Coordinate Systems and Vectors Kinematics in Two-Dimensions: Projectile Motion

8.01
W02D1

Announcements

Problem Set 1 Due Week 02 Tuesday at 9 pm

Math Review Week Tuesday at 9 pm in 26-152

Register your clicker on the website

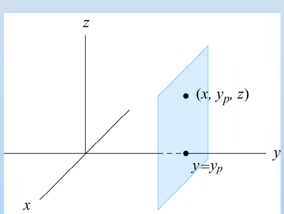
Register your group on the website.

Exam 1 Thursday September 18 from 7:30-9:30 pm

Cartesian Coordinate System

Coordinate system: used to describe the position of a point in space and consists of

1. An origin as the reference point
2. A set of coordinate axes with scales and labels
3. Choice of positive direction for each axis



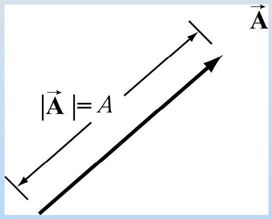
Cartesian Coordinate System

Vector

A vector $\vec{\mathbf{A}}$ is a quantity that has both direction and magnitude.

The magnitude of is denoted by

$$|\vec{\mathbf{A}}| \equiv A$$



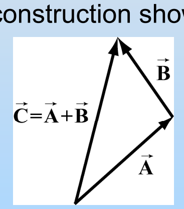
Spatial Properties of Vectors

Vectors can exist at any point *P* in space.

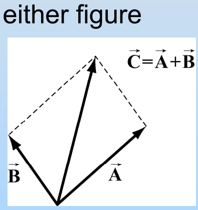
Vector equality: Any two vectors that have the same direction and magnitude are equal no matter where in space they are located.

Vector Addition

Let $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ be two vectors. Define a new vector $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, the “vector addition” of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, by the geometric construction shown in either figure



“head-to-tail”

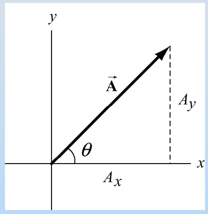


“parallelogram”

Vector Decomposition in Two Dimensions

Consider a vector

$\vec{\mathbf{A}} = (A_x, A_y, 0)$



x- and *y*- components:

$A_x = A\cos(\theta), \quad A_y = A\sin(\theta)$

Magnitude: $A = \sqrt{A_x^2 + A_y^2}$

Direction: $\dfrac{A_y}{A_x} = \dfrac{A\sin(\theta)}{A\cos(\theta)} = \tan(\theta)$

$\theta = \tan^{-1}(A_y / A_x)$

Group Problem: Displacement Vector

A person runs 250 m along the Infinite Corridor at MIT from Mass Ave to the end of Building 8, turns right at the end of the corridor and runs 178 m to the end of Building 2, and then turns right and runs 30 m down the hall.

What is the direction and magnitude of the displacement vector between start and finish?

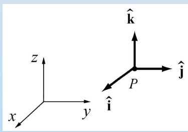
Concept Question: Magnitudes and Components

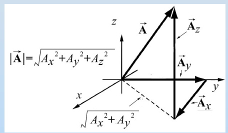
Can a component of a vector have a magnitude greater than the magnitude of the vector?

1. Yes.
2. No.
3. Depends on the vector in question.

Unit Vectors and Components

The idea of multiplication by real numbers allows us to define a set of unit vectors at each point in space $(\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}})$ with $|\hat{\mathbf{i}}|=1, |\hat{\mathbf{j}}|=1, |\hat{\mathbf{k}}|=1$





Then vector decomposition $\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y + \vec{\mathbf{A}}_z$

with components vectors: $\vec{\mathbf{A}}_x = A_x\,\hat{\mathbf{i}},\ \vec{\mathbf{A}}_y = A_y\,\hat{\mathbf{j}},\ \ \vec{\mathbf{A}}_z = A_z\,\hat{\mathbf{k}}$

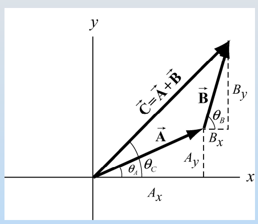
and components: $\vec{\mathbf{A}} = (A_x, A_y, A_z)$

Thus $\vec{\mathbf{A}} = A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}} + A_z\,\hat{\mathbf{k}}$

