e–h, 3–7, 9–12

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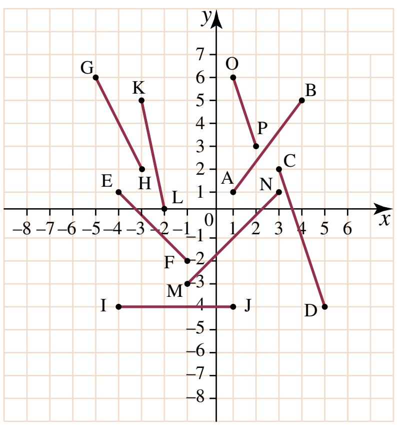
REFLECTION
How could you use the distance formula to show that a series of points lay on the circumference of a circle with centre *C*?

FLUENCY

1 WE12 Find the distance between each pair of points shown at right.



2 WE13 Find the distance between the following pairs of points.

a $(2, 5), (6, 8)$ **b** $(-1, 2), (4, 14)$
c $(-1, 3), (-7, -5)$ **d** $(5, -1), (10, 4)$
e $(4, -5), (1, 1)$ **f** $(-3, 1), (5, 13)$
g $(5, 0), (-8, 0)$ **h** $(1, 7), (1, -6)$
i $(a, b), (2a, -b)$ **j** $(-a, 2b), (2a, -b)$

UNDERSTANDING

3 MC If the distance between the points $(3, b)$ and $(-5, 2)$ is 10 units, then the value of b is:

A -8 **B** -4 **C** 4 **D** 0 **E** 2

4 **MC** A rhombus has vertices A (1, 6), B (6, 6), C (−2, 2) and D (x, y). The coordinates of D are:

A (2, −3) **B** (2, 3) **C** (−2, 3) **D** (3, 2) **E** (3, −2)

5 The vertices of a quadrilateral are A (1, 4), B (−1, 8), C (1, 9) and D (3, 5).

a Find the lengths of the sides.

b Find the lengths of the diagonals.

c What type of quadrilateral is it?

REASONING

6 **WE14** Prove that the points A (0, −3), B (−2, −1) and C (4, 3) are the vertices of an isosceles triangle.

7 The points P (2, −1), Q (−4, −1) and R $(-1, 3\sqrt{3} - 1)$ are joined to form a triangle. Prove that triangle PQR is equilateral.

8 Prove that the triangle with vertices D (5, 6), E (9, 3) and F (5, 3) is a right-angled triangle.

9 A rectangle has vertices A (1, 5), B $(10.6, z)$, C (7.6, −6.2) and D (−2, 1). Find:

a the length of CD

b the length of AD

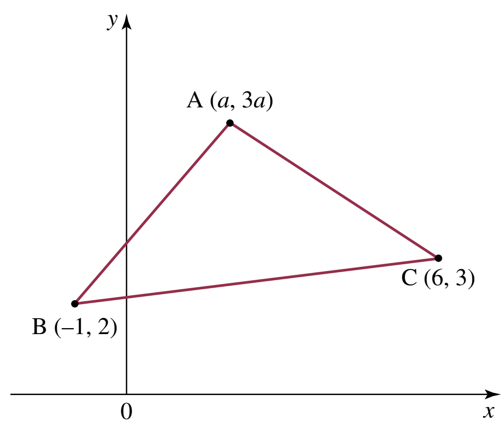
c the length of the diagonal AC

d the value of z.

10 Show that the triangle ABC with coordinates A (a, a), B $(m, -a)$ and C $(-a, m)$ is isosceles.

PROBLEM SOLVING

11 Triangle ABC is an isosceles triangle where AB = AC, B is the point (−1, 2), C is the point (6, 3) and A is the point $(a, 3a)$. Find the value of the integer constant a.

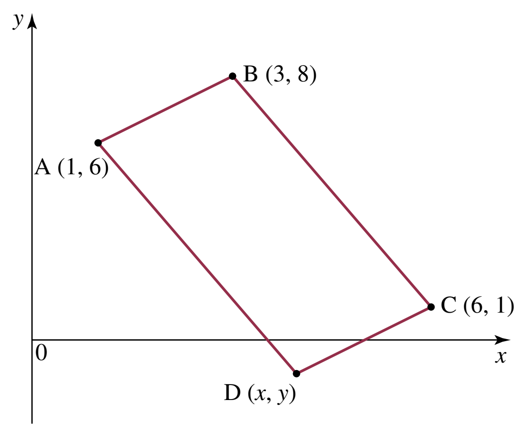


12 ABCD is a parallelogram.

a Find the gradients of AB and BC.

b Find the coordinates of the point D (x, y).

c Show that the diagonals AC and BD bisect each other.



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Distance between two points
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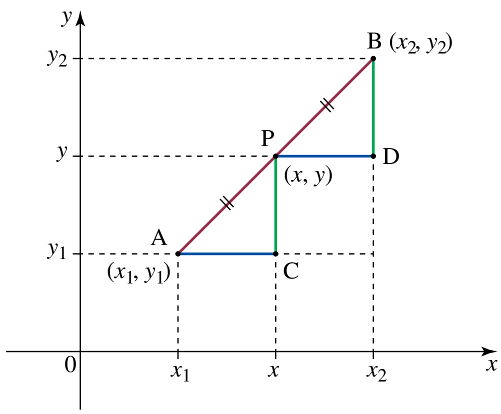
3.5 The midpoint of a line segment

Midpoint of a line segment

- The **midpoint** of a **line segment** is the halfway point.
- The x- and y-coordinates of the midpoint are halfway between those of the coordinates of the end points.
- The following diagram shows the line interval AB joining points A (x_1, y_1) and B (x_2, y_2).

The midpoint of AB is P, so AP = PB.
Points C (x, y_1) and D (x_2, y) are added to the diagram and are used to make the two right-angled triangles ΔAPC and ΔPBD.
The two triangles are congruent:



AP = PB	(given)
∠APC = ∠PBD	(corresponding angles)
∠CAP = ∠DPB	(corresponding angles)
So ΔAPC ≡ ΔPBD	(ASA)

This means that AC = PD;

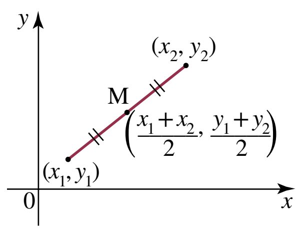
i.e. $x - x_1 = x_2 - x$ (solve for x)
i.e. $2x = x_1 + x_2$

$$x = \frac{x_1 + x_2}{2}$$

In other words x is simply the average of x_1 and x_2.

Similarly, $y = \dfrac{y_1 + y_2}{2}$.

In general, the coordinates of the midpoint of a line segment joining the points (x_1, y_1) and (x_2, y_2) can be found by averaging the x-and y-coordinates of the end points, respectively.



The coordinates of the midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) are: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

WORKED EXAMPLE 15 TI CASIO

Find the coordinates of the midpoint of the line segment joining (−2, 5) and (7, 1).

THINK	WRITE
1 Label the given points (x_1, y_1) and (x_2, y_2).	Let $(x_1, y_1) = (-2, 5)$ and $(x_2, y_2) = (7, 1)$
2 Find the x-coordinate of the midpoint.	$x = \frac{x_1 + x_2}{2}$ $= \frac{-2 + 7}{2}$ $= \frac{5}{2}$ $= 2\frac{1}{2}$
3 Find the y-coordinate of the midpoint.	$y = \frac{y_1 + y_2}{2}$ $= \frac{5 + 1}{2}$ $= \frac{6}{2}$ $= 3$
4 Give the coordinates of the midpoint.	The midpoint is $\left(2\frac{1}{2}, 3\right)$.

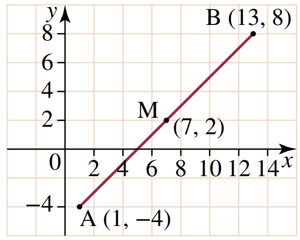
WORKED EXAMPLE 16

The coordinates of the midpoint, M, of the line segment AB are (7, 2). If the coordinates of A are (1, −4), find the coordinates of B.

