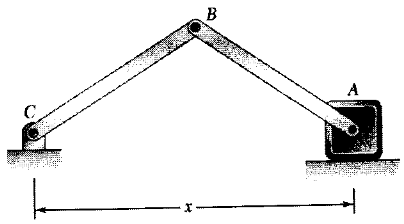
PROBLEM 11.27



The acceleration of slider A is defined by the relation $a = -2k\sqrt{k^2 - v^2}$, where a and v are expressed in ft/s^2 and ft/s, respectively, and k is a constant. The system starts at time $t = 0$ with $x = 1.5$ ft and $v = 0$. Knowing that $x = 1.2$ ft when $t = 0.2$ s, determine the value of k.

SOLUTION

$$a = \frac{dv}{dt} = -2k\sqrt{k^2 - v^2}, \qquad \text{and} \qquad v = 0 \text{ when } t = 0$$

$$-2k\,dt = \frac{dv}{\sqrt{k^2 - v^2}} = d\left[\sin^{-1}\left(\frac{v}{k}\right)\right]$$

$$-2k\int_0^t dt = \left[\sin^{-1}\left(\frac{v}{k}\right)\right]_0^v$$

$$\sin^{-1}\left(\frac{v}{k}\right) = -2kt$$

$$v = k\sin(-2kt) = -k\sin(2kt)$$

$$dx = v\,dt = -k\sin(2kt)\,dt$$

Integrating, using $x = 1.5$ ft at $t = 0$, and $x = 1.2$ ft at $t = 0.2$ s,

$$\int_{1.5}^{1.2} dx = -\int_0^{0.2}\left[k\sin(2kt)\right]dt$$

$$x\Big|_{1.5}^{1.2} = \frac{1}{2}\cos(2kt)\Big|_0^t \qquad 1.2 - 1.5 = \frac{1}{2}\cos[(2)k(0.2)] - \frac{1}{2}$$

$$\cos(0.4k) = 0.4$$

$$0.4k = \cos^{-1}(0.4) = 1.1593 \text{ rad}$$

$$k = \frac{1.1593}{0.4} \qquad\qquad k = 2.90 \text{ ft/s}$$

SOLUTION

x as a function of v.

$$\frac{v}{154} = \sqrt{1 - e^{-0.00057x}}$$

$$e^{-0.00057x} = 1 - \left(\frac{v}{154}\right)^2$$

$$-0.00057x = \ln\left[1 - \left(\frac{v}{154}\right)^2\right]$$

$$x = -1754.4 \ln\left[1 - \left(\frac{v}{154}\right)^2\right] \qquad (1)$$

a as a function of x.

$$v^2 = 23716\left(1 - e^{-0.00057}\right)$$

$$a = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{v^2}{2}\right) = (11858)(0.00057)e^{-0.0005x}$$

$$a = 6.75906\, e^{-0.00057x} = 6.75906\left[1 - \left(\frac{v}{154}\right)^2\right] \qquad (2)$$

(*a*) $v = 20$ m/s.

From (1), $x = 29.843$ $\qquad x = 29.8 \text{ m}$ ◀

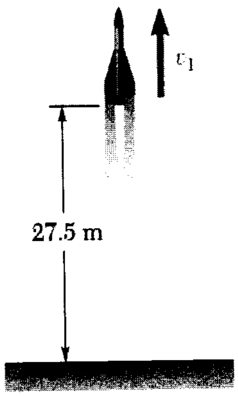
From (2), $a = 6.64506$ $\qquad a = 6.65 \text{ m/s}^2$ ◀

(*b*) $v = 40$ m/s.

From (1), $x = 122.54$ $\qquad x = 122.5 \text{ m}$ ◀

From (2), $a = 6.30306$ $\qquad a = 6.30 \text{ m/s}^2$ ◀

PROBLEM 11.40



A group of students launches a model rocket in the vertical direction. Based on tracking data, they determine that the altitude of the rocket was 27.5 m at the end of the powered portion of the flight and that the rocket landed 16 s later. Knowing that the descent parachute failed to deploy so that the rocket fell freely to the ground after reaching its maximum altitude and assuming that $g = 9.81\text{ m/s}^2$, determine (*a*) the speed v_1 of the rocket at the end of powered flight, (*b*) the maximum altitude reached by the rocket.

SOLUTION

Constant acceleration. Choose $t = 0$ at end of powered flight.

Then, $y_1 = 27.5\text{ m} \qquad a = -g = -9.81\text{ m/s}^2$

(*a*) When y reaches the ground, $y_f = 0$ and $t = 16\text{ s}$.

$$y_f = y_1 + v_1 t + \frac{1}{2}at^2 = y_1 + v_1 t - \frac{1}{2}gt^2$$

$$v_1 = \frac{y_f - y_1 + \frac{1}{2}gt^2}{t} = \frac{0 - 27.5 + \frac{1}{2}(9.81)(16)^2}{16} = 76.76\text{ m/s}$$

$v_1 = 76.8\text{ m/s}$ ◀

(*b*) When the rocket reaches its maximum altitude y_{max},

$$v = 0$$

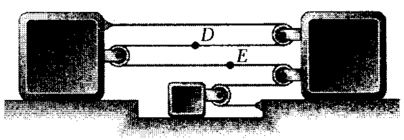
$$v^2 = v_1^2 + 2a(y - y_1) = v_1^2 - 2g(y - y_1)$$

$$y = y_1 - \frac{v^2 - v_1^2}{2g}$$

$$y_{max} = 27.5 - \frac{0 - (76.76)^2}{(2)(9.81)}$$

$y_{max} = 328\text{ m}$ ◀

PROBLEM 11.57



Slider block B moves to the left with a constant velocity of 50 mm/s. At $t = 0$, slider block A is moving to the right with a constant acceleration and a velocity of 100 mm/s. Knowing that at $t = 2$ s slider block C has moved 40 mm to the right, determine (*a*) the velocity of slider block C at $t = 0$, (*b*) the velocity of portion D of the cable at $t = 0$, (*c*) the accelerations of A and C.

SOLUTION

Let x be position relative to the anchor, positive to the right.

Constraint of cable: $$-x_B + (x_C - x_B) + 3(x_C - x_A) = \text{constant}$$

$$4v_C - 2v_B - 3v_A = 0 \qquad 4a_C - 2a_B - 3a_A = 0 \qquad (1, 2)$$

When $t = 0$, $\quad v_B = -50$ mm/s and $(v_a)_0 = 100$ mm/s

(*a*) $$(v_C)_0 = \frac{1}{4}\left[2v_B + 3(v_A)_0\right] = \frac{1}{4}\left[(2)(-50) + (3)(100)\right]$$

$(v_C)_0 = 50$ mm/s $\longrightarrow$ ◀

Constraint of point D: $$(x_D - x_A) + (x_C - x_A) + (x_C - x_B) - x_B = \text{constant}$$

$$v_D + 2v_C - 2v_A - 2v_B = 0$$

(*b*) $$(v_D)_0 = 2(v_A)_0 + 2v_B - 2(v_C)_0 = (2)(100) + (2)(-50) - (2)(50)$$

$(v_D)_0 = 0$ ◀

$$x_C - (x_C)_0 = (v_C)_0 t + \frac{1}{2}a_C t^2$$

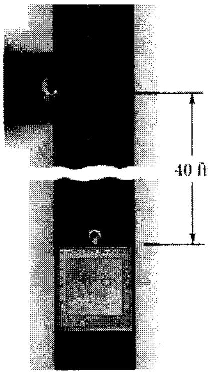
(*c*) $$a_C = \frac{2\left[x_C - (x_C)_0 - (v_C)_0 t\right]}{t^2} = \frac{2\left[40 - (50)(2)\right]}{(2)^2} = -30 \text{ mm/s}^2$$

