Advances in Industrial Control

Cosmin Copot
Clara Mihaela Ionescu
Cristina I. Muresan

Image-Based and Fractional-Order Control for Mechatronic Systems

Theory and Applications with MATLAB®

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Advances in Industrial Control

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Professor **Michael J. Grimble**
Department of Electronic and Electrical Engineering, Royal College Building, 204 George Street, Glasgow G1 1XW, United Kingdom
e-mail: m.j.grimble@strath.ac.uk

Professor **Antonella Ferrara**
Department of Electrical, Computer and Biomedical Engineering, University of Pavia, Via Ferrata 1, 27100 Pavia, Italy
e-mail: antonella.ferrara@unipv.it

or the

In-house Editor

Mr. **Oliver Jackson**
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Cosmin Copot
Department of Electromechanics
University of Antwerp
Antwerpen, Belgium

Clara Mihaela Ionescu
Department of Electromechanics,
Systems and Metals Engineering
Ghent University
Ghent, Oost-Vlaanderen, Belgium

Cristina I. Muresan
Department of Automation
Technical University of Cluj-Napoca
Cluj Napoca, Romania

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Series Editor's Foreword

The subject of control engineering is viewed very differently by researchers and those that must implement and maintain systems. The former group develops general algorithms with a strong underlying mathematical basis, whilst the latter has more local concerns over the limits of equipment, quality of control and plant downtime. The series *Advances in Industrial Control* attempts to bridge this divide and hopes to encourage the adoption of more advanced control techniques when they are likely to be beneficial.

The rapid development of new control theory and technology has an impact on all areas of control engineering and applications. This monograph series encourages the development of more targeted control theory that is driven by the needs and challenges of applications. A focus on applications is important if the different aspects of the "control design" problem are to be explored with the same dedication that "control synthesis" problems have received in the past. The series provides an opportunity for researchers to present an extended exposition of new work on industrial control and applications. The series raises awareness of the substantial benefits that advanced control can provide but should also temper that enthusiasm with a discussion of the challenges that can arise.

This monograph covers some areas of control systems design that are not so well known outside of specialist communities. Image-based control is an emerging area with applications in areas like robotics, and with rather different control design problems because of the nature of the visual sensing system. For example, the selection of visual features is important in vision-based robot control systems, since these features determine the performance and accuracy of the robot control system. The second important topic covered is fractional-order control, based on ideas from fractional calculus—also an area of current research where there is a rapidly growing number of research papers.

The early chapters provide an introduction to the different topics of visual servo systems and there is useful introductory material on modelling and control problems in robotics. Chapter 3 describes detection algorithms for image feature extraction and evaluation. This is an important aspect of the use of visual control architectures.

The subject of fractional-order systems sometimes evokes strong reactions for or against this approach to modelling and control. Fractional-order control problems are discussed in Chap. 4, where there is a basic introduction to the design of position- and velocity-based control systems. Tuning procedures for the ubiquitous PI/PD/PID forms of controller are of course always of interest and are described here for fractional-order control systems. Although the mathematics may not be entirely familiar, the ideas are illustrated by simple examples. Potential benefits including performance and robustness are suggested from the application examples.

Chapter 5 takes a small step into the problems of multivariable systems by dealing with processes that have two inputs and two outputs. One of the more familiar topics covered is the relative gain array often used for structure assessment in the process industries.

In the second part of the text typical applications of vision-based robot control and of fractional-order control are discussed. Simulators for image-based control architectures are fairly essential for most robotic control application design studies, and some of the usual tools are described in Chap. 6. The real-time implementation of industrial robot manipulators is also covered and should be particularly valuable for applications engineers.

The application of fractional-order control to real-time targets is described in Chap. 7. The claim is that a fractional-order control system will outperform a classical control algorithm and this is illustrated using experimental results. The implementation of a fractional-order controller on a field-programmable gate array is also described. For implementation, the fractional-order controllers, which theoretically have infinite memory, are approximated by integer-order transfer functions. The experimental results suggest the fractional-order controllers can outperform a classical controller, which is, of course, the main justification for accepting the added theoretical complexity—complexity that might otherwise affect its potential for use in real applications.