THINK	WRITE/DRAW
1 Let the start of the line segment be (x_1, y_1) and the midpoint be (x, y).	Let $(x_1, y_1) = (1, -4)$ and $(x, y) = (7, 2)$
2 The average of the x-coordinates is 7. Find the x-coordinate of the end point.	$x = \frac{x_1 + x_2}{2}$ $7 = \frac{1 + x_2}{2}$ $14 = 1 + x_2$ $x_2 = 13$
3 The average of the y-coordinates is 2. Find the y-coordinate of the end point.	$y = \frac{y_1 + y_2}{2}$ $2 = \frac{-4 + y_2}{2}$ $4 = -4 + y_2$ $y_2 = 8$

4 Give the coordinates of the end point. The coordinates of the point B are (13, 8).

5 Check that the coordinates are feasible by drawing a diagram.



Exercise 3.5 The midpoint of a line segment

assess on

INDIVIDUAL PATHWAYS

PRACTISE	CONSOLIDATE	MASTER
Questions: 1a–d, 2, 3a, 4, 9, 11	Questions: 1a–d, 2–6, 9, 11	Questions: 1a–f, 2–12

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REFLECTION
If the midpoint of a line segment is the origin, what are the possible values of the *x*- and *y*-coordinates of the end points?

FLUENCY

1 **WE15** Use the formula method to find the coordinates of the midpoint of the line segment joining the following pairs of points.

a $(-5, 1), (-1, -8)$
b $(4, 2), (11, -2)$
c $(0, 4), (-2, -2)$
d $(3, 4), (-3, -1)$
e $(a, 2b), (3a, -b)$
f $(a + 3b, b), (a - b, a - b)$

2 **WE16** The coordinates of the midpoint, M, of the line segment AB are $(2, -3)$. If the coordinates of A are $(7, 4)$, find the coordinates of B.

UNDERSTANDING

3 A square has vertices A $(0, 0)$, B $(2, 4)$, C $(6, 2)$ and D $(4, -2)$. Find:

a the coordinates of the centre
b the length of a side
c the length of a diagonal.

4 **MC** The midpoint of the line segment joining the points $(-2, 1)$ and $(8, -3)$ is:

A $(6, -2)$ B $(5, 2)$ C $(6, 2)$ D $(3, -1)$ E $(5, -2)$

5 **MC** If the midpoint of AB is $(-1, 5)$ and the coordinates of B are $(3, 8)$, then A has coordinates:

A $(1, 6.5)$ B $(2, 13)$ C $(-5, 2)$ D $(4, 3)$ E $(7, 11)$

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Spreadsheet
Midpoint of a segment
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6 a The vertices of a triangle are A $(2, 5)$, B $(1, -3)$ and C $(-4, 3)$. Find:

i the coordinates of P, the midpoint of AC
ii the coordinates of Q, the midpoint of AB
iii the length of PQ.

b Show that BC = 2 PQ.

7 a A quadrilateral has vertices A $(6, 2)$, B $(4, -3)$, C $(-4, -3)$ and D $(-2, 2)$. Find:

i the midpoint of the diagonal AC
ii the midpoint of the diagonal BD.

b What can you infer about the quadrilateral?

8 **a** The points A $(-5, 3.5)$, B $(1, 0.5)$ and C $(-6, -6)$ are the vertices of a triangle. Find:

  i the midpoint, P, of AB
  ii the length of PC
  iii the length of AC
  iv the length of BC.

b Describe the triangle. What does PC represent?

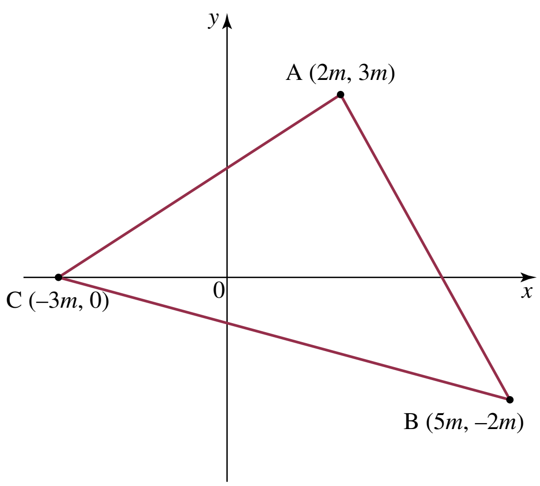
REASONING

9 Find the equation of the straight line that passes through the midpoint of A $(-2, 5)$ and B $(-2, 3)$, and has a gradient of -3.

10 Find the equation of the straight line that passes through the midpoint of A $(-1, -3)$ and B $(3, -5)$, and has a gradient of $\frac{2}{3}$.

PROBLEM SOLVING

11 The points A $(2m, 3m)$, B $(5m, -2m)$ and C $(-3m, 0)$ are the vertices of a triangle. Show that this is a right-angled triangle.



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12 Write down the coordinates of the midpoint of the line joining the points $(3k - 1, 4 - 5k)$ and $(5k - 1, 3 - 5k)$. Show that this point lies on the line with equation $5x + 4y = 9$.

3.6 Parallel and perpendicular lines

Parallel lines

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Interactivity
Parallel and perpendicular lines
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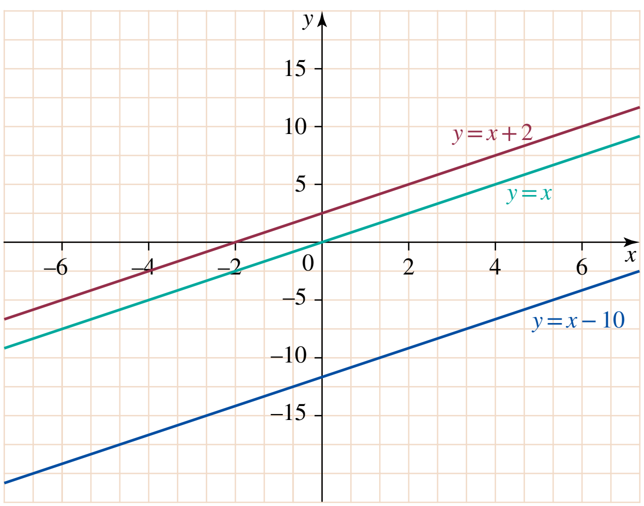
- Lines that have the same gradient are **parallel** lines. The three lines on the graph at right all have a gradient of 1 and are parallel to each other.



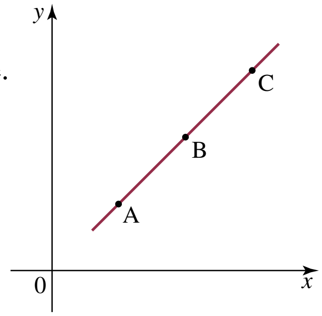
WORKED EXAMPLE 17

Show that AB is parallel to CD given that A has coordinates (−1, −5), B has coordinates (5, 7), C has coordinates (−3, 1) and D has coordinates (4, 15).

THINK	WRITE
1 Find the gradient of AB by applying the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.	Let A $(-1, -5) = (x_1, y_1)$ and B $(5, 7) = (x_2, y_2)$ Since $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{AB} = \frac{7 - (-5)}{5 - (-1)}$ $= \frac{12}{6}$ $= 2$
2 Find the gradient of CD.	Let C $(-3, 1) = (x_1, y_1)$ and D $(4, 15) = (x_2, y_2)$ $m_{CD} = \frac{15 - 1}{4 - (-3)}$ $= \frac{14}{7}$ $= 2$
3 Draw a conclusion. (*Note*: ∥ means 'is parallel to'.)	Since $m_{AB} = m_{CD} = 2$, then AB ∥ CD.

Collinear points

- **Collinear points** are points that all lie on the same straight line.
- If A, B and C are collinear, then $m_{AB} = m_{BC}$.



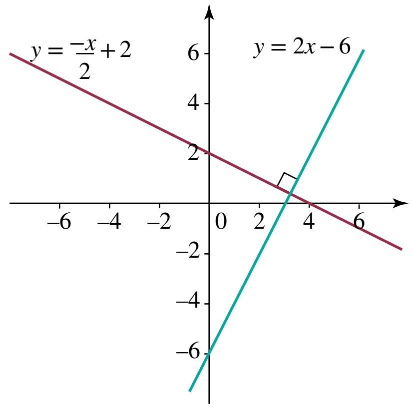
WORKED EXAMPLE 18

Show that the points A (2, 0), B (4, 1) and C (10, 4) are collinear.

