HL Triangle Congruence

Common Core Math Standards

The student is expected to:

COMMON CORE **G-SRT.B.5**

Use congruence ... criteria for triangles to solve problems and to prove relationships in geometric figures.

Mathematical Practices

COMMON CORE **MP.7 Using Structure**

Language Objective

Explain the HL Congruence Theorem in your own words.

ENGAGE

Essential Question: What does the HL Triangle Congruence Theorem tell you about two triangles?

If a leg and the hypotenuse of one right triangle are congruent to the corresponding leg and hypotenuse another right triangle, the triangles are congruent.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo, asking students to describe the shape of the kite in terms of angles, triangles, and any other geometrical terms that seem relevant. Then preview the Lesson Performance Task.

Name ______________________ Class ____________ Date ________

6.3 HL Triangle Congruence

Essential Question: What does the HL Triangle Congruence Theorem tell you about two triangles?

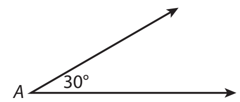
Resource Locker

Explore Is There a Side-Side-Angle Congruence Theorem?

You have already seen several theorems for proving that triangles are congruent. In this Explore, you will investigate whether there is a SSA Triangle Congruence Theorem.

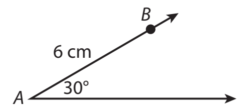
Follow these steps to draw $\triangle ABC$ such that $m\angle A = 30°$, $AB = 6$ cm, and $BC = 4$ cm. The goal is to determine whether two side lengths and the measure of a non-included angle (SSA) determine a unique triangle.

(A) Use a protractor to draw a large 30° angle on a separate sheet of paper. Label it $\angle A$.



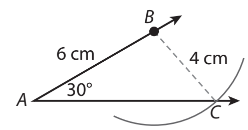
(B) Use a ruler to locate point B on one ray of $\angle A$ so that $AB = 6$ cm.

(C) Now draw $\overline{BC}$ so that $BC = 4$ cm. To do this, open a compass to a distance of 4 cm. Place the point of the compass on point B and draw an arc. Plot point C where the arc intersects the side of $\angle A$. Draw $\overline{BC}$ to complete $\triangle ABC$.



(D) What do you notice? Is it possible to draw only one $\triangle ABC$ with the given side length? Explain.

If extended, the arc would intersect $\overrightarrow{AC}$ in two places. So, it is possible to draw two different triangles with side length BC.



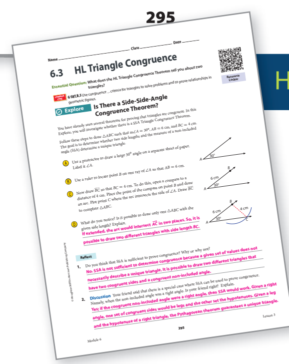
Reflect

1. Do you think that SSA is sufficient to prove congruence? Why or why not?

No. SSA is not sufficient to determine congruence because a given set of values does not necessarily describe a unique triangle. It is possible to draw two different triangles that have two congruent sides and a congruent non-included angle.

2. **Discussion** Your friend said that there is a special case where SSA can be used to prove congruence. Namely, when the non-included angle was a right angle. Is your friend right? Explain.

Yes; if the congruent non-included angle were a right angle, then SSA would work. Given a right angle, one set of congruent sides would be legs and the other set the hypotenuses. Given a leg and the hypotenuse of a right triangle, the Pythagorean theorem guarantees a unique triangle.

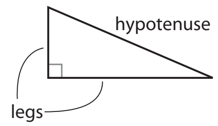


HARDCOVER PAGES 255–262

Turn to these pages to find this lesson in the hardcover student edition.

Explain 1 Justifying the Hypotenuse-Leg Congruence Theorem

In a right triangle, the side opposite the right angle is the **hypotenuse**. The two sides that form the sides of the right angle are the **legs**.



You have learned four ways to prove that triangles are congruent.

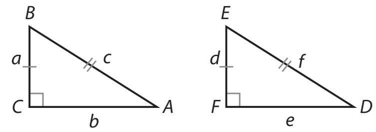
- Angle-Side-Angle (ASA) Congruence Theorem
- Side-Side-Side (SSS) Congruence Theorem
- Side-Angle-Side (SAS) Congruence Theorem
- Angle-Angle-Side (AAS) Congruence Theorem

The Hypotenuse-Leg (HL) Triangle Congruence Theorem is a special case that allows you to show that two right triangles are congruent.

