## CHAPTER 3

## COST OF CAPITAL

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## LEARNING OUTCOMES

After completing this chapter, you will be able to do the following:

- Determine and interpret the weighted average cost of capital (WACC) of a company, and explain the adjustments to it that an analyst should make in developing a cost of capital for a specific project.
- Describe the role of taxes in the cost of capital from the different capital sources.
- Describe alternative methods of calculating the weights used in the WACC, including the use of the company's target capital structure.
- Explain the analyst's concern with the marginal cost of capital in evaluating investment projects, and explain the use of the marginal cost of capital and the investment opportunity schedule in determining the optimal capital budget for a company.
- Explain the marginal cost of capital's role in determining the net present value of a project.
- Calculate and analyze the cost of fixed-rate debt capital using the yield-to-maturity approach and the debt-rating approach.
- Calculate the cost of noncallable, nonconvertible preferred stock.
- Calculate and analyze the cost of equity capital using the capital asset pricing model approach, the dividend discount approach, and the bond yield plus risk premium approach.
- Calculate an unlevered beta using the pure-play method and use this unlevered beta to estimate a levered beta for a project or company.
- Explain the country risk premium in the estimation of the cost of equity for a company situated in a developing market.
- Describe the marginal cost of capital schedule, explain why it may be upward sloping with respect to additional capital, and calculate and interpret its break points.
- Explain and demonstrate the correct treatment of flotation costs.


## 1. INTRODUCTION

A company grows by making investments that are expected to increase revenues and profits. The company acquires the capital or funds necessary to make such investments by borrowing or using funds from owners. By applying this capital to investments with long-term benefits, the company is producing value today, but how much value? The answer depends not only on the investments' expected future cash flows but also on the cost of the funds. Borrowing is not costless. Neither is using owners' funds.

The cost of this capital is an important ingredient both in investment decision making by the company's management and in the valuation of the company by investors. If a company invests in projects that produce a return in excess of the cost of capital, the company has created value; in contrast, if the company invests in projects whose returns are less than the cost of capital, the company has actually destroyed value. Therefore, the estimation of the cost of capital is a central issue in corporate financial management. For the analyst seeking to evaluate a company's investment program and its competitive position, an accurate estimate of a company's cost of capital is important as well.

Cost of capital estimation is a challenging task. As we have already implied, the cost of capital is not observable but rather must be estimated. Arriving at a cost of capital estimate requires a host of assumptions and estimates. Another challenge is that the cost of capital that is appropriately applied to a specific investment depends on the characteristics of that investment: The riskier the investment's cash flows, the greater its cost of capital will be. In reality, a company must estimate project-specific costs of capital. What is often done, however, is to estimate the cost of capital for the company as a whole and then adjust this overall corporate cost of capital upward or downward to reflect the risk of the contemplated project relative to the company's average project.

This chapter is organized as follows: In the next section, we introduce the cost of capital and its basic computation. Section 3 presents a selection of methods for estimating the costs of the various sources of capital, and Section 4 discusses issues an analyst faces in using the cost of capital. Section 5 summarizes the chapter.

## 2. COST OF CAPITAL

The cost of capital is the rate of return that the suppliers of capital-bondholders and ownersrequire as compensation for their contribution of capital. Another way of looking at the cost of capital is that it is the opportunity cost of funds for the suppliers of capital: A potential supplier
of capital will not voluntarily invest in a company unless its return meets or exceeds what the supplier could earn elsewhere in an investment of comparable risk.

A company typically has several alternatives for raising capital, including issuing equity, debt, and instruments that share the characteristics of debt and equity. Each source selected becomes a component of the company's funding and has a cost (required rate of return) that may be called a component cost of capital. Because we are using the cost of capital in the evaluation of investment opportunities, we are dealing with a marginal cost-what it would cost to raise additional funds for the potential investment project. Therefore, the cost of capital that the investment analyst is concerned with is a marginal cost.

Let us focus on the cost of capital for the entire company (later we will address how to adjust that for a specific project). The cost of capital of a company is the required rate of return that investors demand for the average-risk investment of a company. The most common way to estimate this required rate of return is to calculate the marginal cost of each of the various sources of capital and then calculate a weighted average of these costs. This weighted average is referred to as the weighted average cost of capital (WACC). The WACC is also referred to as the marginal cost of capital (MCC) because it is the cost that a company incurs for additional capital. The weights in this weighted average are the proportions of the various sources of capital that the company uses to support its investment program. Therefore, the WACC, in its most general terms, is

$$
\begin{equation*}
\text { WACC }=w_{d} r_{d}(1-t)+w_{p} r_{p}+w_{e} r_{e} \tag{3-1}
\end{equation*}
$$

where
$w_{d}=$ proportion of debt that the company uses when it raises new funds
$r_{d}=$ before-tax marginal cost of debt
$t=$ company's marginal tax rate
$w_{p}=$ proportion of preferred stock the company uses when it raises new funds
$r_{p}=$ marginal cost of preferred stock
$w_{e}=$ proportion of equity that the company uses when it raises new funds
$r_{e}=$ marginal cost of equity

## EXAMPLE 3-1 Computing the Weighted Average Cost of Capital

Assume that ABC Corporation has the following capital structure: 30 percent debt, 10 percent preferred stock, and 60 percent common stock. ABC Corporation wishes to maintain these proportions as it raises new funds. Its before-tax cost of debt is 8 percent, its cost of preferred stock is 10 percent, and its cost of equity is 15 percent. If the company's marginal tax rate is 40 percent, what is ABC's weighted average cost of capital?

## Solution

The weighed average cost of capital is

$$
\begin{aligned}
\mathrm{WACC} & =(0.3)(0.08)(1-0.40)+(0.1)(0.1)+(0.6)(0.15) \\
& =11.44 \text { percent }
\end{aligned}
$$

There are important points concerning the calculation of the WACC, as shown in Equation 3-1, that the analyst must be familiar with. The next two sections address two key issues: taxes and the selection of weights.

### 2.1. Taxes and the Cost of Capital

Notice that in Equation 3-1 we adjust the expected before-tax cost on new debt financing, $r_{d}$, by a factor of $(1-t)$. In the United States and many other tax jurisdictions, the interest on debt financing is a deduction to arrive at taxable income. Taking the tax deductibility of interest as the base case, we adjust the pretax cost of debt for this tax shield. Multiplying $r_{d}$ by ( $1-t$ ) results in an estimate of the after-tax cost of debt.

For example, suppose a company pays $€ 1$ million in interest on its $€ 10$ million of debt. The cost of this debt is not $€ 1$ million because this interest expense reduces taxable income by $€ 1$ million, resulting in a lower tax. If the company is subject to a tax rate of 40 percent, this $€ 1$ million of interest costs the company ( $€ 1$ million) $(1-0.4)=€ 0.6$ million because the interest reduces the company's tax bill by $€ 0.4$ million. In this case, the before-tax cost of debt is 10 percent, whereas the after-tax cost of debt is ( $€ 0.6$ million)/( $€ 10$ million) $=6$ percent.

Estimating the cost of common equity capital is more challenging than estimating the cost of debt capital. Debt capital involves a stated legal obligation on the part of the company to pay interest and to repay the principal on the borrowing. Equity entails no such obligation. Estimating the cost of conventional preferred equity is rather straightforward because the dividend is generally stated and fixed, but estimating the cost of common equity is challenging. There are several methods available for estimating the cost of common equity, and we discuss two in this chapter. The first method uses the capital asset pricing model (CAPM), and the second method uses the dividend discount model, which is based on discounted cash flows. No matter the method, there is no need to make any adjustment in the cost of equity for taxes because the payments to owners, whether in the form of dividends or the return on capital, are not tax deductible for the company.

## EXAMPLE 3-2 Incorporating the Effect of Taxes on the Costs of Capital

Jorge Ricard, a financial analyst, is estimating the costs of capital for the Zeale Corporation. In the process of this estimation, Ricard has estimated the before-tax costs of capital for Zeale's debt and equity as 4 percent and 6 percent, respectively. What are the after-tax costs of debt and equity if Zeale's marginal tax rate is

1. 30 percent?
2. 48 percent?

## Solutions

|  | Marginal Tax Rate | After-Tax Cost of Debt | After-Tax Cost of Equity |
| :--- | :---: | :---: | :---: |
| 1. | 30 percent | $0.04(1-0.30)=2.80$ percent | 6 percent |
| 2. | 48 percent | $0.04(1-0.48)=2.08$ percent | 6 percent |

Note: There is no adjustment for taxes in the case of equity; the before-tax cost of equity is equal to the after-tax cost of equity.

### 2.2. Weights of the Weighted Average

How do we determine what weights to use? Ideally, we want to use the proportion of each source of capital that the company would use in the project or company. If we assume that a company has a target capital structure and raises capital consistent with this target, we should use this target capital structure. The target capital structure is the capital structure that a company is striving to obtain. ${ }^{1}$ If we know the company's target capital structure, then, of course, we should use this in our analysis. Someone outside the company, however, such as an analyst, typically does not know the target capital structure and must estimate it using one of several approaches:

1. Method 1: Assume the company's current capital structure, at market value weights for the components, represents the company's target capital structure.
2. Method 2: Examine trends in the company's capital structure or statements by management regarding capital structure policy to infer the target capital structure.
3. Method 3: Use averages of comparable companies' capital structures as the target capital structure.

In the absence of knowledge of a company's target capital structure, we may take Method 1 as the baseline. Note that in applying Method 3, we use unweighted, arithmetic average, as is often done for simplicity. An alternative is to calculate a weighted average, which would give more weight to larger companies.

Suppose we are using the company's current capital structure as a proxy for the target capital structure. In this case, we use the market value of the different capital sources in the calculation of these proportions. For example, assume a company has the following market values for its capital:

| Bonds outstanding | $\$ 5$ million |
| :--- | ---: |
| Preferred stock | 1 million |
| Common stock | 14 million |
|  | $\$ 20$ million |

The weights that we apply would be

$$
\begin{aligned}
& w_{d}=0.25 \\
& w_{p}=0.05 \\
& w_{e}=0.70
\end{aligned}
$$

Example 3-3 illustrates the estimation of weights. Note that a simple way of transforming a debt-to-equity ratio $(D / E)$ into a weight-that is, $D /(D+E)$-is to divide $D / E$ by $(1+D / E)$.

[^0]
## EXAMPLE 3-3 Estimating the Proportions of Capital

Fin Anziell is a financial analyst with Analytiker Firma. Anziell is in the process of estimating the cost of capital of Gewicht GmbH . The following information is provided:

| Gewicht GmbH |  |
| :--- | :--- |
| Market value of debt | $€ 50$ million |
| Market value of equity | $€ 60$ million |

Primary competitors and their capital structures (in millions):
Competitor Market Value of Debt Market Value of Equity

A
B
C
€ 25
€ 101
$£ 40$
$€ 50$
€ 190
£60

What are Gewicht's proportions of debt and equity that Anziell would use if estimating these proportions using the company's

1. Current capital structure?
2. Competitors' capital structure?

Suppose Gewicht announces that a debt-to-equity ratio of 0.7 reflects its target capital structure.
3. What weights should Anziell use in the cost-of-capital calculations?

## Solution to 1

Current capital structure

$$
\begin{aligned}
& w_{d}=\frac{€ 50 \text { million }}{€ 50 \text { million }+€ 60 \text { million }}=0.4545 \\
& w_{e}=\frac{€ 60 \text { million }}{€ 50 \text { million }+€ 60 \text { million }}=0.5454
\end{aligned}
$$

## Solution to 2

Competitors' capital structure ${ }^{2}$

$$
w_{d}=\frac{\left(\frac{€ 25}{€ 25+€ 50}\right)+\left(\frac{€ 101}{€ 101+€ 190}\right)+\left(\frac{£ 40}{£ 40+£ 60}\right)}{3}=0.3601
$$

[^1]$$
w_{e}=\frac{\left(\frac{€ 50}{€ 25+€ 50}\right)+\left(\frac{€ 190}{€ 101+€ 190}\right)+\left(\frac{£ 60}{£ 40+£ 60}\right)}{3}=0.6399
$$

## Solution to 3

A debt-to-equity ratio of 0.7 represents a weight on debt of $0.7 / 1.7=0.4118$ so that $w_{d}=0.4118$ and $w_{e}=1-0.4118=0.5882$. These would be the preferred weights to use in a cost-of-capital calculation.