$a_C = 30$ mm/s^2 $\longleftarrow$ ◀

Solving (2) for a_A

$$a_A = \frac{1}{3}(4a_C - 2a_B) = \frac{1}{3}\left[(4)(-30) - (2)(0)\right] = -40 \text{ mm/s}^2$$

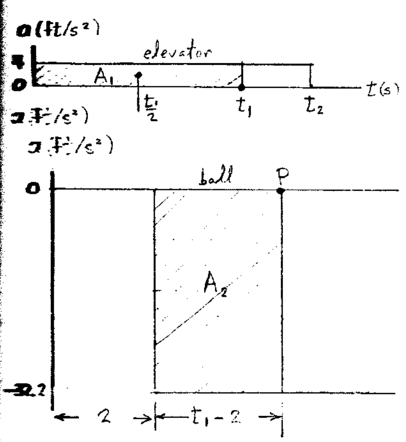
$a_A = 40$ mm/s^2 $\longleftarrow$ ◀



PROBLEM 11.76

An elevator starts from rest and moves upward, accelerating at a rate of $4\ \text{ft/s}^2$ until it reaches a speed of 24 ft/s, which it then maintains. Two seconds after the elevator begins to move, a man standing 40 ft above the initial position of the top of the elevator throws a ball upward with an initial velocity of 64 ft/s. Determine when the ball will hit the elevator.

SOLUTION



Construct the a–t curves for the elevator and the ball.

Limit on A_1 is 24 ft/s. Using $A_1 = 4t$

$$4t_2 = 24 \qquad t_2 = 6\text{ s}$$

Motion of elevator.

For $0 \le t_1 \le 6$ s, $\qquad (x_E)_0 = 0 \qquad (v_E)_0 = 0$

Moment of A_1 about $t = t_1$: $\qquad 4t_1 \dfrac{t_1}{2} = 2t_1^2$

$$x_E = (x_E)_0 + (v_E)_0 t_1 + 2t_1^2 = 2t_1^2$$

Motion of ball. At $t = 2$, $\qquad (x_B)_0 = 40\text{ ft} \qquad (v_B)_0 = 64\text{ ft/s}$

For $t_1 > 2$ s, $\qquad A_2 = -32.2(t_1 - 2)\text{ft/s}$

Moment of A_2 about $t = t_2$: $\quad -32.2(t_1 - 2)\left(\dfrac{t_1 - 2}{2}\right) = -16.1(t_1 - 2)^2$

$$x_B = (x_B)_0 + (v_B)_0 (t_1 - 2) - 16.1(t_1 - 2)^2$$

$$= 40 + 64(t_1 - 2) - 16.1(t_1 - 2)^2$$

When ball hits elevator, $\qquad x_B = x_E$

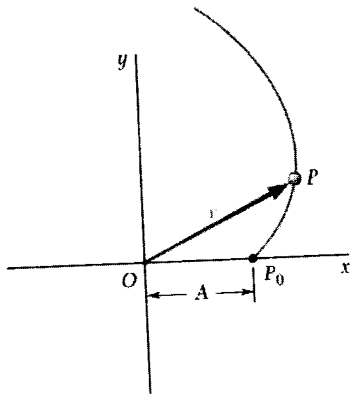
$$40 + 64(t_1 - 2) - 16.1(t_1 - 2)^2 = 2t_1^2 \qquad \text{or}$$

$$18.1t_1^2 - 128.4t_1 + 152.4 = 0$$

Solving the quadratic equation, $\qquad t_1 = 1.507\text{ s} \qquad \text{and} \qquad 5.59\text{ s}$

The smaller root is out of range, hence $\qquad t_1 = 5.59\text{ s}$ ◀

Since this is less than 6 s, the solution is within range.



PROBLEM 11.95

The motion of a particle is defined by the position vector $\mathbf{r} = A(\cos t + t\sin t)\mathbf{i} + A(\sin t - t\cos t)\mathbf{j}$, where t is expressed in seconds. Determine the values of t for which the position vector and the acceleration vector are (*a*) perpendicular, (*b*) parallel.

SOLUTION

Given: $\mathbf{r} = A(\cos t + t\sin t)\mathbf{i} + A(\sin t - t\cos t)\mathbf{j}$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = A(-\sin t + \sin t + t\cos t)\mathbf{i} + A(\cos t - \cos t + t\sin t)\mathbf{j}$$

$$= A(t\cos t)\mathbf{i} + A(t\sin t)\mathbf{j}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = A(\cos t - t\sin t)\mathbf{i} + A(\sin t + t\cos t)\mathbf{j}$$

(*a*) When $\mathbf{r}$ and $\mathbf{a}$ are perpendicular, $\mathbf{r}\cdot\mathbf{a} = 0$

$$A\left[(\cos t + t\sin t)\mathbf{i} + (\sin t - t\cos t)\mathbf{j}\right]\cdot A\left[(\cos t - t\sin t)\mathbf{i} + (\sin t + t\cos t)\mathbf{j}\right] = 0$$

$$A^2\left[(\cos t + t\sin t)(\cos t - t\sin t) + (\sin t - t\cos t)(\sin t + t\cos t)\right] = 0$$

$$\left(\cos^2 t - t^2\sin^2 t\right) + \left(\sin^2 t - t^2\cos^2 t\right) = 0$$

$$1 - t^2 = 0$$

$t = 1\text{ s}$ ◀

(*b*) When $\mathbf{r}$ and $\mathbf{a}$ are parallel, $\mathbf{r}\times\mathbf{a} = 0$

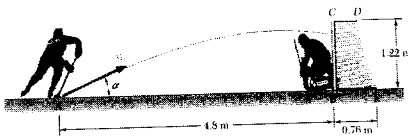
$$A\left[(\cos t + t\sin t)\mathbf{i} + (\sin t - t\cos t)\mathbf{j}\right]\times A\left[(\cos t - t\sin t)\mathbf{i} + (\sin t + t\cos t)\mathbf{j}\right] = 0$$

$$A^2\left[(\cos t + t\sin t)(\sin t + t\cos t) - (\sin t - t\cos t)(\cos t - t\sin t)\right]\mathbf{k} = 0$$

$$\left(\sin t\cos t + t\sin^2 t + t\cos^2 t + t^2\sin t\cos t\right) - \left(\sin t\cos t - t\cos^2 t - t\sin^2 t + t^2\sin t\cos t\right) = 0$$

$$2t = 0$$

$t = 0$ ◀



PROBLEM 11.114

The initial velocity $\mathbf{v}_0$ of a hockey puck is 170 km/h. Determine (*a*) the largest value (less than 45°) of the angle α for which the puck will enter the net, (*b*) the corresponding time required for the puck to reach the net.

