
Clarendon

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CHAPTER 8

Rotational Equilibrium and Dynamics

PHYSICS IN ACTION

One of the most popular early bicycles was the penny-farthing, first introduced in 1870. The bicycle was named for the relative sizes of its two wheels compared to the relative sizes of the penny and the farthing, two English coins. Early bicycles had no gears, just pedals attached directly to the wheel axle. This meant that the wheel turned once for every revolution of the pedals. For the penny-farthing, a gear was developed that allowed the wheel to turn twice for every turn of the pedals. More stable bicycles with gears and chains soon replaced the penny-farthing.

- *What makes a wheel difficult to rotate?*
- *How much does a wheel accelerate for a given applied force?*

CONCEPT REVIEW

Work **(Section 5-1)**

Energy **(Section 5-2)**

Momentum **(Section 6-1)**

Angular speed and acceleration (Section 7-1)

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8-1
Torque

8-1 SECTION OBJECTIVES

- Recognize the difference between a point mass and an extended object.
- Distinguish between torque and force.
- Calculate the magnitude of a torque on an object.
- Identify the lever arm associated with a torque on an object.

THE MAGNITUDE OF A TORQUE

You have been chosen to judge a race involving three objects: a solid sphere, a solid cylinder, and a hollow cylinder. The spectators for the race demand that the race be fair, so you make sure that all of the objects have the same mass and radius and that they all start from rest. Then you let the three objects roll down a long ramp. Is there a way to predict which one will win and which one will lose?

If you really performed such a race (see the Quick Lab on the next page), you would discover that the sphere would come in first and that the hollow cylinder would come in last. This is a little surprising because in the absence of friction the acceleration due to gravity is the same for all objects near the Earth's surface. Yet the acceleration of each of these objects is different.

In earlier chapters, the motion of an object was described by assuming the object was a point mass. This description, however, does not account for the differences in the motion of the objects in the race. This is because these objects are *extended objects.* An extended object is an object that has a definite, finite size and shape. Although an extended object can be treated as a point mass to describe the motion of its center of mass, a more sophisticated model is required to describe its rotational motion.

Figure 8-1
In general, an extended object, such as one of these pins or this bowling ball, can exhibit rotational and translational motion.

Rotational and translational motion can be separated

Imagine that you roll a strike while bowling. What happens when the bowling ball strikes the pins, as shown in **Figure 8-1?** The pins fly backward, spinning in the air. The complicated motion of each pin can be separated into a translational motion and a rotational motion, each of which can be analyzed separately. For now, we will concentrate on an object's rotational motion. Then we will combine the rotational motion of an object with its translational motion.

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Net torque produces rotation

Imagine a cat trying to leave a house by pushing perpendicularly on a cat-flap door. **Figure 8-2** shows a cat-flap door hinged at the top. In this configuration, the door is free to rotate around a line that passes through the hinge. This is the door's *axis of rotation.* When the cat pushes at the outer edge of the door with a force that is perpendicular to the door, the door opens. The ability of a force to rotate an object around some axis is measured by a quantity called **torque.**

torque

a quantity that measures the ability of a force to rotate an object around some axis

Torque depends on a force and a lever arm

If a cat pushed on the door with the same force but at a point closer to the hinge, the door would be more difficult to rotate. How easily an object rotates depends not only on how much force is applied but also on where the force is applied. The farther the force is from the axis of rotation, the easier it is to rotate the object and the more torque is produced. The perpendicular distance from the axis of rotation to a line drawn along the direction of the force is called the **lever arm,** or moment arm.

lever arm

the perpendicular distance from the axis of rotation to a line drawn along the direction of the force

Figure 8-3 shows a diagram of the force F applied by the pet perpendicular to the cat-flap door. If you examine the definition of *lever arm,* you will see that in this case the lever arm is the distance d shown in the figure, the distance from the pet's nose to the hinge. That is, d is the perpendicular distance from the axis of rotation to the line along which the applied force acts. If the pet pressed on the door at a higher point, the lever arm would be shorter. A smaller torque would be exerted for the shorter lever arm than for the one shown in **Figure 8-3.**

Figure 8-2
The cat-flap door rotates on a hinge, allowing pets to enter and leave a house at will.

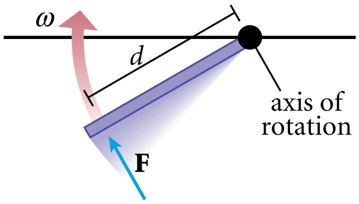

Figure 8-3
A force applied to an extended object can produce a torque. This torque, in turn, causes the object to rotate.

Quick Lab

Two-Object Races

MATERIALS LIST

- ✔ various solid cylinders, such as unopened soup cans
- ✔ various hollow cylinders, such as empty soup cans with the tops and bottoms removed or PVC pipes of different diameters and lengths
- ✔ various spheres, such as a golf ball, tennis ball, and baseball
- ✔ an incline about 1 m long

Place any two objects from the above list at the top of the incline, and release them simultaneously. Note which object reaches the bottom first. Repeat the race with various combinations of objects. See if you can discover a general rule to predict which object will win. (Hint: You may wish to consider factors such as mass, size, and shape.)

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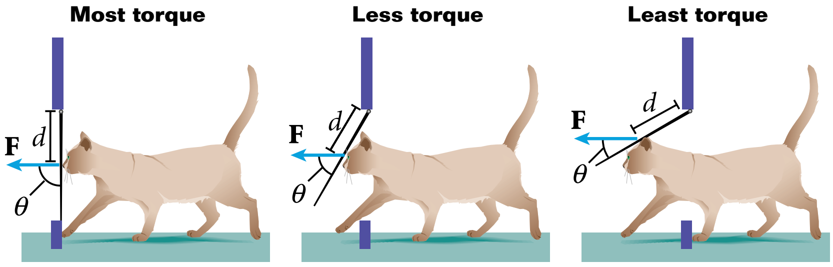

Figure 8-4
In each example, the cat is pushing on the same door at the same distance from the axis and with the same amount of force, but it is producing different amounts of torque.

Torque also depends on the angle between a force and a lever arm

Forces do not have to be perpendicular to an object to cause the object to rotate. Imagine the cat-flap door again. What would happen if the cat pushed on the door at an angle to the door, rather than perpendicular to it as shown in **Figure 8-4**? The door would still rotate, but not as easily.

The symbol for torque is the Greek letter *tau* (τ), and the magnitude of the torque is given by the following equation:

TORQUE

$$\tau = Fd(\sin\theta)$$

torque = force × lever arm

The SI unit of torque is the N•m. Notice that the inclusion of the angle θ, the angle between the force and the distance from the axis, in this equation takes into account the changes in torque shown in **Figure 8-4.**

Figure 8-5 shows a wrench pivoted around a bolt. In this case, the applied force acts at an angle to the wrench. The quantity d is the distance from the axis of rotation to the point where force is applied. The quantity $d(\sin\theta)$, however, is the *perpendicular* distance from the axis of rotation to a line drawn along the direction of the force, so it is the lever arm.

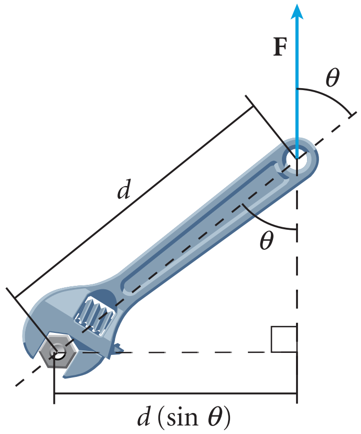

Figure 8-5
The direction of the lever arm is always perpendicular to the direction of the applied force.

PHYSICS INTERACTIVE TUTOR

Module 9
"Torque"
provides an interactive lesson with guided problem-solving practice to teach you about many aspects of rotational motion, including torque.

THE SIGN OF A TORQUE

Torque, like displacement and force, is a vector quantity. However, for the purposes of this book we will primarily deal with torque as a scalar. Therefore, we will assign each torque a positive or negative sign, depending on the direction the force tends to rotate an object. We will use the convention that the sign of the torque resulting from a force is positive if the rotation is counterclockwise and negative if the rotation is clockwise. In calculations, remember to assign positive and negative values to forces and displacements according to the sign convention established in Chapter 2.

To determine the sign of a torque, imagine that it is the only torque acting on the object and that the object is free to rotate. Visualize the direction the object would rotate under these conditions. If more than one force is acting, then each force has a tendency to produce a rotation and should be treated separately. Be careful to associate the correct sign with each torque.

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For example, imagine that you are pulling on a wishbone with a perpendicular force F_1 and that a friend is pulling in the opposite direction with a force F_2. If you pull the wishbone so that it would rotate counterclockwise, then you exert a positive torque of magnitude F_1d_1. Your friend, on the other hand, exerts a negative torque, $-F_2d_2$. To find the net torque acting on the wishbone, simply add up the individual torques.

$$\tau_{net} = \Sigma\tau = \tau_1 + \tau_2 = F_1d_1 + (-F_2d_2)$$

When you properly apply the sign convention, the sign of the net torque will tell you which way the object will rotate, if at all.

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SAMPLE PROBLEM 8A

Torque

PROBLEM

A basketball is being pushed by two players during tip-off. One player exerts a downward force of 11 N at a distance of 7.0 cm from the axis of rotation. The second player applies an upward force of 15 N at a perpendicular distance of 14 cm from the axis of rotation. Find the net torque acting on the ball.

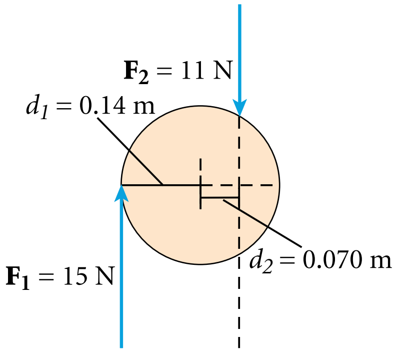
SOLUTION

1. DEFINE

Given: $F_1 = 15$ N $F_2 = 11$ N
$d_1 = 0.14$ m $d_2 = 0.070$ m

Unknown: $\tau_{net} = ?$

Diagram:



2. PLAN

Choose an equation(s) or situation: Apply the definition of torque to each force and add up the individual torques.

$$\tau = Fd$$
$$\tau_{net} = \tau_1 + \tau_2 = F_1d_1 + F_2d_2$$

3. CALCULATE

Substitute the value(s) into the equation(s) and solve: First, determine the torque produced by each force. Each force produces clockwise rotation, so both torques are negative.

$$\tau_1 = F_1d_1 = -(15\ \text{N})(0.14\ \text{m}) = -2.1\ \text{N}\bullet\text{m}$$

$$\tau_2 = F_2d_2 = -(11\ \text{N})(0.070\ \text{m}) = -0.77\ \text{N}\bullet\text{m}$$

$$\tau_{net} = -2.1\ \text{N}\bullet\text{m} - 0.77\ \text{N}\bullet\text{m}$$

$$\boxed{\tau_{net} = -2.9\ \text{N}\bullet\text{m}}$$

CALCULATOR SOLUTION

Your calculator will give the answer as 2.87. Because of the significant figure rule for addition, the answer should be rounded to 2.9.

4. EVALUATE

The net torque is negative, so the ball rotates in a clockwise direction.

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PRACTICE 8A

Torque

1. Find the magnitude of the torque produced by a 3.0 N force applied to a door at a perpendicular distance of 0.25 m from the hinge.

2. A simple pendulum consists of a 3.0 kg point mass hanging at the end of a 2.0 m long light string that is connected to a pivot point.
   a. Calculate the magnitude of the torque (due to the force of gravity) around this pivot point when the string makes a 5.0° angle with the vertical.
   b. Repeat this calculation for an angle of 15.0°.

3. If the torque required to loosen a nut on the wheel of a car has a magnitude of 40.0 N•m, what *minimum* force must be exerted by a mechanic at the end of a 30.0 cm wrench to loosen the nut?