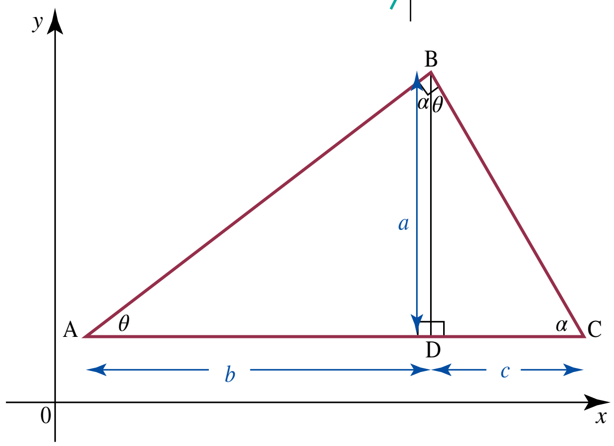
THINK	WRITE
1 Find the gradient of AB.	Let A $(2, 0) = (x_1, y_1)$ and B $(4, 1) = (x_2, y_2)$ Since $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{AB} = \frac{1 - 0}{4 - 2}$ $= \frac{1}{2}$

2	Find the gradient of BC.	Let B (4, 1) $= (x_1, y_1)$ and C (10, 4) $= (x_2, y_2)$ $m_{BC} = \frac{4-1}{10-4}$ $= \frac{3}{6}$ $= \frac{1}{2}$
3	Show that A, B and C are collinear.	Since $m_{AB} = m_{BC} = \frac{1}{2}$ and B is common to both line segments, A, B and C are collinear.

Perpendicular lines



- There is a special relationship between the gradients of two **perpendicular** lines. The graph at right shows two perpendicular lines. What do you notice about their gradients?
- Consider the diagram shown below, in which the line segment AB is perpendicular to the line segment BC, AC is parallel to the *x*-axis, and BD is the perpendicular height of the resulting triangle ABC.



In ΔABD, let $m_{AB} = m_1$

$$= \frac{a}{b}$$

$$= \tan(\theta)$$

In ΔBCD, let $m_{BC} = m_2$

$$= -\frac{a}{c}$$

$$= -\tan(\alpha)$$

In ΔABD, $\tan(\alpha) = \frac{b}{a}$

So $m_2 = -\frac{b}{a}$

$$= \frac{-1}{m_1}$$

Hence $m_2 = -\frac{1}{m_1}$

or $m_1 m_2 = -1$

- Hence, if two lines are perpendicular to each other, then the product of their gradients is -1. Two lines are perpendicular if and only if:

$$m_1 m_2 = -1$$

- If two lines are perpendicular, then their gradients are $\frac{a}{b}$ and $-\frac{b}{a}$ respectively.

WORKED EXAMPLE 19

Show that the lines $y = -5x + 2$ and $5y - x + 15 = 0$ are perpendicular to one another.

THINK	WRITE
1 Find the gradient of the first line.	$y = -5x + 2$ Hence $m_1 = -5$
2 Find the gradient of the second line.	$5y - x + 15 = 0$ Rewrite in the form $y = mx + c$: $5y = x - 15$ $y = \frac{x}{5} - 3$ Hence $m_2 = \frac{1}{5}$ $m_1 m_2 = -5 \times \frac{1}{5}$ $= -1$
3 Test for perpendicularity. (The two lines are perpendicular if the product of their gradients is −1.)	Hence, the two lines are perpendicular.

Determining the equation of a line parallel or perpendicular to another line

- The gradient properties of parallel and perpendicular lines can be used to solve many problems.

WORKED EXAMPLE 20

Find the equation of the line that passes through the point (3, −1) and is parallel to the straight line with equation $y = 2x + 1$.

THINK	WRITE
1 Write the general equation.	$y = mx + c$
2 Find the gradient of the given line. The two lines are parallel, so they have the same gradient.	$y = 2x + 1$ has a gradient of 2 Hence $m = 2$
3 Substitute for m in the general equation.	so $y = 2x + c$
4 Substitute the given point to find c.	$(x, y) = (3, -1)$ $\therefore -1 = 2(3) + c$ $-1 = 6 + c$ $c = -7$
5 Substitute for c in the general equation.	$y = 2x - 7$ or $2x - y - 7 = 0$

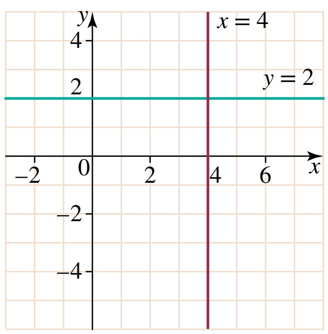
WORKED EXAMPLE 21

Find the equation of the line that passes through the point (0, 3) and is perpendicular to a straight line with a gradient of 5.

THINK	WRITE
1 For perpendicular lines, $m_1 \times m_2 = -1$. Find the gradient of the perpendicular line.	Given $m_1 = 5$ $m_2 = -\frac{1}{5}$
2 Use the equation $y - y_1 = m(x - x_1)$ where $m = -\frac{1}{5}$ and $(x_1, y_1) = (0, 3)$.	Since $y - y_1 = m(x - x_1)$ and $(x_1, y_1) = (0, 3)$ then $y - 3 = -\frac{1}{5}(x - 0)$ $y - 3 = -\frac{x}{5}$ $5(y - 3) = -x$ $5y - 15 = -x$ $x + 5y - 15 = 0$

Horizontal and vertical lines

- Horizontal lines are parallel to the x-axis, have a gradient of zero, are expressed in the form $y = c$ and have no x-intercept.
- Vertical lines are parallel to the y-axis, have an undefined (infinite) gradient, are expressed in the form $x = a$ and have no y-intercept.



WORKED EXAMPLE 22

Find the equation of:

a the vertical line that passes through the point (2, −3)

b the horizontal line that passes through the point (−2, 6).

THINK	WRITE
a The equation of a vertical line is $x = a$. The x-coordinate of the given point is 2.	a $x = 2$
b The equation of a horizontal line is $y = c$. The y-coordinate of the given point is 6.	b $y = 6$

WORKED EXAMPLE 23

Find the equation of the perpendicular bisector of the line joining the points (0, −4) and (6, 5). (A bisector is a line that crosses another line at right angles and cuts it into two equal lengths.)

THINK	WRITE
1 Find the gradient of the line joining the given points by applying the formula. $m = \frac{y_2 - y_1}{x_2 - x_1}$.	Let $(0, -4) = (x_1, y_1)$. Let $(6, 5) = (x_2, y_2)$. $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$ $m_1 = \frac{5 - (-4)}{6 - 0}$ $= \frac{9}{6}$ $= \frac{3}{2}$
2 Find the gradient of the perpendicular line. $m_1 \times m_2 = -1$	$m_1 = \frac{3}{2}$ $m_2 = -\frac{2}{3}$
3 Find the midpoint of the line joining the given points. $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ where $(x_1, y_1) = (0, -4)$ and $(x_2, y_2) = (6, 5)$.	$x = \frac{x_1 + x_2}{2}$ $= \frac{0 + 6}{2}$ $= 3$ $y = \frac{y_1 + y_2}{2}$ $= \frac{-4 + 5}{2}$ $= \frac{1}{2}$ Hence $\left(3, \frac{1}{2}\right)$ are the coordinates of the midpoint.
4 Find the equations of the line with gradient $-\frac{2}{3}$ that passes through $\left(3, \frac{1}{2}\right)$.	Since $y - y_1 = m(x - x_1)$, then $y - \frac{1}{2} = -\frac{2}{3}(x - 3)$
5 Simplify by removing the fractions. Multiply both sides by 3. Multiply both sides by 2.	$3\left(y - \frac{1}{2}\right) = -2(x - 3)$ $3y - \frac{3}{2} = -2x + 6$ $6y - 3 = -4x + 12$ $4x + 6y - 15 = 0$

Exercise 3.6 Parallel and perpendicular lines

assess on

INDIVIDUAL PATHWAYS

■ PRACTISE	■ CONSOLIDATE	■ MASTER
Questions: 1a–d, 2, 5, 6a–c, 7, 8, 9a–c, 12, 13, 16a–b, 18, 20a, 21, 23, 26a, 27	Questions: 1a–d, 2–5, 6c–d, 7, 8, 9a–c, 12, 13, 15, 16a–b, 17a, 18, 20a, 21–23, 26–28, 30, 32	Questions: 1c–f, 2, 3, 4, 5, 6e–f, 7–19, 20b, 21, 22, 24–31, 33–37

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How could you use coordinate geometry to design a logo for an organisation?

