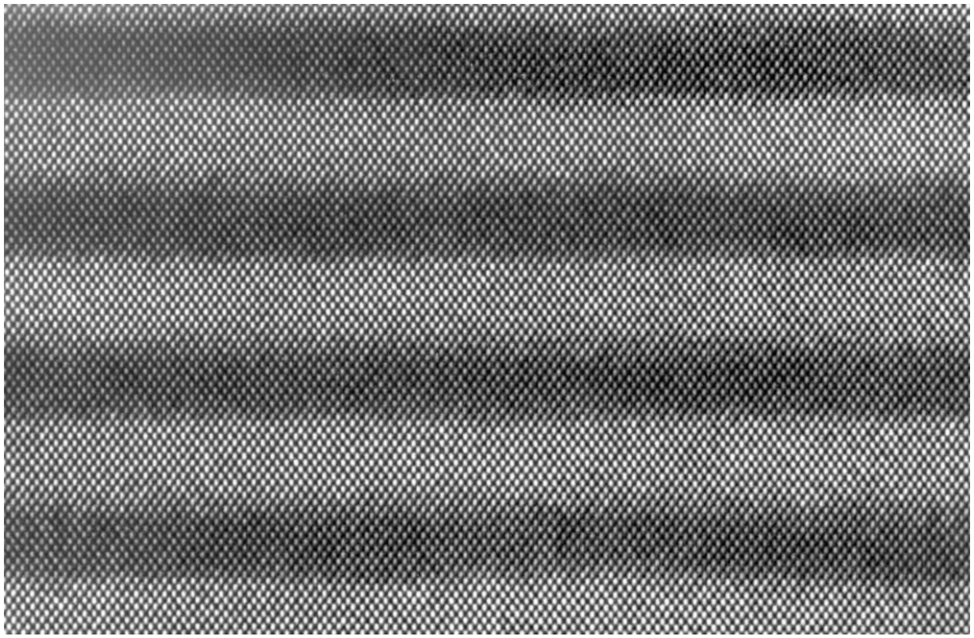
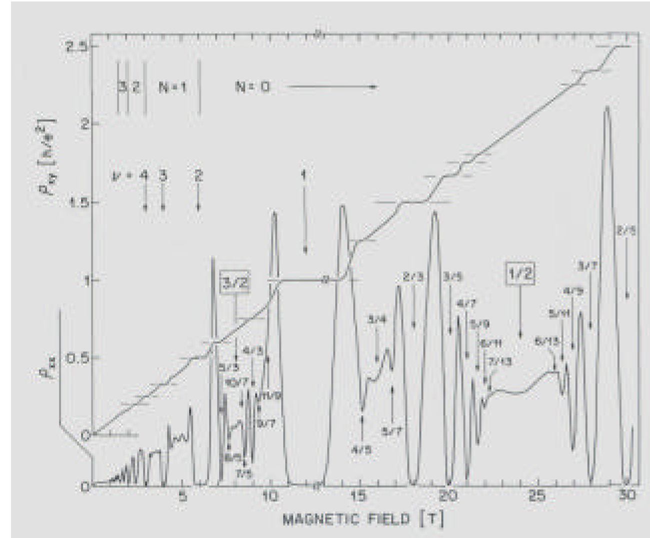
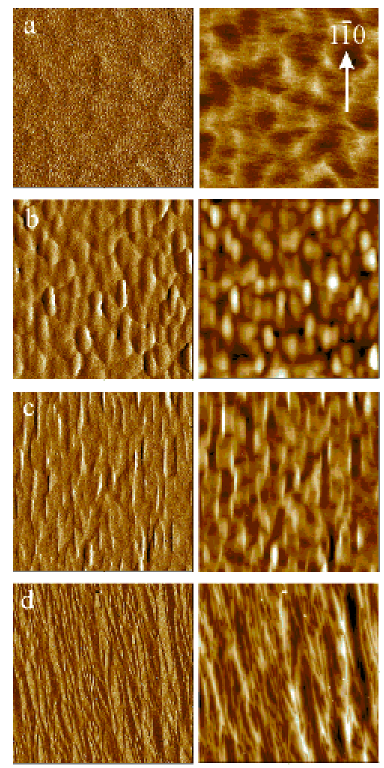
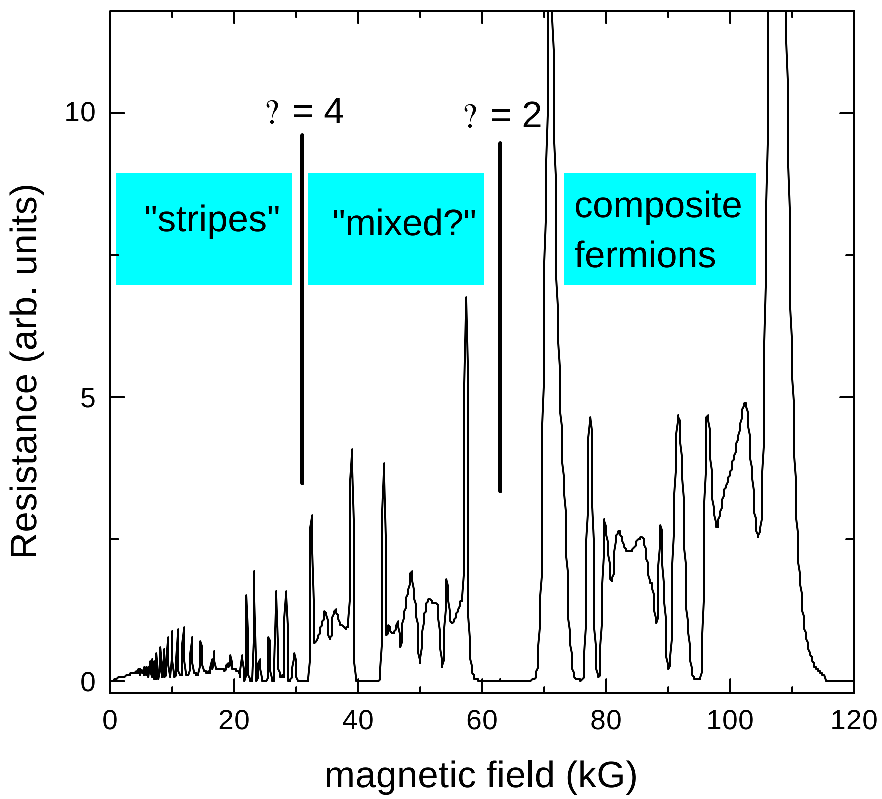
Correlated 2D Electron Aspects of the Quantum Hall Effect







Magnetic field spectrum of the correlated 2D electron system:
Electron interactions lead to a range of manifestations



Lectures Outline:

I. Introduction: materials, transport, Hall effects

II. Composite particles – FQHE, statistical transformations

III. Quasiparticle charge and statistics

IV. Higher Landau levels

V. Other parts of spectrum: non-equilibrium effects, electron solid?

VI. Multicomponent systems: Bilayers

Outline:

I. Introduction: materials, transport, Hall effects

   A. General 2D physics
   B. Materials – MBE
   C. Measurements – quantum Hall effect
   D. Correlations
   E. Fractional quantum Hall effect

II. Composite particles – composite fermions

III. Quasiparticle charge and statistics

IV. Higher Landau levels

V. Other parts of spectrum: non-equilibrium effects, electron solid?

VI. Multicomponent systems: Bilayers

I. Introduction: materials, transport, Hall effects

A. General 2D physics

No B-field

$n =$ electron density, N/ℓ^2

$$2N(\lambda/2)^2 = \pi\ell^2$$

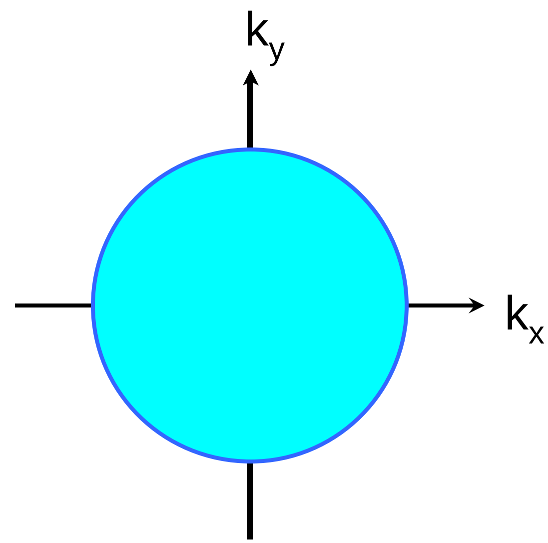
$$k = 2\pi/\lambda$$

filled Fermi sea up to

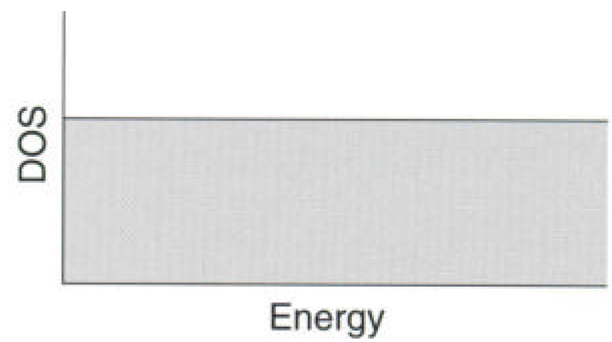
$$k_F = (2\pi n)^{1/2}$$

$$\text{DOS} = dn/dE, \quad n = mE_F/\pi\hbar^2$$

$$dn/dE = m/\pi\hbar^2 = \text{constant}$$



$k_F = (2?n)^{1/2}$



I. Introduction: materials, transport, Hall effects

A. General 2D physics

With B-field

Hamiltonian

$$H = \frac{(\vec{p} + e\vec{A})^2}{2m^*} + \frac{1}{2} g \mu_B \vec{\sigma} \cdot \vec{B} + V(z)$$

energy eigenvalues

$$E_n = (N + 1/2)\hbar\omega_c + \frac{1}{2} g \mu_B B + E_0$$

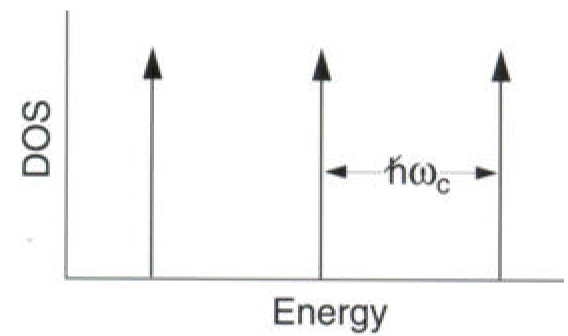
$m^*/m_0 = 0.067$, $g = -0.44$

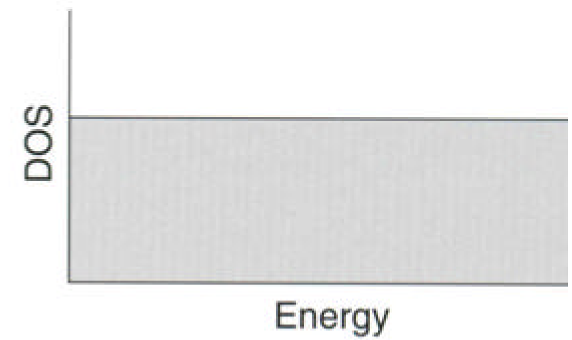
cyclotron frequency $\omega_c = eB/m^*$

density of states $D = eB/h$

Bohr magneton $\mu_B = e\hbar/2m_0$

$\hbar\omega_c = 20\,K$ at $B = 1T$; $g\mu_B B \sim \hbar\omega_c/70$





I. Introduction: materials, transport, Hall effects

A. General 2D physics

With B-field (cont.)

$$H = \frac{(\bar{p} + e\bar{A})^2}{2m^*}$$

symmetric gauge: $\vec{A} = \frac{1}{2}(\bar{r} \times \bar{B})$

$$\psi_{0,j} = z^j e^{-|z|^2/4} u(j), \quad j = 0, 1, 2 \ldots$$

↗ N=0

$$z = (x + iy)$$

Landau gauge: $\vec{A} = -yB\hat{x}$

$$\psi_{N,k} = e^{ikx} v_N(y - y_k)$$

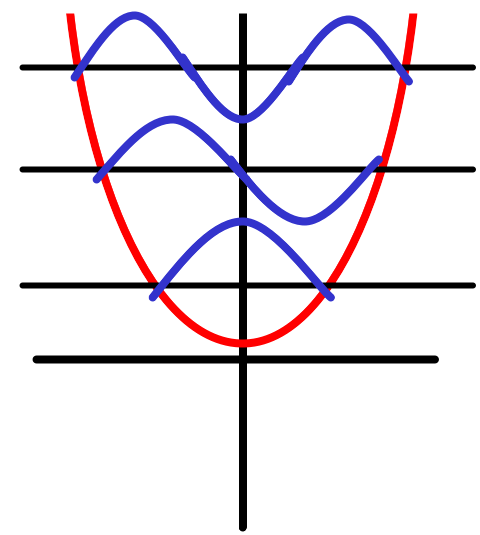
$$v_N(\alpha) = e^{-\alpha^2/2\ell^2} H_N(\alpha)$$

↖ note nodal structure for $N > 0$

$$y_k = k\ell^2$$

$\ell^2 = \hbar/eB$, magnetic length

Higher Landau levels have more nodes



Magnetic length l_0

I. Introduction: materials, transport, Hall effects

A. General 2D physics

<u>With B-field and disorder</u>

scattering times

a) level broadening $\Gamma = \hbar/\tau$

relaxation time $\tau \equiv$

time to be scattered into a different state

$\omega_c \tau \gg 1 \Rightarrow$ resolved Landau levels

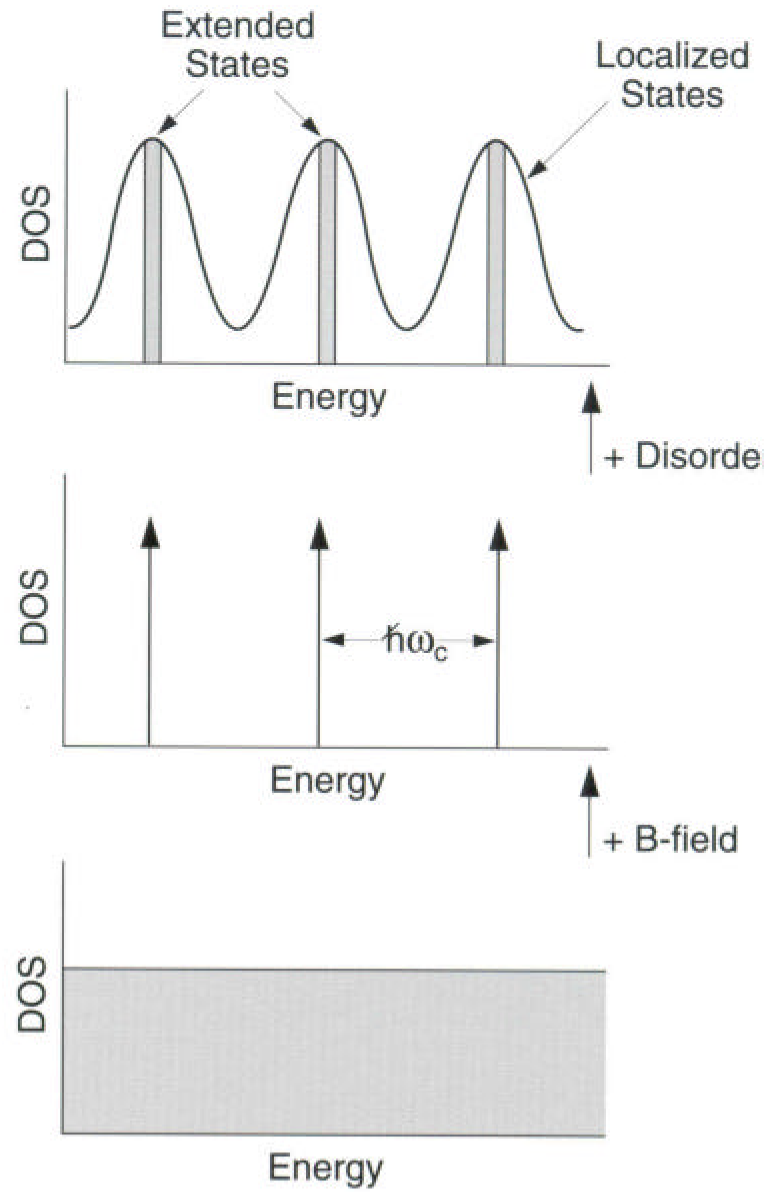
b) transport scattering time

mean-free-path $\ell_{mfp} = v_F \tau_{tr}$

mobility $\mu \equiv v/E$; $\mu = \frac{1}{ne\rho}$

ρ = resistivity , n = areal density

$\sigma = ne\mu = ne^2 \tau_{tr}/m$



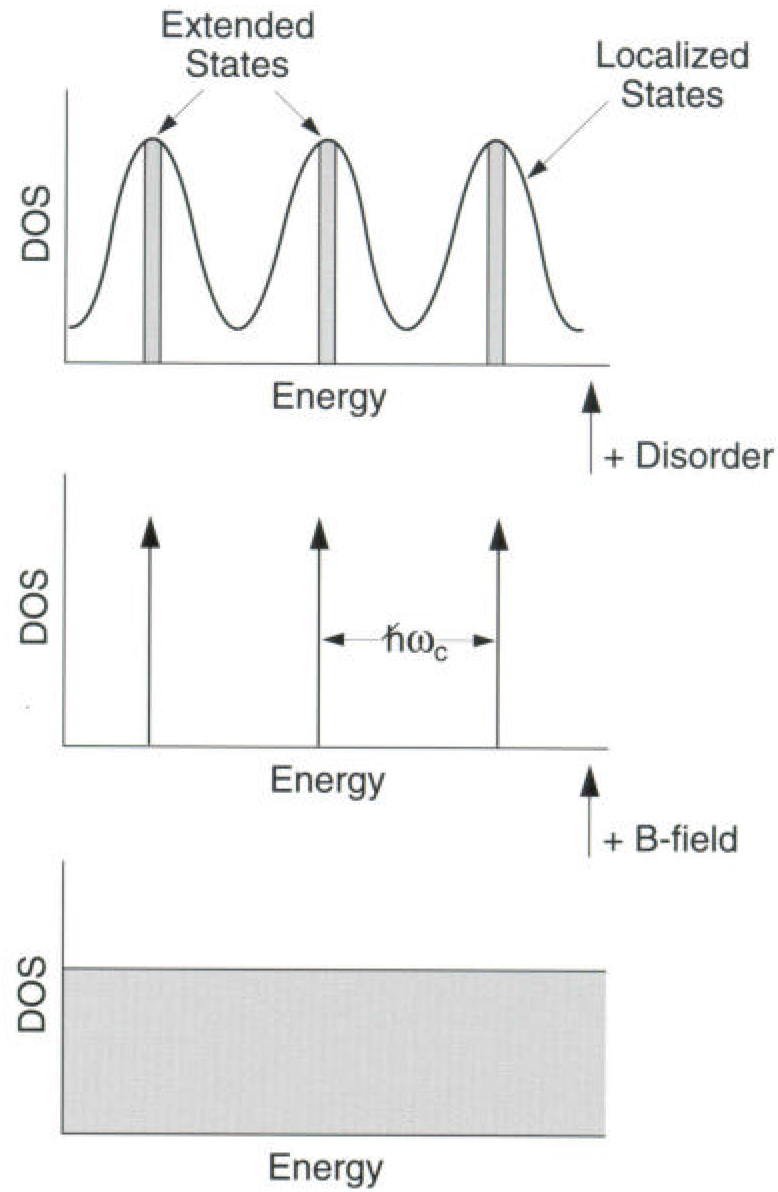
I. Introduction: materials, transport, Hall effects

A. General 2D physics

What are the sources of scattering in a real 2D electron system?

How do you make a real 2D electron system?

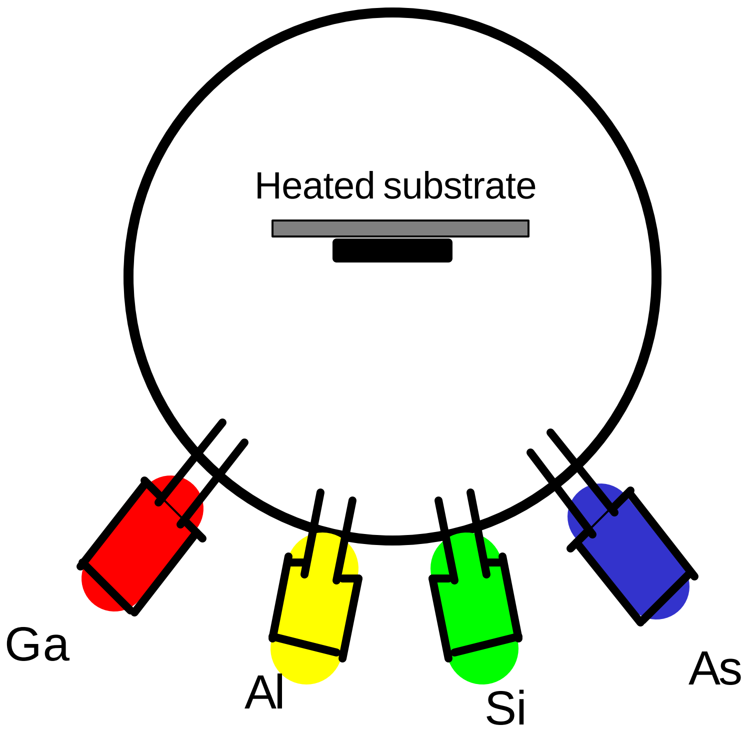
Molecular beam epitaxy

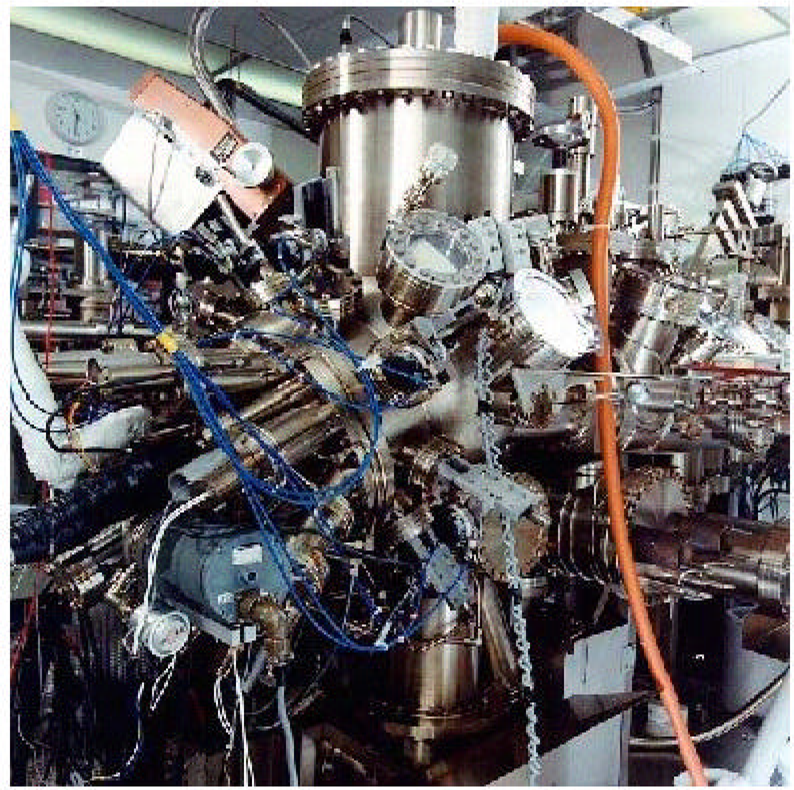


I. Introduction: materials, transport, Hall effects

B. Materials – molecular beam epitaxy

In ultra-high vacuum, sources provide material that is evaporated onto a heated substrate

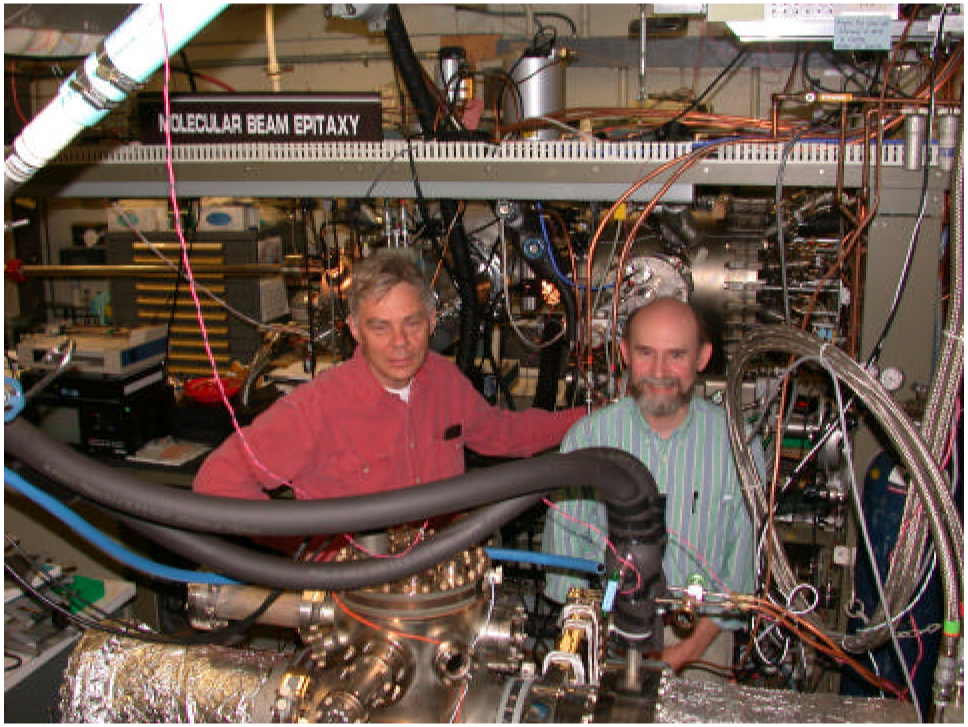




The material is deposited at ~ monolayers / second

I. Introduction: materials, transport, Hall effects

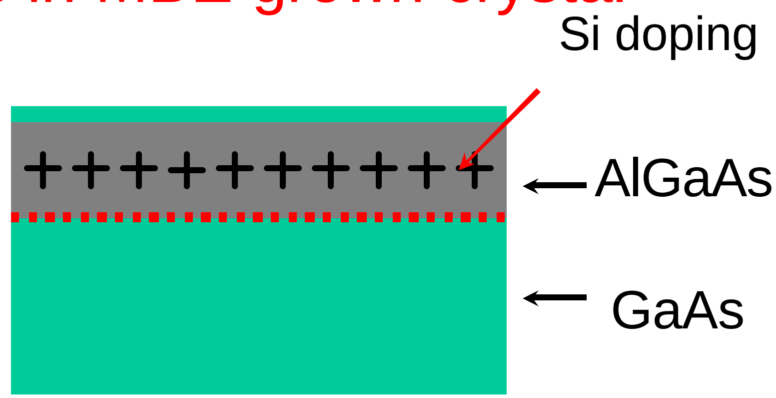
B. Materials – molecular beam epitaxy



Loren Pfeiffer
Ken West

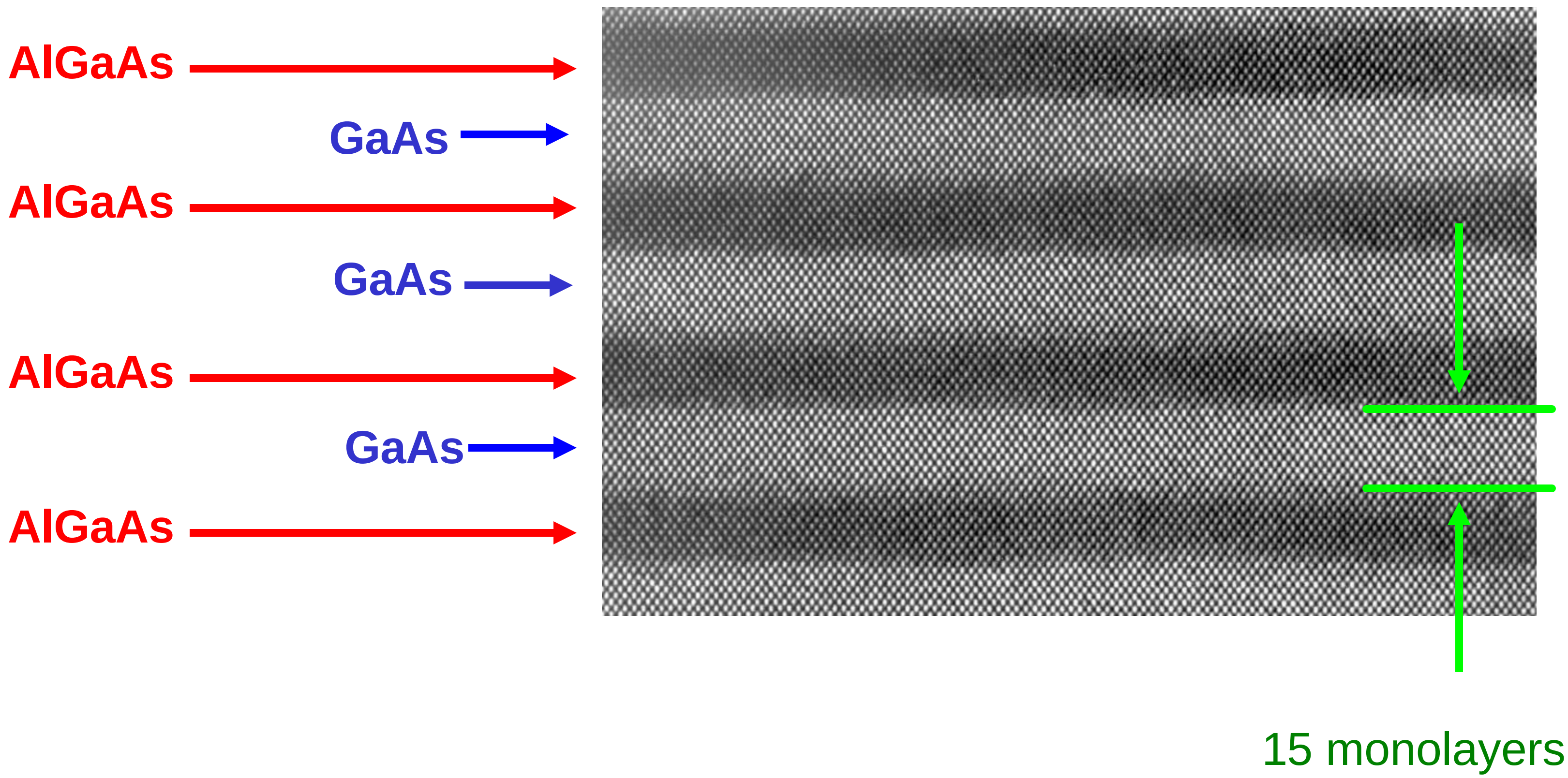
2D electron gas forms at interface of AlGaAs/GaAs in MBE grown crystal

2D electrons in high purity AlGaAs/GaAs interface



I. Introduction: materials, transport, Hall effects

B. Materials – molecular beam epitaxy



The material is deposited at ~ monolayers / second

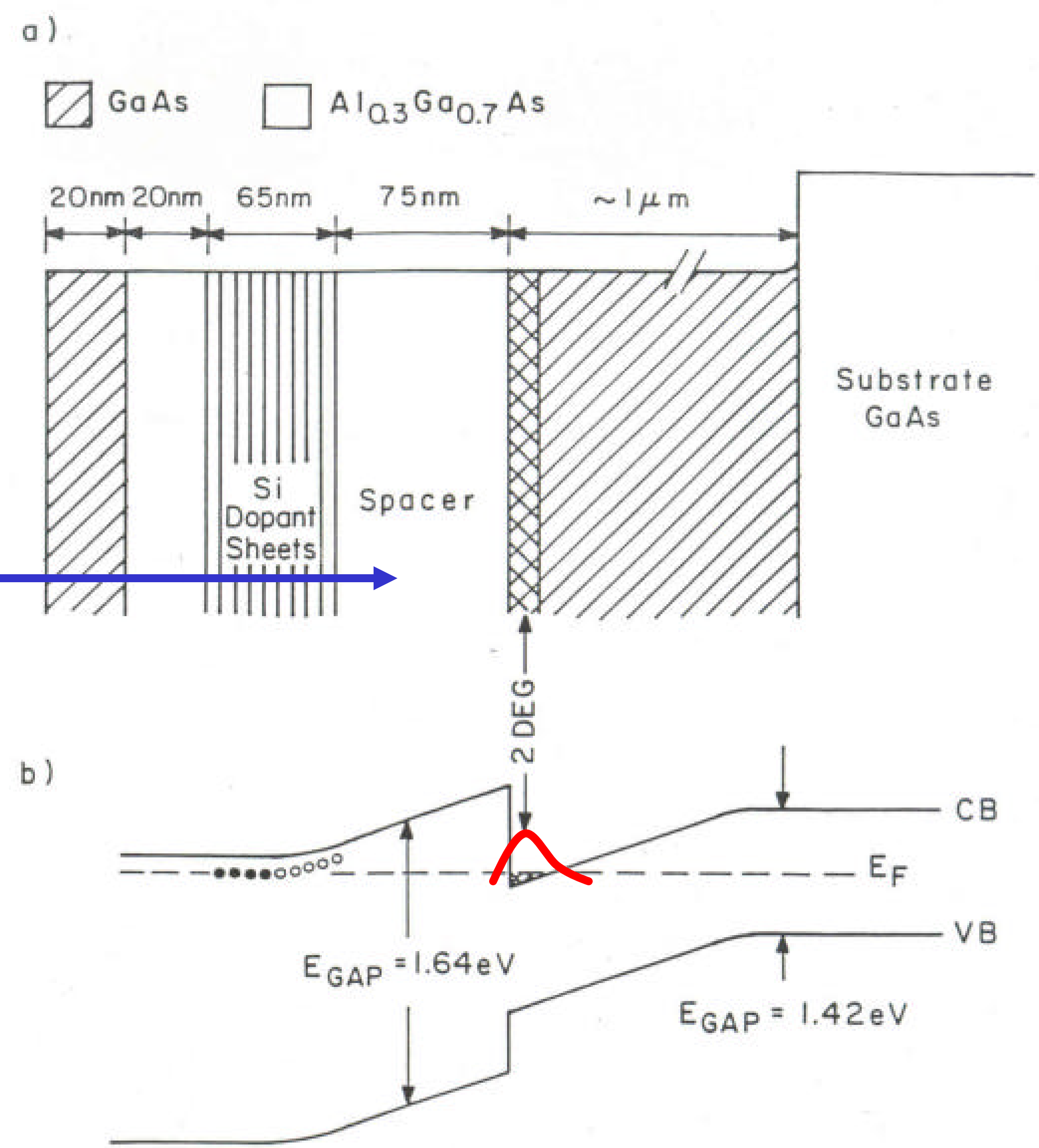
Shuttering different sources layers the materials

I. Introduction: materials, transport, Hall effects

B. Materials – molecular beam epitaxy

a) layering: electrons from Si layer reside at AlGaAs/GaAs interface – ionized dopants isolated from electron layer