Section Review

1. In which of the following situations should the object(s) be treated as a point mass? In which should the object(s) be treated as an extended object?
   a. a baseball dropped from the roof of a house
   b. a baseball rolling toward third base
   c. a pinwheel in the wind
   d. Earth traveling around the sun

2. What is the rotational analog of a force? How does it differ from a force? On what quantities does it depend?

3. Calculate the torque for each force acting on the bar in **Figure 8-6.** Assume the axis is perpendicular to the page and passes through point *O*. In what direction will the object rotate?

4. How would the force needed to open a door change if you put the handle in the middle of the door?

5. **Physics in Action** How does the length of the pedal arm on a penny-farthing bicycle affect the amount of torque applied to the front wheel?

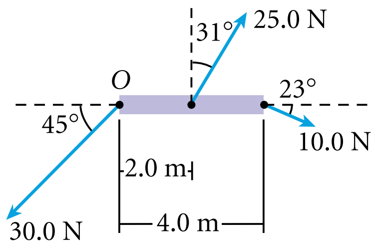

Figure 8-6

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8-2
Rotation and inertia

8-2 SECTION OBJECTIVES

- **Identify the center of mass of an object.**
- **Distinguish between mass and moment of inertia.**
- **Define the second condition of equilibrium.**
- **Solve problems involving the first and second conditions of equilibrium.**

CENTER OF MASS

Locating the axis of rotation for the cat-flap door is simple: it rotates on its hinges because the house applies a force that keeps the hinges in place. Now imagine you are playing fetch with your dog, and you throw a stick up into the air for the dog to retrieve. How can you determine the point around which the stick will rotate as it travels through the air? Unlike the cat-flap door, the stick is not attached to anything. There is a special point around which the stick rotates if gravity is the only force acting on the stick. This point is called the stick's **center of mass.**

center of mass

the point at which all the mass of the body can be considered to be concentrated when analyzing translational motion

Rotational and translational motion can be combined

The center of mass is also the point at which all the mass of the body can be considered to be concentrated. This means that the complete motion of the stick is a combination of both translational and rotational motion. The stick rotates in the air around its center of mass. The center of mass, in turn, moves as if the stick were a point mass, with all of its mass concentrated at that point for purposes of analyzing its translational motion.

Note that the hammer in **Figure 8-7** rotates about its center of mass as it moves through the air. As the rest of the hammer spins, the center of mass moves along the path of a projectile.

For regularly shaped objects, such as a sphere or a cube, the center of mass is at the geometric center of the object. For more complicated objects, calculating the location of the center of mass is more difficult and is beyond the scope of this book. While the center of mass is the position at which an extended object's mass can be treated as a point mass, the *center of gravity* is the position at which the gravitational force acts on the extended object as if it were a point mass. For most situations in this book, the center of mass and the center of gravity are equivalent.

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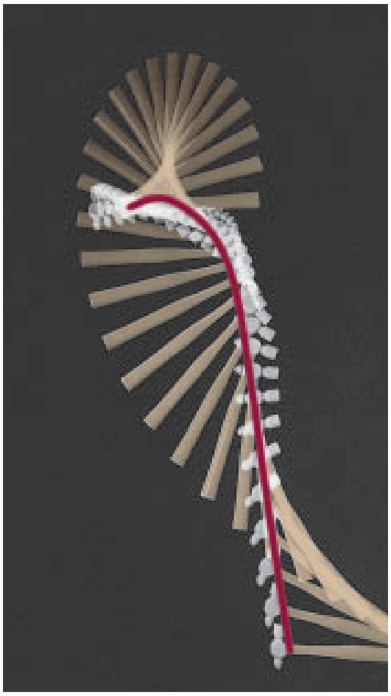

Figure 8-7
The point around which this object rotates is the center of mass. The center of mass traces out a parabola.

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MOMENT OF INERTIA

moment of inertia

the tendency of a body rotating about a fixed axis to resist a change in rotational motion

Imagine you are rotating a baseball bat. There are many axes around which the bat can be rotated. But it is easier to rotate the bat around some axes than others, even though the bat's mass has not changed. The resistance of an object to changes in rotational motion is measured by a quantity called the **moment of inertia.** The term *moment* has a meaning in physics that is different from its everyday meaning. The moment of inertia is a measure of the object's resistance to a change in its rotational motion about some axis.

Moment of inertia is the rotational analog of mass

The moment of inertia is similar to mass because they are both forms of inertia. However, there is an important difference between the inertia that resists changes in translational motion (mass) and the inertia that resists changes in rotational motion (moment of inertia). Mass is an intrinsic property of an object, and the moment of inertia is not. It depends on the object's mass and the distribution of that mass around the axis of rotation. The farther the mass of an object is, on average, from the axis of rotation, the greater is the object's moment of inertia and the more difficult it is to rotate the object. This is why, in the race on page 278, the solid sphere came in first and the hollow cylinder came in last. The mass of the hollow cylinder is all concentrated around its rim (large moment of inertia), while the mass of the sphere is more evenly distributed throughout its volume (small moment of inertia).

Calculating the moment of inertia

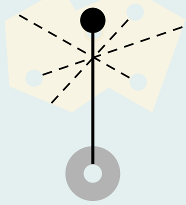
According to Newton's second law, when a net force acts on an object, the resulting acceleration of the object depends on the object's mass. Similarly, when a net torque acts on an object, the resulting change in the rotational motion of the object depends on the object's moment of inertia.

Quick Lab

Finding the Center of Mass Experimentally

MATERIALS LIST

- ✔ cardboard
- ✔ scissors
- ✔ hole punch
- ✔ pushpin or nail
- ✔ corkboard or tackboard
- ✔ length of string, about 40 cm
- ✔ straightedge
- ✔ pencil or pen
- ✔ weight, such as a washer



Cut out an irregular shape from the cardboard, and punch 3–5 holes around the edge of the shape. Put the pushpin through one of the holes, and tack the shape to a corkboard so that the shape can rotate freely. (You may also hang the shape from a nail in the wall.)

Attach the weight to the end of the string and hang the string from the pushpin or nail. When the string stops moving, trace a line on the cardboard that follows the string.

Repeat for each of the holes in the cardboard. The point where the lines intersect is the center of mass.

The calculation of the moment of inertia is a straightforward but often tedious process. Fortunately, some simple formulas are available for common shapes. **Table 8-1** gives the moments of inertia for some common shapes. When the need arises, you can use this table to determine the moment of inertia of a body having one of the listed shapes.

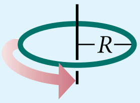
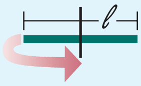
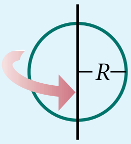
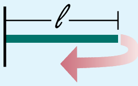
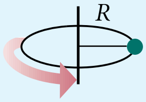
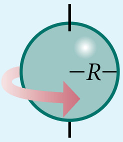
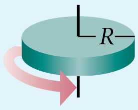
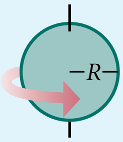
The units for moment of inertia are $kg \bullet m^2$. To get an idea of the size of this unit, note that bowling balls typically have moments of inertia about an axis through their centers ranging from about $0.7\ kg \bullet m^2$ to $1.8\ kg \bullet m^2$, depending on the mass and size of the ball.

Notice that the moment of inertia for the solid sphere is indeed smaller than the moment of inertia for the thin hoop, as expected. In fact, the moment of inertia for the thin hoop about the symmetry axis through the center of mass is the largest moment of inertia that is possible for any shape.

Also notice that a point mass in a circular path, such as a ball on a string, has the same moment of inertia as the thin hoop if the distance of the point mass from its axis of rotation is equal to the hoop's radius. This shows that only the distance of a mass from the axis of rotation is important in determining the moment of inertia for a shape. At a given radius from an axis, it does not matter how the mass is distributed around the axis.

Finally, recall the example of the rotating baseball bat that began this section. A bat can be modeled as a rotating thin rod. **Table 8-1** shows that the moment of

Table 8-1 The moment of inertia for a few shapes

Shape	Moment of inertia	Shape	Moment of inertia
 thin hoop about symmetry axis	MR^2	 thin rod about perpendicular axis through center	$\frac{1}{12}M\ell^2$
 thin hoop about diameter	$\frac{1}{2}MR^2$	 thin rod about perpendicular axis through end	$\frac{1}{3}M\ell^2$
 point mass about axis	MR^2	 solid sphere about diameter	$\frac{2}{5}MR^2$
 disk or cylinder about symmetry axis	$\frac{1}{2}MR^2$	 thin spherical shell about diameter	$\frac{2}{3}MR^2$

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inertia of a thin rod is larger if the rod is longer or more massive. When a bat is held at its end, its length is greatest with respect to the rotation axis, and so its moment of inertia is greatest. The moment of inertia decreases, and the bat is easier to swing if you hold the bat closer to the center. Baseball players sometimes do this either because a bat is too heavy (large M) or is too long (large ℓ). In both cases, the player decreases the bat's moment of inertia.

ROTATIONAL EQUILIBRIUM

Figure 8-8
The two forces exerted on this table are equal and opposite, yet the table moves. How is this possible?

Imagine that you and a friend are trying to move a piece of heavy furniture and that you are both a little confused. Instead of pushing from the same side, you push on opposite sides, as shown in **Figure 8-8.** The two forces acting on the furniture are equal in magnitude and opposite in direction. Your friend thinks the condition for equilibrium is satisfied because the two forces balance each other. He says the piece of furniture shouldn't move. But it does; it rotates in place.

Equilibrium requires zero net force and zero net torque

The piece of furniture can move even though the net force acting on it is zero because the net torque acting on it is not zero. If the net force on an object is zero, the object is in *translational equilibrium.* If the net torque on an object is zero, the object is in *rotational equilibrium.* For an object to be completely in equilibrium, both rotational and translational, there must be both zero net force and zero net torque, as summarized in **Table 8-2.** The dependence of equilibrium on the absence of net torque is called the *second condition for equilibrium.*

To apply the first condition for equilibrium to an object, it is necessary to add up all of the forces acting on the object (see Chapter 4). To apply the second condition for equilibrium to an object, it is also necessary to choose an axis of rotation around which to calculate the torque. Which axis should be chosen? The answer is that it does not matter. The resultant torque acting on an object in rotational equilibrium is independent of where the axis is placed. This fact is useful in solving rotational equilibrium problems because an unknown force that acts along a line passing through this axis of rotation will not produce any torque. Beginning a diagram by arbitrarily setting an axis where a force acts can eliminate an unknown in the problem.

Table 8-2 Conditions for equilibrium

Type of equilibrium	Symbolic equation	Meaning
translational	$\sum F = 0$	The net force on an object must be zero.
rotational	$\sum \tau = 0$	The net torque on an object must be zero.

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SAMPLE PROBLEM 8B

Rotational equilibrium

PROBLEM

A uniform 5.00 m long horizontal beam that weighs 315 N is attached to a wall by a pin connection that allows the beam to rotate. Its far end is supported by a cable that makes an angle of 53° with the horizontal, and a 545 N person is standing 1.50 m from the pin. Find the force in the cable, F_T, and the force exerted on the beam by the wall, *R*, if the beam is in equilibrium.

SOLUTION

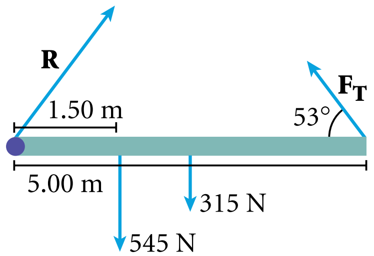
1. DEFINE

Given: $L = 5.00$ m $\quad F_{g,b} = 315$ N $\quad \theta = 53°$

$F_{g,p} = 545$ N $\quad d = 1.50$ m

Unknown: $F_T = ?$ $\quad R = ?$

Diagram: The weight of a uniform extended object is assumed to be concentrated at the object's center of mass.



2. PLAN

Choose an equation(s) or situation: The unknowns are R_x, R_y, and F_T. The first condition of equilibrium for the *x* and *y* directions gives:

x component equation: $\quad F_x = R_x - F_T(\cos\theta) = 0$

y component equation: $\quad F_y = R_y + F_T(\sin\theta) - F_{g,p} - F_{g,b} = 0$

Because there are three unknowns and only two equations, we cannot find the solutions from only the first condition of equilibrium.