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Digital docs
Spreadsheet
Perpendicular checker
doc-5209
Spreadsheet
Equation of a straight line
doc-5210

FLUENCY

1 **WE17** Find whether AB is parallel to CD given the following sets of points.

- **a** A (4, 13), B (2, 9), C (0, −10), D (15, 0)
- **b** A (2, 4), B (8, 1), C (−6, −2), D (2, −6)
- **c** A (−3, −10), B (1, 2), C (1, 10), D (8, 16)
- **d** A (1, −1), B (4, 11), C (2, 10), D (−1, −5)
- **e** A (1, 0), B (2, 5), C (3, 15), D (7, 35)
- **f** A (1, −6), B (−5, 0), C (0, 0), D (5, −4)

2 Which pairs of the following straight lines are parallel?

- **a** $2x + y + 1 = 0$
- **b** $y = 3x - 1$
- **c** $2y - x = 3$
- **d** $y = 4x + 3$
- **e** $y = \frac{x}{2} - 1$
- **f** $6x - 2y = 0$
- **g** $3y = x + 4$
- **h** $2y = 5 - x$

3 **WE18** Show that the points A (0, −2), B (5, 1) and C (−5, −5) are collinear.

4 Show that the line that passes through the points (−4, 9) and (0, 3) also passes through the point (6, −6).

5 **WE19** Show that the lines $y = 6x - 3$ and $x + 6y - 6 = 0$ are perpendicular to one another.

6 Determine whether AB is perpendicular to CD, given the following sets of points.

- **a** A (1, 6), B (3, 8), C (4, −6), D (−3, 1)
- **b** A (2, 12), B (−1, −9), C (0, 2), D (7, 1)
- **c** A (1, 3), B (4, 18), C (−5, 4), D (5, 0)
- **d** A (1, −5), B (0, 0), C (5, 11), D (−10, 8)
- **e** A (−4, 9), B (2, −6), C (−5, 8), D (10, 14)
- **f** A (4, 4), B (−8, 5), C (−6, 2), D (3, 11)

7 **WE20** Find the equation of the line that passes through the point (4, −1) and is parallel to the line with equation $y = 2x - 5$.

8 **WE21** Find the equation of the line that passes through the point (−2, 7) and is perpendicular to a line with a gradient of $\frac{2}{3}$.

9 Find the equations of the following lines.

- **a** Gradient 3 and passing through the point (1, 5)
- **b** Gradient −4 and passing through the point (2, 1)
- **c** Passing through the points (2, −1) and (4, 2)
- **d** Passing through the points (1, −3) and (6, −5)
- **e** Passing through the point (5, −2) and parallel to $x + 5y + 15 = 0$
- **f** Passing through the point (1, 6) and parallel to $x - 3y - 2 = 0$
- **g** Passing through the point (−1, −5) and perpendicular to $3x + y + 2 = 0$

10 Find the equation of the line that passes through the point (−2, 1) and is:

- **a** parallel to the line with equation $2x - y - 3 = 0$
- **b** perpendicular to the line with equation $2x - y - 3 = 0$.

11 Find the equation of the line that contains the point (1, 1) and is:

- **a** parallel to the line with equation $3x - 5y = 0$
- **b** perpendicular to the line with equation $3x - 5y = 0$.

12 WE22 Find the equation of:

a the vertical line that passes through the point $(1, -8)$

b the horizontal line that passes through the point $(-5, -7)$.

13 MC **a** The vertical line passing through the point $(3, -4)$ is given by:

A $y = -4$ **B** $x = 3$ **C** $y = 3x - 4$

D $y = -4x + 3$ **E** $x = -4$

b Which of the following points does the horizontal line given by the equation $y = -5$ pass through?

A $(-5, 4)$ **B** $(4, 5)$ **C** $(3, -5)$

D $(5, -4)$ **E** $(5, 5)$

c Which of the following statements is true?

A Vertical lines have a gradient of zero.

B The y-coordinates of all points on a vertical line are the same.

C Horizontal lines have an undefined gradient.

D The x-coordinates of all points on a vertical line are the same.

E A horizontal line has the general equation $x = a$.

d Which of the following statements is false?

A Horizontal lines have a gradient of zero.

B The line joining the points $(1, -1)$ and $(-7, -1)$ is vertical.

C Vertical lines have an undefined gradient.

D The line joining the points $(1, 1)$ and $(-7, 1)$ is horizontal.

E A horizontal line has the general equation $y = c$.

14 The triangle ABC has vertices A $(9, -2)$, B $(3, 6)$, and C $(1, 4)$.

a Find the midpoint, M, of BC.

b Find the gradient of BC.

c Show that AM is the perpendicular bisector of BC.

d Describe triangle ABC.

15 WE23 Find the equation of the perpendicular bisector of the line joining the points $(1, 2)$ and $(-5, -4)$.

16 Find the equation of the perpendicular bisector of the line joining the points $(-2, 9)$ and $(4, 0)$.

17 ABCD is a parallelogram. The coordinates of A, B and C are $(4, 1)$, $(1, -2)$ and $(-2, 1)$ respectively. Find:

a the equation of AD

b the equation of DC

c the coordinates of D.

UNDERSTANDING

18 In each of the following, show that ABCD is a parallelogram.

a A $(2, 0)$, B $(4, -3)$, C $(2, -4)$, D $(0, -1)$

b A $(2, 2)$, B $(0, -2)$, C $(-2, -3)$, D $(0, 1)$

c A $(2.5, 3.5)$, B $(10, -4)$, C $(2.5, -2.5)$, D $(-5, 5)$

19 In each of the following, show that ABCD is a trapezium.

a A $(0, 6)$, B $(2, 2)$, C $(0, -4)$, D $(-5, -9)$

b A $(26, 32)$, B $(18, 16)$, C $(1, -1)$, D $(-3, 3)$

c A $(2, 7)$, B $(1, -1)$, C $(-0.6, -2.6)$, D $(-2, 3)$

20 MC The line that passes through the points $(0, -6)$ and $(7, 8)$ also passes through:

A $(4, 3)$ B $(5, 4)$ C $(-2, 10)$
D $(1, -8)$ E $(1, 4)$

21 MC The point $(-1, 5)$ lies on a line parallel to $4x + y + 5 = 0$. Another point on the same line as $(-1, 5)$ is:

A $(2, 9)$ B $(4, 2)$ C $(4, 0)$
D $(-2, 3)$ E $(3, -11)$

22 Find the equation of the straight line given the following conditions.

a Passes through the point $(-1, 3)$ and parallel to $y = -2x + 5$
b Passes through the point $(4, -3)$ and parallel to $3y + 2x = -3$

23 Determine which pairs of the following lines are perpendicular.

a $x + 3y - 5 = 0$ b $y = 4x - 7$ c $y = x$
d $2y = x + 1$ e $y = 3x + 2$ f $x + 4y - 9 = 0$
g $2x + y = 6$ h $x + y = 0$

24 Find the equation of the straight line that cuts the x-axis at 3 and is perpendicular to the line with equation $3y - 6x = 12$.