Hypotenuse-Leg (HL) Triangle Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

Example 1 **Prove the HL Triangle Congruence Theorem.**



Given: $\triangle ABC$ and $\triangle DEF$ are right triangles; $\angle C$ and $\angle F$ are right angles.

$\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$

Prove: $\triangle ABC \cong \triangle DEF$

By the Pythagorean Theorem, $a^2 + b^2 = c^2$ and **d**^{2} + **e**^{2} $= f^2$. It is given that $\overline{AB} \cong \overline{DE}$, so $AB = DE$ and $c = f$. Therefore, $c^2 = f^2$ and $a^2 + b^2 =$ **d**^{2} + **e**^{2}. It is given that $\overline{BC} \cong \overline{EF}$, so $BC = EF$ and $a = d$. Substituting a for d in the above equation, $a^2 + b^2 =$ **a**^{2} + **e**^{2}.

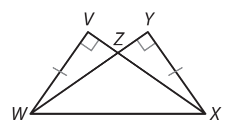
Subtracting a^2 from each side shows that $b^2 =$ **e**^{2}, and taking the square root of each side, $b =$ **e**.

This shows that $\overline{AC} \cong$ **$\overline{DF}$**.

Therefore, $\triangle ABC \cong \triangle DEF$ by **the SSS Triangle Congruence Theorem**.

Your Turn

3. Determine whether there is enough information to prove that triangles $\triangle VWX$ and $\triangle YXW$ are congruent. Explain.



Yes. $\triangle VWX$ and $\triangle YXW$ are right triangles that share hypotenuse $\overline{WX}$. $\overline{WX} \cong \overline{WX}$ by the Reflexive Property of Congruence. It is given that $\overline{WV} \cong \overline{XY}$, therefore $\triangle VWX \cong \triangle YXW$ by the HL Triangle Congruence Theorem.

EXPLORE

Is there a Side-Side-Angle Congruence Theorem?

INTEGRATE TECHNOLOGY

Students can use geometry software to explore what you can do with two sides and a non-included angle.

QUESTIONING STRATEGIES

Could you use SAS on the triangles we are discussing here? Explain. **No; SAS requires the angle to be included between the two pairs of congruent sides.**

EXPLAIN 1

Justifying the Hypotenuse-Leg Congruence Theorem

AVOID COMMON ERRORS

Students may use *hypotenuse* to describe the longest side of any triangle. Remind them that the term is used only with right triangles.

QUESTIONING STRATEGIES

With what kind of triangles can you use the Pythagorean Theorem? **right triangles only**

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices

This lesson provides an opportunity to address Mathematical Practice **MP.7**, which calls for students to "look for and make use of structure." Students look at pairs of triangles that have two congruent sides and congruent non-included angles. They analyze these relationships to determine that this information is sufficient only to prove right triangles congruent.

EXPLAIN 2

Applying the Hypotenuse-Leg Congruence Theorem

QUESTIONING STRATEGIES

Are all right angles congruent? Explain. **Yes; a right angle is an angle that measures 90°, so all right angles have the same measure. This means that all right angles are congruent.**

AVOID COMMON ERRORS

Students may assume that the hypotenuses of two given right triangles are congruent. Remind them not to assume anything is true, and to use only the information that is given.

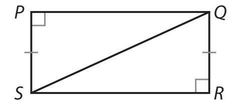
CONNECT VOCABULARY EL

Have students label the parts of a right triangle, identifying the hypotenuse, the legs, and the right angle, and then measure and write the measures of the other two angles.

Explain 2 Applying the HL Triangle Congruence Theorem

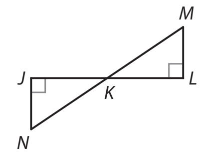
Example 2 **Use the HL Congruence Theorem to prove that the triangles are congruent.**

Ⓐ Given: $\angle P$ and $\angle R$ are right angles. $\overline{PS} \cong \overline{RQ}$
Prove: $\triangle PQS \cong \triangle RSQ$



Statements	Reasons
1. $\angle P$ and $\angle R$ are right angles.	1. Given
2. $\overline{PS} \cong \overline{RQ}$	2. Given
3. $\overline{SQ} \cong \overline{SQ}$	3. Reflexive Property of Congruence
4. $\triangle PQS \cong \triangle RSQ$	4. HL Triangle Congruence Theorem