Doping modulation



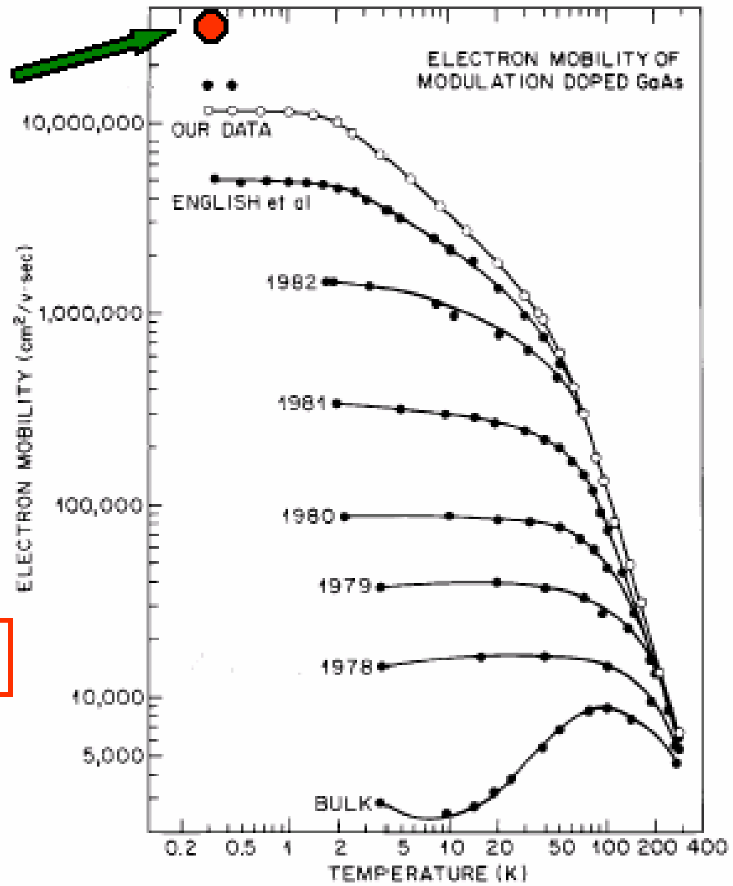
b) Energy level diagram: electron wavefunction traverses interface plane, has finite z-extent – only lowest bound state used

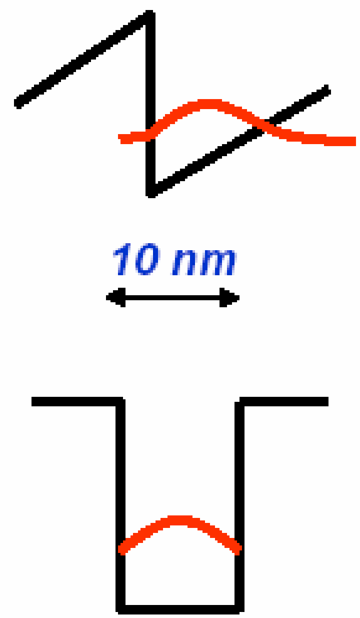
I. Introduction: materials, transport, Hall effects

B. Materials – molecular beam epitaxy

Mobility of Electrons in GaAs

μ = 31 x 10^6 cm^2/Vs





mean free path ~ ¼ mm

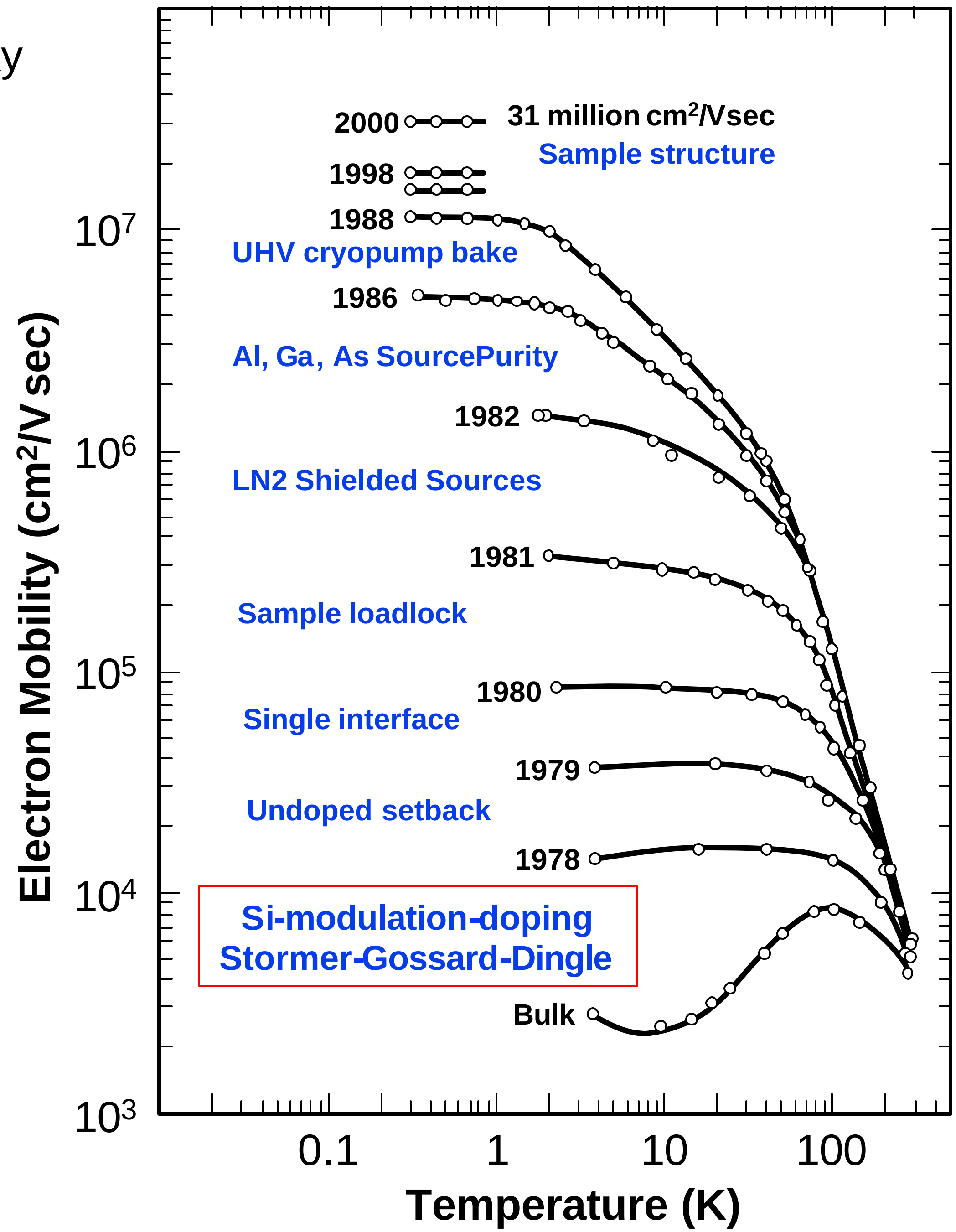
(<1nm in normal metal)

I. Introduction: materials, transport, Hall effects

B. Materials – molecular beam epitaxy

Historical landmarks of 2DEG mobility in GaAs.

Annotated with the specific MBE innovation that caused the mobility improvement.

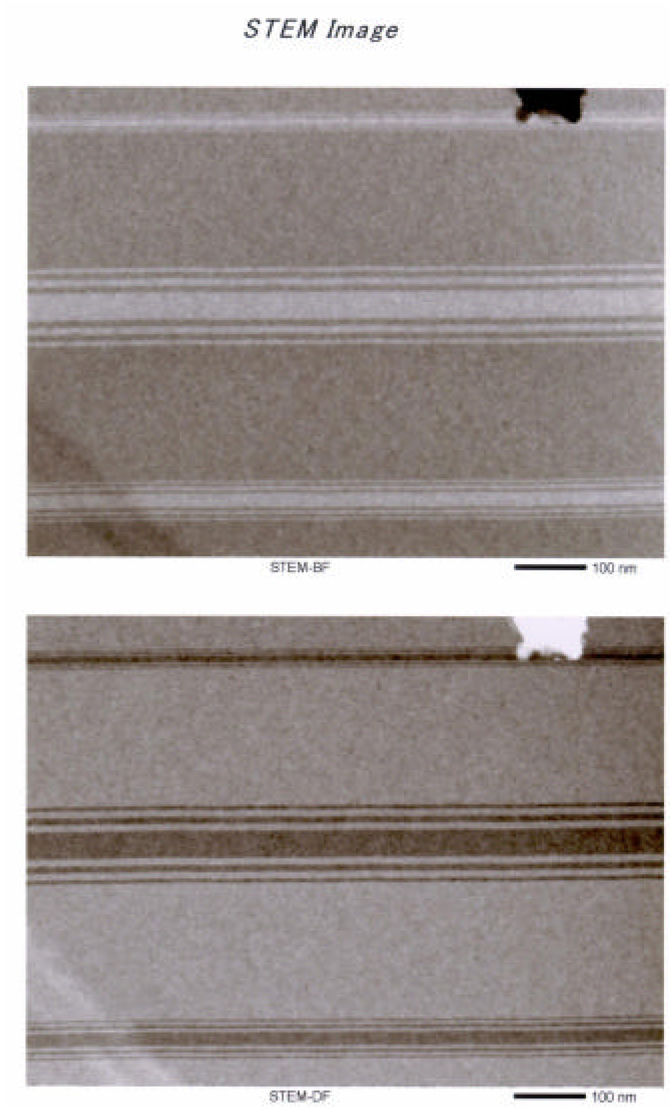


I. Introduction: materials, transport, Hall effects

B. Materials – molecular beam epitaxy

Scattering mechanisms:

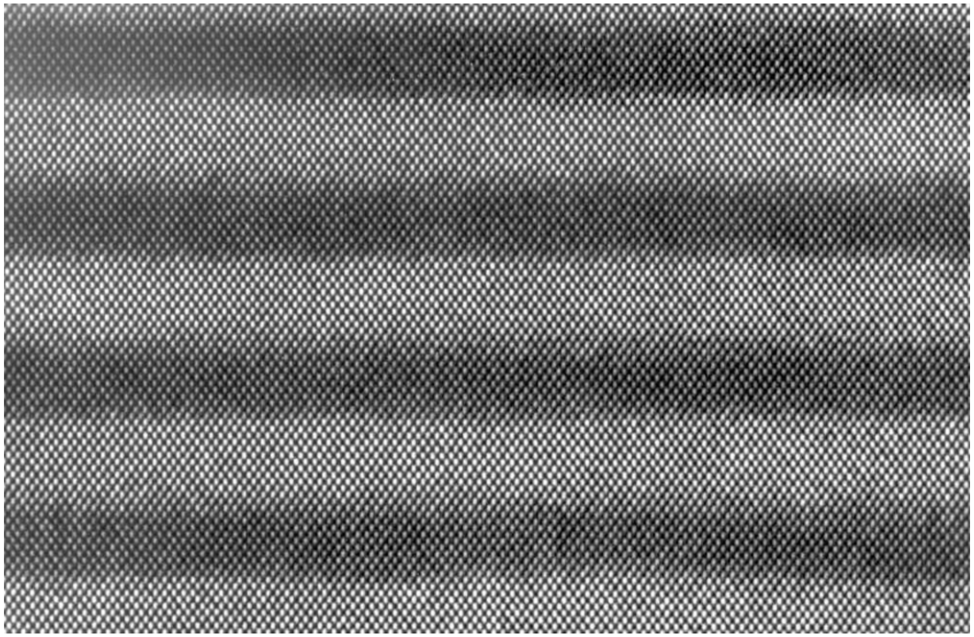
- interface roughness
- alloy scattering
- ionized impurities
- residual disorder
- “systematic” disorder



I. Introduction: materials, transport, Hall effects

B. Materials – molecular beam epitaxy

Numerous “tricks” used to provide clean layer interfaces to reduce the scattering probabilities

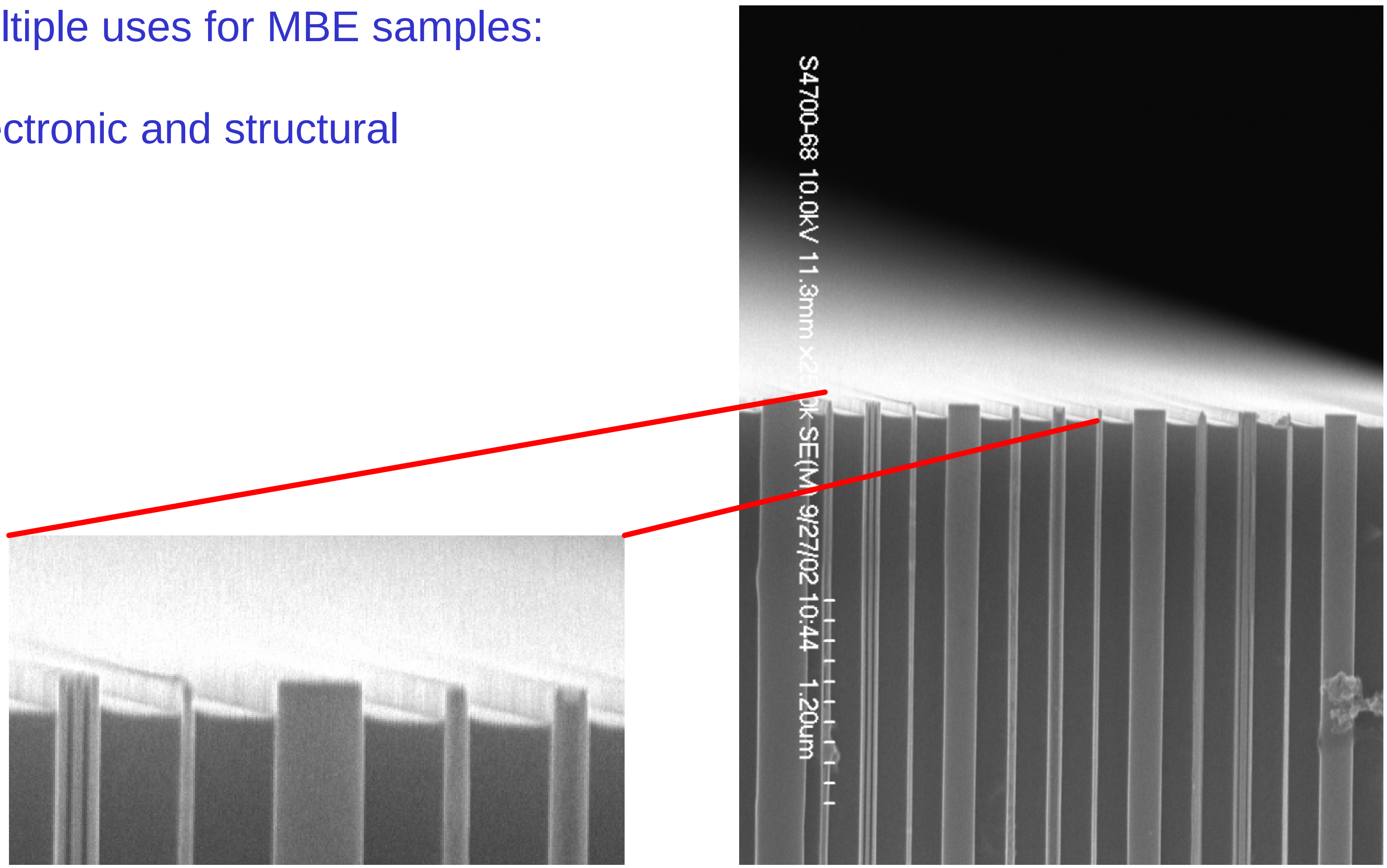


I. Introduction: materials, transport, Hall effects

B. Materials – molecular beam epitaxy

Multiple uses for MBE samples:

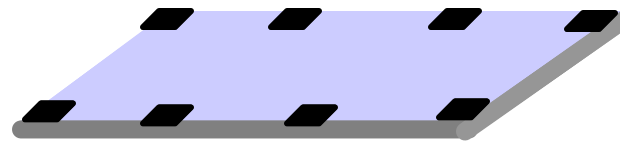
Electronic and structural



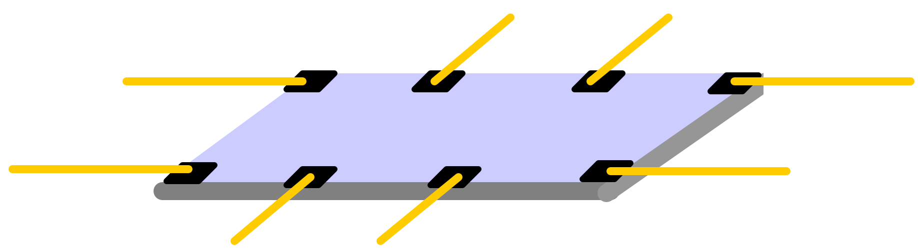
I. Introduction: materials, transport, Hall effects

C. Measurements – Hall effect

Cleave piece of wafer

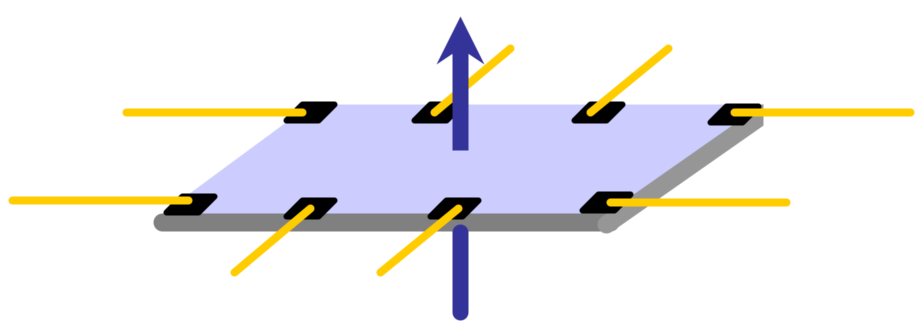


Diffuse contacts into wafer piece



Cool down:

- He4
- He3
- Dilution process

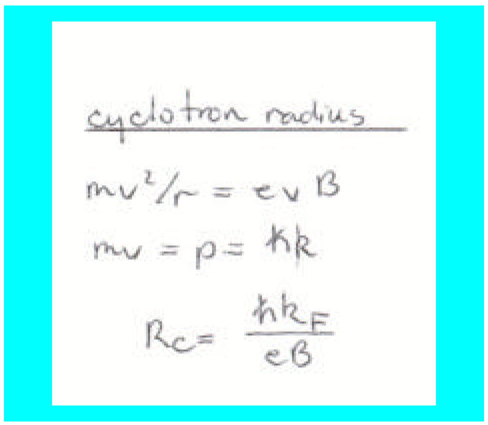


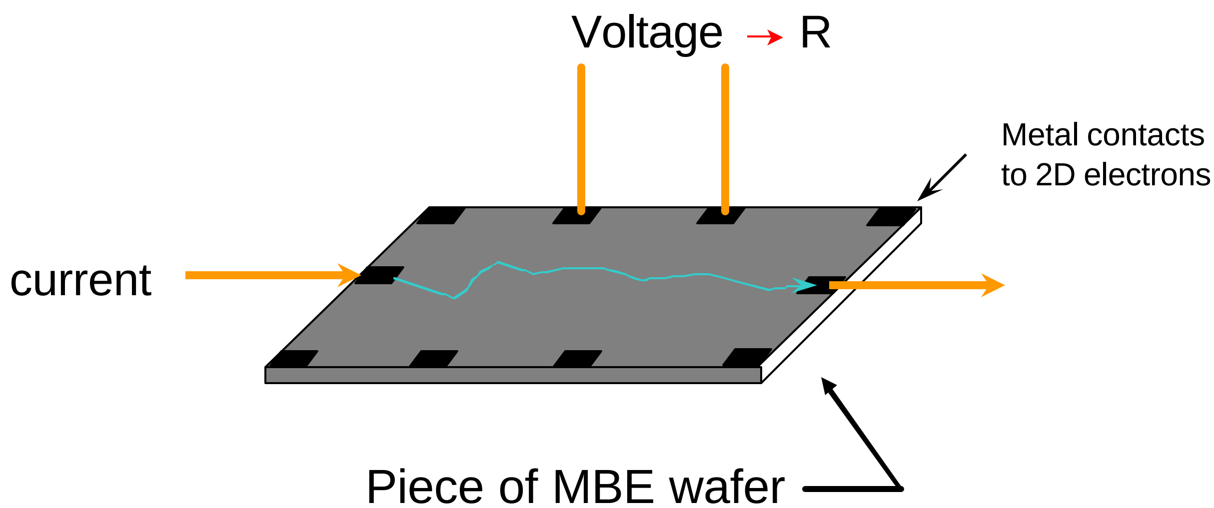
Apply B-field orthogonal to sample

I. Introduction: materials, transport, Hall effects

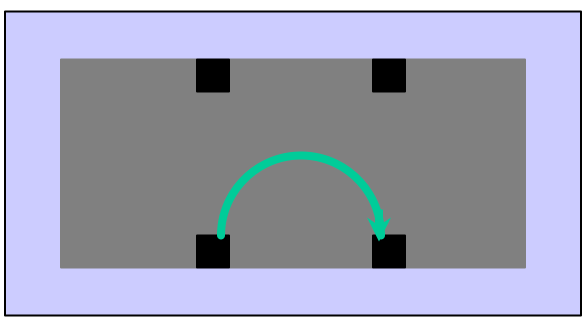
B. Materials – molecular beam epitaxy

characterizing the electron system





Measure scattering length (mean-free-path)



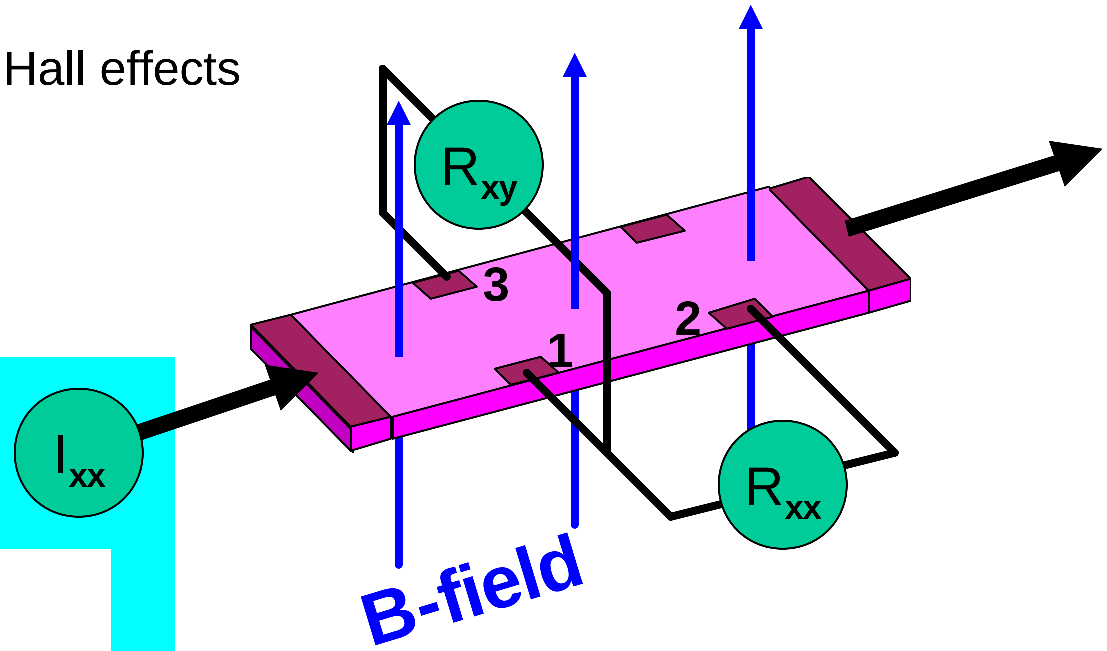
direct current from one contact to another with magnetic field applied perpendicular to layers

300 ?m mean-free-path in best samples:
<1nm in normal metal

Scattering length is curved path length

I. Introduction: materials, transport, Hall effects

C. Measurements – Hall effect

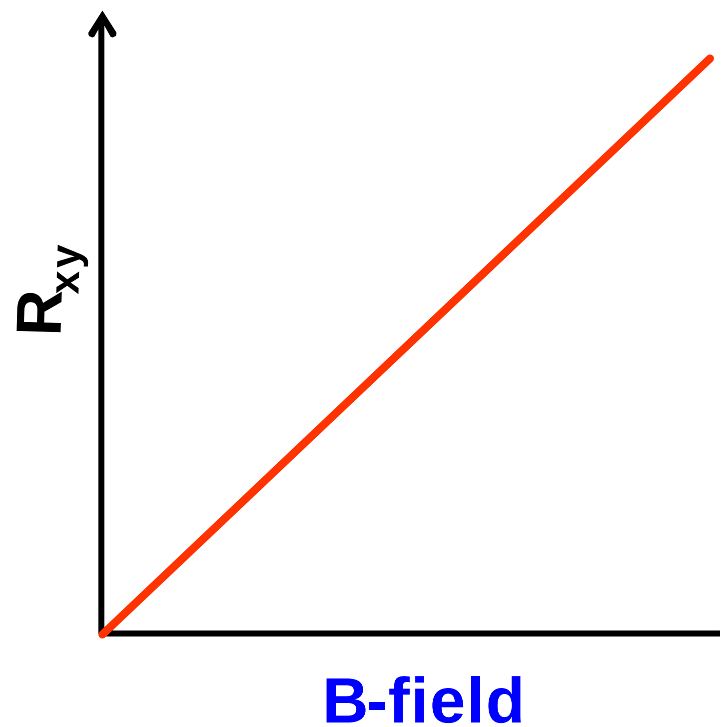


$$R_{xx} = \frac{V_{12}}{I}, \quad R_{xy} = \frac{V_{13}}{I}$$

electron motion is a superposition of circular motion with frequency ω_c and uniform drift motion ⊥ to both E_x and B_z

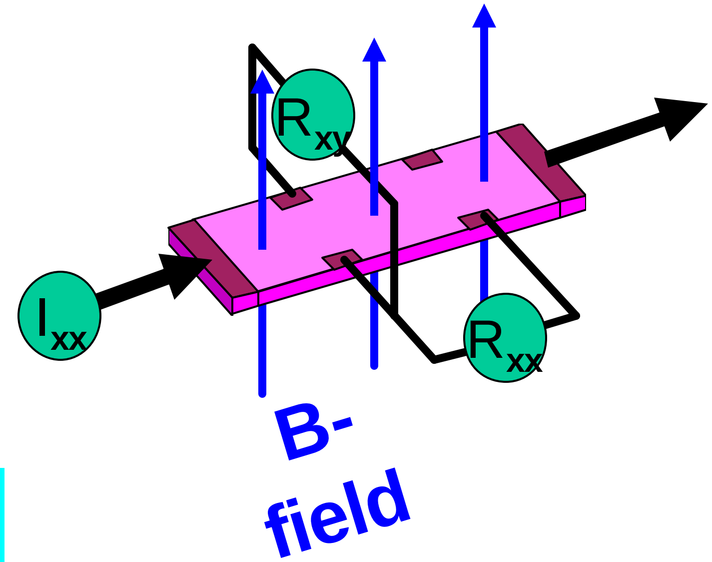
$$eE_x = ev_y B_z \Rightarrow \frac{V_{13}}{I} = \frac{B_z}{ne}$$

$(I = nlev_y, \; E_x l = V_{13})$



I. Introduction: materials, transport, Hall effects

C. Measurements – Hall effect



filling factor ≡ # electrons / # flux quanta

$= \nu$

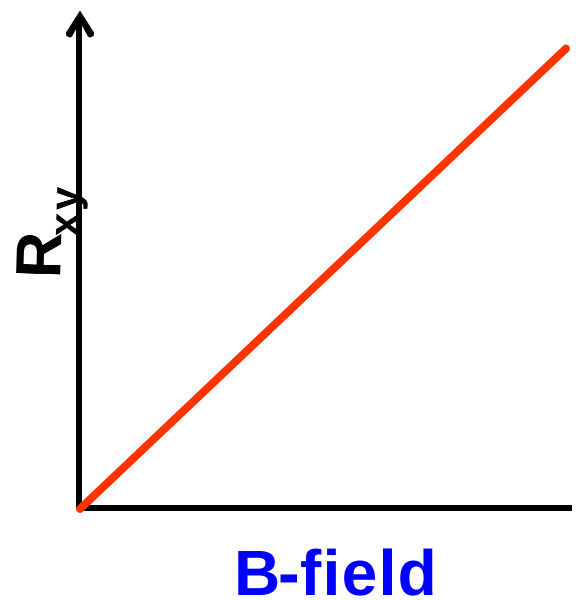
electrons/area $= n$

flux quanta/area $= B/(h/e) = Be/h$

$\Rightarrow \nu = nh/eB$

with $R_{xy} = B/ne$, $R_{xy} = \frac{1}{\nu}\left(\frac{h}{e^2}\right)$

$h/e^2 = 25812.807$ Ohms

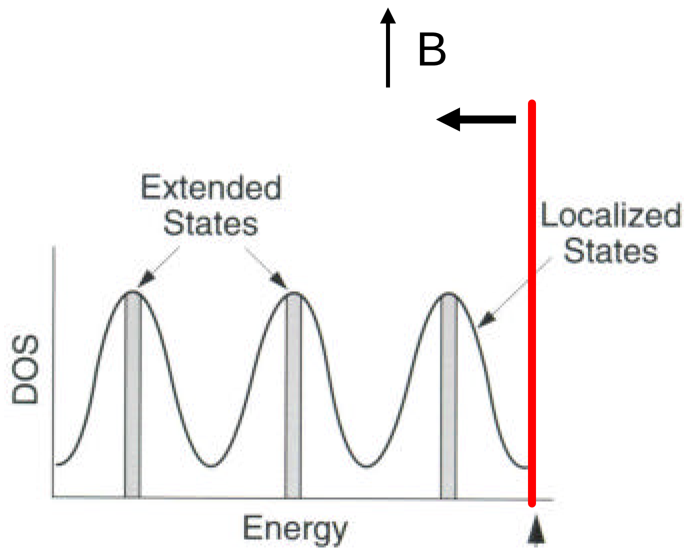


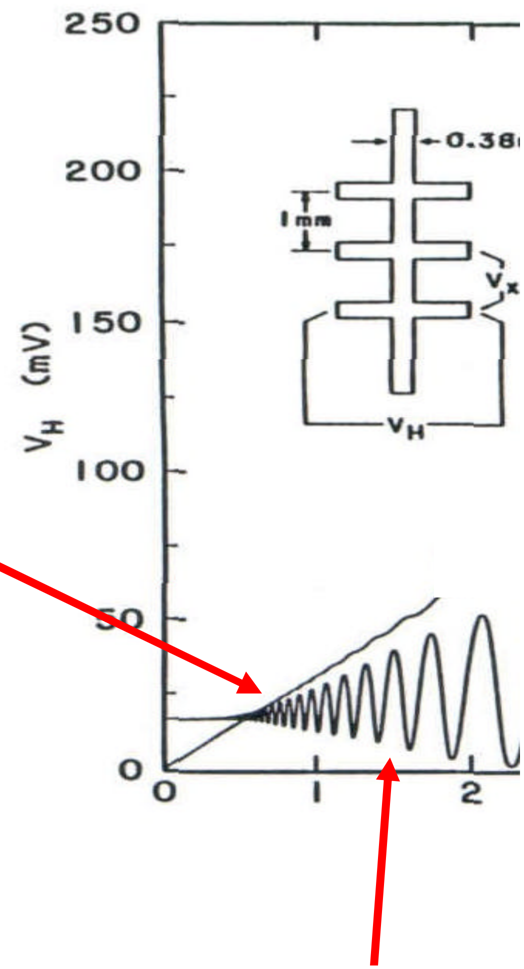
I. Introduction: materials, transport, Hall effects

C. Measurements – Hall effect

With increasing B, degeneracy of LL increases and Fermi level is swept through spectrum (constant density n)

Landau levels resolved when ? c?>>1



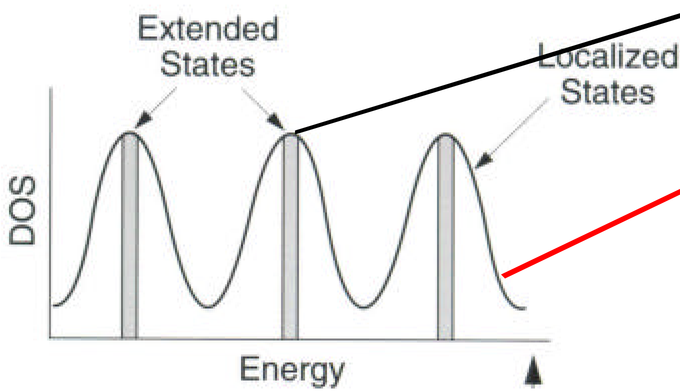


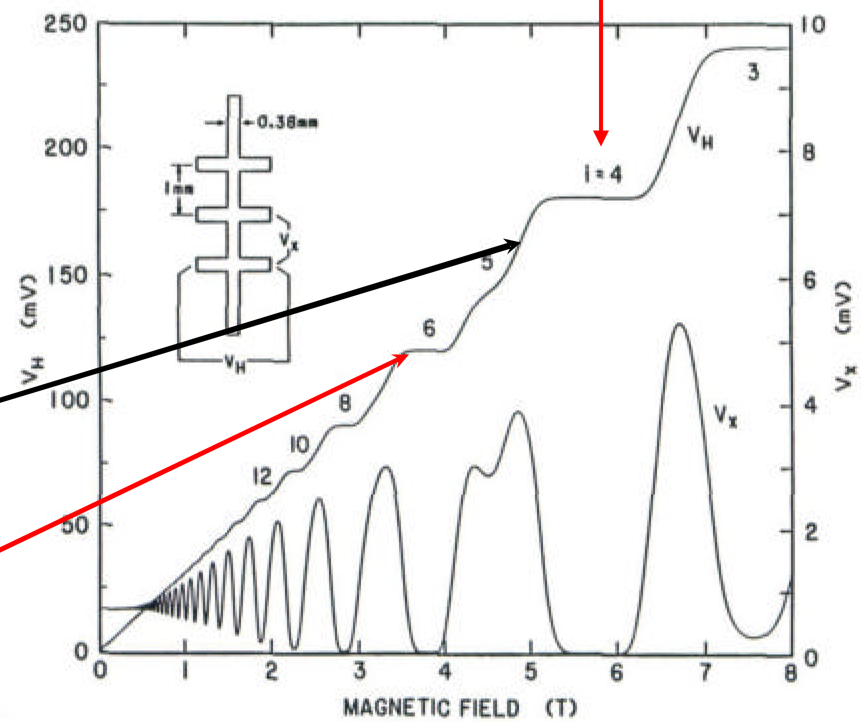
Shubnikov-deHaas oscillations

I. Introduction: materials, transport, Hall effects

C. Measurements – **quantum Hall effect**

At extended states, the Hall voltage increases

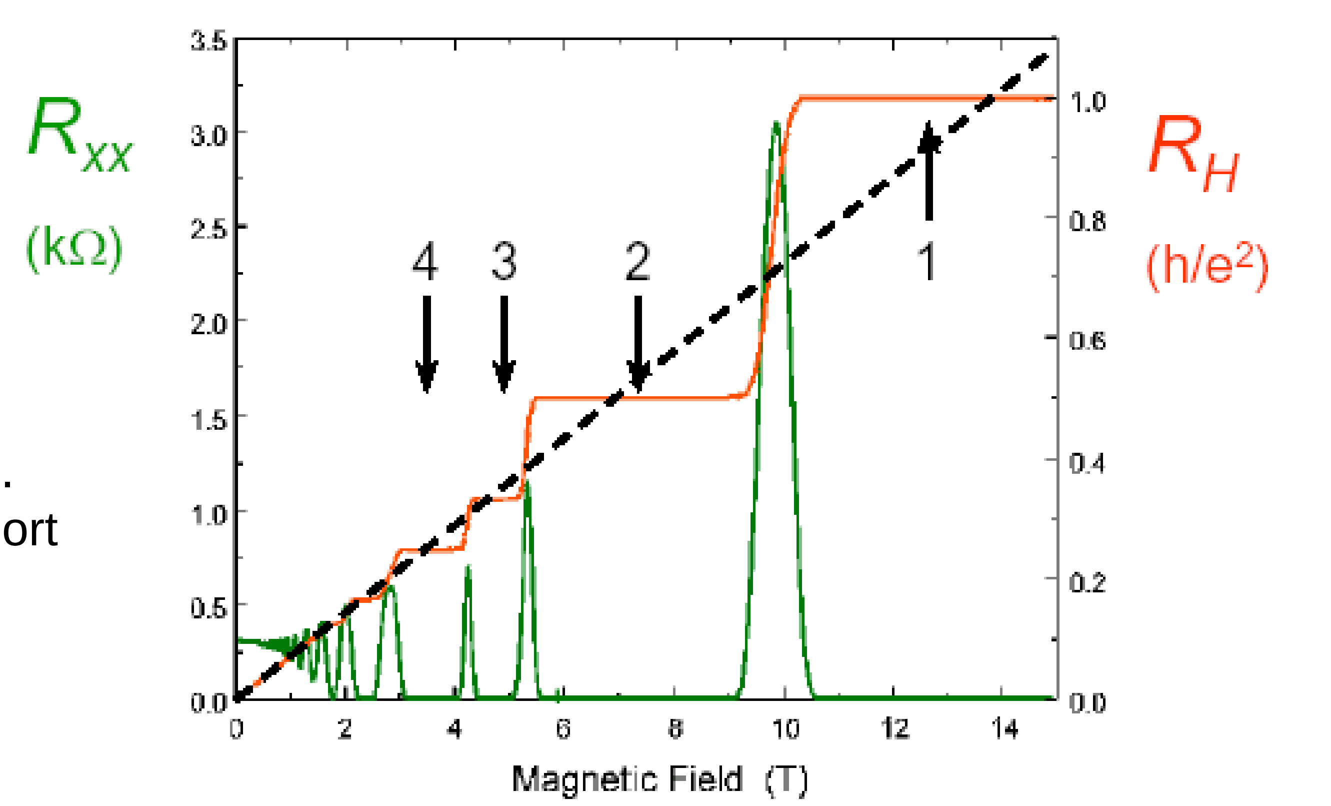




Localization of single electrons between extended states produces plateaus in Hall resistance

I. Introduction: materials, transport, Hall effects

C. Measurements – quantum Hall effect



Minima at n = 2, 4, 6,..
show activated transport

$R = R_0\, e^{(-?/kT)}$

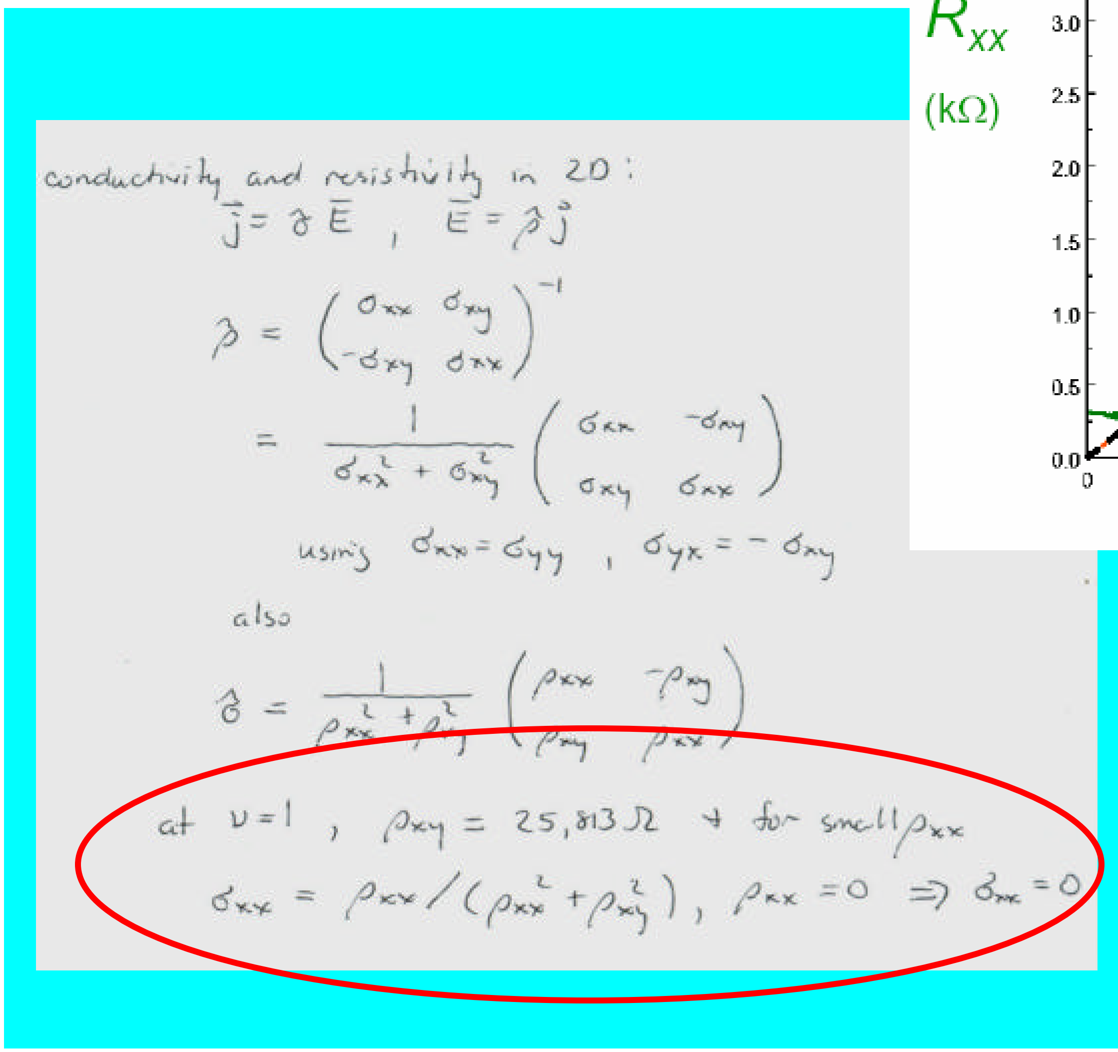
? = cyclotron gap

$R_H = (1/?)(h/e^2)$

(Spin gaps at filling factors 1,3,5,…..)