SOLUTION

Horizontal motion: $x = (v_0 \cos\alpha)t$ or $t = \dfrac{x}{v_0 \cos\alpha}$

Vertical motion: $y = (v_0 \sin\alpha)t - \dfrac{1}{2}gt^2$

$$= x\tan\alpha - \frac{gx^2}{2v_0^2\cos^2\alpha}$$

$$= x\tan\alpha - \frac{gx^2}{2v_0^2}\left(1 + \tan^2\alpha\right)$$

$$\tan^2\alpha - \frac{2v_0^2}{gx}\tan\alpha + \left(1 + \frac{2v_0^2 y}{gx^2}\right) = 0$$

Data: $v_0 = 170$ km/h $= 47.222$ m/s, $x = 4.8$ m at point C,

$y = 1.22$ m at point C.

$$\frac{2v_0^2}{gx} = \frac{(2)(47.222)^2}{(9.81)(4.8)} = 94.712$$

$$\frac{2v_0^2 y}{gx^2} = \frac{(94.712)(1.22)}{4.8} = 24.073$$

(*a*) $\tan^2\alpha - 94.712\alpha + 25.073 = 0$

$\tan\alpha = 0.26547$ and 94.45

$\alpha = 14.869°$ or $89.4°$ $\qquad \alpha = 14.9°$ ◀

(*b*) $t = \dfrac{x}{v_0\cos\alpha} = \dfrac{4.8}{(47.222)\cos 14.869°}$ $\qquad t = 0.1052$ s ◀

PROBLEM 11.139

A motorist is traveling on a curved portion of highway of radius 350 m at a speed of 72 km/h. The brakes are suddenly applied, causing the speed to decrease at a constant rate of 1.25 m/s^2. Determine the magnitude of the total acceleration of the automobile (*a*) immediately after the brakes have been applied, (*b*) 4 s later.

SOLUTION

Initial speed. $v_0 = 72 \text{ km/h} = 20 \text{ m/s}$

Tangential acceleration. $a_t = -1.25 \text{ m/s}^2$

(*a*) Total acceleration at $t = 0$.

$$a_n = \frac{v_0^2}{\rho} = \frac{(20)^2}{350} = 1.14286 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-1.25)^2 + (1.14286)^2}$$

$a = 1.694 \text{ m/s}^2$ ◀

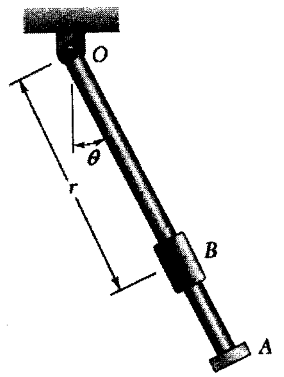
(*b*) Total acceleration at $t = 4$ s.

$$v = v_0 + a_t t = 20 + (-1.25)(4) = 15 \text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{(15)^2}{350} = 0.6426 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(-1.25)^2 + (0.6426)^2}$$

$a = 1.406 \text{ m/s}^2$ ◀



PROBLEM 11.164

The oscillation of rod OA about O is defined by the relation $\theta = (4/\pi)(\sin \pi t)$, where θ and t are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is $r = 10/(t+6)$, where r and t are expressed in mm and seconds, respectively. When $t = 1$ s, determine (*a*) the velocity of the collar, (*b*) the total acceleration of the collar, (*c*) the acceleration of the collar relative to the rod.

SOLUTION

Differentiate the expressions for r and θ with respect to time.

$$r = \frac{10}{t+6} \text{ mm}, \qquad \dot{r} = -\frac{10}{(t+6)^2} \text{ mm/s}, \qquad \ddot{r} = \frac{20}{(t+6)^3} \text{ mm/s}^2$$

$$\theta = \frac{4}{\pi}\sin \pi t \text{ rad}, \qquad \dot{\theta} = 4\cos \pi t \text{ rad/s} \qquad \ddot{\theta} = 4\pi \sin \pi t \text{ rad/s}^2$$

At $t = 1$ s,

$$r = \frac{10}{7} \text{ mm}; \qquad \dot{r} = -\frac{10}{49} \text{ mm/s}, \qquad \ddot{r} = \frac{20}{343} \text{ mm/s}^2$$

$$\theta = 0, \qquad \dot{\theta} = -4 \text{ rad/s}, \qquad \ddot{\theta} = 0$$

(*a*) Velocity of the collar.

$$v_r = \dot{r} = 0.204 \text{ mm/s}, \qquad v_\theta = r\dot{\theta} = -5.71 \text{ mm/s}$$

$$\mathbf{v}_B = (0.204 \text{ mm/s})\mathbf{e}_r - (5.71 \text{ mm/s})\mathbf{e}_\theta \blacktriangleleft$$

(*b*) Acceleration of the collar.

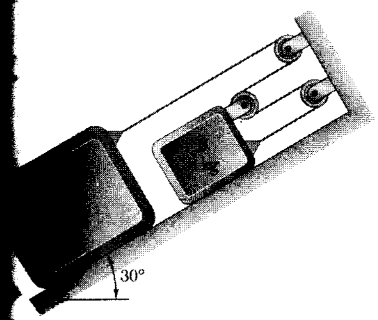
$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{20}{343} - \left(\frac{10}{7}\right)(-4)^2 = -22.8 \text{ mm/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \left(\frac{10}{7}\right)(0) + (2)\left(-\frac{10}{49}\right)(-4) = 1.633 \text{ mm/s}^2$$

$$\mathbf{a}_B = -\left(22.8 \text{ mm/s}^2\right)\mathbf{e}_r + \left(1.633 \text{ mm/s}^2\right)\mathbf{e}_\theta \blacktriangleleft$$

(*c*) Acceleration of the collar relative to the rod.

$$\mathbf{a}_{B/OA} = \ddot{r}\mathbf{e}_r = \frac{20}{343}\mathbf{e}_r \qquad \mathbf{a}_{B/OA} = \left(0.0583 \text{ mm/s}^2\right)\mathbf{e}_r \blacktriangleleft$$



PROBLEM 12.13

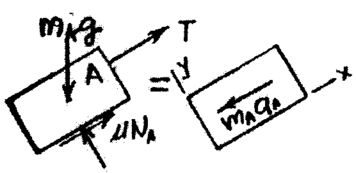
The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and assuming that the coefficients of friction between both blocks and the incline are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the acceleration of each block, (*b*) the tension in the cable.

SOLUTION

Let the positive directions of x_A and x_B be down the incline.

Constraint of the cable: $x_A + 3x_B = \text{constant}$

$$a_A + 3a_B = 0 \qquad a_B = -\frac{1}{3}a_A$$

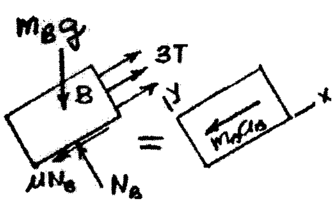


Block A: $+\nwarrow \Sigma F_y = 0$: $N_A - m_A g\cos 30° = 0$

$$+\swarrow \Sigma F_x = ma: \quad m_A g\sin 30° - \mu N_A - T = m_A a_A$$

Eliminate N_A.

$$m_A g(\sin 30° - \mu\cos 30°) - T = m_A a_A$$



Block B: $+\nwarrow \Sigma F_y = 0$: $N_B - m_B g\cos 30° = 0$

$$+\swarrow \Sigma F = ma: \quad m_B g\sin 30° + \mu N_B - 3T = m_B a_B = -\frac{m_B a_A}{3}$$

Eliminate N_B.

$$m_B g(\sin 30° + \mu\cos 30°) - 3T = -\frac{m_B a_A}{3}$$

Eliminate T.

$$(3m_A g - m_B g)\sin 30° - \mu(3m_A g + m_B g)\cos 30° = \left(3m_A + \frac{m_B}{3}\right)a_A$$

Check the value of μ_s required for static equilibrium. Set $a_A = 0$ and solve for μ.

$$\mu = \frac{(3m_A - m_B)\sin 30°}{(3m_A + m_B)\cos 30°} = \frac{(75-20)}{(75+20)}\tan 30° = 0.334.$$

Since $\mu_s = 0.25 < 0.334$, sliding occurs.