Bringing the different ideas together the use of factional-order controllers for visual servoing is considered in Chap. 8. The experimental results and the design processes follow traditional lines but of course using fractional-order descriptions. The experiments involve a ball-and-plate system and also an image-based control law for a vision-based robot control system.

Sliding-mode control methods, covered in Chap. 9, are of course of wide interest in robot applications and are of growing importance in a range of others. Chapter 9 covers sliding-mode control for a class of robotic arm problems. It is shown that the algorithms require relatively few operations and can be implemented very efficiently on microcontrollers, programmable logic controllers, etc.

This text covers some areas of control engineering that are not very well known in most industrial sectors but may have a valuable role in specialist applications. This text is, therefore, a very timely and a very welcome addition to the series on *Advances in Industrial Control*.

Glasgow, UK
December 2019

Michael J. Grimble

Contents

Chapter 1
Introduction

1.1 Visual Servoing System

Visual servoing systems represents a branch of research that combines results from different fields such as: artificial vision, robotics, as well as real-time application design, which becomes a major area of interest for the latest research decade. Servoing systems refer to the use of visual features to control the movement of a manipulator robot. Visual features are defined as properties of the objects that make up an image. Images can be purchased using a visual sensor that is mounted either in a fixed position in the work environment or on the last joint of the robot. The first configuration is called eye-to-hand, and the second configuration is called eye-in-hand. In [1], for the first time, a complete description of the two fundamental architectures of the visual servoing systems is made: the position-based architecture and the image-based architecture. Each of the two architectures presents advantages and disadvantages when dealing with real-time processes [20, 49].

A position-based architecture consists of calculating an error represented in the Cartesian system and requires both an object model (usually a CAD type) and a perfectly calibrated camera in order to estimate the position and orientation of an object [2]. For the image-based architecture, the use of an object model is avoided by measuring an error signal in the image plane, i.e. a signal mapped directly to the controls of the execution element of the robot [20].

Both architectures use visual features in order to describe the properties of an object in the image plane. If for position-based architecture the features are used to characterize positions by correlating the image plane with the 3D space, in the case of image-based architecture these features lead to the formation of the Jacobian matrix, which represents the mapping between the velocities of the objects projected in the image plane and the working environment of the robot. The peculiarities that may occur in the case of the Jacobian matrix plus the impossibility to have direct control over the speeds developed by the robot in the 3D space lead to difficulties in designing the loop regulator. For image-based architecture, the approach is to reduce the error between a set of current features and a set of desired features. An error function is

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defined from the point of view of quantities that can be directly measured in an image, and the control law is constructed so that this error directly maps the robot's motion. Depending on the type of control architecture used, an object can be characterized by its position or based on the visual features extracted from the image. As one of the main goals of the overall stability of servoing systems, it has been found that position-based architecture suffers from massive limitations in terms of robustness and mathematical description of the models required for physical implementations. Hence, so-called hybrid methods were created to combine the advantages of the two architectures [3, 4].

1.2 Visual Features

Choosing an appropriate set of visual features is necessary to ensure the most accurate correlation between the dynamics in the image space and the dynamics in the task space, which leads to different entities: point features (centroid, corners), image moments, the areas of the projected regions, the orientation of the lines joining two points, the lengths of the edges, the parameterization of the lines, etc. [23, 38, 39]. These types of visual features can be used to generate image-based control law. The commonly visual features encountered in practice are the point features and those based on image moments. The point features can be 1D (edge) and 2D (corner). The main advantage of using point features is that the calculation of the interaction matrix is relatively simple because the point coordinates are known. The disadvantage of using point features in servoing applications is that of the reduced stability (they are not invariant to transformations of the object in the working scene).

The problem with point features can be eliminated by using image moments to generate the control law. The idea of using moments in servoing applications is relatively old, but the bottleneck was the fact that the interaction matrix was not available for any type of the object.

In [23] a new method was proposed which allows the calculation of the interaction matrix related to the image moments and this type of visual features became popular in visual servoing applications.