Ⓑ Given: $\angle J$ and $\angle L$ are right angles. K is the midpoint of $\overline{JL}$ and $\overline{MN}$.
Prove: $\triangle JKN \cong \triangle LKM$



Statements	Reasons
1. $\angle J$ and $\angle L$ are right angles.	1. **Given**
2. K is the midpoint of $\overline{JL}$ and $\overline{MN}$.	2. **Given**
3. $\overline{JK} \cong \overline{LK}$ and $\overline{MK} \cong \overline{NK}$	3. **Definition of midpoint**
4. $\triangle JKN \cong \triangle LKM$	4. **HL Triangle Congruence Theorem**

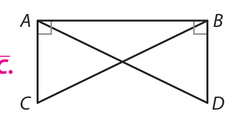
Reflect

4. Is it possible to write the proof in Part B without using the HL Triangle Congruence Theorem? Explain.
Yes, you can use the SAS Triangle Congruence Theorem ($\angle J \cong \angle L$, $\overline{JK} \cong \overline{LK}$, and vertical angles JKN and LKM are congruent) or the SAS Triangle Congruence Theorem ($\overline{JK} \cong \overline{LK}$ and $\overline{MK} \cong \overline{NK}$, and vertical angles JKN and LKM are congruent).

Your Turn

Use the HL Congruence Theorem to prove that the triangles are congruent.

5. Given: $\angle CAB$ and $\angle DBA$ are right angles. $\overline{AD} \cong \overline{BC}$
Prove: $\triangle ABC \cong \triangle BAD$
It is given that and $\angle CAB$ and $\angle DBA$ are right angles and $\overline{AD} \cong \overline{BC}$. $\overline{AB} \cong \overline{AB}$ by the Reflexive Property of Congruence. Then $\triangle ABC \cong \triangle BAD$ by the HL Triangle Congruence Theorem.



COLLABORATIVE LEARNING

Small Group Activity

Have students work in small groups to illustrate the differences between the HL and SAS Triangle Congruence Theorems. They may convey the information in any way, including making a poster, writing an essay, or creating a model. Have each group present their project to the class.

Elaborate

6. You draw a right triangle with a hypotenuse that is 5 inches long. A friend also draws a right triangle with a hypotenuse that is 5 inches long. Can you conclude that the triangles are congruent using the HL Congruence Theorem? If not, what else would you need to know in order to conclude that the triangles are congruent?

No; you cannot apply the HL Triangle Congruence Theorem if you only know the hypotenuses are congruent. You also need to know that a leg in one triangle is congruent to a leg in the other triangle.

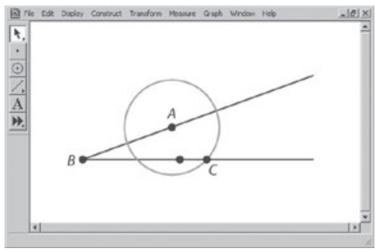
7. **Essential Question Check-In** How is the HL Triangle Congruence Theorem similar to and different from the ASA, SAS, SSS, and AAS Triangle Congruence Theorems?

Possible answer: The HL Triangle Congruence Theorem is similar to the other theorems because it provides criteria you can use to prove that two triangles are congruent. It is different from the other theorems because it only applies to right triangles.

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

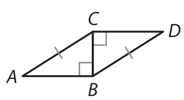
1. Tyrell used geometry software to construct $\angle ABC$ so that $m\angle ABC = 20°$. Then he dragged point A so that $AB = 6$ cm. He used the software's compass tool to construct a circle centered at point A with radius 3 cm. Based on this construction, is there a unique $\triangle ABC$ with $m\angle ABC = 20°$, $AB = 6$ cm, and $AC = 3$ cm? Explain.



No. The circle intersects $\overrightarrow{BC}$ at two different points. Either of those intersection points can be vertex C of the triangle, so there is a not a unique triangle with these side and angle measures.

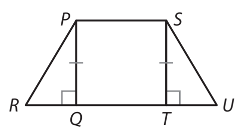
Determine whether enough information is given to prove that the triangles are congruent. Explain your answer.

2. $\triangle ABC$ and $\triangle DCB$



Yes. $\triangle ABC$ and $\triangle DCB$ are right triangles that share leg $\overline{BC}$. $\overline{BC} \cong \overline{BC}$ by the Reflexive Property of Congruence. It is given that $\overline{AC} \cong \overline{BD}$, therefore $\triangle ABC \cong \triangle DCB$ by the HL Triangle Congruence Theorem.

