Cost of Capital

Estimation and Applications

SECOND EDITION

Shannon P. Pratt, CFA, FASA, MCBA

JOHN WILEY & SONS, INC.

Cost of Capital

Critical Praise for
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Roger G. Ibbotson
Ibbotson Associates
Chicago, IL

Other Wiley books by Shannon P. Pratt include:

Cost of Capital Workbook
Business Valuation Body of Knowledge: Exam Review and Professional Reference, Second Edition
Business Valuation Body of Knowledge Workbook
The Market Approach to Valuing Businesses
Business Valuation Discounts and Premiums

Cost of Capital

Estimation and Applications

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JOHN WILEY & SONS, INC.

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To my family
(expanded since the first edition)

Millie

Son Mike Pratt
Daughter-in-law Barbara Brooks
  Randall
  Kenneth
Portland, Oregon

Daughter Susie Wilder
Son-in-law Tim Wilder
  John
  Calvin
  Meg
Springfield, Virginia

Daughter Georgie Senor
Son-in-law Tom Senor
  Elisa
  Katie
  Graham
Fayetteville, Arkansas

Son Steve Pratt
Daughter-in-law Jenny Pratt
  Adeline
  Zeph
Tecate, Mexico

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Dr. Shannon P. Pratt is a founder and a managing director of Willamette Management Associates. Founded in 1969, Willamette is one of the oldest and largest independent valuation consulting, economic analysis, and financial advisory services firms, with offices in principal cities across the United States. He is also a member of the board of directors of Paulson Capital Corp., an investment banking firm.

Over the last 35 years, Dr. Pratt has performed valuation engagements for mergers and acquisitions, employee stock ownership plans (ESOPs), fairness opinions, gift and estate taxes, incentive stock options, buy-sell agreements, corporate and partnership dissolutions, dissenting stockholder actions, damages, marital dissolutions, and many other business valuation purposes. He has testified in a wide variety of federal and state courts across the country and frequently participates in arbitration and mediation proceedings.

He holds an undergraduate degree in business administration from the University of Washington and a doctorate in business administration, majoring in finance, from Indiana University. He is a Fellow of the American Society of Appraisers, a Master Certified Business Appraiser, a Chartered Financial Analyst, a Certified Business Counselor, and a Certified Financial Planner, and a Certified in Mergers and Acquisitions Advisor.

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Dr. Pratt is author of *Business Valuation Discounts and Premiums*, *Business Valuation Body of Knowledge*, *Cost of Capital: Estimation and Applications*, 2nd edition, and *The Market Approach to Valuing Businesses* (all published by John Wiley & Sons, Inc.) and *The Lawyer's Business Valuation Handbook* (published by the American Bar Association). He is coauthor of *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th edition, and *Valuing Small Businesses and Professional Practices*, 3rd edition (both published by McGraw-Hill). He is also coauthor of *Guide to Business Valuations*, 12th edition (published by Practitioners Publishing Company).

He is editor-in-chief of the monthly newsletter *Shannon Pratt's Business Valuation Update*®. He oversees *BVLibrary.com*sm, which includes papers, regulations, court case decisions, and many other resources. He also oversees *Pratt's Stats*™, the official completed transaction database of the International Business Brokers Association, and *BVMarketData.com*sm, which includes the online version of *Pratt's Stats*™ as well as *BIZCOMPS*®, *Mergerstat/Shannon Pratt's Control Premium Study*™, *The FMV Restricted Stock Study*™, and *The Valuation Advisors Lack of Marketability Discount Study*™.

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Mr. Mercer is the author of *Quantifying Marketability Discounts: Developing and Supporting Marketability Discounts in the Appraisal of Closely Held Business Interests* (published by Peabody Publishing, LP) and *Valuing Financial Institutions* (published by Business One Irwin, now Irwin Professional Publishing).

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Foreword

Many of us have been anxiously awaiting Shannon Pratt's second edition of *Cost of Capital: Estimation and Applications*, following the successful first edition. The current edition includes a totally rewritten and expanded chapter on how to use Ibbotson Associates' new *Stocks, Bonds, Bills, and Inflation® Valuation Edition Yearbook*, emphasizing the easy-to-use build-up method, as well as providing clarifying links to many of our other methods and products throughout this book. Shannon also has added a chapter on the cost of capital in Economic Value Added (EVA)®, included new sections and data on lack of marketability, control, and minority interests, and provided results from new studies on micro-stocks, sold companies, and price valuation multiples.

Shannon Pratt has been a leader in the valuation field for decades, writing numerous books, operating a consulting and valuation firm, and producing such industry resources as *Shannon Pratt's Business Valuation Update®* and *Pratt's Stats*™. He has been a collector and provider of data and information on prices, ratios, deals, and sales, as well as legal and tax developments in the industry. He has been a developer and compiler of theoretical approaches and practical procedures. It is particularly helpful that he has turned his attention to the cost of capital.

The cost of capital is a critical component of both the valuation and the corporate decision-making processes. Yet the theory is much less understood than the theory of forecasting expected cash flows. For example, increasing leverage may increase the cost of equity and the cost of debt without necessarily affecting the weighted average cost of capital. Cost of capital procedures are a frequent source of major logical errors, not just judgment errors. Mistakes of this type can leave the decision maker or appraiser vulnerable, inasmuch as he or she can actually be proven wrong. This is an area where practitioners badly need a guide such as *Cost of Capital*, so they understand what they are doing.

The cost of capital is one of the key components in valuation. But it is rarely observed directly. Instead, it must be estimated. Numerous models can be used to estimate the cost of capital, such as the build-up models, the Capital Asset Pricing Model, the discounted cash flow model, and the arbitrage pricing theory. These models may require adjustments for risk, capital structure, size of company, and so forth. There are also many ways to estimate the parameters in these models. All of them may be combined in the weighted average cost of capital. Ibbotson Associates is the provider of many of these estimates. I certainly welcome the second edition of *Cost of Capital* as a publication that can help to educate practitioners about what the data mean and how they can use them.

This book is beginning to serve as the standard reference on cost of capital. It will join Shannon Pratt's set of valuation books in providing the theoretical foundations and practical procedures in valuation, capital budgeting, and investment decision making. However, cost of capital is the most challenging subject in valuation, with the richest data and most complex issues. I am personally enthusiastic about adding this book to my reference library.

Roger G. Ibbotson
Chairman, Ibbotson Associates
Professor in Practice, Yale School of Management

Preface

Cost of capital estimation is at once the most critical and the most difficult element of most business valuations and capital expenditure decisions. This book provides a primer for both the neophyte and the experienced financial analyst in making or assessing the cost of capital estimate.