I. Introduction: materials, transport, Hall effects

C. Measurements – quantum Hall effect



conductivity and resistivity in 2D:

$\vec{j} = \hat{\sigma} \vec{E}$, $\vec{E} = \hat{\rho} \vec{j}$

$$\hat{\rho} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}^{-1}$$

$$= \frac{1}{\sigma_{xx}^2 + \sigma_{xy}^2} \begin{pmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{xx} \end{pmatrix}$$

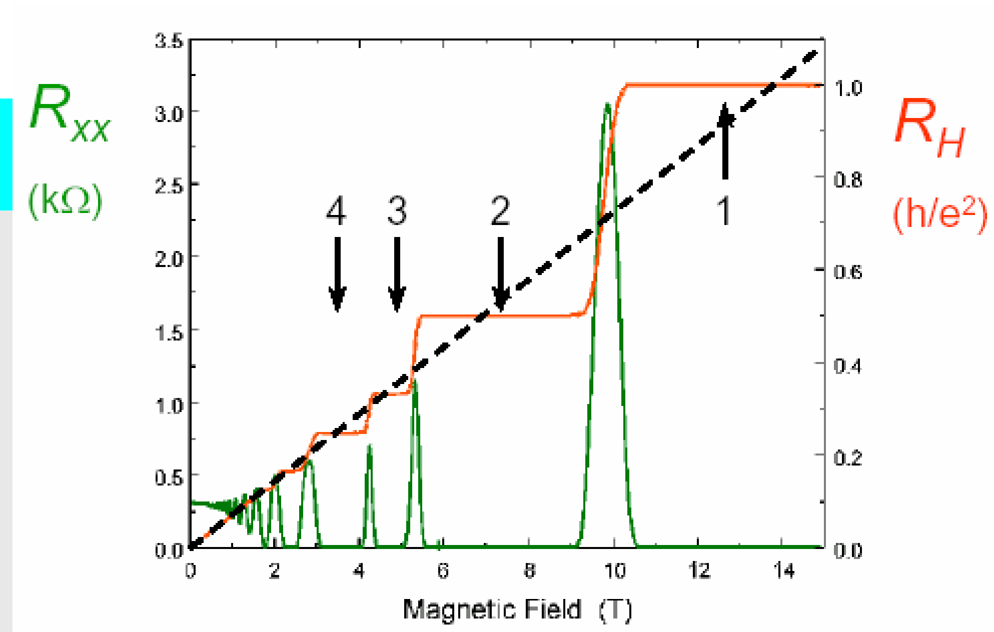
using $\sigma_{xx} = \sigma_{yy}$, $\sigma_{yx} = -\sigma_{xy}$

also

$$\hat{\sigma} = \frac{1}{\rho_{xx}^2 + \rho_{xy}^2} \begin{pmatrix} \rho_{xx} & -\rho_{xy} \\ \rho_{xy} & \rho_{xx} \end{pmatrix}$$

at $\nu = 1$, $\rho_{xy} = 25{,}813\,\Omega$ + for small ρ_{xx}

$\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2)$, $\rho_{xx} = 0 \Rightarrow \sigma_{xx} = 0$



$R_H = (1/?)(h/e^2)$

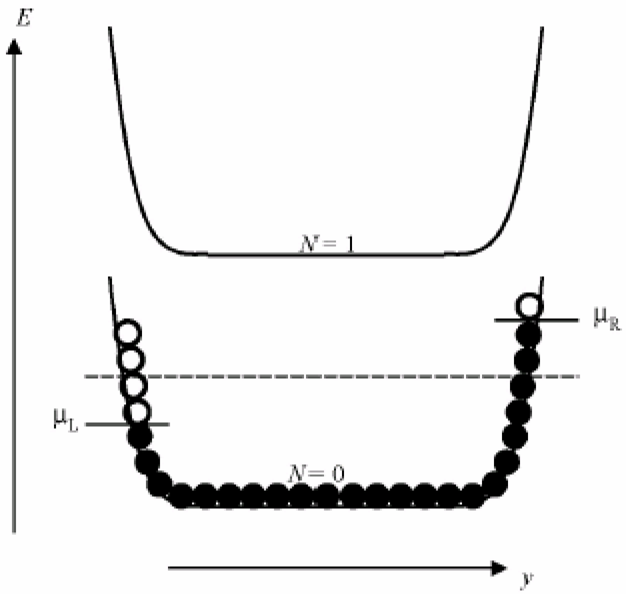
Integral quantum Hall effect represents single particle localization process

I. Introduction: materials, transport, Hall effects

C. Measurements – quantum Hall effect

Integer QHE and Edge States

Edge conduction



No backscattering along same edge

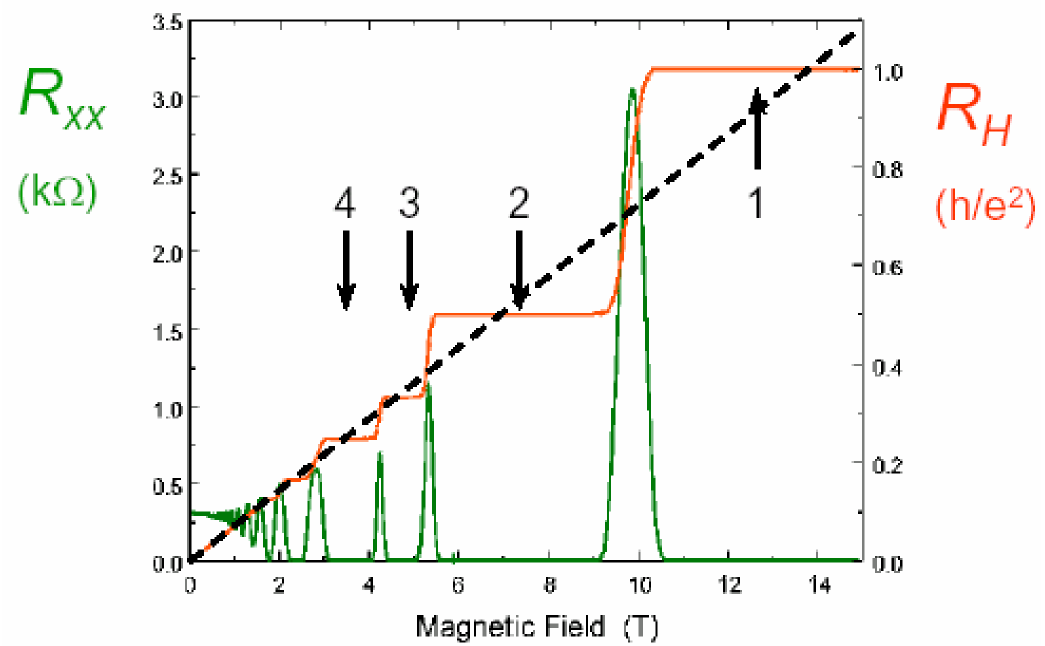
$$I = \sum_{occ.} i_k = \int \frac{L_x dy_k}{2\pi \ell^2} \left(\frac{\partial \varepsilon_N}{\partial y_k} \frac{e\ell^2}{\hbar L_x} \right) = \frac{e}{h}(\mu_R - \mu_L) = \frac{e^2}{h} V_H$$

I. Introduction: materials, transport, Hall effects

C. Measurements – quantum Hall effect

- samples must have sufficiently low disorder that the Landau levels can be resolved (?$_c$?>>1)

- decreasing disorder further will unveil correlations



$R_H = (1/?)(h/e^2)$

I. Introduction: materials, transport, Hall effects

D. Correlations

Correlations

4 competing energies

① thermal: $E_T = KT$

② disorder: $E_D = \Gamma$

③ potential: $E_p \sim e^2/r \sim e^2 n^{1/2}$

④ kinetic: $E_K \sim k_F^2 \sim n$

in order to observe correlations, $E_p > E_T, E_D$

and ratio

$$E_p/E_K > 1 \; ; \quad E_p/E_K = r_s \sim n^{-1/2}$$

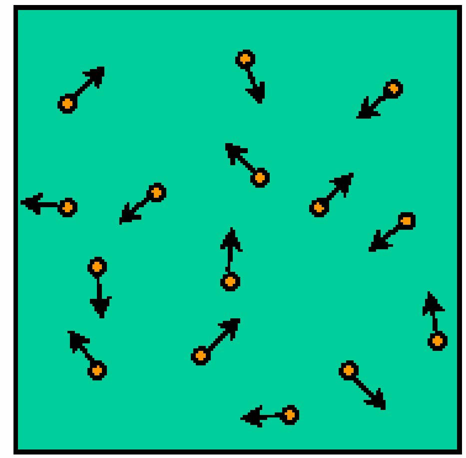
kinetic energy can be quenched by applying a magnetic field

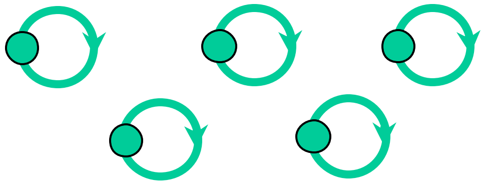
$E_p \sim e^2/l_0$, l_0 magnetic length

I. Introduction: materials, transport, Hall effects

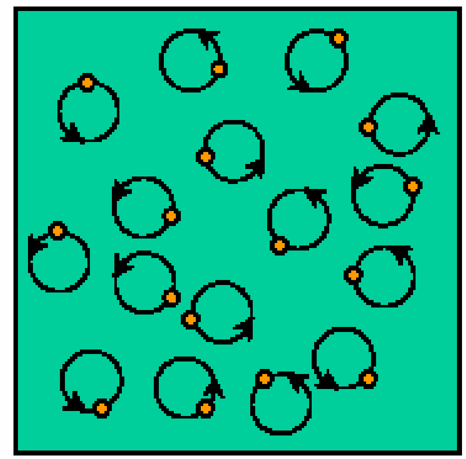
D. Correlations

Magnetic field quenches kinetic energy:
If low intrinsic disorder, correlations manifest





At high magnetic fields, electron orbits smaller than electron separation

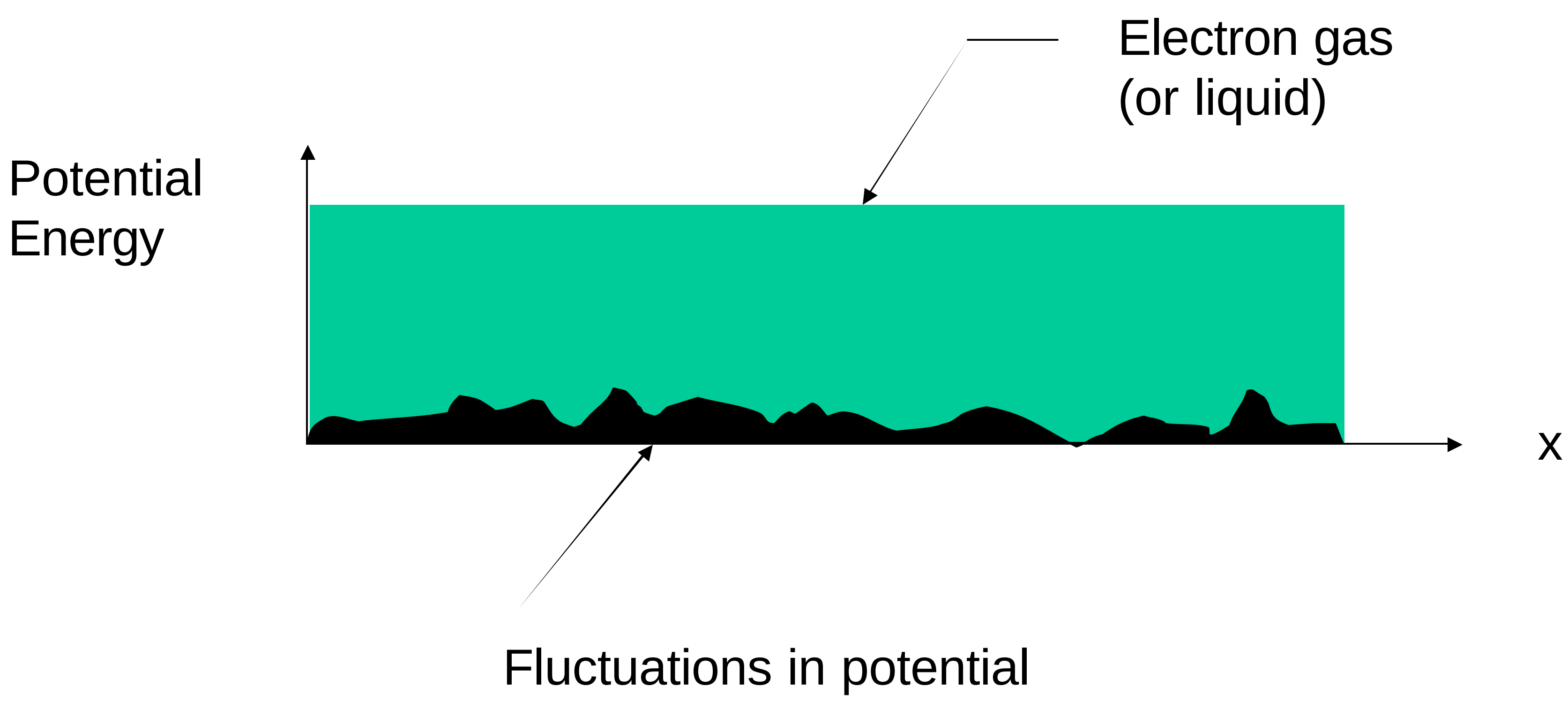


I. Introduction: materials, transport, Hall effects

D. Correlations

Correlation effects may be seen IF:

1) Temperature is low enough
2) Disorder is low enough



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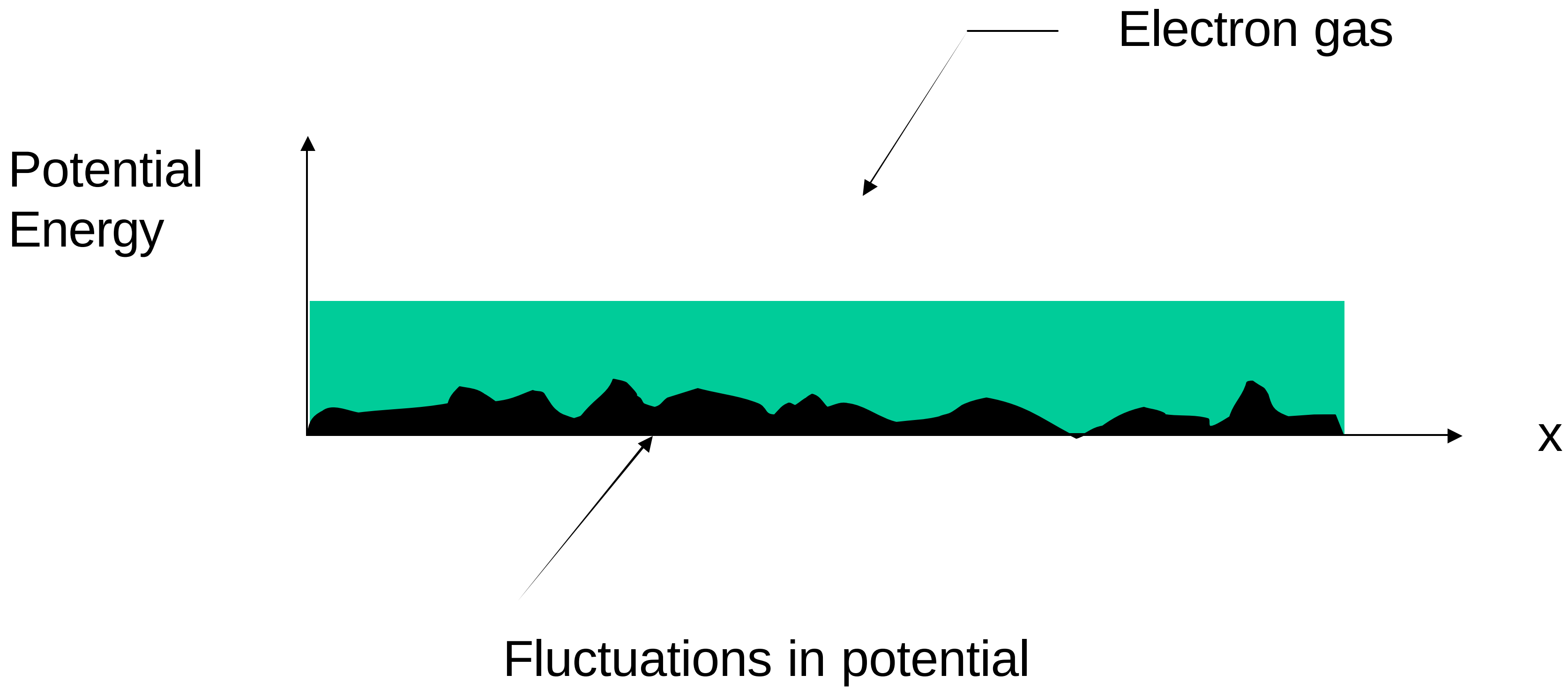


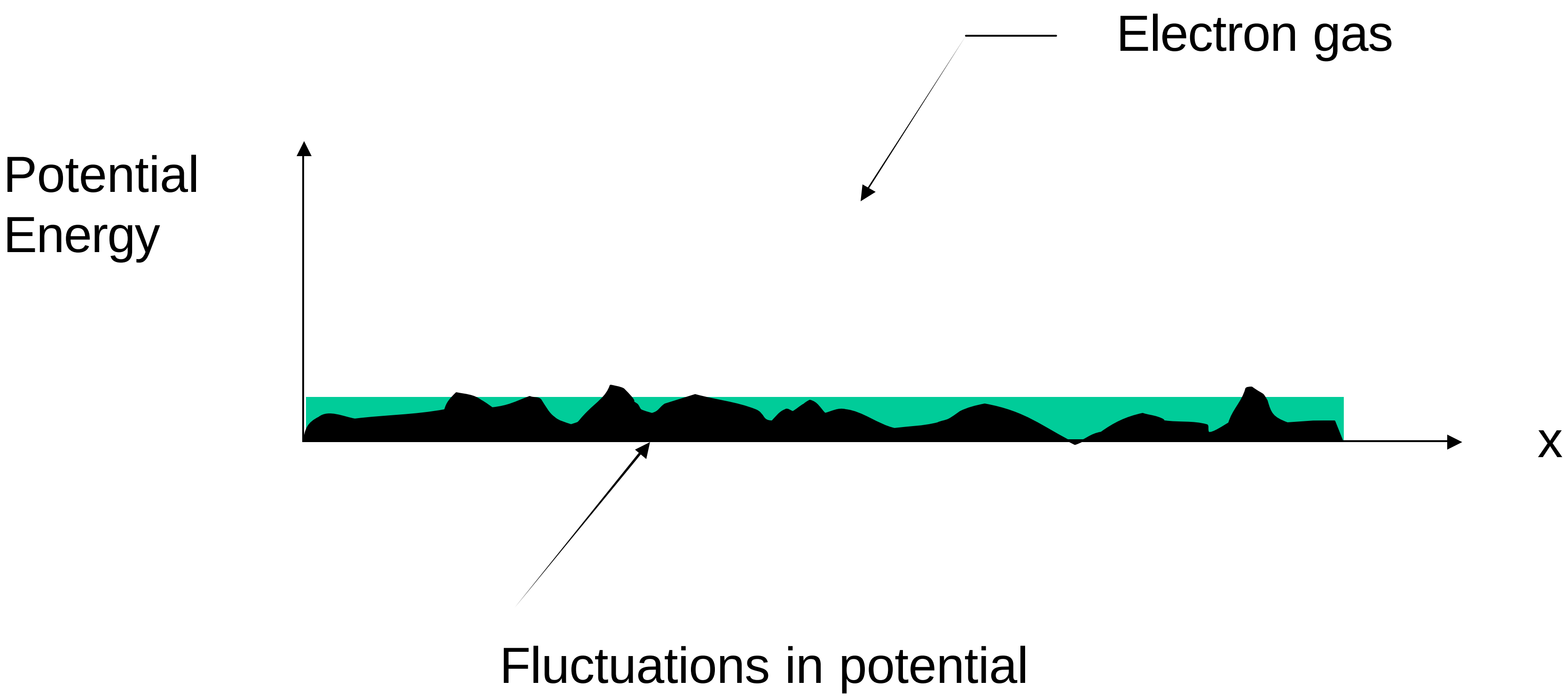
Lower density = Larger influence of disorder on electron gas

I. Introduction: materials, transport, Hall effects

D. Correlations

Correlation effects may be seen IF:

1) Temperature is low enough
2) Disorder is low enough



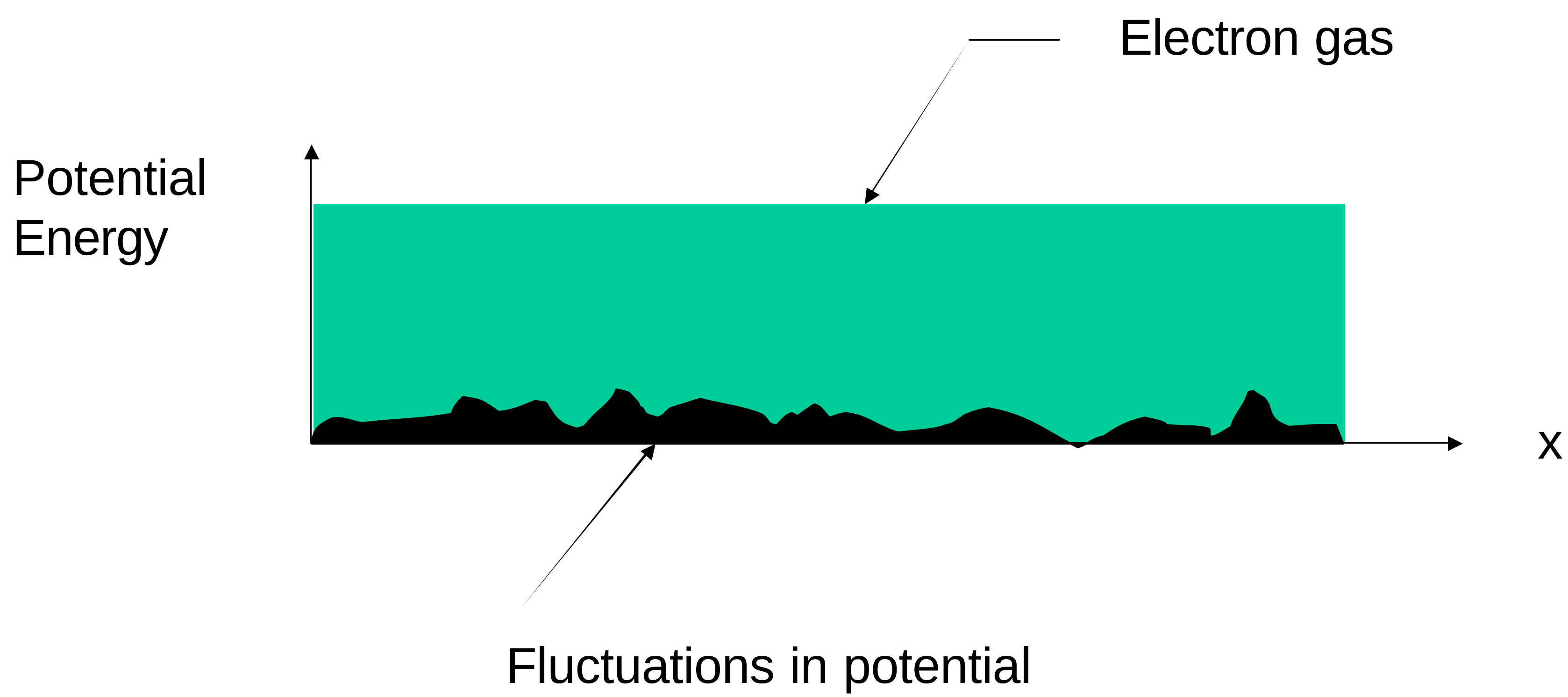
Disorder can destroy correlations

I. Introduction: materials, transport, Hall effects

D. Correlations

Correlation effects may be seen IF:

1) Temperature is low enough
2) Disorder is low enough



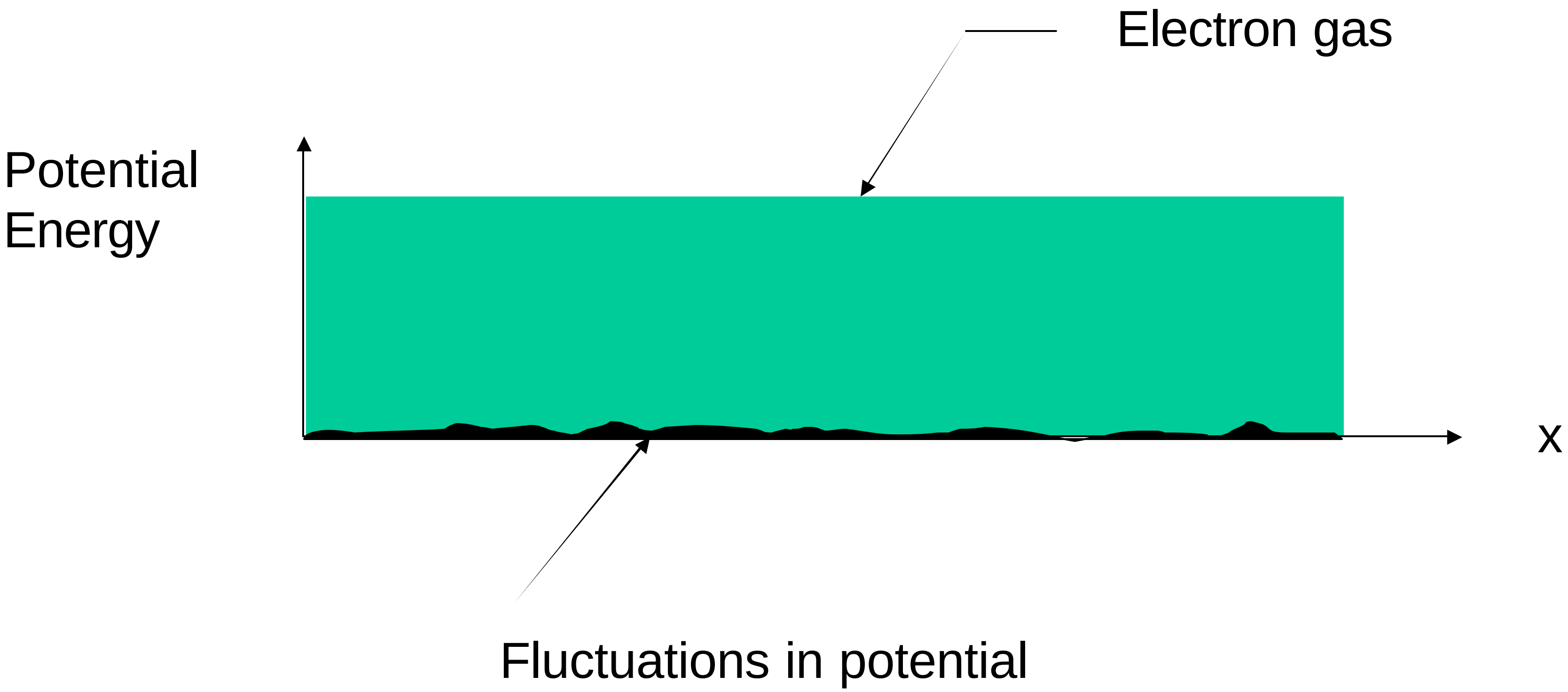
MINIMIZE DISORDER and USE HIGH ELECTRON DENSITY

I. Introduction: materials, transport, Hall effects

D. Correlations

Correlation effects may be seen IF:

1) Temperature is low enough
2) <u>Disorder is low enough</u>



MINIMIZE DISORDER and USE HIGH ELECTRON DENSITY

I. Introduction: materials, transport, Hall effects

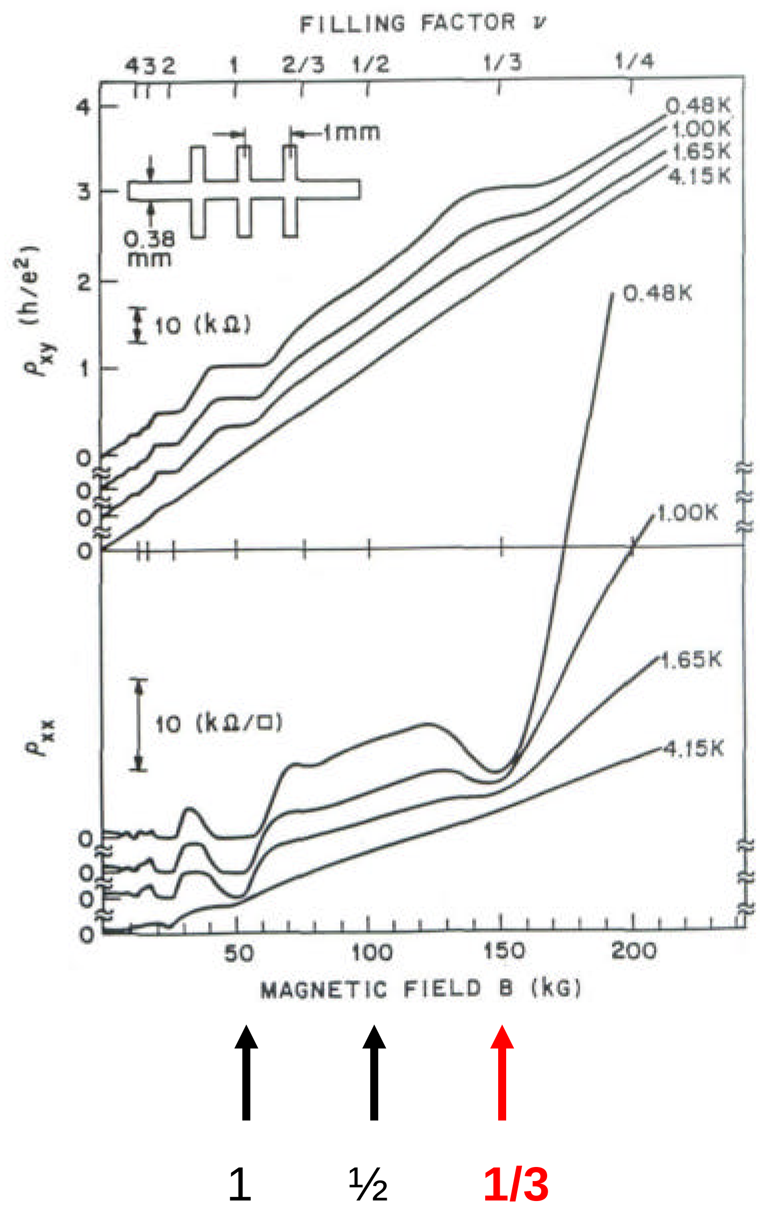
D. Correlations

Back to transport measurements:

Higher mobility (lower disorder) samples produced – AlGaAs/GaAs heterostructures, **modulation doped**

new quantum Hall state found at fractional filling factor **1/3**

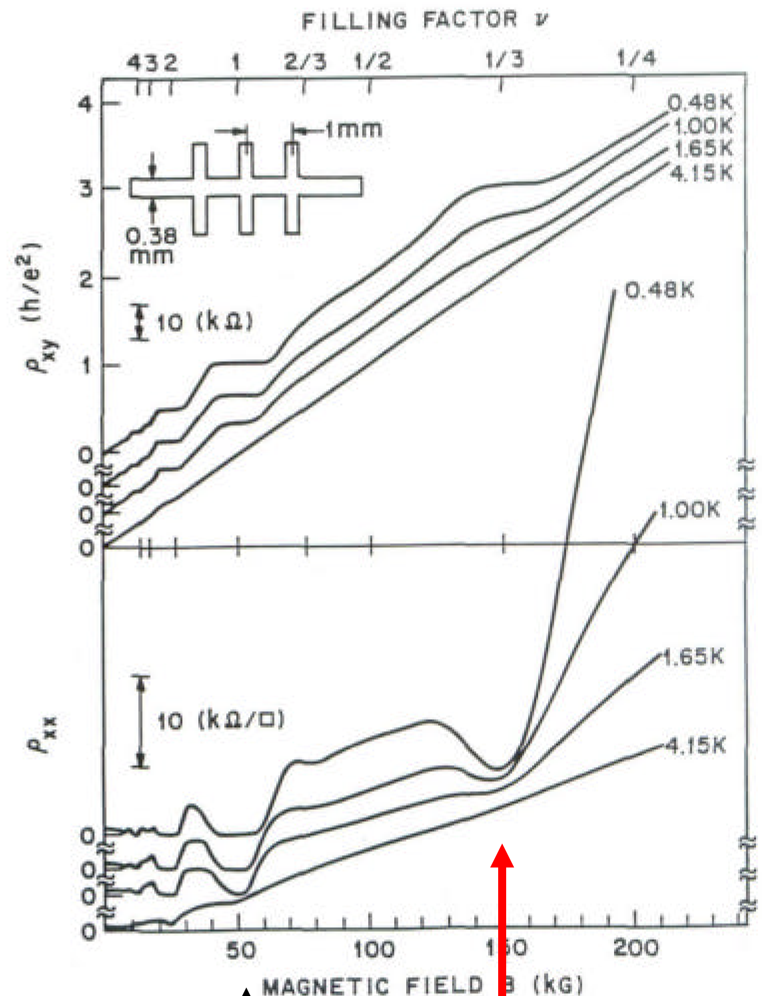
(note: high densities, high B – fields)

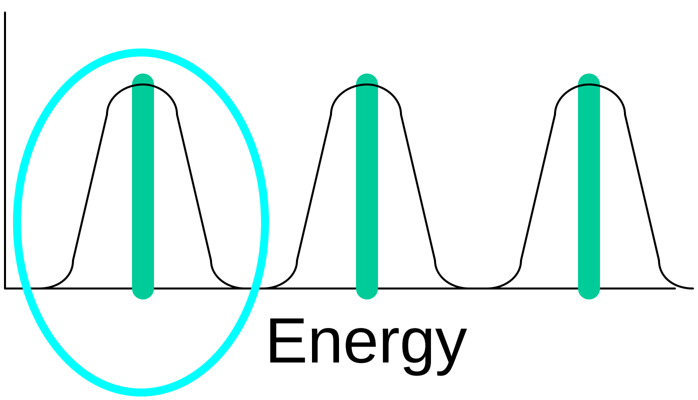


I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect

Higher mobility (lower disorder) samples produced –
new quantum Hall state found at fractional filling factor: shouldn't be there





? = 1

? = 1/3

I. Introduction: materials, transport, Hall effects

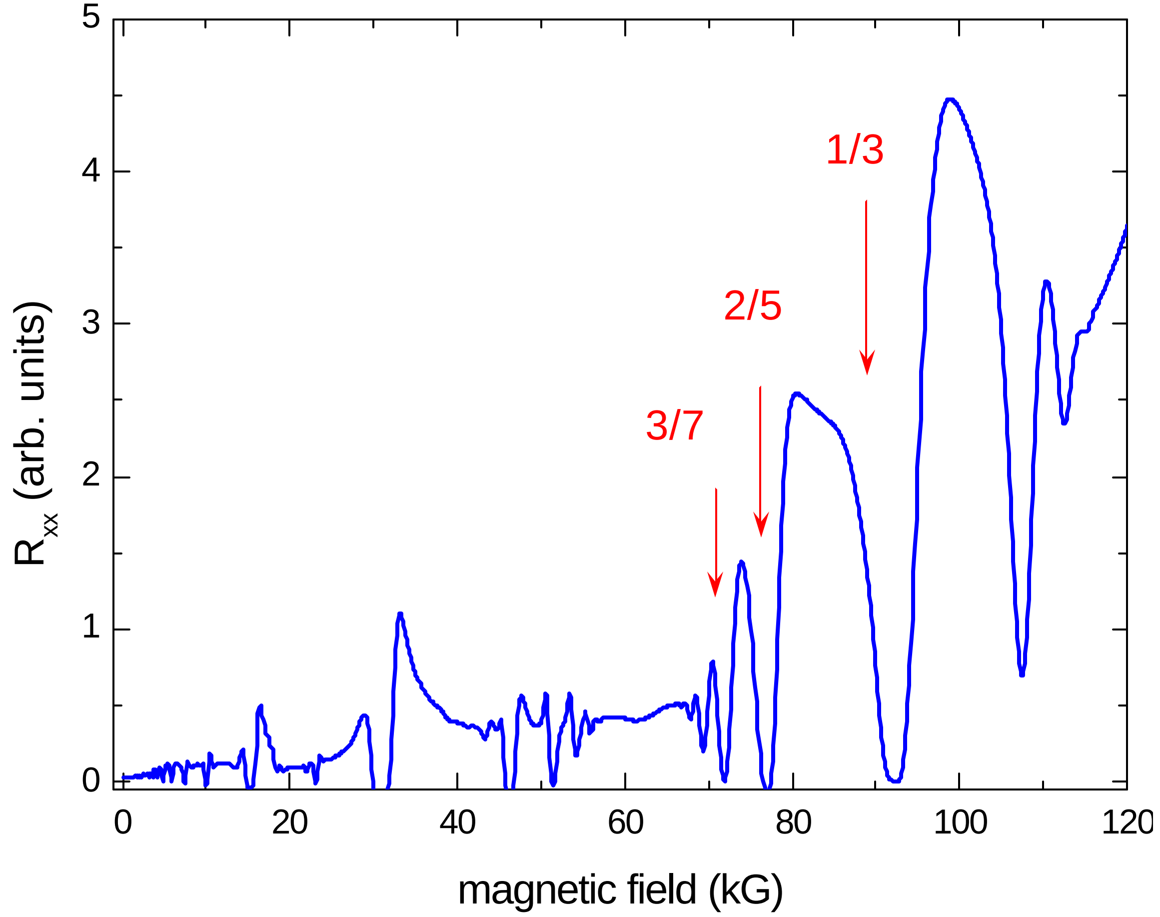
E. The Fractional quantum Hall effect

Higher mobility samples show more FQHE states

1) a hierarchy of fractions observed:

New "quantum numbers" at 1/3, 2/5, 3/7, 4/9,….

Filling factor ? = p / (2p+1)



I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect

Higher mobility samples show more FQHE states

1) a hierarchy of fractions observed:

New "quantum numbers" at 1/3, 2/5, 3/7, 4/9,….

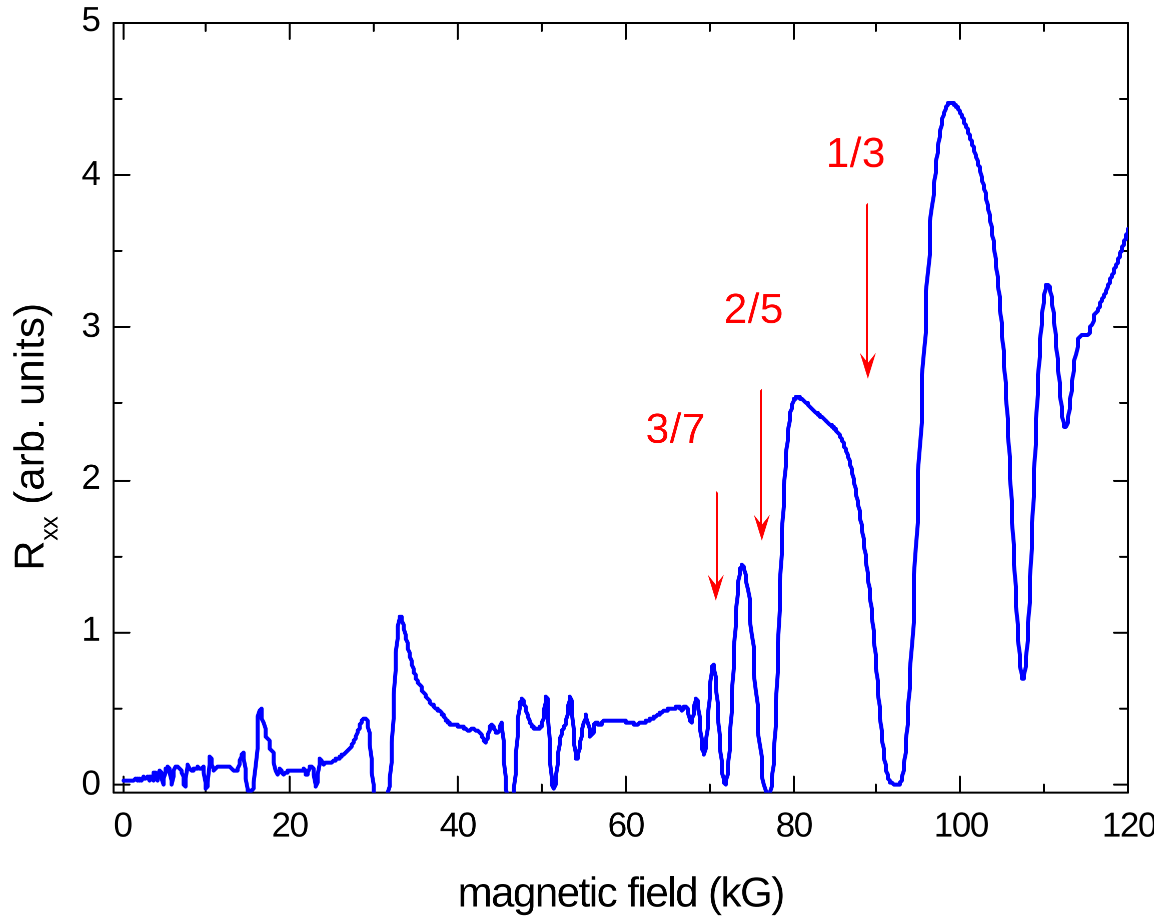
Filling factor ? = p / (2p+1)

2) The minima display activated transport:

$$R = R_0\, e^{(-?/kT)}$$

? here should reflect the Coulomb energy

$$e^2/?l_0$$



I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect

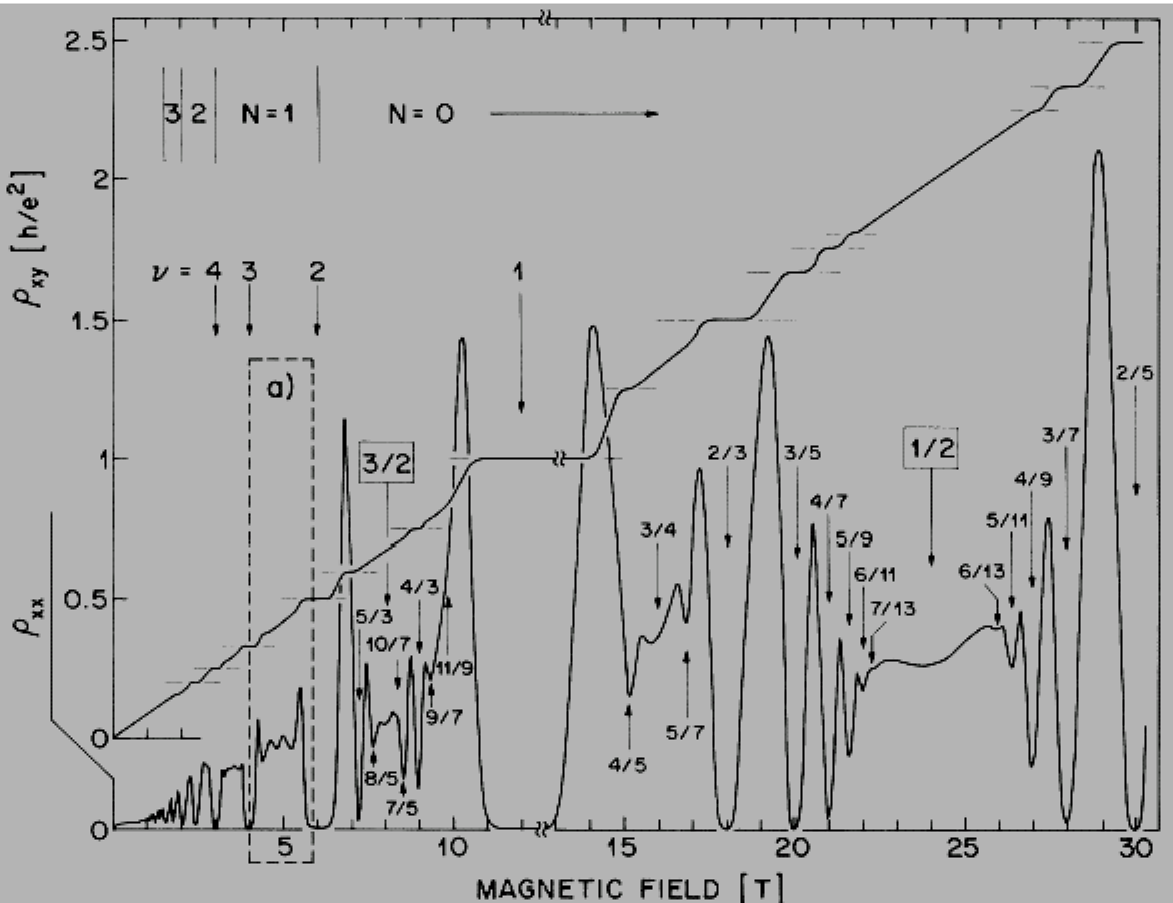


FIG. 1. Overview of diagonal resistivity ρ_{xx} and Hall resistance ρ_{xy} of sample described in text. The use of a hybrid magnet with fixed base field required composition of this figure from four different traces (breaks at $\simeq 12$ T). Temperatures were ≈ 150 mK except for the high-field Hall trace at $T = 85$ mK. The high-field ρ_{xx} trace is reduced in amplitude by a factor 2.5 for clarity. Filling factor ν and Landau levels N are indicated.

How can this all be explained?

I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect

generalization from few particle form

$$\psi = (z_1 - z_2)^m (z_1 + z_2)^n \exp[-\tfrac{1}{4}(|z_1|^2 + |z_2|^2)]$$

where $z = x + iy$

generates

$$\psi = \{\prod_{j<k} f(z_j - z_k)\} \exp(-\tfrac{1}{4} \textstyle\sum_\ell |z_\ell|^2)$$

with constraints

a) $|\psi_m|^2 = |\{\prod_{j<k} (z_j - z_k)^m\} \exp(-\tfrac{1}{4} \sum_\ell |z_\ell|^2)|^2$

$= e^{-\phi/m}$, m odd

ϕ the classical potential energy of one-component plasma of charge $Q = m$ particles

and a) used to determine which m minimizes the energy

for $\nu = 1/3$

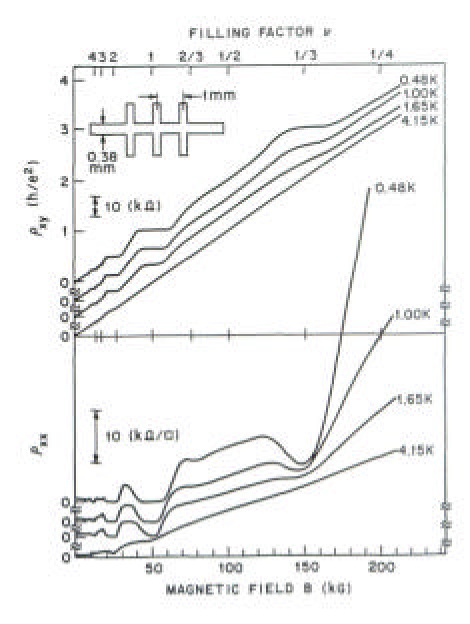
$$\psi(z_1, .. z_n) = \prod_{j<k} (z_j - z_k)^3 \exp(-\tfrac{1}{4} \textstyle\sum_\ell |z_\ell|^2)$$

Laughlin, PRL 50, 1395 (1983).

The Laughlin wave function

$$\Psi(z_1, z_2, \ldots, z_n) \sim \prod_{i<j} (z_i - z_j)^3$$

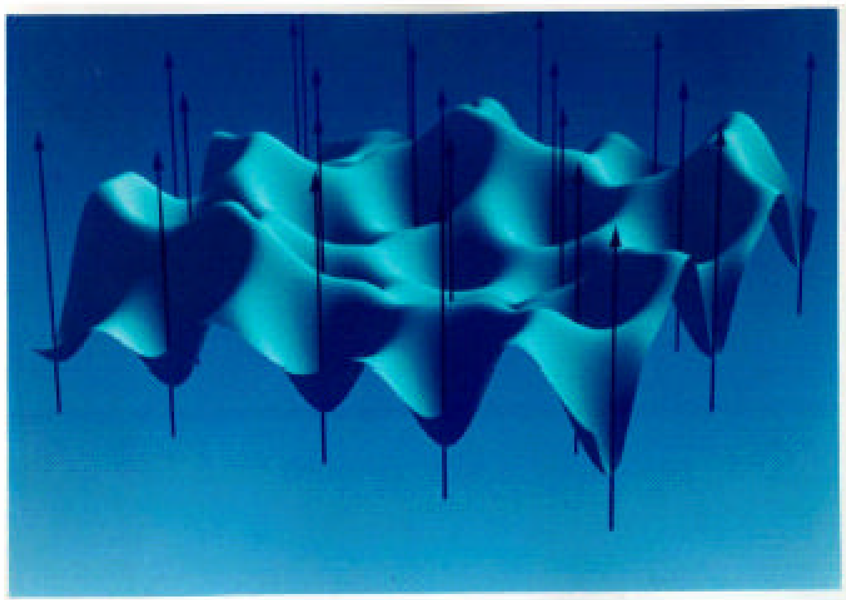
Describes an incompressible quantum liquid at ?=1/3



I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect - pictures

Single electron in the lowest Landau level

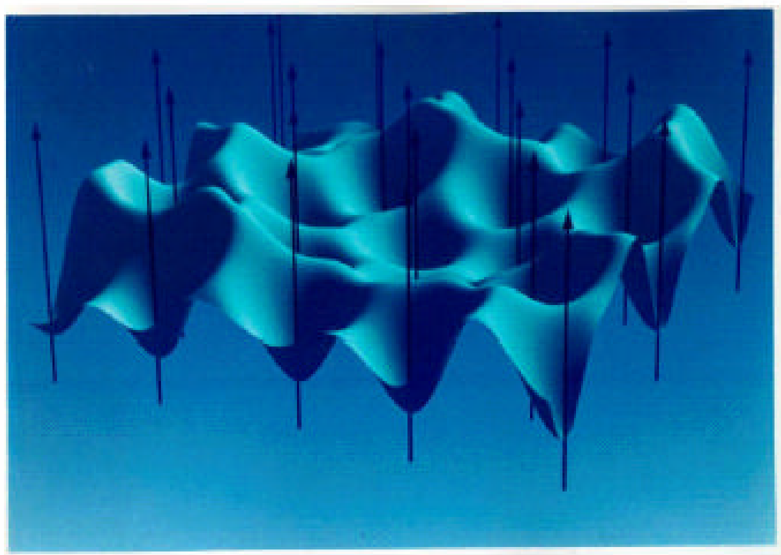


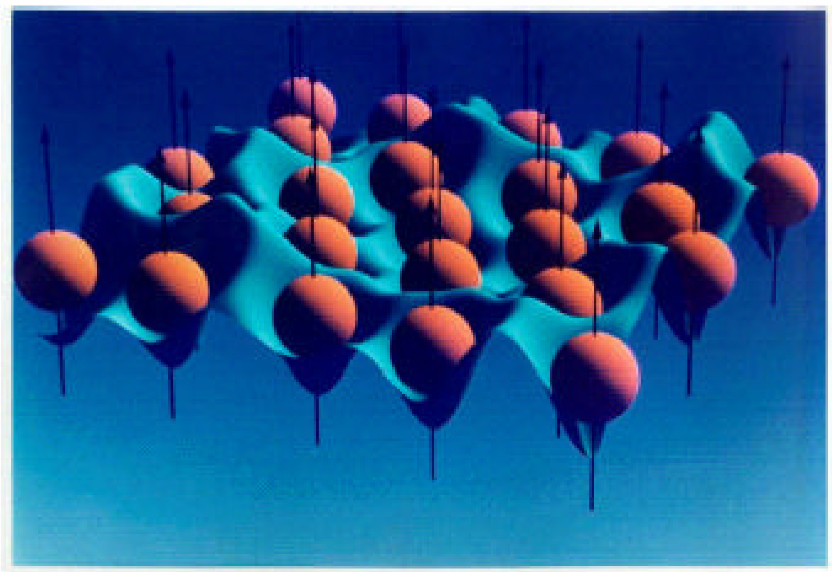
$$\psi(z_1) \sim \sum_{j=0}^{n_\phi - 1} a_j \, z_1^j \; exp\left(\frac{-|z_1|^2}{4}\right)$$

I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect - pictures

Single electron in the lowest Landau level → Filled lowest Landau level





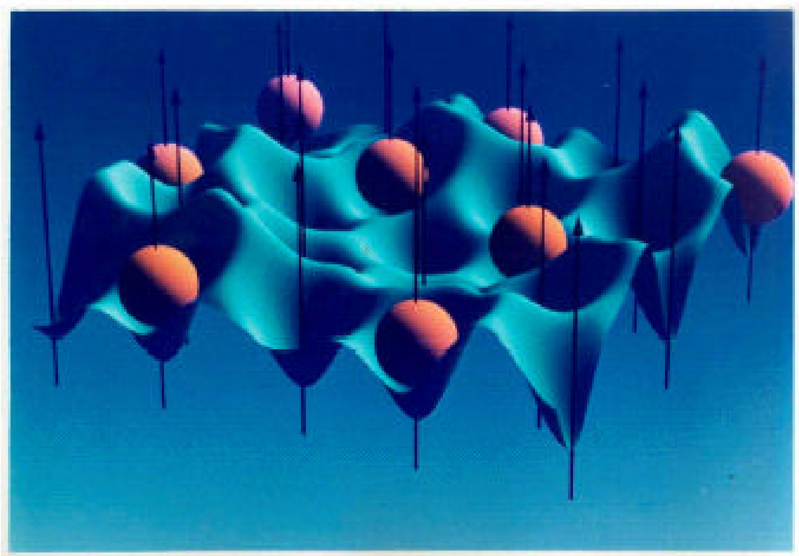
$$\psi(z_1) \sim \sum_{j=0}^{n_\phi - 1} a_j \, z_1^j \; exp\left(\frac{-|z_1|^2}{4}\right)$$

$$\Psi(z_1, z_2, \ldots, z_n) \sim \prod_{i<j} (z_i - z_j)$$

I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect - pictures

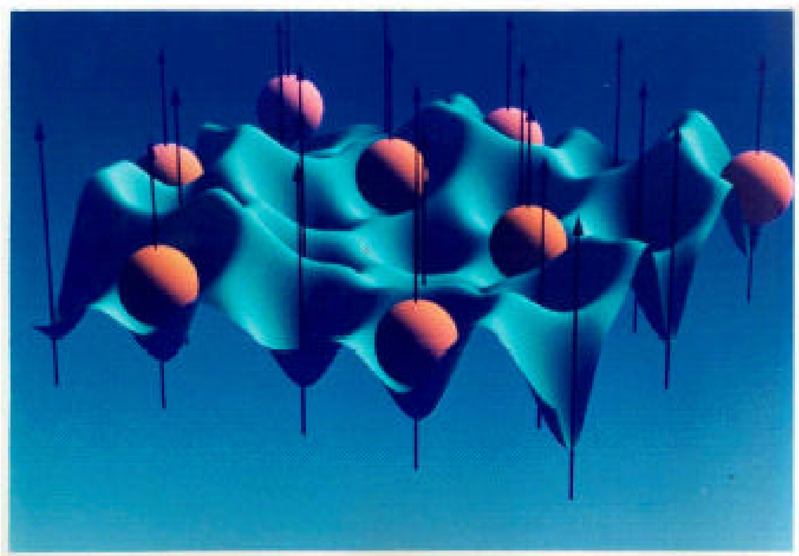
Uncorrelated ? = 1/3 state



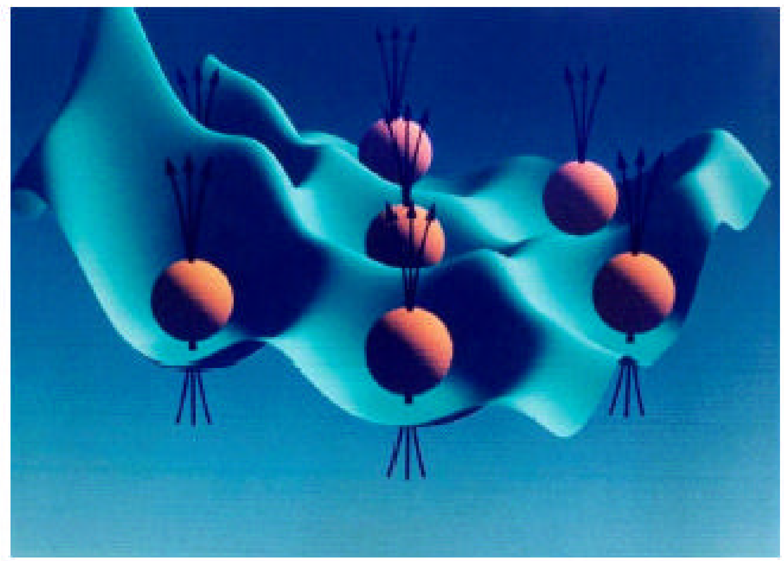
I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect - pictures

Uncorrelated ? = 1/3 state



Correlated ? = 1/3 state



$$\Psi(z_1, z_2, \ldots, z_n) \sim \prod_{i<j} (z_i - z_j)^3$$

E. The Fractional quantum Hall effect

The Laughlin wave function and its excitations

$$\Psi^{*}(z_1, z_2, \ldots, z_n) \sim \prod_k (z_k - w) \prod_{i<j} (z_i - z_j)^3$$

A *quasi-hole* with charge $q = +e/3$

elementary excitations of Ψ_m generated by piercing the fluid at z_0 with solenoid & passing through flux quantum h/e adiabatically

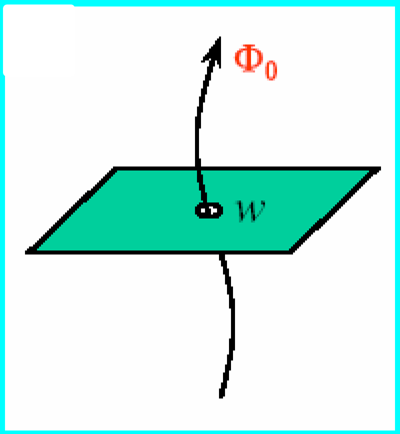
quasi-hole:

$$\Psi_m^{+z_0} = \left\{\prod_i (z_i - z_0)\right\}\left\{\prod_{j<k} (z_j - z_k)^m\right\} \exp\left(-\tfrac{1}{4}\sum_l |z_l|^2\right)$$

excitation is a particle of charge $1/m$

with finite (gap) energy to produce

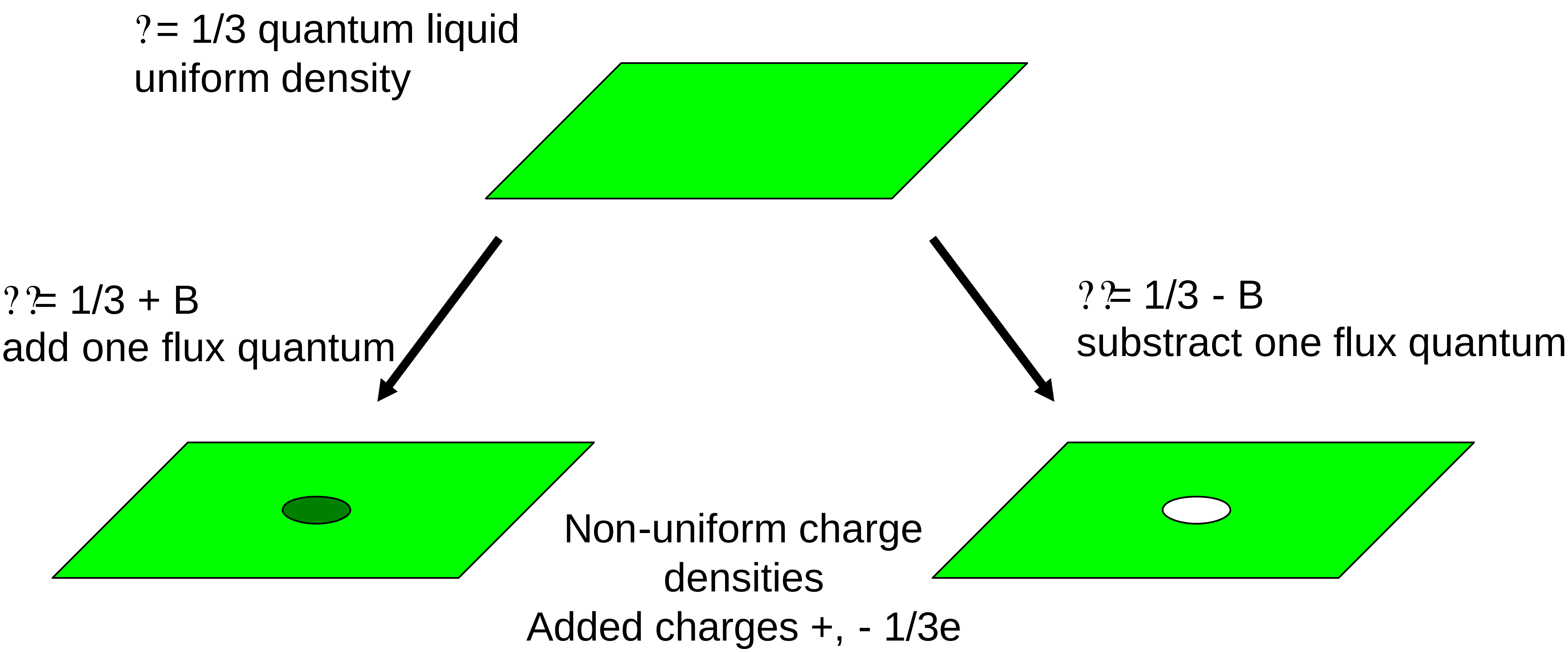
Add one flux quantum



I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect

The Laughlin liquid and its excitations

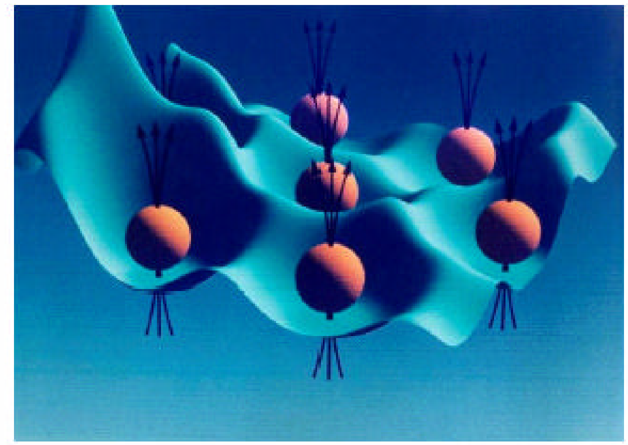


Energy required to change charge is ? ?~ $e^2/?l_0$

l_0 the magnetic length

I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect



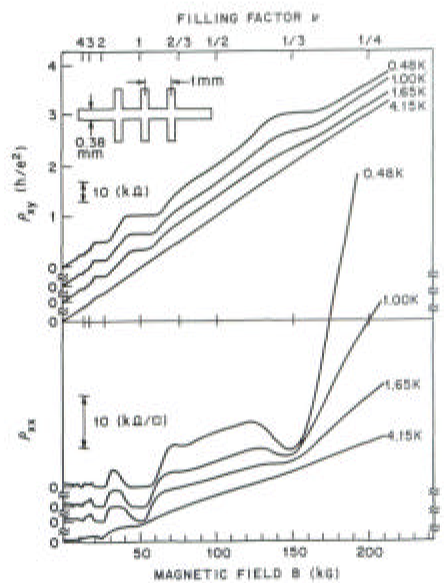
Laughlin state describes:

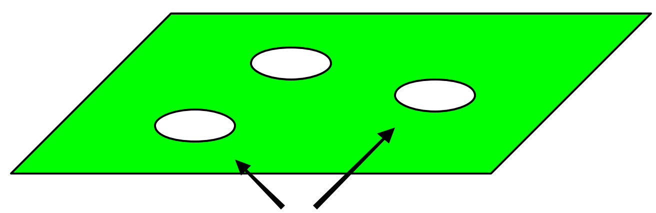
1) New incompressible liquid state

2) Excitations of the liquid, charge 1/m

3) Consistency with experimental results at 1/3, 1/5 (found later)

$$\Psi(z_1, z_2, \ldots, z_n) \sim \prod_{i<j} (z_i - z_j)^3$$



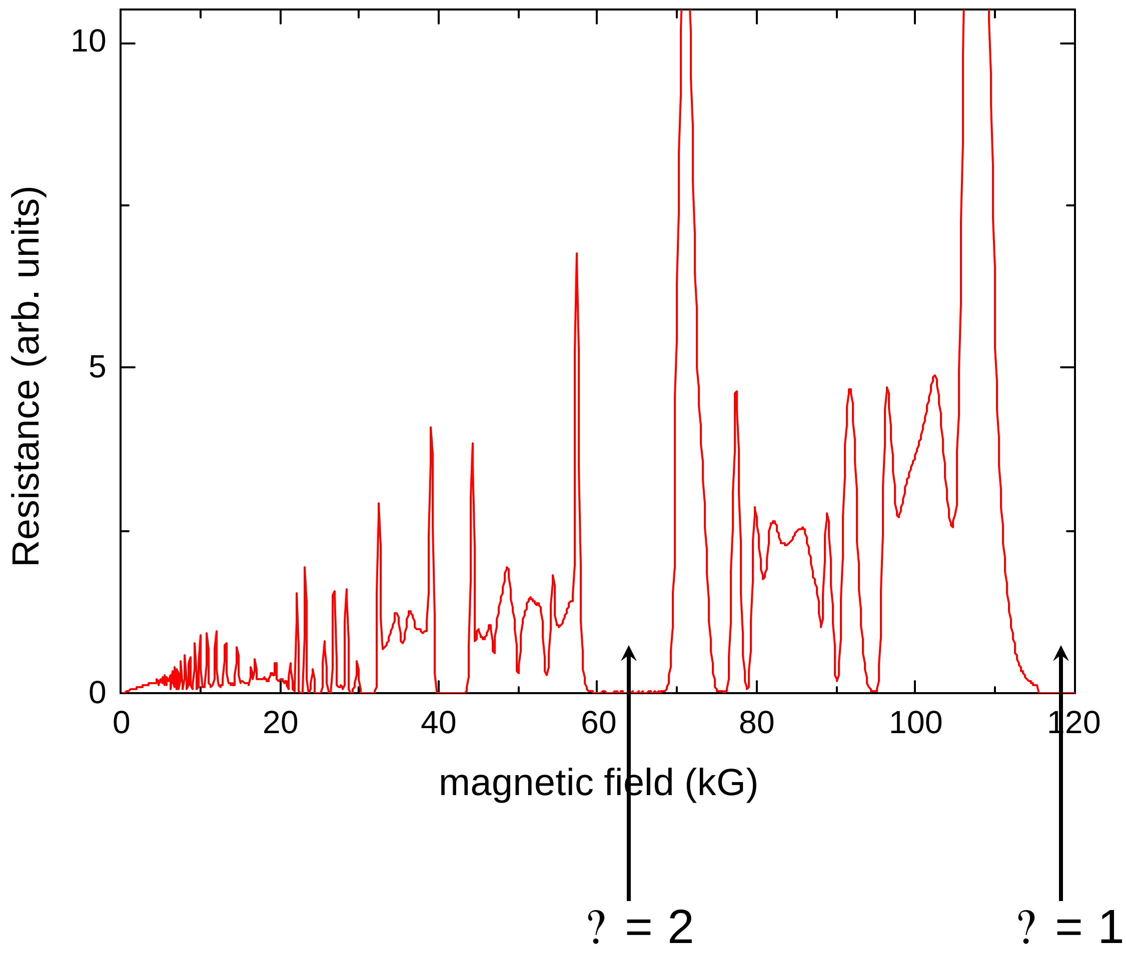


Fractionally charged excitations

I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect

Higher mobilities result in more fractional quantum Hall states



I. Introduction: materials, transport, Hall effects

E. The Fractional quantum Hall effect

Even higher mobilities result in even more fractional quantum Hall states

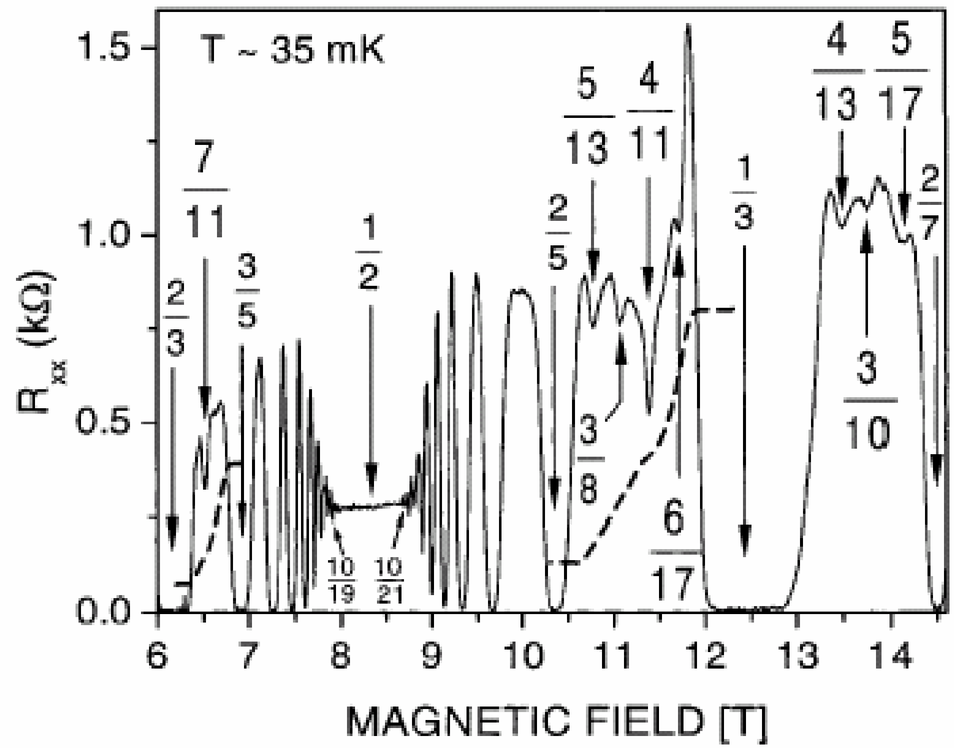


FIG. 1. R_{xx} in the regime $2/3 > \nu > 2/7$ at $T \sim 35$ mK. Major fractions are marked by arrows. Dashed traces are the Hall resistance R_{xy} around $\nu = 7/11$ and $\nu = 4/11$.