3. $\triangle PQR$ and $\triangle STU$



No. The triangles are right triangles and a pair of legs are congruent $\left(\overline{PQ} \cong \overline{ST}\right)$, but it is not known whether the hypotenuses are congruent.

ELABORATE

QUESTIONING STRATEGIES

Will a right angle ever be the included angle between the hypotenuse and a leg? **No; the right angle is always opposite the hypotenuse, so it will always be non-included.**

SUMMARIZE THE LESSON

How is the HL Congruence Theorem different from the other congruence theorems we have studied? **It can be used only with right triangles; it uses a non-included angle with two sides.**

EVALUATE

Personal Math Trainer

ASSIGNMENT GUIDE

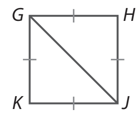
Concepts and Skills	Practice
Explore Is there a Side-Side-Angle Congruence Theorem?	Exercise 1
Example 1 Justifying the Hypotenuse-Leg Congruence Theorem	Exercises 2–5
Example 2 Applying the Hypotenuse-Leg Congruence Theorem	Exercises 6–9

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
1	**2** Skills/Concepts	**MP.5** Using Tools
2–5	**1** Recall information	**MP.2** Reasoning
6–9	**2** Skills/Concepts	**MP.3** Logic
10–13	**2** Skills/Concepts	**MP.4** Modeling
14–20	**2** Skills/Concepts	**MP.4** Modeling
21	**3** Strategic Thinking H.O.T.	**MP.2** Reasoning

QUESTIONING STRATEGIES

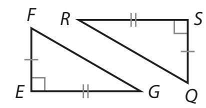
What type of triangle must be given to use HL as a method of proof? **It must be a right triangle.**

4. $\triangle GKJ$ and $\triangle JHG$



Yes. You cannot use the HL Triangle Congruence Theorem since it is not known whether the triangles are right triangles, but you can use the SSS Triangle Congruence Theorem.

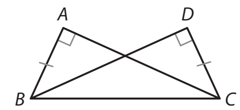
5. $\triangle EFG$ and $\triangle SQR$



Yes. You can use the Pythagorean Theorem to show that $\overline{FG} \cong \overline{QR}$ and then use the HL Triangle Congruence Theorem, or you can use the SAS Triangle Congruence Theorem with the given information.

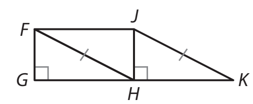
Write a two-column proof, using the HL Congruence Theorem, to prove that the triangles are congruent.

6. Given: $\angle A$ and $\angle D$ are right angles. $\overline{AB} \cong \overline{DC}$
Prove: $\triangle ABC \cong \triangle DCB$



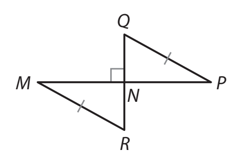
Statements	Reasons
1. $\angle A$ and $\angle D$ are right angles.	1. Given
2. $\overline{AB} \cong \overline{DC}$	2. Given
3. $\overline{BC} \cong \overline{BC}$	3. Reflexive Property of Congruence
4. $\triangle ABC \cong \triangle DCB$	4. HL Triangle Congruence Theorem

7. Given: $\angle FGH$ and $\angle JHK$ are right angles.
H is the midpoint of $\overline{GK}$. $\overline{FH} \cong \overline{JK}$
Prove: $\triangle FGH \cong \triangle JHK$



Statements	Reasons
1. $\angle FGH$ and $\angle JHK$ are right angles.	1. Given
2. H is the midpoint of $\overline{GK}$.	2. Given
3. $\overline{GH} \cong \overline{HK}$	3. Definition of midpoint
4. $\overline{FH} \cong \overline{JK}$	4. Given
5. $\triangle FGH \cong \triangle JHK$	5. HL Triangle Congruence Theorem

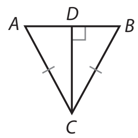
8. Given: $\overline{MP}$ is perpendicular to $\overline{QR}$.
N is the midpoint of $\overline{MP}$. $\overline{QP} \cong \overline{RM}$
Prove: $\triangle MNR \cong \triangle PNQ$



$\overline{MP} \perp \overline{QR}$ so $\angle QNP$ and $\angle MNR$ are right angles (definition of perpendicular). N is the midpoint of $\overline{MP}$, so $\overline{MN} \cong \overline{PN}$ (definition of midpoint). Then, since it is given that $\overline{QP} \cong \overline{RM}$, $\triangle MNR \cong \triangle PNQ$ by the HL Triangle Congruence Theorem.

