CHAOS IN CHUA'S CIRCUIT

by

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Abstract

The simplest autonomous electronic circuit which can become chaotic was proposed recently by Chua. The objective of this paper is to describe a wide variety of bifurcation and chaotic phenomena observed experimentally from this circuit.

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1. Introduction

Chaotic phenomena is ubiquitous in nature [1]. It has now been widely observed in numerous physical (e.g., chemical, mechanical, electrical systems), biological, and economic systems. Research on chaotic systems is now widespread and growing rapidly. Yet almost all published results on this subject are based on computer simulations of discrete equations [e.g., 2,3] or ordinary differential equations [e.g., 4,5]. Moreover, while some of these equations represent greatly simplified models of physical systems [e.g., 4], most are merely contrived equations. Since trajectories of chaotic systems are extremely sensitive to initial conditions and other parameters, the trajectories obtained from computer simulation are not correct solutions in view of inevitable roundoff and local truncation errors: they are in fact pseudo-orbits [6]. However, the attractors derived from computer simulation are generally regarded as a meaningful characterization of the chaotic system. A clarification of proper interpretation of all these simulation results are not yet in sight.

Our goal in this paper is to build a simple physical system and actually measure the chaotic phenomena in the laboratory. The system we built in a simple electronic circuit conceived by Chua [7], as shown in Fig. 1(a). It has 5 elements: a linear resistor, a linear inductor, 2 linear capacitors, and a nonlinear 2-terminal resistor described by the 5-segment v_R-i_R characteristic shown in Fig. 1(b). Since this nonlinear element is not available as an off-the-shelf component, we have designed the op amp circuit in Fig. 1(c) whose measured v_R-i_R characteristic in Fig. 1(d) is virtually identical to that of Fig. 1(b). The v_R-i_R characteristic in Fig. 1(b) can be described exactly by the equation [8]:

$$i_R = v_R - \frac{3}{4}|v_R + 7.5| - \frac{3}{20}|v_R + 2.5| + \frac{3}{20}|v_R - 2.5| + \frac{3}{4}|v_R - 7.5| \triangleq g(v_R) \tag{1}$$

Using this analytic expression, Chua's circuit is described by the following autonomous system of 3 ordinary differential equations:

$$\boxed{\begin{aligned} \dot{v}_{C1} &= \frac{1}{C1}\left[\frac{1}{R}(v_{C2} - V_{C1}) - g(v_{C1})\right] \\ \dot{v}_{C2} &= \frac{1}{C2}\left[\frac{1}{R}(v_{C1} - v_{C2}) - i_L\right] \\ \dot{i}_L &= \frac{1}{L} v_{C2} \end{aligned}} \tag{2}$$

Rather than simulating these equations on the computer, we carried out a careful experimental study of this circuit in the laboratory. A summary of the various bifurcation and chaotic phenomena we have so far observed will be presented in the following sections in the form of actual scope pictures. We hope that these pictures will be useful in developing a theoretical explanation of the chaotic phenomena and the structure of the attractor in Chua's circuit.

2. Chua's Attractor

Consider Chua's circuit in Fig. 1(a) with the following element values:

$$R = 1.53\ k\Omega\ , \quad L = 8\ mH\ , \quad C_1 = .005\ \mu F\ , \quad C_2 = .1\ \mu F$$

The nonlinear resistor is described by the v-i characteristic in Fig. 1(d). Figures 2(b)-(d) display the same chaotic attractor in (v_{C2}, v_{C1})-plane, (i_L, v_{C1})-plane, and (i_L, v_{C2})-plane, respectively. Figure 2(a) is a blown up version of Fig. 2(b) inside the limit cycle. The chaotic attractor consists of two rings joined at the upper and the lower edges by a thin sheet of ribbon made of trajectories. Note that depending on the initial conditions, all trajectories are either attracted to the stable limit cycle or to the chaotic attractor. The boundary set points, which divide the two attractors, appears from our measurement to be extremely complicated. It is definitely not a 2-dimensional surface, but one with a fractal [9] dimension. The existence of the stable limit cycle is quite natural, since the domain of attraction for the chaotic attractor is bounded, there must exist an entity which separates the domain of attraction from other initial conditions.

3. Bifurcation Phenomena with Respect to R

Consider the circuit in Fig. 1(a) with the following element values:

$$C_1 = .005\ \mu F\ , \quad C_2 = .05\ \mu F\ , \quad L = 12.45\ mH$$

Figures 3(a)-(e) display the period-doubling route to chaos in Chua's circuit as R is varied. The single-loop limit cycle in Fig. 3(a) represents a well-defined periodic waveform spawned by a Hopf bifurcation process when a pair of complex-conjugate eigenvalues associated with an equilibrium point of this circuit (described by equations (2)) crosses the imaginary axis and enters the right-half plane. A slight decrease in the value of R leads to the sequence of Lissajous figures shown in Figs. 3(b)-(e). This doubling of period is clearly seen by comparing the spectra fo Fig. 3(a) and Fig. 3(b). Note that new frequency components corresponding to one half of the frequency components in Fig. 3(a) have appeared in the spectrum of the double-loop limit cycle (Fig. 3(b)).

As we continue to decrease R in small amounts, we observe successive period-doubling giving rise to 4 and 8-loop limit cycles. Again note the appearance of new frequency components in the corresponding spectra. A further small decrease in R leads to the chaotic attractor in Fig. 3(e). Since the range of R between Figs. 3(a) and 3(e) is very narrow (1.7 kΩ to 1.5 kΩ), as is typical of the convergence property of the period-doubling bifurcation parameter, we were unable to observe periodic waveforms with order higher than 8. Our results, however, suggest strongly that the chaotic attractor in Fig. 4(e) is spawned by a period-doubling mechanism.

4. Bigurcation Phenomena With Respect to C_1

Consider the circuit in Fig. 1(a) with the following element values:

$R = 1.43 \Omega k$, $C_2 = .05\ \mu F$, $L = 7.2$ mH

Figures 4(a)-(d) display the period doubling phenomena in Chua's circuit as C_1 is varied. The single-loop limit cycle in Fig. 4(a) represents a well-defined periodic waveform in (V_{C_2}, V_{C_1})-plane. A slight decrease in the value of C_1 leads to the doubling of period as shown in Fig. 4(b). Further decreases in the value of C_1 by small amounts result in the Lissajous figures shown in Figs. 4(c)-(d). This period-doubling phenomenon is also observed in the corresponding spectra. These pictures are not shown since they are qualitatively similar to those in Section 3. Since the range of C_1 between Figs. 4(a)-(d) is very narrow (6.20 nF to 5.88 nF), we were unable to observe periodic waveforms with order higher than 4.

5. Bifurcation Phenomena With Respect to L

Consider the circuit in Fig. 1(a) with the following element values:

$R = 1.43$ kΩ, $C_1 = .005\ \mu F$, $C_2 = 0.05\ \mu F$

Figures 5(a)-(d) display the period-doubling phenomena in Chua's circuit as L is varied. The single-loop limit cycle in Fig. 5(a) represents a well-defined periodic waveform in (V_{C_2},V_{C_1})-plane. A slight increase in the value of L leads to the sequence of Lissagous figures shown in Figs. 5(b)-(d). Figures 5(b)-(d) show 3 successive period-doublings giving rise to 2, 4, and 8-loop limit cycles as we continue to increase L. As in Sections 3 and 4 one expects that a further increase in L results in the appearance of a chaotic attractor. However, we discovered a new phenomenon <u>not</u> found in Sections 3 and 4, namely, the appearance of limit cycles with <u>odd</u> periods. Figures 6(a)-(c) display 5, 3, and 4-loop limit cycles along with the steady-state waveform for V_{C_1}, respectively. As we continue to increase L, the limit cycles in Figs. 6(d)-(f) are obtained. Note that as L increases, the attractor changes from a single ring-shaped spiral (Figs. 6(a)-(c)) to two ring-shaped spirals connected by two outer threads (Figs. 6(d)-(f)).