Calculate $\frac{a_A}{g}$ for sliding. Use $\mu = \mu_k = 0.20$.

continued

PROBLEM 12.13 CONTINUED

$$\frac{a_A}{g} = \frac{(3m_A - m_B)\sin 30° - \mu(3m_A + m_B)\cos 30°}{3m_A + m_B/3}$$

$$= \frac{(30 - 8)\sin 30° - (0.20)(30 + 8)\cos 30°}{30 + 2.667} = 0.13525$$

(*a*) $a_A = (0.13525)(9.81) = 1.327 \text{ m/s}^2$ $\qquad \mathbf{a}_A = 1.327 \text{ m/s}^2 \measuredangle 30°$ ◀

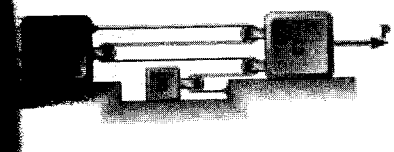
$a_B = -\left(\frac{1}{3}\right)(1.327) = -0.442 \text{ m/s}^2$ $\qquad \mathbf{a}_B = 0.442 \text{ m/s}^2 \measuredangle 30°$ ◀

(*b*) $T = m_A g(\sin 30° - \mu\cos 30°) - m_A a_A$

$= (10)(9.81)(\sin 30° - 0.20\cos 30°) - (10)(1.327)$

$T = 18.79$ **N** ◀

PROBLEM 12.32



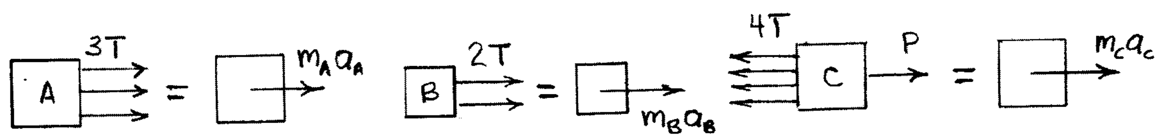
The weights of blocks A, B, and C are $W_A = W_C = 20$ lb, and $W_B = 10$ lb. Knowing that $P = 50$ lb and neglecting the masses of the pulleys and the effect of friction, determine (*a*) the acceleration of each block, (*b*) the tension in the cable.

SOLUTION

Let the positive direction for position coordinates, velocities, and accelerations be to the right. Let the origin lie at the fixed anchor.

Constraint of cable: $3(x_C - x_A) + (x_C - x_B) + (-x_B) = \text{constant}$

$$4a_C - 2a_B - 3a_A = 0 \qquad (1)$$



$\xrightarrow{+} \Sigma F_x = ma_x$:

Block A: $\quad 3T = m_A a_A \quad$ or $\quad a_A = \dfrac{3T}{m_A} = \dfrac{3T}{20}g \qquad (2)$

Block B: $\quad 2T = m_B a_B \quad$ or $\quad a_B = \dfrac{2T}{m_B} = \dfrac{2T}{10}g \qquad (3)$

Block C: $\quad P - 4T = m_C a_C \quad$ or $\quad a_C = \dfrac{P-4T}{20} = \dfrac{P-4T}{20}g \qquad (4)$

Substituting (2), (3), and (4) into (1),

$$4\left(\frac{P-4T}{20}\right) - 2\left(\frac{2T}{10}\right) - 3\left(\frac{3T}{20}\right) = 0$$

$$\left(\frac{16}{20} + \frac{4}{10} + \frac{9}{20}\right)T = \frac{4P}{20}$$

$$T = (0.12121)P = (0.12121)(50) = 6.0605 \text{ lb}$$

(*a*) From (2), $\quad a_A = \dfrac{(3)(6.0605)(32.2)}{20} = 29.3 \text{ ft/s}^2 \qquad \mathbf{a}_A = 29.3 \text{ ft/s}^2 \longrightarrow$ ◀

From (3), $\quad a_B = \dfrac{(2)(6.0605)(32.2)}{10} = 39.0 \text{ ft/s}^2 \qquad \mathbf{a}_B = 39.0 \text{ ft/s}^2 \longrightarrow$ ◀

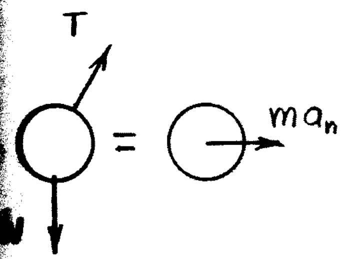
From (4), $\quad a_C = \dfrac{[50 - (4)(6.0605)](32.2)}{20} = 41.5 \text{ ft/s}^2 \qquad \mathbf{a}_C = 41.5 \text{ m/s}^2 \longrightarrow$ ◀

(*b*) As determined above, $\qquad T = 6.06 \text{ lb}$ ◀

PROBLEM 12.39

During a hammer thrower's practice swings, the 16-lb head A of the hammer revolves at a constant speed v in a horizontal circle as shown. If $\rho = 3$ ft and $\theta = 60°$, determine (*a*) the tension in wire BC, (*b*) the speed of the hammer's head.

LUTION



(*a*) $\Sigma F_y = 0: \quad T\sin\theta - W = 0$

$$T = \frac{W}{\sin\theta} = \frac{16}{\sin 60°}$$

or $T = 18.48$ lb ◀

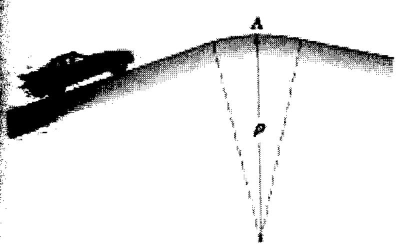
(*b*) $\xrightarrow{+} \Sigma F_x = ma_n: \quad T\cos\theta = m\dfrac{v^2}{\rho}$

$$v^2 = \frac{\rho T\cos\theta}{m} = \frac{\rho W\cos\theta}{m\sin\theta}$$

$$= \frac{\rho g}{\tan\theta} = \frac{(3)(32.2)}{\tan 60°} = 55.77 \text{ ft}^2/\text{s}^2$$

$v = 7.47$ ft/s ◀

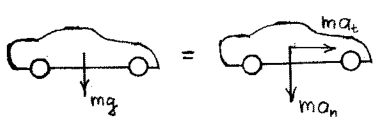
PROBLEM 12.48



During a high-speed chase, an 1100-kg sports car traveling at a speed of 160 km/h just loses contact with the road as it reaches the crest A of a hill. (a) Determine the radius of curvature ρ of the vertical profile of the road at A. (b) Using the value of ρ found in part a, determine the force exerted on a 70-kg driver by the seat of his 1400-kg car as the car, traveling at a constant speed of 80 km/h, passes through A.