Exercise	Depth of Knowledge (D.O.K.)	COMMON CORE Mathematical Practices
22	3 Strategic Thinking H.O.T.	MP.2 Reasoning
23	3 Strategic Thinking H.O.T.	MP.2 Reasoning

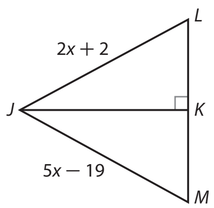
9. Given: $\angle ADC$ and $\angle BDC$ are right angles. $\overline{AC} \cong \overline{BC}$
Prove: $\overline{AD} \cong \overline{BD}$



Statements	Reasons
1. $\angle ADC$ and $\angle BDC$ are right angles.	**1. Given**
2. $\overline{AC} \cong \overline{BC}$	**2. Given**
3. $\overline{DC} \cong \overline{DC}$	**3. Reflexive Property of Congruence**
4. $\triangle ADC \cong \triangle BDC$	**4. HL Triangle Congruence Theorem**
5. $\overline{AD} \cong \overline{BD}$	**5. Corresponding parts of congruent triangles are congruent.**

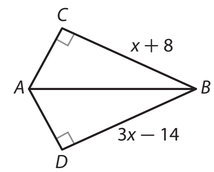
Algebra **What value of x will make the given triangles congruent? Explain.**

10. $\triangle JKL$ and $\triangle JKM$



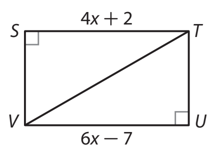
$\overline{JK} \cong \overline{JK}$ by the Reflexive Property of Congruence. If $\overline{JL} \cong \overline{JM}$, then $\triangle JKL \cong \triangle JKM$ by the HL Triangle Congruence Theorem.
If $JL = JM$, then $2x + 2 = 5x - 19$, so $x = 7$.

11. $\triangle ABC$ and $\triangle ABD$



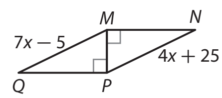
$\overline{AB} \cong \overline{AB}$ by the Reflexive Property of Congruence. If $\overline{BC} \cong \overline{BD}$, then $\triangle ABC \cong \triangle ABD$ by the HL Triangle Congruence Theorem.
If $BC = BD$, then $x + 8 = 3x - 14$, so $x = 11$.

12. $\triangle STV$ and $\triangle UVT$



$\overline{TV} \cong \overline{TV}$ by the Reflexive Property of Congruence. If $\overline{ST} \cong \overline{UV}$, then $\triangle STV \cong \triangle UVT$ by the HL Triangle Congruence Theorem.
If $ST = UV$, then $4x + 2 = 6x - 7$, so $x = 4.5$.

13. $\triangle MPQ$ and $\triangle PMN$



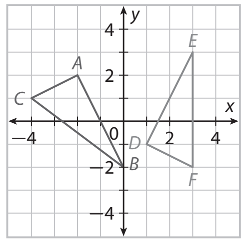
$\overline{MP} \cong \overline{MP}$ by the Reflexive Property of Congruence. If $\overline{MQ} \cong \overline{PN}$, then $\triangle MPQ \cong \triangle PMN$ by the HL Triangle Congruence Theorem.
If $MQ = PN$, then $7x - 5 = 4x + 25$, so $x = 10$.

VISUAL CUES

Use colored pencils to trace over congruent legs and hypotenuses in different colors to help students better visualize the sides that are congruent.

Algebra **Use the HL Triangle Congruence Theorem to show that $\triangle ABC \cong \triangle DEF$. (*Hint*: Use the Distance Formula to show that appropriate sides are congruent. Use the slope formula to show that appropriate angles are right angles.)**

14.



By the Distance Formula,

$AB = \sqrt{(0-(-2))^2 + (-2-2)^2} = \sqrt{20}$,

$DE = \sqrt{(3-1)^2 + (3-(-1))^2} = \sqrt{20}$,

$BC = \sqrt{(4-0)^2 + (1-(-2))^2} = 5$,

and $EF = 5$ by counting units along the vertical segment.