6. <u>Concluding Remarks</u>

In this paper we have presented a wide variety of bifurcation and chaotic pehnomena observed experimentally from a simple physical system, namely, Chua's circuit. In this section we would like to make some comments on the difficulties that any interested readers might encounter in building such circuits and observing these phenomena.

Firstly, since these phenomena are extremely sensitive to parameter changes one should not expect that a circuit, built with the element values given here, to reproduce the exact behavior reported in this paper. In other words, some fine tuning of the circuit parameters might be necessary. For example, in Section 2 the value of R would most probably be different than the one given here, if one attempted to obtain chaotic attractors as in Figs. 2(a)-(d). Furthermore, since in building the nonlinear resistor we have used the saturation regions of the op amps in Fig. 1(c), one might have to adjust some of the resistance values (given in the figure captions) to obtain the desired v-i characteristic as in Fig. 1(b).

Secondly, since the range of the values for which these bifurcation and chaotic phenomena occur (e.g. range of R in Section 3, range of C_1 in Section 4, and range of L in Section 5) is very narrow, one must proceed slowly and patiently to be able to observe these phenomena. It should also be noted that in order to obtain Figs. 2(c) and 2(d) a small current sensing resistor (5.6 Ω) was connected in series with L in the circuit shown in Fig. 1(a).

Finally, we mention that any 5-segment odd-symmetric v-i characteristic with any prescribed set of parameters as in Fig. 1(b) can be easily synthesized by using two op amps and at most 8 linear resistors. For details see [10].

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Figure Captions

Fig. 1. (a) Chua's circuit.

(b) V-i characteristic of nonlinear resistor.

(c) op amp circuit realization of the nonlinear resistor.

(d) Measured V-i characteristic obtained with V_{CC} = 15 V, R_1 = 3.67 kΩ, R_2 = 1.09 kΩ, R_3 = 5.43 kΩ, R_4 = 104 Ω, R_5 = 5.36 KΩ, and R_6 = 128 Ω. op amp: National/8035 741 LN.

Fig. 2. Chaotic attractor measured from Chua's circuit.

(a) Chaotic attractor in (V_{C_2}, V_{C_1})-plane. Scale: V_{C_1} = 2V/div, V_{C_2} = .4 V/div.

(b) Chaotic attractor in (a) along with stable limit cycle. Scale: V_{C_1} = 5V/div, V_{C_2} = 1V/div.

(c) Chaotic attractor and stable limit cycle in (i_L, V_{C_1})-plane. Scale: V_{C_1} = 5V/div, i_L = 3.39 mA/div.

(d) Chaotic attractor and stable limit cycle in (i_L, V_{C_2})-plane. Scale: V_{C_2} = 2V/div, i_L = 3.39 mA/div.

Fig. 3. Waveforms (left side) and spectra (right side) exhibiting the period-doubling route to chaos as R in varied in Chua's circuit.

(a) Single-loop cycle. Scale: V_{C_1} = 2V/div, V_{C_2} = .4V/div.

(b) Double-loop cycle. Scale: V_{C_1} = 2V/div, V_{C_2} = .4V/div.

(c) 4-loop cycle. Scale: V_{C_1} = 1V/div, V_{C_2} = .4V/div.

(d) 8-loop cycle. Scale: V_{C_1} = 1V/div, V_{C_2} = .4V/div.

(e) Chaotic attractor in (V_{C_2}, V_{C_1})-plane. Scale: V_{C_1} = 2V/div, V_{C_2} = .4V/div.

Fig. 4. Period-doubling in Chua's circuit as C_1 in varied. Scale: V_{C_1} = 2V/div, V_{C_2} = .4V/div.

(a) Single-loop cycle.

(b) Double-loop cycle.

(c) 4-loop cycle.

(d) chaotic attractor in (V_{C_2}, V_{C_1})-plane.

Fig. 5. Period-doubling in Chua's circuit as L in varied. Scale: V_{C_1} = 2V/div, V_{C_2} = .4V/div.

(a) Single-loop cycle.

(b) Double-loop cycle.

(c) 4-loop cycle.

(d) 8-loop cycle.

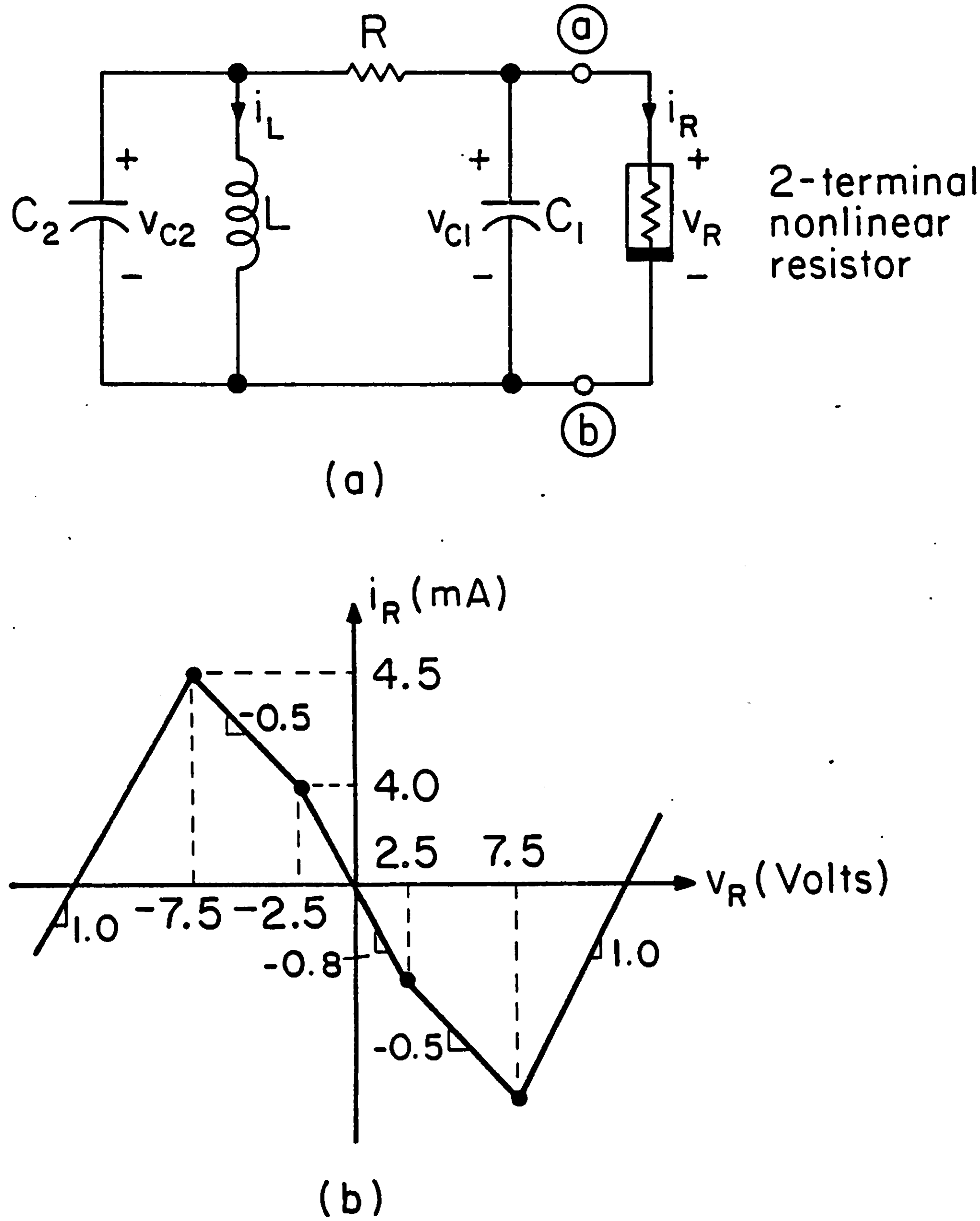
Fig. 6. Limit cycles (left side) and steady-state waveforms for V_{C_1} (right side) as we continue to increase L. Scale: V_{C_1} = 5V/div, V_{C_2} = .4V/div, t = .2 ms/div.