### 2.3. Applying the Cost of Capital to Capital Budgeting and Security Valuation

With some insight now into the calculation of the cost of capital, let us continue to improve our understanding of the roles it plays in financial analysis. A chief use of the marginal cost of capital estimate is in capital budgeting decision making. What role does the marginal cost of capital play in a company's investment program, and how do we adapt it when we need to evaluate a specific investment project?

A company's marginal cost of capital (MCC) may increase as additional capital is raised, whereas returns to a company's investment opportunities are generally believed to decrease as the company makes additional investments, as represented by the investment opportunity schedule (IOS). ${ }^{3}$ We show this relation in Exhibit 3-1, graphing the upward-sloping marginal cost of capital schedule against the downward-sloping investment opportunity schedule. In the context of a company's investment decision, the optimal capital budget is the amount of capital raised and invested at which the marginal cost of capital is equal to the marginal return from investing. In other words, the optimal capital budget occurs when the marginal cost of capital intersects with the investment opportunity schedule as seen in Exhibit 3-1.

EXHIBIT 3-1 Optimal Investment Decision


[^2]The relation between the MCC and the IOS provides a broad picture of the basic decision-making problem of a company. However, we are often interested in valuing an individual project or even a portion of a company, such as a division or product line. In these applications, we are interested in the cost of capital for the project, product, or division as opposed to the cost of capital for the company overall. The cost of capital in these applications should reflect the riskiness of the future cash flows of the project, product, or division. For an average-risk project, the opportunity cost of capital is the company's WACC. If the systematic risk of the project is above or below average relative to the company's current portfolio of projects, an upward or downward adjustment, respectively, is made to the company's WACC. Companies may take an ad hoc or a systematic approach to making such adjustments. The discussion of a systematic approach is a somewhat advanced topic that we defer to Section 4.1.

The WACC or MCC corresponding to the average risk of the company, adjusted appropriately for the risk of a given project, plays a role in capital budgeting decision making based on the net present value (NPV) of that project. Recall from the capital budgeting chapter that the NPV is the present value of all the project cash flows. It is useful to think of it as the difference between the present value of the cash inflows, discounted at the opportunity cost of capital applicable to the specific project, and the present value of the cash outflows, discounted using that same opportunity cost of capital:

$$
\text { NPV }=\text { Present value of inflows }- \text { Present value of outflows }
$$

If an investment's NPV is positive, the company should undertake the project. If we choose to use the company's WACC in the calculation of the NPV of a project, we are assuming that the project

- Has the same risk as the average-risk project of the company, and
- Will have a constant target capital structure throughout its useful life. ${ }^{4}$

These may not be realistic or appropriate assumptions and are potential drawbacks to using the company's WACC in valuing projects. However, alternative approaches are subject to drawbacks as well, and the approach outlined has wide acceptance. ${ }^{5}$

For the analyst, the second key use of the marginal cost of capital is in security valuation using any one of several discounted cash flow valuation models available. ${ }^{6}$ For a particular valuation model, if these cash flows are cash flows to the company's suppliers of capital (that is, free cash flow to the firm), the analyst uses the weighted average cost of capital of the company in the valuation. ${ }^{7}$ If these cash flows are strictly those belonging to the company's

[^3]owners, such as the free cash flow to equity, or dividends, the analyst uses the cost of equity capital to find the present value of these flows. ${ }^{8}$

In the next section, we discuss how an analyst may approach the calculation of the component costs of capital, focusing on debt, preferred stock, and common equity.

## 3. COSTS OF THE DIFFERENT SOURCES OF CAPITAL

Each source of capital has a different cost because of the differences among the sources, such as seniority, contractual commitments, and potential value as a tax shield. We focus on the costs of three primary sources of capital: debt, preferred equity, and common equity.

### 3.1. Cost of Debt

The cost of debt is the cost of debt financing to a company when it issues a bond or takes out a bank loan. We discuss two methods to estimate the before-tax cost of debt, $r_{d}$ the yield-to-maturity approach and debt-rating approach.
3.1.1. Yield-to-Maturity Approach The yield to maturity (YTM) is the annual return that an investor earns on a bond if the investor purchases the bond today and holds it until maturity. In other words, it is the yield, $r_{d}$, that equates the present value of the bond's promised payments to its market price:
$P_{0}=\frac{P M T_{1}}{\left(1+\frac{r_{d}}{2}\right)}+\ldots+\frac{P M T_{n}}{\left(1+\frac{r_{d}}{2}\right)^{n}}+\frac{F V}{\left(1+\frac{r_{d}}{2}\right)^{n}}=\left(\sum_{t=1}^{n} \frac{P M T_{i}}{\left(1+\frac{r_{d}}{2}\right)^{t}}\right)+\frac{F V}{\left(1+\frac{r_{d}}{2}\right)^{n}}$
where
$P_{0}=$ current market price of the bond
$P M T_{t}=$ interest payment in period $t$
$r_{d}=$ yield to maturity ${ }^{9}$
$n=$ number of periods remaining to maturity
$F V=$ maturity value of the bond
This valuation equation assumes that the bond pays semiannual interest and that any intermediate cash flows (in this case, the interest prior to maturity) are reinvested at the rate $r_{d} / 2$.

Example 3-4 illustrates the calculation of the after-tax cost of debt.

[^4]
## EXAMPLE 3-4 Calculating the After-Tax Cost of Debt

Valence Industries issues a bond to finance a new project. It offers a 10 -year, 5 percent semiannual coupon bond. Upon issue, the bond sells at $\$ 1,025$. What is Valence's beforetax cost of debt? If Valence's marginal tax rate is 35 percent, what is Valence's after-tax cost of debt?

## Solution

Given:

$$
\begin{aligned}
& P V=\$ 1,025 \\
& F V=\$ 1,000 \\
& P M T=5 \text { percent of } 1,000 \div 2=\$ 25 \\
& n=10 \times 2=20 \\
& \$ 1,025=\left(\sum_{t=1}^{20} \frac{\$ 25}{(1+i)}\right)+\frac{\$ 1,000}{(1+i)^{20}}
\end{aligned}
$$

Use a financial calculator to solve for $i$, the six-month yield. Because $i=2.342$ percent, the before-tax cost of debt is $r_{d}=2.342$ percent $\times 2=4.684$ percent, and Valence's after-tax cost of debt is $r_{d}(1-t)=0.04684(1-0.35)=0.03045$, or 3.045 percent.
3.1.2. Debt-Rating Approach When a reliable current market price for a company's debt is not available, the debt-rating approach can be used to estimate the before-tax cost of debt. Based on a company's debt rating, we estimate the before-tax cost of debt by using the yield on comparably rated bonds for maturities that closely match that of the company's existing debt.

Suppose a company's capital structure includes debt with an average maturity (or duration) of 10 years and the company's marginal tax rate is 35 percent. If the company's rating is AAA and the yield on debt with the same debt rating and similar maturity (or duration) is 4 percent, the company's after-tax cost of debt is ${ }^{10}$

$$
\begin{aligned}
r_{d}(1-t) & =4 \text { percent }(1-0.35) \\
& =2.6 \text { percent }
\end{aligned}
$$

A consideration when using this approach is that debt ratings are ratings of the debt issue itself, with the issuer being only one of the considerations. Other factors, such as debt seniority and security, also affect ratings and yields; so care must be taken to consider the likely type of debt to be issued by the company in determining the comparable debt rating and yield. The debt-rating approach is a simple example of pricing on the basis of valuation-relevant characteristics, which in bond markets has been known as evaluated pricing or matrix pricing.

[^5]
### 3.1.3. Issues in Estimating the Cost of Debt

3.1.3.1. Fixed-Rate Debt Versus Floating-Rate Debt Up to now, we have assumed that the interest on debt is a fixed amount each period. We can observe market yields of the company's existing debt or market yields of debt of similar risk in estimating the before-tax cost of debt. However, the company may also issue floating-rate debt in which the interest rate adjusts periodically according to a prescribed index, such as the prime rate or LIBOR, over the life of the instrument.

Estimating the cost of a floating-rate security is difficult because the cost of this form of capital over the long term depends not only on the current yields but also on the future yields. The analyst may use the current term structure of interest rates and term structure theory to assign an average cost to such instruments.
3.1.3.2. Debt with Optionlike Features How should an analyst determine the cost of debt when the company uses debt with optionlike features, such as call, conversion, or put provisions? Clearly, options affect the value of debt. For example, a callable bond would have a yield greater than a similar noncallable bond of the same issuer because bondholders want to be compensated for the call risk associated with the bond. In a similar manner, the put feature of a bond, which provides the investor with an option to sell the bond back to the issuer at a predetermined price, has the effect of lowering the yield on a bond below that of a similar nonputable bond.

If the company already has outstanding debt that incorporates optionlike features that the analyst believes are representative of the future debt issuance of the company, the analyst may simply use the yield to maturity on such debt in estimating the cost of debt.

If the analyst believes that the company will add or remove option features in future debt issuance, the analyst can make market value adjustments to the current YTM to reflect the value of such additions and/or deletions. The technology for such adjustments is an advanced topic that is outside the scope of this chapter. ${ }^{11}$
3.1.3.3. Nonrated Debt If a company does not have any debt outstanding or if the yields on the company's existing debt are not available, the analyst may not always be able to use the yield on similarly rated debt securities. It may be the case that the company does not have rated bonds. Though researchers offer approaches for estimating a company's "synthetic" debt rating based on financial ratios, these methods are imprecise because debt ratings incorporate not only financial ratios but also information about the particular bond issue and the issuer that are not captured in financial ratios.
3.1.3.4. Leases A lease is a contractual obligation that can substitute for other forms of borrowing. This is true whether the lease is an operating lease or a capital lease, though only the capital lease is represented as a liability on the company's balance sheet. ${ }^{12}$ If the company uses leasing as a source of capital, the cost of these leases should be included in the cost of capital. The cost of this form of borrowing is similar to that of the company's other longterm borrowing.

[^6]
### 3.2. Cost of Preferred Stock

The cost of preferred stock is the cost that a company has committed to pay preferred stockholders as a preferred dividend when it issues preferred stock. In the case of nonconvertible, noncallable preferred stock that has a fixed dividend rate and no maturity date (fixed rate perpetual preferred stock), we can use the formula for the value of a preferred stock:

$$
P_{p}=\frac{D_{p}}{r_{p}}
$$

where
$P_{p}=$ current preferred stock price per share
$D_{p}=$ preferred stock dividend per share
$r_{P}=$ cost of preferred stock
We can rearrange this equation to solve for the cost of preferred stock:

$$
\begin{equation*}
r_{p}=\frac{D_{p}}{P_{p}} \tag{3-3}
\end{equation*}
$$

Therefore, the cost of preferred stock is the preferred stock's dividend per share divided by the current preferred stock's price per share. Unlike interest on debt, the dividend on preferred stock is not tax deductible by the company; therefore, there is no adjustment to the cost for taxes. ${ }^{13}$

A preferred stock may have a number of features that affect the yield and hence the cost of preferred stock. These features include a call option, cumulative dividends, participating dividends, adjustable-rate dividends, or convertibility into common stock. When estimating a yield based on current yields of the company's preferred stock, we must make appropriate adjustments for the effects of these features on the yield of an issue. For example, if the company has callable, convertible preferred stock outstanding, yet it is expected that the company will issue only noncallable, nonconvertible preferred stock in the future, we would have to either use the current yields on comparable companies' noncallable, nonconvertible preferred stock or estimate the yield on preferred equity using methods outside the scope of this chapter. ${ }^{14}$
distinguishes between operating and finance leases, another term for capital-type leases.) These two forms of leases are distinguished on the basis of ownership transference, the existence of a bargain purchase option, the term of the lease relative to the economic life of the asset, and the present value of the lease payments relative to the value of the asset. In either case, however, the lease obligation is a form of borrowing, even though it is only in the case of a capital lease that the obligation appears as a liability on the company's balance sheet. The discount rate applied in the valuation of a capital lease is the rate of borrowing at the time of the lease commencement; therefore, it is reasonable to apply the company's long-term borrowing rate when estimating the cost of capital for leasing. ${ }^{13}$ This is not to be confused, however, with the dividends received deduction, which reduces the effective tax on intercorporate preferred dividends received.
${ }^{14} \mathrm{~A}$ method for estimating this yield involves first estimating the option-adjusted spread (OAS). For further information on the OAS, see, for example, Fabozzi (2004).

## EXAMPLE 3-5 Calculating the Cost of Preferred Equity

Alcoa, Inc. has one class of preferred stock outstanding, a $\$ 3.75$ cumulative preferred stock, for which there are 546,024 shares outstanding. ${ }^{15}$ If the price of this stock is $\$ 72$, what is the estimate of Alcoa's cost of preferred equity?