The book is fully indexed and designed to be both a straightforward tutorial and a handy desk reference for:

- Business valuation analysts
- Corporate finance analysts
- CPAs
- Judges and attorneys
- Investment bankers and business sale intermediaries
- Academicians and students

WHAT'S NEW IN THIS EDITION

The second edition is not only updated with current data and references since the first edition in 1998, but is also greatly expanded with additional material:

- A new chapter on cost of capital in Economic Value Added (EVA)®.
- A new appendix detailing the iterative process in calculating the cost of equity component in the weighted average cost of capital (WACC).
- A totally new and expanded chapter on using Ibbotson data, with emphasis on the new *Stocks, Bonds, Bills, and Inflation® (SBBI) Valuation Edition Yearbook*, which was inaugurated in 1999 and has been updated annually.
- The chapter on the build-up method has been modified to reflect use of additional data available in the *SBBI Valuation Edition*.
- Two new sections have been added to the minority versus control implications chapter. One is a study conducted on the *Mergerstat/Shannon Pratt's Control Premium Study*™ database showing, among other things, that 16% of takeovers of public companies occur at prices below their public trading prices! The other is a "tale of two markets," making the point that the merger market is a separate market

from the public stock market. These sections provide support for Roger Ibbotson's contention that cost of capital is not influenced by control or minority status.

- Additional studies on the small stock phenomenon by Roger Grabowski and David King, as well as updates of their original studies.
  - In addition to the 25-sector-size total returns for the groups plus the "financially distressed" group, they have done a parallel study on premiums over CAPM for the same size categories.
  - They have added a new study on costs of capital related to three risk factors derived from company financial statements.
- A new study on the *Pratt's Stats*™ sold company database comparing median price/EBITDA multiples and price/sales multiples for transactions from \$10 million to \$50 million in deal size, with transactions from \$1 million to \$10 million, and under \$10 million for eight broad industry groups, giving evidence that the size effect does continue below \$10 million market value.
- The chapter on handling the discount for lack of marketability has been expanded to include summary results of all major discount for lack of marketability studies. In addition, details of two studies that have been newly developed since the first edition are presented.
- The common errors chapter has been expanded.
- The chapter on cost of capital in the courts has been more than doubled, reflecting cases since the first edition and some previous landmark cases.
- The bibliography and data resources appendixes have been updated and expanded.
- The index has been completely rewritten and expanded, making it much more user-friendly and helpful.

SCOPE AND CONTENT OF THE BOOK

My goal has been to make this book a state-of-the-art treatise on cost of capital estimation, while still making it understandable to the nonprofessional. To this end, the organization of the book starts with a layperson's understanding of the basic concepts and then moves from simpler applications to some of the more complex applications regularly found in the marketplace. The presentation is generously supplemented with tables, graphical diagrams, and examples.

This book addresses the following applications:

- Valuation
  - Businesses and business interests
  - Intangible assets, including intellectual properties
  - Other income-generating assets
  - Ad valorem (property) taxation

- Capital budgeting, feasibility studies, and corporate finance decisions
    - Capital budgeting and allocation
    - Feasibility studies

It lays out basic tools that anyone can use immediately either in estimating the cost of capital or in reviewing someone else's cost of capital estimate:

- Basic cost of capital theory
- How cost of capital is used in business and in business asset valuation and capital expenditure decision making:
    - In the income approach
    - In the market approach
    - In the excess earnings method
- The basic mathematical formulas used, with clear explanations
- Comprehensive sources of information
- Clear and complete definitions of commonly used terminology
- Common errors—how to identify them in other people's work products and how to avoid them
- A comprehensive bibliography

CPE CREDIT

A self-study mail-in quiz at the back of the book will entitle the reader to eight hours of CPE credit.

COST OF CAPITAL WORKBOOK

We have also prepared a *Cost of Capital Workbook* in conjunction with this second edition. Section One of the workbook has questions and computational problems based on each chapter in this text, and Section Two has answers to the questions and solutions to the problems. This will provide hands-on experience for those who desire to practice or test their understanding of the concepts in this book. It will also be valuable preparation for those taking examinations in the AIMR, ASA, AICPA, IBA, NACVA, or CICBV programs. The workbook also contains a mail-in quiz for eight hours of CPE credit as well.

COST OF CAPITAL IS DYNAMIC

Cost of capital is dynamic, in terms of both current market statistics and theoretical development. There has been an acceleration of research and literature on cost of capital in recent years. While this book draws heavily on Ibbotson Associates data, many are challenging its applications today, including both the general equity risk premium and data on the size effect. There is growing emphasis on what in this book we call the "DCF method" of estimating the cost of equity capital (Chapter 12). As noted in that chapter, the DCF method consistently produces lower estimates of the cost of equity capital than either the build-up model or the Capital Asset Pricing Model (CAPM). A few references to recent views are presented at the end of Chapter 9 on CAPM, and others are scattered throughout the Bibliography.

Readers can keep up-to-date on both market and theoretical development through the monthly "Cost of Capital Update" and "Market Data Corner" sections in *Shannon Pratt's Business Valuation Update®*. Please contact us with any comments or questions on the book, and/or for a complimentary current issue of the newsletter, at the following address or at (888) BUS-VALU [(888) 287-8258], fax (503) 291-7955, (800) 846-2291.

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Shannon Pratt
Portland, Oregon

Introduction

PURPOSE AND OBJECTIVE OF THIS BOOK

The purpose of this book is to present both the theoretical development of cost of capital estimation and its practical application to valuation, capital budgeting, and rate-setting problems encountered in current practice. It is intended both as a learning text for those who want to study the subject and as a handy reference for those who are interested in background or seek direction in some specific aspect of cost of capital.

The objective is to serve two primary categories of users:

1. The *practitioner* who seeks a greater understanding of the latest theory and practice in cost of capital estimation
2. The *reviewer* who needs to make an informed evaluation of someone else's methodology and data used to produce a cost of capital estimate

OVERVIEW

The reader can expect the following:

- The *theory* of what drives the cost of capital
- The *models* currently in use to estimate cost of capital
- The *data* available as inputs to the models to estimate cost of capital
- *How to use the cost of capital estimate* in:
  Valuation
  Feasibility studies
  Corporate finance decisions
- *How to reflect minority/control and marketability considerations*
- *Terminology*, with its unfortunately varied and sometimes ambiguous usage in current-day financial analysis

IMPORTANCE OF THE COST OF CAPITAL

The cost of capital estimate is the essential link that enables us to convert a stream of expected income into an estimate of present value. Doing this allows us to make informed pricing decisions for purchases and sales and a comparison of one investment opportunity against another.