Choose a point for calculating the net torque: The pin connection is a convenient place to put the axis because the unknown force, *R*, will not contribute to the net torque about this point.

Apply the second condition of equilibrium: The necessary third equation can be found from the second condition of equilibrium.

$$\tau = F_T L(\sin\theta) - F_{g,b}\frac{L}{2} - F_{g,p}d = 0$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$\tau = F_T(\sin 53°)(5.00 \text{ m}) - (315 \text{ N})(2.50 \text{ m}) - (545 \text{ N})(1.50 \text{ m}) = 0$$

$$\tau = F_T(4.0 \text{ m}) - 788 \text{ N}\bullet\text{m} - 818 \text{ N}\bullet\text{m} = 0$$

$$F_T = \frac{1606 \text{ N}\bullet\text{m}}{4.0 \text{ m}}$$

$$\boxed{F_T = 4.0 \times 10^2 \text{ N}}$$

continued on next page

This value for the force in the wire is then substituted into the x and y equations to find R.

$$F_x = R_x - F_T(\cos 53°) = 0$$
$$R_x = (400\text{ N})(\cos 53°)$$
$$R_x = 240\text{ N}$$
$$F_y = R_y + F_T(\sin 53°) - 545\text{ N} - 315\text{ N} = 0$$
$$R_y = -3.2 \times 10^2\text{ N} + 8.60 \times 10^2\text{ N}$$
$$R_y = 540\text{ N}$$
$$R = \sqrt{R_x^2 + R_y^2}$$
$$R = \sqrt{(240\text{ N})^2 + (540\text{ N})^2}$$
$$\boxed{R = 5.9 \times 10^2\text{ N}}$$

4. EVALUATE The sum of the y components of the force in the wire and the force exerted by the wall must equal the weight of the beam and the person. Thus, the force in the wire and the force exerted by the wall must be greater than the sum of the two weights.

$$400 + 590 > 545 + 315$$

PRACTICE 8B

Rotational equilibrium

1. Rework the example problem above with the axis of rotation passing through the center of mass of the beam. Verify that the answers do not change even though the axis is different.

2. A uniform bridge 20.0 m long and weighing 4.00×10^5 N is supported by two pillars located 3.00 m from each end. If a 1.96×10^4 N car is parked 8.00 m from one end of the bridge, how much force does each pillar exert?

3. A 700.0 N window washer is standing on a uniform scaffold supported by a vertical rope at each end. The scaffold weighs 200.0 N and is 3.00 m long. What is the force in each rope when the window washer stands 1.00 m from one end?

4. A 400.0 N child and a 300.0 N child sit on either end of a 2.0 m long seesaw.

a. Where along the seesaw should the pivot be placed to ensure rotational equilibrium? Disregard the mass of the seesaw.

b. Suppose a 225 N child sits 0.200 m from the 400.0 N child. Where must a 325 N child sit to maintain rotational equilibrium?

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Section Review

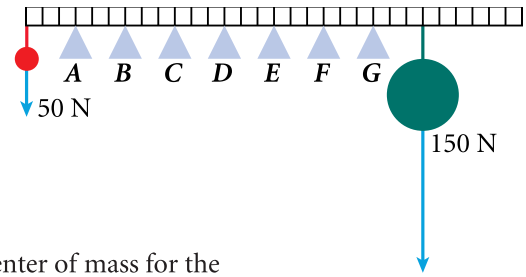

Figure 8-9

1. At which of the seven positions indicated in **Figure 8-9** should the supporting pivot be located to produce the following?
   a. a net positive torque
   b. a net negative torque
   c. no rotation

2. Describe the approximate location of the center of mass for the following objects:
   a. a meterstick
   b. a bowling ball
   c. an ice cube
   d. a doughnut
   e. a banana

3. A student says that moment of inertia and mass are the same thing. Explain what is wrong with this reasoning.

4. Identify which, if any, conditions of equilibrium hold for the following situations:
   a. a bicycle wheel rolling along a level highway at constant speed
   b. a bicycle parked against a curb
   c. the tires on a braking automobile that is still moving
   d. a football traveling through the air

5. A uniform 40.0 N board supports two children, one weighing 510 N and the other weighing 350 N. The support is under the center of mass of the board, and the 510 N child is 1.50 m from the center.
   a. Where should the 350 N child sit to balance the system?
   b. How much force does the support exert on the board?

6. **Physics in Action** Why would it be beneficial for a bicycle to have a low center of mass when the rider rounds a turn?

7. **Physics in Action** A bicycle designer recently modified a bicycle by adding cylindrical weights to the spokes of the wheels. He reasoned that this would make the mass of the wheel, on average, closer to the axle, in turn making the moment of inertia smaller and the wheel easier to rotate. Where did he go wrong?

8. **Physics in Action** The front wheel of a penny-farthing bicycle is three times as large as the rear wheel. How much more massive must the rear wheel be to have the same moment of inertia as the front wheel?

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8-3
Rotational dynamics

8-3 SECTION OBJECTIVES

- **Describe Newton's second law for rotation.**
- **Calculate the angular momentum for various rotating objects.**
- **Solve problems involving rotational kinetic energy.**

NEWTON'S SECOND LAW FOR ROTATION

You learned in Section 8-2 that there is a relationship between the net torque on an object and the angular acceleration given to the object. This is analogous to Newton's second law, which relates the net force on an object to the translational acceleration given to the object. Newton's second law for rotating objects can be written as follows:

NEWTON'S SECOND LAW FOR ROTATING OBJECTS

$$\tau_{net} = I\alpha$$

net torque = moment of inertia × angular acceleration

Figure 8-10
The continuous flow of water exerts a torque on the waterwheel.

Recall that a net positive torque causes an object to rotate counterclockwise. This means that the angular acceleration of the object is also counterclockwise. Similarly, a net negative torque will produce a clockwise angular acceleration. Thus, in calculating an object's angular acceleration, it is important to keep track of the signs of the torques acting on the object.

For example, consider **Figure 8-10,** which shows a continuous stream of water falling on a wheel. The falling water exerts a force on the rim of the wheel, producing a torque that causes the wheel to rotate. Other forces such as air resistance and friction between the axle and the wheel produce counteracting torques. When the net torque on the wheel is zero, the wheel rotates with constant angular velocity.

The relationship between these translational and rotational quantities is summarized in **Table 8-3.**

PHYSICS INTERACTIVE TUTOR®

Module 10
"Rotational Inertia"
provides an interactive lesson with guided problem-solving practice to teach you about rotational motion and Newton's second law for rotating objects.

Table 8-3 Newton's second law for translational and rotational motion

Translation	$F = ma$	force = mass × acceleration
Rotation	$\tau = I\alpha$	torque = moment of inertia × angular acceleration

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SAMPLE PROBLEM 8C

Newton's second law for rotation

PROBLEM

A student tosses a dart using only the rotation of her forearm to accelerate the dart. The forearm rotates in a vertical plane about an axis at the elbow joint. The forearm and dart have a combined moment of inertia of 0.075 kg•m^2 about the axis, and the length of the forearm is 0.26 m. If the dart has a tangential acceleration of 45 m/s^2 just before it is released, what is the net torque on the arm and dart?

SOLUTION

Given: $I = 0.075 \text{ kg} \cdot \text{m}^2$ $\quad a = 45 \text{ m/s}^2$ $\quad d = 0.26 \text{ m}$

Unknown: $\tau = ?$

Use the equation for Newton's second law for rotating objects, given on page 290.

$$\tau = I\alpha \text{ where } \alpha = a/d$$

$$\tau = I(a/d)$$

$$\tau = (0.075 \text{ kg} \cdot \text{m}^2)(45 \text{ m/s}^2)/0.26 \text{ m}$$

$$\boxed{\tau = 13 \text{ N} \cdot \text{m}}$$

PRACTICE 8C

Newton's second law for rotation

1. A potter's wheel of radius 0.50 m and mass 100.0 kg is freely rotating at 50.0 rev/min. The potter can stop the wheel in 6.0 s by pressing a wet rag against the rim.

a. What is the angular acceleration of the wheel?
b. How much torque does the potter apply to the wheel?

2. A bicycle tire of radius 0.33 m and mass 1.5 kg is rotating at 98.7 rad/s. What torque is necessary to stop the tire in 2.0 s?

3. A light string 4.00 m long is wrapped around a solid cylindrical spool with a radius of 0.075 m and a mass of 0.500 kg. A 5.00 kg mass is then attached to the free end of the string, causing the string to unwind from the spool.

a. What is the angular acceleration of the spool?
b. How fast will the spool be rotating after all of the string has unwound?

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MOMENTUM

Have you ever swung a sledgehammer or a similarly heavy object? You probably noticed that it took some effort to start the object rotating and that it also took an effort to stop it from rotating. This is because objects resist changes in their rotational motion as well as in their translational motion.

Rotating objects have angular momentum

angular momentum

the product of a rotating object's moment of inertia and angular speed about the same axis

Because a rotating object has inertia, it also possesses momentum associated with its rotation. This momentum is called **angular momentum.** Angular momentum is defined by the following equation:

ANGULAR MOMENTUM

$$L = I\omega$$

angular momentum = moment of inertia × angular speed

The unit of angular momentum is $kg \bullet m^2/s$. To get an idea of how large this unit is, note that a 35 kg bowling ball rolling at an angular speed of 40 rad/s has an angular momentum of about 80 $kg \bullet m^2/s$. The relationship between these translational and rotational quantities is summarized in **Table 8-4.**

Table 8-4 Translational and angular momentum

Translational	$p = mv$	momentum = mass × speed
Rotational	$L = I\omega$	rotational momentum = moment of inertia × angular speed

Figure 8-11
Angular momentum is conserved as the skater pulls his arms toward his body.

Angular momentum may be conserved

When the net external torque acting on an object or objects is zero, the angular momentum of the object(s) does not change. This is the law of *conservation of angular momentum.*

For example, assuming the friction between the skates and the ice is negligible, there is no torque acting on the skater in **Figure 8-11,** so his angular momentum is conserved. When he brings his hands and feet closer to his body, more of his mass, on average, is nearer his axis of rotation. As a result, the moment of inertia of his body decreases. Because his angular momentum is constant, his angular speed increases to compensate for his smaller moment of inertia.

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Conservation of angular momentum

PROBLEM

A 65 kg student is spinning on a merry-go-round that has a mass of 5.25×10^2 kg and a radius of 2.00 m. She walks from the edge of the merry-go-round toward the center. If the angular speed of the merry-go-round is initially 0.20 rad/s, what is its angular speed when the student reaches a point 0.50 m from the center?