## Solution

The cost of Alcoa's preferred stock $=\$ 3.75 / \$ 72.00=5.21$ percent.

## EXAMPLE 3-6 Choosing the Best Estimate of the Cost of Preferred Equity

Wim Vanistendael is finance director of De Gouden Tulip N.V., a leading Dutch flower producer and distributor. He has been asked by the CEO to calculate the cost of preferred equity and has recently obtained the following information:

- The issue price of preferred stock was $€ 3.5$ million and the preferred dividend is 5 percent.
- If the company issued new preferred stock today, the preferred coupon rate would be 6.5 percent.
- The company's marginal tax rate is 30.5 percent.

What is the cost of preferred equity for De Gouden Tulip N.V.?

## Solution

If De Gouden Tulip were to issue new preferred stock today, the coupon rate would be close to 6.5 percent. The current terms thus prevail over the past terms when evaluating the actual cost of preferred stock. The cost of preferred stock for De Gouden Tulip is therefore 6.5 percent. Because preferred dividends offer no tax shield, there is no adjustment made based on the marginal tax rate.

### 3.3. Cost of Common Equity

The cost of common equity, $r_{e}$, usually referred to simply as the cost of equity, is the rate of return required by a company's common shareholders. A company may increase common equity through the reinvestment of earnings-that is, retained earnings-or through the issuance of new shares of stock.

[^7]As discussed earlier, the estimation of the cost of equity is challenging because of the uncertain nature of the future cash flows in terms of the amount and timing. Commonly used approaches for estimating the cost of equity include the capital asset pricing model, the dividend discount model, and the bond yield plus risk premium method.
3.3.1. Capital Asset Pricing Model Approach In the capital asset pricing model (CAPM) approach, we use the basic relationship from the capital asset pricing model theory that the expected return on a stock, $E\left(R_{i}\right)$, is the sum of the risk-free rate of interest, $R_{F}$, and a premium for bearing the stock's market risk, $\beta_{i}\left(R_{M}-R_{F}\right)$ :

$$
\begin{equation*}
E\left(R_{i}\right)=R_{F}+\beta_{i}\left[E\left(R_{M}\right)-R_{F}\right] \tag{3-4}
\end{equation*}
$$

where
$\beta_{i}=$ return sensitivity of stock $i$ to changes in the market return
$E\left(R_{M}\right)=$ expected return on the market
$E\left(R_{M}\right)-R_{F}=$ expected market risk premium
A risk-free asset is defined here as an asset that has no default risk. A common proxy for the risk-free rate is the yield on a default-free government debt instrument. In general, the selection of the appropriate risk-free rate should be guided by the duration of projected cash flows. If we are evaluating a project with an estimated useful life of 10 years, we may want to use the rate on the 10 -year Treasury bond.

## EXAMPLE 3-7 Using the CAPM to Estimate the Cost of Equity

Valence Industries wants to know its cost of equity. Its chief financial officer (CFO) believes the risk-free rate is 5 percent, equity risk premium is 7 percent, and Valence's equity beta is 1.5 . What is Valence's cost of equity using the CAPM approach?

## Solution

$$
\begin{aligned}
\text { Cost of common stock } & =5 \text { percent }+1.5(7 \text { percent }) \\
& =15.5 \text { percent }
\end{aligned}
$$

The expected market risk premium, $E\left(R_{M}-R_{F}\right)$, is the premium that investors demand for investing in a market portfolio relative to the risk-free rate. When using the CAPM to estimate the cost of equity, in practice we typically estimate beta relative to an equity market index. In that case, the market premium estimate we are using is actually an estimate of the equity risk premium (ERP).

An alternative to the CAPM to accommodate risks that may not be captured by the market portfolio alone is a multifactor model that incorporates factors that may be other sources of priced risk (risk for which investors demand compensation for bearing), including macroeconomic factors and company-specific factors. In general

$$
\begin{align*}
E\left(R_{i}\right)= & R_{F}+\beta_{i 1}(\text { Factor risk premium })_{1}+\beta_{i 2}(\text { Factor risk premium })_{2} \\
& +\ldots+\beta_{i j}(\text { Factor risk premium })_{j} \tag{3-5}
\end{align*}
$$

where
$\beta_{i j}=$ stock $i$ 's sensitivity to changes in the $j$ th factor
$(\text { Factor risk premium })_{j}=$ expected risk premium for the $j$ th factor
The basic idea behind these multifactor models is that the CAPM beta may not capture all the risks, especially in a global context, which include inflation, business cycle, interest rate, exchange rate, and default risks. ${ }^{16,17}$

There are several ways to estimate the equity risk premium, though there is no general agreement as to the best approach. The three we discuss are the historical equity risk premium approach, the dividend discount model approach, and the survey approach.

The historical equity risk premium approach is a well-established approach based on the assumption that the realized equity risk premium observed over a long period of time is a good indicator of the expected equity risk premium. This approach requires compiling historical data to find the average rate of return of a country's market portfolio and the average rate of return for the risk-free rate in that country. For example, an analyst might use the historical returns to the TOPIX Index to estimate the risk premium for Japanese equities. The exceptional bull market observed during the second half of the 1990s and the bursting of the technology bubble that followed during the years 2000-2002 remind us that the time period for such estimates should cover complete market cycles.

Elroy Dimson, Paul Marsh, and Mike Staunton conducted an analysis of the equity risk premiums observed in markets located in 16 countries, including the United States, over the period 1900-2002. ${ }^{18}$ These researchers found that the annualized U.S. equity risk premium relative to U.S. Treasury bills was 5.3 percent (geometric mean) and 7.2 percent (arithmetic mean). They also found that the annualized U.S. equity risk premium relative to bonds was 4.4 percent (geometric mean) and 6.4 percent (arithmetic mean). ${ }^{19}$ Note that the arithmetic mean is greater than the geometric mean as a result of the significant volatility of the observed market rate of return and of the observed risk-free rate. Under the assumption of an unchanging distribution of returns through time, the arithmetic mean is the unbiased estimate of the expected single-period equity risk premium, but the geometric mean better reflects growth rate over multiple periods. ${ }^{20}$ In Exhibit 3-2 we provide historical estimates of the equity risk premium for 16 developed markets from Dimson, Marsh, and Staunton's study.

To illustrate the historical method as applied in the CAPM, suppose that we use the historical geometric mean for U.S. equity of 4.8 percent to value Citibank Inc. (NYSE: C) as

[^8]EXHIBIT 3-2 Equity Risk Premiums Relative to Bonds (1900-2001)

|  | Mean (\%) |  |
| :--- | :---: | :---: |
| Country | Geometric | Arithmetic |
| Australia | 6.3 | 7.9 |
| Belgium | 2.8 | 4.7 |
| Canada | 4.2 | 5.7 |
| Denmark | 1.8 | 3.1 |
| France | 4.6 | 6.7 |
| Germany | 6.3 | 9.6 |
| Ireland | 3.1 | 4.5 |
| Italy | 4.6 | 8.0 |
| Japan | 5.9 | 10.0 |
| The Netherlands | 4.4 | 6.4 |
| South Africa | 5.4 | 7.1 |
| Spain | 2.2 | 4.1 |
| Sweden | 4.9 | 7.1 |
| Switzerland | 2.4 | 3.9 |
| United Kingdom | 4.2 | 5.5 |
| United States | 4.8 | 6.7 |
| World | 4.3 | 5.4 |

Source: Dimson, Marsh, and Staunton (2003).
Note: Germany excludes 1922-1923. Switzerland commences in 1911.
of early January 2006. According to Standard \& Poor's, Citibank had a beta of 1.32 at that time. Using the 10 -year U.S. Treasury bond yield of 4.38 percent to represent the risk-free rate, the estimate of the cost of equity for Citibank is 4.38 percent $+1.32(4.8$ percent $)=$ 10.72 percent.

The historical premium approach has several limitations. One is that the level of risk of the stock index may change over time. Another is that the risk aversion of investors may change over time. And still another limitation is that the estimates are sensitive to the method of estimation and the historical period covered.

## EXAMPLE 3-8 Estimating the Equity Risk Premium Using Historical Rates of Return

Suppose that the arithmetic average T-bond rate observed over the last 100 years is an unbiased estimator for the risk-free rate and amounts to 5.4 percent. Likewise, suppose the arithmetic average of return on the market observed over the last 100 years is an
unbiased estimator for the expected return for the market. The average rate of return of the market was 9.3 percent. Calculate the equity risk premium.

## Solution

$$
\begin{aligned}
\mathrm{ERP} & =\bar{R}_{M}-\bar{R}_{F}=9.3 \text { percent }-5.4 \text { percent } \\
& =3.9 \text { percent }
\end{aligned}
$$

A second approach for estimating the equity risk premium is the dividend discount model based approach or implied risk premium approach, which is implemented using the Gordon growth model (also known as the constant-growth dividend discount model). For developed markets, corporate earnings often meet, at least approximately, the model's assumption of a long-run trend growth rate. We extract the premium by analyzing how the market prices an index. That is, we use the relationship between the value of an index and expected dividends, assuming a constant growth in dividends:

$$
P_{0}=\frac{D_{1}}{r_{e}-g}
$$

where
$P_{0}=$ current market value of the equity market index
$D_{1}=$ dividends expected next period on the index
$r_{e}=$ required rate of return on the market
$g=$ expected growth rate of dividends
We solve for the required rate of return on the market as

$$
\begin{equation*}
r_{e}=\frac{D_{1}}{P_{0}}+g \tag{3-6}
\end{equation*}
$$

Therefore, the expected return on the market is the sum of the dividend yield and the growth rate in dividends. ${ }^{21}$ The equity risk premium thus is the difference between the expected return on the equity market and the risk-free rate.

Suppose the expected dividend yield on an equity index is 5 percent and the expected growth rate of dividends on the index is 2 percent. The expected return on the market according to the Gordon growth model is

$$
\begin{aligned}
E\left(R_{M}\right) & =5 \text { percent }+2 \text { percent } \\
& =7 \text { percent }
\end{aligned}
$$

A risk-free rate of interest of 3.8 percent implies an equity risk premium of 3.2 percent ( $=7$ percent -3.8 percent).

[^9]Another approach to estimate the equity risk premium is quite direct: Ask a panel of finance experts for their estimates and take the mean response. This is the survey approach. For example, one set of U.S. surveys found that the expected U.S. equity risk premium over the next 30 years was 5.5 percent to 7 percent, forecasting from 2001 as the baseline year, and 7.1 percent, using 1998 as the baseline year.

Once we have an estimate of the equity risk premium, we fine-tune this estimate for the particular company or project by adjusting it for the specific systematic risk of the project. We adjust for the specific systematic risk by multiplying the market risk premium by beta to arrive at the company's or project's risk premium, which we then add to the risk-free rate to determine the cost of equity within the framework of the CAPM. ${ }^{22}$
3.3.2. Dividend Discount Model Approach Earlier we used the Gordon growth model to develop an estimate of the equity risk premium for use in the CAPM. We can also use the Gordon growth model directly to obtain an estimate of the cost of equity. To review, the dividend discount model in general states that the intrinsic value of a share of stock is the present value of the share's expected future dividends:

$$
V_{0}=\sum_{t=1}^{\infty}\left(\frac{D_{t}}{\left(1+r_{e}\right)^{t}}\right)=\frac{D_{1}}{\left(1+r_{e}\right)}+\frac{D_{2}}{\left(1+r_{e}\right)^{2}}+\ldots
$$

where

$$
\begin{aligned}
& V_{0}=\text { intrinsic value of a share } \\
& D_{t}=\text { share's dividend at the end of period } t \\
& r_{e}=\text { cost of equity }
\end{aligned}
$$

Based on Gordon's constant growth formulation, we assume dividends are expected to grow at a constant rate, $g .{ }^{23}$ Therefore, if we assume that price reflects intrinsic value ( $V_{0}=P_{0}$ ), we can rewrite the valuation of the stock as

$$
P_{0}=\frac{D_{1}}{r_{e}-g}
$$

We can then rewrite this equation and estimate the cost of equity as we did for Equation 3-6 in Section 3.3.1:

$$
r_{e}=\frac{D_{1}}{P_{0}}+g
$$

Therefore, to estimate $r_{e}$ we need to estimate the dividend in the next period and the assumed constant dividend growth rate. The current stock price, $P_{0}$, is known, and the dividend of the next period, $D_{1}$, can be predicted if the company has a stable dividend policy.

