Corporate Bonds Hedging and a Fat Tailed Structural Model

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Abstract. The aim of this paper is to empirically test the effectiveness of the Merton [1974] model in measuring the sensitivity of corporate bond returns to changes in equity value. Compared to the standard framework the assumption of normally distributed rates of returns is relaxed in order to improve the measurement of the hedge ratios and to allow the use of firm specific elasticities. Despite this, results show that at most only 6.17% of the bonds have an hedge ratio ranging between [-10%; +10%] from the model predicted value.

Keywords: Credit Risk, Hedge Ratios, Corporate Bond Spreads, Spread Sensitivity, Variance Gamma, Normal Inverse Gaussian.

JEL Classification: G12, G13.

1. Introduction

The effectiveness of structural models, pioneered by Black and Scholes [1973] and Merton [1974], in modelling the *credit risk* of a company is still in debate. Indeed, nonetheless the existence of a huge theoretical literature on risky corporate debt pricing, little attention has been paid on the empirical reliability of these models. Among these few attempts Eom et al. [2004] test five different structural models. The main results of their work emphasize a poor job of structural models in predicting credit spreads. In particular the modified Merton model underestimates credit spreads while on average the other structural models overestimate spreads especially for high risk companies.

Simplifying the discussion we can indicate two main motivations of the structural models' failure in predicting bond spreads:

1. failure in measuring the *credit exposure*;
2. influence of other *non credit related variables* on corporate debt spreads.

In order to investigate how much of the spread is related to credit risk, Huang and Huang [2003] test 8 different structural models. Calibrating each model to match historical default loss experience data (default frequency and loss rates given default) they conclude that, for investment grade bonds, credit risk accounts only for a small fraction of the observed corporate-treasury yield spreads. For high yield bonds this fraction is however larger. In their work they do not test the Merton's model due to difficulties in adapting it to coupons (see Huang and Huang [2003], footnote 6). The low size of the default component in the bond spreads is moreover exhibited in numerous other papers as Philip et al. [1984], Elton et al. [2001], Collin-Dufresne et al. [2001] and Chen et al. [2007] among others. On the other hand using a different calibration procedure Longstaff et al. [2005] arrive at a different conclusion and they find that the default component accounts for the majority of the corporate spreads across all rating classes.

The effect played by non credit related variables on bond spreads and the difficulties in identifying explanatory variables (see Collin-Dufresne et al. [2001]), drive some authors to develop different empirical approaches to test

the effectiveness of structural models without focusing primarily on the magnitude of spreads. Following this line, Leland [2004] tests the ability of the structural models developed by Longstaff and Schwartz [1995] and Leland and Toft [1996] in predicting the probability of default. Concentrating on the probability of default instead of spreads, allows us to overcome problems related to the influence of non credit related variables. Leland's results show that structural models could predict the general shape of the default probability for A, Baa and B quite well for time horizons over 5 years. For shorter maturity there are some underestimation problems probably due to the diffusive nature of the stochastic processes (see Zhou [2001] and Duffie and Lando [2001] for possible solutions of this problem). Anyway Leland [2004] results are very sensitive to maturity, asset volatility and default costs.
With a different approach and focusing on *hedge ratios*, Schaefer and Strebulaev [2008] disentangle the credit related part of corporate debt price from the non credit related component. They test the sensitivity of corporate bond returns to changes in equity value using averages of the monthly hedge ratios calculated following the Merton [1974] model. Using a sample of US corporate bonds over the period December 1996 - December 2003 they find that the simple Merton model is able to capture the credit exposure of bond returns except for the AAA rating class. The work of Schaefer and Strebulaev [2008] arises many interesting questions regarding the conditions under which the Merton [1974] model actually produces good estimates of the market observed hedge ratios. First of all due to the presence of high noise in the firm specific hedge ratios, the authors use monthly averages of the hedge ratios (elasticities) belonging to each rating class. The use of

monthly averages, though it reduces the noise of the hedge ratios, it diminishes the capability to identify the motivations underlying the failures of the model. Indeed given the high non-linearities of the hedge ratios, tests of the real effectiveness of of the model would requires the use of firm specific hedge ratios instead of averages. A second question regards the identification of the main characteristics shared by those bonds for which the model produces adequate estimates of the hedge ratios. This last point is particularly of interest both from a theoretical and a practical point of view. Indeed from a pure theoretical standpoint we may be interested in identifying those variables that help in validating the model. From a practical point of view instead, we may want to analyse in what conditions the predicted hedge ratios allow for effective hedging strategies. A third important question relates the validity of the model towards different periods of time. Indeed the Merton [1974] model implies a positive sign of the elasticity of debt value with respect to equity, i.e. the hedge coefficient is always greater than 0. While it is notorious that bonds and equity returns exhibit a positive, though modest positive correlation over the long term, there is a substantial variation over a short term, including periods of negative correlation (Fleming et al. [1998], Hartmann et al. [2001], Chordia et al. [2005] and Connolly et al. [2005]). In period of negative correlation between equity and bonds rates of returns the model indeed fails in predicting the right quantity of equity to buy or sell.

Finally a similar analysis of corporate bond returns is in Collin-Dufresne et al. [2001]: they analyse changes in credit spreads through the study of some variables related to structural models, but without testing any specific model and without making any analysis of the magnitude of the estimated coefficients.

In this paper I follow the approach proposed by Schaefer and Strebulaev [2008] and focusing on hedge ratios I extend their work in the following main directions: I test the validity of the Merton's model using firm specific hedge ratios. This task is made possible once relaxed the assumption of normally distributed rates of returns. In particular following the results of Madan et al. [1998], Madan and Seneta [1990] and Carr et al. [2003] among others, two alternative asymmetric and fat tailed distributions are used: the Variance Gamma (VG) and the Normal Inverse Gaussian (NIG); given the variation over time time of the bond-equity relations, the model is moreover tested through a time varying window from December 31th 2006 to December 31th 2010, and using different proxies for the leverage and the asset value dynamics; finally I analyse the conditions under which the Merton [1974] model works better, relating the absolute distance between the estimated and the theoretical coefficients to various explanatory variables such as liquidity, time to maturity, leverage, analyst coverage and judgements and the volatility of bonds and equities rates of returns.

The sample used in this work includes domestic non-financial US corporate bonds collected in the Merrill Lynch Corporate Index and in the Merrill Lynch High Yield Master II index from January 1997 to January 2011[1]. I consider monthly closing prices from December 31th, 1996 to December 31th, 2010. The entire sample includes 11,909 bonds. From the total sample only bonds with a time to maturity of 4 years and a minimum of 20 consecu-

[1] The final sample is obtained by merging the lists of quoted bonds downloaded every December from 1997 to 2010.

tive price observations for the bond and 56 for the share of the corresponding company are considered in the analysis. After cleaning the data we end up with a final sample of 2,449 bonds issued by 568 different companies. All the bond are initially grouped using the 6 S&P rating codes taken at the time of the issuance. The analysis is subsequently extended by updating the rating classification every year. Data on the historical rating classification and on the US government bond index are downloaded from Datastream while data of the 3-months risk free rate are obtained from the Federal Reserve web site. All the other data used in the paper, i.e. prices, maturities, coupon rates etc., are downloaded from Bloomberg.
The main results of the work are:

1. though the Merton [1974] cannot be rejected for most of the bonds belonging to each rating class, at most only the 6.17% of the bonds have empirical hedge ratios that fall between $[-10\%; +10\%]$ from the theoretical predicted value. Restricting the analysis to the active bonds in the market, we observe an increase in the portion of correctly estimated hedge ratios from December 2006 to December 2010 though the number of those bonds still remain a small fraction of the total sample;
2. the estimated hedge coefficient presents a substantial variation over time with protracted period of over and under estimation. In general the Merton [1974] model overestimates the hedge ratios for investment grade bonds while it underestimates the sensitivity of high yield bonds. An abrupt change in the sensitivity of the bond on the equity

rates of returns is observed during November-December 2008 when the 2007 financial crisis unfolded. For the AAA rated bonds we observe an extended period of negative estimated hedge coefficients from December 2008 to March 2010;

3. the bonds for which the model works better are those with higher liquidity and fundamentals concentrated around their average values. The variables that seem to play a key role are the liquidity of both bonds and equities markets, leverage, the quantity and quality of the information available for a company and the volatilities of the equity and bonds rates of returns;

In line with previous works results indicate that collectively the credit part explains a low portion of the bond spread changes with an explanatory power that increases as we move toward lower rated bonds. There is a high cross correlation in the residuals and not surprisingly correlations of the bond rates of return indicate that there is a spatial relationship between bonds of adjacent rating classes. Like Collin-Dufresne et al. [2001] I find that the principal components analysis applied to the correlation matrix of the residuals indicates that almost the 90% of the variability is explained by a first common component.

The paper is organized as follows: in Section 2. I describe the hedge ratios in the Merton [1974] model along with providing a method for calculating them with alternative probability distributions. In section 3. I describe the sample and show the empirical results along with some robustness tests. Section 4. is dedicated to the analysis of the historical performance of the model. In section 5. the conditions under which the simple Merton model performs better are studied. Finally, Section 6. provides some concluding remarks and suggestions for further extensions.

2. Structural Models of Credit Risk

The idea behind the work of Schaefer and Strebulaev [2008] is to disentangle the debt price as the sum of a credit D_C and non credit D_{NC} related part:

$$D = D_C + D_{NC} \tag{2..1}$$

where D_C is the component of the debt price reflecting the credit exposure and D_{NC} is the component of debt price driven by non credit related variables. Despite pricing errors, assuming the non credit component D_{NC} being unrelated to corporate value and stock returns, bond prices sensitivity to changes in credit risk should be adequately considered in structural models.

In particular D_{NC} contains what is effectively unrelated to credit risk and also a valuation error depending on the model chosen to price the D_C component. The credit related part of debt price D_C should be reflected in credit spreads (part of the spread that depends directly on credit exposure) and is affected by two fundamental features:

- ’162 existence of default risk;
- ’162 recovery rules.

Under the assumption that the non credit related component of the debt price is uncorrelated to firm specific variables, its derivative with respect to equity value should be zero, i.e. $\partial D_{NC}/\partial E = 0$. In such a case, writing the derivative of debt price w.r.t. equity as follows:

$$\frac{\partial D}{\partial E} = \frac{\partial D_C}{\partial E} + \frac{\partial D_{NC}}{\partial E},$$

produces

$$\frac{\partial D}{\partial E} = \frac{\partial D_C}{\partial E}.$$

If a structural model correctly appraises the credit exposure of the company, it should predict a debt price sensitivity $\partial D_C/\partial E$ very close to the one observed in the market.

Given the non-linearity of debt and equity prices in what follows I slightly modify the approach of Schaefer and Strebulaev [2008] and I approximate the variation of debt value with respect to equity using a second order Taylor expansion:

$$\Delta D = \frac{\partial D}{\partial E}\Delta E + \frac{1}{2}\frac{\partial^2 D}{\partial E^2}(\Delta E)^2,$$

that after a bit of manipulation can be rewritten as:

$$r_D = h_E\, r_E + k_E\, r_{E_2}. \tag{2..2}$$

where r_D and r_E are the rates of returns of debt and equity respectively and where h_E is given by:

$$h_E := \left(\frac{1}{\Delta_E} - 1\right)\left(\frac{V}{D} - 1\right), \tag{2..3}$$

whit $\Delta_E = \partial E/\partial V$. And where

$$k_E = \frac{1}{\gamma_E}\left(\frac{V}{D} - 1\right),$$

$$r_{E_2} = \frac{(\Delta E)^2}{E}.$$

The variable γ_E is the second derivative of equity with respect to V (gamma). In order to relax the assumption of normally distributed rates of returns I follow Bakshi and Madan [2000] and I rewrite the hedging coefficient in (2..3) as:

$$h_E = \left(\frac{1}{\Pi_1} - 1\right)\left(\frac{V}{D} - 1\right), \qquad (2..4)$$

where

$$\Pi_1 = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left(\frac{exp(-iu\, log(B))\phi(u-i)}{iu\phi(-i)}\right)du. \qquad (2..5)$$

with $i = \sqrt{-1}$, $Re(x)$ indicates the real part of x and $\phi(u)$ indicates the characteristic function of the distribution considered for the dynamics of the corporate value (see Appendices 3. and 4.).

The ratio V/D in Equation (2..4) represents the market leverage obtained using the market value of the firm and debt. Given the importance of this variable In the sequel of the paper, as a robustness check, I will use three alternatives leverage measures: i) *Total Liabilities/(Market Capitalization+Total Liabilities)* (**LIAB**); ii) *Total Debt/Enterprise Value* (**EV**); iii) *Total Debt/(Book Value Equity + Total Debt)* (**BV**).

3. Sample Description and Numerical Results

Following the results of the previous section and the approximated dynamics of Equation (2..2) we estimate for each bond j and each month t the following equation:

$$\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t} \qquad (3..1)$$

where:

1. $\bar{r}_{D_{j,t}}$ is the excess return of the corporate bond over the monthly yield of the 3-months US constant maturity treasury security (RIFLGFCM03_N.B[2]);

2. $\bar{r}_{E_{j,t}}$ is the excess return of the corporate equity over the monthly yield of the 3-months US constant maturity treasury security;

3. $\bar{r}_{E^2_{j,t}} = \frac{(\Delta E_{j,t})^2}{E_{j,t}} - r_{f_t}$ is a squared excess return over the monthly yield of the 3-months US constant maturity treasury security;

4. $\bar{r}_{f_{10y,t}}$ is the excess return of the over 10 years US government index (TUSGVG5[3]) over the monthly yield of the 3-months US treasury security;

5. $h_{E_{j,t}}$ is the hedge ratio calculated through (2..4) using the indicated three measures of leverage.

The inclusion of the second order term in Equation (3..1) should capture for the non linearity of the ratio between the deltas of the bond and the share price.

[2] Downloaded from the Federal Reserve web site.
[3] Downloaded from Datastream.

Equation (3..1) is estimated for every bonds considered in the sample. The idea is that if the simple Merton model is able to capture bond returns sensitivity, the estimated coefficient $\hat{\beta}_{E_h}$ should be statistically not different from one.