SOLUTION

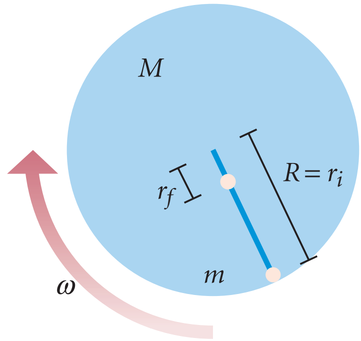
1. DEFINE

Given: $M = 5.25 \times 10^2$ kg $\quad r_i = R = 2.00$ m

$r_f = 0.50$ m $\quad m = 65$ kg $\quad \omega_i = 0.20$ rad/s

Unknown: $\omega_f = ?$

Diagram:



2. PLAN

Choose an equation(s) or situation: Because there are no external torques, the angular momentum of the system (merry-go-round plus student) is conserved.

$$L_i = L_f$$

$$L_{m,i} + L_{s,i} = L_{m,f} + L_{s,f}$$

Determine the moments of inertia. Treat the merry-go-round as a solid disk, and treat the student as a point mass.

$$I_m = \tfrac{1}{2}MR^2$$

$$I_{s,i} = mR^2$$

$$I_{s,f} = mr_f^2$$

3. CALCULATE

Substitute the values into the equation(s) and solve: Determine the initial moments of inertia, I_m and $I_{s,i}$, and the initial angular momentum, L_i.

$$I_m = \left(\frac{1}{2}\right)(5.25 \times 10^2 \text{ kg})(2.00 \text{ m})^2 = 1.05 \times 10^3 \text{ kg}\bullet\text{m}^2$$

$$I_{s,i} = (65 \text{ kg})(2.00 \text{ m})^2 = 260 \text{ kg}\bullet\text{m}^2$$

$$L_i = L_{m,i} + L_{s,i} = I_m\omega_i + I_{s,i}\omega_i$$

$$L_i = (1.05 \times 10^3 \text{ kg}\bullet\text{m}^2)(0.20 \text{ rad/s}) + (260 \text{ kg}\bullet\text{m}^2)(0.20 \text{ rad/s})$$

$$L_i = 260 \text{ kg}\bullet\text{m}^2/\text{s}$$

Determine the final moment of inertia, $I_{s,f}$, and the final angular momentum, L_f.

$$I_{s,f} = (65 \text{ kg})(0.50 \text{ m})^2 = 16 \text{ kg}\bullet\text{m}^2$$

$$L_f = L_{m,f} + L_{s,f} = I_m\omega_f + I_{s,f}\omega_f$$

$$L_f = (1.05 \times 10^3 \text{ kg}\bullet\text{m}^2 + 16 \text{ kg}\bullet\text{m}^2)\omega_f$$

$$L_f = (1.07 \times 10^3 \text{ kg}\bullet\text{m}^2)\omega_f$$

continued on next page

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Equate the initial and final angular momentum.

$$260 \text{ kg}\bullet\text{m}^2/\text{s} = (1.07 \times 10^3 \text{ kg}\bullet\text{m}^2)\ \omega_f$$

$$\boxed{\omega_f = 0.24 \text{ rad/s}}$$

4. EVALUATE Because the total moment of inertia decreases as the student moves toward the axis, the final angular speed should be greater than the initial angular speed.

$$0.24 \text{ rad/s} > 0.20 \text{ rad/s}$$

PRACTICE 8D

Conservation of angular momentum

1. A merry-go-round rotates at the rate of 0.30 rad/s with an 80.0 kg man standing at a point 2.0 m from the axis of rotation. What is the new angular speed when the man walks to a point 1.0 m from the center? Assume that the merry-go-round is a solid 6.50×10^2 kg cylinder with a radius of 2.00 m.
2. A 2.0 kg bicycle wheel with a radius of 0.30 m turns at a constant angular speed of 25 rad/s when a 0.30 kg reflector is at a distance of 0.19 m from the axle. What is the angular speed of the wheel when the reflector slides to a distance of 0.25 m from the axle?
3. A solid, vertical cylinder with a mass of 10.0 kg and a radius of 1.00 m rotates with an angular speed of 7.00 rad/s about a fixed vertical axis through its center. A 0.250 kg piece of putty is dropped vertically at a point 0.900 m from the cylinder's center of rotation and sticks to the cylinder. Determine the final angular speed of the system.
4. As Halley's comet orbits the sun, its distance from the sun changes dramatically, from 8.8×10^{10} m to 5.2×10^{12} m. If the comet's speed at closest approach is 5.4×10^4 m/s, what is its speed when it is farthest from the sun if angular momentum is conserved?
5. The entrance of a science museum features a funnel into which marbles are rolled one at a time. The marbles circle around the wall of the funnel, eventually spiraling down into the neck of the funnel. The internal radius of the funnel at the top is 0.54 m. At the bottom, the funnel's neck narrows to an internal radius of 0.040 m. A 2.5×10^{-2} kg marble begins rolling in a large circular orbit around the funnel's rim at 0.35 rev/s. If it continues moving in a roughly circular path, what will the marble's angular speed be as it passes throught the neck of the funnel? (Consider only the effects of the conservation of angular momentum.)

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KINETIC ENERGY

In Chapter 5 you learned that the mechanical energy of an object includes translational kinetic energy and potential energy, but that was for objects that can be modeled as point masses. In other words, this approach did not consider the possibility that objects could have rotational motion along with translational motion.

Rotating objects have rotational kinetic energy

Rotating objects possess kinetic energy associated with their angular speed. This form of energy is called **rotational kinetic energy** and is expressed by the following equation:

rotational kinetic energy

energy of an object due to its rotational motion

CALCULATING ROTATIONAL KINETIC ENERGY

$$KE_{rot} = \frac{1}{2}I\omega^2$$

$$\textbf{rotational kinetic energy} = \frac{1}{2} \times \textbf{moment of inertia} \times (\textbf{angular speed})^2$$

This is analogous to the translational kinetic energy of a particle, given by the expression $\frac{1}{2}mv^2$, where the moment of inertia replaces the mass and the angular speed replaces the translational speed. The unit of rotational kinetic energy is the joule, the SI unit for energy (see Chapter 5). The relationship between these translational and rotational quantities is summarized in **Table 8-5.**

Table 8-5 Translational and rotational kinetic energy

Translational	$KE_{trans} = \frac{1}{2}mv^2$	translational kinetic energy $= \frac{1}{2}$mass $\times$ (speed)^2
Rotational	$KE_{rot} = \frac{1}{2}I\omega^2$	rotational kinetic energy $= \frac{1}{2}$moment of inertia $\times$ (angular speed)^2

Mechanical energy may be conserved

Recall the race between two objects on page 278. Because it is assumed that gravity is the only external force acting on the cylinders and spheres, the mechanical energy associated with each object is conserved. Unlike the examples of energy conservation in Chapter 5, however, the objects in this example are rotating. Recalling that mechanical energy is the sum of all types of kinetic and potential energy, we must include a rotational kinetic energy term in our formula for mechanical energy, as follows:

$$ME = KE_{trans} + KE_{rot} + PE_g$$

$$ME = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh$$

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SAMPLE PROBLEM 8E

Conservation of mechanical energy

PROBLEM

A solid ball with a mass of 4.10 kg and a radius of 0.050 m starts from rest at a height of 2.00 m and rolls down a 30.0° slope, as shown in Figure 8-15. What is the translational speed of the ball when it leaves the incline?

SOLUTION

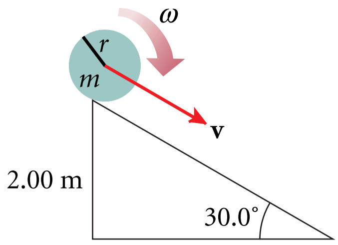
1. DEFINE

Given: $h = 2.00$ m $\quad \theta = 30.0°$ $\quad m = 4.10$ kg

$r = 0.050$ m $\quad v_i = 0.0$ m/s

Unknown: $v_f = ?$

Diagram:



2. PLAN

Choose an equation(s) or situation: Apply the conservation of mechanical energy.

$$ME_i = ME_f$$

Initially, the system possesses only gravitational potential energy. When the ball reaches the bottom of the ramp, this potential energy has been converted to translational and rotational kinetic energy.

$$mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 \quad \text{where } \omega_f = \frac{v_f}{r}$$

The moment of inertia for a solid ball can be found in **Table 8-1,** on page 285.

$$I = \frac{2}{5}mr^2$$

Equate the initial and final mechanical energy.

$$mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v_f}{r}\right)^2 = \frac{1}{2}mv_f^2 + \frac{1}{5}mv_f^2 = \frac{7}{10}mv_f^2$$

Rearrange the equation(s) to isolate the unknown(s):

$$v_f^2 = \frac{10}{7}gh$$

3. CALCULATE

Substitute the values into the equation(s) and solve:

$$v_f^2 = \frac{10}{7}(9.81 \text{ m/s}^2)(2.00 \text{ m})$$

$$\boxed{v_f = 5.29 \text{ m/s}}$$

4. EVALUATE

This speed should be less than the speed of an object undergoing free fall from the same height because part of the energy goes into rotation.

$$v_f(\text{free fall}) = \sqrt{2gh} = 6.26 \text{ m/s}$$

$$5.29 \text{ m/s} < 6.26 \text{ m/s}$$

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PRACTICE 8E

Conservation of mechanical energy

1. Repeat Sample Problem 8E using a solid cylinder of the same mass and radius as the ball and releasing it from the same height. In a race between these two objects on an incline, which would win?

2. A 1.5 kg bicycle tire of radius 0.33 m starts from rest and rolls down from the top of a hill that is 14.8 m high. What is the translational speed of the tire when it reaches the bottom of the hill? (Assume that the tire is a hoop with $I = mr^2$.)

3. A regulation basketball has a 25 cm diameter and may be approximated as a thin spherical shell. How long will it take a basketball starting from rest to roll without slipping 4.0 m down an incline that makes an angle of 30.0° with the horizontal?

Section Review

1. A student holds a 3.0 kg mass in each hand while sitting on a rotating stool. When his arms are extended horizontally, the masses are 1.0 m from the axis of rotation and he rotates with an angular speed of 0.75 rad/s. If the student pulls the masses horizontally to 0.30 m from the axis of rotation, what is his new angular speed? Assume the combined moment of inertia of the student and the stool together is 3.0 kg•m^2 and is constant.

2. A 4.0 kg mass is connected by a massless string over a massless and frictionless pulley to the center of an 8.0 kg wheel. Assume that the wheel has a radius of 0.50 m and a moment of inertia of 2.0 kg•m^2, as shown in **Figure 8-12.** The mass is released from rest at a height of 2.0 m above the ground. What will its speed be just before it strikes the ground? (Hint: Apply conservation of mechanical energy.)

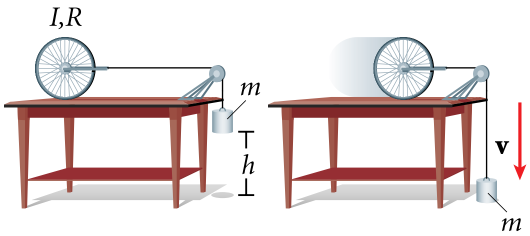

Figure 8-12

3. **Physics in Action** A bicyclist exerts a constant force of 40.0 N on a pedal 0.15 m from the axis of rotation of a penny-farthing bicycle wheel with a radius of 50.0 cm. If his speed is 2.25 m/s 3.0 s after he starts from rest, what is the moment of inertia of the wheel? (Disregard friction and the moment of inertia of the small wheel.)

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8-4
Simple machines

8-4 SECTION OBJECTIVES

- **Identify the six types of simple machines.**
- **Explain how the operation of a simple machine alters the applied force and the distance moved.**
- **Calculate the mechanical advantage of a simple machine.**

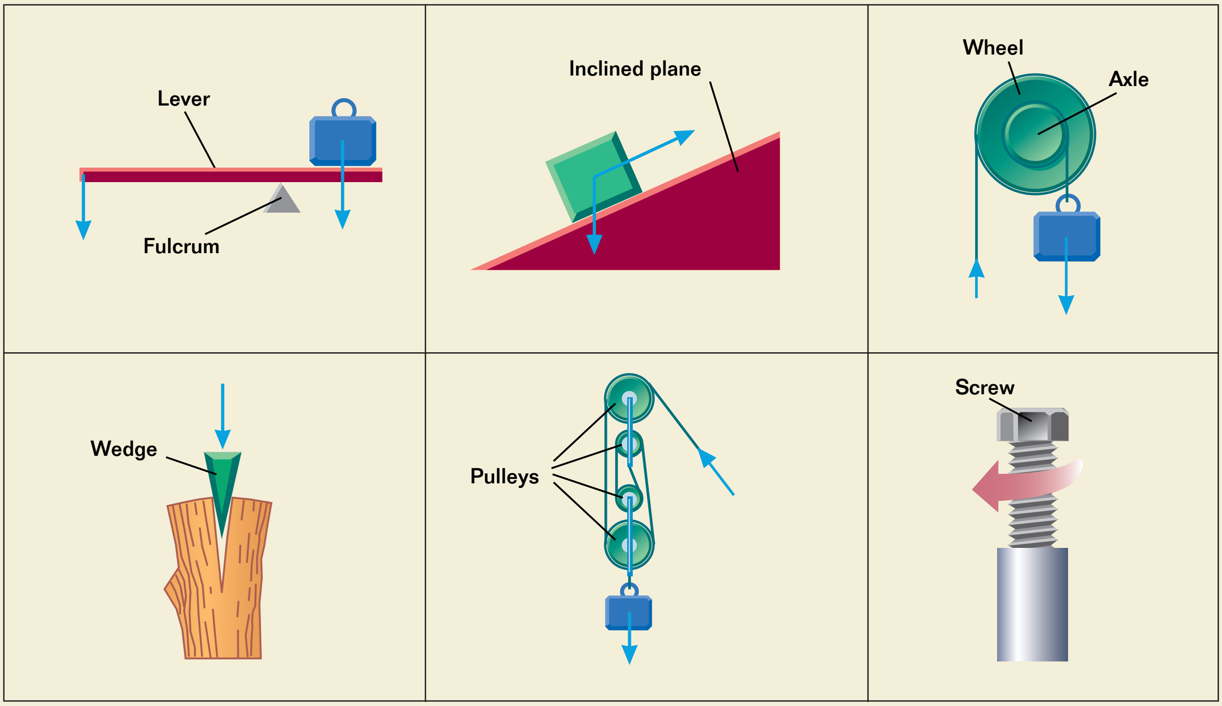
TYPES OF SIMPLE MACHINES

What do you do when you need to drive a nail into a board? You probably hit the nail with a hammer. Similarly, you would probably use scissors to cut paper or a bottle opener to pry a cap off a bottle. All of these devices make your task easier. These devices are *machines.*

The term *machine* may bring to mind intricate systems with multicolored wires and complex gear-and-pulley systems. Compared with internal-combustion engines or airplanes, simple devices such as hammers, scissors, and bottle openers may not seem like machines, but they are.