COST OF CAPITAL ESSENTIAL IN THE MARKET

In valuation and financial decision making, the cost of capital estimate is just as important as the estimate of the expected amounts of income to be discounted or capitalized. Yet we continually see income estimates laboriously developed and then converted to estimated value by a cost of capital that is practically pulled out of thin air.

In the marketplace, better-informed cost of capital estimation will improve literally billions of dollars' worth of financial decisions every day.

SOUND SUPPORT ESSENTIAL IN THE COURTROOM

In the courts, billions of dollars turn on experts' disputed cost of capital estimates in many contexts:

- Gift, estate, and income tax disputes
- Dissenting stockholder suits
- Corporate and partnership dissolutions
- Marital property settlements
- Employee stock ownership plans (ESOPs)
- Ad valorem (property) taxes
- Utility rate-setting
- Damages calculations

Fortunately, courts are becoming unwilling to accept "Trust me, I'm a great expert" in these disputes and instead are carefully weighing the quality of supporting evidence presented by opposing sides. Because cost of capital is critical to the valuation of any ongoing business, the thorough understanding, analysis, and presentation of cost of capital issues will go a long way toward carrying the day in a battle of experts in a legal setting.

ORGANIZATION OF THIS BOOK

Part I. Cost of Capital Basics

The first chapter defines *cost of capital*. The second chapter describes, in a general sense, how it is used in business valuation and capital budgeting. Chapter 3 defines net cash flow and explains why it is the preferred economic income variable for valuation and capital budgeting. Chapter 4 explains the difference between discounting and capitalizing. Chapter 5 addresses the concept of risk and the impact of risk on the cost of capital. From there we move to the various components of a company's capital structure and the concept of a weighted average of the cost of each component (weighted average cost of capital).

Part II. Estimating the Cost of Equity Capital

The second part explores cost of capital estimation. This includes the build-up model, the Capital Asset Pricing Model (CAPM), discounted cash flow (DCF) models, and arbitrage pricing theory (APT) for estimating the cost of equity.

Part III. Other Topics Related to Cost of Capital

The third part addresses commonly encountered variations in cost of capital application:

- Minority versus controlling interest valuations
- Handling discounts for lack of marketability
- Court case examples of cost of capital issues
- How cost of capital relates to the excess earnings valuation method
- Ad valorem applications
- Cost of capital in Economic Value Added (EVA®)
- Common errors

Appendixes

The appendixes provide sources for follow-up to this book, including a detailed bibliography, cost of capital courses and conferences, sources for the current data needed to implement cost of capital estimation, a ValuSource PRO software section, and a detailed explanation of the iterative process for cost of equity capital estimation in the context of the weighted average cost of capital (WACC).

SUMMARY

The book is designed to serve as both a primer and a reference source.

Part I covers cost of capital basics. Part II covers the methods generally used to estimate cost of equity capital. Part III covers a variety of topics commonly encountered in cost of capital applications. The appendixes provide a directory for further study, data sources, a discussion of using ValuSource PRO software, and a detailed explanation and illustration of the iterative process to estimating cost of equity in the WACC.

Notation System Used in This Book

A source of confusion for those trying to understand financial theory and methods is that financial writers have not adopted a standard system of notation. The following notation system is adapted from the fourth edition of *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, by Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs (New York: McGraw-Hill, 2000).

VALUE AT A POINT IN TIME

PV = Present value
FV = Future value
$MVIC$ = Market value of invested capital

COST OF CAPITAL AND RATE OF RETURN VARIABLES

k = Discount rate (generalized)
k_e = Discount rate for common equity capital (cost of common equity capital). Unless otherwise stated, it generally is assumed that this discount rate is applicable to net cash flow available to common equity.
$k_{e(pt)}$ = Cost of equity prior to tax effect
k_p = Discount rate for preferred equity capital
k_d = Discount rate for debt (net of tax effect, if any) (*Note:* For complex capital structures, there could be more than one class of capital in any of the preceding categories, requiring expanded subscripts.)
$k_{d(pt)}$ = Cost of debt prior to tax effect
k_{ni} = Discount rate for equity capital when net income rather than net cash flow is the measure of economic income being discounted
c = Capitalization rate
c_e = Capitalization rate for common equity capital. Unless otherwise stated, it generally is assumed that this capitalization rate is applicable to net cash flow available to common equity.
c_{ni} = Capitalization rate for net income
c_p = Capitalization rate for preferred equity capital

c_d	= Capitalization rate for debt (*Note:* For complex capital structures, there could be more than one class of capital in any of the preceding categories, requiring expanded subscripts.)
t	= Tax rate (expressed as a percentage of pretax income)
R	= Rate of return
R_f	= Rate of return on a risk-free security
$E(R)$	= Expected rate of return
$E(R_m)$	= Expected rate of return on the "market" (usually used in the context of a market for equity securities, such as the New York Stock Exchange [NYSE] or Standard & Poor's [S&P] 500)
$E(R_i)$	= Expected rate of return on security i
B	= Beta (a coefficient, usually used to modify a rate of return variable)
B_L	= Levered beta
B_U	= Unlevered beta
RP	= Risk premium
RP_m	= Risk premium for the "market" (usually used in the context of a market for equity securities, such as the NYSE or S&P 500)
RP_s	= Risk premium for "small" stocks (usually average size of lowest quintile or decile of NYSE as measured by market value of common equity) over and above RP_m
RP_u	= Risk premium for unsystematic risk attributable to the specific company
RP_i	= Risk premium for the ith security
$K_1 \ldots K_n$	= Risk premium associated with risk factor 1 through n for the average asset in the market (used in conjunction with arbitrage pricing theory)
$WACC$	= Weighted averaged cost of capital

INCOME VARIABLES

E	= Expected economic income (in a generalized sense; i.e., could be dividends, any of several possible definitions of cash flows, net income, etc.)
NI	= Net income (after entity-level taxes)
NCF_e	= Net cash flow to equity
NCF_f	= Net cash flow to the firm (to overall invested capital, or entire capital structure, including all equity and long-term debt)
PMT	= Payment (interest and principal payment on debt security)
D	= Dividends
T	= Tax (in dollars)
GCF	= Gross cash flow (usually net income plus noncash charges)
EBT	= Earnings before taxes

EBIT = Earnings before interest and taxes
EBDIT = Earnings before depreciation, interest, and taxes ("Depreciation" in this context usually includes amortization. Some writers use EBITDA to specifically indicate that amortization is included.)
EBITDA = Earnings before interest, taxes, depreciation, and amortization

PERIODS OR VARIABLES IN A SERIES

i = The *i*th period or the *i*th variable in a series (may be extended to the *j*th variable, the *k*th variable, etc.)
n = The number of periods or variables in a series, or the last number in a series
∞ = Infinity
0 = Period_0, the base period, usually the latest year immediately preceding the valuation date

WEIGHTINGS

W = Weight
W_e = Weight of common equity in capital structure
W_p = Weight of preferred equity in capital structure
W_d = Weight of debt in capital structure
(*Note:* For purposes of computing a weighted average cost of capital [WACC], it is assumed that preceding weightings are at market value.)