(a) 5-loop cycle.

(b) 3-loop cycle.

(c) 4-loop cycle.

(d)-(f) double-ring shape cycles.


R
a
b
i_L
i_R
+
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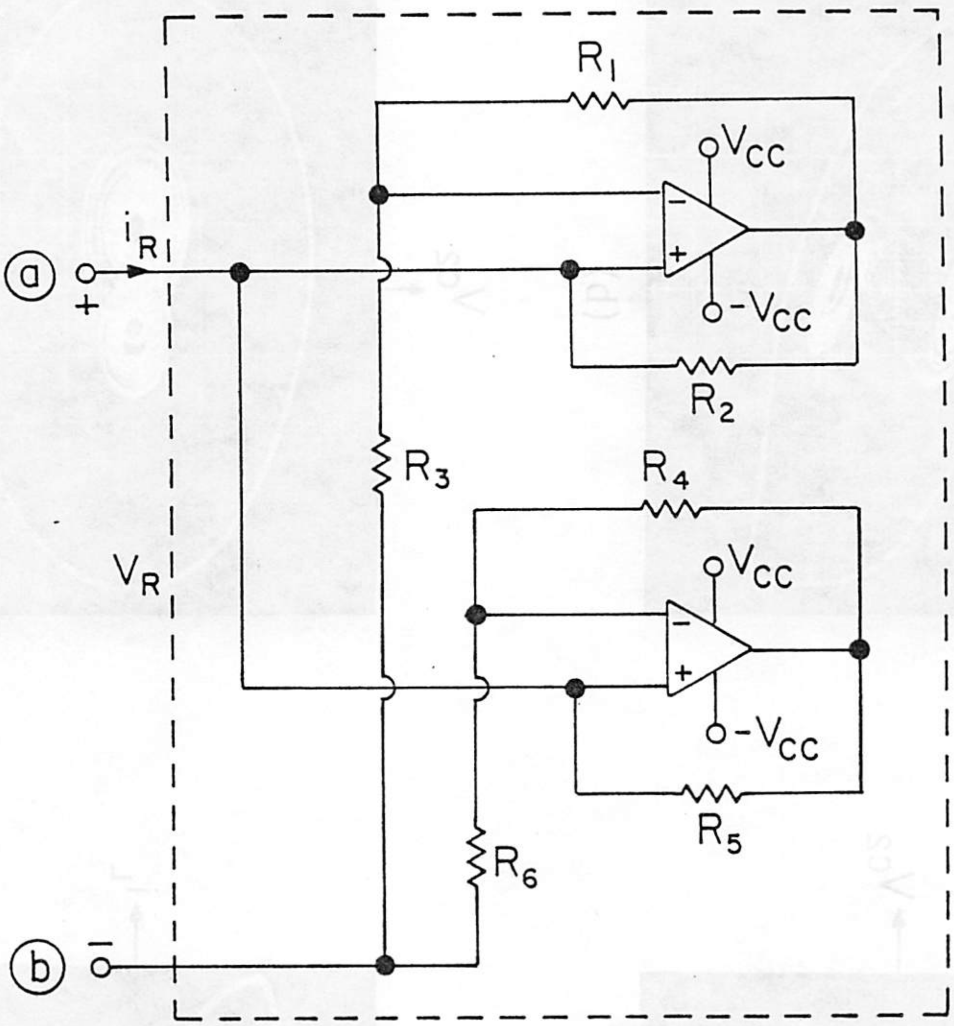
C_2
v_{C2}
L
v_{C1}
C_1
v_R
2-terminal
nonlinear
resistor
(a)
i_R (mA)
4.5
-0.5
4.0
2.5
7.5
v_R (Volts)
1.0
-7.5
-2.5
-0.8
1.0
-0.5
(b)

Fig. 1



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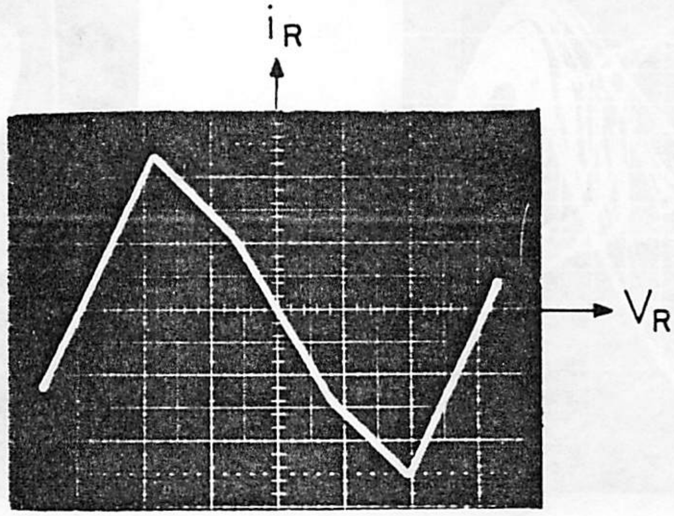


(d)

Fig. I (cont'd)