[^10](The ratio $D_{1} / P_{0}$ may be called the forward annual dividend yield.) The challenge is estimating the growth rate.

There are at least two ways to estimate the growth rate. The first is to use a forecasted growth rate from a published source or vendor. A second is to use a relationship between the growth rate, the retention rate, and the return on equity. In this context, this is often referred to as the sustainable growth rate and is interpretable as the rate of dividend (and earnings) growth that can be sustained over time for a given level of return on equity, keeping the capital structure constant and without issuing additional common stock. The relationship is given in Equation 3-7:

$$
\begin{equation*}
g=(1-D / \mathbb{E p S}) \mathrm{ROE} \tag{3-7}
\end{equation*}
$$

where
$D / E P S=$ assumed stable dividend payout ratio
ROE $=$ historical return on equity
The term ( $1-D / E P S$ ) is the company's earnings retention rate.
Consider Citigroup, Inc. Citigroup has an earnings retention rate of 59 percent. As of early January 2006, Citigroup had a forward annual dividend yield of 3.9 percent, a trailing return on equity of approximately 20 percent, but an estimated average return on equity going forward of approximately 16.6 percent. According to Equation 3-7, Citigroup's sustainable growth rate is 0.59 ( 16.6 percent $)=9.79$ percent. The dividend discount model estimate of the cost of equity is therefore 9.79 percent +3.9 percent $=13.69$ percent.
3.3.3. Bond Yield Plus Risk Premium Approach The bond yield plus risk premium approach is based on the fundamental tenet in financial theory that the cost of capital of riskier cash flows is higher than that of less risky cash flows. In this approach, we sum the before-tax cost of debt, $r_{d}$, and a risk premium that captures the additional yield on a company's stock relative to its bonds. The estimate is, therefore,

$$
\begin{equation*}
r_{e}=r_{d}+\text { Risk premium } \tag{3-8}
\end{equation*}
$$

The risk premium compensates for the additional risk of equity compared with debt. ${ }^{24}$ Ideally, this risk premium is forward looking, representing the additional risk associated with the stock of the company as compared with the bonds of the same company. However, we often estimate this premium using historical spreads between bond yields and stock yields. In developed country markets, a typical risk premium added is in the range of 3 to 5 percent.

Looking again at Citigroup, as of early January 2006, the yield to maturity of the Citigroup 5.3 s bonds maturing in 2016 was approximately 4.95 percent. Adding an arbitrary risk premium of 3.5 percent produces an estimate of the cost of equity of $4.95+3.5=8.45$ percent. This estimate contrasts with the higher estimates of 10.72 percent under the CAPM approach and with 13.69 percent under the dividend discount model approach. Such disparities are not uncommon and reflect the difficulty of cost of equity estimation.

[^11]
## 4. TOPICS IN COST OF CAPITAL ESTIMATION

When calculating a company's weighted average cost of capital (WACC), it is essential to understand the risk factors that have been considered in determining the risk-free rate, the equity risk premium, and beta to ensure a consistent calculation of WACC and to avoid the double-counting or omission of pertinent risk factors.

### 4.1. Estimating Beta and Determining a Project Beta

When using the CAPM to estimate the cost of equity, the analyst must estimate beta. The estimation of beta presents many choices as well as challenges.

One common method of estimating the company's stock beta is to use a market model regression of the company's stock returns $\left(R_{i}\right)$ against market returns $\left(R_{m}\right)$ over $T$ periods: ${ }^{25}$

$$
R_{i t}=\hat{a}+\hat{b} R_{m t} \quad t=1,2, \ldots T
$$

where
$\hat{a}=$ estimated intercept
$\hat{b}=$ estimated slope of the regression that is used as an estimate of beta
However, beta estimates are sensitive to the method of estimation and data used. Consider some of the issues:

- Estimation period: The estimated beta is sensitive to the length of the estimation period, with beta commonly estimated using data over two to nine years. Selection of the estimation period is a trade-off between data richness captured by longer estimation periods and company-specific changes that are better reflected with shorter estimation periods. In general, longer estimation periods are applied to companies with a long and stable operating history, and shorter estimation periods are used for companies that have undergone significant structural changes in the recent past (such as restructuring, recent acquisition, or divestiture) or changes in financial and operating leverage.
- Periodicity of the return interval (e.g., daily, weekly, or monthly): Researchers have observed smaller standard error in beta estimated using smaller return intervals, such as daily returns. ${ }^{26}$
- Selection of an appropriate market index: The choice of market index affects the estimate of beta.
- Use of a smoothing technique: Some analysts adjust historical betas to reflect the tendency of betas to revert to $1 .{ }^{27}$ As an example, the expression $\beta_{i, a d j}=0.333+0.667_{\beta i}$ adjusts betas above and below 1.0 toward 1.0.
- Adjustments for small-capitalization stocks: Small-capitalization stocks have generally exhibited greater risks and greater returns than large-capitalization stocks over the long run. Roger Ibbotson, Paul Kaplan, and James Peterson argue that betas for small-capitalization companies be adjusted upward. ${ }^{28}$

[^12]Arriving at an estimated beta for publicly traded companies is generally not a problem because of the accessibility of stock return data, the ease of use of estimating beta using simple regression, and the availability of estimated betas on publicly traded companies from financial analysis vendors, such as Barra, Bloomberg, Thomson Financial's Datastream, Reuters, and Value Line. The challenge is to estimate a beta for a company that is not publicly traded or to estimate a beta for a project that is not the average or typical project of a publicly traded company. Estimating a beta in these cases requires proxying for the beta by using the information on the project or company combined with a beta of a publicly traded company.

The beta of a company or project is affected by the systematic components of business risk and by financial risk. Both of these factors affect the uncertainty of the cash flows of the company or project. The business risk of a company or project is the risk related to the uncertainty of revenues, referred to as sales risk, and to operating risk, which is the risk attributed to the company's operating cost structure. Sales risk is affected by the elasticity of the demand of the product, the cyclicality of the revenues, and the structure of competition in the industry. Operating risk is affected by the relative mix of fixed and variable operating costs: The greater the fixed operating costs, relative to variable operating costs, the greater the uncertainty of income and cash flows from operations will be.

Financial risk is the uncertainty of net income and net cash flows attributed to the use of financing that has a fixed cost, such as debt and leases. The greater the use of fixedfinancing sources of capital, relative to variable sources, the greater the financial risk. In other words, a company that relies heavily on debt financing instead of equity financing is assuming a great deal of financial risk.

How does a financial analyst estimate a beta for a company or project that is not publicly traded? One common method is the pure-play method, which requires using a comparable publicly traded company's beta and adjusting it for financial leverage differences.

A comparable company is a company that has similar business risk. The reason for the name "pure-play" is that one of the easiest ways of identifying a comparable for a project is to find a company in the same industry that is in that single line of business. For example, if the analyst is examining a project that involves drugstores, appropriate comparables in the United States may be Walgreens, CVS Corporation, and Rite Aid Corporation.

In estimating a beta in this way, the analyst must make adjustments to account for differing degrees of financial leverage. This requires a process of "unlevering" and "levering" the beta. The beta of the comparable is first "unlevered" by removing the effects of its financial leverage. ${ }^{29}$ The unlevered beta is often referred to as the asset beta because it reflects the business risk of the assets. Once we determine the unlevered beta, we adjust it for the capital structure of the company or project that is the focus of our analysis. In other words, we "lever" the asset beta to arrive at an estimate of the equity beta for the project or company of interest.

For a given company, we can unlever its equity beta to estimate its asset beta. To do this, we must determine the relationship between a company's asset beta and its equity beta. Because the company's risk is shared between creditors and owners, we can represent the company's risk, $\beta_{\text {asset }}$, as the weighted average of the company's creditors' market risk, $\beta_{\text {debt }}$, and the market risk of the owners, $\beta_{\text {equity }}$ :

$$
\beta_{\text {asset }}=\beta_{\text {debt }} w_{d}+\beta_{\text {equity }} w_{e}
$$

[^13]or
$$
\beta_{\text {asset }}=\beta_{\mathrm{debt}}\left(\frac{D}{D+E}\right)+\beta_{\text {equity }}\left(\frac{E}{D+E}\right)
$$
where
\[

$$
\begin{aligned}
& E=\text { market value of equity } \\
& D=\text { market value of debt } \\
& w_{d}=\text { proportion of debt }=D /(D+E) \\
& w_{e}=\text { proportion of equity }=E /(D+E)
\end{aligned}
$$
\]

But interest on debt is deducted by the company to arrive at taxable income, so that the claim that creditors have on the company's assets does not cost the company the full amount but rather the after-tax claim; the burden of debt financing is actually less due to interest deductibility. We can represent the asset beta of a company as the weighted average of the betas of debt and equity after considering the effects of the tax deductibility of interest:

$$
\beta_{\text {asset }}=\beta_{\text {debt }} \frac{(1-t) D}{(1-t) D+E}+\beta_{\text {equity }} \frac{E}{(1-t) D+E}
$$

where $t$ is the marginal tax rate.
We generally assume that a company's debt does not have market risk; so $\beta_{\text {debt }}=0$. This means that the returns on debt do not vary with the returns on the market, which we generally assume to be true for most large companies. If $\beta_{\text {debt }}=0$, then ${ }^{30}$

$$
\begin{equation*}
\beta_{\text {asset }}=\beta_{\text {equiry }}\left[\frac{1}{1+\left((1-t) \frac{D}{E}\right)}\right] \tag{3-9}
\end{equation*}
$$

Therefore, the market risk of a company's equity is affected by both the asset's market risk, $\beta_{\text {asser }}$, and a factor representing the nondiversifiable portion of the company's financial risk, $\left.\left[1+(1-t)^{\mathrm{D}} / E\right)\right]$ :

$$
\begin{equation*}
\beta_{\text {equity }}=\beta_{\text {asser }}\left[1+\left((1-t) \frac{D}{E}\right)\right] \tag{3-10}
\end{equation*}
$$

Suppose a company has an equity beta of 1.5 , a debt-to-equity ratio of 0.4 , and a marginal tax rate of 30 percent. Using Equation 3-9, the company's asset beta is 1.1719:

$$
\begin{aligned}
\beta_{\text {asser }} & =1.5\left[\frac{1}{1+((1-0.3)(0.4))}\right] \\
& =1.5(0.7813) \\
& =1.1719
\end{aligned}
$$

In other words, if the company did not have any debt financing, its $\beta_{\text {asset }}=\beta_{\text {equity }}=1.1719$; however, the use of debt financing increases its $\beta_{\text {equity }}$ from 1.1719 to 1.5 . What would the
$\overline{{ }^{30} \text { The first step is } \beta_{\text {asset }}}=\beta_{\text {equity }}\left[\frac{E}{(1-t) D+E}\right]$, which we simplify to arrive at Equation 3-9.
company's equity beta be if the company's debt-to-equity ratio were 0.5 instead of 0.4 ? In this case, we apply Equation 10 , using the debt-to-equity ratio of 0.5 :

$$
\beta_{\text {equity }}=1.1719\{1+[(1-0.3)(0.5)]\}=1.5821
$$

Therefore, the unlevering calculation produces a measure of market risk for the assets of the company-ignoring the company's capital structure. We use the levering calculation in Equation 3-10 to estimate the market risk of a company given a specific asset risk, marginal tax rate, and capital structure.

We can use the same unlevering and levering calculations to estimate the asset risk and equity risk for a project. We start with the equity beta of the comparable company, which is the levered beta, $\beta_{\mathrm{L}, \text { comparable }}$, and then convert it into the equivalent asset beta for the unlevered company, $\beta_{\mathrm{U}, \text { comparable }}$. Once we have the estimate of the unlevered beta, which is the company's asset risk, we then can use the project's capital structure and marginal tax rate to convert this asset beta into an equity beta for the project, $\beta_{\mathrm{L}, \text { project }}$.

## Estimating a Beta Using the Pure-Play Method

## Step 1: Select the comparable.

Determine comparable company or companies (companies with similar business risk).

$$
\sqrt{3}
$$

## Step 2: Estimate comparable's beta.

Estimate the equity beta of the comparable company or companies.

$$
\sqrt{n}
$$

## Step 3: Unlever the comparable's beta.

Unlever the beta of the comparable company or companies, removing the financial risk component of the equity beta, leaving the business risk component of the beta.
ת

## Step 4: Lever the beta for the project's financial risk.

Lever the beta of the project by adjusting the asset beta for the financial risk of the project.