Therefore, $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$. By the Slope Formula,

slope of $\overline{AB} = \frac{-2-2}{0-(-2)} = -2$, slope of $\overline{AC} = \frac{1-2}{-4-(-2)} = \frac{1}{2}$,

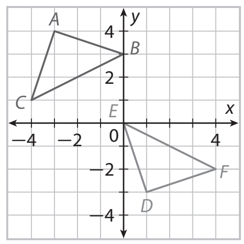
slope of $\overline{DE} = \frac{3-(-1)}{3-1} = 2$, and slope of $\overline{DF} = \frac{-2-(-1)}{3-1} = -\frac{1}{2}$.

Since $\left(\text{slope of } \overline{AB}\right) \cdot \left(\text{slope of } \overline{AC}\right) = -1$, $\overline{AB} \perp \overline{AC}$ and $\angle A$ is a right angle.

Since $\left(\text{slope of } \overline{DE}\right) \cdot \left(\text{slope of } \overline{DF}\right) = -1$, $\overline{DE} \perp \overline{DF}$ and $\angle D$ is a right angle.

So, $\triangle ABC \cong \triangle DEF$ by the HL Triangle Congruence Theorem.

15.



By the Distance Formula,

$AB = \sqrt{(0-(-3))^2 + (3-4)^2} = \sqrt{10}$,

$DE = \sqrt{(1-0)^2 + (-3-0)^2} = \sqrt{10}$,

$BC = \sqrt{(-4-0)^2 + (1-3)^2} = \sqrt{20}$,

and $EF = \sqrt{(4-0)^2 + (-2-0)^2} = \sqrt{20}$.

Therefore, $\overline{AB} \cong \overline{DE}$ and $\overline{BC} \cong \overline{EF}$. By the Slope Formula,

slope of $\overline{AB} = \frac{3-4}{0-(-3)} = -\frac{1}{3}$, slope of $\overline{AC} = \frac{1-4}{-4-(-3)} = 3$,

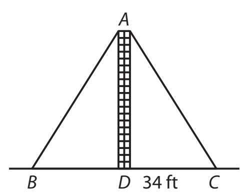
slope of $\overline{DE} = \frac{0-(-3)}{0-1} = -3$, slope of $\overline{DF} = \frac{-2-(-3)}{4-1} = \frac{1}{3}$.

Since $\left(\text{slope of } \overline{AB}\right) \cdot \left(\text{slope of } \overline{AC}\right) = -1$, $\overline{AB} \perp \overline{AC}$ and $\angle A$ is a right angle.

Since $\left(\text{slope of } \overline{DE}\right) \cdot \left(\text{slope of } \overline{DF}\right) = -1$, $\overline{DE} \perp \overline{DF}$ and $\angle D$ is a right angle.

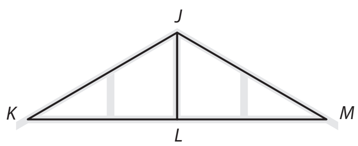
So, $\triangle ABC \cong \triangle DEF$ by the HL Triangle Congruence Theorem.

16. **Communicate Mathematical Ideas** A vertical tower is supported by two guy wires, as shown. The guy wires are both 58 feet long. Is it possible to determine the distance from the bottom of guy wire $\overline{AB}$ to the bottom of the tower? If so, find the distance. If not, explain why not.



Yes. It is given that $\overline{AB} \cong \overline{AC}$, and $\overline{AD} \cong \overline{AD}$ by the Reflexive Property of Congruence. Since the tower is vertical, $\angle ADB$ and $\angle ADC$ are right angles, so $\triangle ADB \cong \triangle ADC$ by the HL Triangle Congruence Theorem. Therefore, $\overline{BD} \cong \overline{CD}$ since corresponding parts of congruent triangles are congruent. This means $BD = CD = 34$ ft.

17. A carpenter built a truss, as shown, to support the roof of a doghouse.



a. The carpenter knows that $\overline{KJ} \cong \overline{MJ}$. Can the carpenter conclude that $\triangle KJL \cong \triangle MJL$? Why or why not?

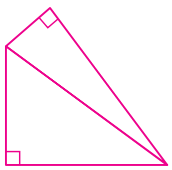
No; there is not enough information to use any of the triangle congruence theorems.

b. **What If?** Suppose the carpenter also knows that $\angle JLK$ is a right angle. Can the carpenter now conclude that $\triangle KJL \cong \triangle MJL$? Explain.