As mentioned in the introductory section the sample used includes domestic US corporate bonds of the non financial industry collected in the Merrill Lynch Corporate Index and in the Merrill Lynch High Yield Master II index from January 1997 to January 2011. I consider monthly closing prices from December 31th, 1996 to December 31th, 2010. All the data with the exception of the 3-months treasury yield and the over 10 years US government index are downloaded from Bloomberg. The time series of the 3-month treasury yield is obtained from the federal reserve web site. The whole sample includes 11,909 bonds. Only bonds with a time to maturity of 4 years and a minimum of 20 consecutive observations for the bond and 56 for the share of the corresponding company are considered in the analysis. After controlling for the erroneous match of the bond and the issuer and for the minimum number of observations above we end up with a sample of 4,967 bonds issued by 766 companies. From the sample are moreover deleted the bonds with leverage of the issuing company, using the three indicated different measures, greater than 1 or equal to zero. I moreover delete bonds with a monthly return exceeding 1,000% and with a percentage of zero returns higher than 10% and 20% for equity and bonds respectively[4]. The final sam-

[4] To make an example if for bond j I have 100 monthly observations, than this bond is dropped from the sample if 20 of the 100 observations are lower in absolute value than 10^{-5}. This should guarantee that the sample does not contains very low liquid bonds.

ple contains 2,449 bonds issued by 568 different companies. Table 1 contains the basic statistics of the final sample.

Summary Statistics of the Sample

	Total Sample	AAA	AA	A	BBB	BB	B
N° Issuers	568	9	32	156	220	129	160
N° Bonds	2,449	52	198	815	845	287	252
Time to Maturity (months)	118.42	206.92	112.27	129.04	123.55	90.73	85.01
Bonds Returns							
Mean	0.0020	0.0014	0.0010	0.0019	0.0024	0.0014	0.0027
Standard Deviation	0.0348	0.0246	0.0207	0.0301	0.0393	0.0372	0.0456
Skewness	0.1138	0.1155	0.1846	0.2161	0.1210	-0.0996	-0.0546
Kurtosis	7.7209	5.1482	5.7224	6.7127	8.3909	8.3706	10.0960
Share Returns							
Mean	0.0147	0.0100	0.0131	0.0105	0.0124	0.0160	0.01928
Standard Deviation	0.1261	0.0731	0.0817	0.0995	0.1135	0.1322	0.1619
Skewness	0.3325	0.0548	0.0153	0.1485	0.2296	0.2142	0.5895
Kurtosis	5.7646	3.5725	4.42783	5.3255	5.8721	4.9161	6.0803
Leverage i) (LIAB)							
Mean	0.4609	0.2664	0.2910	0.3800	0.4611	0.4997	0.5589
Standard Deviation	0.0763	0.0450	0.0553	0.0635	0.0720	0.0816	0.0908
Leverage ii) (EV)							
Mean	0.3421	0.1614	0.1520	0.2419	0.3271	0.3909	0.4710
Standard Deviation	0.0825	0.03888	0.0402	0.0647	0.0779	0.0931	0.1056
Leverage iii) (BV)							
Mean	0.3496	0.2188	0.2633	0.2841	0.3311	0.3733	0.4493
Standard Deviation	0.0476	0.0404	0.0390	0.0430	0.0423	0.0459	0.0584
Jarque-Bera Test							
Bond Returns	0.7938	0.6346	0.7121	0.7325	0.8438	0.8049	0.9087
Share Returns	0.4912	0.2222	0.3750	0.4679	0.5136	0.4031	0.5438

Table 1 **Summary statistics.** This table reports summary of the monthly statistics of the sample over the period December 31th, 1997 - December 31th, 2010. The statistics are calculated considering all bonds belonging to the indicated rating class. The time to maturity is an average (considering all bonds belonging to each rating class) time to maturity and is expressed in average months remaining until maturity. The measures of leverage are calculated as: i) *Total Liabilities/(Market Value of Equity + Total Liabilities)* (**LIAB**); ii) *Total Debt/(Enterprise Value)* (**EV**); iii) *Total Debt/(Book Value Equity + Total Debt)* (**BV**). The Jarque-Bera test indicates the rejection rate of the normality test with a critical value of 5% for the bonds and shares included in each class of rating. In particular for each series the test assigns the value 1 if the normality is rejected and 0 if it cannot be rejected and I then calculate the average of this index inside each rating class.

The rate of return for each bond is calculated as:

$$r_{i,t} = \frac{P_{i,t} + AI_{i,t} + C_i/N_i \, \mathbb{1}_{i,t}}{P_{i,t-1} + AI_{i,t-1}} - 1$$

where $P_{i,t}$ is the clean price of bond i at month t; $AI_{i,t}$ is the accrued interest maturated from the last coupon payment for bond i up to the month t; if the coupon payment falls between time $t-1$ and t then the coupon divided for the periodicity C_i/N_i is added. $\mathbb{1}$ is the indicator function taking the value of 1 if the coupon is paid between $t-1$ and t and zero otherwise. The high rejection rates of the normality and the presence of excess kurtosis and non zero skewness provide further motivations to justify the use of alternative probability distributions.

As previously indicated the hedge ratios of the VG and NIG distributions are calculated following Bakshi and Madan [2000][5] and the estimation of VG and NIG distribution parameters is performed through the Generalized Method of Moment[6] (see Seneta [2004], Tjetjep and Seneta [2006] and Finlay and Seneta [2008]). Details of the parameters estimation are contained in Appendix 1..

In line with Schaefer and Strebulaev [2008] and Collin-Dufresne et al. [2001] Equation (3..1) is estimated separately for each bond in the sample by OLS. Tables 2 and 3 contain the estimated coefficients using firm specific and monthly average hedge ratios when the market leverage of the company is

[5] The integral in 2..5 is approximated numerically using the Simpson's rule. The truncation value of the integral is determined by an iterative algorithm that stops as the value of the integral stabiliezes.

[6] Given the high number of estimations 149,042 and the not completely closed form nature of the VG and NIG densities the use of the ML would have required a much higher computational burden.

(**LIAB**). The coefficients contained in the Tables are averages of the bond specific OLS coefficients in each rating class. Like Schaefer and Strebulaev [2008] the standard errors of the coefficients are estimated by taking into consideration for the cross-variances of the estimations (see Appendix 2.) and the R^2 are obtained by averaging the coefficients of determination of the bond specific regressions in each rating class.

The results in Table 2 indicate that on average we have to reject the hypothesis of the capability of the Merton model in measuring the hedge ratios for the AAA, AA and B rated bonds. Compared to the results of Schaefer and Strebulaev [2008] we could thus conclude that using firm specific hedge ratios, the simple Merton model does a poor job in measuring the hedge ratios for bonds with rating at both extremes. Apparently using NIG distribution the model is able to measure the sensitivity of the AAA rated bonds anyway, the high standard error for this class of rating does not allow to drive any robust conclusion since, as it can be seen, the estimated coefficient is neither statistically different from 0. Unlike Schaefer and Strebulaev [2008] this problem is not extended to the AA rated bonds, indeed the results in Table 2 show that all but the AAA rated bonds have an estimated hedging coefficient statistically different from 0. Due to collinearity problems the coefficients with firm specific hedge ratios and assuming normally distributed rate of returns are not presented. Indeed for bonds in the investment grade classes the simple Merton [1974] generates hedge ratios that approximate to zero and as a consequence we have a multicollinearity problem (see Figure 1).

Table 3 contains the OLS estimated coefficients of equation (3..1) using monthly averages instead of firm specific hedge ratios. All but the AAA classified bonds have an estimated coefficient statistically different from zero but as it can be seen from the Table, the Merton model is rejected only for the AA and B rated bond. Again the high standard error of the AAA bonds does not allow to achieve any robust conclusion about the real effectiveness of the model for this class of rating. On average the results are comparable with Schaefer and Strebulaev [2008] although the coefficients of determination are strongly below their benchmarks[7].

An interesting analysis is to look at the cross dispersion of the estimated coefficients in order to highlight their heterogeneity among bonds. For this reason figure 1 contains the absolute frequencies of the estimated $\hat{\beta}_{E_h}$. As it can be noted the coefficients estimated using firm specific hedge ratios and assuming normally distributed rates of returns are extremely dispersed. At the same time it can be noticed that great part of the estimated coefficient for the hedge ratios are negative. Given the multivariate nature of the regression the motivations underlying this phenomenon may lay on the negative correlation between equity and bonds rates of returns or on the impact of the treasury rates, this point is specifically addressed in Section 4.. However, negative estimated coefficients would induce a speculative rather than a hedging strategy with potentially high gains and loss. For those bonds indeed the Merton [1974] model fails in designing the hedge strategy. The results using firm specific hedge ratios have on average a higher standard

[7] As it can be noted from Table 7, the explanatory power of the regression is strongly affected by the period analysed.

errors for the estimated coefficients and this effect is mainly given by the high cross-variances of the coefficients among bonds (see Figure 1).

To understand how the results are affected by the initial rating classification, the model in Equation (3..1) is moreover estimated by updating every year the rating classification of the bonds. The historical rating classification is downloaded every year from January 1997 to January 2011 from Datastream. I implicitly assume that a bond classified in a particular rating class at the end of a year has remained in the same class from the end of the past year until that date. Given the impossibility to guarantee a sufficient minimum number of observations the estimation is conducted by a pooled regression. The results obtained are contained in Table 4 and as it can be seen are in line with those obtained with the system of regressions although, the lower standard errors, lead to an almost complete rejection of the effectiveness of the model. In particular similar to the previous analysis we observe a worse performance of the Merton model for bonds with rating at both extremes. The relative number of bonds in each rating class and for each year are depicted in Figure 2. Looking at this picture we observe a relative deterioration in the quality of the bonds included in the sample from December 1997 to December 2010. Indeed the percentage of investment grade bonds displays a negative trend over the whole period, while the the high yield bonds we observe the reverse. Given the information content of the rating, these particular trends may actually affect the validity of the model. Section 4. is dedicated to the analysis of the historical performance of the model.

OLS Estimates of $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$
Leverage=Total Liabilities/(Total Liabilities + Market Capitalization)
Firm Specific Hedge Ratios

Variance Gamma

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.111	-0.031	-0.102	0.117	0.150	0.174	0.089
	(2.10E-3)	(1.18E-3)	(8.91E-4)	(1.78E-3)	(2.56E-3)	(2.56E-3)	(3.02E-3)
$\hat{\beta}_{E_h}$	1.138	-0.047^{**}	0.669	0.903	1.387	1.188	1.618^{**}
	(2.30E-1)	(4.64E-1)	(2.24E-1)	(2.43E-1)	(3.00E-1)	(2.19E-1)	(2.44E-1)
$\hat{\beta}_{E_k}$ (×100)	-0.312	3.01E-4	0.433^{**}	-0.531^{*}	-0.337	-0.895^{**}	0.493
	(3.18E-3)	(4.26E-3)	(1.82E-3)	(3.12E-3)	(4.52E-3)	(3.72E-3)	(4.42E-3)
$\hat{\beta}_{rf}$	0.172^{***}	0.438^{***}	0.327^{***}	0.310^{***}	0.142^{**}	-0.035	-0.116
	(5.15E-2)	(2.82E-2)	(2.29E-2)	(4.27E-2)	(6.19E-2)	(6.32E-2)	(7.88E-2)
R^2	0.276	0.391	0.347	0.313	0.253	0.213	0.224

Normal Inverse Gaussian

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.109	-0.028	-0.103	0.113	0.150	0.166	0.087
	(2.10E-3)	(1.20E-3)	(8.95E-4)	(1.77E-3)	(2.57E-3)	(2.55E-3)	(2.98E-3)
$\hat{\beta}_{E_h}$	1.074	0.538	0.596^{*}	0.796	1.312	1.156	1.569^{**}
	(2.47E-1)	(4.38E-1)	(2.22E-1)	(2.08E-1)	(3.64E-1)	(2.09E-1)	(2.39E-1)
$\hat{\beta}_{E_k}$ (×100)	-0.322	-1.60E-1	0.405^{**}	-0.521^{*}	-0.352	-0.861^{**}	0.432
	(3.19E-3)	(4.27E-3)	(1.86E-3)	(3.09E-3)	(4.56E-3)	(3.72E-3)	(4.40E-3)
$\hat{\beta}_{rf}$	0.173^{***}	0.437^{***}	0.328^{***}	0.311^{***}	0.145^{**}	-0.035	-0.113
	(5.15E-2)	(2.87E-2)	(2.30E-2)	(4.26E-2)	(6.21E-2)	(6.28E-2)	(7.81E-2)
R^2	0.278	0.387	0.348	0.313	0.255	0.216	0.229
$\bar{n}$	59.00	73.77	72.62	60.50	57.33	53.13	52.71
N	2,449	52	198	815	845	287	252

Table 2 **OLS estimates with firm specific hedge ratios.** This table reports the results of the system of regressions $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$ with firm specific hedge ratios for the two distributions VG and NIG. With $\bar{n}$ we denote the average number of observations per bond. The reported coefficients are averages of the bond specific OLS estimated coefficients in each rating class. The standard errors are reported in parenthesis and are calculated as indicated in Appendix 2.. The p-values for the $\hat{\beta}_{E_h}$ are calculated with respect to the theoretical value of 1, the others as usual are calculated with respect to zero. The R^2 is an average of the coefficients of determination of every regression in each rating class. The variable $\bar{r}_{D_{j,t}}$ is the excess return of bond j in month t; the variable $h_{E_{j,t}} \bar{r}_{E_{j,t}}$ is the product of the excess return of share j in month t and the theoretically predicted hedge ratio with leverage defined as Total Liabilities/(Total Liabilities+Market Capitalization); the variable $\bar{r}_{E^2_{j,t}}$ is the square of the excess return of share j in month t; finally the variable $\bar{r}_{f_{10y,t}}$ is the excess return of the 10 years treasury bond. The indexes $***$, $**$ and $*$ indicate the statistical significance at 1%, 5% and 10% respectively.