25 Calculate the value of m for which lines with the following pairs of equations are perpendicular to each other.

a $2y - 5x = 7$ and $4y + 12 = mx$ b $5x - 6y = -27$ and $15 + mx = -3y$

26 MC The gradient of the line perpendicular to the line with equation $3x - 6y = 2$ is:

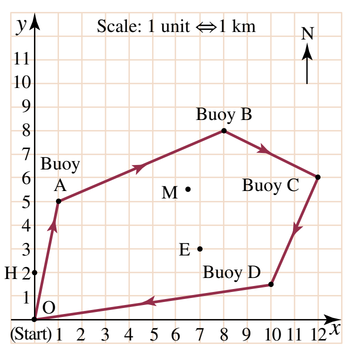
A 3 B -6 C 2 D $\frac{1}{2}$ E -2

27 MC Triangle ABC has a right angle at B. The vertices are $A(-2, 9)$, $B(2, 8)$ and $C(1, z)$. The value of z is:

A $8\frac{1}{4}$ B 4 C 12 D $7\frac{3}{4}$ E -4

REASONING

28 The map shows the proposed course for a yacht race. Buoys have been positioned at A $(1, 5)$, B $(8, 8)$, C $(12, 6)$, and D $(10, w)$.



a How far is it from the start, O, to buoy A?
b The race marshall boat, M, is situated halfway between buoys A and C. What are the coordinates of the boat's position?
c Stage 4 of the race (from C to D) is perpendicular to stage 3 (from B to C). What is the gradient of CD?
d Find the linear equation that describes stage 4.
e Hence determine the exact position of buoy D.
f An emergency boat is to be placed at point E, $(7, 3)$. How far is the emergency boat from the hospital, located at H, 2 km north of the start?

29 Show that the following sets of points form the vertices of a right-angled triangle.

a A $(1, -4)$, B $(2, -3)$, C $(4, -7)$
b A $(3, 13)$, B $(1, 3)$, C $(-4, 4)$
c A $(0, 5)$, B $(9, 12)$, C $(3, 14)$

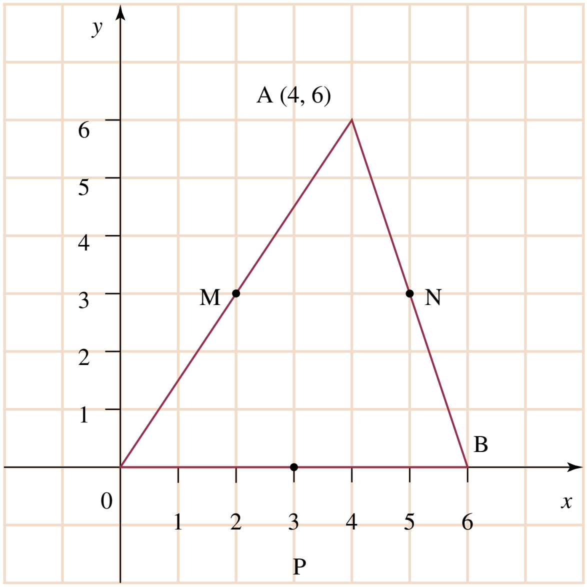
30 Prove that the quadrilateral ABCD is a rectangle when A is $(2, 5)$, $B(6, 1)$, C $(3, -2)$ and D $(-1, 2)$.

31 Prove that the quadrilateral ABCD is a rhombus, given A $(2, 3)$, B $(3, 5)$, C $(5, 6)$ and D $(4, 4)$.
Hint: A rhombus is a parallelogram with diagonals that intersect at right angles.

32 **a** A square has vertices at $(0, 0)$ and $(2, 0)$. Where are the other 2 vertices? (There are 3 sets of answers.)
b An equilateral triangle has vertices at $(0, 0)$ and $(2, 0)$. Where is the other vertex? (There are 2 answers.)
c A parallelogram has vertices at $(0, 0)$ and $(2, 0)$. and $(1, 1)$. Where is the other vertex? (There are 3 sets of answers.)

33 A is the point $(0, 0)$ and B is the point $(0, 2)$.
a Find the perpendicular bisector of AB.
b Show that any point on this line is equidistant from A and B.

Questions 34 and 35 relate to the diagram.
M is the midpoint of OA.
N is the midpoint of AB.
P is the midpoint of OB.

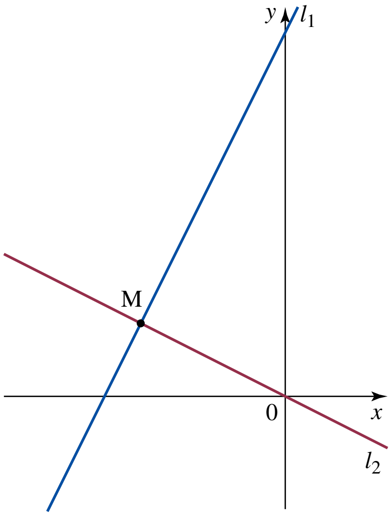


34 A simple investigation:
a Show that MN is parallel to OB.
b Is PN parallel to OA?
c Is PM parallel to AB?
d Will this be true for any triangle OAB? (Show your proof.)

35 A difficult investigation:
a Find the perpendicular bisectors of OA and OB.
b Find the point W where the two bisectors intersect.
c Show that the perpendicular bisector of AB also passes through W.
d Explain why W is equidistant from O, A and B.
e W is called the circumcentre of triangle OAB. Using W as the centre, draw a circle through O, A and B.

PROBLEM SOLVING

36 The lines l_1 and l_2 are at right angles to each other. The line l_1 has the equation $px + py + r = 0$. Show that the distance from M to the origin is given by $\dfrac{r}{\sqrt{p^2 + p^2}}$.



37 Line A is parallel to the line with equation $2x - y = 7$ and passes through the point (2, 3). Line B is perpendicular to the line with equation $4x - 3y + 3 = 0$ and also passes through the point (2, 3). Line C intersects with line A where it cuts the y-axis and intersects with line B where it cuts the x-axis.

- **a** Determine the equations for all three lines. Give answers in the form $ax + by + c = 0$.
- **b** Sketch all three lines on the one set of axes.
- **c** Determine whether the triangle formed by the three lines is scalene, isosceles or equilateral.

CHALLENGE 3.2

The first six numbers of a particular number pattern are 1, 2, 3, 6, 11 and 20. Given that this pattern continues, what will be the next four numbers? Describe the pattern.

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3.7 Review

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The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

Review questions

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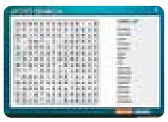
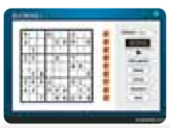
Interactivities

Word search
int-2832



Crossword
int-2833

Sudoku
int-3590



Language

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

axes	horizontal	rise
bisect	linear graph	run
Cartesian plane	midpoint	segment
collinear	origin	substitute
coordinates	parallel	trapezium
diagonal	parallelogram	vertical
general form	perpendicular	vertices
gradient	quadrilateral	x-intercept
gradient–intercept form	rhombus	y-intercept

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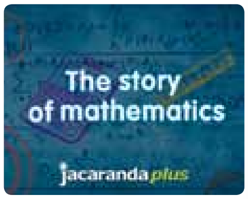


Descartes (eles-1842) tells the story of Rene Descartes, a French philosopher and mathematician who brought the concepts of geometry and algebra together, developing the two-dimensional grid we know as the Cartesian plane.

RICH TASK

What common computer symbol is this?

On computer hardware, and on many different software applications, a broad range of symbols is used. These symbols help us to identify where things need to be plugged into, what buttons we need to push, or what option needs to be selected. The main focus of this task involves constructing a common symbol found on the computer. The instructions are given below. Grid lines have been provided on the opposite page for you to construct the symbol.

The construction part of this task requires you to graph nine lines to reveal a common computer symbol. Draw the scale of your graph to accommodate x- and y-values in the following ranges: $-10 \le x \le 16$ and $-10 \le y \le 16$. Centre the axes on the grid lines provided.

- Line 1 has an equation $y = x - 1$. Graph this line in the range $-7 \le x \le -2$.
- Line 2 is perpendicular to line 1 and has a y-intercept of -5. Determine the equation of this line, and then draw the line in the range $-5 \le x \le -1$.
- Line 3 is parallel to line 1, with a y-intercept of 3. Determine the equation of the line, and then graph the line in the range $-9 \le x \le -4$.
- Line 4 is parallel to line 1, with a y-intercept of -3. Determine the equation of the line, and then graph the line in the range $-1 \le x \le 2$.

- Line 5 has the same length as line 4 and is parallel to it. The point (–2, 3) is the starting point of the line, which decreases in both x- and y-values from there.
- Line 6 commences at the same starting point as line 5, and then runs at right angles to line 5. It has an x-intercept of 1 and is the same length as line 2.
- Line 7 commences at the same starting point as both lines 5 and 6. Its equation is $y = 6x + 15$. The point (–1, 9) lies at the midpoint.
- Line 8 has the equation $y = -x + 15$. Its midpoint is the point (7, 8) and its extremities are the points where the line meets line 7 and line 9.
- Line 9 has the equation $6y - x + 8 = 0$. It runs from the intersection of lines 4 and 6 until it meets line 8.