A machine is any device that transmits or modifies force, usually by changing the force applied to an object. All machines are combinations or modifications of six fundamental types of machines, called *simple machines.* These six simple machines are the lever, pulley, inclined plane, wheel and axle, wedge, and screw, as shown in **Table 8-6.**

Table 8-6 Six simple machines



USING SIMPLE MACHINES

Because the purpose of a simple machine is to change the direction or magnitude of an input force, a useful way of characterizing a simple machine is to compare how large the output force is relative to the input force. This ratio, called the machine's *mechanical advantage,* is written as follows:

$$MA = \frac{\text{output force}}{\text{input force}} = \frac{F_{out}}{F_{in}}$$

A good example of mechanical advantage is the claw hammer, which is a type of lever, as shown in **Figure 8-13.**

Figure 8-13
A hammer makes it easier to pry a nail from a board by multiplying the input force. The hammer swivels around the point marked with a black dot.

A person applies an input force to one end of the handle. The handle, in turn, exerts an output force on the head of a nail stuck in a board. Rotational equilibrium is maintained, so the input torque must balance the output torque. This can be written as follows:

$$\tau_{in} = \tau_{out}$$
$$F_{in}d_{in} = F_{out}d_{out}$$

Substituting this expression into the definition of mechanical advantage gives the following result:

$$MA = \frac{F_{out}}{F_{in}} = \frac{d_{in}}{d_{out}}$$

The longer the input lever arm is compared with the output lever arm, the greater the mechanical advantage is. This in turn indicates the factor by which the input force is amplified. If the force of the board on the nail is 99 N and if the mechanical advantage is 10, then only a force of 10 N is needed to pull out the nail. Without a machine, the nail could not be removed unless the force was greater than 100 N.

Machines can alter the force and the distance moved

You have learned that mechanical energy is conserved in the absence of friction. This law holds for machines as well. A machine can increase (or decrease) the force acting on an object at the expense (or gain) of the distance moved, but the product of the two—the work done on the object—is constant.

For example, imagine an incline. **Figure 8-14** shows two examples of a refrigerator being loaded onto a flatbed truck. In one example, the refrigerator is lifted directly onto the truck. In the other example, an incline is used.

In the first example, a force ($\mathbf{F_1}$) of 1200 N is required to lift the refrigerator, which moves through a distance (d_1) of 1.5 m. This requires 1800 N•m of work. In the second example, a lesser force ($\mathbf{F_2}$) of only 360 N is needed, but the refrigerator must be pushed a greater distance (d_2) of 5.0 m. This also requires 1800 N•m of work. As a result, the two methods require the same amount of energy.

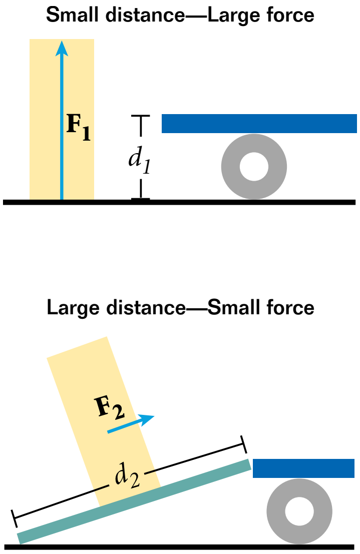


Figure 8-14
Simple machines can alter both the force needed to perform a task and the distance through which the force acts.

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Tomorrow's Technology

Human Extenders

A cyborg, as any science-fiction aficionado knows, is part human and part machine and is able to perform extraordinary tasks. Although cyborgs are still more fiction than science, Dr. Homayoon Kazerooni, of the University of California at Berkeley, has been inventing machines called *human extenders* that can give mere mortals superhuman strength.

"Human extenders are robotic systems worn by a human to move heavy objects," Dr. Kazerooni says. One of the first machines Dr. Kazerooni designed is a 1.5 m (5 ft) long steel arm that weighs thousands of newtons (several hundred pounds) and is attached to a pedestal on the floor. The operator inserts one arm into the device, and an attached computer senses the arm's movement and uses hydraulic pressure to move the extender in conjunction with the operator's arm. With the extender, a person can lift objects weighing as much as 890 N (200 lb) while exerting a force of only 89 N (20 lb). In this case, the extender yields a mechanical advantage of 10 (890 N/89 N = 10).

Dr. Kazerooni is developing a complete suit of human extenders that will be powered by electricity. Controlled completely by the movement of the user, the suit has two arms that sense and respond to both the force applied by the human and the weight of the object being lifted, taking most of the effort away from the operator. The machine's legs are able to balance the weight of the equipment, and they attach at the operator's feet to allow movement around the room.

Dr. Kazerooni envisions human extenders being used primarily as labor aids for factory workers. Approximately 30 percent of all workplace accidents in the United States are related to back injuries, and they are usually the result of a repeated lifting and moving of heavy objects. Human extenders could solve that problem. "The person who is wearing the machine," Dr. Kazerooni says, "will feel less force and less fatigue, and therefore the potential for back injuries or any kind of injury would be less."

Figure 8-15

Back

Efficiency is a measure of how well a machine works

The simple machines we have considered so far are ideal, frictionless machines. Real machines, however, are not frictionless. They dissipate energy. When the parts of a machine move and contact other objects, some of the input energy is dissipated as sound or heat. The *efficiency* of a machine is a measure of how much input energy is lost compared with how much energy is used to perform work on an object. It is defined by the following equation:

$$eff = \frac{W_{out}}{W_{in}}$$

If a machine is frictionless, then mechanical energy is conserved. This means that the work done on the machine (input work) is equal to the work done by the machine (output work) because work is a measure of energy transfer. Thus, the mechanical efficiency of an ideal machine is 1, or 100 percent. This is the best efficiency a machine can have. Because all real machines have at least a little friction, the efficiency of real machines is always less than 1.

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Section Review

1. **Figure 8-16** shows an example of a Rube Goldberg machine. Identify two types of simple machines that are included in this compound machine.

2. The efficiency of a squeaky pulley system is 73 percent. The pulleys are used to raise a mass to a certain height. What force is exerted on the machine if a rope is pulled 18.0 m in order to raise a 58 kg mass a height of 3.0 m?

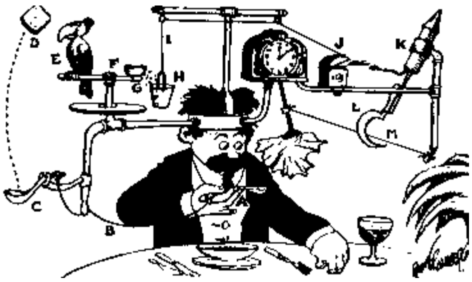


Figure 8-16

3. A person lifts a 950 N box by pushing it up an incline. If the person exerts a force of 350 N along the incline, what is the mechanical advantage of the incline?

4. You are attempting to move a large rock using a long lever. Will the work you do on the lever be greater than, the same as, or less than the work done by the lever on the rock? Explain.

5. **Physics in Action** A bicycle can be described as a combination of simple machines. Identify three types of simple machines that are used to propel a typical bicycle.

PHYSICS ON THE EDGE

Quantum Angular Momentum

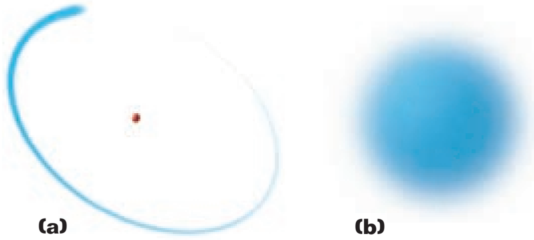
Earlier in this chapter, we discussed angular momentum and its effects in the macroscopic world of your everyday experience. In the early twentieth century, scientists realized that they must modify their ideas about angular momentum when working with the microscopic world of atoms and subatomic particles.

Electron orbital angular momentum

In 1911, Ernest Rutherford proposed a model of the atom in which negatively charged particles called *electrons* orbit a positively charged nucleus containing particles called *protons,* much as the Earth orbits the sun, as shown in **Figure 8-17(a).** Because the electron orbits the nucleus, it has an orbital angular momentum.

Further investigations into the microscopic realm revealed that the electron cannot be precisely located in space and therefore cannot be visualized as orbiting the proton. Instead, in modern theory the electron's location is depicted by an electron cloud, as shown in **Figure 8-17(b),** whose density varies throughout the cloud in proportion to the probability of finding the electron at a particular location in the cloud. Even though the electron does not orbit the nucleus in this model, the electron still has an orbital angular momentum that is very different from the angular momentum discussed earlier in this chapter.

For most of the history of science, it was assumed that angular momentum could have any possible value. But investigations at the atomic level have shown that this is not the case. The orbital angular momentum of the electron can have only certain possible values. Such a quantity is said to be discrete, and the angular momentum is said to be *quantized.* The branch of modern theory that deals with quanta is called *quantum mechanics.*



(a) (b)

Figure 8-17
(a) In Rutherford's model of the atom, electrons orbit the nucleus. **(b)** In quantum mechanics, an electron cloud is used to show the probability that the electron will be at different points. The densest regions of the cloud represent the most probable locations for the electron.

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Electron spin

In addition to the orbital angular momentum of the electron, certain experimental evidence led scientists to postulate another type of angular momentum of the electron. This type of angular momentum is known as *spin* because the effects it explains are those that would result if the electron were to spin on its axis, much like the Earth spins on its axis of rotation. Scientists first imagined that the electron actually spins in this way, but it soon became clear that electron spin is not a literal description. Instead, electron spin is a property that is independent of the electron's motion in space. In this respect, electron spin is very different from the spin of the Earth.

Just as a wheel can turn either clockwise or counterclockwise, there are two possible types of electron spin, *spin up* and *spin down*, as shown in **Figure 8-18.** Thus, like orbital angular momentum, spin is quantized. Because the electron isn't really spinning in space, it should not be assumed that **Figure 8-18** is a physical description of the electron's motion.