OLS Estimates of $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$
Leverage=Total Liabilities/(Total Liabilities + Market Capitalization)
Monthly Average Hedge Ratios

Variance Gamma

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.113	-0.026	-0.108	0.116	0.166	0.172	0.097
	(2.06E-3)	(1.19E-3)	(8.95E-4)	(1.75E-3)	(2.52E-3)	(2.53E-3)	(2.96E-3)
$\hat{\beta}_{e_h}$	0.996	0.638	0.460^{***}	0.826	1.227	1.038	1.153
	(1.80E-1)	(4.46E-1)	(2.04E-1)	(1.92E-1)	(2.23E-1)	(1.53E-1)	(1.51E-1)
$\hat{\beta}_{e_k}$ (×100)	-0.522	-8.95E-2	0.428^{**}	-0.557^{*}	-0.635	-1.002^{***}	-0.046
	(3.28E-3)	(4.33E-3)	(1.88E-3)	(3.07E-3)	(4.65E-3)	(3.75E-3)	(4.87E-3)
$\hat{\beta}_{rf}$	0.173^{***}	0.435^{***}	0.329^{***}	0.311^{***}	0.148^{**}	-0.033	-0.119
	(5.06E-2)	(2.84E-2)	(2.30E-2)	(4.22E-2)	(6.07E-2)	(6.22E-2)	(7.73E-2)
R^2	0.280	0.385	0.349	0.315	0.260	0.224	0.234

Normal Inverse Gaussian

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.121	-0.022	-0.107	0.115	0.166	0.171	0.131
	(2.06E-3)	(1.20E-3)	(8.97E-4)	(1.76E-3)	(2.53E-3)	(2.52E-3)	(2.88E-3)
$\hat{\beta}_{e_h}$	0.999	0.540	0.453^{***}	0.779	1.161	0.980	1.276^{*}
	(1.73E-1)	(3.64E-1)	(1.91E-1)	(1.82E-1)	(2.17E-1)	(1.45E-1)	(1.54E-1)
$\hat{\beta}_{e_k}$ (×100)	-0.542^{*}	-1.69E-1	0.422^{**}	-0.555^{*}	-0.636	-0.992^{***}	-0.313
	(3.28E-3)	(4.35E-3)	(1.88E-3)	(3.07E-3)	(4.67E-3)	(3.74E-3)	(4.81E-3)
$\hat{\beta}_{rf}$	0.177^{***}	0.435^{***}	0.329^{***}	0.311^{***}	0.148^{**}	-0.033	-0.103
	(5.06E-2)	(2.87E-2)	(2.30E-2)	(4.22E-2)	(6.10E-2)	(6.20E-2)	(7.55E-2)
R^2	0.283	0.385	0.349	0.315	0.260	0.225	0.246

Normal

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.121	-0.021	-0.107	0.116	0.166	0.171	0.131
	(2.06E-3)	(1.20E-3)	(8.97E-4)	(1.76E-3)	(2.53E-3)	(2.52E-3)	(2.87E-3)
$\hat{\beta}_{e_h}$	1.017	0.728	0.458^{**}	0.789	1.167	0.994	1.290^{*}
	(1.75E-1)	(8.49E-1)	(2.15E-1)	(1.84E-1)	(2.19E-1)	(1.47E-1)	(1.55E-1)
$\hat{\beta}_{e_k}$ (×100)	-0.542^{*}	-1.98E-1	0.422^{**}	-0.556^{*}	-0.637	-0.988^{***}	-0.308
	(3.28E-3)	(4.33E-3)	(1.89E-3)	(3.07E-3)	(4.67E-3)	(3.73E-3)	(4.80E-3)
$\hat{\beta}_{rf}$	0.177^{***}	0.436^{***}	0.329^{***}	0.311^{***}	0.148^{**}	-0.033	-0.103
	(5.06E-2)	(2.86E-2)	(2.30E-2)	(4.22E-2)	(6.10E-2)	(6.20E-2)	(7.54E-2)
R^2	0.283	0.384	0.348	0.315	0.260	0.226	0.246
$\bar{n}$	59.00	73.77	72.62	60.50	57.33	53.13	52.71
N	2,449	52	198	815	845	287	252

Table 3 **OLS estimates with firm monthly average hedge ratios.** This table reports the results of the system of regressions $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$ with monthly average hedge ratios for the two distributions VG and NIG. With $\bar{n}$ we denote the average number of observations per bond. The reported coefficients are averages of the bond specific OLS estimated coefficients in each rating class. The standard errors are reported in parenthesis and are calculated as indicated in Appendix 2.. The p-values for the $\hat{\beta}_{E_h}$ are calculated with respect to the theoretical value of 1, the others as usual are calculated with respect to zero. The R^2 is an average of the coefficients of determination of every regression in each rating class. The variable $\bar{r}_{D_{j,t}}$ is the excess return of bond j in month t; the variable $h_{E_{j,t}} \bar{r}_{E_{j,t}}$ is the product of the excess return of share j in month t and the theoretically predicted hedge ratio with leverage defined as Total Liabilities/(Total Liabilities+Market Capitalization); the variable $\bar{r}_{E^2_{j,t}}$ is the square of the excess return of share j in month t; finally the variable $\bar{r}_{f_{10y,t}}$ is the excess return of the 10 years treasury bond. The indexes $***$, $**$ and $*$ indicate the statistical significance at 1%, 5% and 10% respectively.

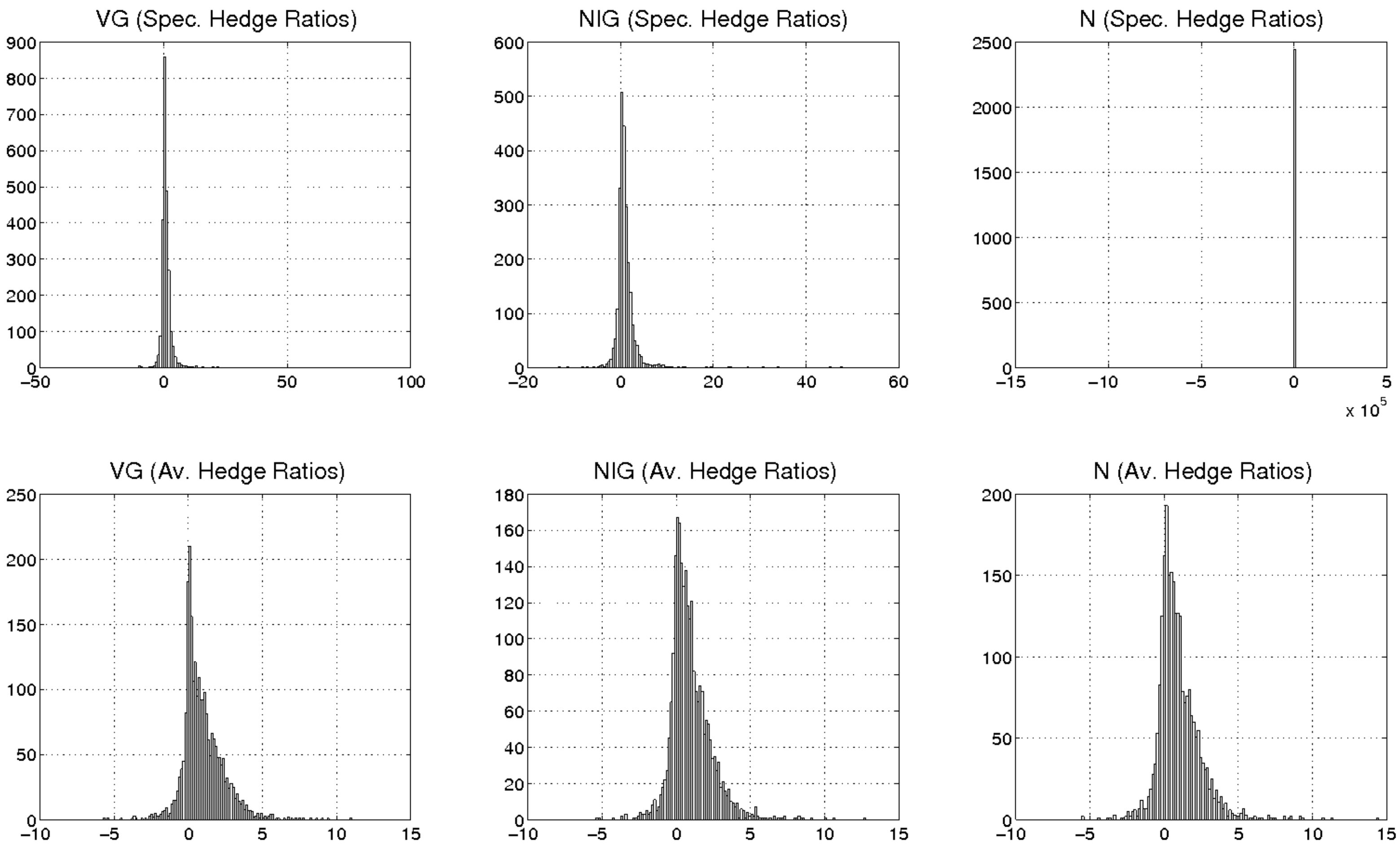

Fig. 1. This picture displays the absolute frequencies of the estimated $\hat{\beta}_{E_h}$ of equation $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$ for the Variance Gamma, Normal Inverse Gaussian and Normal probability distributions. The three histograms in the upper part of the figure are the frequencies of the estimated $\hat{\beta}_{E_h}$ using firm specific hedge ratios. The histograms in the lower part refer to the estimations with monthly average hedge ratios. The leverage used to calculate the theoretical hedge ratios is equal to *Total Liabilities/(Total Liabilities + Market Capitalization).*

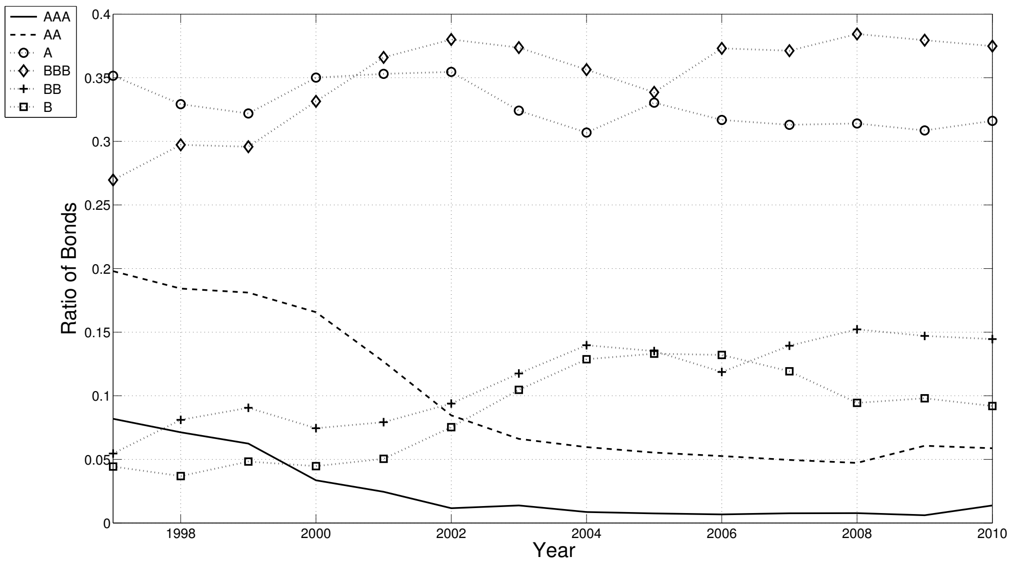

Fig. 2. This picture contains the relative number of bonds classified in each rating class from December 1997 to December 2010.

As a robustness check the model is moreover estimated using different leverage measures. The alternative leverage considered are calculated as:

$$(a) \quad \frac{TD}{EV} = \frac{\text{Total Debt}}{\text{Enterprise Value}}$$

$$(b) \quad \frac{TD}{TD+BE} = \frac{\text{Total Debt}}{\text{Total Debt + Book Value of Equity}}$$

The enterprise value is obtained from Bloomberg and is given by adding the market capitalization of equity and the market values of the traded debt. Tables 8, 9, 10 and 11 contain the results of the estimation performed considering the above alternative leverage measures. As it can be noted the results are very similar to the first leverage parametrization though, we observe a slight reduction of the rejection of the model using the book

Panel Estimates of $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$
Leverage=Total Liabilities/(Total Liabilities + Market Capitalization)
Firm Specific Hedge Ratios

Variance Gamma

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	-0.081^{***}	-0.062	-0.065^{***}	-0.065^{***}	-0.072^{***}	-0.092^{**}	-0.068
	(1.01E-4)	(5.58E-4)	(2.23E-4)	(1.27E-4)	(1.70E-4)	(3.60E-4)	(4.64E-4)
$\hat{\beta}_{e_h}$	0.823^{***}	0.402^{***}	0.150^{***}	0.601^{***}	0.981	1.102^{***}	0.655^{***}
	(1.09E-2)	(1.78E-1)	(3.76E-2)	(1.97E-2)	(2.05E-2)	(3.17E-2)	(2.85E-2)
$\hat{\beta}_{e_k}$ (×100)	-0.001^{***}	-2.97E-1	-0.033	$3.34E-4^{**}$	-0.012^{***}	-0.043^{***}	-0.074^{***}
	(1.80E-6)	(2.22E-3)	(3.56E-4)	(1.32E-6)	(1.19E-5)	(5.14E-5)	(9.74E-5)
$\hat{\beta}_{rf}$	0.182^{***}	0.501^{***}	0.371^{***}	0.362^{***}	0.188^{***}	-0.099^{***}	-0.227^{***}
	(3.06E-3)	(1.60E-2)	(6.92E-3)	(3.85E-3)	(5.05E-3)	(1.07E-2)	(1.45E-2)
R^2	0.058	0.351	0.230	0.172	0.062	0.073	0.063

Normal Inverse Gaussian

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	-0.092^{***}	-0.063	-0.072^{***}	-0.066^{***}	-0.077^{***}	-0.091^{**}	-0.130^{***}
	(1.01E-4)	(5.57E-4)	(2.22E-4)	(1.27E-4)	(1.70E-4)	(3.60E-4)	(4.54E-4)
$\hat{\beta}_{e_h}$	0.979^{*}	0.600^{**}	0.447^{***}	0.544^{***}	0.956^{**}	1.084^{***}	1.252^{***}
	(1.16E-2)	(1.92E-1)	(5.54E-2)	(1.86E-2)	(2.01E-2)	(3.06E-2)	(3.67E-2)
$\hat{\beta}_{e_k}$ (×100)	-0.001^{***}	-3.03E-1	-0.033	$4.78E-4^{***}$	-0.003^{**}	-0.044^{***}	-0.044^{***}
	(1.80E-6)	(2.21E-3)	(3.46E-4)	(1.32E-6)	(1.17E-5)	(5.13E-5)	(9.17E-5)
$\hat{\beta}_{rf}$	0.188^{***}	0.502^{***}	0.371^{***}	0.363^{***}	0.190^{***}	-0.097^{***}	-0.200^{***}
	(3.05E-3)	(1.60E-2)	(6.90E-3)	(3.85E-3)	(5.06E-3)	(1.07E-2)	(1.42E-2)
R^2	0.067	0.352	0.234	0.171	0.061	0.075	0.106