1 What common computer symbol have you drawn?
2 The top section of your figure is a familiar geometric shape. Use the coordinates on your graph, together with the distance formula to determine the necessary lengths to calculate the area of this figure.
3 Using any symbol of interest to you, draw your symbol on grid lines and provide instructions for your design. Ensure that your design involves aspects of coordinate geometry that have been used throughout this task.

CODE PUZZLE

Who won the inaugural 875 km Sydney to Melbourne marathon in 1983?

The equations of the straight lines, in the form $y = mx + c$, that fit the given information and the letter beside each give the puzzle's answer code.

- gradient of –2 and y-intercept of 2 **A**
- gradient of 1 and passes through (5, 4) **C**
- passes through (2, 5) and (0, 3) **D**
- $m = -1$ and $c = 1$ **E**
- $m = 4$ and passes through (–2, –3) **F**
- gradient of 3 and y-intercept of 4 **G**
- passes through (0, –3) and $\left(-\frac{3}{4}, 0\right)$ **H**
- $m = -1$ and $c = -3$ **I**
- passes through (2, 7) and (–3, 2) **L**
- gradient of 2 and passes through (–2, –1) **M**
- passes through (5, 9) and (0, –1) **N**
- $m = 1$ and $c = -2$ **O**
- gradient of –4 and passes through $\left(\frac{1}{4}, 0\right)$ **P**
- passes through (1, 1) and (–1, –5) **R**
- passes through (3, –8) and has a gradient of –1 **S**
- passes through (1, 2) and (3, 10) **T**
- gradient is –3 and passes through (–2, 7) **U**
- $m = -3$ and $c = -1$ **V**
- passes through (2, –3) with a gradient of –2 **X**
- passes through (0, 2) and (2, 0) **Y**

$y = -x - 5$	$y = -x - 3$	$y = -2x + 1$	$y = 4x - 2$	$y = -x + 2$	$y = x - 2$	$y = 2x - 1$	$y = -x + 1$	$y = -x + 2$	$y = -x + 1$	$y = -2x + 2$	$y = 3x - 2$	$y = x - 2$	$y = x + 5$	$y = x + 3$

$y = -4x + 1$	$y = x - 2$	$y = 4x - 2$	$y = -2x + 2$	$y = 4x - 2$	$y = x - 2$	$y = 4x + 5$	$y = -2x + 2$	$y = 3x - 2$	$y = 2x + 3$	$y = -x + 1$	$y = 3x - 2$

$y = x - 1$	$y = x + 5$	$y = -x - 3$	$y = 4x + 5$	$y = 4x + 5$	$y = -x + 2$	$y = x - 2$	$y = -3x + 1$	$y = 2x - 1$	$y = 3x + 4$	$y = -x - 3$	$y = 2x - 1$

$y = 4x + 5$	$y = -x - 3$	$y = -3x - 1$	$y = -x + 1$	$y = x + 3$	$y = -2x + 2$	$y = -x + 2$	$y = -x - 5$	$y = -2x + 2$	$y = 2x - 1$	$y = x + 3$

$y = 4x + 5$	$y = -x - 3$	$y = 4x + 5$	$y = 4x - 2$	$y = -x + 1$	$y = -x + 1$	$y = 2x - 1$	$y = -4x - 3$	$y = x - 2$	$y = -3x + 1$	$y = 3x - 2$	$y = -x - 5$

eBookplus Activities

3.1 Overview

Video

- The story of mathematics (eles-1842)

3.2 Sketching linear graphs

Interactivity

- IP interactivity 3.2 (int-4572): Sketching linear graphs

Digital docs

- SkillSHEET (doc-5197): Describing the gradient of a line
- SkillSHEET (doc-5198): Plotting a line using a table of values
- SkillSHEET (doc-5199): Stating the *y*-intercept from a graph
- SkillSHEET (doc-5200): Solving linear equations that arise when finding *x*- and *y*-intercepts
- SkillSHEET (doc-5201): Using Pythagoras' theorem
- SkillSHEET (doc-5202): Substitution into a linear rule
- SkillSHEET (doc-5203): Transposing linear equations to standard form

3.3 Determining linear equations

Interactivity

- IP interactivity 3.3 (int-4573): Determining linear equations

Digital docs

- SkillSHEET (doc-5196): Measuring the rise and the run
- SkillSHEET (doc-5204): Finding the gradient given two points
- WorkSHEET 3.1 (doc-13849): Gradient

3.4 The distance between two points

Interactivity

- IP interactivity 3.4 (int-4574): The distance between two points

Digital doc

- Spreadsheet (doc-5206): Distance between two points

3.5 The midpoint of a line segment

Interactivity

- IP interactivity 3.5 (int-4575): The midpoint of a line segment

Digital docs

- Spreadsheet (doc-5207): Midpoint of a segment
- WorkSHEET 3.2 (doc-13850): Midpoint of a line segment

3.6 Parallel and perpendicular lines

Interactivities

- Parallel and perpendicular lines (int-2779)
- IP interactivity 3.6 (int-4576): Parallel and perpendicular lines

Digital docs

- Spreadsheet (doc-5209): Perpendicular checker
- Spreadsheet (doc-5210): Equation of a straight line

3.7 Review

Interactivities

- Word search (int-2832)
- Crossword (int-2833)
- Sudoku (int-3590)

Digital docs

- Topic summary (doc-13713)
- Concept map (doc-13714)

To access eBookPLUS activities, log on to www.jacplus.com.au

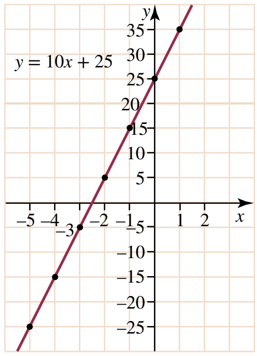
Answers

TOPIC 3 Coordinate geometry

Exercise 3.2 — Sketching linear graphs

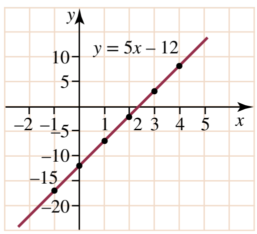
1 a

x	y
–5	–25
–4	–15
–3	–5
–2	5
–1	15
0	25
1	35



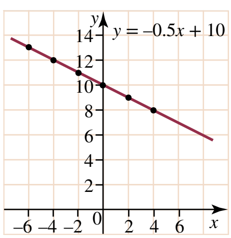
b

x	y
–1	–17
0	–12
1	–7
2	–2
3	3
4	8



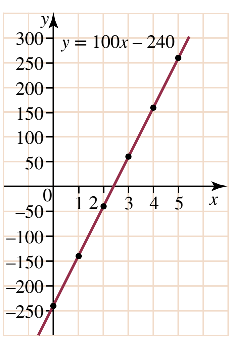
c

x	y
–6	13
–4	12
–2	11
0	10
2	9
4	8



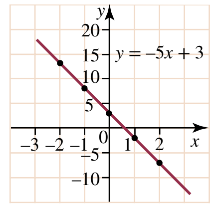
d

x	y
0	–240
1	–140
2	–40
3	60
4	160
5	260



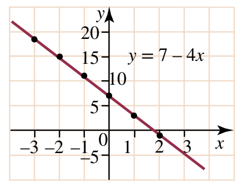
e

x	y
–3	18
–2	13
–1	8
0	3
1	–2
2	–7



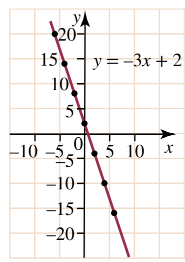
f

x	y
–3	19
–2	15
–1	11
0	7
1	3
2	–1



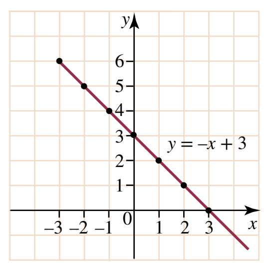
2 a

x	y
–6	20
–4	14
–2	8
0	2
2	–4
4	–10
6	–16



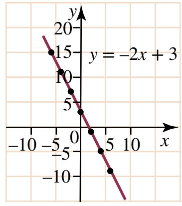
b

x	y
–3	6
–2	5
–1	4
0	3
1	2
2	1
3	0

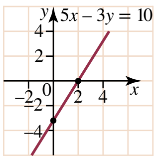


c

x	y
−6	15
−4	11
−2	7
0	3
2	−1
4	−5
6	−9



3 a



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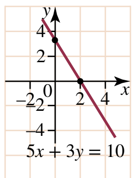