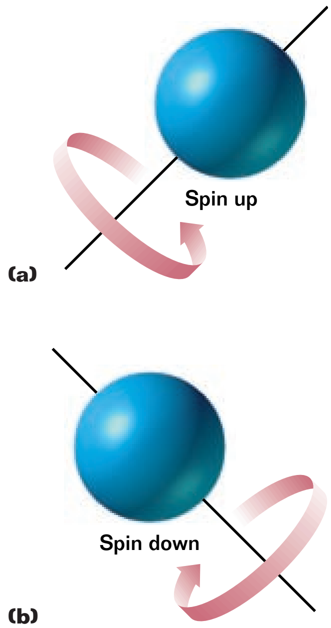

Figure 8-18
In the quantum-mechanical model of the atom, the electron has both an orbital angular momentum and an intrinsic angular momentum, known as spin. **(a)** Spin up and **(b)** spin down are the only possible values for electron spin.

Conservation of angular momentum

Although the quantum-mechanical concept of angular momentum is radically different from the classical concept of angular momentum, there is one fundamental similarity between the two models. Earlier in this chapter, you learned that angular momentum is always conserved. This principle still holds in quantum mechanics, where the total angular momentum, that is, the sum of the orbital angular momentum and spin, is always conserved.

As you have seen, the quantum-mechanical model of the atom cannot be visualized in the same way that previous atomic models could be. Although this may initially seem like a flaw of the modern theory, the accuracy of predictions based on quantum mechanics has convinced many scientists that physical models based on our experiences in the macroscopic realm cannot provide a complete picture of nature. Consequently, mathematical models must be used to describe the microscopic realm of atoms and subatomic particles.

Table 8-8 Angular momentum

Classical angular momentum	Quantum angular momentum
corresponds to a literal rotation	does not correspond to a literal rotation
can have any possible value	can have only certain discrete values
total angular momentum (orbital + rotational) conserved	total angular momentum (orbital angular momentum + spin) conserved

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CHAPTER 8
Summary

KEY TERMS

angular momentum (p. 292)

center of mass (p. 283)

lever arm (p. 279)

moment of inertia (p. 284)

rotational kinetic energy (p. 295)

torque (p. 279)

KEY IDEAS

Section 8-1 Torque

- Torque is a measure of a force's ability to rotate an object.
- The torque on an object depends on the magnitude of the applied force and on the length of the lever arm, according to the following equation:

$$\tau = Fd(\sin\theta)$$

Section 8-2 Rotation and inertia

- The moment of inertia of an object is a measure of the resistance of the object to changes in rotational motion.
- For an extended object to be in complete equilibrium, it must be in both translational and rotational equilibrium.

Section 8-3 Rotational dynamics

- The rotational equation analogous to Newton's second law can be described as follows:

$$\tau = I\alpha$$

- A rotating object possesses angular momentum, which is conserved in the absence of any external forces on the object.
- A rotating object possesses rotational kinetic energy, which is conserved in the absence of any external forces on the object.

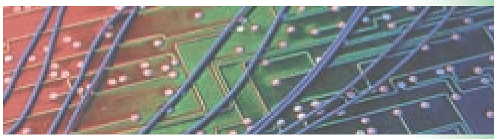
Section 8-4 Simple machines

- A simple machine can alter the force applied to an object or the distance an applied force moves an object.
- Simple machines can provide a mechanical advantage.

Key Symbols

Quantities		Units		Conversions
τ	torque	N•m	newton meter	$= kg \cdot m^2/s^2$
$d(\sin\theta)$	lever arm	m	meter	
I	moment of inertia	$kg \cdot m^2$	kilogram meter squared	
L	angular momentum	$kg \cdot m^2/s$	kilogram meter squared per second	
KE_{rot}	rotational kinetic energy	J	joule	$= N \cdot m$ $= kg \cdot m^2/s^2$

CHAPTER 8
Review and Assess



TORQUE AND MOMENT OF INERTIA

Conceptual questions

1. Explain how an orthodontist uses torque to straighten or realign teeth.

2. Which of the forces acting on the rod in **Figure 8-19** will produce a torque about the axis at the left end of the rod?

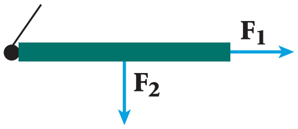


Figure 8-19

3. Two children are rolling automobile tires down a hill. One child claims that the tire will roll faster if one of them curls up in the tire's center. The other child claims that will cause the tire to roll more slowly. Which child is correct?

4. The moment of inertia of Earth was recently measured to be $0.331MR^2$. What does this tell you about the distribution of mass inside Earth? (Hint: Compare this value with the moments of inertia in **Table 8-1.**)

5. The moment of inertia for a regular object can never be larger than MR^2, where M is the mass and R is the size of the object. Why is this so? (Hint: Which object has a moment of inertia of MR^2?)

6. Two forces of equal magnitude act on a wheel, as shown in **Figure 8-20.** Which force will produce the greater torque on the wheel?

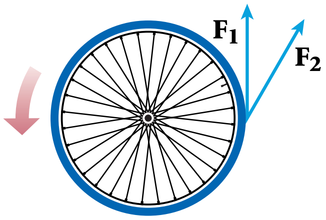


Figure 8-20

7. Two forces equal in magnitude but opposite in direction act at the same point on an object. Is it possible for there to be a net torque on the object? Explain.

8. It is more difficult to do a sit-up with your hands held behind your head than it is to do a sit-up with your arms stretched out in front of you. Explain why this statement is true.

Practice problems

9. A bucket filled with water has a mass of 54 kg and is hanging from a rope that is wound around a 0.050 m radius stationary cylinder. If the cylinder does not rotate and the bucket hangs straight down, what is the magnitude of the torque the bucket produces around the center of the cylinder?
(See Sample Problem 8A.)

10. A mechanic jacks up a car to an angle of 8.0° with the horizontal in order to change the front tires. The car is 3.05 m long and has a mass of 1130 kg. Its center of mass is located 1.12 m from the front end. The rear wheels are 0.40 m from the back end. Calculate the torque exerted by the car around the back wheels.
(See Sample Problem 8A.)

11. The arm of a crane at a construction site is 15.0 m long, and it makes an angle of 20.0° with the horizontal. Assume that the maximum load the crane can handle is limited by the amount of torque the load produces around the base of the arm.
    a. What is the magnitude of the maximum torque the crane can withstand if the maximum load the crane can handle is 450 N?
    b. What is the maximum load for this crane at an angle of 40.0° with the horizontal?

(See Sample Problem 8A.)

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CENTER OF MASS AND ROTATIONAL EQUILIBRIUM

Review questions

12. At a circus performance, a juggler is throwing two spinning clubs. One of the clubs is heavier than the other. Which of the following statements is true?

- **a.** The smaller club is likely to have a larger moment of inertia.
- **b.** The ends of each club will trace out parabolas as the club is thrown.
- **c.** The center of mass of each club will trace out a parabola as the club is thrown.

13. When the juggler in the previous problem stands up straight and holds each club at arm's length, his center of mass will probably be

- **a.** located at a point exactly in the middle of his body
- **b.** slightly to the side where he is holding the light club
- **c.** slightly to the side where he is holding the heavy club

14. What are the conditions for equilibrium? Explain how they apply to children attempting to balance a seesaw.

15. What must be true about the velocity of a moving object in equilibrium?

16. A twirler throws a baton in the air.

- **a.** Describe the motion of the ends of the baton as it moves through the air.
- **b.** Decribe the motion of the center of mass of the baton.

Conceptual questions

17. A projectile is fired into the air and suddenly explodes into several fragments. What can be said about the motion of the center of mass of the fragments after the explosion?

18. Is it possible to balance two objects that have different masses (and therefore weights) on a simple balance beam? Explain.

19. A particle moves in a straight line, and you are told that the torque acting on it is zero about some unspecified origin. Does this necessarily imply that the total force on the particle is zero? Can you conclude that the angular velocity of the particle is constant? Explain.

Practice problems

20. A window washer is standing on a scaffold supported by a vertical rope at each end. The scaffold weighs 205 N and is 3.00 m long. What is the force each rope exerts on the scaffold when the 675 N worker stands 1.00 m from one end of the scaffold? (See Sample Problem 8B.)

21. A floodlight with a mass of 20.0 kg is used to illuminate the parking lot in front of a library. The floodlight is supported at the end of a horizontal beam that is hinged to a vertical pole, as shown in **Figure 8-21.** A cable that makes an angle of 30.0° with the beam is attached to the pole to help support the floodlight. Find the following, assuming the mass of the beam is negligible when compared with the mass of the floodlight:

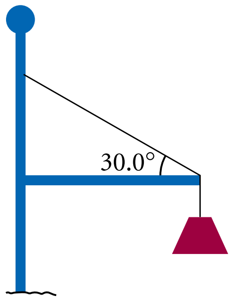

Figure 8-21

- **a.** the force provided by the cable
- **b.** the horizontal and vertical forces exerted on the beam by the pole

(See Sample Problem 8B.)

22. A 1200.0 N uniform boom is supported by a cable, as shown in **Figure 8-22.** The boom is pivoted at the bottom, and a 2000.0 N weight hangs from its top. Find the force applied by the supporting cable and the components of the reaction force on the bottom of the boom. (See Sample Problem 8B.)

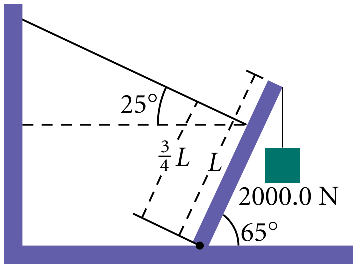

Figure 8-22

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23. A uniform 10.0 N picture frame is supported as shown in **Figure 8-23.** Find the force in the cords and the magnitude of the horizontal force at *P* that are required to hold the frame in this position.
(See Sample Problem 8B.)

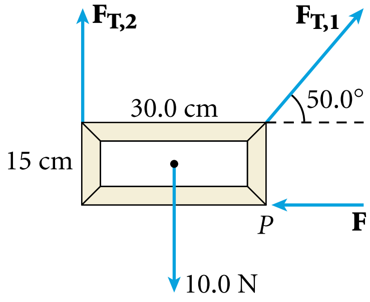

Figure 8-23

NEWTON'S SECOND LAW FOR ROTATION

Conceptual questions

24. An object rotates with a constant angular velocity. Can there be a net torque acting on the object? Explain your answer.

25. If an object is at rest, can you be certain that no external torques are acting on it?

26. Two uniformly solid disks of equal radii roll down an incline without slipping. The first disk has twice the mass of the second disk. How much torque was exerted on the first disk compared with the amount exerted on the second disk?

Practice problems

27. A 30.0 kg uniform solid cylinder has a radius of 0.180 m. If the cylinder accelerates at 2.30×10^{-2} rad/s^2 as it rotates about an axis through its center, how large is the torque acting on the cylinder?
(See Sample Problem 8C.)

28. A 350 kg merry-go-round in the shape of a horizontal disk with a radius of 1.5 m is set in motion by wrapping a rope about the rim of the disk and pulling on the rope. How large a torque would have to be exerted to bring the merry-go-round from rest to an angular speed of 3.14 rad/s in 2.00 s?
(See Sample Problem 8C.)

ANGULAR MOMENTUM AND ROTATIONAL KINETIC ENERGY

Review questions

29. Is angular momentum always conserved? Explain.

30. Is it possible for two objects with the same mass and the same rotational speeds to have different values of angular momentum? Explain.

31. A child on a merry-go-round moves from near the axis to the outer edge of the merry-go-round. What happens to the rotational speed of the merry-go-round? Explain.

32. Is it possible for an ice skater to change her rotational speed without any external torque? Explain.

Conceptual questions

33. Ice skaters use the conservation of angular momentum to produce high-speed spins when they bring their arms close to the rotation axis. Imagine that a skater moves her arms inward, cutting the moment of inertia in half and therefore doubling the angular speed. If we consider the rotational kinetic energy, we see that the energy is *doubled* in this situation. Thus, angular momentum is conserved, but kinetic energy is not. Where does this extra rotational kinetic energy come from?

34. A solid 2.0 kg ball with a radius of 0.50 m starts at a height of 3.0 m and rolls down a 20° slope. A solid disk and a ring start at the same time and the same height. Both the ring and the disk have the same mass and radius as the ball. Which of the three objects will win the race to the bottom if all roll without slipping?