Normal

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	-0.094^{***}	-0.065	-0.064^{***}	-0.069^{***}	-0.077^{***}	-0.100^{***}	-0.131^{***}
	(1.01E-4)	(5.56E-4)	(2.23E-4)	(1.27E-4)	(1.70E-4)	(3.60E-4)	(4.54E-4)
$\hat{\beta}_{e_h}$	1.018	1.041	0.363^{***}	0.597^{***}	0.966^{*}	1.108^{***}	1.305^{***}
	(1.19E-2)	(2.56E-1)	(5.09E-2)	(1.95E-2)	(2.03E-2)	(3.11E-2)	(3.78E-2)
$\hat{\beta}_{e_k}$ (×100)	1.71E-4	-2.27E-1	-0.065^{*}	1.95E-5	-0.003^{***}	-0.023^{***}	-0.039^{***}
	(1.79E-6)	(2.22E-3)	(3.57E-4)	(1.31E-6)	(1.17E-5)	(5.05E-5)	(9.14E-5)
$\hat{\beta}_{rf}$	0.189^{***}	0.504^{***}	0.370^{***}	0.364^{***}	0.190^{***}	-0.097^{***}	-0.198^{***}
	(3.05E-3)	(1.60E-2)	(6.91E-3)	(3.85E-3)	(5.06E-3)	(1.07E-2)	(1.42E-2)
R^2	0.068	0.355	0.233	0.172	0.061	0.076	0.108
N	138,057	1,830	9,627	45,677	50,142	18,000	12,781

Table 4 **Panel estimates with firm specific hedge ratios.** This table reports the results of the system of regressions $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$ with firm specific hedge ratios for the two distributions VG and NIG. The p-values for the $\hat{\beta}_{E_h}$ are calculated with respect to the theoretical value of 1, the others as usual are calculated with respect to zero. The variable $\bar{r}_{D_{j,t}}$ is the excess return of bond j in month t; the variable $h_{E_{j,t}} \bar{r}_{E_{j,t}}$ is the product of the excess return of share j in month t and the theoretically predicted hedge ratio with leverage defined as Total Liabilities/(Total Liabilities + Market Capitalization); the variable $\bar{r}_{E^2_{j,t}}$ is the square of the excess return of share j in month t; finally the variable $\bar{r}_{f_{10y,t}}$ is the excess return of the 10 years treasury bond. The indexes $***$, $**$ and $*$ indicate the statistical significance at 1%, 5% and 10% respectively.

value of equity.

As a further robustness check I consider a different proxy for approximating the corporate value. In Particular given that bond prices are quoted with a normalized unit measure, as a first step we can approximate the market value of debt by multiplying the monthly bond prices divided for 100 for the amount in dollars issued of every bond. After this operation we can calculate the overall company exposure by adding the market values of the bonds belonging to a particular company. We can then calculate the total rate of return by averaging the return of share and the return of the total debt:

$$r_{V_t} = r_{E_t}(1 - L_t) + r_{D_t}\, L_t \tag{3..2}$$

where:

$$L_t = \frac{\text{Total Liabilities}_t}{\text{Total Liabilities}_t + \text{Market Value Equity}_t}$$

r_{E_t} is the month t rate of return of share and r_{D_t} is the month t rate of return of the total bond exposure of a particular company calculated as described above. Since the size of the time series included are different, a value of zero when one of the specific month observation is missing is placed[8]. The approach followed above is different from that one followed by Schaefer and Strebulaev [2008] and in principle could be more affected by the low liquidity of the bond market. In our case anyway this problem is mitigated given that we have controlled for the low liquidity of the bonds eliminating

[8] To make an example if for month t the rate of return of all the the bonds of a company were missing, because for example not yet issued, then I consider $R_{D_t} = 0$. As a consequence the value R_{V_t} is only composed by the rate of return of share. The same applies for the leverage.

the time series for which we observe a number of non trading days above 20%. Compared to the results contained in Tables 2 and 3, the use of the new set of distribution parameters produces on average higher coefficients of the hedge ratio. Indeed given the lower volatility of the bond's rates of returns compared to the equity, the estimated volatility with (3..2) is smaller than in the only equity case. The lower volatility and the concave shape of the first part of the delta of a call option, as a function of σ, produces lower hedge ratios and thus higher coefficients. Thus overall the new set of parameters only produces better estimates for the AAA and AA rated bonds but worsen the others.

4. Historical Performances

The analysis of the previous sections concentrates on the whole sample ranging from December 31th, 1996 to December 31th, 2010. In this section we use a different approach and test the implication of the Merton [1974] model using a moving window from December 31th, 2006 to December 31th, 2010. In particular, starting from the whole sample (December 31th, 1996 - December 31th, 2010), the last month observations of each bond are deleted and the model is estimate another time[9]. Given that we are interested in the ability of the model to generate market observed hedge ratios we restrict the analysis only to bonds that are active at the date considered. To make an example the results at August 2008 are restricted to bonds that are active in that month. This restriction moreover allows us to identify possible

[9] Similar results are obtained using a moving window with a fixed number of observations though in this case, we end up with a smaller sample given the need to guarantee at least 20 monthly observations for each bond.

structural changes in the performance of the model.

Figure 3 shows the results considering this particular time varying window with firm specific hedge ratios. The number of bonds for each month under the analysis along with the coefficient of determination are contained in Table 7. From the mentioned Figure we observe that we cannot reject the model for most of the period and both VG-NIG distributions with the exclusion of the BBB and B rating classes. Similar results still apply using monthly average hedge ratios. For the AAA bonds we observe a general overestimation of the sensitivity measure[10], indicating that the Merton model overestimate the sensitivity of the debt value with respect to equity. This is in line with the results of Huang and Huang [2003] that found a low impact of the credit exposure for high grades bonds. We moreover observe a general underestimation of the sensitivity measures for the non investment grade bonds. Indeed for the bonds included in these classes of rating we could expect that the simplified assumption underlying the Merton model are to binding. For all the rating classes we observe an abrupt increase followed by a strong reduction of the estimated coefficients from August 2008 to February 2008. This particular behaviour may be given by the known effect of market uncertainty in the relation between stock and bond returns (see Connolly et al. [2005]). The correlation between bonds and stock rates of returns is indeed positive if we consider all the sample period but present a high variation through time. In particular in November 2008 we experience an abrupt increase in the correlation between stock and bonds returns

[10] When the estimated coefficient is above (below) 1 it indicates that the theoretical hedge ratios are lower (higher) than those observed in the market.

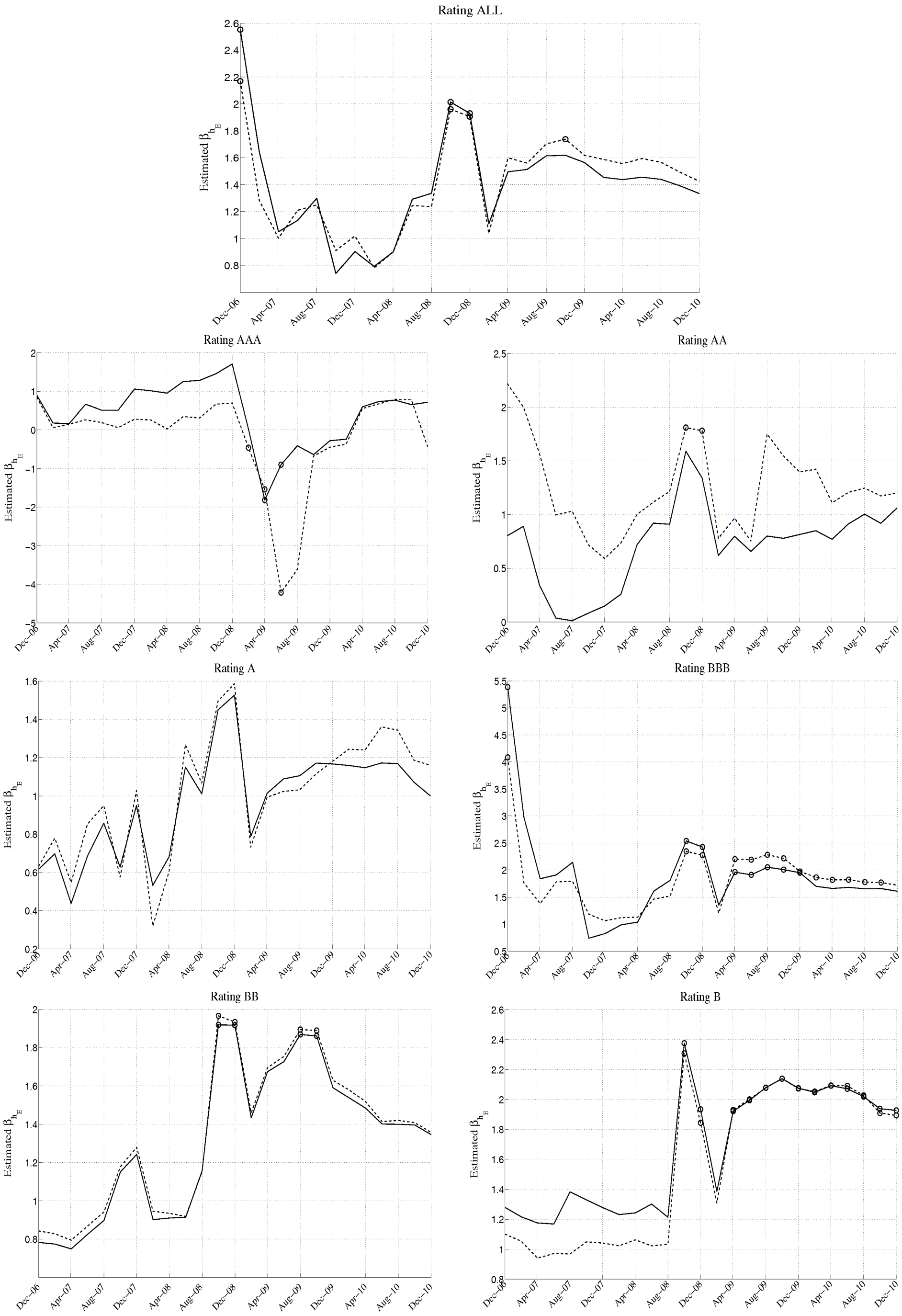

Fig. 3. **Historical dynamics of** $\hat{\beta}_{E_h}$. These plots display the estimated $\hat{\beta}_{E_h}$ coefficients of the equation: $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$ assuming a NIG (*continuous line*) and VG (*dashed line*) distributions using a time moving window from December 31th, 2006 to December 31th, 2010. The estimations that are statistically different from the theoretical value of 1 at 5% confidence level are marked with a circle. The theoretical hedge ratios are calculated with a leverage given by *Total Liabilities/(Total Liabilities + Market Capitalization).*

for all but AAA rated bonds. This abrupt phenomenon, that is not captured by the built hedge ratios, translates in the extreme movements of the estimated coefficients. The negative value of the coefficients for the AAA rated bond after December 2008 is mainly driven by the inclusion of the 10 years treasury government index rates of return. This latter effect could be caused by the high pressure on safer bond due to the flight to quality phenomenon along with the crashes in the stock markets due to the financial crisis. Indeed while the correlation between bond and share rates of returns for the AAA rated bonds, has slightly increased but still remained close to zero after the crisis, the correlation between the equity and government bond rates of returns has jumped to positive values leading to the negative sign of the estimated coefficient for this class of rating. From the analysis of the data we moreover find that the correlation between equity and bonds rates of returns for the AA and A rated bonds were negative from December 2006 to approximately August 2008, in the same period we observe a higher distance from 1 of the estimated hedge coefficients for these class of rating. Not surprisingly the highest correlations between bonds and equity is found for the B rated bonds with a maximum value of 0.45. For the AAA and AA rated bonds it remains below 0.1. In line with works of Fleming et al. [1998], Hartmann et al. [2001], Chordia et al. [2005] and Connolly et al. [2005] the results highlight a substantial time variations of the correlations between equity-bond-treasury rates of returns including sustained periods of negative correlations that produce a high time variation of the estimated hedging coefficients.

5. Key Determinants of the Model

The results of the previous sections raises two important considerations one theoretical and the other essentially practical. From a theoretical point of view we have seen that the Merton [1974] model in general cannot be rejected for bonds that belong to the middle classes of rating. This conclusion is anyway strongly affected by the period analysed, as Section 4. outlines, and on the methodology employed to calculate the standard errors of the parameters. Indeed from the results of Table 4 we observe that the model is rejected for almost all the classes of rating.

From a pure practical standpoint, a perfect hedging position would require a coefficient perfectly equal to 1. Indeed, if we only consider the relation between bond and equity rates of returns, an error in the estimation of the hedge ratio would produce a gain/loss of the following magnitude:

$$r_D - h_E r_E = (\hat{\beta}_{E_h} - 1) h_E r_E$$

where $h_E \in [0, \infty]$. For high values of h_E a $\hat{\beta}_{E_h} \neq 1$ could generate high losses/gains.

For this reason we believe that the analysis of the size of the hedging error and of the underlying determinants is of a primary importance. In this spirit this section aims to study the main characteristics that are shared by the corporate bonds for which the Merton model works better. The analysis is conducted by grouping the estimated hedge ratio coefficients of equation (3..1) based on their absolute distance from 1 and then by looking at the following characteristics: 1) average excess return of share (r_E)[11]; 2) standard deviation of the excess return of share ($std(r_E)$); 3) average excess return of bonds (r_D); 4) standard deviation of the excess return of bonds ($std(r_D)$); 5) log of the average time to maturity ($T2M$); 6) average number of analysts following a company ($N.\,An.$); 7) standard deviation of the number of analysts following a company ($std(N.\,An.)$); 8) average rating on the consensus of the analysts ($R.\,An.$); 9) standard deviation of the rating on the consensus of the analysts ($std(R.\,An.)$); 10) average zero returns of share ($Ill.\,Eq.$); 11) average zero returns of bonds ($Ill.\,D.$). 12) average leverage ($Lev.$) calculated as (*Market Value of Equity + Total Liabilities)/Total Liabilities*; 13) standard deviation of the leverage $std(Lev.)$. In particular we test the following cross-sectional equation:

$$\text{ABS}(\hat{\beta}_{E_h}\text{-}1) = \alpha_0 + \beta\, X + \epsilon \qquad (5..1)$$

Where ABS($\hat{\beta}_{E_h}$-1) is a $N \times 1$ column vector of the absolute value of the distances between the estimated coefficient and 1; α_0 is a $N \times 1$ vector of 1; β is a 1×13 column vector of coefficients; X is a $13 \times N$ matrix of the above mentioned covariates; and ϵ is a $N \times 1$ vector of spherical noise.