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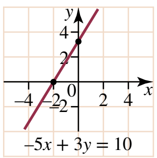

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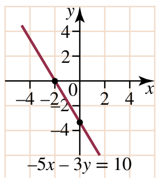

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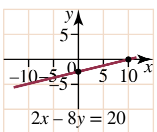

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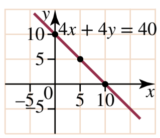
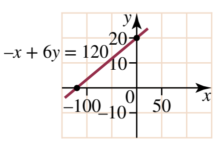


g



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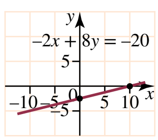

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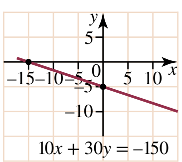

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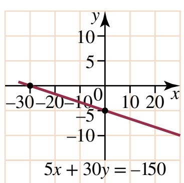

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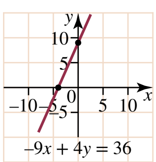

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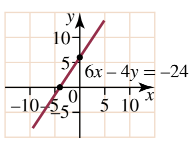

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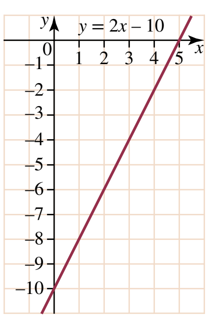

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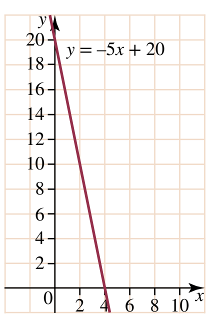

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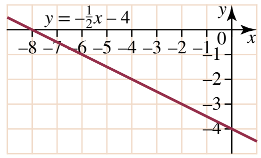

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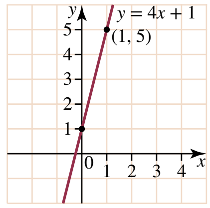

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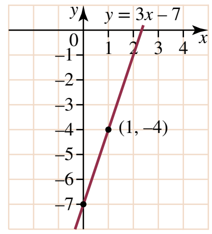

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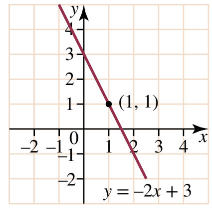

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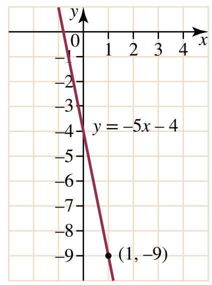

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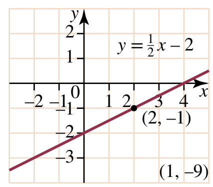

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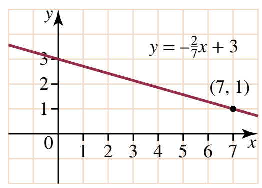

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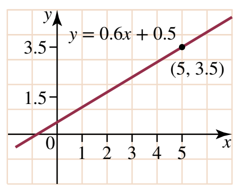

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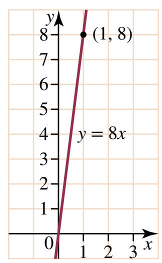

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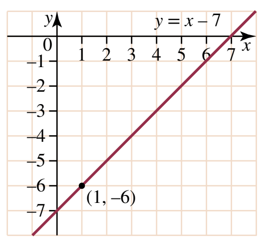

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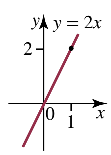

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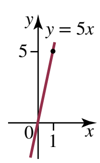

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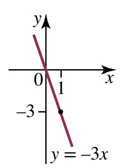

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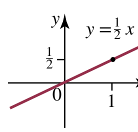
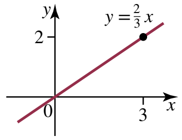
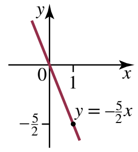
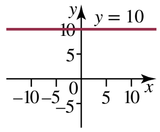
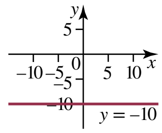
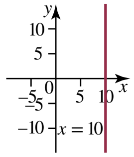


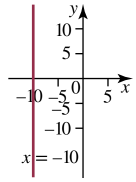
e 

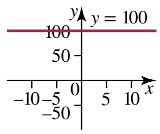
f 

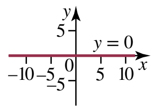
6 a 

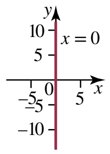
b 

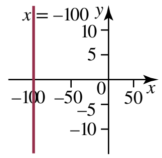
c 

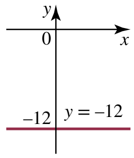
d 

e 

f 

g 

h 

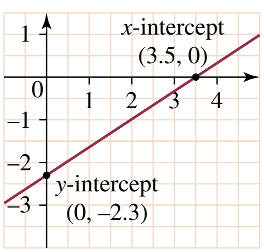
i 

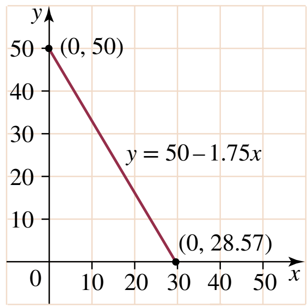
7 a x-intercept: −0.5; y-intercept: 0.4
b x-intercept: 0.5; y-intercept: −0.4
c x-intercept: 0; y-intercept: 0
d x-intercept: −3; y-intercept: 12
e x-intercept: −4; y-intercept: −4
f x-intercept: −1; y-intercept: −0.5
g x-intercept: 2.75; y-intercept: 2.2
h x-intercept: 7; y-intercept: 3.5
i x-intercept: 9.75; y-intercept: −3.9
j x-intercept: $\frac{23}{13} \approx 1.77$; y-intercept: 4.6

8 a (2, 0), (0, −8)
b $(-\frac{1}{2}, 0)$, (0, 3)
c (−5, 0), (0, 25)

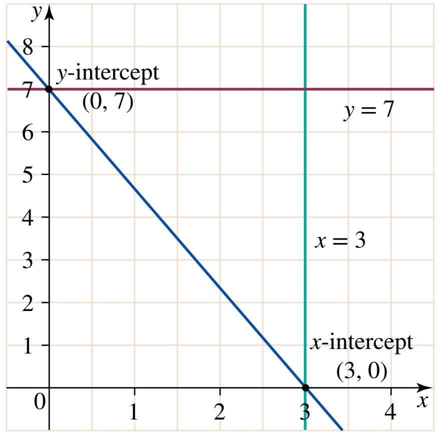
9 Answers will vary.