SOLUTION

(a) $v = 160 \text{ km/h} = 44.44 \text{ m/s}$



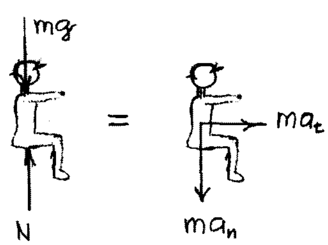
Wheels do not touch the road.

$$\Sigma F_y = -ma_n: \quad -mg = -mv^2/\rho$$

$$\rho = \frac{v^2}{g} = \frac{(44.44)^2}{9.81} = 201.4$$

$\rho = 201 \text{ m}$ ◀

(b) $v = 80 \text{ km/h} = 22.22 \text{ m/s}$



$$m = 70 \text{ kg for passenger}$$

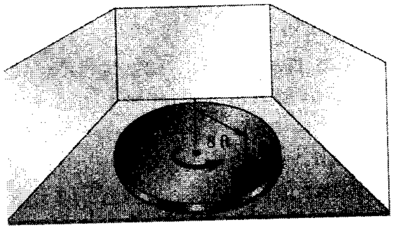
$$\Sigma F_y = -ma_n:$$

$$N - mg = -\frac{mv^2}{\rho}$$

$$N = m\left(g - \frac{v^2}{\rho}\right)$$

$$= (70)\left(9.81 - \frac{22.22^2}{201.4}\right)$$

$N = 515 \text{ N}\uparrow$ ◀

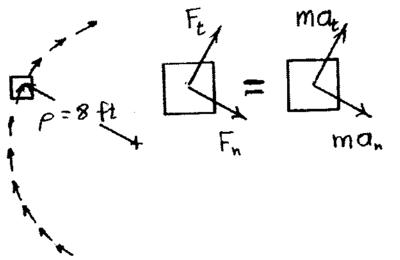


PROBLEM 12.61

A turntable A is built into a stage for use in a theatrical production. It is observed during a rehearsal that a trunk B starts to slide on the turntable 12 s after the turntable begins to rotate. Knowing that the trunk undergoes a constant tangential acceleration of $0.75\ \text{ft/s}^2$, determine the coefficient of static friction between the trunk and the turntable.

SOLUTION

Uniformly accelerated motion on a circular path. $\rho = 8$ ft



$$v = v_0 + a_t t$$

$$= 0 + (0.75)(12) = 9\ \text{ft/s}$$

$$F_t = ma_t = \frac{W}{g}a_t: \quad F_t = W\frac{a_t}{g} = \frac{0.75}{32.2}W = 0.0233\ W$$

$$F_n = ma_n = \frac{Wv^2}{g\rho}: \quad F_n = \frac{W(9)^2}{(32.2)(8)} = 0.3144\ W$$

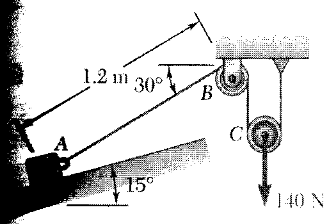
$$F = \sqrt{F_t^2 + F_n^2} = 0.315\ W$$

This is the friction force available to cause the trunk to slide.

The normal force N is calculated from equilibrium of forces in the vertical direction.

$$\Sigma F_y = 0: \quad N - W = 0 \qquad N = W$$

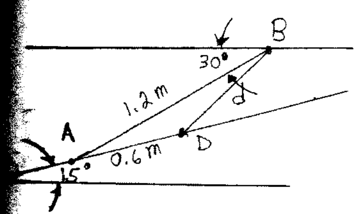
Since sliding is impending, $\mu_s = \dfrac{F}{W} = 0.315$ $\qquad$ $\mu_s = \mathbf{0.315}$ ◀

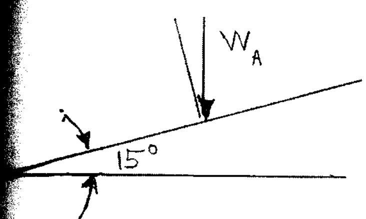


PROBLEM 13.12

The 7-kg block A is released in the position shown with a velocity of 1.5 m/s up the incline. Knowing that the velocity of the block is 3 m/s after it has moved 0.6 m up the incline, determine the work done by the friction force exerted on the block. Neglect the masses of the pulleys.

UTION





$W_A = 7(9.81) = 68.67$ N

Given: Block A is released at the position shown at a velocity of 1.5 m/s up.

After moving 0.6 m the velocity is 3 m/s.

Find: work done by friction force on the block, $V_f J$

From the Law of Cosines

$$d^2 = (1.2)^2 + (0.6)^2 - 2(1.2)(0.6)(\cos 15°)$$

$$d^2 = 0.4091\,\text{m}^2$$

$$d = 0.63958\,\text{m}$$

$$U_C = 140\,\text{N}\left[\frac{1}{2}(1.2 - 0.63958)\right]\text{m} = 39.229\ \text{N}\cdot\text{m}$$

$$U_A = -68.67\ \text{N}(\sin 15°)(0.6\ \text{m}) = -10.664\ \text{N}\cdot\text{m}$$

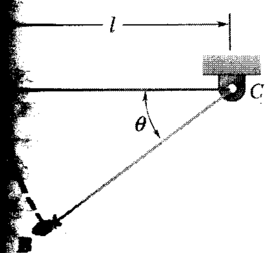
$$U_C + U_A - U_{friction} = T_2 - T_1 = \frac{1}{2}\,m_A[v_2^2 - v_1^2]$$

$$39.229 - 10.664 - U_{friction} = \frac{1}{2}\,(7\ \text{kg})[(3)^2 - (1.5)^2]\ \text{m}^2$$

$$-U_{friction} = -39.229 + 10.664 + 23.625$$

$$= -4.94\ \text{J}$$

$U_{friction} = -4.94$ J ◀



PROBLEM 13.44

A bag is gently pushed off the top of a wall at A and swings in a vertical plane at the end of a rope of length l Determine the angle θ for which the rope will break, knowing that it can withstand a maximum tension equal to twice the weight of the bag.

ION

Use work - energy : position 1 is at A, position 2 is at B.

$$T_1 + U_{1\to 2} = T_2 \qquad (1)$$

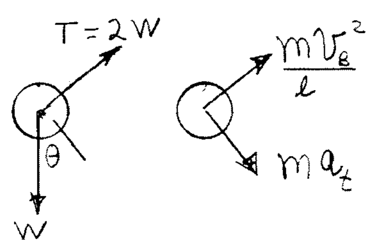
Where $T_1 = 0;\quad U_{1\to 2} = mgl\sin\theta;\quad T_2 = \frac{1}{2}mv_B^2$

Substitute

$$0 + mgl\sin\theta = \frac{1}{2}mv_B^2$$

$$v_B^2 = 2gl\sin\theta \qquad (2)$$

For $T = 2\,W$ use Newtons 2nd law.



$$\nearrow \Sigma F_n = ma_n \Rightarrow 2W - W\sin\theta = \frac{mv_B^2}{l} \qquad (3)$$

Substitute (2) into (3)

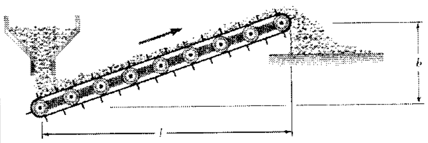
$$2\not{m}g - \not{m}g\sin\theta = 2\not{m}g\frac{\not{l}\sin\theta}{\not{l}}$$

$$2 = 3\sin\theta$$

$$\text{or } \sin\theta = \frac{2}{3} \Rightarrow \theta = 41.81^\circ$$

$\theta = 41.8^\circ$ ◀

PROBLEM 13.50



A power specification formula is to be derived for electric motors which drive conveyor belts moving solid material at different rates to different heights and distances. Denoting the efficiency of the motors by η and neglecting the power needed to drive the belt itself, derive a formula (*a*) in the SI system of units, for the power P in kW, in terms of the mass flow rate m in kg/h, the height b, and the horizontal distance l in meters, and (*b*) in U.S. customary units, for the power in hp, in terms of the mass flow rate m in tons/h, and the height b and horizontal distance l in feet.