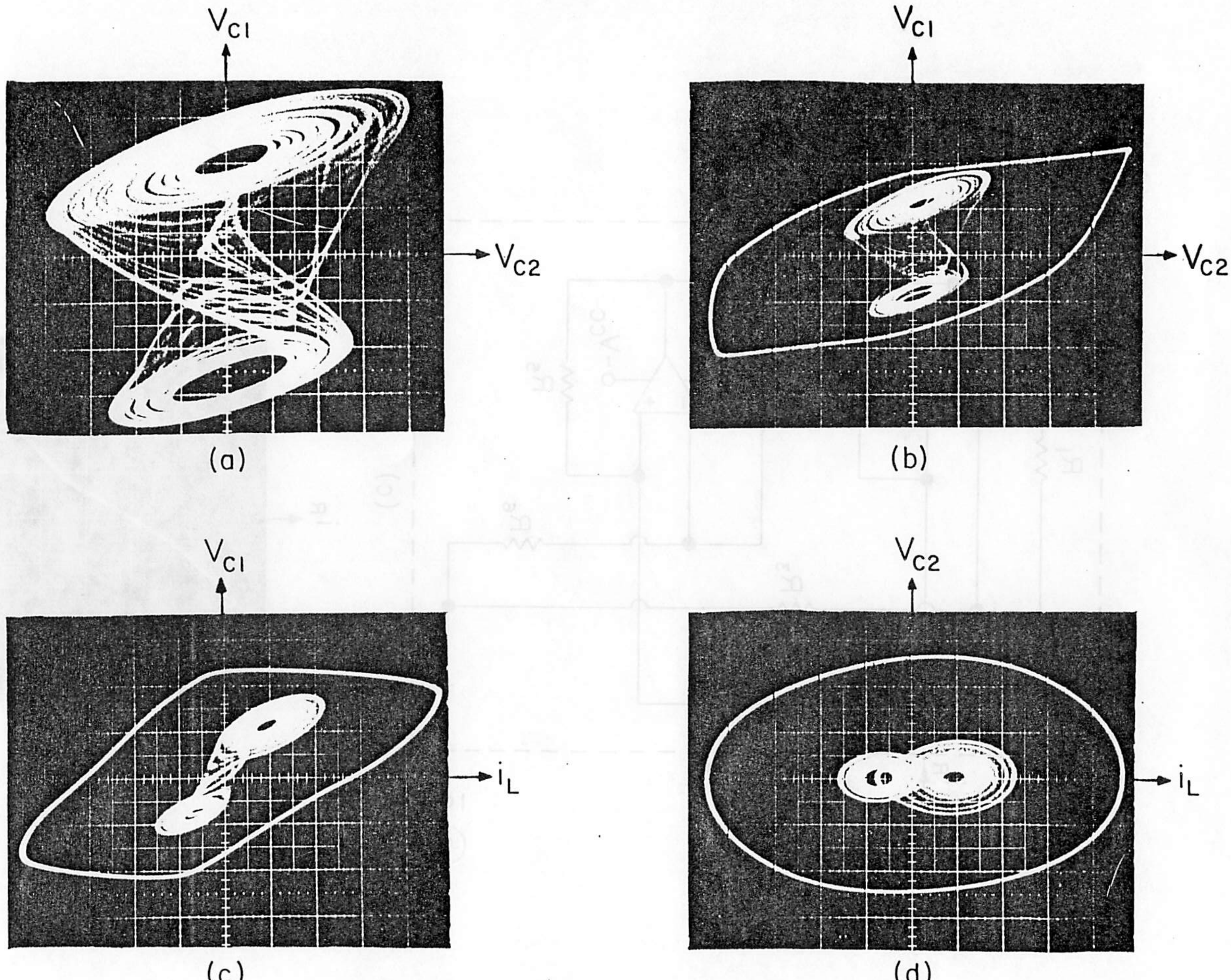


Fig. 2

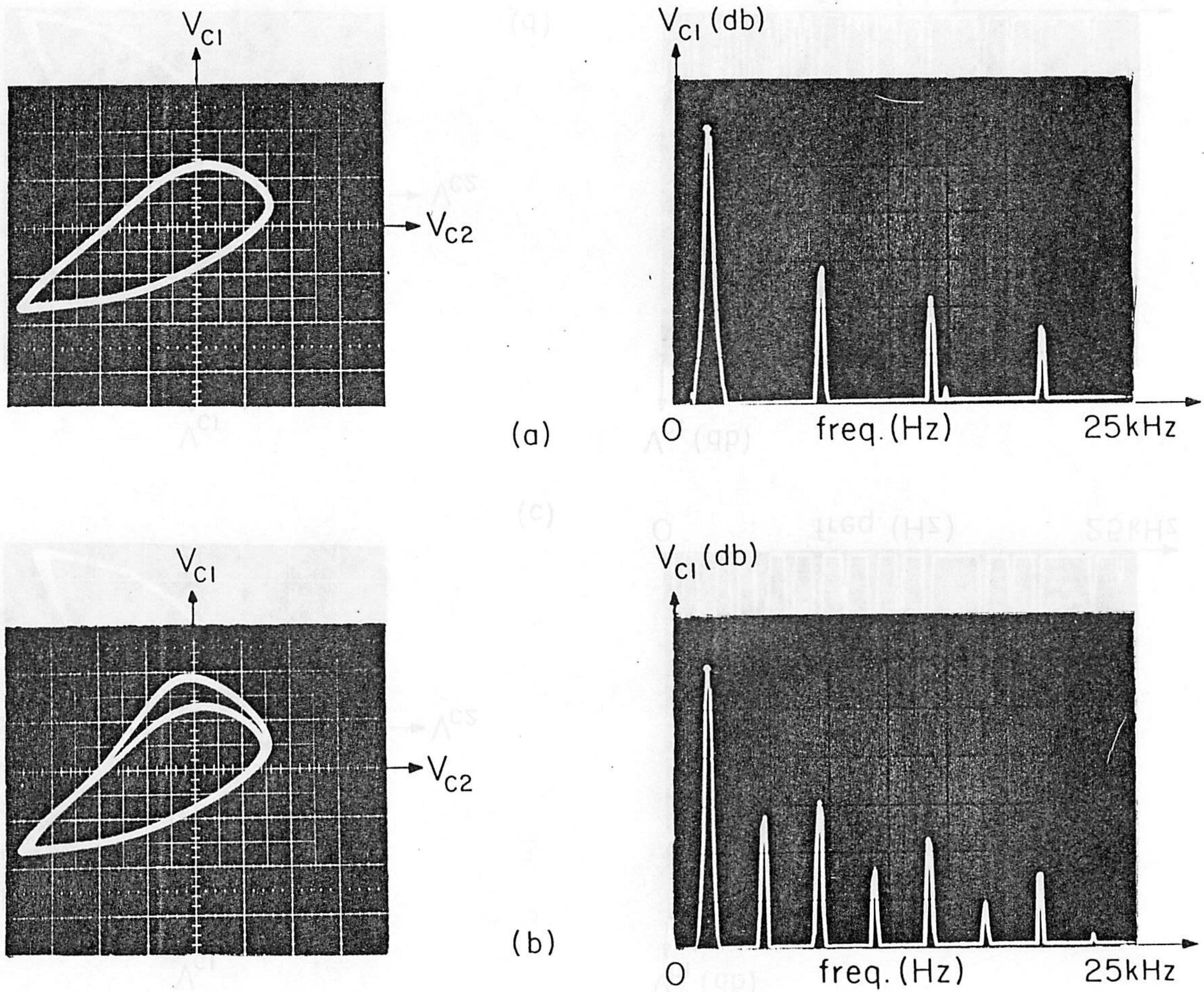
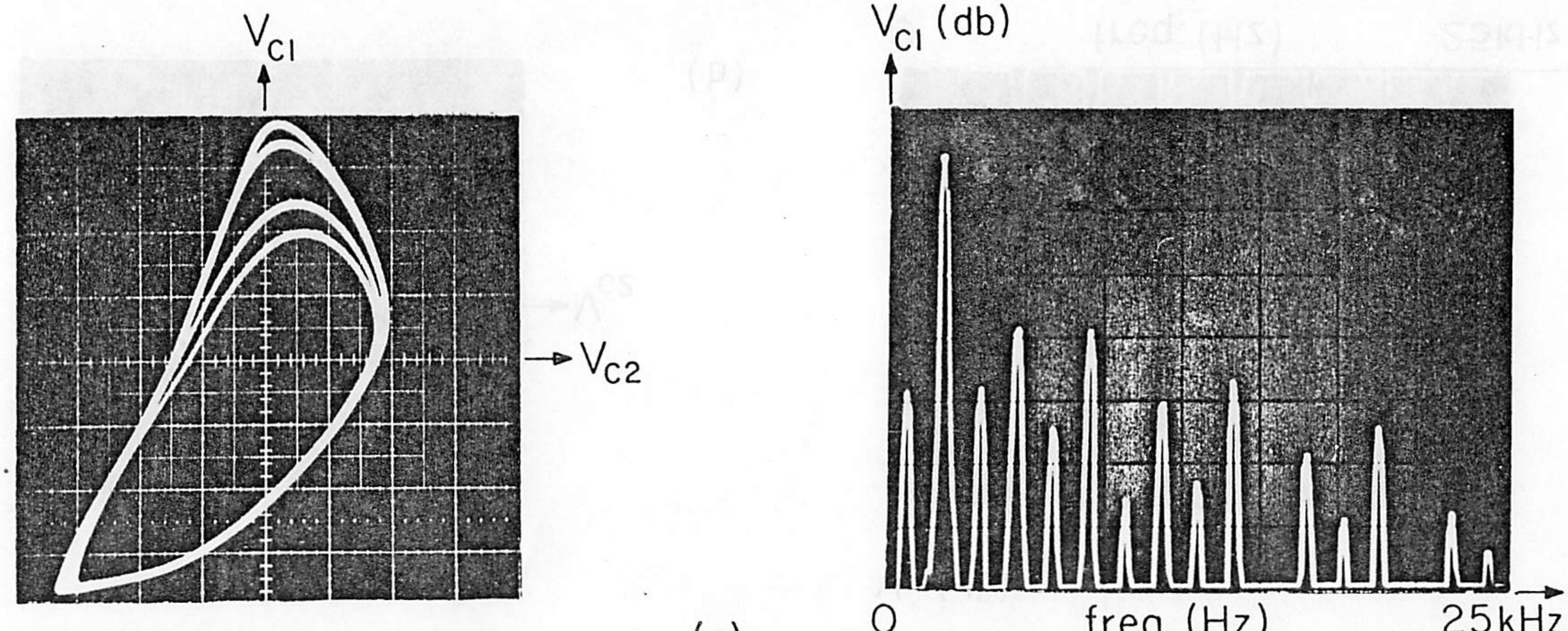
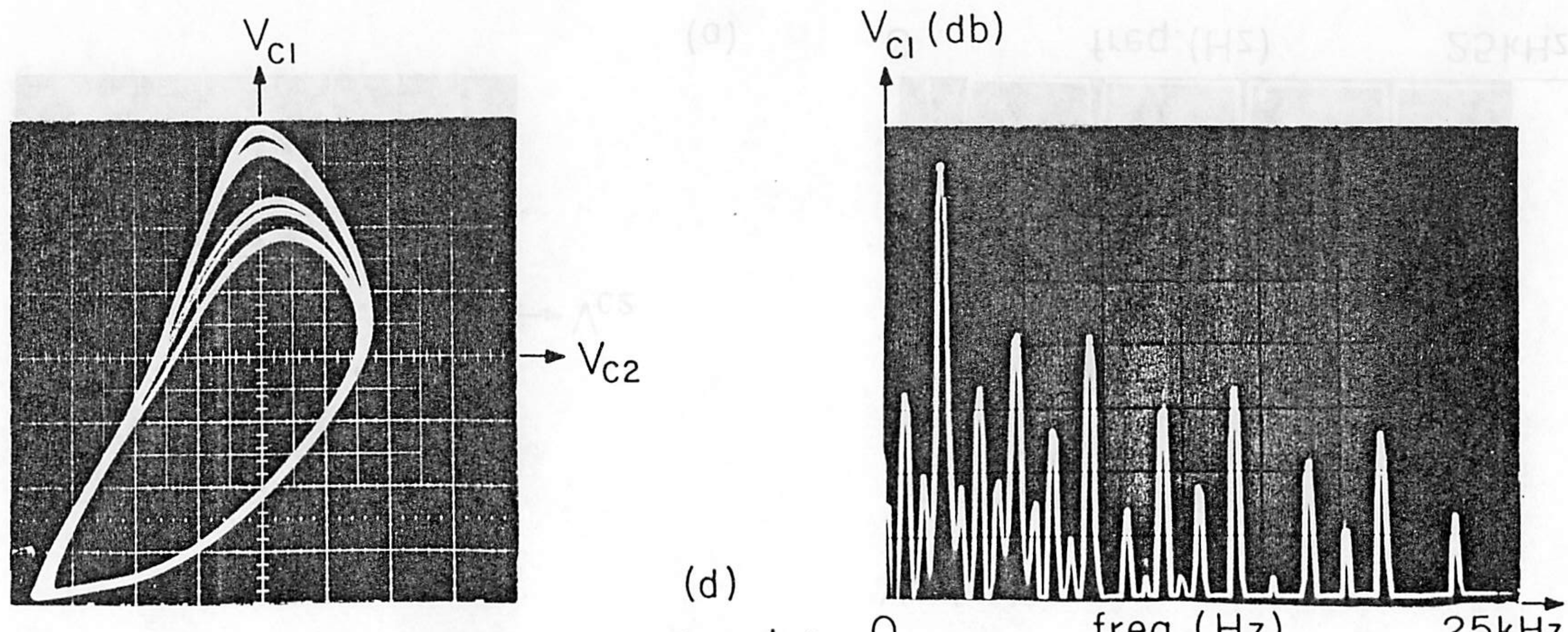