We begin by estimating the levered beta of the comparable company, $\beta_{\mathrm{L}, \text { comparable }}$. Using the capital structure and tax rate of the levered company, we estimate the asset beta for the comparable company, $\beta_{\mathrm{U}, \text { comparable: }}$

$$
\begin{equation*}
\beta_{\mathrm{U}, \text { comparable }}=\frac{\beta_{\mathrm{L}, \text { comparable }}}{\left[1+\left(\left(1-t_{\text {comparable }}\right) \frac{\mathrm{D}_{\text {comparable }}}{\mathrm{E}_{\text {comparable }}}\right)\right]} \tag{3-11}
\end{equation*}
$$

We then consider the financial leverage of the project or company and calculate its equity risk, $\beta_{\text {L,project }}$ :

$$
\begin{equation*}
\beta_{\mathrm{L}, \text { project }}=\beta_{\mathrm{U}, \text { comparable }}\left[1+\left(\left(1-t_{\text {project }}\right) \frac{\mathrm{D}_{\text {project }}}{\mathrm{E}_{\text {project }}}\right)\right] \tag{3-12}
\end{equation*}
$$

To illustrate the use of these equations, suppose we want to evaluate a project that will be financed with debt and equity in a ratio of $0.4: 1$ [a debt-to-equity ratio of 0.4 , corresponding to approximately $0.4 /(0.4+1.0)=€ 0.286$ for each euro of capital needed]. We find a comparable company operating in the same line of business as the project. The marginal tax rate for the company sponsoring the project and the comparable company is 35 percent. The comparable company has a beta of 1.2 and a debt-to-equity ratio of 0.125 . The unlevered beta of the comparable is 1.1098:

$$
\begin{aligned}
\beta_{\mathrm{U}, \text { comparable }} & =\frac{1.2}{[1+((1-0.35) 0.125)]} \\
& =1.1098
\end{aligned}
$$

The levered beta for the project is 1.3983 :

$$
\begin{aligned}
\beta_{\text {L.project }} & =1.1098[1+((1-0.35) 0.4)] \\
& =1.3983
\end{aligned}
$$

We then use the 1.3983 as the beta in our CAPM estimate of the component cost of equity for the project and, combined with the cost of debt in a weighted average, provide an estimate of the cost of capital for the project. ${ }^{31}$

## EXAMPLE 3-9 Inferring an Asset Beta

Suppose that the beta of a publicly traded company's stock is 1.3 and that the market value of equity and debt are, respectively, C $\$ 540$ million and $\mathrm{C} \$ 720$ million. If the marginal tax rate of this company is 40 percent, what is the asset beta of this company?

## Solution

$$
\begin{aligned}
\beta_{U} & =\frac{1.3}{\left[1+\left((1-0.4) \frac{720}{540}\right)\right]} \\
& =0.72
\end{aligned}
$$

[^14]
## EXAMPLE 3-10 Calculating a Beta Using the Pure-Play Method

Raymond Cordier is the business development manager of Aerotechnique S.A., a private Belgian subcontractor of aerospace parts. Although Aerotechnique is not listed on the Belgian stock exchange, Cordier needs to evaluate the levered beta for the company. He has access to the following information:

- The average levered and average unlevered betas for the group of comparable companies operating in different European countries are 1.6 and 1.0, respectively.
- Aerotechnique's debt-to-equity ratio, based on market values, is 1.4.
- Aerotechnique's corporate tax rate is 34 percent.


## Solution

The beta for Aerotechnique is estimated on the basis of the average unlevered beta extracted from the group of comparable companies. On that basis, and applying the financing structure of Aerotechnique, the estimated beta for Aerotechnique is

$$
\begin{aligned}
\beta_{\text {Aerotechnique }} & =1.0[1+((1-0.34)(1.4))] \\
& =1.924
\end{aligned}
$$

## EXAMPLE 3-11 Estimating the Weighted Average Cost of Capital

Georg Schrempp is the CFO of Bayern Chemicals KgaA, a large German manufacturer of industrial, commercial, and consumer chemical products. Bayern Chemicals is privately owned, and its shares are not listed on an exchange. The CFO has appointed Markus Meier, CFA, of Crystal Clear Valuation Advisors, a third-party valuator, to perform a stand-alone valuation of Bayern Chemicals. Meier had access to the following information to calculate Bayern Chemicals' weighted average cost of capital:

- The nominal risk-free rate is represented by the yield on the long-term 10 -year German bund, which at the valuation date was 4.5 percent.
- The average long-term historical equity risk premium in Germany is assumed at 5.7 percent. ${ }^{32}$
- Bayern Chemicals' corporate tax rate is 38 percent.
- Bayern Chemicals' target debt-to-equity ratio is 0.7 . Bayern is operating at its target debt-to-equity ratio.

[^15]- Bayern Chemicals' cost of debt has an estimated spread of 225 basis points over the 10 -year bund.
- Exhibit 3-3 supplies additional information on comparables for Bayern Chemicals.

EXHIBIT 3-3 Information on Comparables

| Comparable Companies | Country | Tax Rate (\%) | Market <br> Capitalization <br> (in millions) | Net Debt (in millions) | D/E | Beta |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| British Chemicals Ltd. | U.K. | 30.0 | 4,500 | 6,000 | 1.33 | 1.45 |
| Compagnie <br> Petrochimique S.A. | France | 30.3 | 9,300 | 8,700 | 0.94 | 0.75 |
| Rotterdam Chemie N.V. | Netherlands | 30.5 | 7,000 | 7,900 | 1.13 | 1.05 |
| Average |  |  |  |  | 1.13 | 1.08 |

Based only on the information given, calculate Bayern Chemicals' WACC.

## Solution

To calculate the cost of equity, the first step is to unlever the betas of the comparable companies and calculate an average for a company with business risk similar to the average of these companies:

| Comparable Companies | Unlevered Beta |
| :--- | :---: |
| British Chemicals Ltd. | 0.75 |
| Compagnie Petrochimique S.A. | 0.45 |
| Rotterdam Chemie N.V. | 0.59 |
| Average $^{33}$ | 0.60 |

Levering the average unlevered beta for the peer group average, applying Bayern Chemicals' target debt-to-equity ratio and marginal tax rate, results in a beta of 0.86 :

$$
\begin{aligned}
\beta_{\text {Bayern Chemical }} & =0.60\{1+[(1-0.38) 0.7]\} \\
& =0.86
\end{aligned}
$$

The cost of equity of Bayern Chemicals, $r_{e}$ can be calculated as follows:

$$
\begin{aligned}
r_{e} & =4.5 \text { percent }+(0.86)(5.7 \text { percent }) \\
& =9.4 \text { percent }
\end{aligned}
$$

[^16]The weights for the cost of equity and cost of debt may be calculated as follows:

$$
\begin{aligned}
w_{d} & =\frac{D / E}{\left(\frac{D}{E}+1\right)} \\
& =\frac{0.7}{1.7} \\
& =0.41 \\
w_{e} & =1-w_{d} \\
& =1-0.41 \\
& =0.59
\end{aligned}
$$

The before-tax cost of debt of Bayern Chemicals, $r_{d}$, is 6.75 percent:

$$
\begin{aligned}
r_{d} & =4.5 \text { percent }+2.25 \text { percent } \\
& =6.75 \text { percent }
\end{aligned}
$$

As a result, Bayern Chemicals' WACC is 7.27 percent:

$$
\begin{aligned}
\mathrm{WACC} & =[(0.41)(0.0675)(1-0.38)]+[(0.59)(0.094)] \\
& =0.0726 \text { or } 7.26 \text { percent }
\end{aligned}
$$

### 4.2. Country Risk

The use of a stock's beta to capture the country risks of a project is well supported in empirical studies that examine developed nations. However, beta does not appear to adequately capture country risk for companies in developing nations. ${ }^{34} \mathrm{~A}$ common approach for dealing with this problem is to adjust the cost of equity estimated using the CAPM by adding a country spread to the market risk premium. ${ }^{35}$ The country spread is also referred to as a country equity premium.

Perhaps the simplest estimate of the country spread is the sovereign yield spread, which is the difference between the government bond yield in that country, denominated in the currency of a developed country, and the Treasury bond yield on a similar maturity bond in the developed country. ${ }^{36}$ However, this approach may be too coarse for the purposes of equity risk premium estimation.

[^17]Another approach is to calculate the country equity premium as the product of the sovereign yield spread and the ratio of the volatility of the developing country equity market to that of the sovereign bond market denominated in terms of the currency of a developed country: ${ }^{37}$
Country equity premium $=$ Sovereign yield spread $\left(\begin{array}{c}\text { Annualized standard deviation } \\ \text { of equity index }\end{array} \frac{\begin{array}{c}\text { Annualized standard deviation } \\ \text { of the sovereign bond market } \\ \text { in terms of the developed } \\ \text { market currency }\end{array}}{}\right)$

The logic of this calculation is that the sovereign yield spread captures the general risk of the country, which is then adjusted for the volatility of the stock market relative to the bond market. This country equity premium is then used in addition to the equity premium estimated for a project in a developed country. Therefore, if the equity risk premium for a project in a developed country is 4.5 percent and the country risk premium is 3 percent, the total equity risk premium used in the CAPM estimation is 7.5 percent. If the appropriate beta is 1.2 and the risk-free rate of interest is 4 percent, the cost of equity is

Equity risk premium $=0.04+1.2(0.045+0.03)=0.13$ or 13 percent

## EXAMPLE 3-12 Estimating the Country Equity Premium

Miles Avenaugh, an analyst with the Global Company, is estimating a country equity premium to include in his estimate of the cost of equity capital for Global's investment in Argentina. Avenaugh has researched yields in Argentina and observed that the Argentinean government's 10 -year bond is 9.5 percent. A similar maturity U.S. Treasury bond has a yield of 4.5 percent. The annualized standard deviation of the Argentina Merval stock index, a market value index of stocks listed on the Buenos Aires Stock Exchange, during the most recent year is 40 percent. The annualized standard deviation of the Argentina dollar-denominated 10-year government bond over the recent period was 28 percent.

What is the estimated country equity premium for Argentina based on Avenaugh's research?

## Solution

$$
\begin{aligned}
\text { Country risk premium } & =0.05\left(\frac{0.40}{0.28}\right) \\
& =0.05(1.4286) \\
& =0.0714, \text { or } 7.14 \text { percent }
\end{aligned}
$$

[^18]Still another approach is to use country credit ratings to estimate the expected rates of returns for countries that have credit ratings but no equity markets. ${ }^{38}$ This method requires estimating reward to credit risk measures for a large sample of countries for which there are both credit ratings and equity markets, then applying this ratio to those countries without equity markets based on the country's credit rating.

### 4.3. Marginal Cost of Capital Schedule

As we noted in Section 2.3, as a company raises more funds, the costs of the different sources of capital may change, resulting in a change in the weighted average cost of capital for different levels of financing. The result is the marginal cost of capital (MCC) schedule, which we often depict in graphical form as the weighted average cost of capital for different amounts of capital raised, as we showed earlier in Exhibit 3-1. ${ }^{39}$

Why would the cost of capital change as more capital is raised? One source of a difference in cost depending on the amount of capital raised is that a company may have existing debt with a bond covenant that restricts the company from issuing debt with similar seniority as existing debt. Or a debt incurrence test may restrict a company's ability to incur additional debt at the same seniority based on one or more financial tests or conditions. For example, if a company issues senior debt such that any additional debt at that seniority violates the debt incurrence test of an existing bond covenant, the company may have to issue less senior debt or even equity, which would have a higher cost.

Another source of increasing marginal costs of capital is a deviation from the target capital structure. In the ideal, theoretical world, a company has a target capital structure, goes to the market each period, and raises capital in these proportions. However, as a practical matter, companies do not necessarily tap the market in these ideal proportions because of considerations for economies of scale in raising new capital and market conditions. Because of such perceived economies of scale, companies tend to issue new securities such that, in any given period, it may deviate from the proportions dictated by any target or optimal capital structure. In other words, these short-run deviations are due to the "lumpiness" of security issuance. As the company experiences deviations from the target capital structure, the marginal cost of capital may increase, reflecting these deviations.

The amount of capital at which the weighted average cost of capital changes-which means that the cost of one of the sources of capital changes-is referred to as a break point. The reality of raising capital is that the marginal cost of capital schedule is not as smooth as we depicted in Exhibit 3-1 but rather is a step-up cost schedule, as shown in Exhibit 3-4.