The results of the regression are contained in Table 5. As it can be noted the market observed and theoretical hedge ratios are closer for those bonds with higher volatility of the equity rates of returns but less volatile bonds prices. An increase of the quantity of the information available for a company, as proxied by the number of analysts and the variation of their judgements,

[11] The average excess return of share and and bond has been multiplied for 100. The standard deviation is calculated on this unit of measure.

reduces the hedging errors. For what concerns the leverage, we can observe that an increase of the leverage and a reduction of its volatility increase the distance between the market and the theoretical ratios. The first effect concerning the leverage, can be explained by the simple assumptions relative the default dynamics in the Merton [1974] model. The standard error of the leverage could indeed indicates a higher market activity reflecting better information quality for those companies.

Among the bonds that belong to the group with lower hedging error, those with $\mathrm{ABS}(\cdot) < 0.5$, particular importance is played by the liquidity of both stock and bond market, the time to maturity and the variation of the analyst judgements. The existence of a significant constant term for this group of bonds may indicate the presence of a systematic error or of missing variables that are group specific. On the other hand, the hedging errors of those bonds for which the model perform worse, those with $\mathrm{ABS}(\cdot) \geq 0.5$, are instead strongly affected by the leverage, the volatility of equity and bonds rates of returns and the quantity/quality of the information available. Restricting the analysis to bonds with an absolute error of 0.1 we obtain that among 2,449 bonds only 151 and 138, using respectively VG and NIG distributions, are between 0.9 and 1.1. Together with the results of Tables 2 and 3 this indicates that while the rejection of the Merton model may be uncommon, depending on the rating class, the empirically estimated hedge ratios are really close to the theoretical value only for a small fraction of the bonds analysed. Similar results are still obtained using monthly average hedge ratios. A cluster analysis, moreover indicates that the bonds for which the model better appraises the hedge ratios are those with main underlying variables concentrated around the average values. This last findings is mainly related to the non-linear shape of the hedge ratios. Indeed even if the average values of the time to maturity, volatilities, zero trading days are similar among bonds with correctly predicted hedge ratios and not, the volatility of the main fundamentals variables are different between groups. In Figure 4 are plotted the kernel densities estimations of the main variables. As it can be seen the bonds with a higher distance of the estimated coefficients from 1 are those with fatter tails. Finally Figure 5 displays the historical dynamics of the ratio of the bonds with observed hedge ratios close to the theoretical one and the total number of active bonds. As it can be seen from those pictures the bonds for which the model perform better are the high yield bonds. Though we observe an increase in the percentage of correctly estimated hedge ratios from December 2006 to December 2010, the portion of correctly appraised sensitivities still remain low and at most 0.21 if we consider an absolute error of 0.3. These results emphasize a systematic error in the estimation of the hedge ratios. Summarizing it seems that there is a large portion of bonds with dynamics disconnected from the equity values at least in the Merton framework and thus the results are in line with Bao and Pan [2010] findings. In particular in line with the findings of Huang and Huang [2003] an analysis of the determinants of the bonds spread changes show that credit risk accounts more for low grade than for high grade bonds. With the inclusion of well known pricing factors (SMB, HML, MARKET, VIX) we are able to explain a higher portion of the bond spread changes in all the rating class though the highest R^2 does not exceed 0.45. Principal component analysis applied to the correlation matrices of the residuals of each rating class, indicates that one common factor drives almost the 90% of the variation. This result is analogous to what has been found by Collin-Dufresne et al. [2001] and indicates that there is one common variable, not captured by the used proxies, that drives almost 50% of the variations of the bond rate of returns.

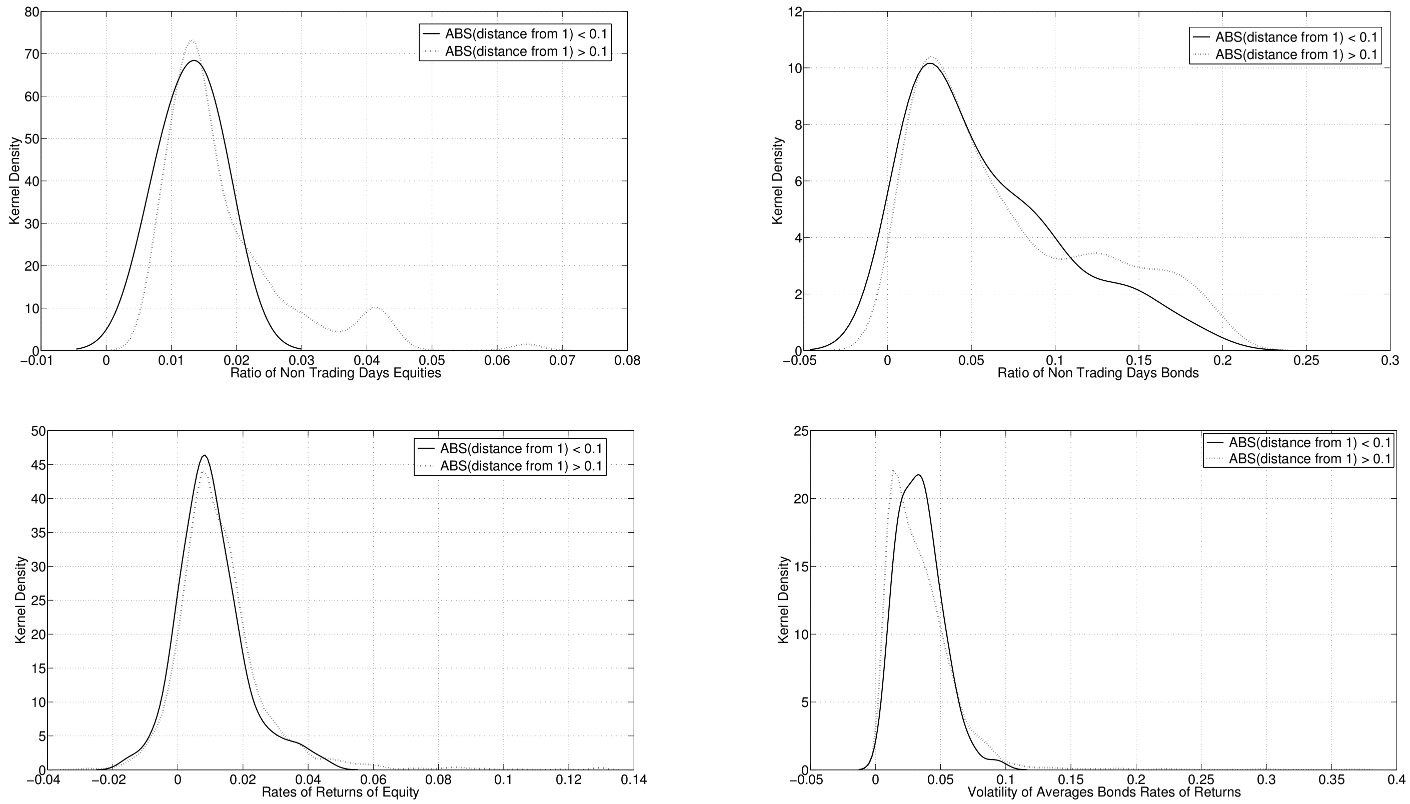

Fig. 4. Gaussian kernel density estimation of the ratio of the number of non trading days of equities, bonds, the rates of returns of equities and the volatility of the rates of returns of bonds. Given the presence of a high number of zeros in the series related the non trading days, the density is calculated only with positive values of non trading days ratios.

6. Conclusions

The results support partly Schaefer and Strebulaev [2008] findings that the simple Merton [1974] model can predict bond returns sensitivity with respect to changes in stock returns (hedge ratios). My findings suggest that the ability of Merton's framework in capturing bond returns sensitivity is strongly affected by the period analysed. Overall only a small fraction of the bonds analysed have estimated hedge ratios close to the theoretically predicted. Possible explanations of the results could be related to the estimation of the underlying variables (Huang and Huang [2003]), liquidity and tax asymmetries and to the framework describing default and loss given default (Black and Cox [1976], Leland [1994] and Leland and Toft [1996] among others). Liquidity, leverage. quality and quantity of company information and the volatility of bond and equity rates of returns seem to be the variables that most affect the empirical validity of the model. In particular I have found that the bonds for which the model perform better are those with higher liquidity, lower leverage, more available information and less dispersed volatilities of equities and bonds rates of return.

The single credit risk accounts only for a small fraction of the variability of credit spreads for high rated bonds, the explanatory power increases with high yields bonds.

Overall the results indicate that the theoretical implications of the Merton [1974] can not be generally rejected, but warn about its capability in building hedging position. A more in deep analysis of this topic would anyway require the comparison with alternative hedging strategies.

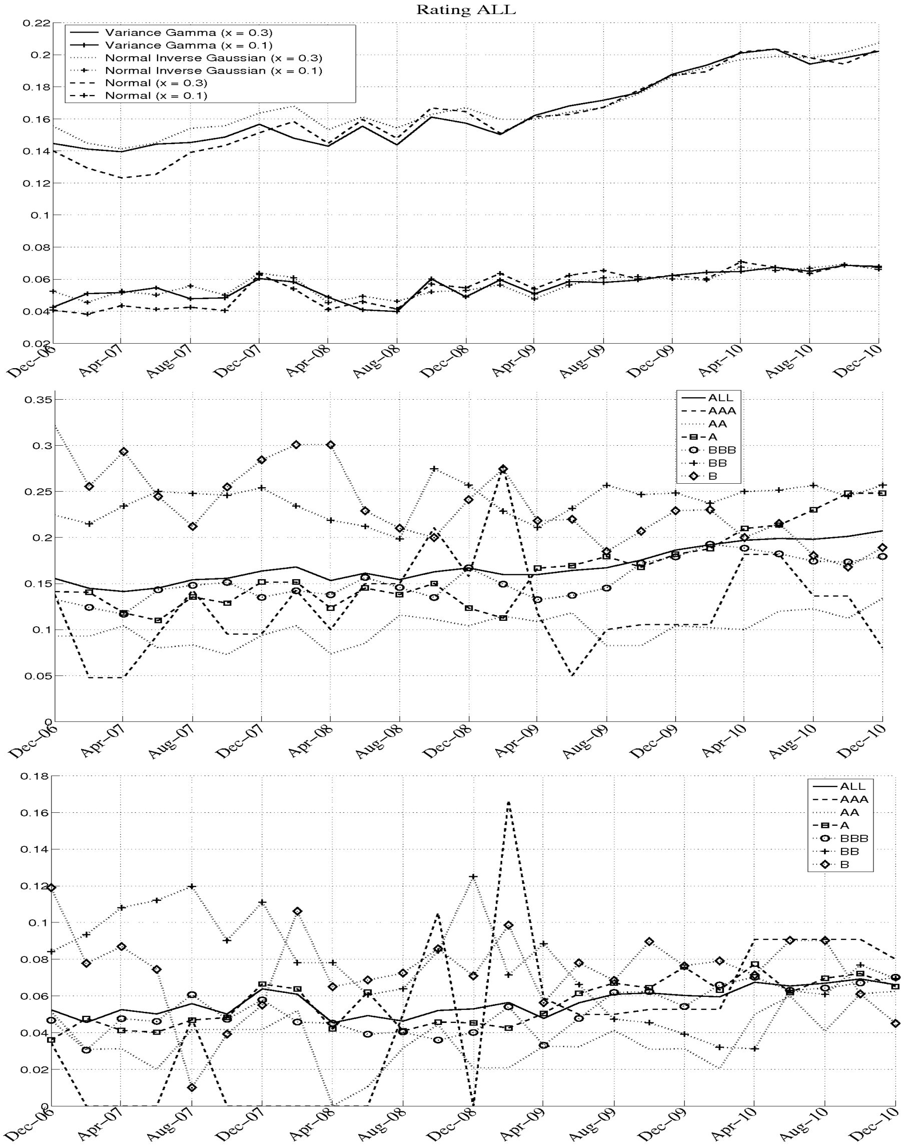

Fig. 5. **Ratios of bonds with ABS** $\left(\hat{\beta}_{E_h} - 1\right) < x$**.** These pictures display the relative number of bonds, over the total number, for which we observe a distance between the estimated and the theoretical hedge coefficients lower than $x = 0.3$ (*non marked upper lines of the first plot and second plot*) and $x = 0.1$ (*marked lower lines of the first plot and third plot*). The regressed equation is $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$. The theoretical hedge coefficients are calculated with leverage equal to *Total Liabilities/(Total Liabilities + Market Capitalization)*.