Vector Addition

$$\vec{\mathbf{A}} = A\cos(\theta_A)\ \hat{\mathbf{i}} + A\sin(\theta_A)\ \hat{\mathbf{j}}$$

$$\vec{\mathbf{B}} = B\cos(\theta_B)\ \hat{\mathbf{i}} + B\sin(\theta_B)\ \hat{\mathbf{j}}$$



Vector Sum: $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$

Components

$$C_x = A_x + B_x, \quad C_y = A_y + B_y$$

$$C_x = C\cos(\theta_C) = A\cos(\theta_A) + B\cos(\theta_B)$$

$$C_y = C\sin(\theta_C) = A\sin(\theta_A) + B\sin(\theta_B)$$

$$\vec{\mathbf{C}} = (A_x + B_x)\,\hat{\mathbf{i}} + (A_y + B_y)\,\hat{\mathbf{j}} = C\cos(\theta_C)\,\hat{\mathbf{i}} + C\sin(\theta_C)\,\hat{\mathbf{j}}$$

Vector Description of Two-Dimensional Motion

Position $\vec{\mathbf{r}}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$

Displacement $\Delta\vec{\mathbf{r}}(t) = \Delta x(t)\,\hat{\mathbf{i}} + \Delta y(t)\,\hat{\mathbf{j}}$

Velocity $\vec{\mathbf{v}}(t) = \dfrac{dx(t)}{dt}\hat{\mathbf{i}} + \dfrac{dy(t)}{dt}\hat{\mathbf{j}} \equiv v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}}$

Acceleration $\vec{\mathbf{a}}(t) = \dfrac{dv_x(t)}{dt}\hat{\mathbf{i}} + \dfrac{dv_y(t)}{dt}\hat{\mathbf{j}} \equiv a_x(t)\hat{\mathbf{i}} + a_y(t)\hat{\mathbf{j}}$

Position, Velocity, and Accelerations in Different Reference Frames

Reference Systems

Use coordinate system as a ‘*reference frame*’ to describe the position, velocity, and acceleration of objects.

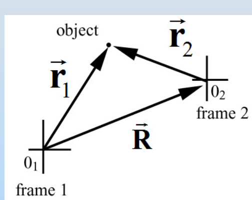
Position Vectors in Different Reference Frames

Two reference frames, Frame 1 denoted by O_1 and Frame 2 denoted by O_2.

Origins do not coincide.

Object has different position vectors in different frames

$$\vec{r}_1 = \vec{R} + \vec{r}_2$$



Law of Addition of Velocities

Suppose an object is moving; then, observers in different reference frames will measure different velocities

Velocity of Frame 2 with respect to Frame 1: $\vec{\mathbf{V}} = d\vec{\mathbf{R}} / dt$

Velocity of the object in Frame 1: $\vec{\mathbf{v}}_1 = d\vec{\mathbf{r}}_1 / dt$

Velocity of the object in Frame 2: $\vec{\mathbf{v}}_2 = d\vec{\mathbf{r}}_2 / dt$

Velocity of an object in two different reference frames

$$\frac{d\vec{\mathbf{r}}_1}{dt} = \frac{d\vec{\mathbf{R}}}{dt} + \frac{d\vec{\mathbf{r}}_2}{dt} \Rightarrow \vec{\mathbf{v}}_1 = \vec{\mathbf{V}} + \vec{\mathbf{v}}_2$$

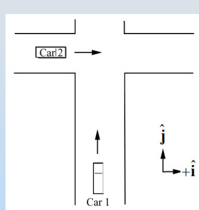
Concept Question: Relatively Inertial Reference Frames

Suppose Frames 1 and 2 are relatively inertial reference frames.

1) An object that is at rest in Frame 2 is moving at a constant velocity in reference Frame 1.
2) An object that is accelerating in Frame 2 has the same acceleration in reference Frame 1.
3) An object that is moving at constant velocity in Frame 2 is accelerating in reference Frame 1.
4) An object that is accelerating in Frame 2 is moving at constant velocity in reference Frame 1.
5) Two of the above
6) None of the above

Group Problem: Law of Addition of Velocities

Suppose two cars, Car 1, and Car 2, are traveling along roads that are perpendicular to each other. Reference Frame A is at rest with respect to the ground. Reference Frame B is at rest with respect to Car 1. Choose unit vectors such that Car 1 is moving in the positive y-direction, and Car 2 is moving in the positive x-direction in reference Frame A.



a) What is the vector description of the velocity of Car 2 in Reference Frame B?

b) What is the magnitude of the velocity of Car 2 as observed in Reference Frame B?

c) What angle does the velocity of Car 2 make with respect to the positive x-direction as observed in Reference Frame B?