Consider the case of a company facing the costs of capital given in Exhibit 3-5.

[^19]EXHIBIT 3-4 Marginal Cost of Capital Schedule


EXHIBIT 3-5 Schedule of the Costs of Debt and Equity

| Amount of New Debt <br> (in millions) | After-tax Cost <br> of Debt (\%) | Amount of New Equity <br> (in millions) | Cost of Equity (\%) |
| :--- | :---: | :--- | :---: |
| New debt $\leq € 2$ | 2.0 | New equity $\leq € 6$ | 5.0 |
| $€ 2<$ New debt $\leq € 5$ | 2.5 | $€ 6<$ New equity $\leq € 8$ | 7.0 |
| $€ 5<$ New debt | 3.0 | $€ 8<$ New equity | 9.0 |

If the company raises capital according to its target capital structure proportions of 40 percent debt and 60 percent equity, this company faces a marginal cost of capital schedule that is upward sloping, with break points at $€ 5$ million, $€ 10$ million, $€ 12.5$ million, and $€ 13.3$ million, as depicted in Exhibit 3-4. These break points are determined from the amounts of capital at which the cost changes, calculated as

Break point $=\frac{\text { Amount of capital at which the source's cost of capital changes }}{\text { Proportion of new capital raised from the source }}$
For example, the first break point for debt financing is reached with $€ 2$ million/0.4 = € 5 million of new capital raised. The first break point attributed to a change in equity cost occurs at $€ 6$ million $/ 0.6=€ 10$ million. Example 3-13 illustrates a marginal cost of capital schedule with break points and also how the WACC figures in the choice of an optimal capital structure.

## EXAMPLE 3-13 Marginal Cost of Capital Schedule

Alan Conlon is the CFO of Allied Canadian Breweries Ltd. He wants to determine the capital structure that will result in the lowest cost of capital for Allied. He has access to the following information:

- The minimum rate at which the company can borrow for 12 months is the 12 -month LIBOR +200 basis points until it reaches a debt-to-total-capital ratio of 30 percent. For debt-to-total-capital ratios above 30 percent, the spread over 12-month LIBOR increases according to the schedule given in Exhibit 3-6.

EXHIBIT 3-6 Spreads over LIBOR for Alternative Debt-to-Equity Ratios

| $\frac{D}{D+E}$ | Spread (BPS) |
| :--- | :---: |
| 0.4 | 300 |
| 0.5 | 400 |
| 0.6 | 600 |
| 0.7 | 800 |
| 0.8 | 1,000 |

- The current 12 -month LIBOR is 4.5 percent.
- The market risk premium is 4 percent, the risk-free rate is 4.25 percent, and unleveraged beta is 0.9 .
- The company's tax rate is 36 percent.

1. Determine the WACC for levels of the debt-to-equity ratio given in Exhibit 3-6.
2. Recommend a target capital structure, given that the company is concerned with achieving the lowest possible cost of capital.

## Solution to 1

The WACC expressed as a function of the capital structure is shown in Exhibit 3-7.
EXHIBIT 3-7 WACC for Alternative Capital Structures

| $\frac{D}{D+E}$ | $\beta$ | $R_{D}(\%)$ | $R_{E}(\%)$ | WACC (\%) |
| :---: | :---: | :---: | :---: | :---: |
| 0.1 | 0.96 | 6.5 | 8.1 | 7.7 |
| 0.2 | 1.04 | 6.5 | 8.4 | 7.6 |
| 0.3 | 1.15 | 6.5 | 8.8 | 7.4 |
| 0.4 | 1.28 | 7.5 | 9.4 | 7.6 |
| 0.5 | 1.48 | 8.5 | 10.2 | 7.8 |
| 0.6 | 1.76 | 10.5 | 11.3 | 8.6 |
| 0.7 | 2.24 | 12.5 | 13.2 | 9.6 |
| 0.8 | 3.20 | 14.5 | 17.1 | 10.8 |
| 0.9 | 6.08 | 16.5 | 28.6 | 12.4 |

## Solution to 2

The optimal capital structure is 30 percent debt, which corresponds to an optimal D/E of 0.43 .

### 4.4. Flotation Costs

When a company raises new capital, it generally seeks the assistance of investment bankers. Investment bankers charge the company a fee based on the size and type of offering. This fee is referred to as the flotation cost. In the case of debt and preferred stock, we do not usually incorporate flotation costs into the estimated cost of capital because the amount of these costs is quite small, often less than 1 percent. ${ }^{40}$

However, with equity issuance, the flotation costs may be substantial; so we should consider these when estimating the cost of external equity capital. For example, Inmoo Lee, Scott Lochhead, Jay Ritter, and Quanshui Zhao observe average flotation costs for new equity in the United States of 7.11 percent. ${ }^{41}$ The flotation costs in other countries differ from the U.S. experience: Thomas Bühner and Christoph Kaserer observe flotation costs around 1.65 percent in Germany, Seth Armitage estimates an average issuance cost of 5.78 percent in the United Kingdom, and Christoph Kaserer and Fabian Steiner observe an average cost of 4.53 for Swiss capital offerings. ${ }^{42}$ A large part of the differences in costs among these studies is likely attributed to the type of offering; cash underwritten offers, typical in the United States, are generally more expensive than rights offerings, which are common in Europe.

Should we incorporate flotation costs into the cost of capital? There are two views on this topic. One view, which you can find often in textbooks, is to incorporate the flotation costs into the cost of capital. The other view is that flotation costs should not be included in the cost of capital but rather incorporated into any valuation analysis as an additional cost of the project.

Consistent with the first view, we can specify flotation costs in monetary terms, as an amount per share or as a percentage of the share price. With flotation costs in monetary terms on a per-share basis, $F$, the cost of external equity is

$$
\begin{equation*}
r_{e}=\left(\frac{D_{1}}{P_{0}-F}\right)+g \tag{3-15}
\end{equation*}
$$

As a percentage applied against the price per share, the cost of external equity is

$$
\begin{equation*}
r_{e}=\left(\frac{D_{1}}{P_{0}(1-f)}\right)+g \tag{3-16}
\end{equation*}
$$

where $f$ is the flotation cost as a percentage of the issue price.

[^20]Suppose a company has a current dividend of $\$ 2$ per share, a current price of $\$ 40$ per share, and an expected growth rate of 5 percent. The cost of internally generated equity would be 10.25 percent:

$$
\begin{aligned}
r_{e} & =\left(\frac{\$ 2(1+0.05)}{\$ 40}\right)+0.05 \\
& =0.0525+0.05 \\
& =0.1025 \text { or } 10.25 \text { percent }
\end{aligned}
$$

If the flotation costs are 4 percent of the issuance, the cost of externally generated equity would be slightly higher at 10.469 percent:

$$
\begin{aligned}
r_{e} & =\left(\frac{\$ 2(1+0.05)}{\$ 40(1-0.04)}\right)+0.05 \\
& =0.05469+0.05 \\
& =0.1047, \text { or } 10.47 \text { percent }
\end{aligned}
$$

The problem with this approach is that the flotation costs are a cash flow at the initiation of the project and affect the value of any project by reducing the initial cash flow. Adjusting the cost of capital for flotation costs is incorrect because by doing so we are adjusting the present value of the future cash flows by a fixed percentage-in the preceding example, a difference of 22 basis points, which does not necessarily equate to the present value of the flotation costs. ${ }^{43}$

The alternative and recommended approach is to make the adjustment to the cash flows in the valuation computation. For example, consider a project that requires a $€ 60,000$ initial cash outlay and is expected to produce cash flows of $€ 10,000$ each year for 10 years. Suppose the company's marginal tax rate is 40 percent and that the before-tax cost of debt is 5 percent. Furthermore, suppose that the company's dividend next period is $€ 1$, the current price of the stock is $€ 20$, and the expected growth rate is 5 percent, so that the cost of equity using the dividend discount model is $(€ 1 / € 20)+0.05=0.10$, or 10 percent. Assume the company will finance the project with 40 percent debt and 60 percent equity. Exhibit 3-8 summarizes the information on the component costs of capital.

The weighted average cost of capital is 7.2 percent, calculated as $0.40(3$ percent $)+$ 0.60 (10 percent). Ignoring flotation costs for the moment, the net present value (NPV) of this project is

$$
\begin{aligned}
\mathrm{NPV} & =€ 69,591-€ 60,000 \\
& =€ 9,591
\end{aligned}
$$

EXHIBIT 3-8 After-Tax Costs of Debt and Equity

| Source of Capital | Amount Raised $(€)$ | Proportion | Marginal After-Tax Cost |
| :--- | :---: | :---: | :--- |
| Debt | 24,000 | 0.40 | $0.05(1-0.4)=0.03$ |
| Equity | 36,000 | 0.60 | 0.10 |

[^21]If the flotation costs are, say, 5 percent of the new equity capital, the amount is $€ 1,800$. The net present value considering flotation costs is

$$
\begin{aligned}
\mathrm{NPV} & =€ 69,591-€ 60,000-€ 1,800 \\
& =€ 7,791
\end{aligned}
$$

If, instead of considering the flotation costs as part of the cash flows, we adjust the cost of equity, the cost of capital is 7.3578 percent and the NPV is

$$
\begin{aligned}
\mathrm{NPV} & =€ 69,089-€ 60,000 \\
& =€ 9,089
\end{aligned}
$$

As you can see, we arrive at different assessments of value using these two methods.
So, if it is preferred to deduct the flotation costs as part of the net present value calculation, why do we see the adjustment in the cost of capital so often in textbooks? The first reason is that it is often difficult to identify particular financing associated with a project. Using the adjustment for the flotation costs in the cost of capital may be useful if specific project financing cannot be identified. Second, by adjusting the cost of capital for the flotation costs, it is easier to demonstrate how costs of financing a company change as a company exhausts internally generated equity (i.e., retained earnings) and switches to externally generated equity (i.e., a new stock issue).

### 4.5. What Do Chief Financial Officers Do?

In this chapter, we have introduced you to methods that may be used to estimate the cost of capital for a company or a project. What do companies actually use when making investment decisions? In a survey of a large number of U.S. company CFOs, John Graham and Campbell Harvey asked about the methods that companies actually use. ${ }^{44}$ Their survey revealed the following:

- The most popular method for estimating the cost of equity is the capital asset pricing model.
- Few companies use the dividend cash flow model to estimate a cost of equity.
- Publicly traded companies are more likely to use the capital asset pricing model than are private companies.
- In evaluating projects, the majority use a single-company cost of capital, but a large portion apply some type of risk adjustment for individual projects.

The survey also reveals that the single-factor capital asset pricing model is the most popular method for estimating the cost of equity, though the next most popular methods, respectively, are average stock returns and multifactor return models. The lack of popularity of the dividend discount model indicates that this approach, which was once favored, has lost its following in practice. ${ }^{45}$

[^22]In a survey of publicly traded multinational European companies, Franck Bancel and Usha Mittoo provide evidence consistent with the Graham and Harvey survey. ${ }^{46}$ They find that over 70 percent of companies use the CAPM to determine the cost of equity; this compares with the 73.5 percent of U.S. companies that use the CAPM. In a survey of both publicly traded and private European companies, Dirk Brounen, Abe de Jong, and Kees Koedijk confirm the result of Graham and Harvey that larger companies are more likely to use the more sophisticated methods, such as CAPM, in estimating the cost of equity. ${ }^{47}$ Brounen, Jong, and Koedijk find that the popularity of the use of CAPM is less for their sample (ranging from 34 percent to 55.6 percent, depending on the country) than for the other two surveys, which may reflect the inclusion of smaller, private companies in the latter sample.

We learn from the survey evidence that the CAPM is a popular method for estimating the cost of equity capital and that it is used less by smaller, private companies. This latter result is not surprising because of the difficulty in estimating systematic risk in cases in which the company's equity is not publicly traded.

## 5. SUMMARY

In this chapter, we provided an overview of the techniques used to calculate the cost of capital for companies and projects. We examined the weighted average cost of capital, discussing the methods commonly used to estimate the component costs of capital and the weights applied to these components. The international dimension of the cost of capital, as well as key factors influencing the cost of capital, were also analyzed.