Cross-Sectional Regression of the Key Determinants

$$\text{ABS}(\hat{\beta}_{E_h}\text{-}1) = \alpha_0 + \beta X + \epsilon$$

	Variance Gamma			Normal Inverse Gaussian		
	Total	ABS(·) < 0.5	ABS (·) ≥ 0.5	Total	ABS(·) < 0.5	ABS (·) ≥ 0.5
α_0	0.1052	0.2993***	-0.4680	0.8280	0.4148***	0.3763
	(1.09E+0)	(6.81E-2)	(1.41E+0)	(7.13E-1)	(7.40E-2)	(8.91E-1)
r_E	0.0622	0.0031	-0.0040	0.0416	0.0025	-0.0179
	(4.37E-2)	(4.92E-3)	(5.97E-2)	(3.95E-2)	(4.90E-3)	(5.59E-2)
$std(r_E)$	-0.0299***	0.0004	-0.0223**	-0.0231***	-0.0016	-0.0162**
	(8.75E-3)	(8.98E-4)	(1.04E-2)	(7.19E-3)	(1.23E-3)	(7.77E-3)
r_D	0.1599	0.0240	0.2166	0.4249	0.0089	0.4976
	(3.81E-1)	(2.13E-2)	(4.11E-1)	(3.00E-1)	(2.44E-2)	(3.20E-1)
$std(r_D)$	0.3348***	-0.0012	0.3312***	0.3041***	0.0018	0.3021***
	(8.42E-2)	(4.74E-3)	(9.22E-2)	(7.19E-2)	(4.93E-3)	(7.98E-2)
$T2M$	0.1220	-0.0130	0.3112	-0.0659	-0.0278***	0.0692
	(2.21E-1)	(9.82E-3)	(2.85E-1)	(1.71E-1)	(1.03E-2)	(2.17E-1)
$N.\,An.$	-0.0283***	-5.69E-5	-0.0327**	-0.0275***	3.11E-5	-0.0314**
	(1.02E-2)	(9.51E-4)	(1.39E-2)	(9.41E-3)	(8.99E-4)	(1.30E-2)
$std(N.\,An.)$	0.0219	0.0035	-0.0093	0.0597*	0.0019	0.0452
	(3.67E-2)	(4.23E-3)	(4.62E-2)	(3.27E-2)	(3.72E-3)	(4.01E-2)
$R.\,An.$	0.0540	0.0008	0.1551	0.0400	-0.0034	0.1432
	(1.16E-1)	(1.29E-2)	(1.43E-1)	(8.05E-2)	(1.28E-2)	(1.06E-1)
$std(R.\,An.)$	-1.5780***	-0.0545*	-1.7080***	-1.3385***	-0.0533	-1.3744***
	(3.53E-1)	(3.30E-2)	(4.81E-1)	(2.85E-1)	(3.32E-2)	(3.80E-1)
$Ill.\,Eq.$	0.3719*	0.0227*	0.3708	0.4072**	0.0259**	0.4519**
	(2.02E-1)	(1.23E-2)	(2.35E-1)	(1.89E-1)	(1.07E-2)	(2.30E-1)
$Ill.\,D.$	0.0179	0.0021*	0.0115	0.0111	0.0023**	0.0056
	(1.69E-2)	(1.18E-3)	(2.16E-2)	(1.10E-2)	(1.16E-3)	(1.42E-2)
$Lev.$	0.2262***	-0.0011	0.2321**	0.1690***	-0.0063	0.1677**
	(7.76E-2)	(4.72E-3)	(1.01E-1)	(5.59E-2)	(6.55E-3)	(7.57E-2)
$std(Lev.)$	-0.3881***	-0.0005	-0.4442**	-0.3101***	-0.0005	-0.3563**
	(1.28E-1)	(4.57E-3)	(2.10E-1)	(1.07E-1)	(6.29E-3)	(1.78E-1)
R^2	0.0967	0.0187	0.1051	0.1368	0.0321	0.1504
N	2,449	733	1,716	2,449	759	1,690

Table 5 This table contains the results of the regression of the absolute value of the distance between the estimated hedge coefficients and 1 with a series of explanatory variables (equation (5..1)). The column Total contains the results relative to the whole sample while the columns ABS(·) $\gtreqless$ 0.5 contain the results of two different groups with distance lower and higher to 0.5. The variable used are: 1) average excess return of share (r_E); 2) standard deviation of the excess return of share ($std(r_E)$); 3) average excess return of bonds (r_D); 4) standard deviation of the excess return of bonds ($std(r_D)$); 5) log of the average time to maturity ($T2M$); 6) average number of analysts following a company ($N.\,An.$); 7) standard deviation of the number of analysts following a company ($std(N.\,An.)$); 8) average rating on the consensus of the analysts ($R.\,An.$); 9) standard deviation of the rating on the consensus of the analysts ($std(R.\,An.)$); 10) average of zero returns of share ($Ill.\,Eq.$); 11) average zero returns of bonds ($Ill.\,D..$). 12) average leverage ($Lev.$) calculated as (*Market Value of Equity + Total Liabilities)/Total Liabilities*; 13) standard deviation of the leverage $std(Lev.)$. The indexes $***$, $**$ and $*$ indicate the statistical significance at 1%, 5% and 10% respectively.

1. Parameters Estimation

The set of parameters for the Variance Gamma and Normal Inverse Gaussian distributions has been estimated by GMM. The orthogonality conditions are calculated by matching the theoretical and empirical first fourth moments. The theoretical moments are obtained from the characteristic functions of the two distributions that are detailed in Appendices 3. and 4.. In particular the characteristic function ϕ of a random variable X is the Fourier-Stieltjes transform of the distribution function $F(X) = P(X \leq x)$:

$$\phi_X(u) = E[exp(iuX)] = \int_{-\infty}^{+\infty} exp(iux)dF(x)$$

where $i^2 = -1$. From the characteristic function we can easily obtain the k-th moment under the condition that $E[|X|^k] < \infty$:

$$E[X^k] = i^{-k} \frac{d\phi(u)}{du^k}\Big|_{u=0}$$

Alternatively we can recover the moment generating function simply by evaluating $\nu(u) = \phi(-iu)$ and then calculate every moments by:

$$E[X^k] = \frac{d\nu(u)}{du^k}\Big|_{u=0}$$

Given that the third and fourth central moments can be rewritten as:

$$E[(x - E[x])^3] = E[x^3] - 3E[x]E[x^2] + 2E[x]^3$$

$$E[(x - E[x])^4] = E[x^4] + 6E[x]^2E[x^2] - 4E[x]E[x^3] - 3E[x]^4$$

The parameters are then estimated by solving:

$$\hat{\theta} = \arg\min_{\theta} \left(g(\theta)' \, W \, g(\theta) \right)$$

where $g(\theta)$ is a $K \times 1$ column vector that contains the K orthogonality conditions. In order to speed up the calculation and given the better quality of the estimation we use an alternative optimal weighting matrix that is given by $W = \text{diag}\,(\text{inv}(W^*))$ where

$$W^* = \sum_{i=1}^{T} \left(g(\hat{\theta})_i \, g(\hat{\theta})'_i \right)$$

The matrix $B = \text{diag}(A)$ is a matrix with diagonal elements $B_{i,i} = A_{i,i}$ and $B_{i,j \neq i} = 0$. The weighting matrix for the first iteration is set equal to the identity matrix. The estimation of the parameters has been performed for an initial sample of 1,216 different shares and a total of 149,042 monthly observations[12]. Figure 6 depicts the frequencies of the J statistic of Hansen [1982] under which:

$$T \times g(\hat{\theta})' W^* g(\hat{\theta}) \sim \chi^2_{K-L}$$

where T is the number of observations, K is the number of orthogonality conditions imposed and L is the number of the parameters. In our case the K=4 and L=3, the critical value at 95% confidence level is $\chi^2_{1;\,95\%} = 3.8415$ and $\chi^2_{1;\,97.5\%} = 5.0239$. We report in Table 6 the average correlations between the estimated and empirical moments. The standard errors of the parameters are available from the Author upon request.

[12] The number of shares included here is higher that the number of shares effectively included in the analysis since we do not take into consideration for the minimum number of bond observations and errors in the data regarding the leverage and maturity.

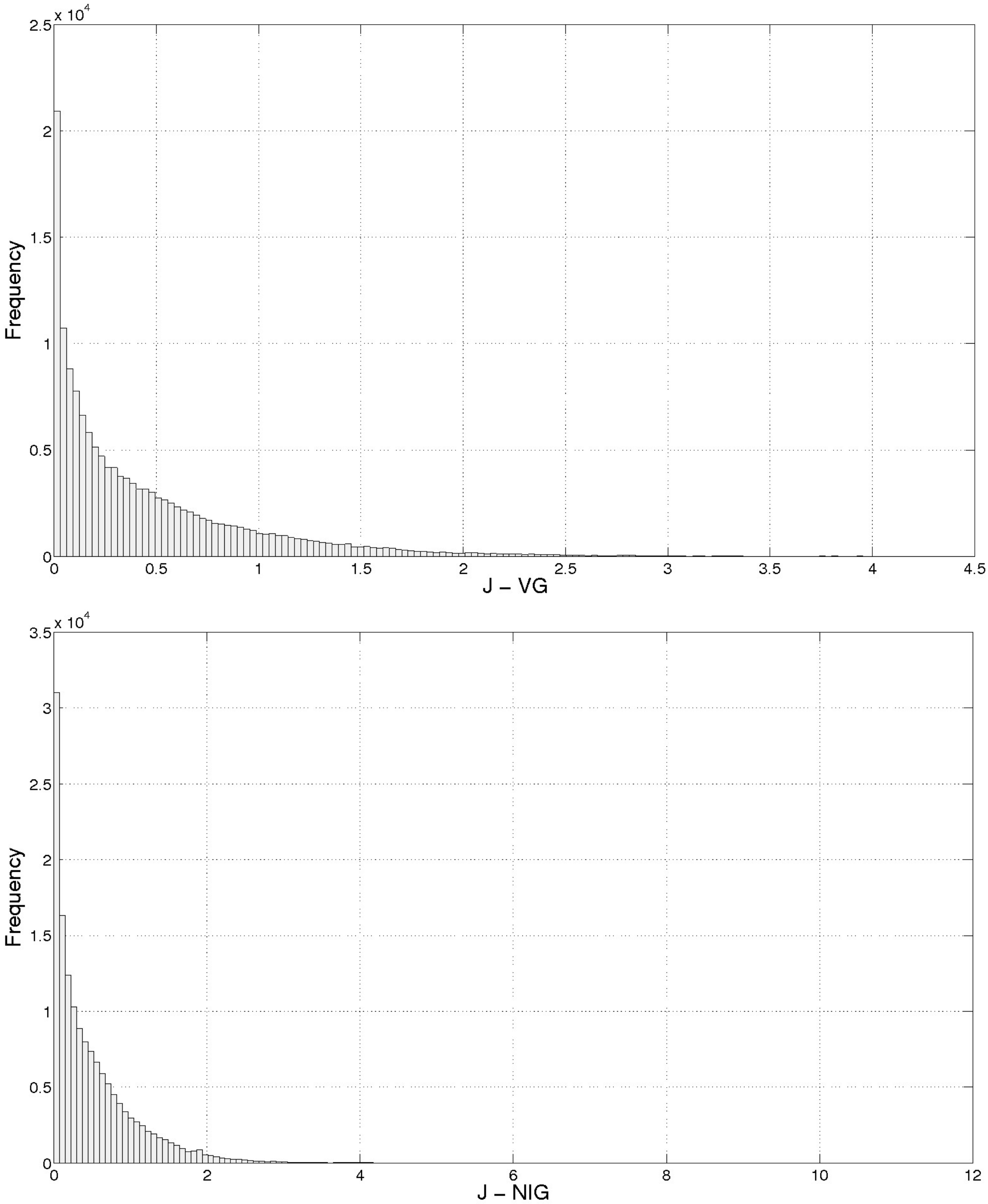

Fig. 6. This figure contains the plots of the frequency of the J statistics for the over-identification restrictions for the estimation of the parameters of the distributions. The critical χ^2 values are $\chi^2_{1;\,95\%} = 3.8415$ and $\chi^2_{1;\,97.5\%} = 5.0239$.

Dist.	M_1	M_2	M_3	M_4	$Kur.$	$Skew.$
VG	0.98	0.99	0.64	0.99	0.81	0.65
NIG	0.98	0.99	0.54	0.98	0.81	0.55

Table 6 This table contains the average correlation coefficients between the estimated and empirical moments. $M_i,\ i = 1:4$ are the first fourth central moments. Kur and $Skew$ are the Kurtosys and Skewness respectively.

2. Variance and Covariance of the Parameters

For every class of rating let $\bar{r}_{D_j}$ be the $T_j \times 1$ vector of monthly excess returns for the jth bond, $X_j = [\mathbf{1};\, h_{E_j}\bar{r}_{E_j};\, \bar{r}_{E_j^2};\, \bar{r}_{f10y,t}]$ be the $T_j \times 4$ matrix of covariates for the jth bond, where $\mathbf{1}$ is a $T_j \times 1$ column vector of ones. For every equation the coefficients are estimated through OLS:

$$\hat{\beta}_j = \left(X_j^{'} X_j\right)^{-1} X_j^{'} \bar{r}_{D_j}$$

where $\hat{\beta}$ is the $K \times 1$ vector of the estimated parameters from equation (3..1). The variance and covariance matrix of the coefficients of the N bonds in the sample is then obtained by:

$$\text{Est. Cov.} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}}{N^2} \hat{\sigma}^2_{i_s,j_s} A_{i_s} A_{j_s}^{'} \tag{A1}$$

where:

$$A_{i_s} = \left(X_{i_s}^{'} X_{i_s}\right)^{-1} X_{i_s}^{'}$$

i_s and j_s indicates that for a couple of bonds the length of the time series is homogeneous. In other words suppose that for bond i we have the observations from t_i to T_i and for bond j we have the observations from t_j to T_j, where $t_i > t_j$ and $T_i < T_j$, then in order to calculate the value in (A1) for bond i and j we first calculate $t_{i_s} = t_{j_s} = \max(t_i,\, t_j)$ and

$T_{i_s} = T_{j_s} = \min(T_i, T_j)$. In order to calculate the covariance between two series we require a minimum of 21 month observations, in other words covariances for which $T_{i_s} - t_{i_s} < 21$ are not calculated.

Finally:

$$\hat{\sigma}_{i,j} = \frac{e_{i_s}^{'} e_{j_s}}{M - t_{i_s} + 1 - K}$$

where e_{i_s} is the $T_{i_s} \times 1$ vector of the residuals from the OLS estimation.