SOLUTION

(*a*) Material is lifted to a height b at a rate, $\qquad (m\text{ kg/h})(g\text{ m/s}^2) = [mg\,(\text{N/h})]$

Thus,

$$\frac{\Delta U}{\Delta t} = \frac{[mg\,(\text{N/h})][b(m)]}{(3600\text{ s/h})} = \left(\frac{mgb}{3600}\right)\text{N}\cdot\text{m/s}$$

$$1000\text{ N}\cdot\text{m/s} = 1\text{ kw}$$

Thus, including motor efficiency, η

$$P(\text{kw}) = \frac{mgb(\text{N}\cdot\text{m/s})}{(3600)\left(\dfrac{1000\text{ N}\cdot\text{m/s}}{\text{kw}}\right)(\eta)}$$

$$P(\text{kw}) = 0.278\times 10^{-6}\frac{mgb}{\eta}$$

(*b*)

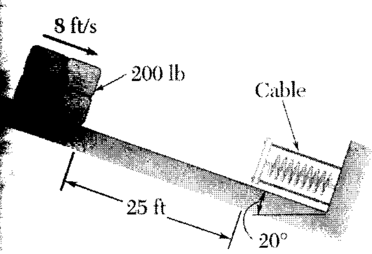
$$\frac{\Delta U}{\Delta t} = \frac{[W(\text{tons/h})(2000\text{ lb/ton})][b(\text{ft})]}{3600\text{ s/h}}$$

$$= \frac{Wb}{1.8}\text{ ft}\cdot\text{lb/s};\ 1\text{hp} = 550\text{ ft}\cdot\text{lb/s}$$

With η,

$$hp = \left[\frac{Wb}{1.8}(\text{ft}\cdot\text{lb/s})\right]\left[\frac{1\text{hp}}{550\text{ ft}\cdot\text{lb/s}}\right]\left[\frac{1}{\eta}\right]$$

$$hp = \frac{1.010\times 10^{-3}Wb}{\eta}$$



PROBLEM 13.65

A spring is used to stop a 200-lb package which is moving down a 20° incline. The spring has a constant $k = 125$ lb/in. and is held by cables so that it is initially compressed 6 in. Knowing that the velocity of the package is 8 ft/s when it is 25 ft from the spring and neglecting friction, determine the maximum additional deformation of the spring in bringing the package to rest.

SOLUTION

Conservation of energy

Position (1) is at the top of the incline; position (2) is when the spring has maximum deformation

$$k = 1500 \text{ lb/ft}$$

Where $T_1 + V_1 = T_2 + V_2$

At (1)

$$T_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}\left(\frac{200}{32.2}\right)(8)^2 = 198.76 \text{ ft}\cdot\text{lb}$$

$$V_1 = V_{g1} + V_{e1} = mgz_1 = \frac{1}{2}kx_1^2 \text{ (datum at point 2)}$$

$$= 200(25 - x)\sin 20° + \frac{1}{2}(1500)(0.5)^2$$

x = Deformation of the spring

$$V_1 = 1710.1 + 68.404x + 187.5$$

At (2) $\quad T_2 = 0; \quad V_2 = \cancel{V_{g2}}^{\,0} + V_{e2} = \frac{1}{2}kx_2^2 = \frac{1}{2}(1500)(0.5 + x)^2$

Substituting into (1) $\quad 198.78 + 1710.1 + 68.404x + 187.5 = 750(0.5 + x)^2$

Solve

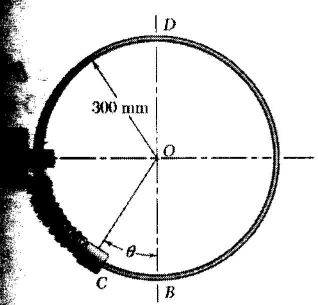
$$750x^2 + 681.596x - 1908.9 = 0$$

$$x = -2.11 \text{ or } +1.2044 \text{ ft}$$

$x = 1.204$ ft ◀

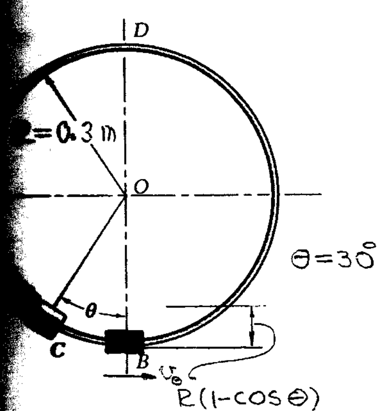
$= 14.45$ in. ◀

PROBLEM 13.69



A thin circular rod is supported in a *vertical plane* by a bracket at A. Attached to the bracket and loosely wound around the rod is a spring of constant $k = 40$ N/m and undeformed length equal to the arc of circle AB. A 200-g collar C is unattached to the spring and can slide without friction along the rod. Knowing that the collar is released from rest when $\theta = 30°$, determine (*a*) the velocity of the collar as it passes through point B, (*b*) the force exerted by the rod on the collar as is passes through B.

SOLUTION



(*a*)

$$v_C = 0, \qquad T_C = 0$$

$$T_B = \frac{1}{2}mv_B^2$$

$$T_B = \frac{1}{2}(0.2\text{ kg})v_B^2$$

$$T_B = 0.1v_B^2 \qquad V_C = (V_C)_e + (V_C)_g$$

$$\text{arc } BC = \Delta L_{BC} = R\theta$$

$$\Delta L_{BC} = (0.3\text{ m})(30°)\frac{(\pi)}{180°}$$

$$\Delta L_{BC} = 0.15708\text{ m}$$

$$(V_C)_e = \frac{1}{2}k(\Delta L_{BC})^2 = \frac{1}{2}(40\text{ N/m})(0.15708\text{ m})^2 = 0.49348\text{ J}$$

$$(V_C)_g = WR(1-\cos\theta) = (0.2\text{ kg})(9.81\text{ m/s}^2)(0.3\text{ m})(1-\cos 30°)$$