Pan,et al. PRL 04

I. Introduction: materials, transport, Hall effects

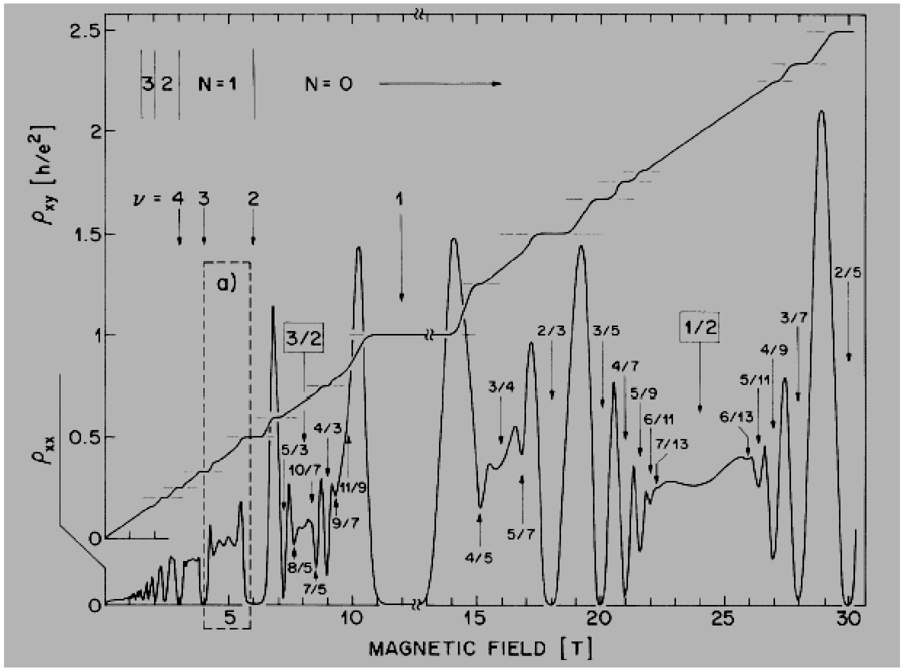
Summary:

✍ 2D electron samples show 2D physics in transport: Shubnikov-deHaas oscillations

✍integer quantum Hall effect: resolved Landau levels with localization between centers of Landau levels

✍low disorder 2D electron systems show fractional quantum Hall effect – correlations of electrons as described by the Laughlin wave function

✍what about **many** fractional quantum Hall states?



Outline:

I. Introduction: materials, transport, Hall effects

II. Composite particles – composite fermions

   A. Experiments - hierarchy of fractions
   B. Composite fermions and their Landau levels
   C. Composite fermions and experiments
   D. Fermi surface picture - SAW
   E. Other Fermi surface experiments
   F. Composite fermion effective mass
   G. Other composite fermions

III. Quasiparticle charge and statistics

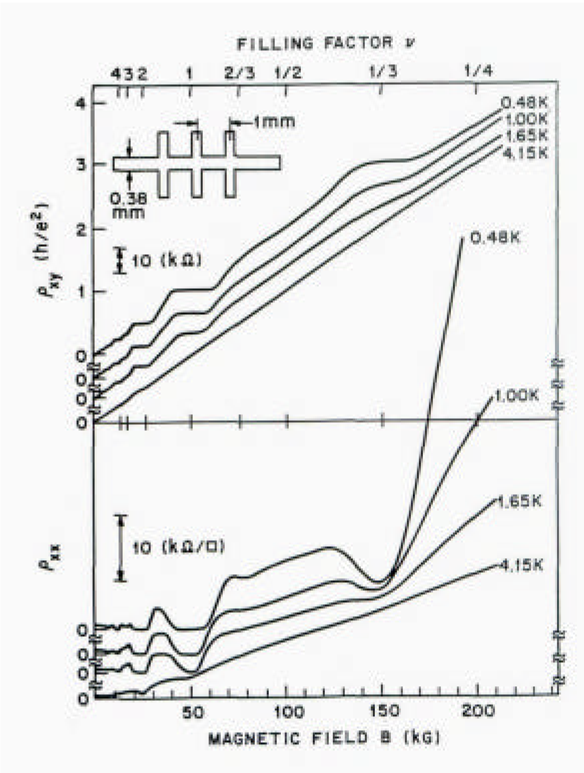
IV. Higher Landau levels

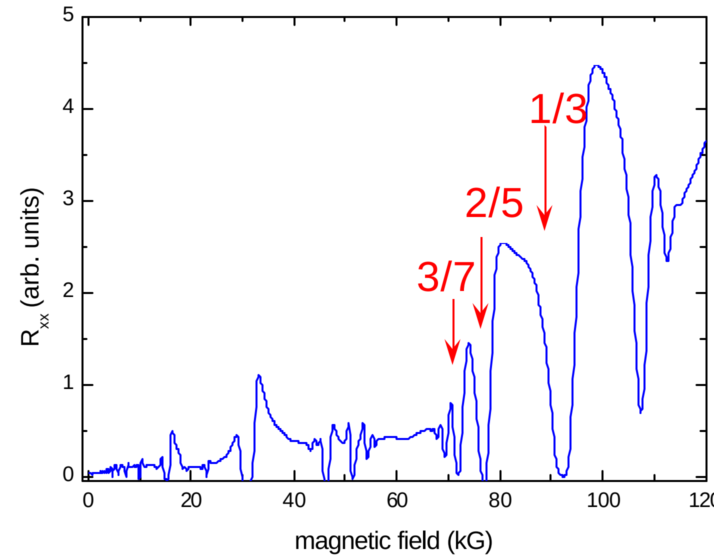
V. Other parts of spectrum: non-equilibrium effects, electron solid?

VI. Multicomponent systems: Bilayers

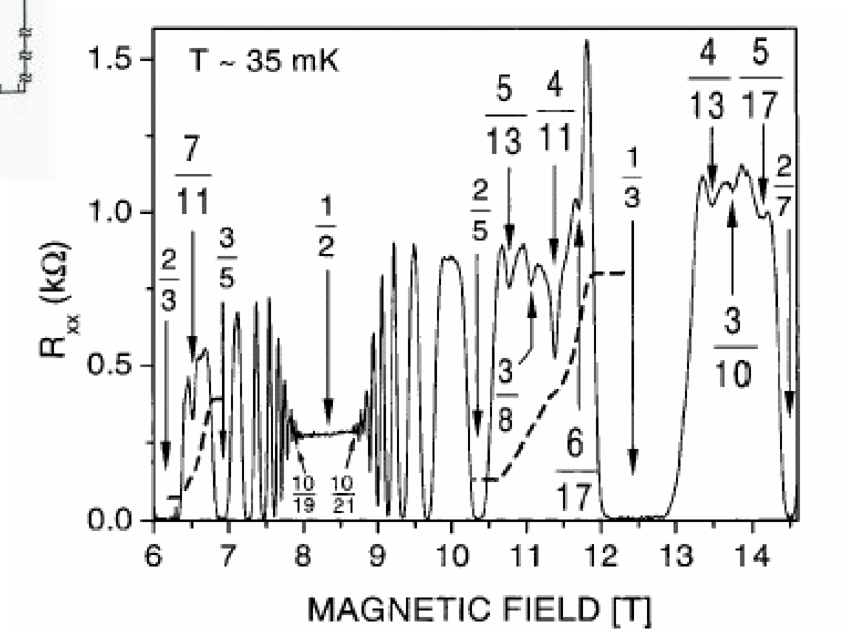
II. Composite particles – composite fermions

A. Experiments - hierarchy of fractions





With lower disorder samples, more fractional states observed



Better samples, larger activation energies

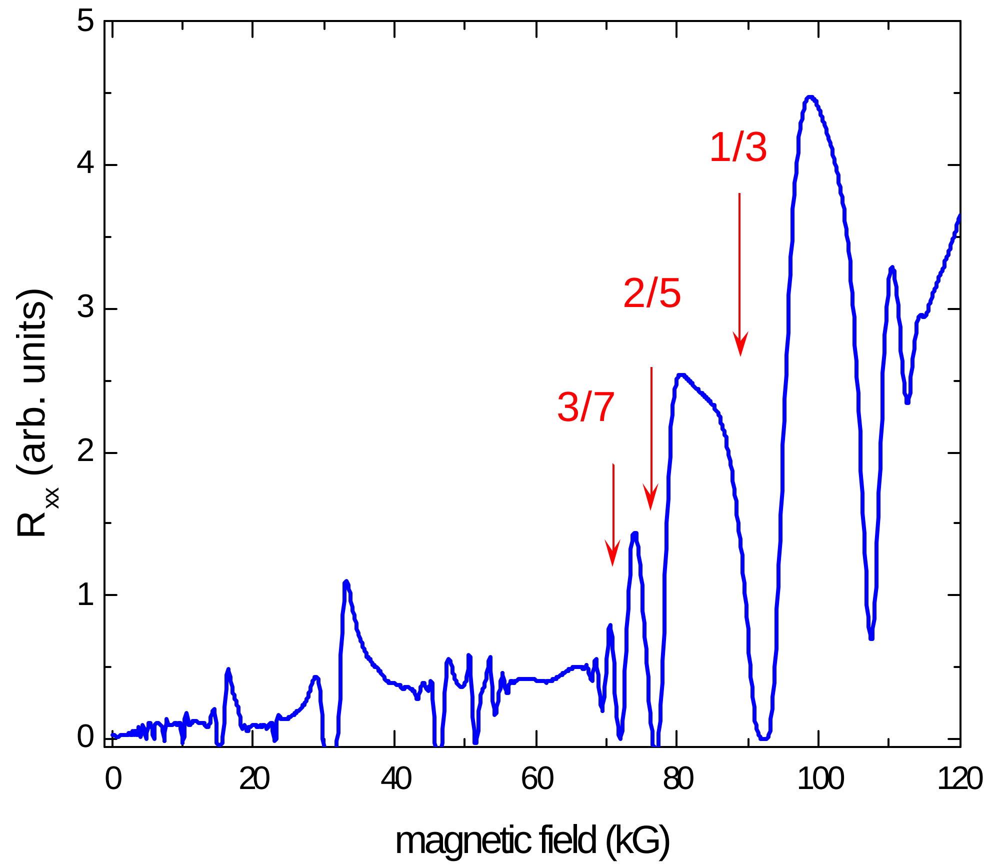
II. Composite particles – composite fermions

A. Experiments – hierarchy of fractions

FQHE states found at filling factors ? ?= p/(2p+1), p=1, 2, 3...

Or more generally, at filling factors ? ?= p/(2np ± 1) and at ?= 1 - p/(2np ± 1)

This includes series of 2/3, 3/5, 4,7… and the series around filling factor 1/4

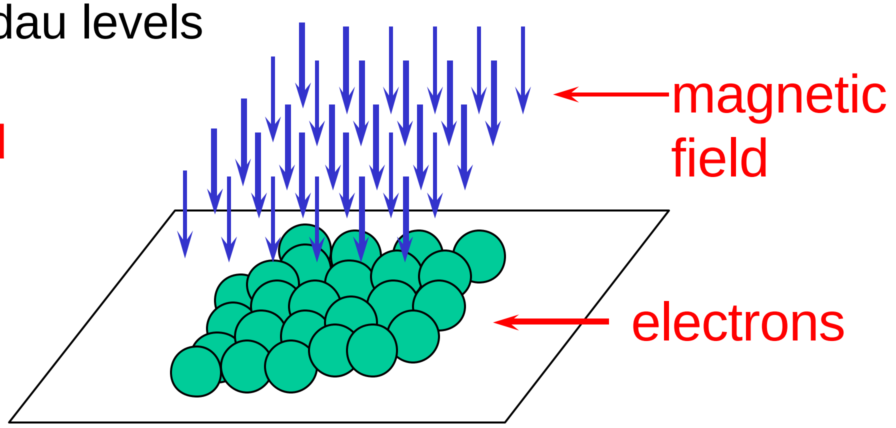


1/3 filling factor understood from Laughlin state: what are the others?

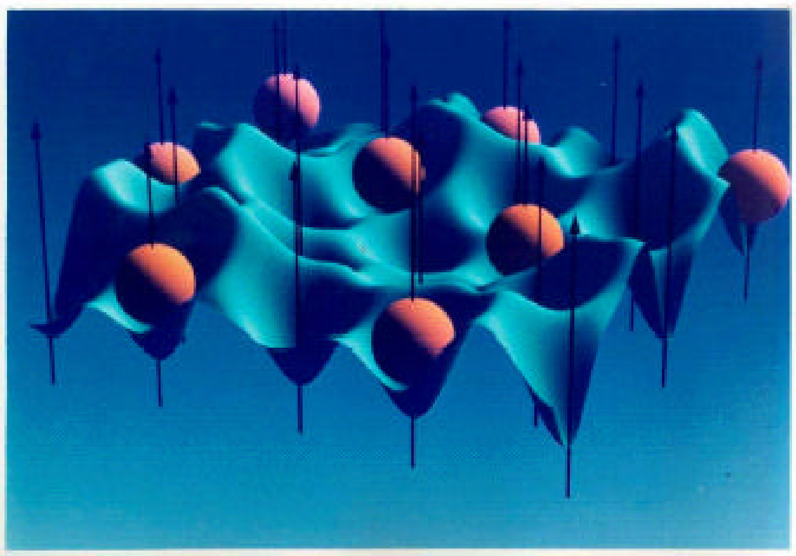
II. Composite particles – composite fermions

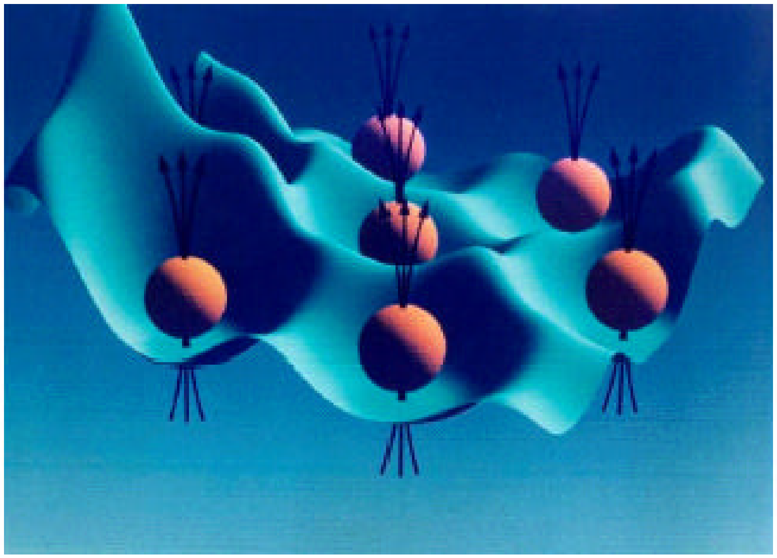
B. Composite fermions and their Landau levels

Composite particles of charge and magnetic flux:



From previous description of FQHE liquid we saw that a "correlation hole" is energetically favorable spot for electron to reside: this "associates" flux line(s) to the electrons - example 1/3

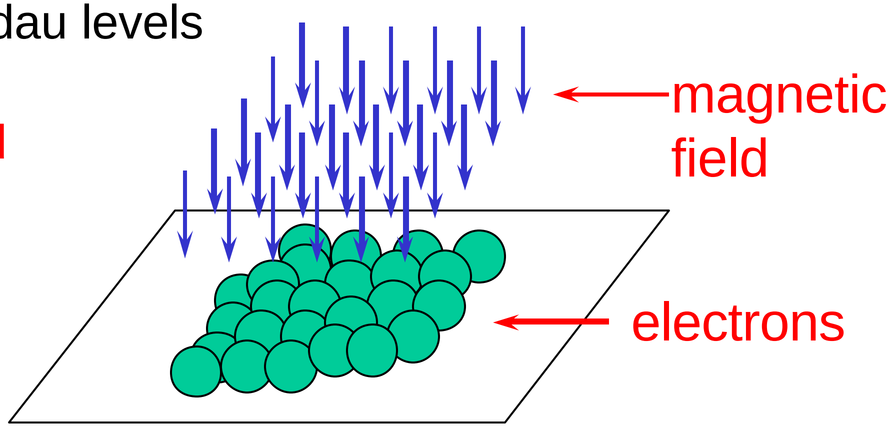




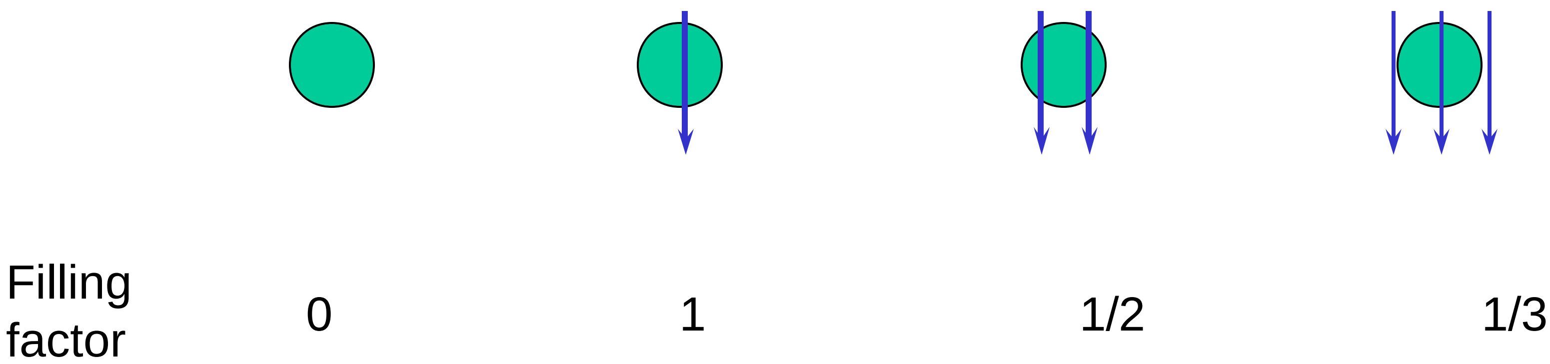
II. Composite particles – composite fermions

B. Composite fermions and their Landau levels

Composite particles of charge and magnetic flux:



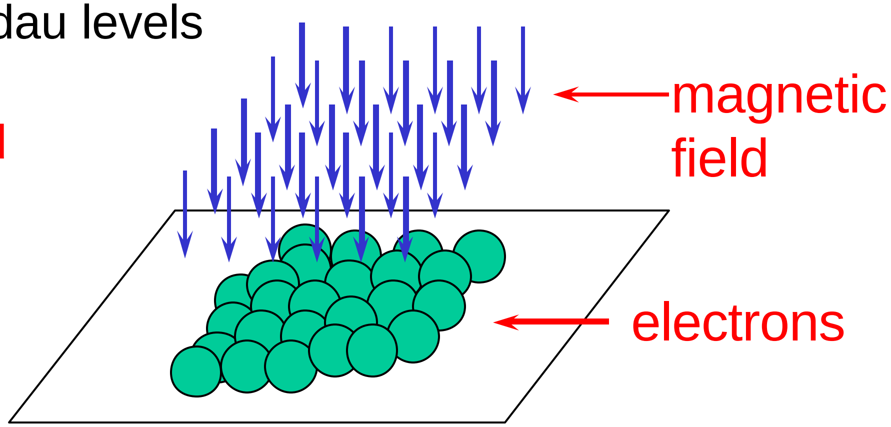
Any number of magnetic flux may be associated with charge to produce a quasiparticle of the flux/charge composite



II. Composite particles – composite fermions

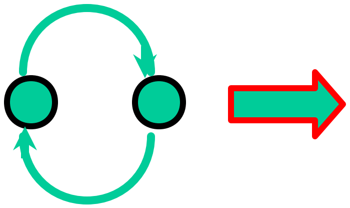
B. Composite fermions and their Landau levels

Composite particles of charge and magnetic flux:



Given that flux may be associated with charge, now examine the statistics of these quasipartles:
An electron wave function upon particle exchange gains phase change

FERMIONS

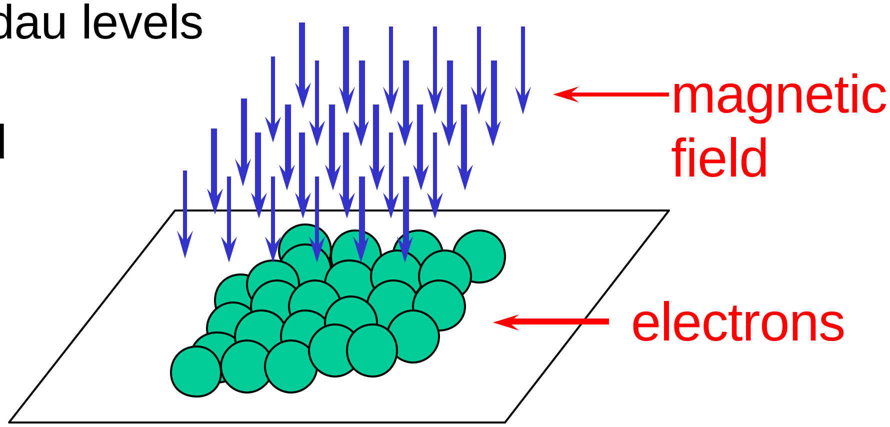


$e^{-i?}$

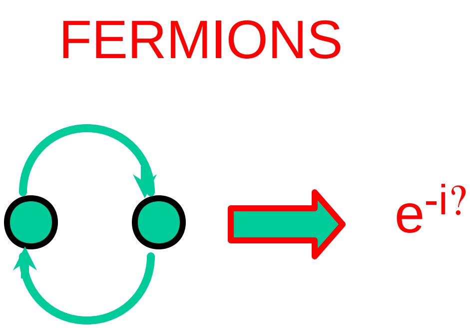
II. Composite particles – composite fermions

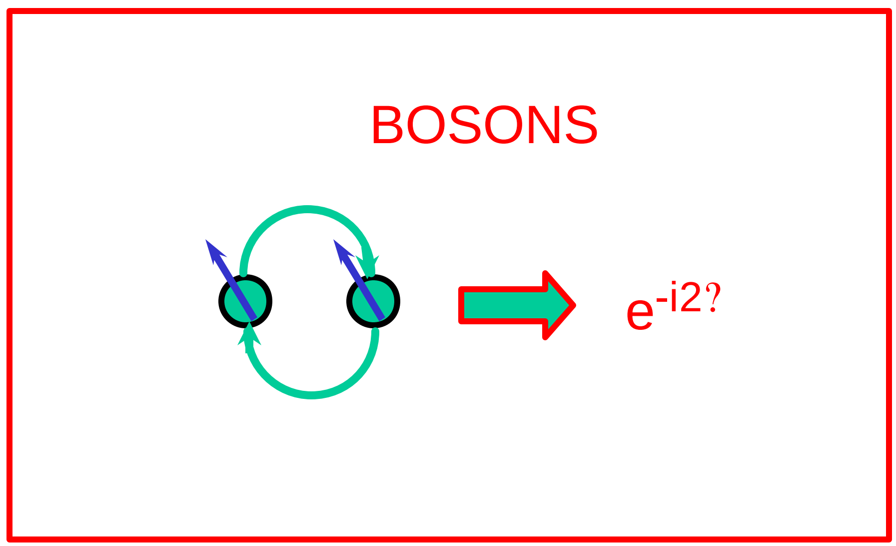
B. Composite fermions and their Landau levels

Composite particles of charge and magnetic flux:



Given that flux may be associated with charge, now examine the statistics of these quasipartles:
An electron with one associated flux quantum obeys bosonic statistics

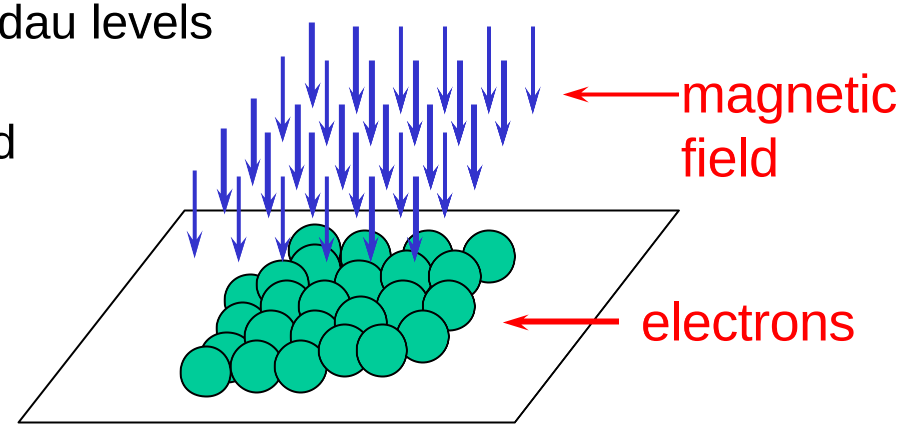




II. Composite particles – composite fermions

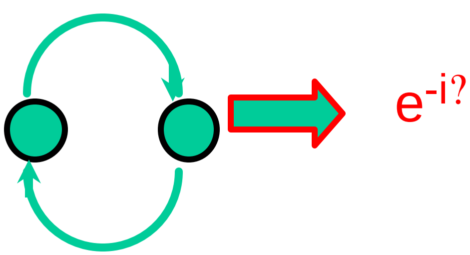
B. Composite fermions and their Landau levels

Composite particles of charge and magnetic flux:

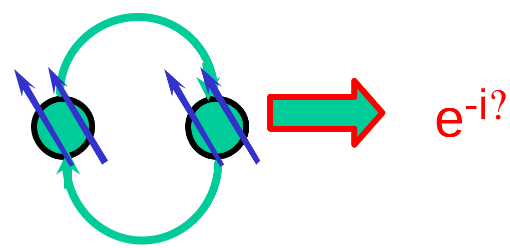


Given that flux may be associated with charge, now examine the statistics of these quasipartles:
An electron with two associated flux quanta obeys fermionic statistics

FERMIONS



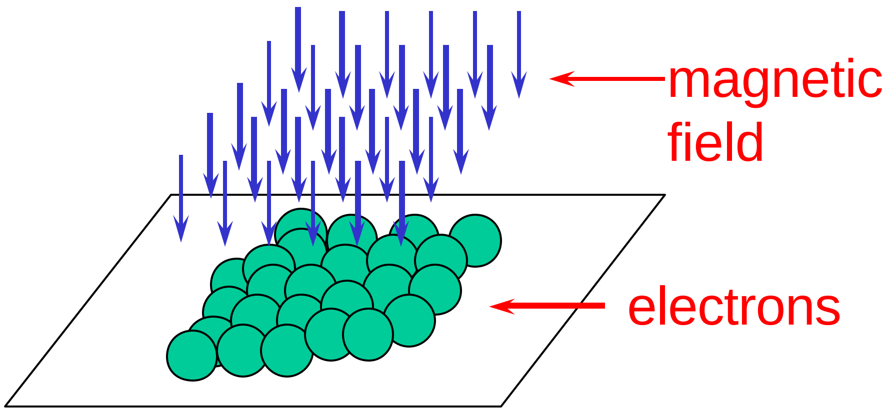
COMPOSITE FERMIONS



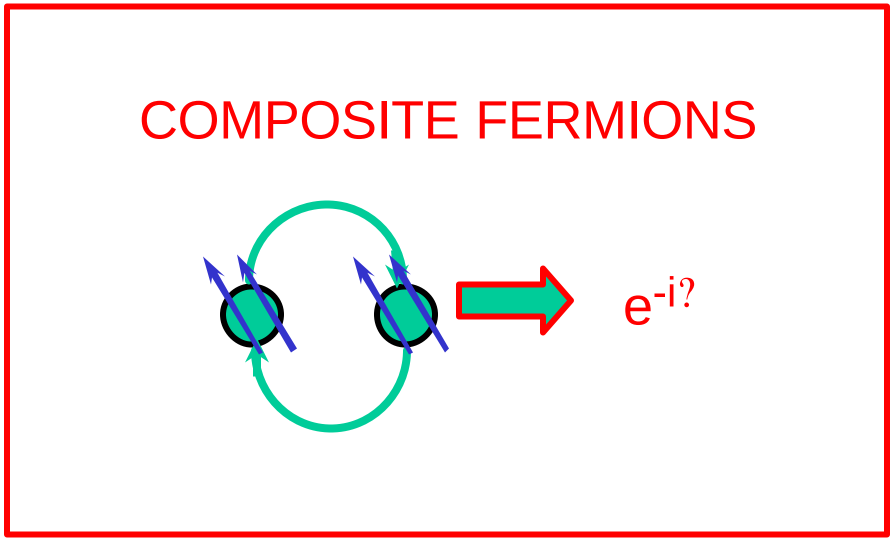
II. Composite particles – composite fermions

B. Composite fermions and their Landau levels

Composite particles of charge and magnetic flux:



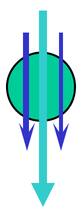
J. Jain examined the correlated 2DES using the quasiparticle, the composite fermion, with the rational that the FQHE states are due to Landau levels of this quasiparticle



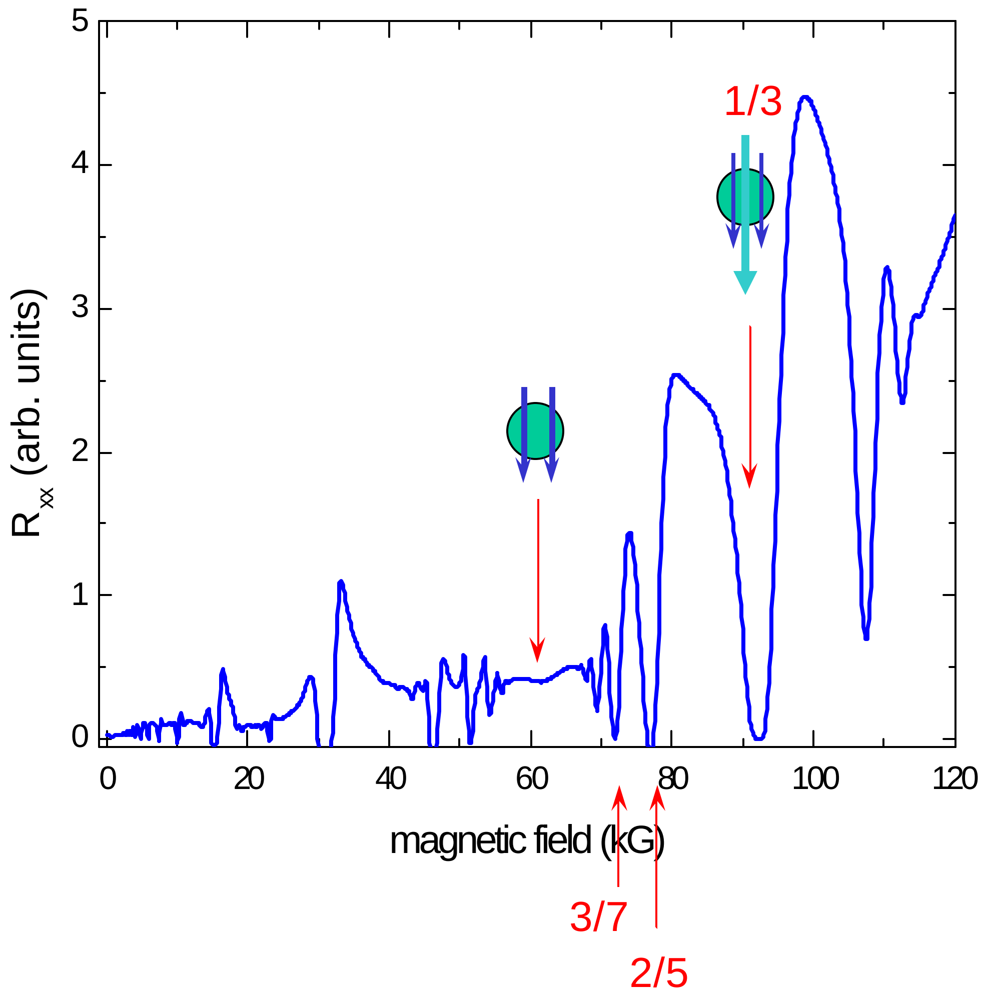
II. Composite particles – composite fermions