3. Variance Gamma Distribution

The Variance Gamma (VG) process can be seen as a Gamma time changed Brownian Motion with constant drift rate (Schoutens [2003]). In particular let $G = G_t,\, t \geq 0$ be a Gamma process, that is a process starting at zero and having stationary and independent Gamma distributed increments, with

$$f_G(x; t/\nu, 1/\nu) = \frac{(1/\nu)^{(t/\nu)}}{\Gamma(t/\nu)} x^{(t/\nu - 1)} exp(-x/\nu), \quad x > 0, \tag{A1}$$

and let $W = W_t,\, t \geq 0$ be a standard Brownian motion. Assuming $\sigma > 0$ and $\theta \in \Re$, then X_t

$$X_t = \theta\, G_t + \sigma W_{G_t} \tag{A2}$$

follows a Variance Gamma process $VG(\sigma, \nu, \theta)$. Suppose that I model the continuously compounded rate of return of shares as:

$$\log\Big(\frac{V_t}{V_0}\Big) = \mu t + X_t \tag{A3}$$

where X_t is a Variance Gamma process with characteristic function

$$\Phi_X^P(\omega) = \frac{1}{\left(1 - i\theta\nu\omega + \frac{1}{2}\sigma^2\nu\omega^2\right)^{\frac{t}{\nu}}}. \tag{A4}$$

The continuously compounded firm's value rate of return has the following characteristic function:

$$\Phi_R^P(\omega) = \frac{e^{i\omega\mu\, t}}{\left(1 - i\theta\nu\omega + \frac{1}{2}\sigma^2\nu\omega^2\right)^{\frac{t}{\nu}}} \tag{A5}$$

In order to obtain the risk neutral measure I consider the mean correction procedure proposed by Schoutens [2003] leading to the following characteristic function:

$$\Phi^{\mathbb{Q}}_{\frac{\log V_T}{\log V_0}}(\omega) = \Phi^{P}_{\frac{\log V_T}{\log V_0}}(\omega) e^{i\omega\, m}, \tag{A6}$$

where m is the correction for the mean necessary to obtain an expected firm's value rate of return equal to the risk free rate r. In particular I have:

$$m = r\, t - \mu\, t + \frac{t}{\nu}\log\left(1 - \theta\nu - \frac{1}{2}\sigma^2\nu\right) \tag{A7}$$

Substituting (A7) into (A6) and rearranging terms, the risk neutral characteristic function of firm's value rate of return becomes:

$$\Phi^{\mathbb{Q}}_{\log V_t}(\omega) = \frac{e^{i\omega(\log V_0 + rt)} e^{i\omega\frac{t}{\nu}\log\left(1-\theta\nu-\frac{1}{2}\sigma^2\nu\right)}}{\left(1 - i\omega\theta\nu - \frac{1}{2}\sigma^2\nu\omega^2\right)^{\frac{t}{\nu}}}. \tag{A8}$$

Equity value thus becomes:

$$E_T = \max\big(0, V_T - K\big) \tag{A9}$$

$$E_0 = V_0 \Pi_1 - K e^{-rT} \Pi_2 \tag{A10}$$

$$\Pi_j = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} Re \left[\frac{e^{-i\omega \log K} f_j(\omega)}{i\omega} \right] d\omega, \quad j = 1, 2 \tag{A11}$$

$$f_1 = \frac{f_2 \Big(1 - i\omega\theta\nu - \frac{1}{2}\sigma^2\nu\omega^2\Big)^{\frac{t}{\nu}}}{\Big(1 - \theta\nu(1 + i\omega) + \frac{1}{2}\sigma^2\nu(\omega^2 - 1 - 2\omega\, i)\Big)^{\frac{t}{\nu}}} \tag{A12}$$

$$f_2 = \Phi^{\mathbb{Q}}_{\log V_t}(\omega) \tag{A13}$$

Under the assumption of VG distributed rate of returns, hedge ratios are obtained by substituting (A11)-(A13) into (2..4):

$$h_E = \left(\frac{1}{\Pi_1} - 1\right)\left(\frac{V}{D} - 1\right).$$

4. Normal Inverse Gaussian Distribution

The Normal Inverse Gaussian (NIG) process is an Inverse Gaussian (IG) time changed Brownian motion. Let $W = W_t$, $t \geq 0$ be a standard Brownian motion and let $I = I_t$, $t \geq 0$ be an Inverse Gaussian (IG) process starting at zero and having independent and stationary Inverse Gaussian distributed increments with:

$$f_{IG}(x;\, t,\, b) = \frac{t}{\sqrt{2\pi}} exp(tb) x^{-3/2} exp\Big(-\frac{1}{2}(t^2 x^{-1} + b^2 x)\Big), \quad x > 0, \tag{A1}$$

and $b = \delta\sqrt{\alpha^2 - \beta^2}$. Assuming $\alpha > 0$, $-\alpha < \beta < \alpha$ and $\delta > 0$ the process:

$$X_t = \beta\delta^2\, I_t + \delta\, W_{I_t} \tag{A2}$$

follows a $NIG(\alpha,\, \beta,\, \delta)$ distribution.

The characteristic function of a NIG random variable is:

$$\Phi_{NIG}(\omega) = exp\Big(- t\delta \left(\sqrt{\alpha^2 - (\beta + i\omega)^2} - \sqrt{\alpha^2 - \beta^2} \right) \Big). \tag{A3}$$

As a consequence, the characteristic function of share's return becomes:

$$\Phi_R^P(\omega) = exp\Big(i\omega\mu - t\delta \big(\sqrt{\alpha^2 - (\beta + i\omega)^2} - \sqrt{\alpha^2 - \beta^2} \big) \Big). \tag{A4}$$

In order to obtain the risk neutral measure I follow the same scheme shown in Appenix A. The mean correcting term is:

$$m = rt - \mu\, t + t\delta\Big(\sqrt{\alpha^2 - (\beta + 1)^2} - \sqrt{\alpha^2 - \beta^2} \Big), \tag{A5}$$

allowing to write equity value as:

$$E_T = \max\big(0, V_T - K\big) \tag{A6}$$

$$E_0 = V_0 \Pi_1 - K e^{-rT} \Pi_2 \tag{A7}$$

$$\Pi_j = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty Re \left[\frac{e^{-i\omega \log K} f_j(\omega)}{i\omega} \right] d\omega, \quad j = 1, 2 \tag{A8}$$

$$\begin{aligned} f_1 = f_2 \exp \Big[& t\delta\big(\sqrt{\alpha^2 - (\beta + i\omega)^2} - \sqrt{\alpha^2 - \beta^2} \big) + \\ & + t\delta\big(\sqrt{\alpha^2 - (\beta + 1)^2} - \sqrt{\alpha^2 - \beta^2} \big) + \\ & - t\delta\big(\sqrt{\alpha^2 - (\beta + i(\omega - i))^2} - \sqrt{\alpha^2 - \beta^2} \big) \Big] \end{aligned} \tag{A9}$$

$$\begin{aligned} f_2 = \exp \Big[& i\omega\big(rt + \log V_0 \big) - t\delta\big(\sqrt{\alpha^2 - (\beta + i\omega)^2} - \sqrt{\alpha^2 - \beta^2} \big) + \\ & + i\omega\, t\delta\big(\sqrt{\alpha^2 - (\beta + 1)^2} - \sqrt{\alpha^2 - \beta^2} \big) \Big] \end{aligned} \tag{A10}$$

Under NIG distributed stock price return, I can obtain the hedge ratios by substituting (A8)-(A10) into (2..4):

$$h_E = \left(\frac{1}{\Pi_1} - 1\right)\left(\frac{V}{D} - 1\right).$$

Number of Bonds

	Dec. 2006	Apr. 2007	Aug. 2007	Dec. 2008	Apr. 2008	Aug. 2008	Dec. 2008	Apr. 2009	Aug. 2009	Dec. 2009	Apr. 2010	Aug. 2010	Dec. 2010
ALL	1106	1104	1129	1143	1168	1231	1246	1277	1346	1427	1497	1509	1573
AAA	29	21	21	21	20	20	19	17	20	19	22	22	25
AA	97	96	96	96	95	95	96	92	97	96	100	98	112
A	361	364	384	376	381	391	397	396	418	462	491	517	568
BBB	428	420	412	415	421	446	449	483	517	553	584	591	613
BB	107	111	117	126	128	141	144	147	148	153	160	148	144
B	84	92	99	109	123	138	141	142	146	144	140	133	111

R^2 for the VG distribution

	Dec. 2006	Apr. 2007	Aug. 2007	Dec. 2008	Apr. 2008	Aug. 2008	Dec. 2008	Apr. 2009	Aug. 2009	Dec. 2009	Apr. 2010	Aug. 2010	Dec. 2010
ALL	0.42	0.41	0.39	0.37	0.35	0.32	0.40	0.30	0.29	0.28	0.28	0.27	0.26
AAA	0.55	0.55	0.53	0.51	0.51	0.47	0.49	0.38	0.39	0.38	0.39	0.38	0.37
AA	0.57	0.56	0.55	0.53	0.51	0.49	0.48	0.36	0.35	0.34	0.33	0.32	0.33
A	0.49	0.48	0.46	0.44	0.41	0.38	0.44	0.32	0.30	0.30	0.30	0.29	0.28
BBB	0.42	0.40	0.39	0.36	0.33	0.29	0.38	0.28	0.26	0.25	0.25	0.24	0.23
BB	0.26	0.26	0.25	0.23	0.21	0.20	0.36	0.28	0.28	0.27	0.27	0.26	0.24
B	0.17	0.18	0.17	0.17	0.18	0.18	0.35	0.29	0.29	0.28	0.28	0.27	0.26

R^2 for the NIG distribution

	Dec. 2006	Apr. 2007	Aug. 2007	Dec. 2008	Apr. 2008	Aug. 2008	Dec. 2008	Apr. 2009	Aug. 2009	Dec. 2009	Apr. 2010	Aug. 2010	Dec. 2010
ALL	0.42	0.41	0.40	0.37	0.35	0.32	0.41	0.30	0.29	0.28	0.28	0.27	0.26
AAA	0.55	0.55	0.53	0.51	0.51	0.47	0.49	0.37	0.39	0.38	0.38	0.38	0.36
AA	0.57	0.56	0.55	0.53	0.51	0.49	0.48	0.36	0.35	0.34	0.33	0.32	0.33
A	0.49	0.48	0.46	0.44	0.42	0.39	0.44	0.32	0.30	0.30	0.30	0.29	0.28
BBB	0.42	0.40	0.39	0.36	0.33	0.29	0.38	0.28	0.26	0.25	0.25	0.24	0.23
BB	0.26	0.27	0.25	0.23	0.21	0.20	0.37	0.28	0.28	0.28	0.27	0.26	0.24
B	0.17	0.18	0.17	0.17	0.18	0.19	0.36	0.29	0.29	0.29	0.29	0.28	0.27

Table 7 Number of active bonds and coefficients of determination from December 2006 to December 2010. The dynamics of the estimated coefficients for the hedge ratios are displayed in Figure 3.

OLS Estimates of $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$
Leverage=Total Debt/Enterprise Value
Firm Specific Hedge Ratios

Variance Gamma

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.118	-0.020	-0.100	0.123	0.157	0.181	0.097
	(2.10E-3)	(1.18E-3)	(9.11E-4)	(1.78E-3)	(2.57E-3)	(2.54E-3)	(3.03E-3)
$\hat{\beta}_{e_h}$	1.049	0.832	0.522^{**}	0.696	1.314	1.257	1.531^{**}
	(2.40E-1)	(3.57E-1)	(2.00E-1)	(2.78E-1)	(2.94E-1)	(2.08E-1)	(2.31E-1)
$\hat{\beta}_{e_k}$ (×100)	-0.349	-1.70E-1	0.450^{**}	-0.554^{*}	-0.390	-0.933^{**}	0.451
	(3.18E-3)	(4.20E-3)	(1.87E-3)	(3.11E-3)	(4.52E-3)	(3.72E-3)	(4.40E-3)
$\hat{\beta}_{rf}$	0.170^{***}	0.437^{***}	0.327^{***}	0.307^{***}	0.141^{**}	-0.036	-0.117
	(5.16E-2)	(2.84E-2)	(2.33E-2)	(4.28E-2)	(6.18E-2)	(6.27E-2)	(7.90E-2)
R^2	0.276	0.385	0.346	0.310	0.255	0.218	0.226

Normal Inverse Gaussian

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.118	-0.019	-0.101	0.121	0.159	0.176	0.101
	(2.10E-3)	(1.19E-3)	(9.10E-4)	(1.78E-3)	(2.56E-3)	(2.54E-3)	(3.02E-3)
$\hat{\beta}_{e_h}$	1.054	0.628	0.405^{***}	0.825	1.299	1.187	1.418^{**}
	(2.42E-1)	(3.79E-1)	(1.94E-1)	(2.59E-1)	(3.17E-1)	(2.06E-1)	(2.09E-1)
$\hat{\beta}_{e_k}$ (×100)	-0.349	-3.07E-1	0.430^{**}	-0.545^{*}	-0.394	-0.896^{**}	0.437
	(3.19E-3)	(4.16E-3)	(1.88E-3)	(3.10E-3)	(4.52E-3)	(3.73E-3)	(4.39E-3)
$\hat{\beta}_{rf}$	0.172^{***}	0.436^{***}	0.327^{***}	0.308^{***}	0.144^{**}	-0.038	-0.115
	(5.14E-2)	(2.86E-2)	(2.33E-2)	(4.28E-2)	(6.17E-2)	(6.25E-2)	(7.87E-2)
R^2	0.278	0.384	0.343	0.312	0.257	0.220	0.229
$\bar{n}$	59.00	73.77	72.62	60.50	57.33	53.13	52.71
N	2,449	52	198	815	845	287	252

Table 8 **OLS estimates with firm specific hedge ratios.** This table reports the results of the system of regressions $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$ with monthly average hedge ratios for the two distributions VG and NIG. With $\bar{n}$ we denote the average number of observations per bond. The reported coefficients are averages of the bond specific OLS estimated coefficients in each rating class. The standard errors are reported in parenthesis and are calculated as indicated in Appendix 2.. The p-values for the $\hat{\beta}_{E_h}$ are calculated with respect to the theoretical value of 1, the others as usual are calculated with respect to zero. The R^2 is an average of the coefficients of determination of every regression in each rating class. The variable $\bar{r}_{D_{j,t}}$ is the excess return of bond j in month t; the variable $h_{E_{j,t}} \bar{r}_{E_{j,t}}$ is the product of the excess return of share j in month t and the theoretically predicted hedge ratio with leverage defined as Total Debt/Enterprise Value; the variable $\bar{r}_{E^2_{j,t}}$ is the square of the excess return of share j in month t; finally the variable $\bar{r}_{f_{10y,t}}$ is the excess return of the 10 years treasury bond. The indexes $***$, $**$ and $*$ indicate the statistical significance at 1%, 5% and 10% respectively.