Practice problems

35. A 15.0 kg turntable with a radius of 25 cm is covered with a uniform layer of dry ice that has a mass of 9.0 kg. The angular speed of the turntable and dry ice is initially 0.75 rad/s, but it increases as the dry ice evaporates. What is the angular speed of the turntable once all the dry ice has evaporated?
(See Sample Problem 8D.)

Back

36. A 65 kg woman stands at the rim of a horizontal turntable with a moment of inertia of 1.5×10^3 kg•m^2 and a radius of 2.0 m. The system is initially at rest, and the turntable is free to rotate about a frictionless vertical axle through its center. The woman then starts walking clockwise (when viewed from above) around the rim at a constant speed of 0.75 rad/s relative to Earth. In what direction and with what angular speed does the turntable rotate?
(See Sample Problem 8D.)

37. A 35 kg bowling ball with a radius of 13 cm starts from rest at the top of an incline 3.5 m in height. Find the translational speed of the bowling ball after it has rolled to the bottom of the incline. (Assume the ball is a uniform solid sphere.)
(See Sample Problem 8E.)

38. A solid 240 N ball with a radius of 0.20 m rolls 6.0 m down a ramp that is inclined at 37° with the horizontal. If the ball starts from rest at the top of the ramp, what is the angular speed of the ball at the bottom of the ramp?
(See Sample Problem 8E.)

SIMPLE MACHINES

Review questions

39. Why is it easier to loosen the lid from the top of a paint can with a long-handled screwdriver than with a short-handled screwdriver?

40. If a machine cannot multiply the amount of work, what is the advantage of using such a machine?

Conceptual questions

41. You are attempting to move a large rock using a long lever. Is it more effective to place the lever's axis of rotation nearer to your hands or nearer to the rock? Explain.

42. A perpetual motion machine is a machine that, when set in motion, will never come to a halt. Why is such a machine not possible?

43. If you were to use a machine to increase the output force, what factor would have to be sacrificed? Give an example.

MIXED REVIEW PROBLEMS

44. Two spheres look identical and have the same mass. One is hollow, and the other is solid. Which method would determine which is which?

- **a.** roll them down an incline
- **b.** drop them from the same height
- **c.** weigh them on a scale

45. A wooden bucket filled with water has a mass of 75 kg and is attached to a rope that is wound around a cylinder with a radius of 0.075 m. A crank with a turning radius of 0.25 m is attached to the end of the cylinder. What minimum force directed perpendicularly to the crank handle is required to raise the bucket?

46. If the torque required to loosen a nut that holds a wheel on a car has a magnitude of 58 N•m, what force must be exerted at the end of a 0.35 m lug wrench to loosen the nut when the angle is 56°?

47. In a canyon between two mountains, a spherical boulder with a radius of 1.4 m is just set in motion by a force of 1600 N. The force is applied at an angle of 53.5° measured with respect to the radius of the boulder. What is the magnitude of the torque on the boulder?

48. A 23.0 cm screwdriver is used to pry open a can of paint. If the axis of rotation is 2.00 cm from the end of the screwdriver blade and a force of 84.3 N is exerted at the end of the screwdriver's handle, what force is applied to the lid?

49. The net work done in accelerating a propeller from rest to an angular speed of 220 rad/s is 3000.0 J. What is the moment of inertia of the propeller?

50. A 0.100 kg meterstick is supported at its 40.0 cm mark by a string attached to the ceiling. A 0.700 kg mass hangs vertically from the 5.00 cm mark. A mass is attached somewhere on the meterstick to keep it horizontal and in *both* rotational and translational equilibrium. If the force applied by the string attaching the meterstick to the ceiling is 19.6 N, determine the following:

- **a.** the value of the unknown mass
- **b.** the point where the mass attaches to the stick

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51. A uniform ladder 8.00 m long and weighing 200.0 N rests against a smooth wall. The coefficient of static friction between the ladder and the ground is 0.600, and the ladder makes a 50.0° angle with the ground. How far up the ladder can an 800.0 N person climb before the ladder begins to slip?

52. A 0.0200 m diameter coin rolls up a 15.0° inclined plane. The coin starts with an initial angular speed of 45.0 rad/s and rolls in a straight line without slipping. How much vertical distance does it gain before it stops rolling?

53. In a circus performance, a large 4.0 kg hoop with a radius of 2.0 m rolls without slipping. If the hoop is given an angular speed of 6.0 rad/s while rolling on the horizontal and is allowed to roll up a ramp inclined at 15° with the horizontal, how far (measured along the incline) does the hoop roll?

54. A 12 kg mass is attached to a cord that is wrapped around a wheel with a radius of 10.0 cm, as shown in **Figure 8-24.** The acceleration of the mass down the frictionless incline is measured to be 2.0 m/s^2. Assuming the axle of the wheel to be frictionless, determine

a. the force in the rope.
b. the moment of inertia of the wheel.
c. the angular speed of the wheel 2.0 s after it begins rotating, starting from rest.

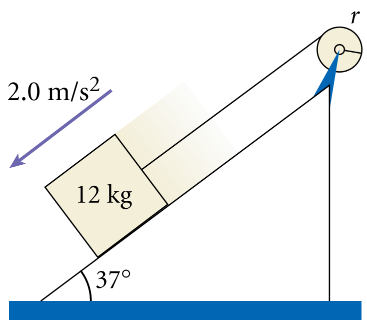

Figure 8-24

55. A person is standing on tiptoe, and the person's total weight is supported by the force on the toe. A mechanical model for the situation is shown in **Figure 8-25,** where T is the force in the Achilles tendon and R is the force on the foot due to the tibia. Assume the total weight is 700.0 N, and find the values of T and R, if the angle labeled θ is 21.2°.

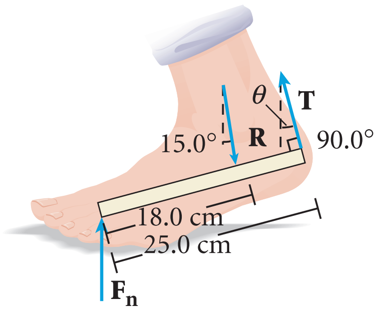

Figure 8-25

56. A cylindrical fishing reel has a mass of 0.85 kg and a radius of 4.0 cm. A friction clutch in the reel exerts a restraining torque of 1.3 N•m if a fish pulls on the line. The fisherman gets a bite, and the reel begins to spin with an angular acceleration of 66 rad/s^2. Find the following:

a. the force of the fish on the line
b. the amount of line that unwinds from the reel in 0.50 s

57. The combination of an applied force and a frictional force produces a constant torque of 36 N•m on a wheel rotating about a fixed axis. The applied force acts for 6.0 s, during which time the angular speed of the wheel increases from 0 to 12 rad/s. The applied force is then removed, and the wheel comes to rest in 65 s. Answer the following questions:

a. What is the moment of inertia of the wheel?
b. What is the frictional torque?
c. How many revolutions does the wheel make during the entire 71 s time interval?

58. A cable passes over a pulley. Because of friction, the force in the cable is not the same on opposite sides of the pulley. The force on one side is 120.0 N, and the force on the other side is 100.0 N. Assuming that the pulley is a uniform disk with a mass of 2.1 kg and a radius of 0.81 m, determine the angular acceleration of the pulley.

59. As part of a kinetic sculpture, a 5.0 kg hoop with a radius of 3.0 m rolls without slipping. If the hoop is given an angular speed of 3.0 rad/s while rolling on the horizontal and then rolls up a ramp inclined at 20.0° with the horizontal, how far does the hoop roll along the incline?

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60. A cylindrical 5.00 kg pulley with a radius of 0.600 m is used to lower a 3.00 kg bucket into a well. The bucket starts from rest and falls for 4.00 s.

a. What is the linear acceleration of the falling bucket?

b. How far does it drop?

c. What is the angular acceleration of the cylindrical pulley?

61. The hands of the clock in the famous Parliament Clock Tower in London are 2.7 m and 4.5 m long and have masses of 60.0 kg and 100.0 kg, respectively. Calculate the torque around the center of the clock due to the weight of these hands at 5:20. (Model the hands as thin rods.)

62. A coin with a diameter of 3.00 cm rolls up a 30.0° inclined plane. The coin starts with an initial angular speed of 60.0 rad/s and rolls in a straight line without slipping. How far does it roll up the inclined plane?

63. A solid sphere rolls along a horizontal, smooth surface at a constant linear speed without slipping. Show that the rotational kinetic energy about the center of the sphere is two-sevenths of its total kinetic energy.

64. A horizontal 800.0 N merry-go-round with a radius of 1.5 m is started from rest by a constant horizontal force of 50.0 N applied tangentially to the merry-go-round. Find the kinetic energy of the merry-go-round after 3.0 s. Assume it is a solid cylinder.

65. A top has a moment of inertia of 4.00×10^{-4} kg•m^2 and is initially at rest. It is free to rotate about a vertical stationary axis. A string around a peg along the axis of the top is pulled, maintaining a constant tension of 5.57 N in the string. If the string does not slip while it is wound around the peg, what is the angular speed of the top after 80.0 cm of string has been pulled off the peg? (Hint: Consider the work done.)

66. Calculate the following:

a. the angular momentum of Earth that arises from its spinning motion on its axis

b. the angular momentum of Earth that arises from its orbital motion about the sun

(Hint: See item 4 on page 305 and Appendix E.)

67. A skater spins with an angular speed of 12.0 rad/s with his arms outstretched. He lowers his arms, decreasing his moment of inertia from 41 kg•m^2 to 36 kg•m^2.

a. Calculate his initial and final rotational kinetic energy.

b. Is the rotational kinetic energy increased or decreased?

c. How do you account for this change in kinetic energy?

68. A pulley has a moment of inertia of 5.0 kg•m^2 and a radius of 0.50 m. A cord is wrapped over the pulley and attached to a hanging object on either end. Assume the cord does not slip, the axle is frictionless, and the two hanging objects have masses of 2.0 kg and 5.0 kg.

a. Find the acceleration of each mass.

b. Find the force in the cord supporting each mass. (Note that they are different.)

69. A 4.0 kg mass is connected by a light cord to a 3.0 kg mass on a smooth surface as shown in **Figure 8-26.** The pulley rotates about a frictionless axle and has a moment of inertia of 0.50 kg•m^2 and a radius of 0.30 m. Assuming that the cord does not slip on the pulley, answer the following questions:

a. What is the acceleration of the two masses?

b. What are the forces in the string F_1 and F_2?

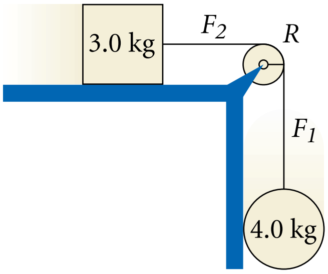

Figure 8-26

70. A car is designed to get its energy from a rotating flywheel with a radius of 2.00 m and a mass of 500.0 kg. Before a trip, the disk-shaped flywheel is attached to an electric motor, which brings the flywheel's rotational speed up to 1000.0 rev/min.

a. Find the kinetic energy stored in the flywheel.

b. If the flywheel is to supply as much energy to the car as a 7457 W motor would, find the length of time the car can run before the flywheel has to be brought back up to speed again.

71. **Figure 8-27** shows a system of point masses that rotates at an angular speed of 2.0 rev/s. The masses are connected by light, flexible spokes that can be lengthened or shortened. What is the new angular speed if the spokes are shortened to 0.50 m? (An effect similar to this occurred in the early stages of the formation of our galaxy. As the massive cloud of gas and dust contracted, an initially small rotation increased with time.)

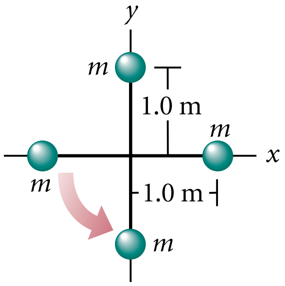

Figure 8-27

Technology & Learning

Graphing calculators

Refer to Appendix B for instructions on downloading programs for your calculator. The program "Chap8" allows you to analyze a graph of torque versus angle of applied force.