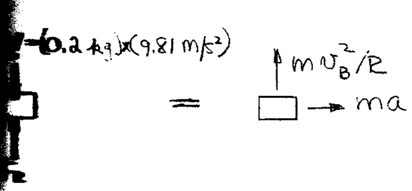
$$(V_C)_g = 0.078857\text{ J}$$

$$V_C = (V_C)_e + (V_C)_g = 0.49348\text{ J} + 0.078857\text{ J} = 0.57234\text{ J}$$

$$V_B = (V_B)_e + (V_B)_g = 0 + 0 = 0$$

$$T_C + V_C = T_B + V_B; \qquad 0 + 0.57234 = 0.1v_B^2$$

$$v_B^2 = 5.7234\text{ m}^2/\text{s}^2 \qquad v_B = 2.39\text{ m/s} \blacktriangleleft$$

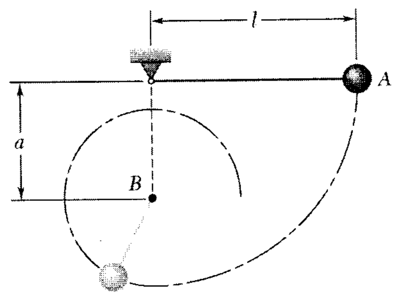


(*b*)

$$+\!\uparrow \Sigma F = F_R - W = \frac{mv_B^2}{R}$$

$$F_R = 1.962\text{ N} + (0.2\text{ kg})\frac{(5.7234\text{ m}^2/\text{s}^2)}{(0.3\text{ m})}$$

$$F_R = 1.962\text{ N} + 3.8156\text{ N} = 5.7776\text{ N} \qquad F_R = 5.78\text{ N}\uparrow \blacktriangleleft$$



PROBLEM 13.75

The pendulum shown is released from rest at A and swings through **90°** before the cord touches the fixed peg B. Determine the smallest value **of** a for which the pendulum bob will describe a circle about the peg.

SOLUTION

Use conservation of energy from the point of release (A) and the top of the circle.

$$T_1 + V_1 = T_2 + V_2 \qquad \text{(1) (datum at lowest point)}$$

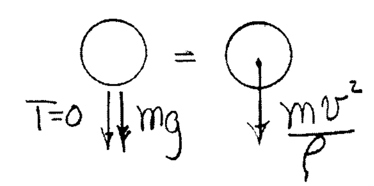
where

$$T_1 = 0; \quad V_1 = mg\ell$$

At 2 $$T_2 = \frac{1}{2}mv^2; \; V_2 = mgz = mg(2)(\ell - a)$$

Substituting into (1) $$0 + mg\ell = \frac{1}{2}mv^2 + 2mg(\ell - a) \qquad (2)$$

We need another equation – use Newton's 2[nd] law at the top. (Tension, $T_0 = 0$ at top)



$$+\!\downarrow \sum F_n = ma_n \Rightarrow \not{m}g = \frac{\not{m}v^2}{\rho}$$

$$v^2 = g\rho = g(\ell - a)$$

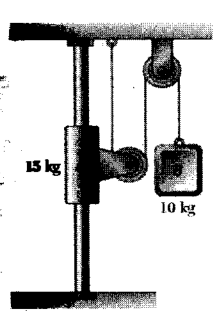
Substituting into (2)

$$mg\ell = \frac{1}{2}mg(\ell - a) + 2mg(\ell - a)$$

$$2\ell = \ell - a + 4\ell - 4a$$

$$5a = 3\ell$$

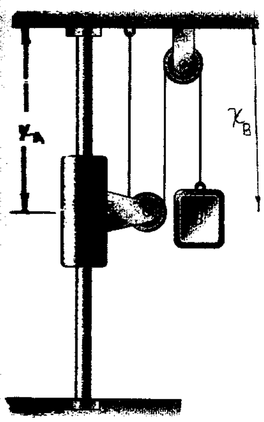
$$a = \frac{3}{5}\ell$$



PROBLEM 13.134

The system shown is released from rest. Determine the time it takes for the velocity of A to reach 0.6 m/s. Neglect friction and the mass of the pulleys.

UTION



Kinematics

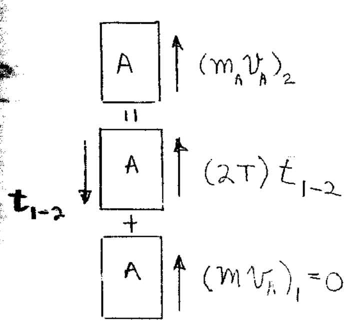
Length of cable is constant.

$$L = 2X_A + X_B$$

$$\frac{dL}{dt} = 2v_A + v_B = 0$$

$$v_B = -2v_A$$

$$(v_A)_2 = 0.6 \text{ m/s}$$



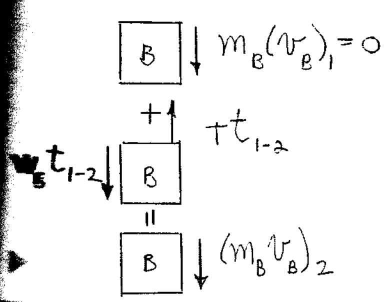
Collar A

$$m_A = 15 \text{ kg}$$

$$(m_A v_A)_1 + (2T)(t_{1-2}) - W_A t_{1-2} = m(v_A)_2$$

$$0 + [2T - (15 \times 9.81)]t_{1-2} = (15)(0.6)$$

$$(T - 73.575)t_{1-2} = 4.5 \qquad (1)$$



Collar B

$$m_B = 10 \text{ kg}$$

$$(v_B)_2 = 2(v_A)_2 = 1.2 \text{ m/s}$$

$$+\downarrow\ (m_B v_A)_1 - T(t_{1-2}) + W_B(t_{1-2}) = (m_B v_B)_2$$

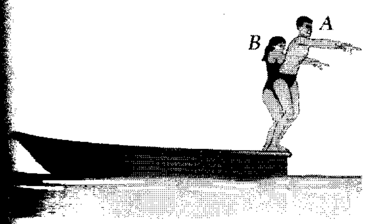
$$0 + [(10 \times 9.81) - T](t_{1-2}) = 10(1.2) \qquad (2)$$

Add Equation (1) and Equation (2) (eliminating T)

$$(98.1 - 73.575)t_{1-2} = 4.5 + 12$$

$$t_{1-2} = \frac{16.5}{24.52} = 0.673 \text{ s}$$

$t = 0.673$ s ◀



PROBLEM 13.149

Two swimmers A and B, of mass 75 kg and 50 kg, respectively, dive off the end of a 200-kg boat. Each swimmer has a relative horizontal velocity of 3 m/s when leaving the boat. If the boat is initially at rest, determine its final velocity, assuming that (*a*) the two swimmers dive simultaneously, (*b*) swimmer A dives first, (*c*) swimmer B dives first.