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OLS Estimates of $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y},t} + \epsilon_{j,t}$
Leverage=Total Debt/Enterprise Value
Monthly Average Hedge Ratios

Variance Gamma

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.122	-0.025	-0.108	0.118	0.167	0.171	0.129
	(2.05E-3)	(1.19E-3)	(8.98E-4)	(1.76E-3)	(2.52E-3)	(2.53E-3)	(2.85E-3)
$\hat{\beta}_{e_h}$	1.169	0.480	0.478**	0.981	1.364	1.042	1.249
	(2.00E-1)	(3.96E-1)	(2.29E-1)	(2.29E-1)	(2.48E-1)	(1.52E-1)	(1.62E-1)
$\hat{\beta}_{e_k}$ (×100)	−0.561*	-5.62E-2	0.430**	−0.576*	-0.674	−0.999***	-0.273
	(3.28E-3)	(4.24E-3)	(1.87E-3)	(3.07E-3)	(4.66E-3)	(3.74E-3)	(4.78E-3)
$\hat{\beta}_{rf}$	0.177***	0.435***	0.329***	0.311***	0.147**	-0.033	-0.100
	(5.03E-2)	(2.86E-2)	(2.30E-2)	(4.22E-2)	(6.07E-2)	(6.21E-2)	(7.54E-2)
R^2	0.284	0.384	0.348	0.315	0.262	0.227	0.240

Normal Inverse Gaussian

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.123	-0.029	-0.107	0.119	0.169	0.171	0.131
	(2.06E-3)	(1.19E-3)	(9.01E-4)	(1.76E-3)	(2.53E-3)	(2.52E-3)	(2.90E-3)
$\hat{\beta}_{e_h}$	1.115	0.754	0.462**	0.932	1.309	0.989	1.253*
	(1.93E-1)	(3.54E-1)	(2.30E-1)	(2.19E-1)	(2.42E-1)	(1.45E-1)	(1.52E-1)
$\hat{\beta}_{e_k}$ (×100)	−0.565*	-3.18E-1	0.426**	−0.577*	-0.680	−0.996***	-0.276
	(3.29E-3)	(4.06E-3)	(1.89E-3)	(3.08E-3)	(4.68E-3)	(3.74E-3)	(4.83E-3)
$\hat{\beta}_{rf}$	0.177***	0.435***	0.329***	0.312***	0.148**	-0.033	-0.102
	(5.06E-2)	(2.86E-2)	(2.31E-2)	(4.23E-2)	(6.09E-2)	(6.20E-2)	(7.60E-2)
R^2	0.283	0.386	0.347	0.315	0.261	0.228	0.246

Normal

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.122	-0.020	-0.106	0.118	0.168	0.171	0.131
	(2.06E-3)	(1.19E-3)	(9.00E-4)	(1.76E-3)	(2.53E-3)	(2.52E-3)	(2.90E-3)
$\hat{\beta}_{e_h}$	1.190	1.415	0.695	1.005	1.349	1.011	1.267*
	(2.01E-1)	(2.09E+0)	(3.84E-1)	(2.32E-1)	(2.49E-1)	(1.48E-1)	(1.53E-1)
$\hat{\beta}_{e_k}$ (×100)	−0.569*	-2.61E-1	0.415**	−0.583*	-0.686	−0.993***	-0.269
	(3.29E-3)	(4.32E-3)	(1.90E-3)	(3.07E-3)	(4.68E-3)	(3.73E-3)	(4.82E-3)
$\hat{\beta}_{rf}$	0.176***	0.435***	0.330***	0.311***	0.148**	-0.032	-0.102
	(5.05E-2)	(2.84E-2)	(2.31E-2)	(4.22E-2)	(6.08E-2)	(6.19E-2)	(7.59E-2)
R^2	0.284	0.384	0.347	0.316	0.261	0.228	0.246
$\bar{n}$	59.00	73.77	72.62	60.50	57.33	53.13	52.71
N	2,449	52	198	815	845	287	252

Table 9 **OLS estimates with monthly average hedge ratios.** This table reports the results of the system of regressions $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y},t} + \epsilon_{j,t}$ with monthly average hedge ratios for the two distributions VG and NIG. With $\bar{n}$ we denote the average number of observations per bond. The reported coefficients are averages of the bond specific OLS estimated coefficients in each rating class. The standard errors are reported in parenthesis and are calculated as indicated in Appendix 2.. The p-values for the $\hat{\beta}_{E_h}$ are calculated with respect to the theoretical value of 1, the others as usual are calculated with respect to zero. The R^2 is an average of the coefficients of determination of every regression in each rating class. The variable $\bar{r}_{D_{j,t}}$ is the excess return of bond j in month t; the variable $h_{E_{j,t}} \bar{r}_{E_{j,t}}$ is the product of the excess return of share j in month t and the theoretically predicted hedge ratio with leverage defined as Total Debt/Enterprise Value; the variable $\bar{r}_{E^2_{j,t}}$ is the square of the excess return of share j in month t; finally the variable $\bar{r}_{f_{10y},t}$ is the excess return of the 10 years treasury bond. The indexes $***$, $**$ and $*$ indicate the statistical significance at 1%, 5% and 10% respectively.

OLS Estimates of $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$
Leverage=Total Debt/(Total Debt + Book Value Equity)
Firm Specific Hedge Ratios

	All	AAA	AA	A	BBB	BB	B
Variance Gamma							
$\hat{\alpha}_0$ (×100)	0.128	-0.027	-0.104	0.125	0.169	0.187	0.142
	(2.07E-3)	(1.18E-3)	(8.97E-4)	(1.77E-3)	(2.53E-3)	(2.54E-3)	(2.91E-3)
$\hat{\beta}_{e_h}$	1.104	0.610	0.402**	0.985	1.405	1.052	1.194
	(2.33E-1)	(3.89E-1)	(2.35E-1)	(2.80E-1)	(2.82E-1)	(1.83E-1)	(1.71E-1)
$\hat{\beta}_{e_k}$ (×100)	-0.482	-6.30E-2	0.493***	−0.580*	-0.542	−1.050***	-0.172
	(3.22E-3)	(4.25E-3)	(1.86E-3)	(3.06E-3)	(4.56E-3)	(3.78E-3)	(4.71E-3)
$\hat{\beta}_{rf}$	0.172***	0.436***	0.327***	0.308***	0.143**	-0.036	-0.112
	(5.09E-2)	(2.83E-2)	(2.30E-2)	(4.25E-2)	(6.11E-2)	(6.24E-2)	(7.63E-2)
R^2	0.280	0.387	0.348	0.312	0.259	0.222	0.238
Normal Inverse Gaussian							
	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.126	-0.019	-0.107	0.121	0.171	0.176	0.145
	(2.07E-3)	(1.18E-3)	(8.99E-4)	(1.76E-3)	(2.54E-3)	(2.52E-3)	(2.89E-3)
$\hat{\beta}_{e_h}$	1.065	0.700	0.568*	0.841	1.356	1.103	1.237
	(2.35E-1)	(4.02E-1)	(2.25E-1)	(2.54E-1)	(3.11E-1)	(1.78E-1)	(1.58E-1)
$\hat{\beta}_{e_k}$ (×100)	-0.487	-2.30E-1	0.460**	−0.565*	-0.546	−1.004***	-0.247
	(3.22E-3)	(4.21E-3)	(1.89E-3)	(3.05E-3)	(4.58E-3)	(3.76E-3)	(4.72E-3)
$\hat{\beta}_{rf}$	0.173***	0.437***	0.327***	0.308***	0.146**	-0.037	-0.106
	(5.08E-2)	(2.84E-2)	(2.30E-2)	(4.24E-2)	(6.12E-2)	(6.21E-2)	(7.56E-2)
R^2	0.282	0.386	0.347	0.314	0.260	0.225	0.245
$\bar{n}$	59.00	73.77	72.62	60.50	57.33	53.13	52.71
N	2,449	52	198	815	845	287	252

Table 10 **OLS estimates with firm specific hedge ratios.** This table reports the results of the system of regressions $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$ with monthly average hedge ratios for the two distributions VG and NIG. With $\bar{n}$ we denote the average number of observations per bond. The reported coefficients are averages of the bond specific OLS estimated coefficients in each rating class. The standard errors are reported in parenthesis and are calculated as indicated in Appendix 2.. The p-values for the $\hat{\beta}_{E_h}$ are calculated with respect to the theoretical value of 1, the others as usual are calculated with respect to zero. The R^2 is an average of the coefficients of determination of every regression in each rating class. The variable $\bar{r}_{D_{j,t}}$ is the excess return of bond j in month t; the variable $h_{E_{j,t}} \bar{r}_{E_{j,t}}$ is the product of the excess return of share j in month t and the theoretically predicted hedge ratio with leverage defined as Total Debt/(Total Debt + Book Value Equity); the variable $\bar{r}_{E^2_{j,t}}$ is the square of the excess return of share j in month t; finally the variable $\bar{r}_{f_{10y,t}}$ is the excess return of the 10 years treasury bond. The indexes $***$, $**$ and $*$ indicate the statistical significance at 1%, 5% and 10% respectively.

ence in Venice and of the International Risk Management Conference 2010 in Florence. All errors are of my responsibility.

OLS Estimates of $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$
Leverage=Total Debt/(Total Debt + Book Value Equity)
Monthly Average Hedge Ratios

Variance Gamma

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.117	-0.021	-0.108	0.118	0.167	0.174	0.132
	(2.06E-3)	(1.19E-3)	(8.94E-4)	(1.76E-3)	(2.51E-3)	(2.52E-3)	(2.88E-3)
$\hat{\beta}_{e_h}$	1.065	0.699	0.483**	0.941	1.365	1.003	1.074
	(1.91E-1)	(4.10E-1)	(2.13E-1)	(2.19E-1)	(2.45E-1)	(1.48E-1)	(1.37E-1)
$\hat{\beta}_{e_k}$ (×100)	−0.551*	-1.91E-1	0.426**	−0.568*	-0.659	−1.034***	-0.374
	(3.28E-3)	(4.20E-3)	(1.88E-3)	(3.07E-3)	(4.65E-3)	(3.76E-3)	(4.99E-3)
$\hat{\beta}_{rf}$	0.173***	0.435***	0.329***	0.311***	0.147**	-0.033	-0.108
	(5.05E-2)	(2.84E-2)	(2.29E-2)	(4.22E-2)	(6.06E-2)	(6.20E-2)	(7.56E-2)
R^2	0.282	0.385	0.349	0.315	0.261	0.228	0.242

Normal Inverse Gaussian

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.124	-0.024	-0.108	0.117	0.168	0.174	0.144
	(2.06E-3)	(1.19E-3)	(8.98E-4)	(1.76E-3)	(2.53E-3)	(2.52E-3)	(2.86E-3)
$\hat{\beta}_{e_h}$	1.069	0.646	0.457***	0.892	1.297	0.953	1.132
	(1.84E-1)	(3.97E-1)	(2.05E-1)	(2.08E-1)	(2.39E-1)	(1.40E-1)	(1.33E-1)
$\hat{\beta}_{e_k}$ (×100)	−0.564*	-2.04E-1	0.426**	−0.565*	-0.663	−1.025***	-0.481
	(3.29E-3)	(4.30E-3)	(1.89E-3)	(3.07E-3)	(4.67E-3)	(3.75E-3)	(4.98E-3)
$\hat{\beta}_{rf}$	0.177***	0.436***	0.329***	0.311***	0.148**	-0.033	-0.100
	(5.05E-2)	(2.86E-2)	(2.30E-2)	(4.22E-2)	(6.09E-2)	(6.18E-2)	(7.51E-2)
R^2	0.284	0.385	0.348	0.315	0.261	0.229	0.250

Normal

	All	AAA	AA	A	BBB	BB	B
$\hat{\alpha}_0$ (×100)	0.124	-0.020	-0.107	0.117	0.169	0.174	0.144
	(2.06E-3)	(1.20E-3)	(8.98E-4)	(1.76E-3)	(2.52E-3)	(2.52E-3)	(2.86E-3)
$\hat{\beta}_{e_h}$	1.107	0.791	0.487**	0.926	1.333	0.967	1.141
	(1.89E-1)	(5.28E-1)	(2.25E-1)	(2.16E-1)	(2.45E-1)	(1.43E-1)	(1.34E-1)
$\hat{\beta}_{e_k}$ (×100)	−0.566*	-2.32E-1	0.419**	−0.566*	-0.671	−1.023***	-0.470
	(3.29E-3)	(4.35E-3)	(1.89E-3)	(3.07E-3)	(4.67E-3)	(3.75E-3)	(4.97E-3)
$\hat{\beta}_{rf}$	0.177***	0.435***	0.329***	0.311***	0.148**	-0.033	-0.101
	(5.05E-2)	(2.85E-2)	(2.31E-2)	(4.22E-2)	(6.08E-2)	(6.19E-2)	(7.51E-2)
R^2	0.284	0.385	0.348	0.315	0.261	0.229	0.250
$\bar{n}$	59.00	73.77	72.62	60.50	57.33	53.13	52.71
N	2,449	52	198	815	845	287	252

Table 11 **OLS estimates with monthly average hedge ratios.** This table reports the results of the system of regressions $\bar{r}_{D_{j,t}} = \alpha_0 + \beta_{E_h} h_{E_{j,t}} \bar{r}_{E_{j,t}} + \beta_{E_k} \bar{r}_{E^2_{j,t}} + \beta_{rf} \bar{r}_{f_{10y,t}} + \epsilon_{j,t}$ with monthly average hedge ratios for the two distributions VG and NIG. With $\bar{n}$ we denote the average number of observations per bond. The reported coefficients are averages of the bond specific OLS estimated coefficients in each rating class. The standard errors are reported in parenthesis and are calculated as indicated in Appendix 2.. The p-values for the $\hat{\beta}_{E_h}$ are calculated with respect to the theoretical value of 1, the others as usual are calculated with respect to zero. The R^2 is an average of the coefficients of determination of every regression in each rating class. The variable $\bar{r}_{D_{j,t}}$ is the excess return of bond j in month t; the variable $h_{E_{j,t}} \bar{r}_{E_{j,t}}$ is the product of the excess return of share j in month t and the theoretically predicted hedge ratio with leverage defined as Total Debt/(Total Debt + Book Value Equity); the variable $\bar{r}_{E^2_{j,t}}$ is the square of the excess return of share j in month t; finally the variable $\bar{r}_{f_{10y,t}}$ is the excess return of the 10 years treasury bond. The indexes $***$, $**$ and $*$ indicate the statistical significance at 1%, 5% and 10% respectively.

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