Fig. 3

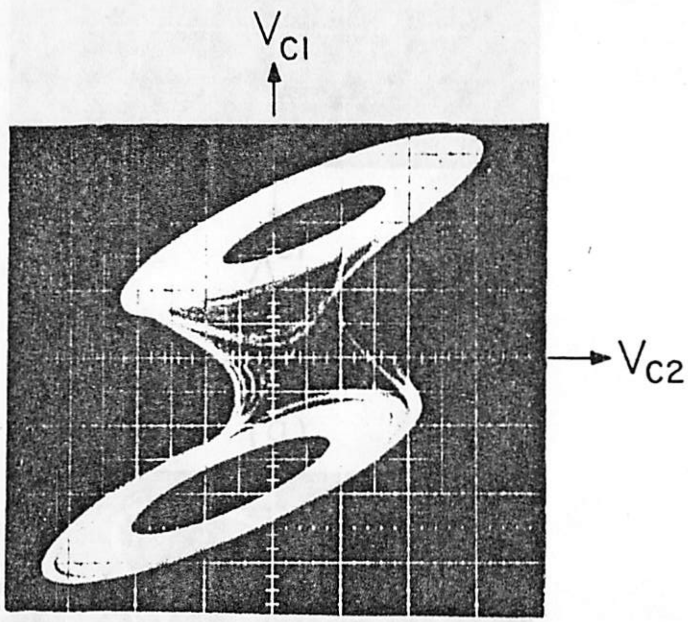


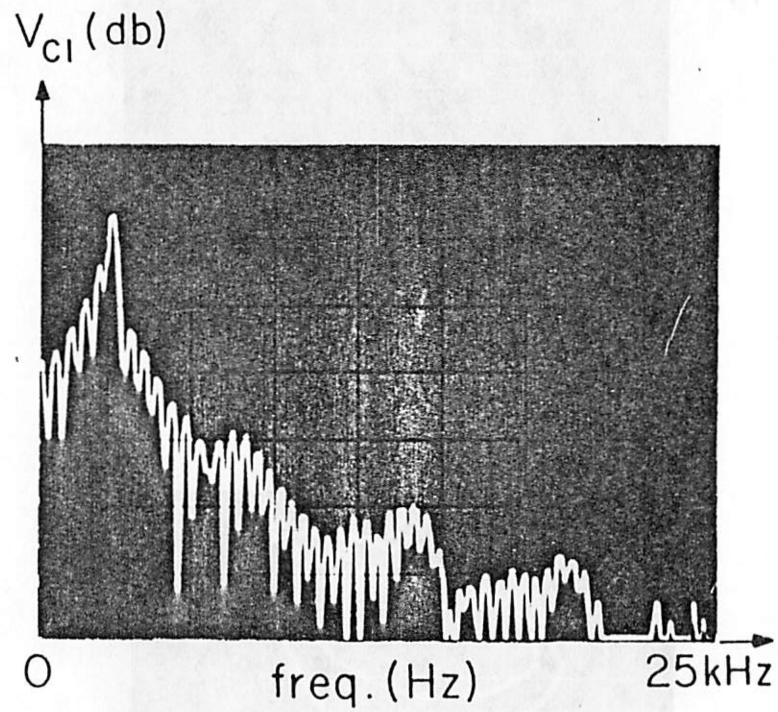
(c)