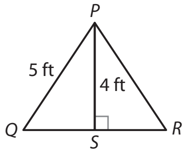
Yes; $\triangle KJL \cong \triangle MJL$ by the HL Triangle Congruence Theorem since $\overline{KJ} \cong \overline{MJ}$ and $\overline{JL} \cong \overline{JL}$.

18. **Counterexamples** Denise said that if two right triangles share a common hypotenuse, then the triangles must be congruent. Sketch a figure that serves as a counterexample to show that Denise's statement is not true.

Sample figure:



19. **Multi-Step** The front of a tent is covered by a triangular flap of material. The figure represents the front of the tent, with $\overline{PS} \perp \overline{QR}$ and $\overline{PQ} \cong \overline{PR}$. Jonah needs to determine the perimeter of $\triangle PQR$ so that he can replace the zipper on the tent. Find the perimeter. Explain your steps.



Since $\overline{PQ} \cong \overline{PR}$ and $\overline{PS} \cong \overline{PS}$, $\triangle PSQ \cong \triangle PSR$ by the HL Triangle Congruence Theorem. Therefore, $\overline{QS} \cong \overline{RS}$ since corresponding parts of congruent triangles are congruent. By the Pythagorean Theorem, $QS^2 + 4^2 = 5^2$, so $QS^2 = 9$, and $QS = 3$ ft. The perimeter of $\triangle PQR$ is 16 ft.

AVOID COMMON ERRORS

Do not assume an angle is a right angle just because it looks "square." A right angle should be so marked with its symbol or mentioned as such in the problem statement's given information.

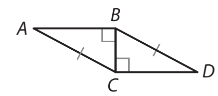
JOURNAL

Have students describe how proving two triangles congruent by HL is different from using the SAS method.

20. A student is asked to write a two-column proof for the following.

Given: $\angle ABC$ and $\angle DCB$ are right angles. $\overline{AC} \cong \overline{BD}$

Prove: $\overline{AB} \cong \overline{DC}$



Assuming the student writes the proof correctly, which of the following will appear as a statement or reason in the proof? Select all that apply.

A. ASA Triangle Congruence Theorem

B. $\overline{BC} \cong \overline{BC}$

C. $\angle A \cong \angle D$

D. Reflexive Property of Congruence

E. CPCTC

F. HL Triangle Congruence Theorem

Statements	Reasons
1. $\angle ABC$ and $\angle DCB$ are right angles.	**1. Given**
2. $\overline{AC} \cong \overline{BD}$	**2. Given**
3. $\overline{BC} \cong \overline{BC}$	**3. Reflexive Property of Congruence**
4. $\triangle ABC \cong \triangle DCB$	**4. HL Triangle Congruence Theorem**
5. $\overline{AB} \cong \overline{DC}$	**5. CPCTC**

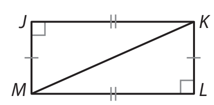
Answer: B, D, E, F

H.O.T. Focus on Higher Order Thinking

21. Analyze Relationships Is it possible for a right triangle with a leg that is 10 inches long and a hypotenuse that is 26 inches long to be congruent to a right triangle with a leg that is 24 inches long and a hypotenuse that is 26 inches long? Explain.

Yes. Let the remaining leg of the first triangle have a length of x inches. Then by the Pythagorean Theorem, $x^2 + 10^2 = 26^2$. So, $x^2 = 576$, and $x = 24$. Therefore, the hypotenuse and a leg of the first right triangle are congruent to the hypotenuse and a leg of the second right triangle, so the triangles are congruent by the HL Triangle Congruence Theorem.

22. Communicate Mathematical Ideas In the figure, $\overline{JK} \cong \overline{LM}$, $\overline{JM} \cong \overline{LK}$, and $\angle J$ and $\angle L$ are right angles. Describe how you could use three different congruence theorems to prove that $\triangle JKM \cong \triangle LMK$.



Sample answer: (1) Since $\angle J$ and $\angle L$ are right angles, $\overline{JM} \cong \overline{LK}$, and $\overline{MK} \cong \overline{MK}$, $\triangle JKM \cong \triangle LMK$ by the HL Triangle Congruence Theorem.