Law of Addition of Accelerations

Suppose an object is moving; then, observers in different reference frames will measure different accelerations

Acceleration of Frame 2 with respect to Frame 1: $\vec{\mathbf{A}} = d\vec{\mathbf{V}} \,/\, dt$

Acceleration of the object in Frame 1: $\vec{\mathbf{a}}_1 = d\vec{\mathbf{v}}_1 \big/ dt$

Acceleration of the object in Frame 2: $\vec{\mathbf{v}}_2 = d\vec{\mathbf{r}}_2 \big/ dt$

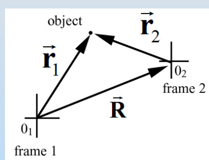
Acceleration of an object in two different reference frames

$$\frac{d\vec{\mathbf{v}}_1}{dt} = \frac{d\vec{\mathbf{V}}}{dt} + \frac{d\vec{\mathbf{v}}_2}{dt} \Rightarrow \vec{\mathbf{a}}_1 = \vec{\mathbf{A}} + \vec{\mathbf{a}}_2$$

Relatively Inertial Reference Frames

Two reference frames with the zero relative acceleration

$\vec{\mathbf{V}} = d\vec{\mathbf{R}} \big/ dt \qquad \vec{\mathbf{A}} = d\vec{\mathbf{V}} \big/ dt = \vec{\mathbf{0}}$



One moving object has different position vectors in different frames

$$\vec{\mathbf{r}}_1 = \vec{\mathbf{R}} + \vec{\mathbf{r}}_2$$

Law of addition of velocities

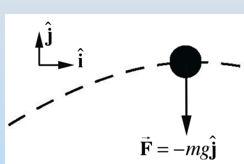
$$\vec{\mathbf{v}}_1 = \vec{\mathbf{V}} + \vec{\mathbf{v}}_2$$

Acceleration in either reference frame is the same

$$\vec{\mathbf{a}}_1 = \vec{\mathbf{A}} + \vec{\mathbf{a}}_2 = \vec{\mathbf{a}}_2$$

Projectile Motion: Components of Acceleration

Force: $\vec{\mathbf{F}} = -mg\,\hat{\mathbf{j}}$



Newton's Second Law:

$$-mg\,\hat{\mathbf{j}} = m\vec{\mathbf{a}}$$

Acceleration: $\vec{\mathbf{a}} = -g\,\hat{\mathbf{j}}$

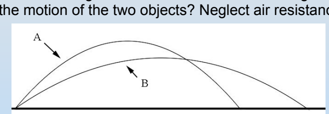
$$a_x = 0 \qquad a_y = -g \qquad g = 9.8\ \text{m/s}^2$$

Demo:
Relative Motion Gun A7
Video Link:

http://tsgphysics.mit.edu/front/index.php?page=demo.php?letnum=A%207&show=0

Concept Q.: Which Hits First?

A person simultaneously throws two objects in the air. The objects leave the person's hands at different angles and travel along the parabolic trajectories indicated by A and B in the figure below. Which of the following statements best describes the motion of the two objects? Neglect air resistance.



1. The object moving along the trajectory A returns to the initial height before the object moving along the trajectory B.
2. The object moving along the higher trajectory A returns to the initial height after the object moving along the lower trajectory B.
3. Both objects return to the initial height at the same time.
4. There is not enough information specified in order to determine which object returns to the initial height first.

Demo:
Stuffed Animal and Gun A6

Group Problem: Stuffed Animal and the Gun

A stuffed animal is suspended at a height *h* above the ground. A physics demo instructor has set up a projectile gun a horizontal distance *d* away from the stuffed animal. The projectile is initially a height *s* above the ground. The demo instructor fires the projectile with an initial velocity of magnitude v_0 just as the stuffed animal is released. Find the angle the projectile gun must be aimed in order for the projectile to strike the stuffed animal. Ignore air resistance.