1.3 Control Architectures for Visual Servoing System

Robot control studies the structure and operation of the robot control system. Based on the geometric and dynamic model, of the robot task which is further converted into the trajectory to be followed, the necessary commands are established for the actuating elements and the hardware and software elements using the feedback signals obtained from the visual sensor [5]. This results in the complexity of the control system of a robot. It will frequently have a hierarchical organization, where on the upper level is the decision part regarding the action to be taken, and on the lower level the control and action elements of the joints. These commands will also have to take into account the desired performance for the robot, here the dynamic model of the robot intervenes. The typical structure of the control system will contain a computer on the upper level and a system with one or more microcontrollers controlling the actuating elements in the joints.

Selecting an appropriate set of visual features and designing the visual control law is the main aspect to achieve a high-performance control architecture. The most common problems in servoing systems that influence the process of selecting the visual features are the problems related to local minimum, to singularity and to visibility. In general, local-minimum problems occur only in certain configurations [23]. The basic notion for a local minimum is when the camera velocity is zero ($\mathbf{v_c} = 0$) while the error between the current features $\mathbf{f}$ and the desired features $\mathbf{f}^*$ is different from zero, and thus the system converges to a final configuration that is different from the desired configuration.

If the vector of visual features $\mathbf{f}$ is composed of three points of interest, the same image of the three points can be seen from different hypotheses such that for different configurations of the camera we have $\mathbf{f} = \mathbf{f}^*$, which correspond to several global minimums. To obtain a unique position it is necessary to use at least four points. By using four points, the control law tries to impose eight trajectory restrictions on the image plane while the system has only six degrees of freedom. In this case, the control law can generate unattainable movements in the image plane indicating a local minimum [142]. There are different control strategies for visual servoing systems proposed to eliminate the local minimum problem. For example, in [6] it was proposed to use a hybrid architecture, and in [2] the use of a planning strategy.

The visual features used in the servoing systems can leave the view of the camera during the servoing task [156]. Therefore, it is required that the control laws to be used are able to maintain the visual features in the field of view of the camera in order to obtain the correct feedback during the servoing process. To minimize the likelihood of visual features leaving the camera view, planning strategies [2] or marking strategies [7] can be used. The increase in the number of degrees of freedom of the robots and the increased complexity of the objects in the working environment have led to the need to implement new methods for designing the control law [55]. Thus, one of the proposed solutions aims at predictive control [54] in order to increase the convergence and the stability of the servoing system.

1.4 Fractional-Order Control

The non-integer-order derivative was a topic of discussion for more than 300 years, and now it is known as fractional calculus. It corresponds to the generalization of ordinary differentiation and integration to arbitrary (non-integer) order [64].

In the past decade, fractional-order control (FOC) has received extensive attention from the research community in several fields including robotics, mechatronics, biology, physics, among others. Compared with the classical integer-order controllers, FOC techniques have achieved more impressive results in many control systems in terms of improving the robustness during wind gusts, payload variations, disturbances due to friction, modelling uncertainties, etc. [8, 9].

The robot motion tracking systems represent one of the most challenging control applications in the field of manipulator robots due to the highly non-linear and time-varying dynamics. Recently, FOC of non-linear systems has started to attract interest in different kinds of applications [10]. A FOC strategy for visual servoing systems is presented in [158], where the image-based control is designed using a point features based on fractional-order PI controller. The system to evaluate the performance of the proposed control strategy is composed of a manipulator robot with 6 degrees of freedom with an eye-in-hand camera.

In [11], a two-degree of freedom fractional-order proportional–integral–derivative (2-DOF FOPID) controller structure for a two-link planar rigid robotic manipulator with payload for trajectory tracking task is proposed. The tuning of the controller parameters is realized using cuckoo search algorithm (CSA). The performance of the proposed 2-DOF FOPID controllers is compared with those of their integer-order designs, i.e. 2-DOF PID controllers, and with the classical PID controllers. Another work for robot manipulators with continuous fractional-order nonsingular terminal sliding-mode (CFONTSM) based on time delay estimation (TDE) is presented in [12]. The simulation and experiment results show that the proposed control design can ensure higher tracking precision and faster convergence compared with TDE-based continuous integer-order NTSM (CIONTSM) design in a wide range of speed. Meanwhile, better performance is also observed compared with TDE-based IONTSM and FONTSM control designs using boundary layer technique.