- The weighted average cost of capital is a weighted average of the after-tax marginal costs of each source of capital: WACC $=w_{d} r_{d}(1-t)+w_{p} r_{p}+w_{e} r_{e}$
- An analyst uses the WACC in valuation. For example, the WACC is used to value a project using the net present value method:
NPV $=$ Present value of inflows - Present value of the outflows
- The before-tax cost of debt is generally estimated by means of one of the two methods: yield to maturity or bond rating.
- The yield-to-maturity method of estimating the before-tax cost of debt uses the familiar bond valuation equation. Assuming semiannual coupon payments, the equation is

$$
\begin{aligned}
P_{0} & =\frac{P M T_{1}}{\left(1+\frac{r_{d}}{2}\right)}+\ldots+\frac{P M T_{n}}{\left(1+\frac{r_{d}}{2}\right)^{n}}+\frac{F V}{\left(1+\frac{r_{d}}{2}\right)^{n}} \\
& =\left[\sum_{t=1}^{n} \frac{P M T_{i}}{\left(1+\frac{r_{d}}{2}\right)^{t}}\right]+\frac{F V}{\left(1+\frac{r_{d}}{2}\right)^{n}}
\end{aligned}
$$

We solve for the six-month yield $\left(r_{d} / 2\right)$ and then annualize it to arrive at the before-tax cost of debt, $r_{d}$.

[^23]- Because interest payments are generally tax deductible, the after-tax cost is the true, effective cost of debt to the company. If a current yield or bond rating is not available, such as in the case of a private company without rated debt or a project, the estimate of the cost of debt becomes more challenging.
- The cost of preferred stock is the preferred stock dividend divided by the current preferred stock price:

$$
r_{p}=\frac{D_{p}}{P_{p}}
$$

- The cost of equity is the rate of return required by a company's common stockholders. We estimate this cost using the CAPM (or its variants) or the dividend discount method.
- The CAPM is the approach most commonly used to calculate the cost of common stock. The three components needed to calculate the cost of common stock are the risk-free rate, the equity risk premium, and beta:

$$
E\left(R_{i}\right)=R_{F}+\beta_{i}\left[E\left(R_{M}\right)-R_{F}\right]
$$

- When estimating the cost of equity capital using the CAPM when we do not have publicly traded equity, we may be able to use the pure-play method in which we estimate the unlevered beta for a company with similar business risk, $\beta_{\mathrm{U}}$,

$$
\beta_{U, \text { comparable }}=\frac{\beta_{\mathrm{L}, \text { comparable }}}{\left[1+\left(\left(1-t_{\text {comparable }}\right) \frac{\mathrm{D}_{\text {comparable }}}{\mathrm{E}_{\text {comparable }}}\right)\right]}
$$

and then lever this beta to reflect the financial risk of the project or company:

$$
\beta_{\mathrm{L}, \text { project }}=\beta_{\mathrm{U}, \text { comparable }}\left[1+\left(\left(1-t_{\text {project }}\right) \frac{\mathrm{D}_{\text {project }}}{\mathrm{E}_{\text {project }}}\right)\right]
$$

- It is often the case that country and foreign exchange risk are diversified so that we can use the estimated $\beta$ in the CAPM analysis. However, in the case in which these risks cannot be diversified away, we can adjust our measure of systematic risk by a country equity premium to reflect this nondiversified risk:

Country equity premium $=$ Sovereign yield spread

Annualized standard deviation
$\overline{\text { Annualized standard deviation }}$ of the sovereign bond market in terms of the developed market currency

- The dividend discount model approach is an alternative approach to calculating the cost of equity, whereby the cost of equity is estimated as follows:

$$
r_{e}=\frac{D_{1}}{P_{0}}+g
$$

- We can estimate the growth rate in the dividend discount model by using published forecasts of analysts or by estimating the sustainable growth rate:

$$
g=(1-D / E P S) \mathrm{ROE}
$$

- In estimating the cost of equity, an alternative to the CAPM and dividend discount approaches is the bond yield plus risk premium approach. In this approach, we estimate the before-tax cost of debt and add a risk premium that reflects the additional risk associated with the company's equity.
- The marginal cost of capital schedule is a graph plotting the new funds raised by a company on the $x$-axis and the cost of capital on the $y$-axis. The cost of capital is level to the point at which one of the costs of capital changes, such as when the company bumps up against a debt covenant, requiring it to use another form of capital. We calculate a break point using information on when the different sources' costs change and the proportions that the company uses when it raises additional capital:

$$
\text { Break point }=\frac{\text { Amount of capital at which the source's cost of capital changes }}{\text { Proportion of new capital raised from the source }}
$$

- Flotation costs are costs incurred in the process of raising additional capital. The preferred method of including these costs in the analysis is as an initial cash flow in the valuation analysis.
- Survey evidence tells us that the CAPM method is the most popular method used by companies in estimating the cost of equity. The CAPM is more popular with larger, publicly traded companies, which is understandable considering the additional analyses and assumptions required in estimating systematic risk for a private company or project.


## PRACTICE PROBLEMS

1. The cost of equity is equal to the
A. Expected market return.
B. Rate of return required by stockholders.
C. Cost of retained earnings plus dividends.
D. Risk the company incurs when financing.
2. Which of the following statements is correct?
A. The appropriate tax rate to use in the adjustment of the before-tax cost of debt to determine the after-tax cost of debt is the average tax rate because interest is deductible against the company's entire taxable income.
B. For a given company, the after-tax cost of debt is less than both the cost of preferred equity and the cost of common equity.
C. For a given company, the investment opportunity schedule is upward sloping because, as a company invests more in capital projects, the returns from investing increase.
D. The target capital structure is the average ratio of debt to equity for the most recent fiscal years.
3. Using the dividend discount model, what is the cost of equity capital for Zeller Mining if the company will pay a dividend of $\mathrm{C} \$ 2.30$ next year, has a payout ratio of 30 percent, a return on equity of 15 percent, and a stock price of $\mathrm{C} \$ 45$ ?
A. 5.11 percent.
B. 9.61 percent.
C. 10.50 percent.
D. 15.61 percent.
4. Dot.Com has determined that it could issue $\$ 1,000$ face value bonds with an 8 percent coupon paid semiannually and a five-year maturity at $\$ 900$ per bond. If Dot.Com's marginal tax rate is 38 percent, its after-tax cost of debt is closest to
A. 6.2 percent.
B. 6.4 percent.
C. 6.6 percent.
D. 6.8 percent.
5. The cost of debt can be determined using the yield-to-maturity and the bond rating approaches. If the bond rating approach is used, the
A. Coupon is the yield.
B. Yield is based on the interest coverage ratio.
C. Company is rated and the rating can be used to assess the credit default spread of the company's debt.
D. After-tax cost of the debt is not known.
6. Morgan Insurance Ltd. issued a fixed-rate perpetual preferred stock three years ago and placed it privately with institutional investors. The stock was issued at $\$ 25$ per share with a $\$ 1.75$ dividend. If the company were to issue preferred stock today, the yield would be 6.5 percent. The stock's current value is
A. $\$ 25.00$.
B. $\$ 26.92$.
C. $\$ 37.31$.
D. $\$ 40.18$.
7. A financial analyst at Buckco Ltd. wants to compute the company's weighted average cost of capital (WACC) using the dividend discount model. The analyst has gathered the following data:

| Before-tax cost of new debt | 8 percent |
| :--- | :--- |
| Tax rate | 40 percent |
| Target debt-to-equity ratio | 0.8033 |
| Stock price | $\$ 30$ |
| Next year's dividend | $\$ 1.50$ |
| Estimated growth rate | 7 percent |

Buckco's WACC is closest to
A. 8 percent.
B. 9 percent.
C. 12 percent.
D. 20 percent.
8. The Gearing Company has an after-tax cost of debt capital of 4 percent, a cost of preferred stock of 8 percent, a cost of equity capital of 10 percent, and a weighted average cost of capital of 7 percent. Gearing intends to maintain its current capital structure as it raises additional capital. In making its capital budgeting decisions for the average-risk project, the relevant cost of capital is
A. 4 percent.
B. 7 percent.
C. 8 percent.
D. 10 percent.
9. Fran McClure of Alba Advisers is estimating the cost of capital of Frontier Corporation as part of her valuation analysis of Frontier. McClure will be using this estimate, along with projected cash flows from Frontier's new projects, to estimate the effect of these new projects on the value of Frontier. McClure has gathered the following information on Frontier Corporation:

|  | Current Year | Forecasted for <br> Next Year |
| :--- | :---: | :---: |
| Book value of debt | $\$ 50$ | $\$ 50$ |
| Market value of debt | $\$ 62$ | $\$ 63$ |
| Book value of shareholders' equity | $\$ 55$ | $\$ 58$ |
| Market value of shareholders' equity | $\$ 210$ | $\$ 220$ |

The weights that McClure should apply in estimating Frontier's cost of capital for debt and equity are, respectively,
A. $w_{d}=0.200 ; w_{e}=0.800$.
B. $w_{d}=0.185 ; w_{e}=0.815$.
C. $w_{d}=0.223 ; w_{e}=0.777$.
D. $w_{d}=0.228 ; w_{e}=0.772$.
10. Wang Securities had a long-term stable debt-to-equity ratio of 0.65 . Recent bank borrowing for expansion into South America raised the ratio to 0.75 . The increased leverage has what effect on the asset beta and equity beta of the company?

Asset Beta Equity Beta
A. Same Higher
B. Same Lower
C. Lower Higher
D. Lower Lower
11. Brandon Wiene is a financial analyst covering the beverage industry. He is evaluating the impact of DEF Beverage's new product line of flavored waters. DEF currently has a debt-to-equity ratio of 0.6 . The new product line would be financed with $\$ 50$ million of debt and $\$ 100$ million of equity. In estimating the valuation impact of this new product line on DEF's value, Wiene has estimated the equity beta and asset beta
of comparable companies. In calculating the equity beta for the product line, Wiene is intending to use DEF's existing capital structure when converting the asset beta into a project beta. Which of the following statements is correct?
A. Using DEF's debt-to-equity ratio of 0.6 is appropriate in calculating the new product line's equity beta.
B. Using DEF's debt-to-equity ratio of 0.6 is not appropriate; rather, the debt-to-equity ratio of the new product, 0.5 , is appropriate to use in calculating the new product line's equity beta.
C. Wiene should use the new debt-to-equity ratio of DEF that would result from the additional $\$ 50$ million debt and $\$ 100$ million equity in calculating the new product line's equity beta.
D. Wiene should use the asset beta determined from the analysis of comparables as the equity beta in evaluating the new product line.
12. Trumpit Resorts Company currently has 1.2 million common shares of stock outstanding and the stock has a beta of 2.2. It also has $\$ 10$ million face value of bonds that have five years remaining to maturity and 8 percent coupon with semiannual payments, and they are priced to yield 13.65 percent. Trumpit has learned that it can issue new common stock at $\$ 10$ a share. The current risk-free rate of interest is 3 percent and the expected market return is 10 percent. If Trumpit issues up to $\$ 2.5$ million of new bonds, the bonds will be priced at par and have a yield of 13.65 percent; if it issues bonds beyond $\$ 2.5$ million, the expected yield will be 16 percent. Trumpit's marginal tax rate is 30 percent. If Trumpit raises $\$ 7.5$ million of new capital while maintaining the same debt-to-equity ratio, its weighted average cost of capital is closest to
A. 14.5 percent.
B. 15.5 percent.
C. 16.5 percent.
D. 17.5 percent.

The following information relates to Questions 13 through 18.
Jurgen Knudsen has been hired to provide industry expertise to Henrik Sandell, CFA, an analyst for a pension plan managing a global large-cap fund internally. Sandell is concerned about one of the fund's larger holdings, auto parts manufacturer Kruspa AB. Kruspa currently operates in 80 countries, with the previous year's global revenues at $€ 5.6$ billion. Recently, Kruspa's CFO announced plans for expansion into China. Sandell worries that this expansion will change the company's risk profile and wonders if he should recommend a sale of the position.

Sandell provides Knudsen with the basic information. Kruspas global annual free cash flow to the firm is $€ 500$ million and earnings are $€ 400$ million. Sandell estimates that cash flow will level off at a 2 percent rate of growth. Sandell also estimates that Kruspa's after-tax free cash flow to the firm on the China project for next three years is, respectively, $€ 48$ million, $€ 52$ million, and $€ 54.4$ million. Kruspa recently announced a dividend of $€ 4.00$ per share of stock. For the initial analysis, Sandell requests that Knudsen ignore possible currency fluctuations. He expects the Chinese plant to sell only to customers within China for the first three years. Knudsen is asked to evaluate Kruspa's planned financing of the required $€ 100$ million with a $€ 80$ public offering of 10 -year debt in Sweden and the remainder with an equity offering.