B. Composite fermions and their Landau levels

Composite particles of charge and magnetic flux:

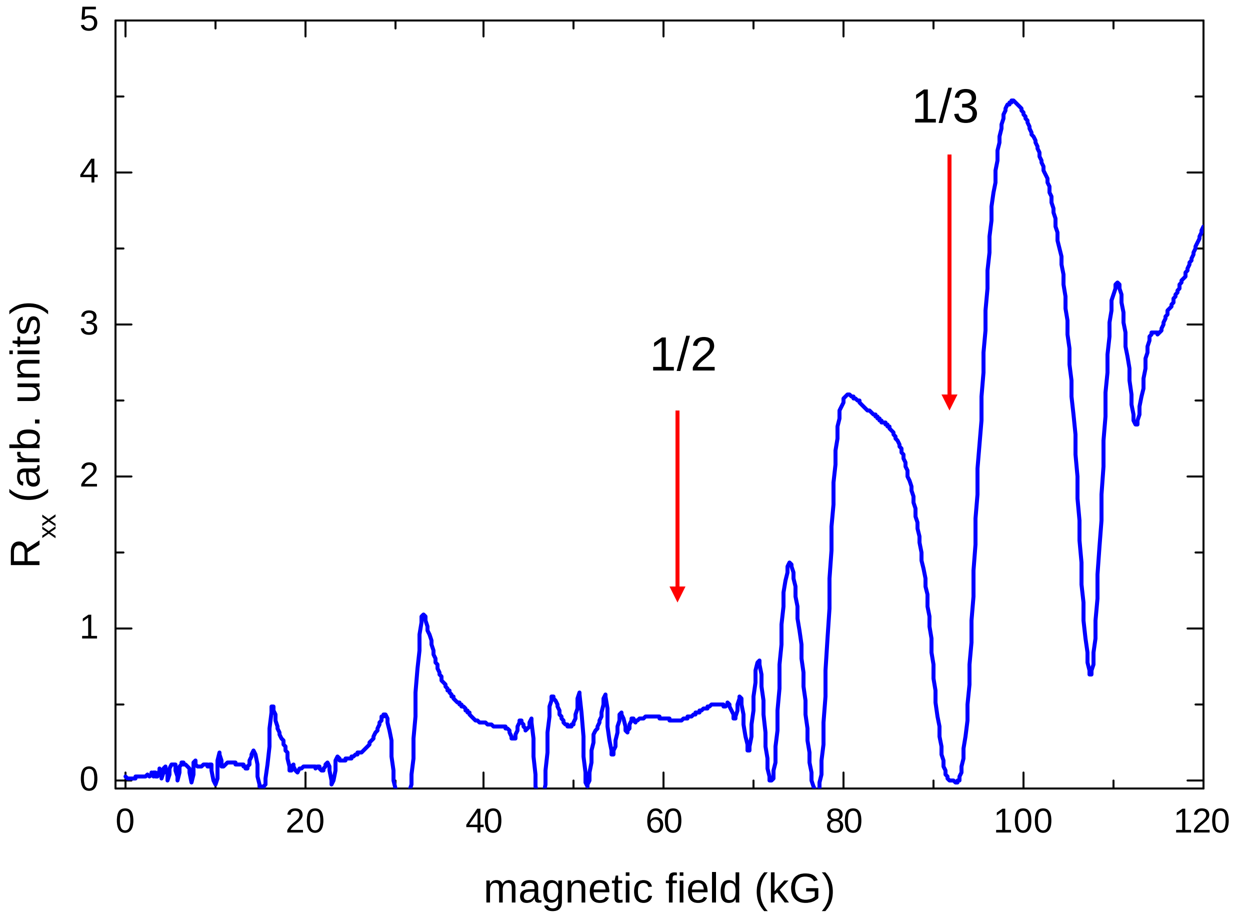


Composite fermion at its filling factor of 1



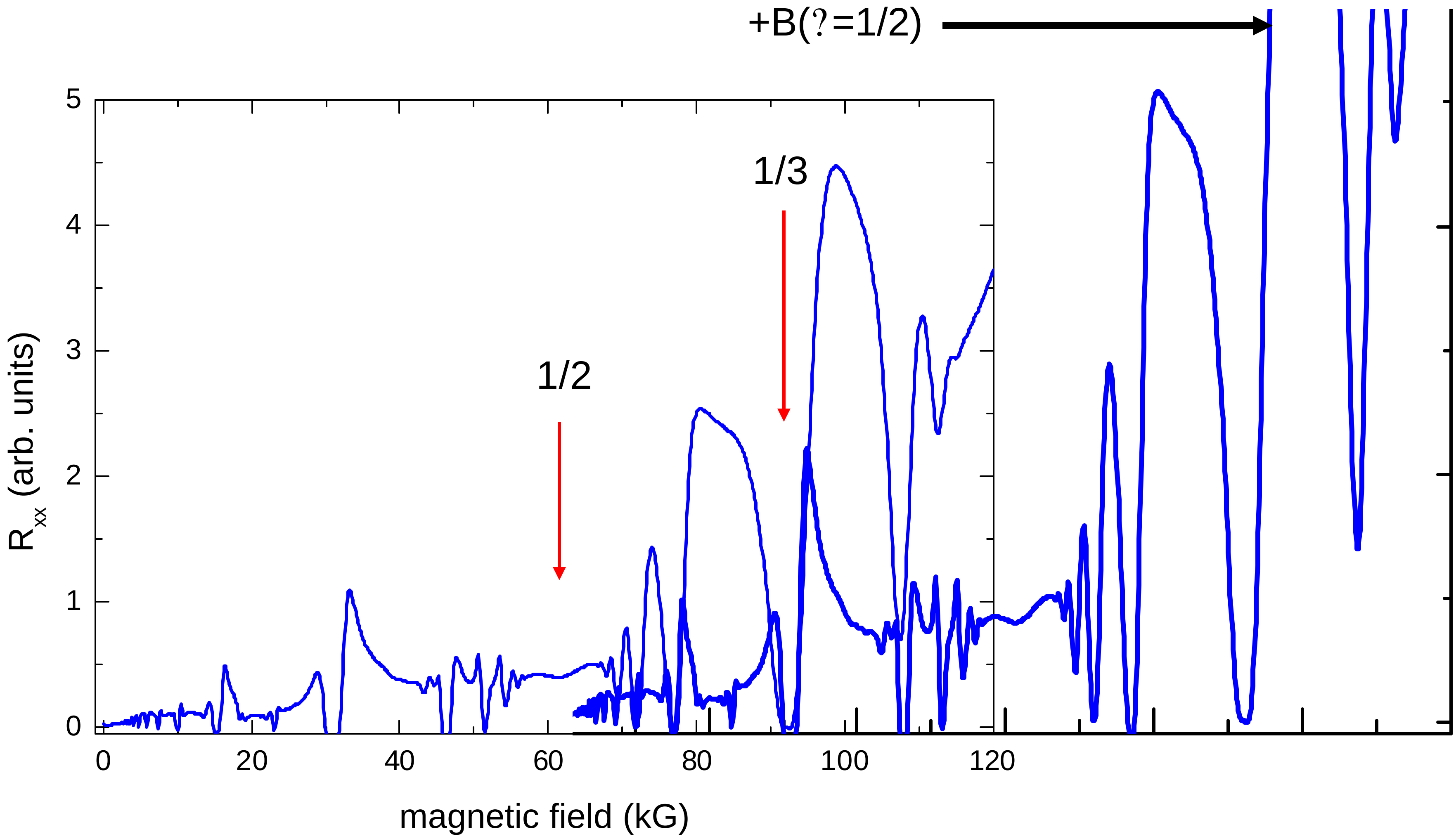
II. Composite particles – composite fermions

B. Composite fermions and their Landau levels



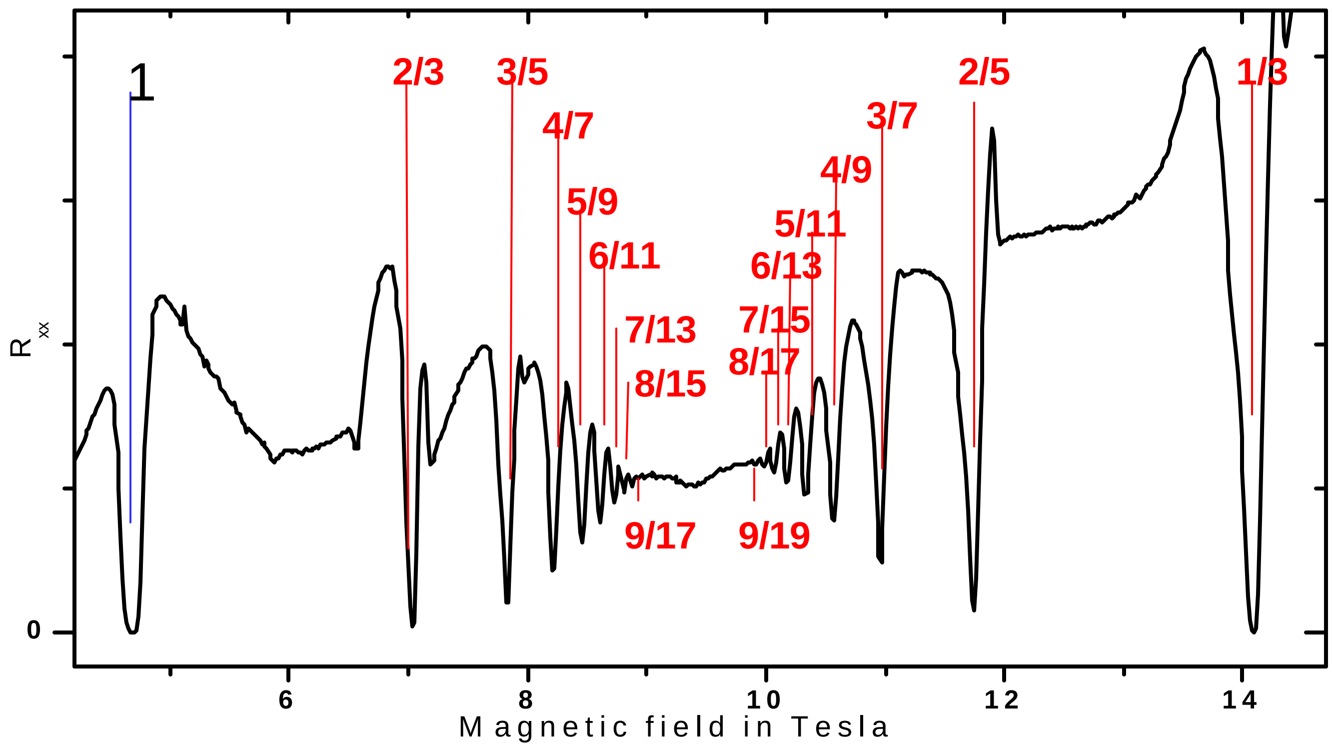
II. Composite particles – composite fermions

B. Composite fermions and their Landau levels



II. Composite particles – composite fermions

B. Composite fermions and their Landau levels



By enumeration, the hierarchy of observed fractions now become integer Landau levels for this quasiparticle

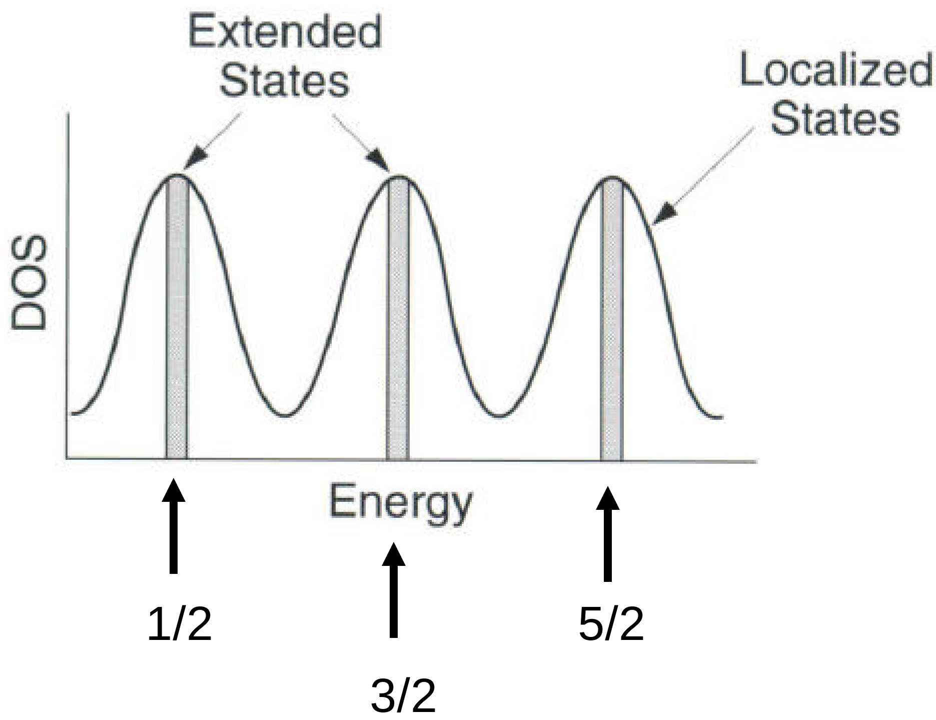
true filling factor	composite fermion filling factor
1/3	1
2/5	2
3/7	3
4/9	4

II. Composite particles – composite fermions

B. Composite fermions and their Landau levels

While this picture developed, new findings at an “odd” location –

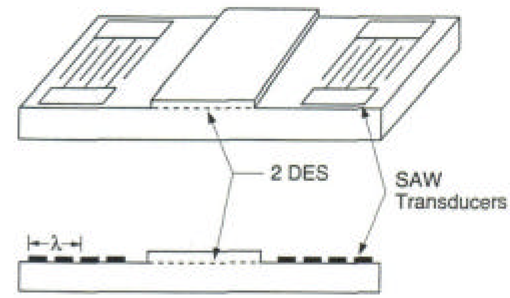
Filling factor 1/2 :
should be the center of the Landau level –
non-localized electrons following the classical Hall trace – **metallic?**

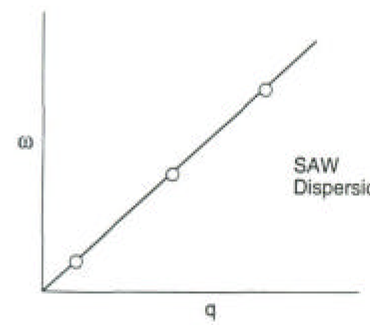


II. Composite particles – composite fermions

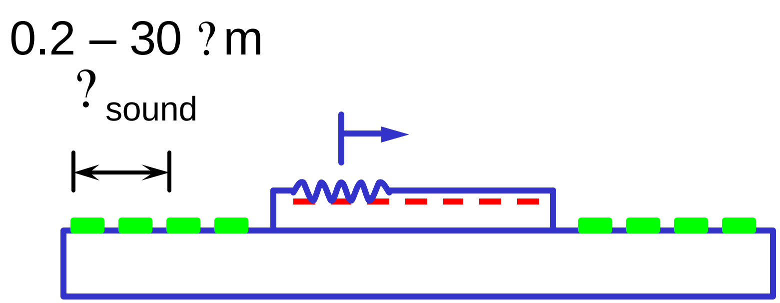
C. Composite fermions and experiments

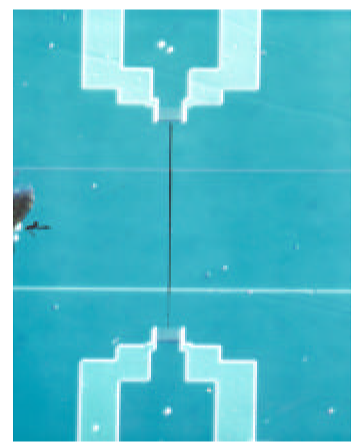
Surface acoustic wave experiments:
Measure conductivity over short distances





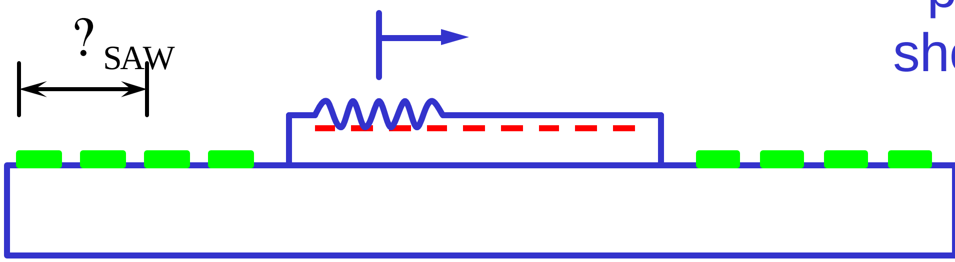
Propagating sound wave
applies electric field:
electrons respond to this field



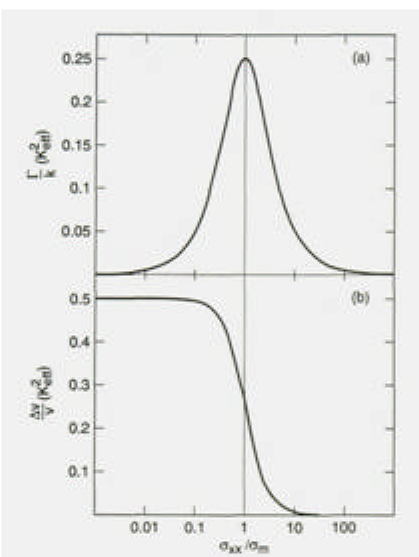


II. Composite particles – composite fermions

C. Composite fermions and experiments



SAW device launches longitudinal wave with E-field in direction of propagation: conducting layer can short this piezoelectric field, changing the propagation properties



$$\frac{\Delta v}{v} = \left(\frac{\alpha^2}{2}\right) \frac{1}{1 + (\sigma_{xx}(\omega, q)/\sigma_{\mathrm{m}})^2},$$

$$\Gamma = q\left(\frac{\alpha^2}{2}\right) \frac{\sigma_{xx}(\omega, q)/\sigma_{\mathrm{m}}}{1 + (\sigma_{xx}(\omega, q)/\sigma_{\mathrm{m}})^2}$$

SAW measures conductivity of the 2DES at the wavelength and frequency of the SAW

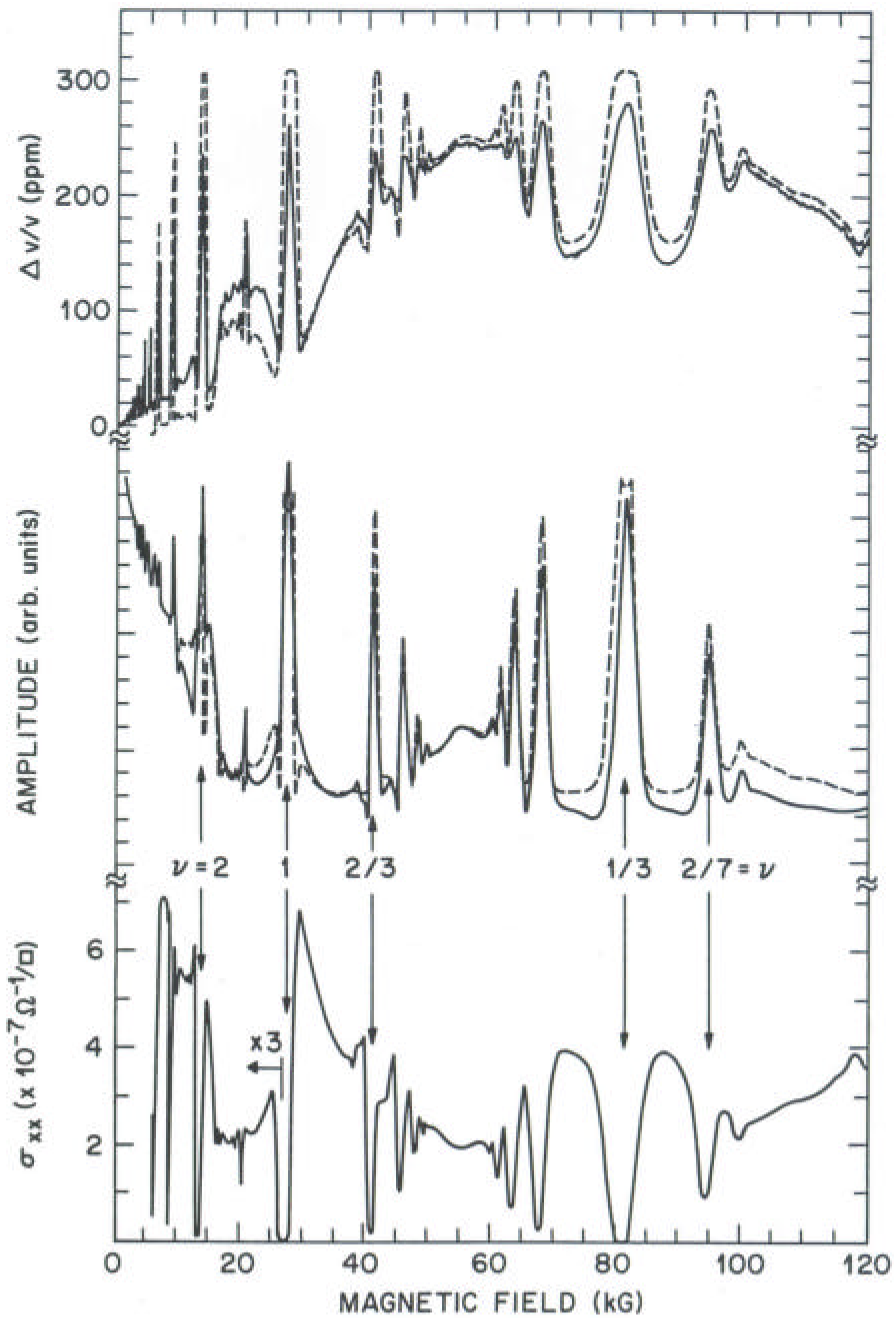
? xx(q,?) <=> ? v/v

SAW ? down to 0.2? m

II. Composite particles – composite fermions

C. Composite fermions and experiments

Results:
At low frequencies, large wavelengths see all the features of a standard transport measurement

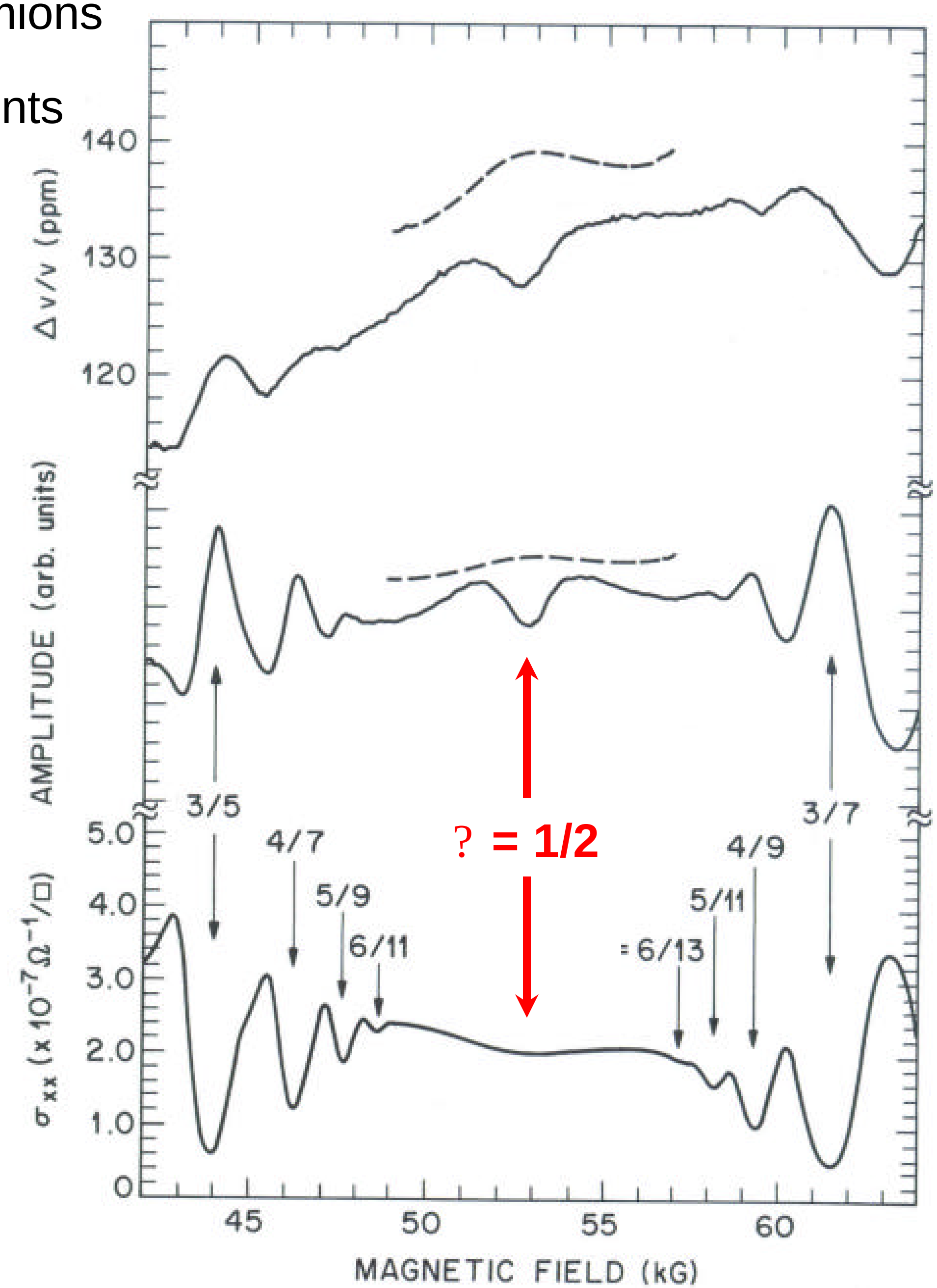


SAW measures conductivity of the 2DES at the wavelength and frequency of the SAW

? $_{xx}$(q,?) <=> ? v/v

II. Composite particles – composite fermions

C. Composite fermions and experiments



Results:

At higher frequencies, smaller wavelengths (<3?m),

see new features at ½ filling factor

Marks a new quantum number

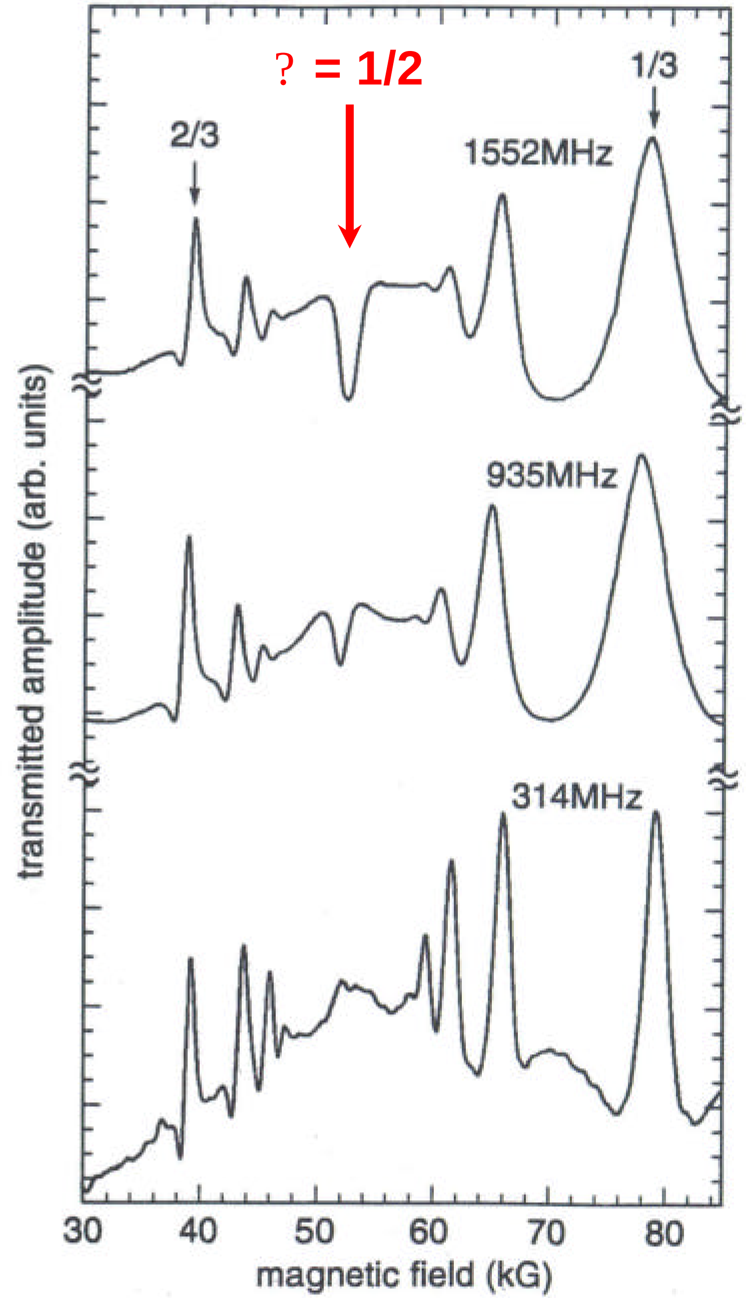
II. Composite particles – composite fermions

C. Composite fermions and experiments

Results:

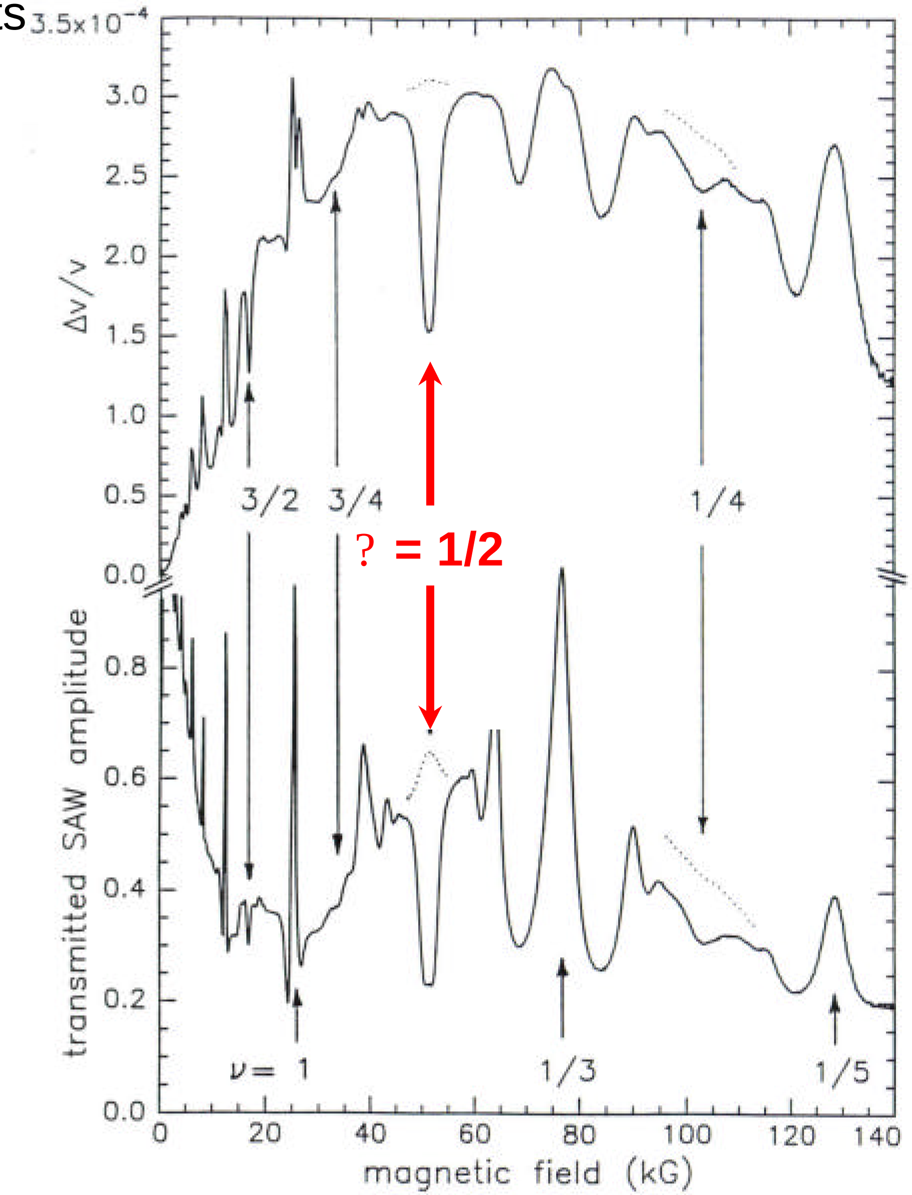
The smaller the wavelength, the larger the feature at 1/2

Feature corresponds to enhanced conductivity



II. Composite particles – composite fermions

C. Composite fermions and experiments

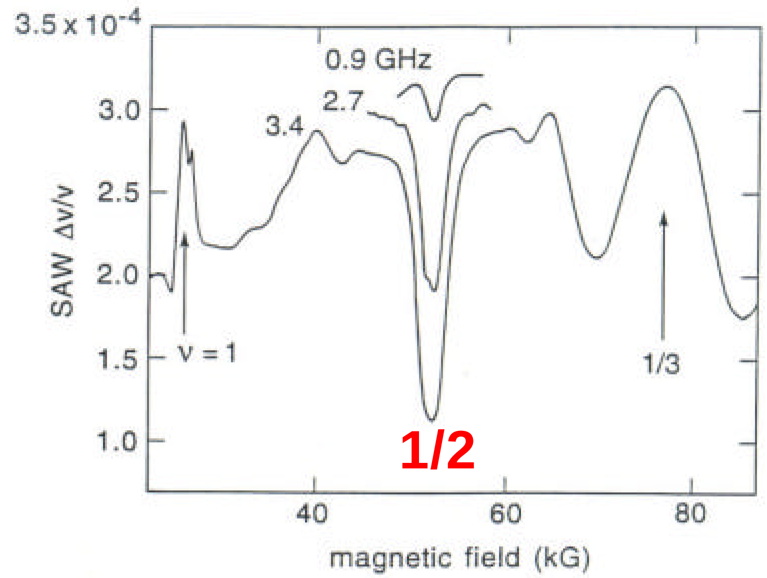


Results:

At 3GHz, 1? m ?, dominant feature is enhanced conductivity at ½

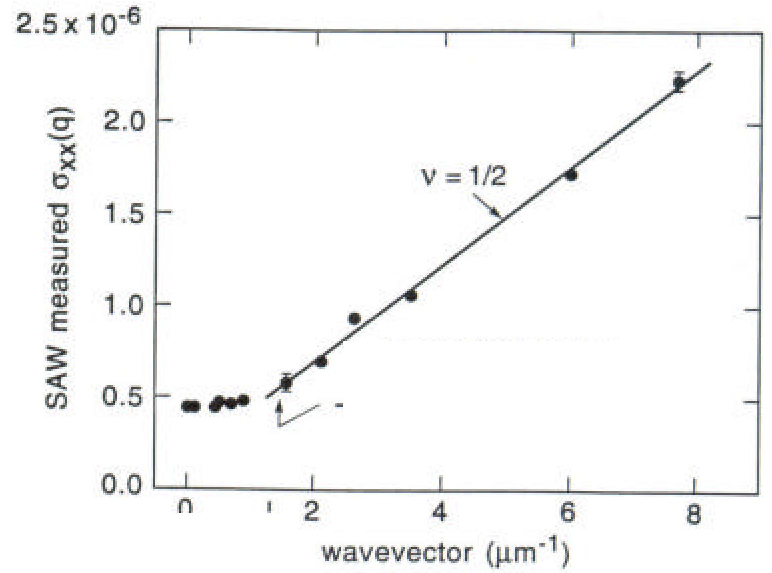
II. Composite particles – composite fermions

C. Composite fermions and experiments



For higher frequencies, smaller wavelengths, the enhanced conductivity grows

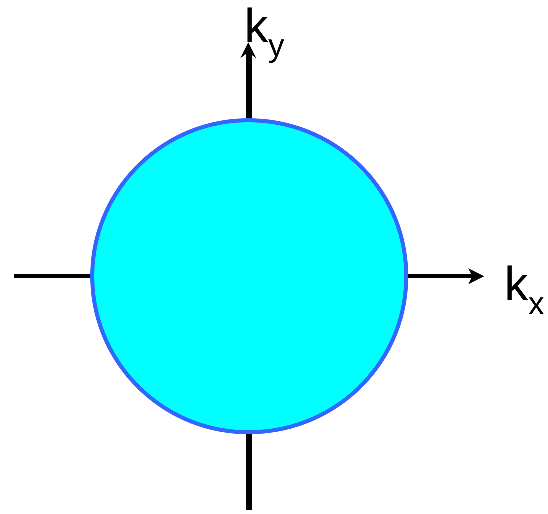
What is causing this?