SOLUTION

$$m_A = 75 \text{ kg}, \quad m_B = 50 \text{ kg}, \quad m_C = 200 \text{ kg (Boat)}$$

(*a*) Swimmers dive simultaneously

$$0 = m_C v_C + (m_A + m_B) v_2 \quad (1)$$

Relative velocity of swimmers with respect to the boat is 3 m/s

$$v_2 - v_C = 3 \text{ m/s} \Rightarrow v_2 = v_C + 3$$

Substitute into (1)

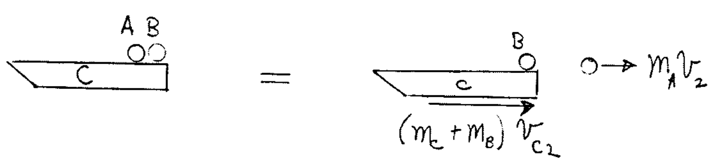
$$0 = m_C v_C + (m_A + m_B)(v_C + 3)$$

Solve

$$v_C = \frac{-3(m_A + m_B)}{m_A + m_B + m_C} = \frac{-3(75 + 50)}{(75 + 50 + 200)}$$

$v_C = 1.154$ m/s ← ◀

(*b*) A dives first and then B



x-dir

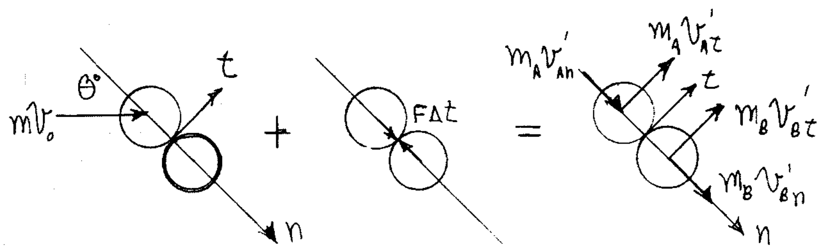
$$0 = (m_C + m_B) v_{C_2} - m_A v_2 \quad (2)$$

continued

PROBLEM 13.165

Two identical billiard balls can move freely on a horizontal table. Ball A has a velocity $\mathbf{v}_0$ as shown and hits ball B, which is at rest, at a point C defined by $\theta = 45°$. Knowing that the coefficient of restitution between the two balls is $e = 0.8$ and assuming no friction, determine the velocity of each ball after impact.

[cut off]LUTION



$m_A = m_B = m$

Ball *A t-dir*

$$\not{m}v_0 \sin\theta = \not{m}v'_{At} \Rightarrow v'_{At} = v_0 \sin\theta$$

Ball *B t-dir*

$$0 = m_B v'_{Bt} \Rightarrow v'_{Bt} = 0$$

Ball *A* + *B n-dir*

$$\not{m}v_0 \cos\theta + 0 = \not{m}\, v'_{An} + \not{m}\, v'_{Bn} \qquad (1)$$

Coefficient of restitution

$$v'_{Bn} - v'_{An} = e(v_{An} - v_{Bn})$$

$$v'_{Bn} - v'_{An} = e(v_0 \cos\theta - 0) \qquad (2)$$

Solve (1) and (2)

$$v'_{An} = v_0\left(\frac{1-e}{2}\cos\theta\right); \quad v'_{Bn} = v_0\left(\frac{1+e}{2}\right)\cos\theta$$

With numbers

$$e = 0.8; \quad \theta = 45°$$

$$v'_{At} = v_0 \sin 45° = 0.707\, v_0$$

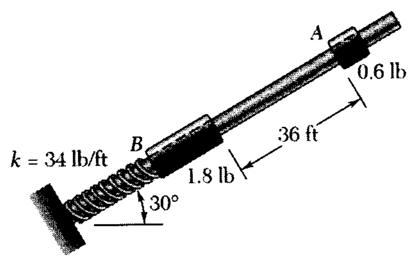
$$v'_{An} = v_0\left(\frac{1-0.8}{2}\cos 45°\right) = 0.0707\ v_0$$

$$v'_{Bt} = 0$$

$$v'_{Bn} = v_0\left(\frac{1+0.8}{2}\right)\cos 45° = 0.6364\, v_0$$

continued

PROBLEM 13.183



A 0.6-lb collar A is released from rest, slides down a frictionless rod, and strikes a 1.8-lb collar B which is at rest and supported by a spring of constant 34 lb/ft. Knowing that the velocity of collar A is zero immediately after impact, determine (*a*) the coefficient of restitution between the two collars, (*b*) the energy lost in the impact, (*c*) the maximum distance collar B moves down the rod after impact.

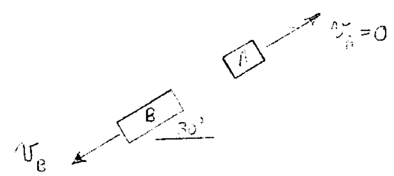
SOLUTION

Velocity of A just before impact, v_0

$$v_0 = \sqrt{2gh} = \sqrt{2\left(32.2\text{ ft/s}^2\right)(3.6\text{ ft})\sin 30°}$$

$$v_0 = \sqrt{2(32.2)(3.6)(0.5)} = 10.7666\text{ ft/s}$$

After impact



Conservation of momentum

$$\overset{+}{\swarrow}{}_{30°}\; m_A v_0 = m_B v_B - m_A v_A;\quad \left(\frac{0.6}{g}\right)v_0 = \left(\frac{1.8}{g}\right)v_B \qquad (1)$$

g's cancel

Restitution

$$\left(v_A + v_B\right) = e\left(v_0 + 0\right);\quad v_B = e v_0 \qquad (2)$$

From (1) $\quad v_B = \left(\frac{0.6}{1.8}\right)v_0 = \left(\frac{0.6}{1.8}\right)(10.7666\text{ ft/s}) = 3.5889\text{ ft/s}$

From (2) $\quad e = \left(v_B/v_0\right), \quad e = \frac{1}{3}$

(*a*) $\quad e = 0.333$ ◀

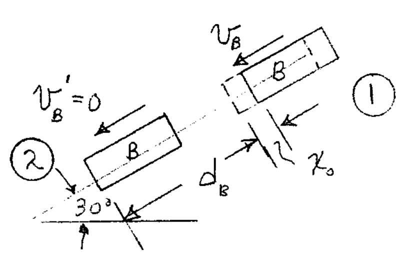
(*b*) Energy loss

$$\Delta\text{ Energy} = m_A g(3.6)\sin 30° - \frac{1}{2}m_B {v_B}^2$$

$$= (0.6\text{ lb})(3.6\text{ ft})(0.5) - \frac{1}{2}\left(\frac{1.8}{32.2}\right)(3.5889\text{ ft/s})^2$$

$$= 1.08 - 0.36 = 0.72\text{ ft}\cdot\text{lb}$$

Loss = 0.720 ft·lb ◀



(*c*) Static deflection = x_0, B moves down d_B $\swarrow_{30°}$

Conservation of energy ① *to* ②

Position ①-spring deflected, x_0

$$kx_0 = m_B g\sin 30°$$

PROBLEM 13.183 CONTINUED

$$T_1 + V_1 = T_2 + V_2; \qquad T_1 = \frac{1}{2}m_B v_B^2, \qquad T_2 = 0$$

$$V_1 = V_e + V_g = \frac{1}{2}kx_0^2 + m_B g d_B \sin 30°$$

$$V_2 = V_e' + V_g' = \int_0^{x_0 + d_B} kx dx = \frac{1}{2}k\left(d_B^2 + 2d_B x_0 + x_0^2\right)$$

$$\frac{1}{2}kx_0^2 + mgd_B \sin 30° + \frac{1}{2}m_B v_B^2 = \frac{1}{2}k\left(d_B^2 + 2d_B x_0 + x_0^2\right) + 0 + 0$$

$$\therefore\ kd_B^2 = m_B v_B^2; \qquad 34d_B^2 = \left(\frac{1.8}{32.2}\right)(3.5889)^2$$

$$d_B = 0.1455 \text{ ft}$$

$$d_B = 1.746 \text{ in.} \blacktriangleleft$$