(d)

Fig. 3 (cont'd)





(e)

Fig. 3 (cont'd)

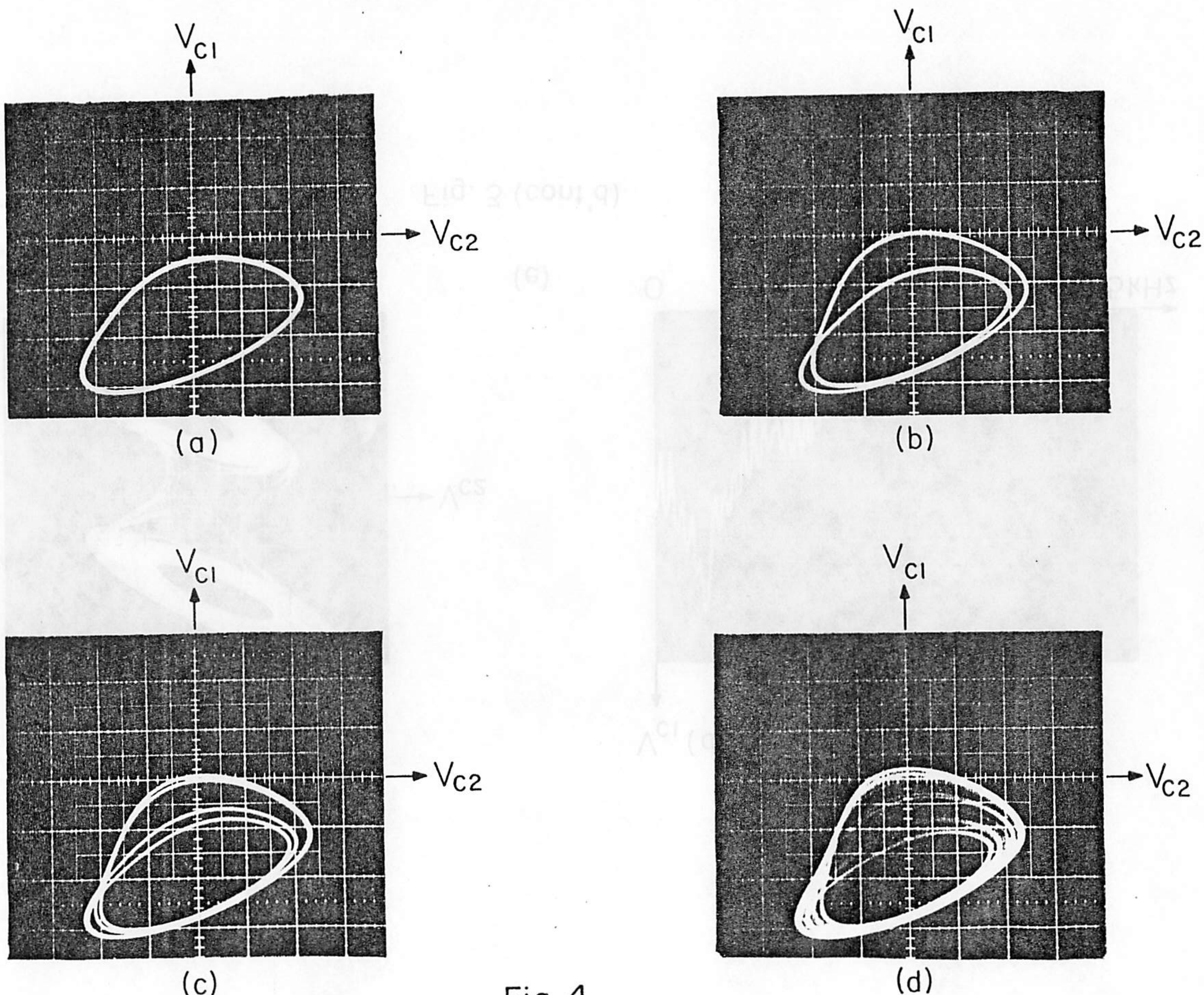


Fig. 4