A non-linear adaptive fractional-order fuzzy proportional–integral–derivative (NLA-FOFPID) controller to control a 2-link planar rigid robotic manipulator with payload is studied in [13]. The gains of the controllers are optimized using a Backtracking Search Algorithm. Several simulations were performed to assess the performances of NLA–FPID (integer case) and NLA–FOFPID controllers for servo and regulatory mode. It has been observed that NLA–FOFPID controller outperforms NLA–FPID controller by offering much better performance. Particularly, in an uncertain environment it offered very robust behaviour as compared to NLA–FPID controller. Similarly, a fractional-order self-organizing fuzzy controller (FOSOFC) to control a two-link electrically-driven rigid robotic (EDRR) manipulator system is proposed in [14]. The simulation results show the effective behaviour of the FOSOFC, the obtained performance is compared with fractional-order fuzzy proportional–integral and derivative (FOFPID) controller for trajectory tracking as well as the disturbance rejection study. A recent work about the fuzzy fractional-order control of robotic manipulators with PID error manifolds is proposed in [15]. The simulation results demonstrated the reliability of the proposed structure, and comparisons with respect to classical, discontinuous and continuous, sliding-mode controllers highlighted the superiority of the proposed method.

On the other hand, the mechatronic systems represent one of the most challenging control applications due to their interdisciplinary nature. For example, in [91] a method for tuning and designing fractional-order controllers for a class of second-order unstable processes, using stability analysis principles is studied. The experimental results suggest that the fractional-order controller is able to reduce the oscillatory behaviour and achieve a fast settling time and a zero steady-state error. Similarly, the performance of a magnetic levitation system (MLS) with fractional-order proportional–integral–derivative (FO-PID) controller and integer-order PID controller for a particular position of the levitated object is studied [16]. The controller parameters are tuned using dynamic particle swarm optimization (dPSO) technique. Effectiveness of the proposed control scheme is verified by simulation and experimental results.

A recent article about a new model-free fractional-order sliding-mode control (MFFOSMC) based on an extended state observer (ESO) for a quarter car active suspension systems is presented [17]. In which, the main goal is to increase the ride comfort while the dynamic wheel load and the suspension deflection remain within safety-critical bounds. The simulation results demonstrate the effectiveness of the proposed controller, a comparison with classical PID, time delay estimation control, and intelligent PID controller has been performed.

1.5 Book Summary

In this book, we present a comprehensive and unified approach to use vision-based feedback in fractional-order control design algorithms to achieve intrinsic loop robustness and enable loop performance specifications. The book is structured in 10 Chapters ordered as follows after this introductory chapter.

Chapter 2
In this chapter, an overview of the visual servoing concept is presented. The first part shows the representation of position and orientation of an object in 2- and 3-dimensional environment. A coordinate frame is used to describe a set of points attached to a specific object. The pose of a coordinate frame can be described with respect to other coordinate frames and the transformation between two frames is given by a homogeneous transformation matrix. In the second part of this chapter, the fundamentals of visual-based control architecture were presented. From the perspective of visual sensor location, we can identify two configurations: (i) eye-in-hand, the camera is mounted on the robot's TCP; (ii) eye-to-hand, the camera is fixed in the working environment. Based on the type of the control structure there are two main architectures: (i) Image-Based Visual Servoing, the controller is designed directly in the image plane and (ii) Position-Based Visual Servoing, the controller is designed using pose estimation based on camera calibration and 3D target model.

Chapter 3
The performance of the visual servoing system is highly connected with the type and the performance of the visual features. In this chapter, the geometric and photometric visual features were considered. In order to extract the geometric visual features two point features operator have been presented: Harris operator and SIFT descriptor,

while photometric features were described by image moments. From the class of geometric features, point features are mostly used in visual servoing applications. One of the main disadvantages of this type of features is that the point features operator does not provide stable features for these types of applications. The number of point features from the first image should be the same on the entire sequence of images. Apart from the stability of point features are other criteria (repeatability, accuracy, features spread) that need to be fulfilled in order to obtain good performance. This problem of point features can be eliminated by using image moments which allow a general representation of an image and also allow the description of a more complex object. The performance evaluation of this type of features can be realized using a criterion based on Hausdorff distance.