II. Composite particles – composite fermions

D. Fermi surface picture: composite fermions form Fermi surface at filling factor 1/2

at ? = 1/2



$\mathbf{k_F = (2)^{1/2}\, k_{F\ electrons}}$

Halperin, Lee, and Read (1993):

Quasiparticle composite fermions produce not only FQHE by filling Landau levels, they also form true filled Fermi sea at filling factor ½

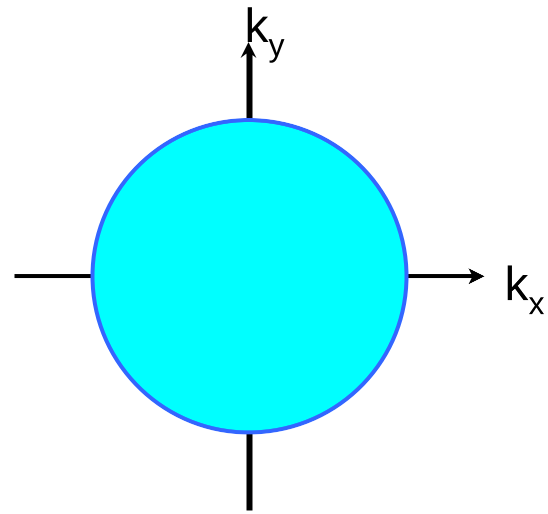
Near ½, quasiparticles move in effective magnetic field

$$\mathbf{B_{effective} = B_{applied} - B\,(1/2)}$$

II. Composite particles – composite fermions

D. Fermi surface picture: composite fermions form Fermi surface at filling factor 1/2

at ? = 1/2



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Away from ½ the quasiparticles move in cyclotron orbits with radius

$$\mathbf{R_c = h\, k_F / 2?\, e\, B_{effective}}$$

II. Composite particles – composite fermions

D. Fermi surface picture: composite fermions form Fermi surface at filling factor 1/2

Halperin, Lee, and Read (1993):

Quasiparticle composite fermions produce not only FQHE by filling Landau levels, they also form true filled Fermi sea at filling factor ½

Near ½, quasiparticles move in effective magnetic field

$\mathbf{B_{effective} = B_{applied} - B\ (1/2)}$

$H = K + V$

$K = \frac{1}{2m^*}\int d^2r\, \psi_e^+ (-i\bar{\nabla} + e\bar{A})^2 \psi_e$

transformation to quasiparticle operators

$\psi^+(\bar{r}) \equiv \psi_e^+(\bar{r}) \exp\left(2\pi i \tilde{\varphi} \int d^2r' \arg(r - r')\rho(\bar{r}')\right)$

$\rho(\bar{r}) = \psi_e^+(\bar{r})\psi_e(\bar{r})$

$= \psi^+(\bar{r})\,\psi(\bar{r})$

local electron density

arg: angle $\bar{r} - \bar{r}'$ to x

⇓

$K = \frac{1}{2m^*}\int d^2\bar{r}\, \psi^+(\bar{r}) \left[-i\bar{\nabla} + e\bar{A} - \bar{a}(\bar{r})\right]^2 \psi(\bar{r})$

where $\bar{a}(\bar{r}) \equiv \tilde{\varphi} \int d^2\bar{r}\, \frac{\hat{z} \times (\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^2}\rho(\bar{r}')$

Chern-Simons gauge field

attaches $\tilde{\varphi}$ flux quanta to each electron

⇓

composite particles are produced that reside in fictitious magnetic field $\bar{b}(\bar{r})$

$\bar{b}(\bar{r}) \equiv \bar{\nabla} \times a(\bar{r}) = 2\pi\rho(\bar{r})\tilde{\varphi}$

for $\tilde{\varphi} = 2$, 2 flux quanta on each quasiparticle

quasiparticle in effective magnetic field for φ even

$\Delta B = B - 2\pi\tilde{\varphi}\, n_e$

II. Composite particles – composite fermions

D. Fermi surface picture: composite fermions form Fermi surface at filling factor 1/2

Wavevector dependence of conductivity derived in HLR

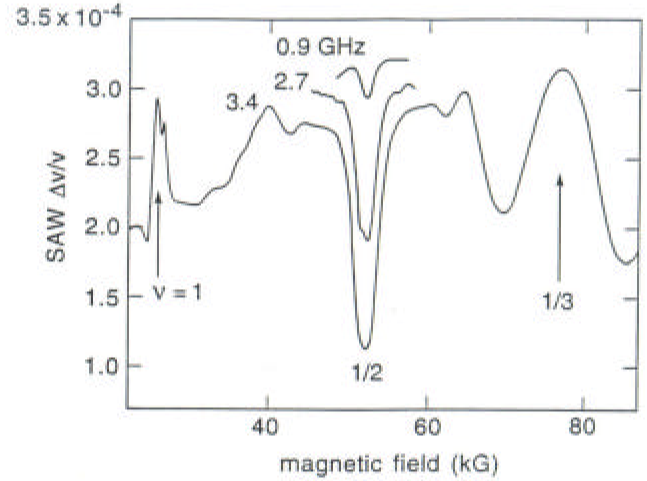
H.L.R.

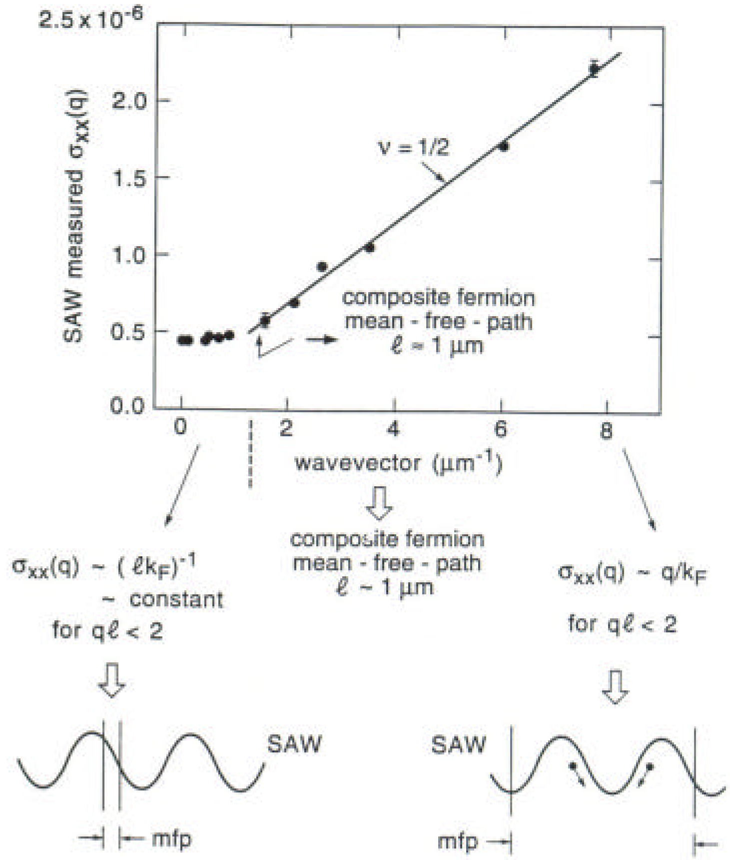
for $q\ell \gg 2$, $\sigma_{xx}(q) = \frac{e^2}{8\pi\hbar}\frac{q}{k_F}$

$q\ell < 2$, $\sigma_{xx}(q) = \frac{e^2}{4\pi\hbar}\frac{1}{k_F\ell}$

ℓ quasiparticle mean-free path

For SAW wavelength less than the composite fermion mean-free-path, enhanced conductivity observed

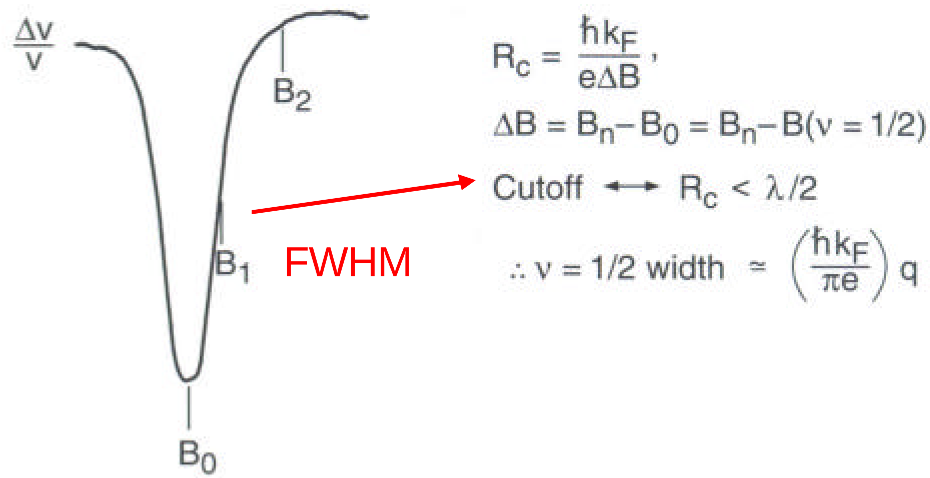


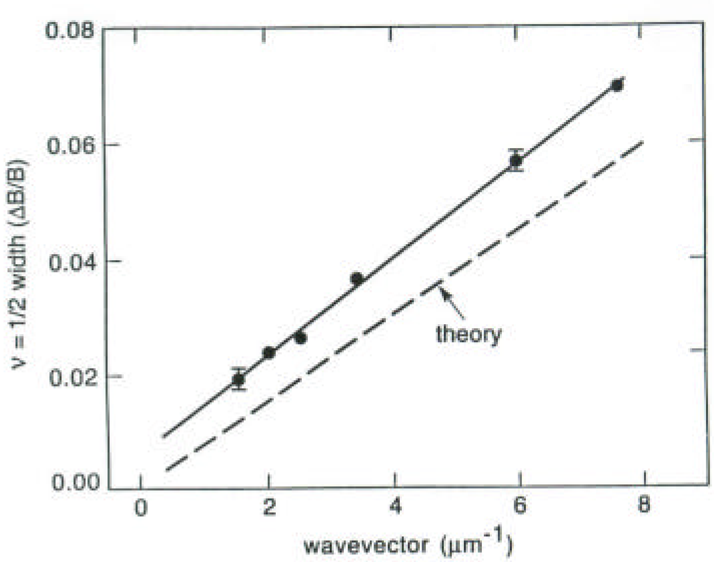


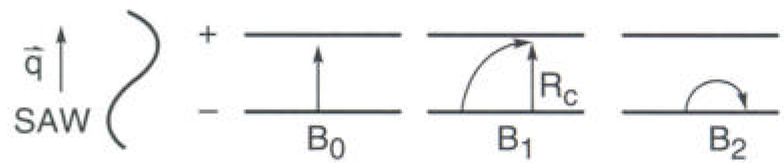
II. Composite particles – composite fermions

D. Fermi surface picture: composite fermions form Fermi surface at filling factor 1/2

Width of enhanced conductivity at ½ used to extract k_F of composite fermions

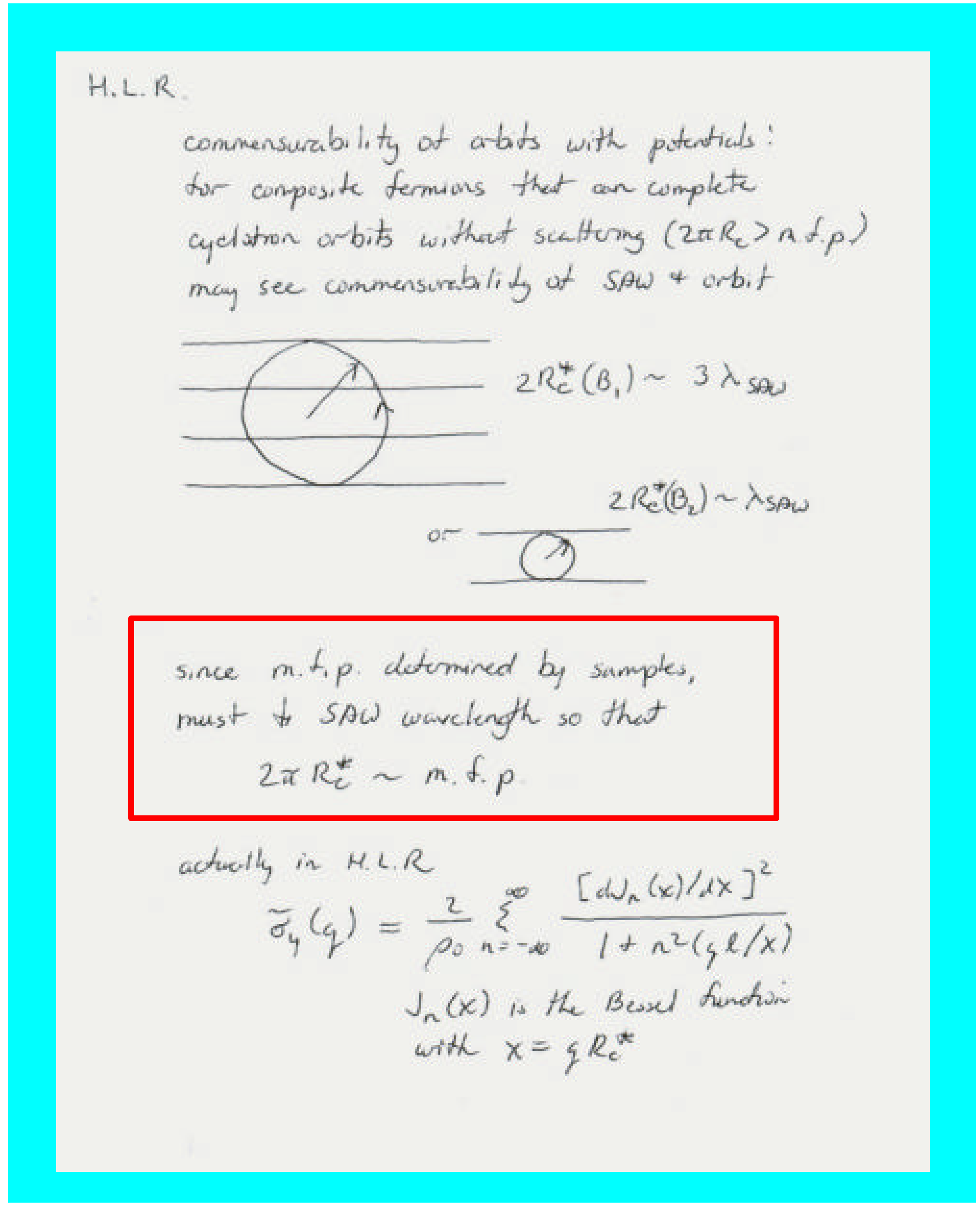






II. Composite particles – composite fermions

D. Fermi surface picture: composite fermions form Fermi surface at filling factor 1/2



H.L.R. :

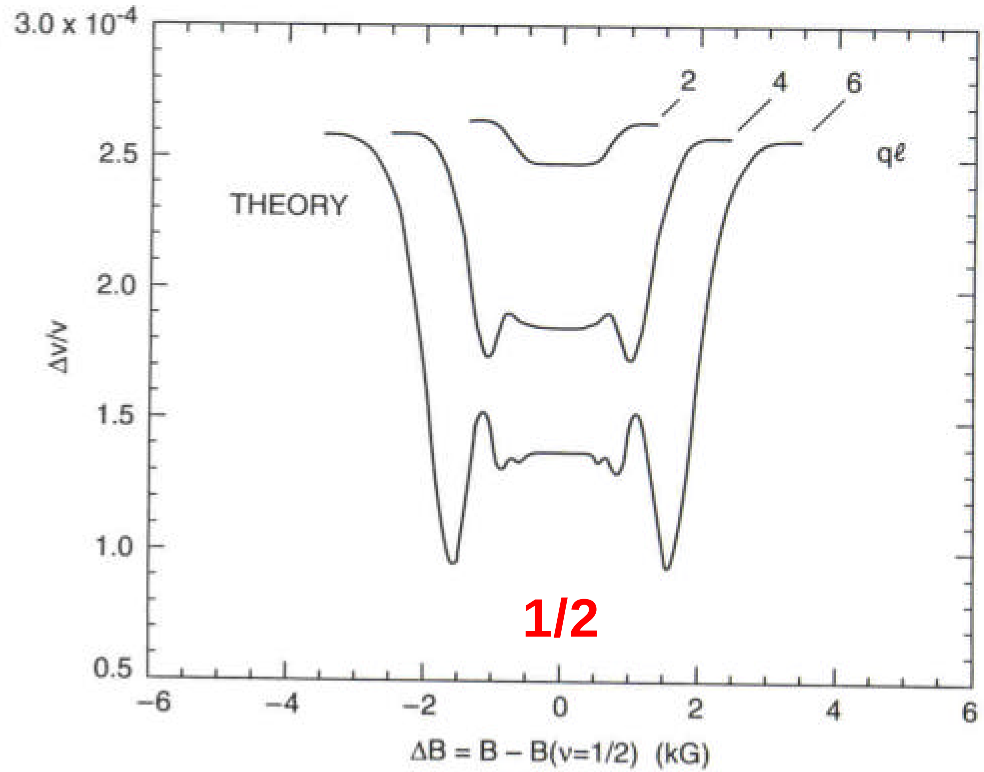
Commensurability of composite fermion cyclotron orbit and potential (SAW or lithographically defined) should be observed as has been done for electrons

Need composite fermion mean-free-path > 2?R_c

II. Composite particles – composite fermions

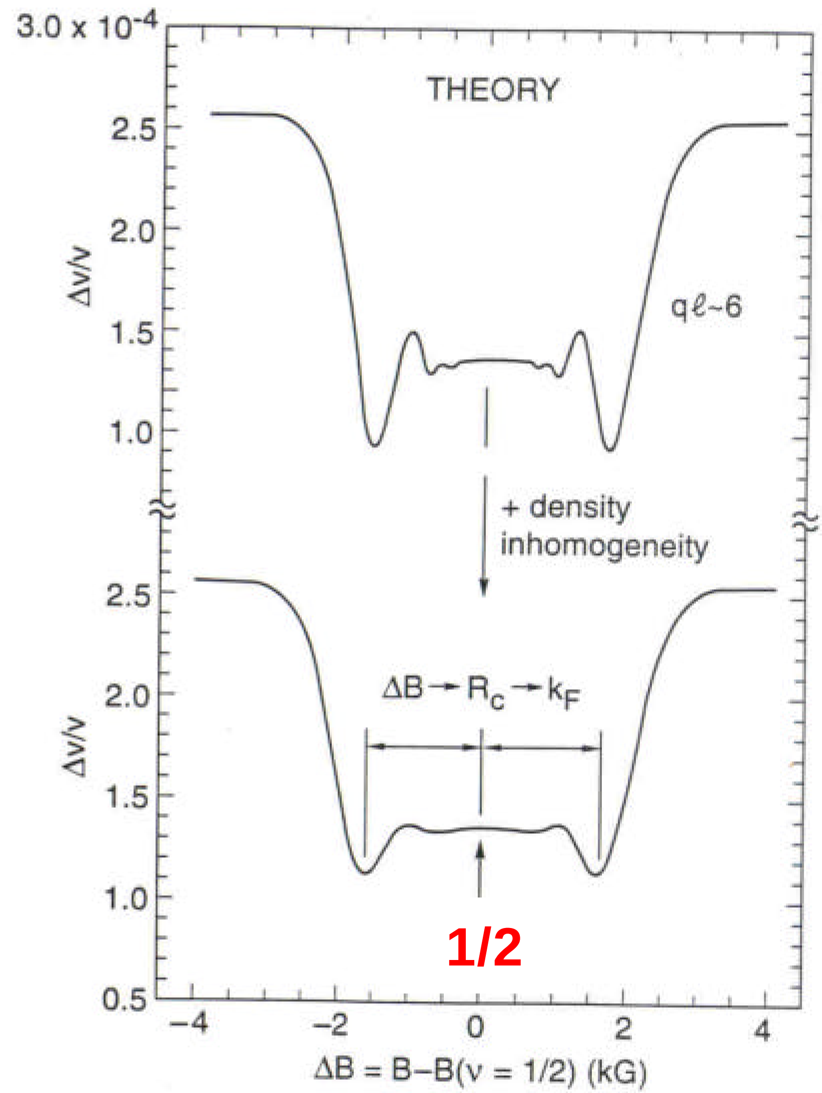
D. Fermi surface picture: composite fermions form Fermi surface at filling factor 1/2

H.L.R. :
Explicit predictions for SAW results if commensurability present –
Use large SAW q and large composite fermion m.f.p



II. Composite particles – composite fermions

D. Fermi surface picture: composite fermions form Fermi surface at filling factor 1/2

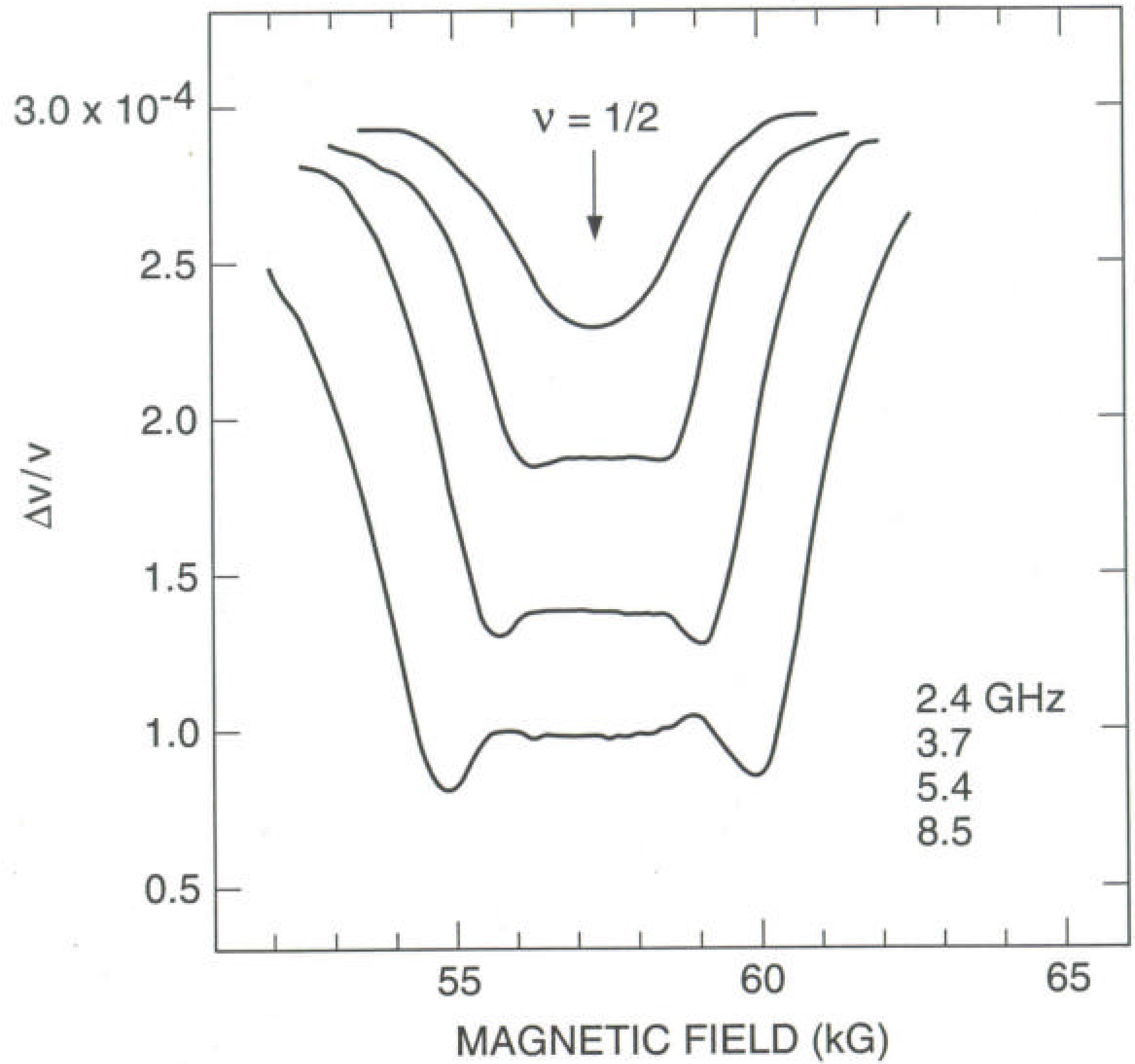


For large SAW q, large composite particle m.f.p. must also consider denstiy inhomogeneities

II. Composite particles – composite fermions

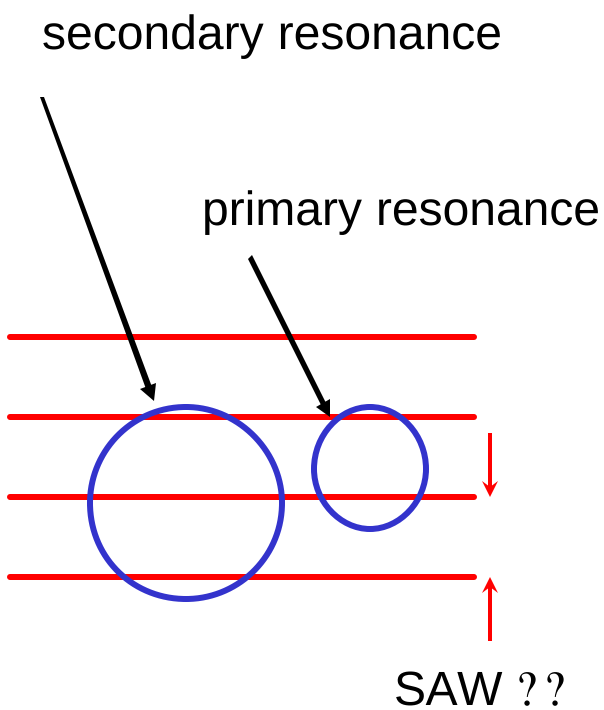
D. Fermi surface picture: composite fermions form Fermi surface at filling factor 1/2

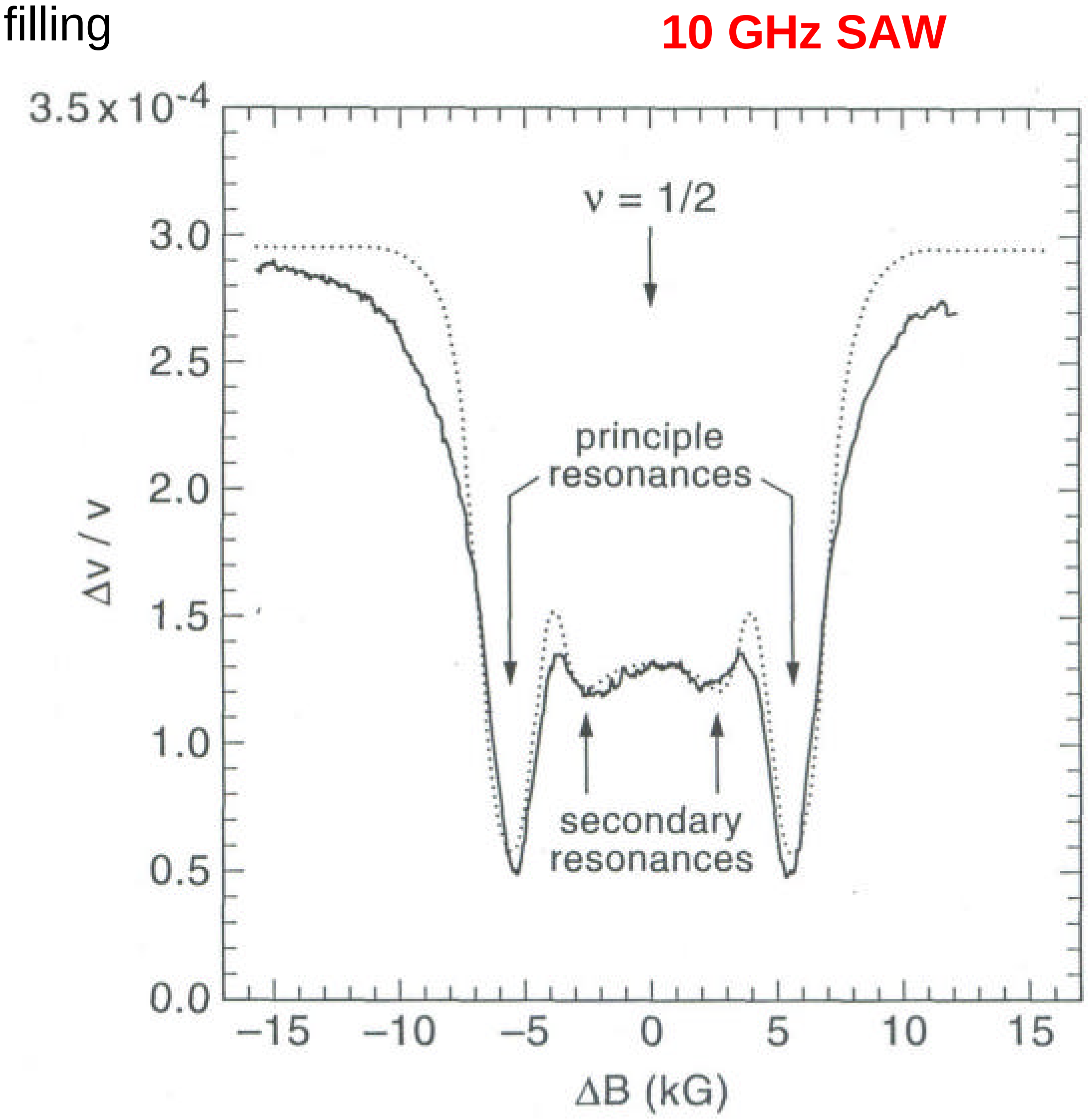
Commensurability experimentally observed



II. Composite particles – composite fermions

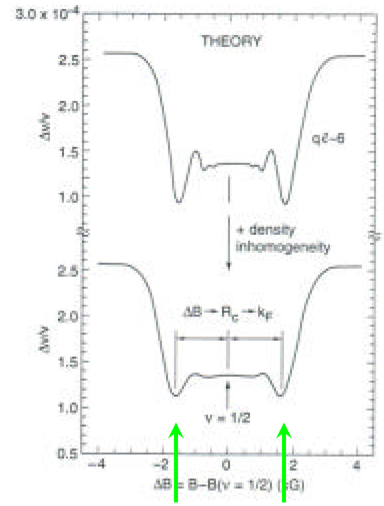
D. Fermi surface picture: composite fermions form Fermi surface at filling factor 1/2



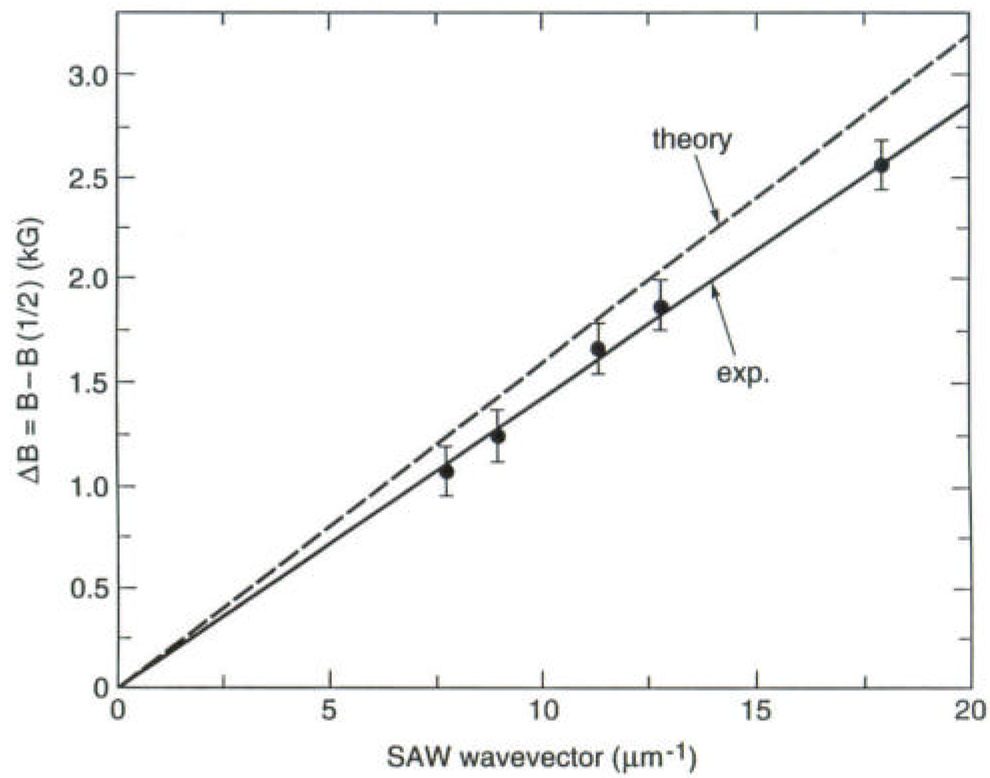


II. Composite particles – composite fermions

D. Fermi surface picture: composite fermions form Fermi surface at filling factor 1/2



Experimental magnetic field positions of resonances for different SAW wavevectors can measure k_F.