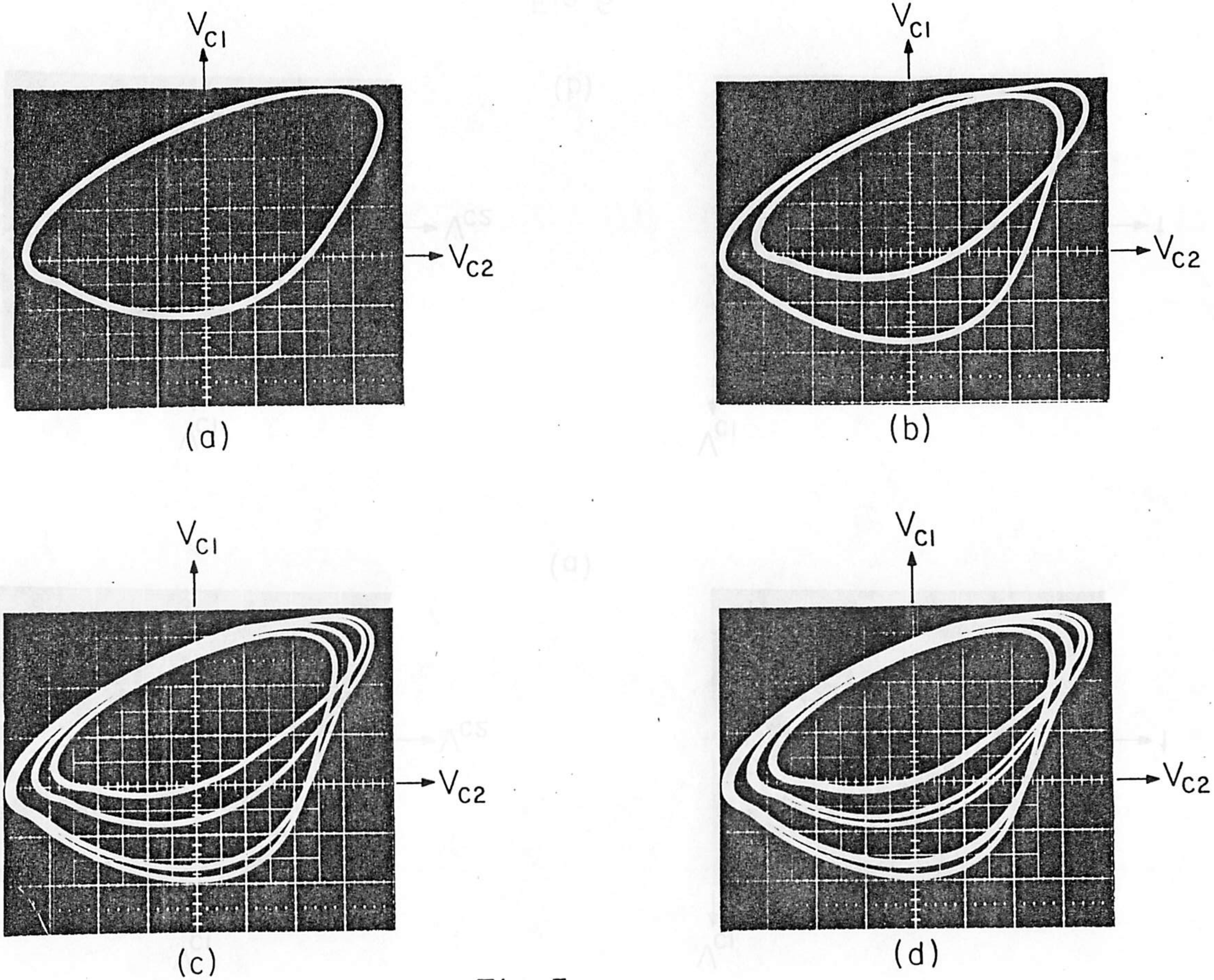


Fig. 5

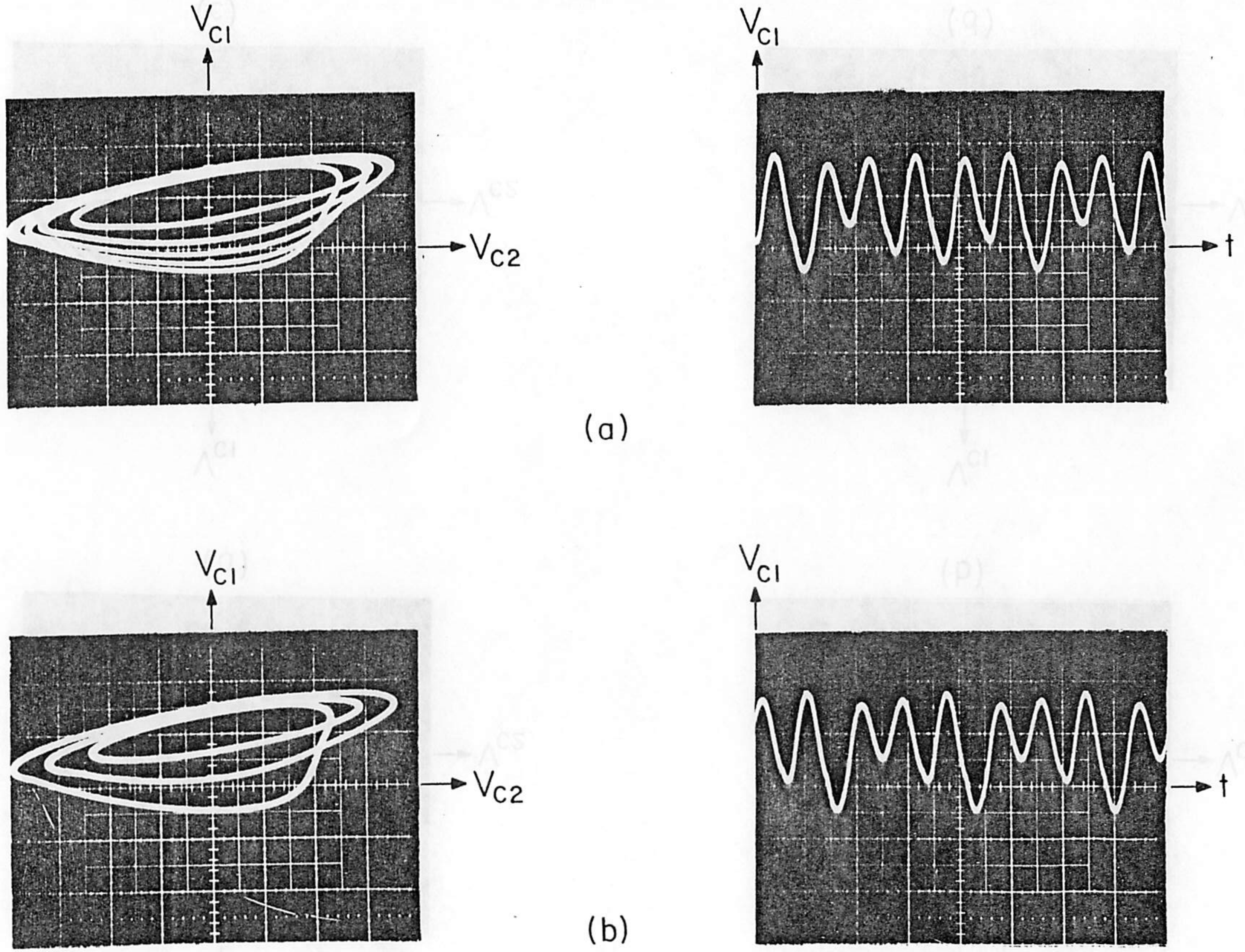
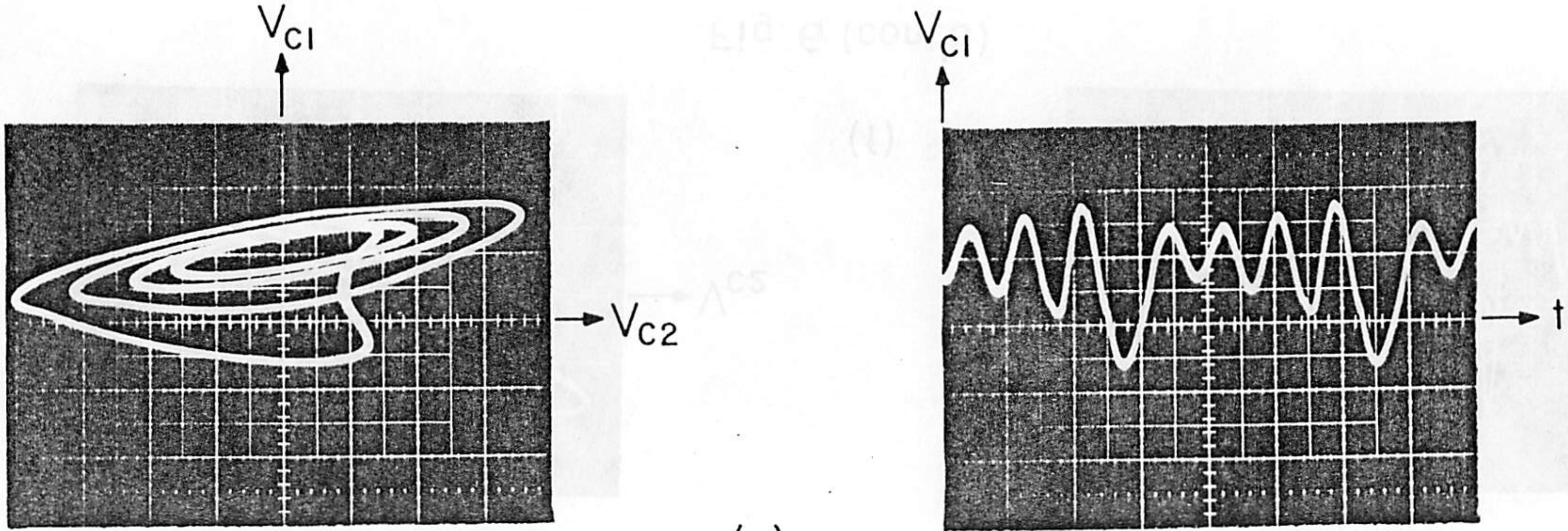