10 $y = \frac{2}{3}x - \frac{7}{3}$

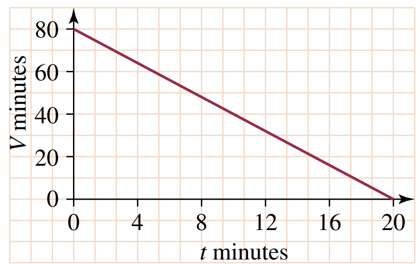


11 a x represents the number of songs she buys online.
b 

c 14 songs

12 a 

b 7
c $-\frac{7}{3}$
d B

13 a Initially there are 80 litres of water.
b Time cannot be negative.
c 4 litres per minute
d 20 minutes
e 

Exercise 3.3 — Determining linear equations

1 a $y = 2x + 4$ b $y = -3x + 12$ c $y = -x + 5$
d $y = 2x - 8$ e $y = \frac{1}{2}x + 3$ f $y = -\frac{1}{4}x - 4$
g $y = 7x - 5$ h $y = -3x - 15$

2 a $y = 2x$ b $y = -3x$ c $y = \frac{1}{2}x$ d $y = -\frac{3}{4}x$

3 a $y = x + 3$ b $y = 2x - 1$ c $y = -\frac{1}{2}x + \frac{7}{2}$
d $y = \frac{1}{2}x + \frac{1}{2}$ e $y = -2x - 2$ f $y = -x - 8$

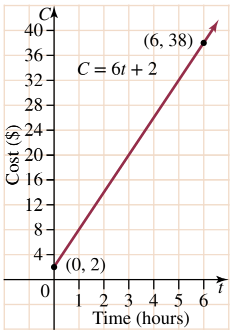
4 a $y = 3x + 3$ b $y = -3x + 4$
c $y = -4x + 2$ d $y = 4x + 2$
e $y = -x - 4$ f $y = 0.5x - 4$
g $y = 5x + 2.5$ h $y = -6x + 3$
i $y = -2.5x + 1.5$ j $y = 3.5x + 6.5$

5 a $y = 5x - 19$ **b** $y = -5x + 31$ **c** $y = -4x - 1$
d $y = 4x - 34$ **e** $y = 3x - 35$ **f** $y = -3x + 6$
g $y = -2x + 30$ **h** $y = 2x - 4.5$ **i** $y = 0.5x - 19$
j $y = -0.5x + 5.5$

6 a

t	0	1	2	3
C	2	8	14	20

b



c **i** (0, 2)
ii The y-intercept represents the initial cost of bowling at the alley, which is the shoe rental.
d $m = 6$
e $C = 6t + 2$
f \$32
g Answers will vary.

7 It does not matter if you rise before you run or run before you rise, as long as you take into account whether the rise or run is negative.

8 a $m = \dfrac{y - c}{x}$ **b** $y = mx + c$

9 Teacher to check

10 $m_{AB} = m_{CD} = 2$ and $m_{BC} = m_{AD} = \dfrac{1}{2}$. As opposite sides have the same gradients, this quadrilateral is a parallelogram.

Challenge 3.1

$y = \frac{3}{5}x + 2$

Exercise 3.4 — The distance between two points

1 AB = 5, CD = $2\sqrt{10}$ or 6.32, EF = $3\sqrt{2}$ or 4.24, GH = $2\sqrt{5}$ or 4.47, IJ = 5, KL = $\sqrt{26}$ or 5.10, MN = $4\sqrt{2}$ or 5.66, OP = $\sqrt{10}$ or 3.16

2 a 5 **b** 13 **c** 10
d 7.07 **e** 6.71 **f** 14.42
g 13 **h** 13 **i** $\sqrt{a^2 + 4b^2}$
j $3\sqrt{a^2 + b^2}$

3 B

4 D

5 a AB = 4.47, BC = 2.24, CD = 4.47, DA = 2.24
b AC = 5, BD = 5
c Rectangle

6, **7** and **8** Answers will vary.

9 a 12 **b** 5 **c** 13 **d** −2.2

10 Answers will vary.

11 $a = 2$

12 a $m_{AB} = 1$ and $m_{BC} = -\frac{7}{3}$
b D (4, −1)
c Teacher to check

Exercise 3.5 — The midpoint of a line segment

1 a $(-3, -3\frac{1}{2})$ **b** $(7\frac{1}{2}, 0)$ **c** $(-1, 1)$
d $(0, 1\frac{1}{2})$ **e** $(2a, \frac{1}{2}b)$ **f** $(a + b, \frac{1}{2}a)$

2 (−3, −10)

3 a (3, 1) **b** 4.47 **c** 6.32

4 D

5 C

6 a **i** (−1, 4) **ii** $(1\frac{1}{2}, 1)$ **iii** 3.91
b BC = 7.8 = 2 PQ

7 a **i** (1, −0.5) **ii** (1, −0.5)
b The diagonals bisect each other, so it is a parallelogram.

8 a **i** (−2, 2) **ii** 8.94 **iii** 9.55 **iv** 9.55
b Isosceles. PC is the perpendicular height of the triangle.

9 $y = -3x - 2$

10 $3y - 2x + 14 = 0$

11 Teacher to check

12 $(4k - 1, 3.5 - 5k)$

Exercise 3.6 — Parallel and perpendicular lines

1 a No **b** Yes **c** No
d No **e** Yes **f** No

2 **b**, **f**; **c**, **e**

3 Answers will vary.

4 Answers will vary.

5 Answers will vary.

6 a Yes **b** Yes **c** No
d Yes **e** Yes **f** No

7 $y = 2x - 9$

8 $3x + 2y - 8 = 0$

9 a $y = 3x + 2$ **b** $y = -4x + 9$
c $3x - 2y - 8 = 0$ **d** $5y + 2x + 13 = 0$
e $x + 5y + 5 = 0$ **f** $x - 3y + 17 = 0$
g $x - 3y - 14 = 0$

10 a $2x - y + 5 = 0$ **b** $x + 2y = 0$

11 a $3x - 5y + 2 = 0$ **b** $5x + 3y - 8 = 0$

12 a $x = 1$ **b** $y = -7$

13 a B **b** C **c** D **d** B

14 a (2, 5) **b** 1
c Answers will vary. **d** Isosceles triangle

15 $y = -x - 3$

16 $4x - 6y + 23 = 0$

17 a $y = -x + 5$ **b** $y = x + 3$ **c** (1, 4)

18 Answers will vary.

19 Answers will vary.

20 B

21 E

22 a $y = -2x + 1$ **b** $3y + 2x + 1 = 0$

23 **a**, **e**; **b**, **f**; **c**, **h**; **d**, **g**

24 $y = -\frac{1}{2}x + \frac{3}{2}$

25 a $m = -\frac{8}{5}$ **b** $m = \frac{18}{5}$

26 E

27 B

28 a 5.10 km **b** (6.5, 5.5) **c** 2
d $y = 2x - 18$ **e** (10, 2) **f** 7.07 km

29, **30**, **31** Answers will vary.

32 a (2, 0), (2, 2) or (−2, 0), (−2, 2) or (1, 1), (1, −1)
b $(1, \sqrt{3})$ or $(1, -\sqrt{3})$
c (3, 1), (−1, 1) or (1, −1)

33 a $x = 1$ **b** Answers will vary.

34 a Answers will vary. **b** Yes
c Yes **d** Answers will vary.

35 a OA: $2x + 3y - 13 = 0$; OB: $x = 3$

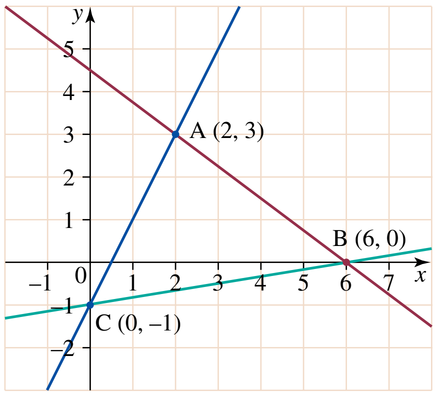
b $(3, \frac{7}{3})$

c, d Answers will vary.

36 Teacher to check

37 a Line A: $2x - y - 1 = 0$, Line B: $3x + 4y - 18 = 0$, Line C: $x - 6y - 6 = 0$

b

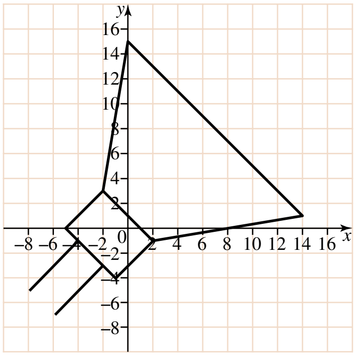


c Scalene

Challenge 3.2

37, 68, 125, 230. To find the next number, add the three preceding numbers.

Investigation — Rich task



1 The symbol is the one used to represent a speaker.

2 The shape is a trapezium.

Area $= \frac{1}{2}$(length line 6 + length line 8) $\times$ perpendicular distance between these lines.

$= \frac{1}{2}(4\sqrt{2} + 14\sqrt{2}) \times 7\sqrt{2}$

$= 126$ units^2

3 Teacher to check

Code puzzle

Sixty-one-year-old potato farmer Cliff Young in five days and fifteen hours