GROWTH

g = Rate of growth in a variable (e.g., net cash flow)

MATHEMATICAL FUNCTIONS

$\sum$ = Sum of (add all the variables that follow)
$\prod$ = Product of (multiply together all the variables that follow)
$\bar{x}$ = Mean average (the sum of the values of the variables divided by the number of variables)
G = Geometric mean (the product of the values of the variables taken to the root of the number of variables)

Part I

Cost of Capital Basics

Chapter 1

Defining Cost of Capital

Components of a Company's Capital Structure
Cost of Capital Is a Function of the Investment
Cost of Capital Is Forward Looking
Cost of Capital Is Based on Market Value, Not Book Value
Cost of Capital Is Usually Stated in Nominal Terms
Cost of Capital Equals Discount Rate
Discount Rate Is Not the Same as Capitalization Rate
Summary

Cost of capital is *the expected rate of return that the market requires in order to attract funds to a particular investment.* In economic terms, the cost of capital for a particular investment is an *opportunity cost*—the cost of forgoing the next best alternative investment. In this sense, it relates to the economic *principle of substitution*—that is, an investor will not invest in a particular asset if there is a more attractive substitute.

The "market" refers to the universe of investors who are reasonable candidates to provide funds for a particular investment. Capital or funds are usually provided in the form of cash, although in some instances capital may be provided in the form of other assets. The cost of capital usually is expressed in percentage terms, that is, the annual amount of dollars that the investor requires or expects to realize, expressed as a percentage of the dollar amount invested.

Put another way:

> Since the cost of anything can be defined as the price one must pay to get it, the cost of capital is the return a company must promise in order to get capital from the market, either debt or equity. A company does not set its own cost of capital; it must go into the market to discover it. Yet meeting this cost is the financial market's one basic yardstick for determining whether a company's performance is adequate.[1]

As the preceding quote suggests, most of the information for estimating the cost of capital for any company, security, or project comes from the investment markets. The cost of capital is always an *expected* return. Thus, analysts and would-be investors never actually observe it. We analyze many types of market data to estimate the cost of capital for a company, security, or project in which we are interested.

As Roger Ibbotson put it, "The Opportunity Cost of Capital is equal to the return that could have been earned on alternative investments at a specific level of risk."[2] In other words, it is the competitive return available in the market on a comparable investment, risk being the most important component of comparability.

COMPONENTS OF A COMPANY'S CAPITAL STRUCTURE

The term "capital" in this context means the components of an entity's capital structure. The primary components of a capital structure include:

- Long-term debt
- Preferred equity (stock or partnership interests with preference features, such as seniority in receipt of dividends or liquidation proceeds)
- Common equity (stock or partnership interests at the lowest or residual level of the capital structure)

There may be more than one subcategory in any or all of the above categories of capital. Also, there may be related forms of capital, such as warrants or options. Each component of an entity's capital structure has its unique cost, depending primarily on its respective risk.

Simply and cogently stated, "The cost of equity is the rate of return investors require on an equity investment in a firm."[3]

Recognizing that the cost of capital applies to both debt and equity investments, a well-known text states, "Both creditors and shareholders expect to be compensated for the opportunity cost of investing their funds in one particular business instead of others with equivalent risk."[4]

The next quote explains how the cost of capital can be viewed from three different perspectives:

> The cost of capital (sometimes called the expected or required rate of return or the discount rate) can be viewed from three different perspectives. On the asset side of a firm's balance sheet, it is the rate that should be used to discount to a present value the future expected cash flows. On the liability side, it is the economic cost to the firm of attracting and retaining capital in a competitive environment, in which investors (capital providers) carefully analyze and compare all return-generating opportunities. On the investor's side, it is the return one expects and requires from an investment in a firm's debt or equity. While each of these perspectives might view the cost of capital differently, they are all dealing with the same number.[5]

When we talk about the cost of ownership capital (i.e., the expected return to a stock or partnership investor), we usually use the phrase "cost of equity capital." When we talk about the cost of capital to the firm overall (i.e., the average cost of capital for both ownership interests and debt), we usually use the phrase "weighted average cost of capital" (WACC) or "blended cost of capital."

COST OF CAPITAL IS A FUNCTION OF THE INVESTMENT

As Ibbotson puts it, "The cost of capital is a function of the investment, not the investor."[6] The cost of capital comes from the marketplace. The marketplace is the universe of investors for a particular asset.

Brealey and Myers state the same concept: "The true cost of capital depends on the use to which the capital is put."[7] They make the point that it would be an error to evaluate a potential investment on the basis of a company's overall cost of capital if that investment were more or less risky than the company's existing business. "Each project should be evaluated at its *own* opportunity cost of capital."[8]

When a company uses the cost of capital to evaluate a commitment of capital to an investment or project, it often refers to that cost of capital as the "hurdle rate." The "hurdle rate" means the minimum expected rate of return that the company would be willing to accept to justify making the investment. As noted in the previous paragraph, the "hurdle rate" for any given prospective investment may be at, above, or below the company's overall cost of capital, depending on the degree of risk of the prospective investment compared to the company's overall risk.

The most popular theme of contemporary corporate finance is that companies should be making investments, either capital investments or acquisitions, from which the returns will exceed the cost of capital for that investment. Doing so creates *economic value added*, *economic profit*, or *shareholder value added*.[9]

COST OF CAPITAL IS FORWARD LOOKING

The cost of capital represents investors' *expectations*. There are three elements to these expectations:

1. The "real" rate of return—the amount investors expect to obtain in exchange for letting someone else use their money on a riskless basis
2. Expected inflation—the expected depreciation in purchasing power while the money is tied up
3. Risk—the uncertainty as to when and how much cash flow or other economic income will be received

It is the combination of the first two items above that is sometimes referred to as the "time value of money." While these expectations may be different for different investors, the market tends to form a consensus with respect to a particular investment or category of investments. That consensus determines the cost of capital for investments of varying levels of risk.