Additional information:

| Equity risk premium, Sweden | 4.82 percent |
| :--- | :--- |
| Risk-free rate of interest, Sweden | 4.25 percent |
| Industry debt-to-equity ratio | 0.3 |
| Market value of Kruspa's debt | $€ 900$ million |
| Market value of Kruspảs equity | $€ 2.4$ billion |
| Kruspa's equity beta | 1.3 |
| Kruspás before-tax cost of debt | 9.25 percent |
| China credit A2 country risk premium | 1.88 percent |
| Corporate tax rate | 37.5 percent |
| Interest payments each year | Level |

13. Using the capital asset pricing model, Kruspa's cost of equity capital for its typical project is closest to
A. 7.62 percent.
B. 10.52 percent.
C. 12.40 percent.
D. 14.84 percent.
14. Sandell is interested in the weighted average cost of capital of Kruspa AB prior to its investing in the China project. This weighted average cost of capital (WACC) is closest to
A. 7.65 percent.
B. 9.23 percent.
C. 10.17 percent.
D. 10.52 percent.
15. In his estimation of the project's cost of capital, Sandell would like to use the asset beta of Kruspa as a base in his calculations. The estimated asset beta of Kruspa prior to the China project is closest to
A. 1.053 .
B. 1.110 .
C. 1.140 .
D. 1.327 .
16. Sandell is performing a sensitivity analysis of the effect of the new project on the company's cost of capital. If the China project has the same asset risk as Kruspa, then the estimated project beta for the China project, if it is financed 80 percent with debt, is closest to
A. 1.053 .
B. 1.300 .
C. 2.635 .
D. 3.686 .
17. As part of the sensitivity analysis of the effect of the new project on the company's cost of capital, Sandell is estimating the cost of equity of the China project considering that the China project requires a country equity premium to capture the risk of the project. The cost of equity for the project in this case is closest to
A. 9.23 percent.
B. 10.52 percent.
C. 19.91 percent.
D. 28.95 percent.
18. In his report, Sandell would like to discuss the sensitivity of the project's net present value to the estimation of the cost of equity. The China project's net present values, calculated using the equity beta without and with the country risk premium, are, respectively
A. €26 million and € $€ 24$ million.
B. $€ 28$ million and $€ 25$ million.
C. $€ 30$ million and $€ 27$ million.
D. $€ 32$ million and $€ 31$ million.

The following information relates to Questions 19 through 22.
Boris Duarte, CFA, covers initial public offerings for Zellweger Analytics, an independent research firm specializing in global small-cap equities. He has been asked to evaluate the upcoming new issue of TagOn, a U.S.-based business intelligence software company. The industry has grown at 26 percent per year for the previous three years. Large companies dominate the market, but sizable "pure-play" companies such as Relevant, Ltd., ABJ, Inc., and Opus Software Pvt. Ltd. also compete. Each of these competitors is domiciled in a different country, but they all have shares of stock that trade on the U.S. NASDAQ. The debt ratio of the industry has risen slightly in recent years.

|  | Sales in <br> Millions | Market <br> value equity <br> in Billions | Market <br> value debt <br> in Millions | Equity <br> Beta | Tax <br> Rate | Share <br> Price |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Relevant Ltd. | $\$ 752$ | $\$ 3.8$ | $\$ 0.0$ | 1.702 | 23 percent | $\$ 42$ |
| ABJ, Inc. | $\$ 843$ | $\$ 2.15$ | $\$ 6.5$ | 2.800 | 23 percent | $\$ 24$ |
| Opus Software Pvt. Ltd. | $\$ 211$ | $\$ 0.972$ | $\$ 13.0$ | 3.400 | 23 percent | $\$ 13$ |

Duarte uses the information from the preliminary prospectus for TagOn's initial offering. The company intends to issue 1 million new shares. In his conversation with the investment bankers for the deal, he concludes the offering price will be between $\$ 7$ and $\$ 12$. The current capital structure of TagOn consists of a $\$ 2.4$ million five-year noncallable bond issue and 1 million common shares. Other information that Duarte has gathered:

| Currently outstanding bonds | $\$ 2.4$ million five-year bonds, coupon of 12.5 percent, <br> with a market value of $\$ 2.156$ million |
| :--- | :--- |
| Risk-free rate of interest | 5.25 percent |
| Estimated equity risk premium | 7 percent |
| Tax rate | 23 percent |

19. The asset betas for Relevant, ABJ, and Opus, respectively, are
A. $1.70,2.52,2.73$.
B. $1.70,2.79,3.37$.
C. $1.70,2.81,3.44$.
D. 2.634 for each.
20. The weighted average asset beta for the pure players in this industry—Relevant, ABJ, and Opus-weighted by market value, is closest to
A. 1.37 .
B. 1.67 .
C. 1.97 .
D. 2.27.
21. Using the capital asset pricing model, the cost of equity capital for a company in this industry, with a debt-to-equity ratio of 0.01 and a marginal tax rate of 23 percent, is closest to
A. 17 percent.
B. 21 percent.
C. 24 percent.
D. 31 percent.
22. The marginal cost of capital for TagOn, based on the average asset beta for the industry and assuming that new stock can be issued at $\$ 8$ per share, is closest to
A. 20.0 percent.
B. 20.5 percent.
C. 21.0 percent.
D. 21.5 percent.

[^0]:    ${ }^{1}$ In the chapter on capital structure and leverage, we will discuss the capital structure decision in greater detail, including a look at how it relates to the value of the company.

[^1]:    ${ }^{2}$ These weights represent the arithmetic average of the three companies' debt proportion and equity proportion, respectively. If instead we chose to use a weighted average, we would calculate the debt proportion as the sum of the debt for all three companies, divided by the sum of the total capital for all three; we would calculate the equity proportion in the same manner. The weighted average proportions are 0.3562 and 0.6438 , respectively.

[^2]:    ${ }^{3}$ The investment opportunity schedule originates with Fisher's production opportunities (Fisher, 1930) and was adapted to capital budgeting by John Hirshleifer (1958).

[^3]:    ${ }^{4}$ WACC is estimated using fixed proportions of equity and debt. The NPV method assumes a constant required rate of return, whereas a fluctuating capital structure would cause WACC to fluctuate. The importance of this issue is demonstrated by Miles and Ezzell (1980).
    ${ }^{5}$ See the chapter on capital budgeting for a discussion.
    ${ }^{6}$ See Stowe, Robinson, Pinto, and McLeavey (2002) for a presentation of such models.
    ${ }^{7}$ Free cash flow to the firm (FCFF) is the cash flow available to the company's suppliers of capital after all operating expenses (including taxes) have been paid and necessary investments in working capital (e.g., inventory) and fixed capital (e.g., plant and equipment) have been made.

[^4]:    ${ }^{8}$ Free cash flow to equity (FCFE) is the cash flow available to holders of the company's common equity after all operating expenses, interest, and principal payments have been paid and necessary investments in working capital and fixed capital have been made. See Stowe, et al. for more details on FCFF and FCFE and valuation models based on those concepts.
    ${ }^{9} r_{d}$ is expressed as an annual rate and is divided by the number of payment periods per year. Because most corporate bonds pay semiannual interest, we divide $r_{d}$ by 2 in this calculation. The interest payment for each period thus corresponds with the bond's semiannual coupon payment.

[^5]:    ${ }^{10}$ Duration is a more precise measure of a bond's interest rate sensitivity than maturity.

[^6]:    ${ }^{11}$ See, for example, Fabozzi (2004), for an introduction. Fabozzi discusses the estimation of an optionadjusted spread (OAS) to price the call option feature of a callable bond.
    ${ }^{12}$ In the United States, an operating lease is distinguished from a capital lease in Statement of Financial Accounting Standards No. 13, Accounting for Leases (FASB, November 1976). (IAS No. 17 similarly

[^7]:    ${ }^{15}$ Alcoa Annual Report 2004, footnote R, p. 56.

[^8]:    ${ }^{16}$ An example of the multifactor model is the three-factor Fama and French model [Fama and French (1992)], which includes factors for the market, equity capitalization, and the ratio of book value of equity to the market value of equity.
    ${ }^{17}$ These models are discussed in more detail by Bruner, Conroy, Li, O'Halloran, and Palacios Lleras (2003) and by Fama and French (2004).
    ${ }^{18}$ Dimson, Marsh, and Staunton (2003).
    ${ }^{19}$ Siegel (2005) presents a longer time series of market returns, covering the period from 1802 through 2004, and observes an equity return of 6.82 percent and an equity risk premium in the range of 3.31 to 5.36 percent. The range depends on the method of calculation (compounded or arithmetic) and the benchmark (bonds or bills).
    ${ }^{20}$ Aside from the method of averaging (geometric versus arithmetic), estimates of the historical equity risk premium differ depending on the assumed investment horizon (short versus intermediate versus long), whether conditional on some variable or unconditional, whether U.S. or global markets are examined, the source of the data, the period observed, and whether nominal or real returns are estimated.

[^9]:    ${ }^{21}$ We explain Equation 3-6 in more detail in Section 3.3.2.

[^10]:    ${ }^{22}$ Some researchers argue that the equity risk premium should reflect a country risk premium. For example, a multinational company or project may have a higher cost of capital than a comparable domestic company because of political risk, foreign exchange risk, or higher agency costs. In most cases, this risk is unsystematic and hence does not affect the cost of capital estimate.
    ${ }^{23}$ Gordon (1962).

[^11]:    ${ }^{24}$ This risk premium is not to be confused with the equity risk premium. The equity risk premium is the difference between the cost of equity and the risk-free rate of interest. The risk premium in the bond yield plus risk premium approach is the difference between the cost of equity and the company's cost of debt.

[^12]:    ${ }^{25}$ This equation is commonly referred to as the market model and was first introduced by Jensen (1969).
    ${ }^{26}$ Daves, Ehrhardt, and Kunkel (2000).
    ${ }^{27}$ Blume (1971).
    ${ }^{28}$ Ibbotson, Kaplan, and Peterson (1997).

[^13]:    ${ }^{29}$ The process of unlevering and levering a beta was developed by Hamada (1972) and is based on the capital structure theories of Franco Modigliani and Merton Miller.

[^14]:    ${ }^{31}$ In this example, the weights are $w_{d}=0.4 / 1.4=0.2857$ and $w_{e}=1 / 1.4=0.7143$.

[^15]:    ${ }^{32}$ Dimson, Marsh, and Staunton, ibid.

[^16]:    ${ }^{33}$ An analyst must apply judgment and experience to determine a representative average for the comparable companies. This example uses a simple average, but in some situations a weighted average based on some factor such as market capitalization may be more appropriate.

[^17]:    ${ }^{34}$ Harvey (2001).
    ${ }^{35}$ Adding the country spread to the market risk premium for a developing country and then multiplying this sum by the market risk of the project is making the assumption that the country risk premium varies according to market risk. An alternative method calculates the cost of equity as the sum of three terms: (1) the risk-free rate of interest, (2) the product of the beta and the developed market risk premium, and (3) the country risk premium. This latter method assumes that the country risk premium is the same, regardless of the project's market risk.
    ${ }^{36}$ Mariscal and Lee (1993).

[^18]:    ${ }^{37}$ Damodaran (1999 and 2003).

[^19]:    ${ }^{38}$ Erb, Harvey, and Viskanta (1996).
    ${ }^{39}$ In the section on capital structure and leverage, we will discuss cases where a company's WACC may actually decrease as additional capital is raised. For example, if a company financed solely with common equity raises additional capital via debt, the tax advantages provided by debt will result in a lower WACC under the new capital structure. For this discussion, we are assuming that the company is already operating at or near its optimum balance of debt versus equity.

[^20]:    ${ }^{40}$ We can incorporate them for these sources by simply treating the flotation costs as an outlay, hence reducing proceeds from the source.
    ${ }^{41}$ Lee, Lochhead, Ritter, and Zhao (1996).
    ${ }^{42}$ Bühner and Kaserer (2002); Armitage (2000); and Kaserer and Steiner (2004).

[^21]:    ${ }^{43}$ This argument is made by Ezzell and Porter (1976). They argue that the correct treatment is to deduct flotation costs as part of the valuation as one of the initial-period cash flows.

[^22]:    ${ }^{44}$ Graham and Harvey (2002).
    ${ }^{45} \mathrm{~A}$ survey published by Gitman and Mercurio (1982) indicated that fewer than 30 percent used the CAPM model in the estimation of the cost of equity.

[^23]:    ${ }^{46}$ Bancel and Mittoo (2004).
    ${ }^{47}$ Brounen, de Jong, and Koedijk (2004).