(2) Since $\angle J$ and $\angle L$ are right angles, $\angle J \cong \angle L$. Also, $\overline{JK} \cong \overline{LM}$ and $\overline{JM} \cong \overline{LK}$, so $\triangle JKM \cong \triangle LMK$ by the SAS Triangle Congruence Theorem.

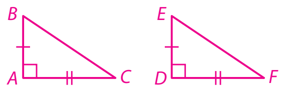
(3) By the Reflexive Property of Congruence, $\overline{MK} \cong \overline{MK}$. Also, $\overline{JK} \cong \overline{LM}$ and $\overline{JM} \cong \overline{LK}$, so $\triangle JKM \cong \triangle LMK$ by the SSS Triangle Congruence Theorem.

23. **Justify Reasoning** Do you think there is an LL Triangle Congruence Theorem? That is, if the legs of one right triangle are congruent to the legs of another right triangle, are the triangles necessarily congruent? If so, write a proof of the theorem. If not, provide a counterexample.

There is an LL Triangle Congruence Theorem.

Given: $\triangle ABC$ $\triangle DEF$ are right triangles;
$\angle A$ and $\angle D$ are right angles.
$\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$

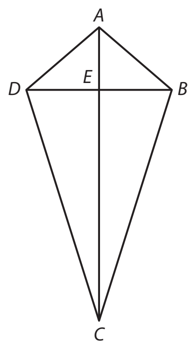
Prove: $\triangle ABC \cong \triangle DEF$



Statements	Reasons
1. $\angle A$ and $\angle D$ are right angles.	1. Given
2. $\angle A \cong \angle D$	2. All right angles are congruent.
3. $\overline{AB} \cong \overline{DE}$	3. Given
4. $\overline{AC} \cong \overline{DF}$	4. Given
5. $\triangle ABC \cong \triangle DEF$	5. SAS Triangle Congruence Theorem

Lesson Performance Task

The figure shows kite *ABCD*.



a. What would you need to know about the relationship between $\overline{AC}$ and $\overline{DB}$ in order to prove that $\triangle ADE \cong \triangle ABE$ and $\triangle CDE \cong \triangle CBE$ by the HL Triangle Congruence Theorem?

b. Can you prove that $\triangle ADC$ and $\triangle ABC$ are congruent using the HL Triangle Congruence Theorem? Explain why or why not.

c. How can you prove that the two triangles named in Part b are in fact congruent, even without the additional piece of information?

a. $\overline{AC}$ is the perpendicular bisector of $\overline{DB}$

b. You cannot prove that $\triangle ADC$ and $\triangle ABC$ are congruent using the HL Triangle Congruence Theorem because you do not know that $\angle ADC$ and $\angle ABC$ are right angles.

c. The SSS Triangle Congruence Theorem $\left(\overline{AD} \cong \overline{AB}, \overline{CD} \cong \overline{CB}, \overline{AC} \cong \overline{AC}\right)$

EXTENSION ACTIVITY

Students have studied five congruence theorems: ASA, SAS, SSS, AAS, and HL. Have students draw five diagrams, each as simple or as complicated as they wish, with each diagram showing at minimum a pair of triangles, and each illustrating one of the five congruence theorems. For each diagram, students should write what is given, what is to be proved, and which congruence theorem is illustrated, then add a formal or informal proof.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 To prove that two triangles are congruent by SSS, you must meet three conditions, showing that the three sides of one triangle are congruent respectively to the three sides of the other triangle. Renaldo said that, in fact, for all five methods of proving triangles congruent, it is necessary to meet three conditions. Do you agree or disagree? Explain. **Agree. Sample answer: For ASA, SAS, SSS, and AAS, the three letters specify that three pairs of angles, sides, or combinations of the two must be shown congruent. HL appears at first to require that only two conditions be met. But in addition to showing that the hypotenuses and one pair of legs are congruent, the theorem requires that a third condition must be met: You must show that the triangles are right triangles.**

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Explain that figure *ABCD* in the Lesson Performance Task not only looks like a kite, it's known geometrically as a *kite*. Ask students to study the kite's features and then write a concise, precise definition of a geometrical kite. **Students will study kites in Module 9. Their definitions will vary but should address the fact that a kite is a quadrilateral with two pairs of adjacent congruent sides, and that the pairs have different lengths.**

Scoring Rubric

2 points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain his/her reasoning.

0 points: Student does not demonstrate understanding of the problem.