The cost of capital, derived from investors' expectations and the market's consensus of those expectations, is applied to *expected economic income, usually measured in terms of cash flows,* in order to estimate present values or to compare investment alternatives of similar or differing levels of risk. "Present value," in this context, refers to the dollar amount that a rational and well-informed investor would be willing to pay today for the stream of expected economic income being evaluated. In mathematical terms, the cost of capital is the percentage rate of return that equates the stream of expected income with its present cash value.

COST OF CAPITAL IS BASED ON MARKET VALUE, NOT BOOK VALUE

The cost of capital is the expected rate of return on some base value. That base value is measured as the market value of an asset, not its book value. For example, the yield to maturity shown in the bond quotations in the financial press is based on the closing market price of a bond, not on its face value. Similarly, the implied cost of equity for a company's stock must be (or should be) based on the market price per share at which its trades, not on the company's book value per share of stock. It was noted earlier that the cost of capital is estimated from market data. This data refers to *expected returns relative to market prices*. By applying the cost of capital derived from market expectations to the expected cash flows (or other measure of economic income) from the investment or project under consideration, the market value can be estimated.

COST OF CAPITAL IS USUALLY STATED IN NOMINAL TERMS

Keep in mind that we have talked about expectations, including inflation. The return an investor requires includes compensation for reduced purchasing power of the dollar over the life of the investment. Therefore, when the analyst or investor applies the cost of capital to expected returns to estimate value, he or she must also include expected inflation in those expected returns.

This obviously assumes that investors have reasonable consensus expectations regarding inflation. For countries subject to unpredictable hyperinflation, it is sometimes more practical to estimate cost of capital in real terms rather than in nominal terms.

COST OF CAPITAL EQUALS DISCOUNT RATE

The essence of the cost of capital is that it is the percentage return that equates expected economic income with present value. The expected rate of return in this context is called a *discount rate*. By a "discount rate," the financial community means an annually compounded rate at which each increment of expected economic income is discounted back to its present value. A discount rate reflects both time value of money and risk and therefore represents the cost of capital. The sum of the discounted present values of each future period's incremental cash flow or other measure of return equals the present value of the investment, reflecting the expected amounts of return over the life of the investment. The terms "discount rate," "cost of capital," and "required rate of return" are often used interchangeably.

The economic income referenced here represents *total expected returns*. In other words, this economic income includes increments of cash flow realized by the investor while holding the investment, as well as proceeds to the investor on liquidation of the investment. The rate at which these expected future total returns are reduced to present value is the discount rate, which is the *cost of capital* (required rate of return) for a particular investment.

DISCOUNT RATE IS NOT THE SAME AS CAPITALIZATION RATE

Discount rate and capitalization rate are two distinctly different concepts. As noted in the previous section, discount rate equates to cost of capital. It is a rate applied to *all* expected incremental returns to convert the expected return stream to a present value.

A capitalization rate, however, is merely a divisor applied to *one single* element of return to estimate a present value. The only instance in which the discount rate is equal to the capitalization rate is when each future increment of expected return is equal (i.e., no growth), and the expected returns are in perpetuity. One of the few examples would be a preferred stock paying a fixed amount of dividend per share in perpetuity.

In the unique case where an amount of return is expected to grow at a constant rate in perpetuity, the capitalization rate applicable to that expected return is equal to the discount rate less the expected rate of growth. The relationship between discount and capitalization rates is discussed further in future chapters, especially in Chapter 4 on "Discounting versus Capitalizing."

SUMMARY

As stated in the Introduction, "The cost of capital estimate is the essential link that enables us to convert a stream of expected income into an estimate of present value."

Cost of capital has several key characteristics:

- It is *market driven*. It is the expected rate of return that the market requires to commit capital to an investment.
- It is a function of the *investment*, not the *investor*.
- It is *forward looking*, based on *expected* returns.
- The base against which cost of capital is measured is *market value*, not book value.
- It is usually measured in *nominal terms*, that is, including expected inflation.

- It is the link, called a *discount rate*, that equates expected future returns for the life of the investment with the present value of the investment at a given date.

Notes

1. Mike Kaufman, "Profitability and the Cost of Capital," Chapter 8 in *Handbook of Budgeting*, 4th ed., Robert Rachlin, ed. (New York: John Wiley & Sons, Inc., 1999), 8-3.
2. Ibbotson Associates, *Cost of Capital Workshop* (Chicago: Ibbotson Associates, 1999), Chapter 1, p. 2.
3. Aswath Damodaran, *Investment Valuation: Tools and Techniques for Determining the Value of* Any *Asset*, 2d ed. (New York: John Wiley & Sons, Inc., 2000), 182.
4. Tom Copeland, Tim Koller, and Jack Murrin, *Valuation: Measuring and Managing the Value of Companies*, 3d ed. (New York: John Wiley & Sons, Inc., 2000), 201.
5. *Stocks, Bonds, Bills and Inflation, Valuation Edition 2002 Yearbook* (Chicago: Ibbotson Associates, 2002), 23.
6. Ibbotson Associates, *Cost of Capital Workshop* (Chicago: Ibbotson Associates, 1999), Chapter 1, p. 7.
7. Richard A. Brealey and Stewart C. Myers, *Principles of Corporate Finance*, 6th ed. (Boston: Irwin McGraw-Hill, 2000), 222.
8. Ibid. at 221.
9. See, for example, Copeland et al., *Valuation*; also see Alfred Rappaport, *Creating Shareholder Value*, rev. ed. (New York: The Free Press, 1998).

Chapter 2

Introduction to Cost of Capital Applications: Valuation and Project Selection

Net Cash Flow Is the Preferred Economic Income Measure
Cost of Capital Is the Proper Discount Rate
Present Value Formula
Example: Valuing a Bond
Relationship of Discount Rate to Capitalization Rate
Applications to Businesses, Business Interests, Projects, and Divisions
Summary

Cost of capital has many applications, the two most common being valuation and capital investment project selection. These two applications are very closely related. This chapter discusses these two applications in very general terms so the reader can quickly understand how the cost of capital is used every day in valuations and financial decisions worth billions of dollars. Later chapters discuss these applications in more detail.