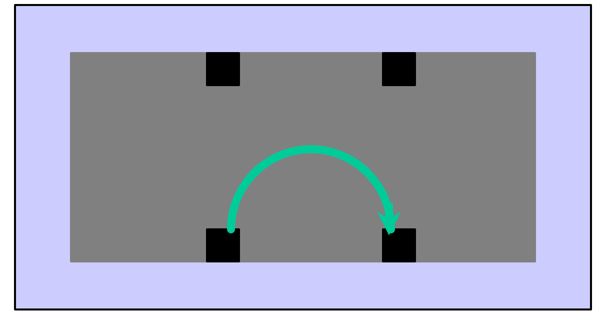


? $B \sim k_F\, q_{SAW}$

$k_F = (2)^{1/2}\, k_{F\ \mathrm{electrons}}$

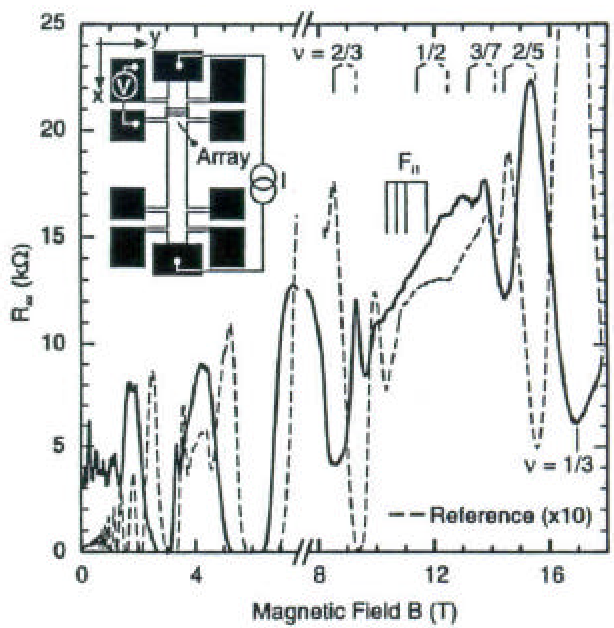
II. Composite particles – composite fermions

E. More experiments on Fermi surfaces: focusing

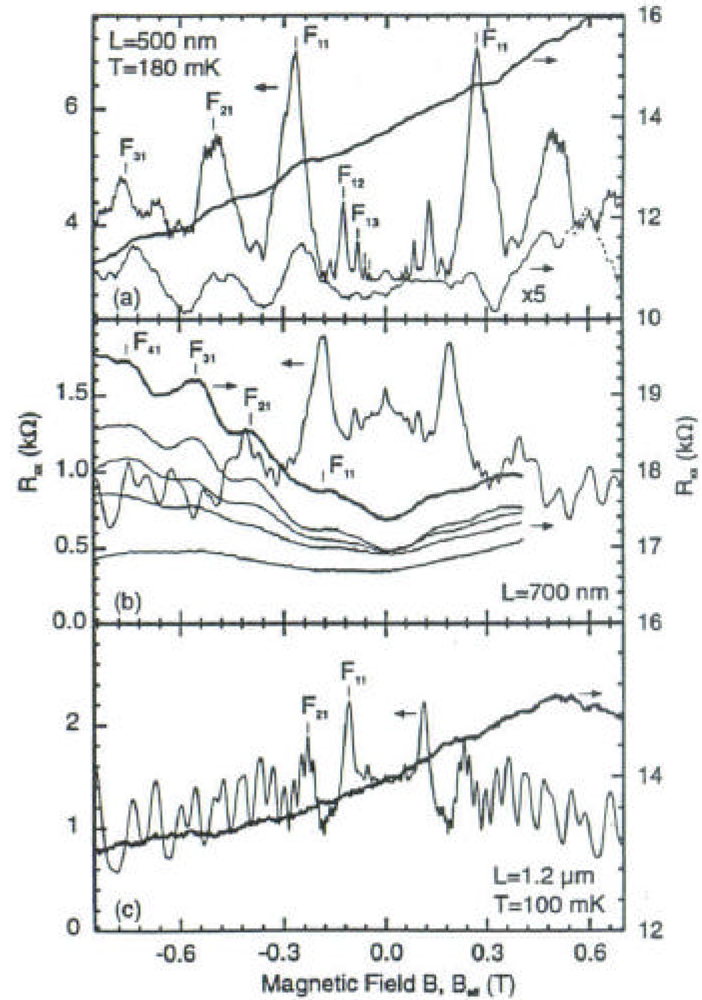


direct current from one contact to another with magnetic field applied perpendicular to layers

Use composite fermion cyclotron radius to focus into contacts

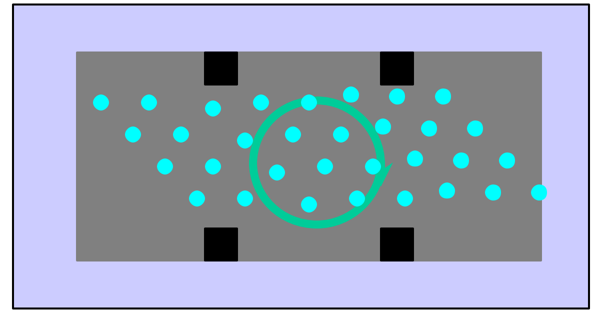


Smet: PRL 94

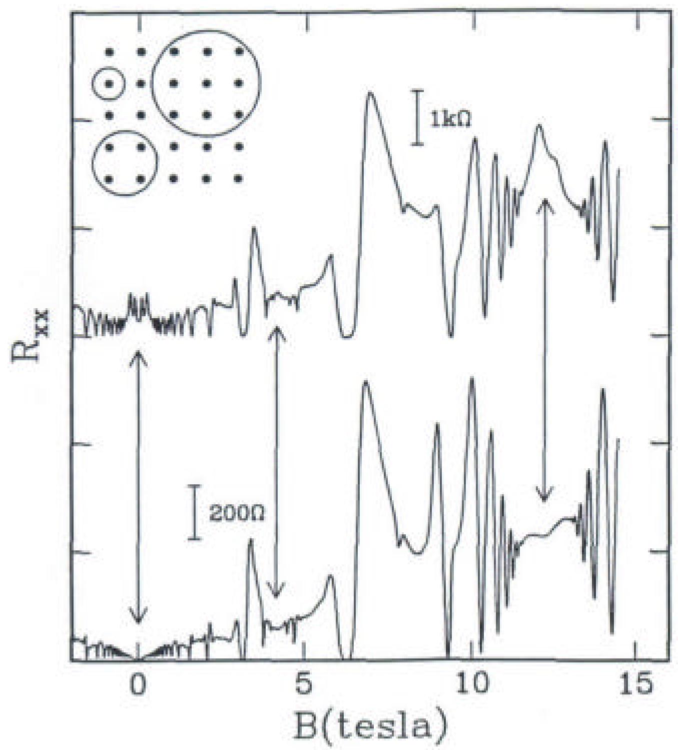


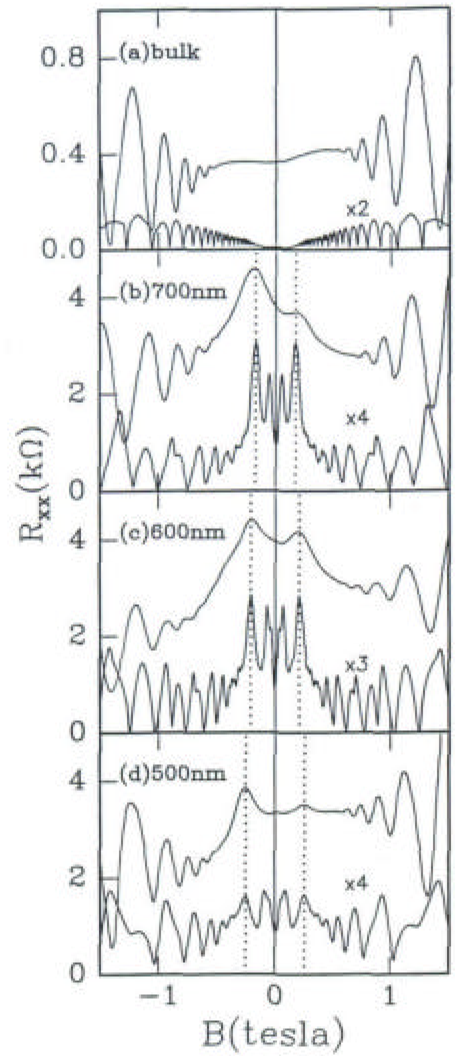
II. Composite particles – composite fermions

E. More experiments on Fermi surfaces: antidots



Encircle holes made in 2D gas with cyclotron orbits and resistance increases as with electrons

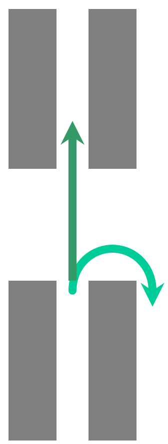


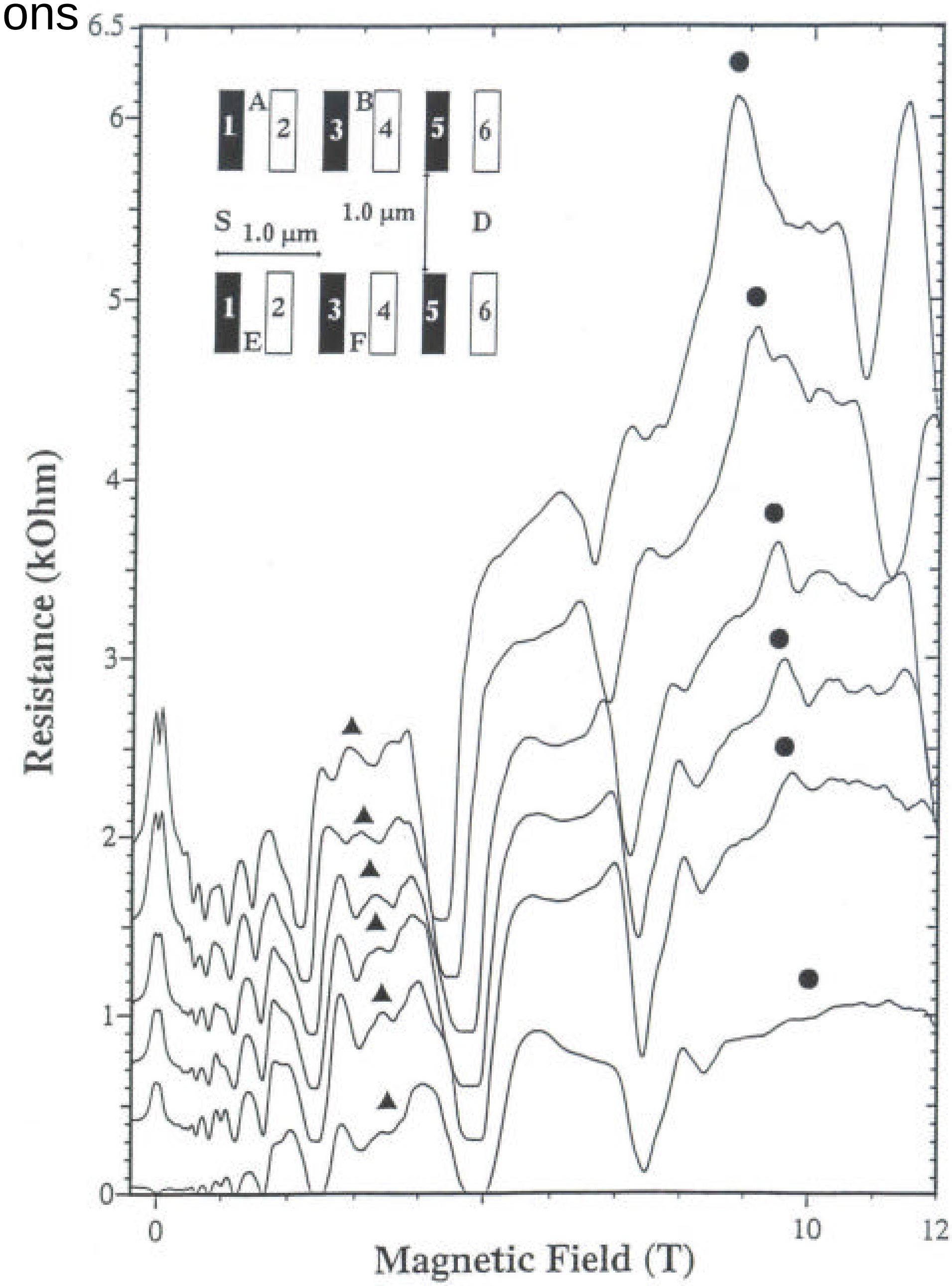


Kang, et al. PRL 93

II. Composite particles – composite fermions

E. More experiments on Fermi surfaces: ballistic shorting





As nanostructure channel is defined, the $B_{effective}$ ballistic charge transport is enhanced for both electrons and composite fermions

II. Composite particles – composite fermions

F. Composite fermion effective mass

H.L.R.:
Cyclotron energy gaps of the composite fermions use the effective mass of the quasiparticle

$$E_g(\nu) = \hbar\omega_c^* = \frac{\hbar e B_{\text{effective}}}{m}$$

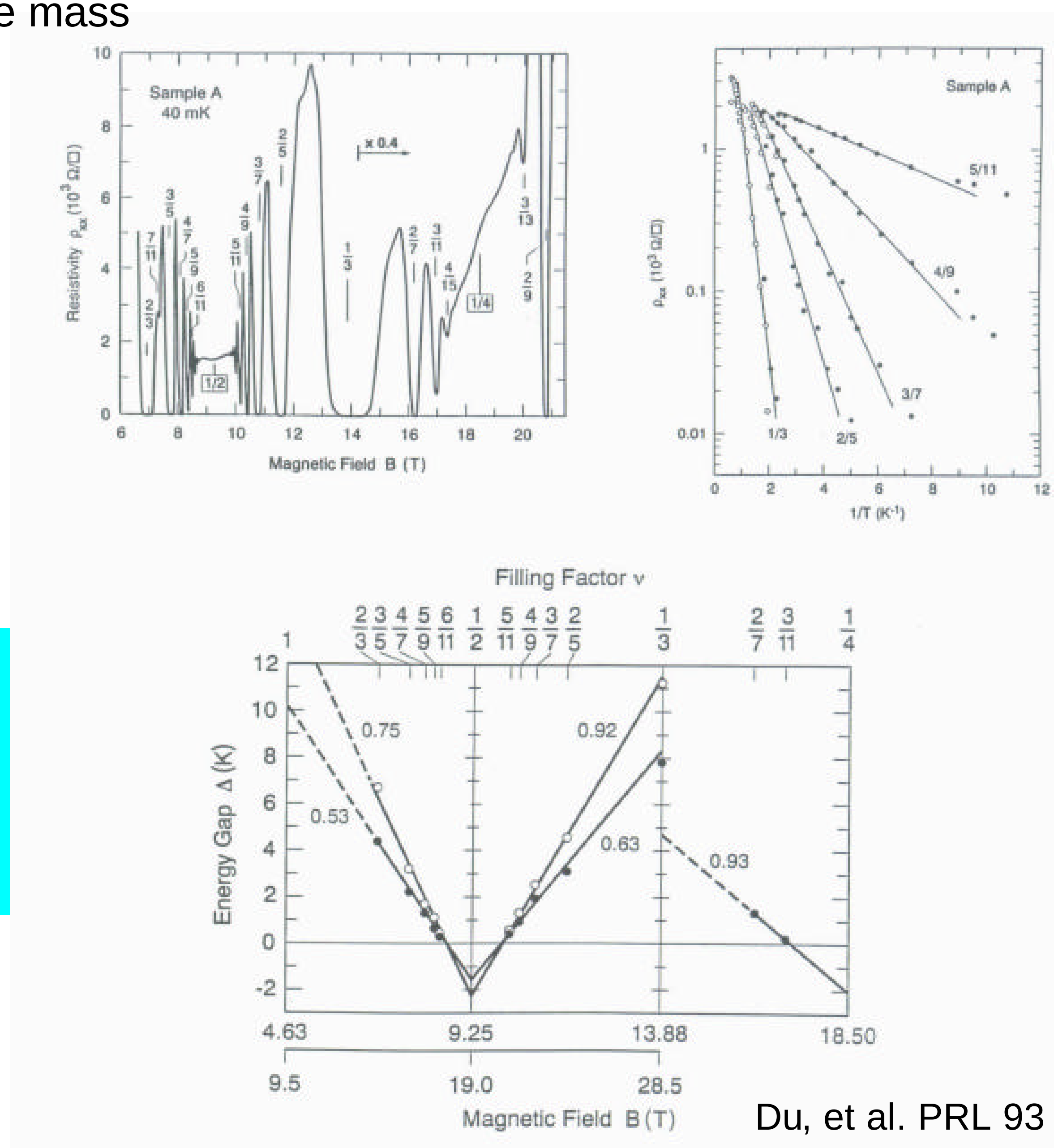
II. Composite particles – composite fermions

F. Composite fermion effective mass

in transport measurements the activation energies of the series of fractions (4/9, 3/7, 2/5, 1/3) corresponding to composite fermion Landau levels 4,3,2,1 indeed increase linearly with $B_{effective}$.

The effective mass derived from this is ~ $0.8m_e$, almost a factor of 10 larger than the free GaAs mass.

A mass divergence toward ½ is expected

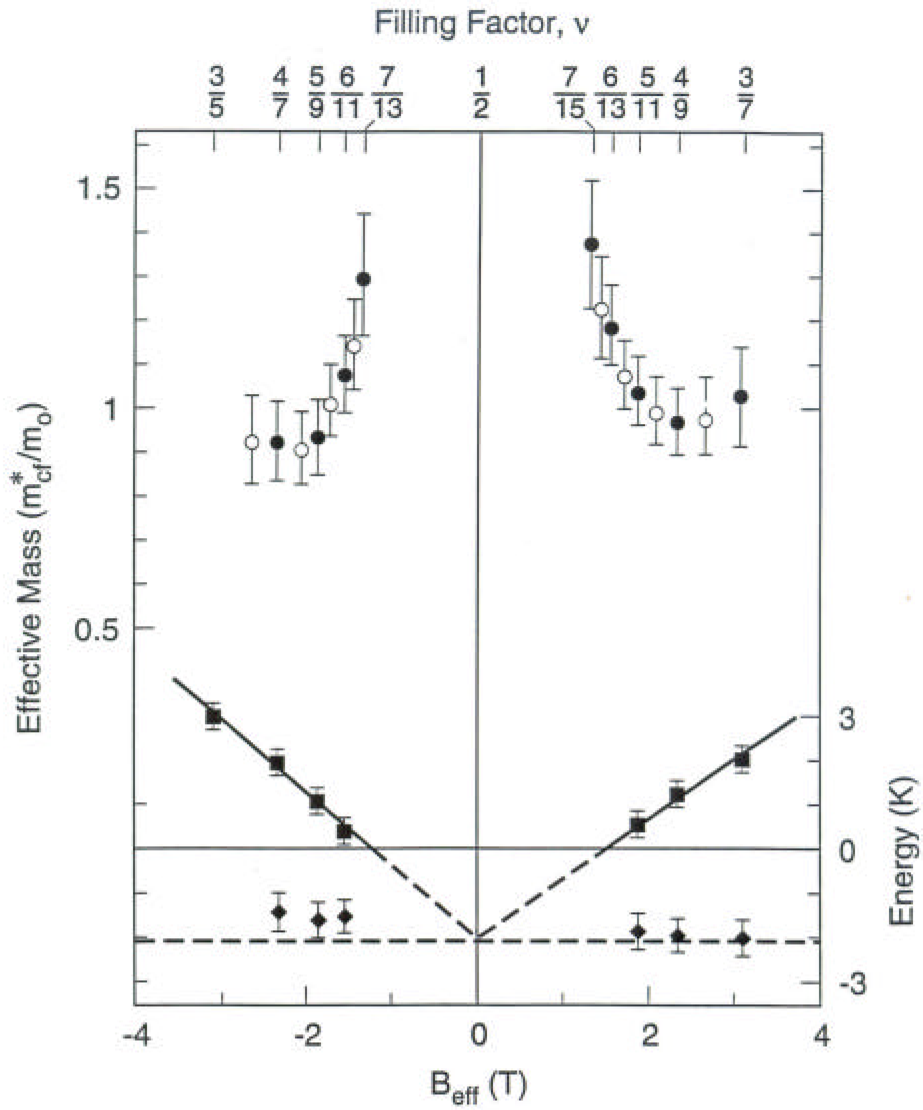


Du, et al. PRL 93

II. Composite particles – composite fermions

F. Composite fermion effective mass

Further transport
measurements support this



Du, et al. PRL 95

II. Composite particles – composite fermions

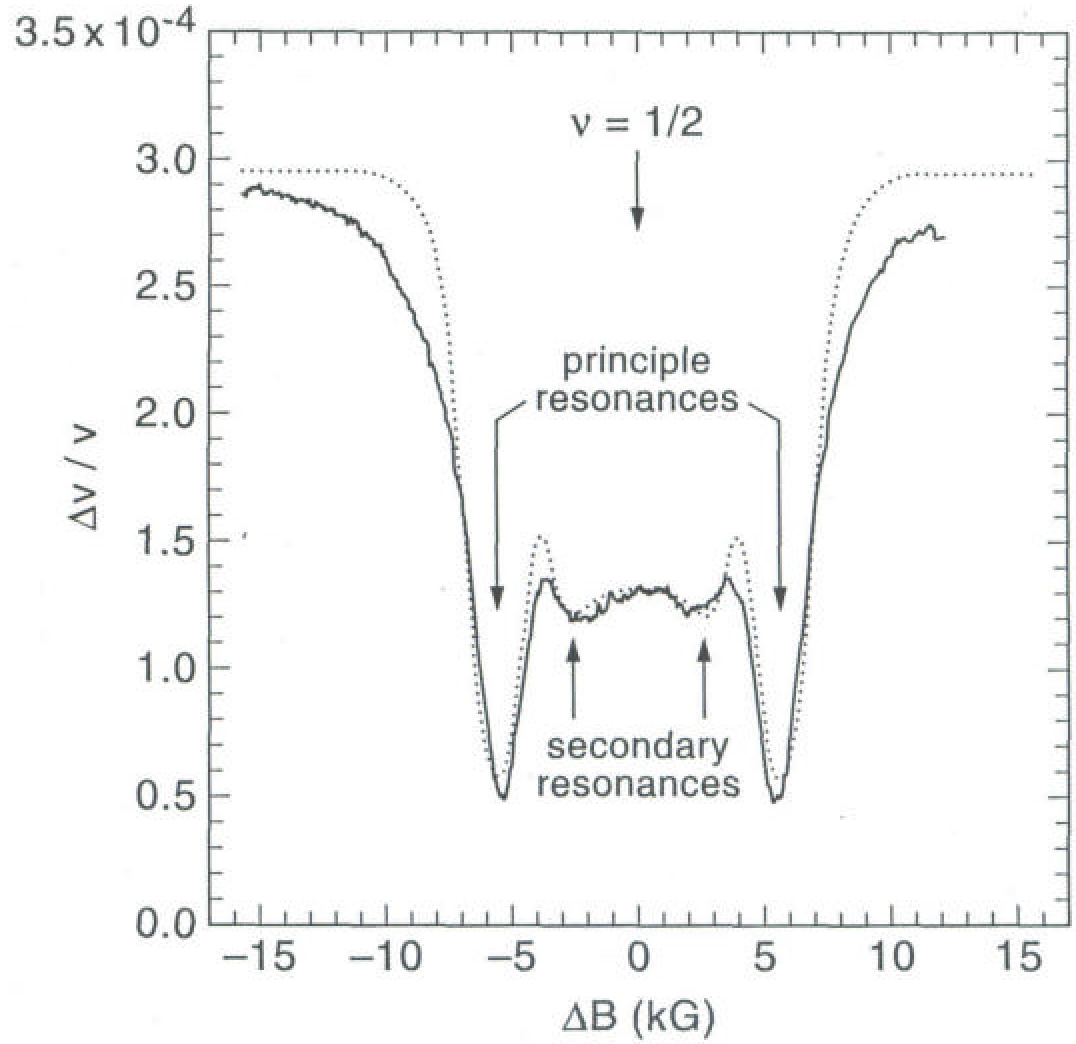
F. Composite fermion effective mass

This effective mass picture is compared with the results from SAW measurements:

The quasiparticle cyclotron orbit frequency must be greater than the SAW frequency to observe resonances

As effective mass increases, the cyclotron frequency will drop

10 GHz SAW

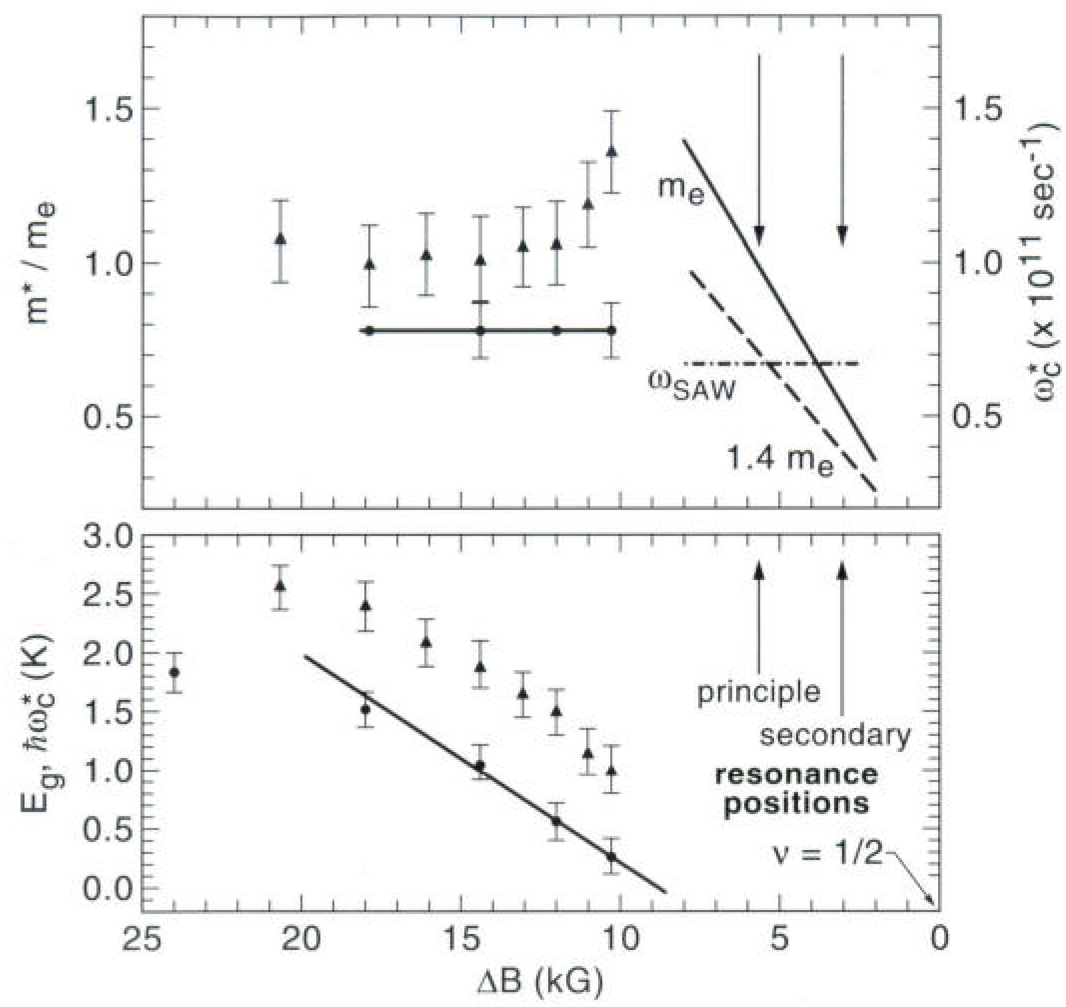


II. Composite particles – composite fermions

F. Composite fermion effective mass

The observation of SAW resonances is not consistent with a diverging composite fermion effective mass

See theory of effective mass, Simon et al 96.



10 GHz SAW

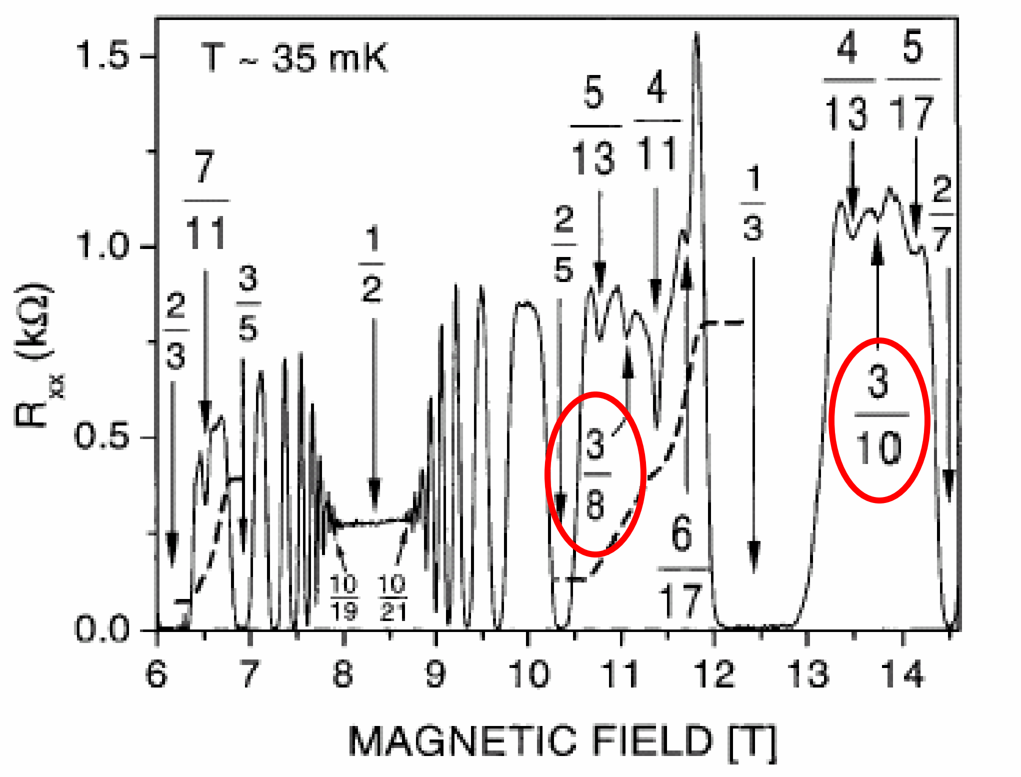
II. Composite particles – composite fermions

G. Other composite fermions

Composite fermions and their Fermi surfaces expected at other even denominator filling factors (1/4, 3/4, 3/2, 3/8, …)

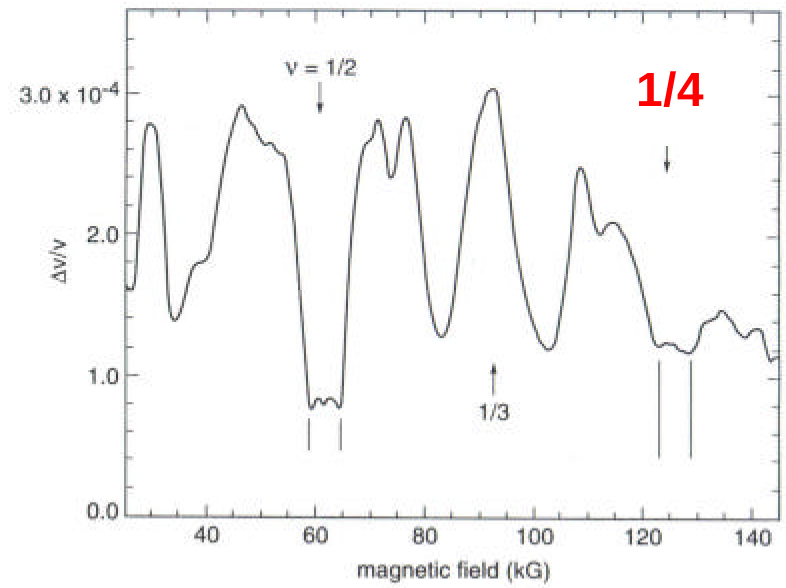
Features observed in transport

Resonances in SAW observed, positions in $B_{effective}$ must be adjusted for the active composite particle number.



II. Composite particles – composite fermions

G. Other composite fermions



Composite fermions and their Fermi surfaces expected at other even denominator filling factors (1/4, 3/4, 3/2, 3/8, …)

Presumably observed in transport

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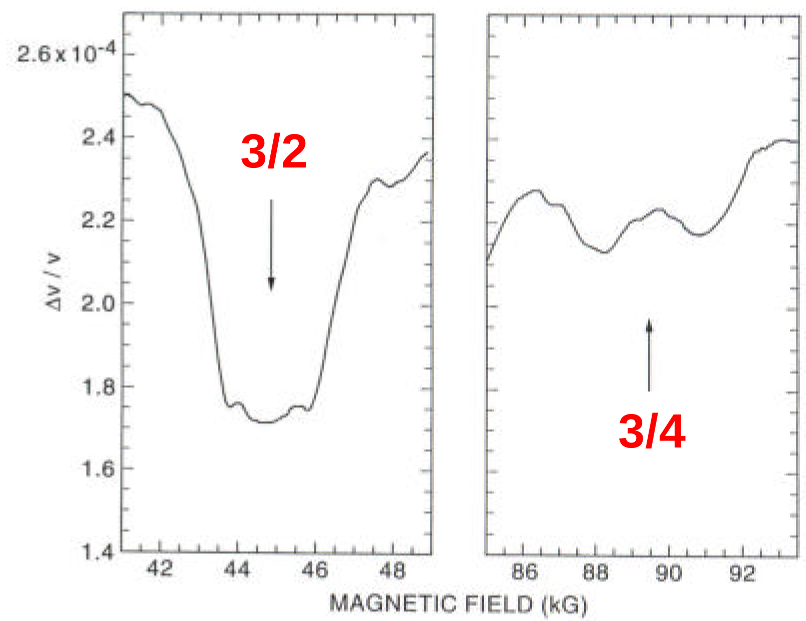
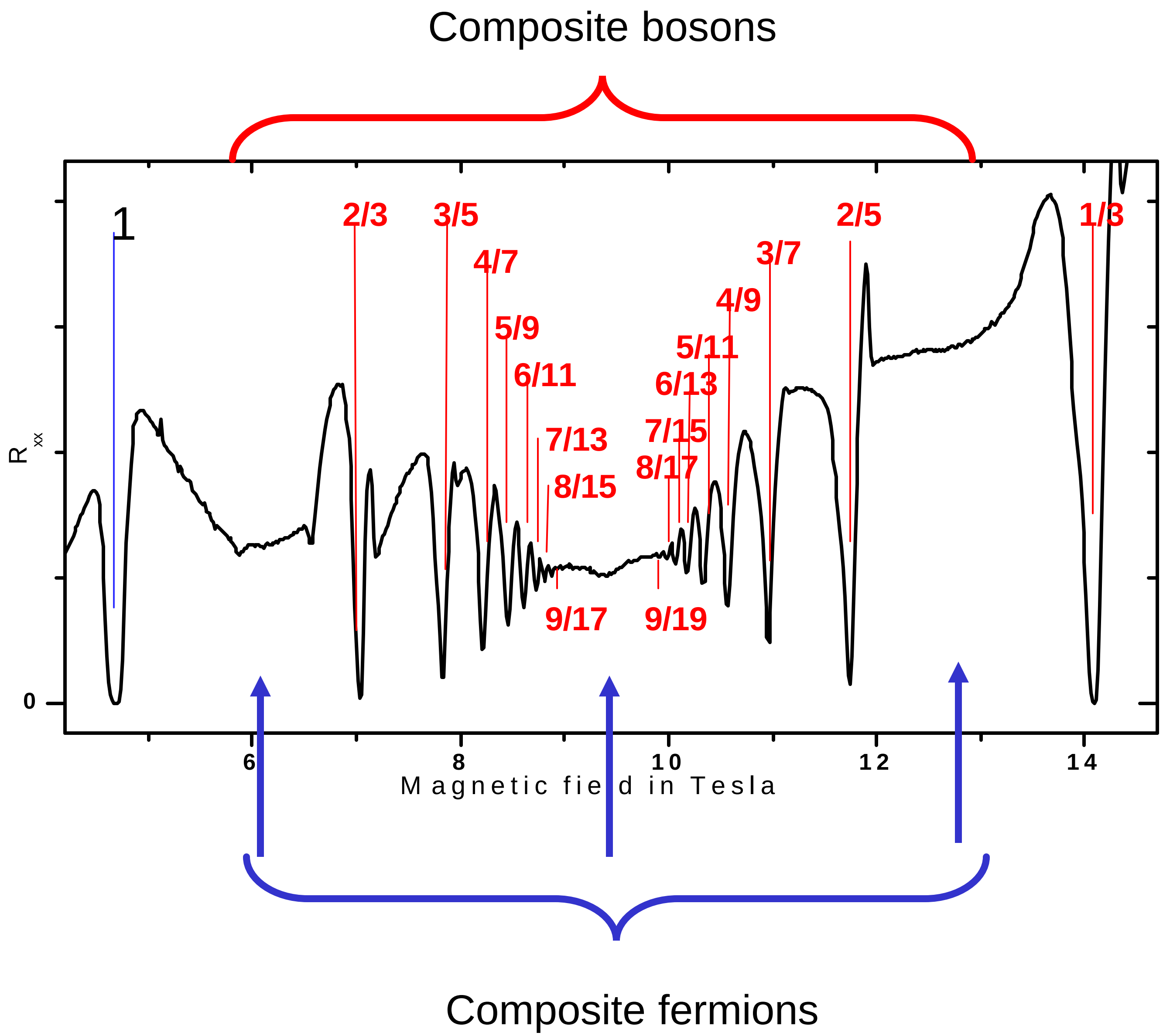


G. Statistical transformations and vortex picture:



Summary:

- 2D electron samples show 2D physics: shubnikov-deHaas oscillations

- integer quantum Hall effect: resolved Landau levels with localization between centers of Landau levels

- low disorder 2D electron systems show fractional quantum Hall effect – correlations of electrons as described by the Laughlin wave function

- composite fermions explain series of fractional quantum Hall states

- statistical transformations an important part of the magnetic field spectrum in 2D