Chapter 4
Fractional-order controllers have seen a rapid increase regarding research interest due to the numerous advantages they offer, including better closed-loop dynamics and robustness. In this chapter, the basic concepts of fractional systems are covered as an introduction to more complex issues tackled in the subsequent chapters of the book. Tuning rules for fractional-order PI/PD/PID controllers are given, as well as illustrative examples, covering applications from mechatronic systems. To demonstrate the advantages of using a fractional-order controller instead of the traditional integer-order controllers three examples are covered: an unstable magnetic levitation system, a velocity system, as well as a position system. The comparative simulation results, in closed-loop, show that indeed the fractional- order controllers provide improved performance and increased robustness.

Chapter 5
Here, the tuning rules detailed and exemplified in Chap. 4 are extended for the case of multivariable systems, more specifically for the case of the two-inputs-two-outputs systems. First of all, a short review of classical control techniques for multivariable processes is presented followed by a brief analysis of interaction and proper pairing of input–output signals. Then, a multivariable fractional-order controller design is presented for a two-inputs-two-outputs process, with the tuning rules and methodology based on a modification of classical integer-order multivariable PID controller design.

Chapter 6
In this chapter, a new proportional control law which includes the dynamic model of the manipulator robot, the robot is modelled as a 'Virtual Cartesian Motion Device', has been developed. Performance evaluation of the servoing system was performed with respect to the type of visual features used to design the visual control law. The implementation, testing and validation of the control algorithm was achieved through development of a simulator for servoing systems. The simulators developed within this chapter allow the use of both types of visual features, point features as well as image moments. Starting from this simulator a control architecture for real-time servoing applications has been developed. In the first stage, the real-time control architecture was designed for an ABB-IRB2400 robot manipulator with an eye-in-hand camera configuration and makes use of point features extracted with Harris and

SIFT operators. In the second phase, a real-time control architecture for a FANUC Arc Mate 120 robot manipulator was implemented. Although the proportional controller is one of the simplest controllers that can be designed, the experimental results indicate that the proportional control law achieves satisfactory performance for servoing applications. In order to improve the performance of the visual servoing system, a more complex control architecture such as predictive controller [147, 150, 159–161] can be designed, but this was out of the scope of this book.

Chapter 7
The chapter presents two case studies regarding the implementation of fractional-order controllers on real-time targets. The chapter also includes the implementation steps for digital control systems and discusses pitfalls on the actual implementation of fractional-order controllers on dedicated real-time devices. The advantages of using fractional-order controllers over the traditional integer-order controllers are explained and validated using two examples, including an open-loop unstable system. A full fractional-order PI controller design is detailed in the last part on a real-life application, i.e. a modular servo system. The experimental results indicate the accuracy, efficiency and robustness of the tuned fractional-order controller which outperforms the classical control. In this chapter, a digital implementation on a field-programmable gate array (FPGA) device is also presented for the control of a velocity system, using a fractional-order PI controller.

Chapter 8
In this chapter two case studies regarding the implementation of a fractional-order controller on servoing system are presented. For the first case, fractional-order PD controllers are implemented onto a ball and plate system. By changing the experimental conditions, i.e. changing the ball, we show that the fractional-order controller is more suitable for this process. This result is very important since in the real-life applications as most of the mechatronic systems, the practical process can be different from the theoretical models. For the second case, an image-based control law based on fractional calculus applied to visual servoing systems has been presented. The considered control architecture consists of a 6 d.o.f. manipulator robot with an eye-in-hand configuration. To evaluate the performances of the proposed fractional-order controller, a real-time implementation using MATLAB® and a Fanuc robot was performed. Different experiments for planar static objects were conducted using both, fractional-order PI^{μ} and integer-order PI controllers. For both case studies considered in this chapter, the experimental results show that the control law based on fractional-order obtains better performances in comparison with classical control law.