NET CASH FLOW IS THE PREFERRED ECONOMIC INCOME MEASURE

For the purpose of this chapter, we will assume that the measure of economic income to which cost of capital will be applied is *net cash flow* (sometimes called *free cash flow*). Net cash flow is discretionary cash available to be paid out to capital stakeholders (e.g., dividends, withdrawals, discretionary bonuses) without jeopardizing the projected ongoing operations of the business. We will provide a more exact definition of net cash flow in Chapter 3.

Net cash flow is the measure of economic income on which most financial analysts today prefer to focus for both valuation and capital investment project selection. We explain the reasons for this preference in more detail in Chapter 3. Net cash flow represents money available to stakeholders. Most analysts prefer this measure of income because it obviates owners' discretionary disposal of company funds. Although the contemporary literature of corporate finance widely embraces a preference for net cash flow as the relevant economic income variable to which to apply cost of capital for valuation and decision making, there is still a contingent of analysts who like to focus on accounting income.[1]

COST OF CAPITAL IS THE PROPER DISCOUNT RATE

At the end of Chapter 1, it was said that the cost of capital is customarily used as a discount rate to convert expected future returns to a present value. This concept is summarized succinctly by Brealey and Myers: "Value today always equals future cash flow discounted at the opportunity cost of capital."[2]

In this context, let us keep in mind critical characteristics of a discount rate:

- *Definition:* A *discount rate* is a yield rate used to convert anticipated future payments or receipts into present value (i.e., a cash value as of today or as of a specified valuation date).
- The discount rate represents the *total rate of return* that the investor expects to realize on the amount invested.

The use of the cost of capital to estimate present value thus requires two sets of estimates:

1. The *numerator:* The expected amount of return on the investment in each future period over the life of the investment
2. The *denominator:* The discount rate, which is the cost of capital

Usually analysts and investors make the simplifying assumption that the cost of capital is constant over the life of the investment and use the same cost of capital to apply to each increment of expected future return. There are, however, special cases in which analysts might choose to estimate a discrete cost of capital to apply to the expected return in each future period. (An example is when the analyst anticipates a changing weighted average cost of capital because of a changing capital structure.) The above notwithstanding, well-known author, professor, and consultant Dr. Alfred Rappaport espouses a constant cost of capital in his 1998 edition of *Creating Shareholder Value:*

> The appropriate rate for discounting the company's cash flow stream is the weighted average of the costs of debt and equity capital. . . . It is important to emphasize that the relative weights attached to debt and equity, respectively, are neither predicated on dollars the firm has raised in the past, nor do they constitute the relative proportions of dollars the firm plans to raise in the current year. Instead, the relevant weights should be based on the proportions of debt and equity that the firm targets for its capital structure over the long-term planning period.[3]

The latter view is most widely accepted.

PRESENT VALUE FORMULA

Converting the foregoing concepts into a mathematical formula, we have the following, which is the essence of using cost of capital to estimate present value:

Formula 2.1

$$PV = \frac{NCF_1}{(1+k)} + \frac{NCF_2}{(1+k)^2} + \ldots + \frac{NCF_n}{(1+k)^n}$$

where:

PV = Present value
$NCF_1 \ldots NCF_n$ = Net cash flow (or other measure of economic income) expected in each of the periods 1 through n, n being the final cash flow in the life of the investment
k = Cost of capital applicable to the defined stream of net cash flow

The critical job for the analyst is to match the cost of capital estimate to the definition of the economic income stream being discounted. This is largely a function of reflecting in the cost of capital estimate the degree of risk inherent in the expected cash flows being discounted. The relationship between risk and the cost of capital is the subject of Chapter 5.

EXAMPLE: VALUING A BOND

A simple example of the use of Formula 2.1 is valuing a bond for which a risk rating has been estimated. Let us make five assumptions:

1. The bond has a face value of $1,000.
2. It pays 8% interest on its face value.
3. The bond pays interest once a year, at the end of the year. (This, of course, is a simplifying assumption. Some bonds and notes pay only annually, but most publicly traded bonds pay interest semiannually.)
4. The bond matures exactly three years from the valuation date.
5. As of the valuation date, the market yield to maturity (i.e., total rate of return, including interest payments and price appreciation) for bonds of the same risk grade as the subject bond is 10%.

Note three important implications of this scenario:

1. The issuing company's *embedded cost of capital* for this bond is only 8%, although the *market cost of capital* at the valuation date is 10%. The discrepancy

may be because the general level of interest rates was lower at the time of issuance of this particular bond, or because the market's rating of the risk associated with this bond increased between the date of issuance and the valuation date.

2. If the issuing company wanted to issue new debt on comparable terms as of the valuation date, it presumably would have to offer investors a 10% yield, the current market-driven cost of capital for bonds of that risk grade, to induce investors to purchase the bonds.
3. For purposes of valuation and capital budgeting decisions, when we refer to cost of capital, we mean market cost of capital, not embedded cost of capital. (Embedded cost of capital is sometimes used in utility rate-making, but this chapter focuses only on valuation and capital budgeting applications of cost of capital.)

Substituting numbers derived from the preceding assumptions into Formula 2.1 gives us:

Formula 2.2

$$PV = \frac{\$80}{(1+.10)} + \frac{\$80}{(1+.10)^2} + \frac{\$80}{(1+.10)^3} + \frac{\$1{,}000}{(1+.10)^3}$$

$$= \frac{\$80}{1.10} + \frac{\$80}{1.21} + \frac{\$80}{1.331} + \frac{\$1{,}000}{1.331}$$

$$= \$72.73 + \$66.12 + \$60.11 + \$751.32$$

$$= \underline{\underline{\$950.28}}$$

In this example, the fair market value of the subject bond as of the valuation date is \$950.28. That is the amount that a willing buyer would expect to pay and a willing seller would expect to receive (before considering any transaction costs).

RELATIONSHIP OF DISCOUNT RATE TO CAPITALIZATION RATE

It is important to distinguish between a discount rate and a capitalization rate and to understand the relationship between the two. Critical characteristics of a capitalization rate include:

- *Definition:* A *capitalization rate* is a yield rate used to convert a single payment or measure of economic income into present value (as opposed to a discount rate, which is used to convert *all* expected future payments to a present value).
- The capitalization rate represents only the *current rate of return*, that is, the return received in a single period (as opposed to a discount rate, which represents the *total rate of return*).