Fig. 6



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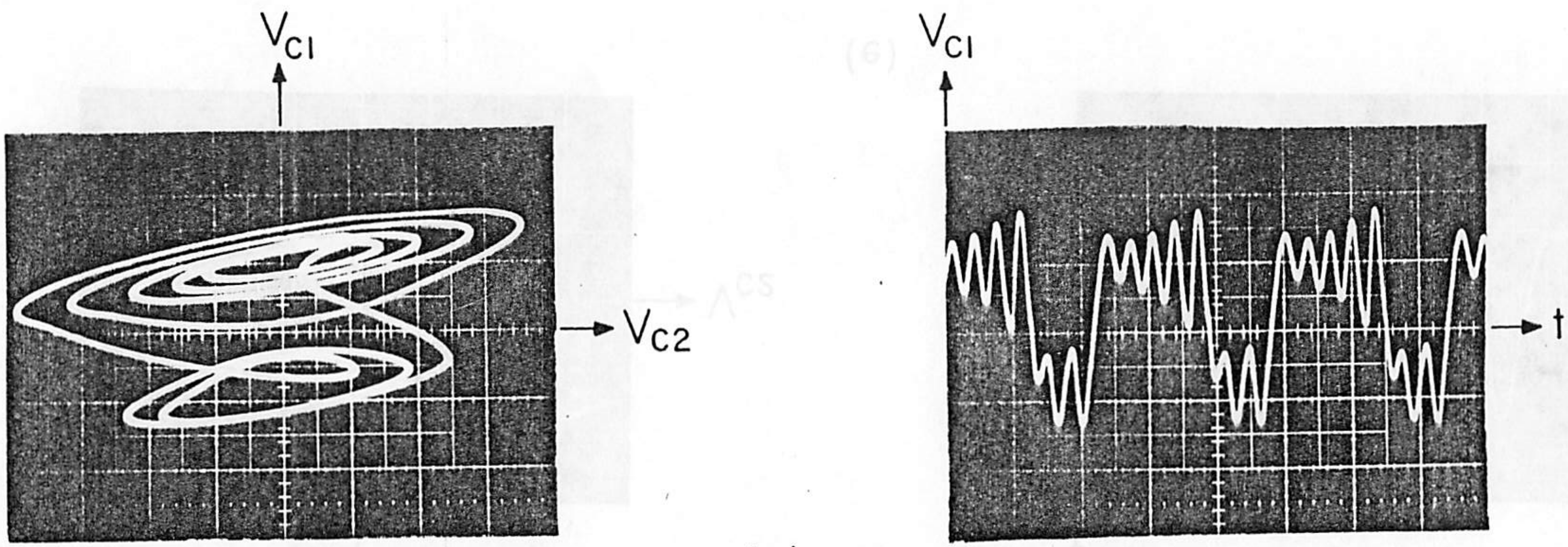

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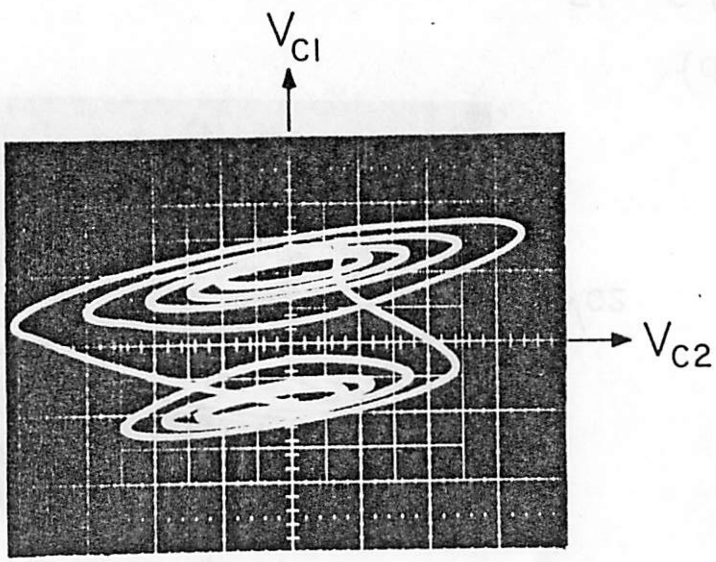
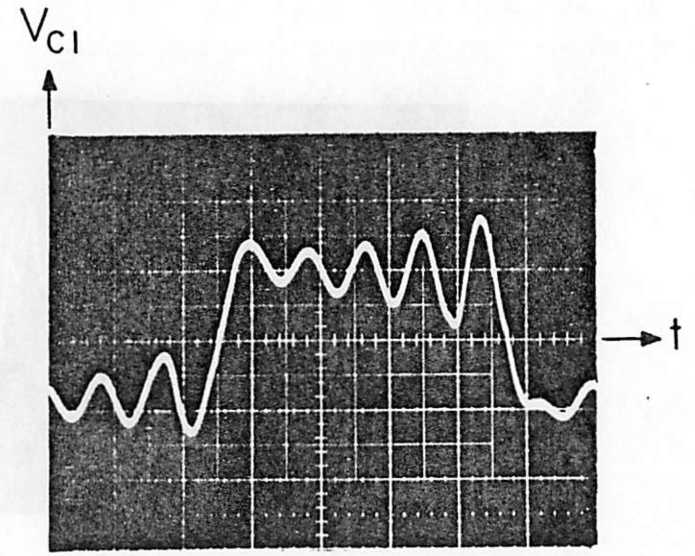
Fig. 6 (cont'd)





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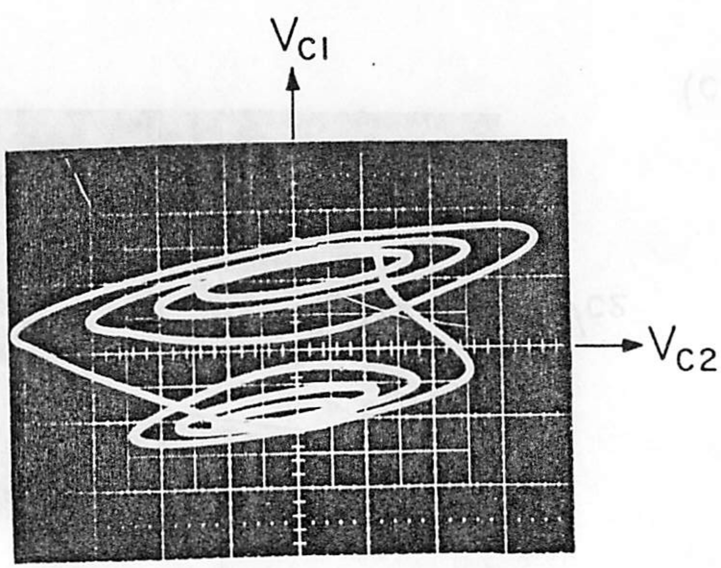

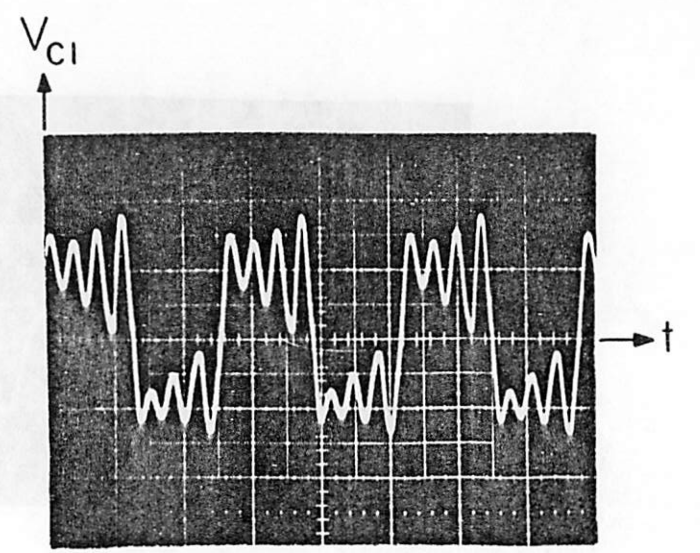

(f)

Fig. 6 (cont'd)