Chapter 9
In this chapter, an application of a sliding-mode control strategy with a generalized dynamic reference trajectory is presented. The results have been tested in realistic simulations from a field test case of combine harvester with spout angle control for optimal filling of crop reservoirs. While the implementation of the sliding-mode control strategy is not (much) more demanding than that of linear controllers, its

performance is much more robust in changing dynamic conditions of the real process. Other applications with similar features can be considered, without loss of generality.

Chapter 10
Here we give a summary of the main features of the book in terms of scientific content. The given theory background and exemplified applications are integrated in a broader concept of bringing forward new emerging tools in the area of mechatronics.

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Part I
Visual Servoing Systems and Fractional-Order Control

Chapter 2
Visual Servoing Systems

2.1 Rigid Body Pose

A fundamental requirement in robotics and computer vision is to represent the position and orientation of objects in an environment whereas the latter can be static or dynamic. Such objects may include robots, cameras, workpieces, obstacles and paths. A frame of coordinates, also known as a Cartesian coordinate system, is a set of orthogonal axes which intersect at a point defined as the origin. A rigid body (under the assumption that for any transformation its constituent points maintain a constant relative position with respect to the object's coordinate frame) is completely described in space by its position and orientation (in short further defined as *pose*) with respect to a reference frame (i.e. a fixed coordinate frame of the environment).

2.1.1 2D Pose Description

A 2-dimensional (2D) environment, or plane, is usually described by a Cartesian coordinate frame with orthogonal axes denoted by X and Y and typically drawn with the X-axis horizontal and the Y-axis vertical. Unit vectors parallel to the axes are denoted by $\vec{i}$ and $\vec{j}$. A point is represented by its X- and Y-coordinates $[p_X \ \ p_Y]^{\mathrm{T}}$ or as a bound vector:

$$\mathbf{p} = p_X \vec{i} + p_Y \vec{j} \tag{2.1}$$

Regarding orientation description, the most common approach is to use rotations that are rigid transformations. For the 2D environment, rotations about the origin can be represented by 2×2 matrices of the form:

$$\mathbf{R} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \tag{2.2}$$

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The main properties of rotation matrices can be summarized as follows:

- a rotation of 0 radians will have no effect because the rotation matrix yields the identity matrix $\mathbf{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- two successive rotations can be decomposed as matrix multiplication;
- the inverse of a rotation matrix is its transpose $\mathbf{R}\mathbf{R}^{\mathrm{T}} = 1$;
- the determinant of rotation matrix is equal to 1 which means that $\mathbf{R}$ belongs to the special orthogonal group of dimension 2 or equivalently $\mathbf{R} \in SO_2 \subset \mathbb{R}^{2\times 2}$.

It is interesting to observe that instead of representing an angle, which is a scalar, we have used a 2×2 matrix that comprises four elements, although these elements are not independent. Each column has a unit magnitude which provides two constraints. The columns are orthogonal which provide another constraint. Four elements and three constraints are effectively one independent value. The rotation matrix is an example of a non-minimum representation and the disadvantages such as the increased memory it requires are outweighed by its advantages such as composability [48, 49].

Regarding position description, we need to analyse translations. A translation can be represented by a vector. If the translation vector is $\mathbf{t} = [t_X \ \ t_Y]$, then

$$\mathbf{p}' = \mathbf{p} + \mathbf{t} \tag{2.3}$$

gives the new position of the point under analysis.

A general rigid body transformation is given by a pair $(\mathbf{R}(\theta), \mathbf{t})$. These pairs have the following effect on the pose of a position vector:

$$\mathbf{p}' = \mathbf{R}(\theta)\mathbf{p} + \mathbf{t} \tag{2.4}$$

or, if we prefer the homogeneous coordinates representation:

$$\begin{bmatrix} p'_X \\ p'_Y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & t_X \\ \sin(\theta) & \cos(\theta) & t_Y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_X \\ p_Y \\ 1 \end{bmatrix} \tag{2.5}$$

The matrix has a very specific structure and belongs to the special Euclidean group of dimension 2.

2.1.2 3D Pose Description

The 3-dimensional (3D) case is an extension of the 2D case discussed in the previous section. We add an extra coordinate axis, typically denoted by Z, that is orthogonal to both the X- and Y-axes. The direction of the Z-axis obeys the right-hand rule and