APPLICATIONS TO BUSINESSES, BUSINESS INTERESTS, PROJECTS, AND DIVISIONS

The same construct can be used to value an equity interest in a company or a company's entire invested capital. One projects the cash flows available to the interest to be valued and discounts those cash flows at a cost of capital discount rate that reflects the risk associated with achieving the particular cash flows. Details of this procedure for valuing entire companies or interests in companies are presented in later chapters.

Similarly, the same construct can be applied to evaluating a capital budgeting decision, such as building a plant or buying equipment. In that case, the cash flows to be discounted are *incremental* cash flows, that is, cash flows resulting from the decision that would not occur absent the decision. The early portions of the cash flow stream may be negative while funds are being invested in the project.

The primary relationship to remember is that cost of capital is a function of the investment, not of the investor. Therefore, the analyst must evaluate the risk of each project under consideration. If the risk of the project is greater or less than the company's overall risk, then the cost of capital by which that project is evaluated should be commensurately higher or lower than the company's overall cost of capital.

Although some companies apply a single "hurdle rate" to all proposed projects or investments, the consensus in the literature of corporate finance is that the rate by which to evaluate any investment should be based on the risk of that investment, not on the company's overall risk that drives the company's cost of capital. I agree with this consensus. If the company invests in something riskier than its normal operations, the company's risk will increase marginally. When this increased risk is recognized and reflected in the market, it will raise the company's cost of capital. If the returns on the riskier new investment are not great enough to achieve higher returns commensurate with this higher cost of capital, the result will be a decrease in the stock price and a loss of shareholder value.

Somewhere between estimating cost of capital for an entire company and cost of capital for a specific project is the matter of divisional cost of capital, or estimating cost of capital for a division of a company. In many respects, estimating cost of capital for a division is akin to estimating cost of capital for an entire privately held company.

SUMMARY

The most common cost of capital applications are valuation of an investment or prospective investment and project selection decisions (the core component of capital budgeting). In both applications, returns expected from the capital outlay are discounted to a present value by a discount rate, which should be the cost of capital applicable to the specific investment or project. The measure of returns generally preferred today is *net cash flow*, as discussed in the next chapter.

Notes

1. See, for example, Z. Christopher Mercer, *Valuing Financial Institutions* (Homewood, IL: Business One Irwin, 1992), Chapter 13; and his article "The Adjusted Capital Asset Pricing Model for Developing Capitalization Rates," *Business Valuation Review* (December 1989): 147 et. seq.
2. Richard A. Brealey and Stewart C. Myers, *Principles of Corporate Finance*, 6th ed. (Boston: Irwin McGraw-Hill, 2000), 73.
3. Alfred Rappaport, *Creating Shareholder Value* (New York: The Free Press, 1998), 37.

Chapter 3

Net Cash Flow: The Preferred Measure of Return

Defining Net Cash Flow
- Net Cash Flow to Equity
- Net Cash Flow to Invested Capital

Net Cash Flows Should Be Probability-Weighted Expected Values

Why Net Cash Flow Is the Preferred Measure of Economic Income
- Conceptual Reason for Preferring Net Cash Flow
- Empirical Reason for Preferring Net Cash Flow

Summary

Cost of capital is a meaningless concept until we define the measure of economic income to which it is to be applied. The variable of choice for most financial decision making based on the tools of modern finance is *net cash flow*. This, obviously, poses two critical questions:

1. How do we define net cash flow?
2. Why is it considered the best economic income variable to use in net present value analysis?

DEFINING NET CASH FLOW

Net cash flow is cash that a business or project does not have to retain and reinvest in itself to sustain the projected levels of cash flows in future years. In other words, it is cash available to be paid out in any year to the owners of capital without jeopardizing the company's expected-cash-flow-generating capability in future years. (*Net cash flow* is sometimes called *free cash flow*. It is also sometimes called *net free cash flow*, although this phrase seems redundant. Finance terminology being as ambiguous as it is, minor variations in the definitions of these terms arise occasionally.)

Net Cash Flow to Equity

In valuing *equity* by discounting or capitalizing expected cash flows (keeping in mind the important difference between discounting and capitalizing, as discussed elsewhere), *net cash flow to equity* (NCF_e in our notation system) is defined as:

	Net income to common stock (after tax)
+	Noncash charges (e.g., depreciation, amortization, deferred revenue, deferred taxes)
–	Capital expenditures*
–	Additions to net working capital*
–	Dividends on preferred stock
±	Changes in long-term debt (add cash from borrowing, subtract repayments)*
=	Net cash flow to equity

*Only amounts necessary to support projected operations

Net Cash Flow to Invested Capital

In valuing the entire *invested capital* of a company or project by discounting or capitalizing expected cash flows, *net cash flow to invested capital* (NCF_f in our notation system) is defined as:

	Net income to common stock (after tax)
+	Noncash charges (e.g., depreciation, amortization, deferred revenue, deferred taxes)
–	Capital expenditures*
–	Additions to net working capital*
+	Dividends on preferred stock
+	Interest expense (net of the tax deduction resulting from interest as a tax-deductible expense)
=	Net cash flow to invested capital

*Only amounts necessary to support projected operations

In other words, NCF_f (net cash flow to invested capital) includes interest (tax-affected, because interest is a deductible expense for tax purposes), because invested capital includes the debt on which the interest is paid, whereas net cash flow to equity does not.

Occasionally, an analyst treats earnings before interest, taxes, depreciation, and amortization (EBITDA) as if it were free cash flow. This error is not a minor matter, since the analyst has added back the noncash charges without deducting the capital expenditure investments, not to mention additions to working capital necessary to keep the operation functioning as expected.

When we discount net cash flow to equity, the appropriate discount rate is the cost of equity capital. When we discount net cash flow to all invested capital, the appropriate discount rate is the weighted average cost of capital (WACC).

NET CASH FLOWS SHOULD BE PROBABILITY-WEIGHTED EXPECTED VALUES

Net cash flows to be discounted or capitalized should be *expected values*, that is, *probability-weighted* cash flows.