Torque, as you learned earlier in this chapter, is described by the following equation:

$$\tau = Fd(\sin\theta)$$

The program "Chap8" stored on your graphing calculator makes use of the equation for torque. Once the "Chap8" program is executed, your calculator will ask for the force and the distance from the axis of rotation. The graphing calculator will use the following equation to create a graph of the torque (Y1) versus the angle (X) at which the force is applied. The relationships in this equation are the same as those in the force equation shown above.

$$Y1 = Fd\sin(X)$$

Recall that the sine function is a periodic function that repeats every 360° and falls below the *x*-axis at 180°. Because the only values necessary for the torque calculation are less than 180°, the *x* values for the viewing window are preset. Xmin and Xmax values are set at 0 and 180, respectively.

a. What is a more straightforward way of saying, "The mechanic applied a force of –8 N at an angle of 200°"?

First, be certain that the calculator is in degree mode by pressing [MODE] [▼] [▼] [▶] [ENTER].

Execute "Chap8" on the [PRGM] menu, and press [ENTER] to begin the program. Enter the values for the force and the distance from the axis of rotation (shown below), and press [ENTER] after each value.

The calculator will provide a graph of the torque versus the angle at which the force is applied. (If the graph is not visible, press [WINDOW] and change the *y*-value settings for the graph window, then press [GRAPH]. Adjusting the *x* values is not necessary.)

Press [TRACE] and use the arrow keys to trace along the curve. The *x* value corresponds to the angle in degrees, and the *y* value corresponds to the torque in newton•meters.

Determine the torque involved in each of the following situations:

b. a force of 15.0 N that is applied 0.45 m from a door's hinges makes an angle of 75° with the door

c. the same force makes an angle of 45° with the door

d. a force of 15.0 N that is applied 0.25 m from a door's hinges makes an angle of 45° with the door

e. the same force makes an angle of 25° with the door

f. At what *x* value do you find the largest torque?

Press [2nd] [QUIT] to stop graphing. Press [ENTER] to input a new value or [CLEAR] to end the program.

72. The efficiency of a pulley system is 64 percent. The pulleys are used to raise a mass of 78 kg to a height of 4.0 m. What force is exerted on the rope of the pulley system if the rope is pulled for 24 m in order to raise the mass to the required height?

73. A crate is pulled 2.0 m at constant velocity along a 15° incline. The coefficient of kinetic friction between the crate and the plane is 0.160. Calculate the efficiency of this procedure.

74. A pulley system has an efficiency of 87.5 percent. How much of the rope must be pulled in if a force of 648 N is needed to lift a 150 kg desk 2.46 m?

75. A pulley system is used to lift a piano 3.0 m. If a force of 2200 N is applied to the rope as the rope is pulled in 14 m, what is the efficiency of the machine? Assume the mass of the piano is 750 kg.

76. A uniform 6.0 m tall aluminum ladder is leaning against a frictionless vertical wall. The ladder has a weight of 250 N. The ladder slips when it makes a 60.0° angle with the horizontal floor. Determine the coefficient of friction between the ladder and the floor.

77. A ladder with a length of 15.0 m and a weight of 520.0 N rests against a frictionless wall, making an angle of 60.0° with the horizontal.

a. Find the horizontal and vertical forces exerted on the base of the ladder by Earth when an 800.0 N firefighter is 4.00 m from the bottom of the ladder.

b. If the ladder is just on the verge of slipping when the firefighter is 9.00 m up, what is the coefficient of static friction between the ladder and the ground?

Alternative Assessment

Performance assessment

1. Imagine a balance with unequal arms. An earring placed in the left basket was balanced by 5.00 g of standard masses on the right. When placed in the right basket, the same earring required 15.00 g on the left to balance. Which was the longer arm? Do you need to know the exact length of each arm to determine the mass of the earring? Explain.

2. You have 32 identical beads, 30 of which are assembled in clusters of 16, 8, 4, and 2. Design and build a mobile that will balance, using all the beads and clusters. Explain your design in terms of torque and rotational equilibrium.

3. A well-known problem in architecture is to stack bricks on top of one another in a way that provides a maximum offset. Design an experiment to determine how much offset you can have by stacking two, three, and four physics textbooks. How many books would be needed to prevent the center of mass of the top book from being directly over the bottom book? (Alternatively, try this with a deck of playing cards.)

Portfolio projects

4. Describe exactly which measurements you would need to make in order to identify the torques at work during a ride on a specific bicycle. (Your plans should include measurements you can make with equipment available to you.) If others in the class analyzed different bicycle models, compare the models for efficiency and mechanical advantage.

5. Prepare a poster or a series of models of simple machines, explaining their use and how they work. Include a schematic diagram next to each sample or picture to identify the fulcrum, lever arm, and resistance. Add your own examples to the following list: nail clipper, wheelbarrow, can opener, nutcracker, electric drill, screwdriver, tweezers, key in lock.

6. Research what architects do, and create a presentation on how they use physics in their work. What studies and training are necessary? What are some areas of specialization in architecture? What associations and professional groups keep architects informed about new developments?

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CHAPTER 8
Laboratory Exercise

MACHINES AND EFFICIENCY

In this experiment, you will raise objects using two different types of machines. You will find the work input and the work output for each machine. The ratio of the useful work output to the work input is called the *efficiency* of the machine. By calculating efficiency, you will be able to compare different machines for different jobs.

SAFETY

- **Tie back long hair, secure loose clothing, and remove loose jewelry to prevent their getting caught in moving parts and pulleys.**
- **Attach string to masses and objects securely. Falling or dropped masses can cause serious injury.**

OBJECTIVES

- Measure the work input and work output of several machines.
- Calculate the efficiency of each machine.
- Compare machines based on their efficiencies, and determine what factors affect efficiency.

MATERIALS LIST

- ✔ balance
- ✔ C-clamp
- ✔ cord
- ✔ dynamics cart
- ✔ inclined plane
- ✔ mass hanger
- ✔ pulleys, single and tandem
- ✔ meterstick
- ✔ set of hooked masses
- ✔ right-angle clamp
- ✔ support stand
- ✔ suspension clamp

PREPARATION

1. Read the entire lab, and plan what measurements you will take.
2. Prepare a data table with six columns and seven rows in your lab notebook. In the first row, label the first through sixth columns *Trial, Machine, mass*$_1$ *(kg)*, Δh *(m)*, *mass*$_2$ *(kg)*, and Δd *(m)*. In the first column, label the second through seventh rows *1, 2, 3, 4, 5,* and *6.*

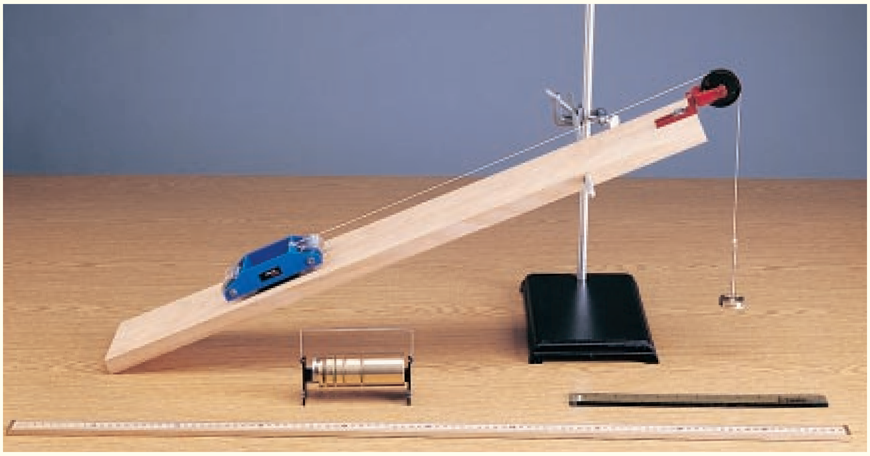

Figure 8-28
Step 3: Choose any angle, but make sure the top of the plane is at least 20 cm above the table.
Step 4: Make sure the string is long enough to help prevent the cart from falling off the top of the plane. Attach the mass hanger securely to the end of the string.
Step 7: Keep the angle the same for all trials.

PROCEDURE

Inclined plane

3. Set up the inclined plane as shown in **Figure 8-28** on page 313. Set the incline securely to any angle. Keep the angle constant during this part of the experiment. Place the inclined plane away from the edge of the table, or clamp its base to the edge of the table.

4. Measure the mass of the cart. Attach a piece of cord through the hole on the body of the cart. The cord should be long enough so that the other end of the cord reaches the table top before the cart reaches the top of the incline. Place the cart on the plane and run the cord over the pulley at the top of the plane. Attach a mass hanger to the free end of the cord.

5. Place a 200 g mass in the cart. Record the total mass of the cart and its contents as $mass_1$. Attach masses to the mass hanger until you find the lowest mass that will allow the cart to move up the plane with a constant velocity. Stop the cart before it reaches the top of the incline. Record the mass of the mass hanger plus the added mass as $mass_2$ in your data table.

6. Measure the distances, and record them. Δh is the *vertical* distance the cart moves, while the mass hanger on the cord moves the distance Δd.

7. Repeat steps 5 and 6 several times, increasing the mass in the cart by 100 g and finding the mass that will allow the cart to move with a constant velocity each time. Record all data for each trial in your data table.

Pulley

8. Set up a pulley system like the one shown in **Figure 8-29.** For the first trial, use five pulleys. Keep the area beneath the pulley system clear throughout the experiment. Measure the mass of the bottom set of pulleys before including them in the setup. Attach a 500 g mass to the bottom, as shown. Record the total mass of the 500 g mass plus the bottom set of pulleys as $mass_1$ in your data table.

9. Starting with 50 g, add enough mass to the mass hanger to prevent the pulleys from moving when released. Place the mass hanger just below the 500 g mass, and measure the initial positions of both masses to the nearest millimeter by measuring the height of each mass above the base.

10. Add masses to the mass hanger until you find the mass that will make the 500 g mass move up with constant velocity once it has been started. Record the mass of the mass hanger plus the added mass as $mass_2$ in your data table.

11. Measure the final positions of both masses, and record the distances (final position – initial position) in your data table. Δh is the *vertical* dis-

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tance through which the mass on the pulley is raised, while the mass on the mass hanger moves down through the distance Δd.

12. Using the same 500 g $mass_1$, perform two more trials using different pulley systems (four pulleys, six pulleys, and so on). Record all data. Be sure to include the mass of the bottom set of pulleys in the total mass that is raised in each trial.

13. Clean up your work area. Put equipment away safely so that it is ready to be used again.

Figure 8-29

Step 8: Clamp a meterstick parallel to the stand to take measurements throughout the lab.

Step 9: Use another ruler as a straight edge to help you measure the positions.

Step 10: The pulleys should not begin moving when the mass is added, but they should move with a constant velocity after a gentle push.

Step 11: Measure and record the distance moved by the mass on the pulley as Δh and the mass hanger distance as Δd.

ANALYSIS AND INTERPRETATION

Calculations and data analysis

1. Organizing data For each trial, make the following calculations:

   a. the weight of the mass being raised

   b. the weight of the mass on the string

   c. the work input and the work output

2. Analyzing results In which trial did a machine perform the most work? In which trial did a machine perform the least work?

3. Analyzing data Calculate the efficiency for each trial.

4. Evaluating data Is the machine that performed the most work also the most efficient? Is the machine that performed the least work also the least efficient? What is the relationship between work and efficiency?

5. Analyzing results Based on your calculations in item 4, which is more efficient, a pulley system or an inclined plane?

Conclusions

6. Evaluating methods Why is it important to calculate the work input and the work output from measurements made when the object is moving with constant velocity?

Extensions

7. Designing experiments Design an experiment to measure the efficiency of different lever setups. If there is time and your teacher approves, test your lever setups in the lab. How does the efficiency of a lever compare with the efficiency of the other types of machines you have studied?

8. Building models Compare the trial with the highest efficiency and the trial with the lowest efficiency. Based on their differences, design a more efficient machine than any you built in the lab. If there is time and your teacher approves, test the machine to test whether it is more efficient.