If the distribution of possible cash flows in each period is symmetrical above and below the most likely cash flow in that period, then the most likely cash flow is equal to the probability-weighted cash flow (the mathematical expected value of the distribution). However, many distributions of possible cash flows are skewed. This is where

Exhibit 3.1 Cash Flow Expectation Tables

Scenario A—Symmetrical Cash Flow Expectation

Midpoint of Range	Probability of Occurrence	Weighted Value
$1,600	0.01	$ 16
1,500	0.09	135
1,300	0.20	260
1,000	0.40	400
700	0.20	140
500	0.09	45
400	0.01	4
	1.00	$1,000

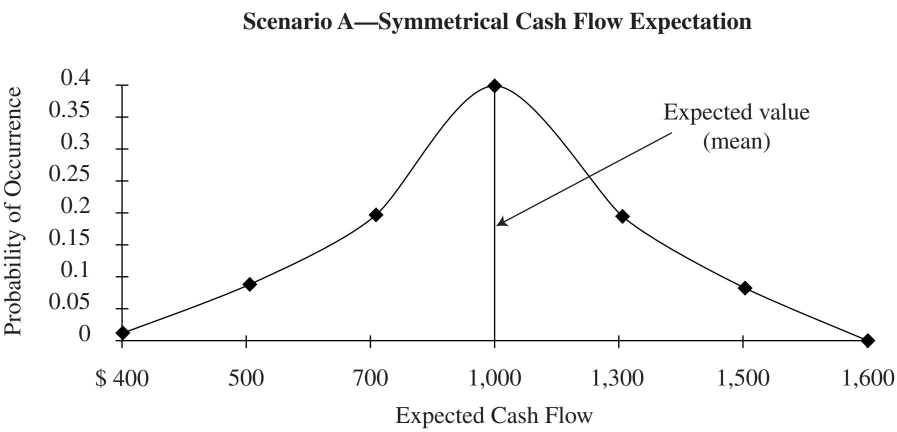
Scenario B—Skewed Cash Flow Expectation

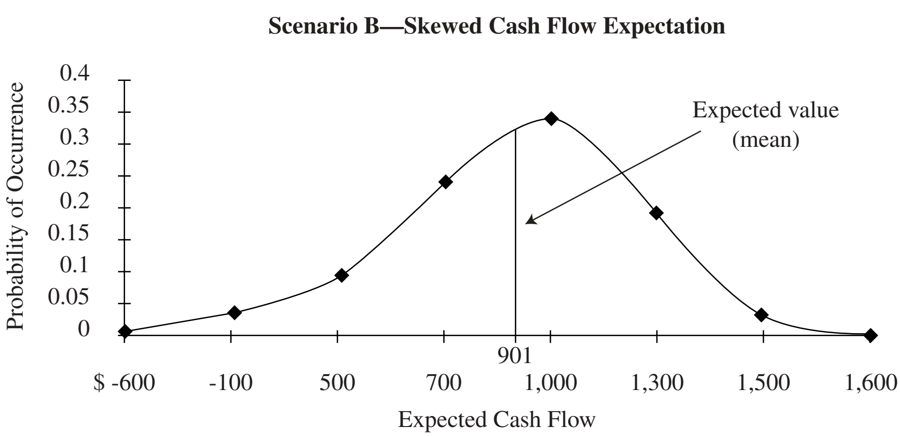
Midpoint of Range	Probability of Occurrence	Weighted Value
$1,600	0.01	$ 16
1,500	0.04	60
1,300	0.20	260
1,000	0.35	350
700	0.25	175
500	0.10	50
(100)	0.04	(4)
(600)	0.01	(6)
	1.00	$901

probability weighting comes into play. Exhibit 3.1 tabulates the probability-weighted expected values of projected cash flows under a symmetrically distributed scenario and a skewed distribution scenario. Exhibit 3.2 portrays the information in Exhibit 3.1 graphically.

In both scenario A and scenario B, the most likely cash flow is $1,000. In scenario A, the expected value (probability weighted) is also $1,000. But in scenario B, the expected value is only $901. In scenario B, $901 is the figure that should appear in the numerator of the discounted cash flow formula, not $1,000. Most analysts do not have the luxury of a probability distribution for each expected cash flow, and it is not a common practice. However, they should be aware of the concept when deciding on the amount of each expected cash flow to be discounted.

Exhibit 3.2 Cash Flow Expectation Graphs





WHY NET CASH FLOW IS THE PREFERRED MEASURE OF ECONOMIC INCOME

There are two reasons why the financial community tends to focus on net cash flow as the preferred measure of economic income to be discounted by the opportunity cost of capital to estimate the net present value of an investment opportunity. They are:

1. *Conceptual:* It is what you really get (i.e., what an investor actually expects to receive).
2. *Empirical:* It is the economic income measure for which we have the best historical data available to estimate a discount rate.

Conceptual Reason for Preferring Net Cash Flow

Net cash flow, as defined earlier, is that portion of the cash flow over which the control owner has total discretion as to its disposal. It is not necessary to retain net cash flow to sustain the business; rather, it is available to be paid out to owners or used for any other desired purposes. This is the measure of economic income of greatest interest to most investors.

Ibbotson clearly states the case for preferring what it terms "free cash flows" (i.e., net cash flows after tax) as the appropriate economic income measure to discount:

> There are several things to note about free cash flow. First, it is an after-tax concept. While the equation starts with earnings before interest and taxes (EBIT), this number is tax-adjusted to get to an after-tax value. The equation starts with tax-adjusted EBIT because we want to focus on cash flows independently of capital structure. We must therefore start with earnings before interest expenses and then tax adjust those earnings. Secondly, pure accounting adjustments need to be added back into the analysis. It is for this reason that depreciation expense and deferred tax expense are added back into the after-tax EBIT. Finally, cash flows necessary to keep the company going forward must be subtracted from the equation. These cash flows represent necessary capital expenditures to maintain plant, property, and equipment or other capital expenditures that arise out of the ordinary course of business. Another common subtraction is reflected in changes in working capital. The assumption in most business valuation settings is that the entity in question will remain a long-term going concern that will grow over time. As companies grow, they accumulate additional accounts receivable and other working capital elements that require additional cash to support.
>
> Free cash flow is the relevant cash flow stream because it represents the broadest level of earnings that can be generated by the asset. With free cash flow as the starting point, the owners of a firm can decide how much of the cash flow stream should be diverted toward new ventures, capital expenditures, interest payments, and dividend payments. It is incorrect to focus on earnings as the cash flow stream to be valued because earnings contain a number of accounting adjustments and already include the impact of the capital structure.[1]

Empirical Reason for Preferring Net Cash Flow

If the Ibbotson Associates data are used to develop an equity discount rate—using either the build-up model or the Capital Asset Pricing Model (CAPM)—the discount rate is applicable to net cash flow available to the equity investor. This is because the Ibbotson return data have two components:

1. Dividends
2. Change in stock price

The investor receives the dividends, so their utilization is entirely at the investor's discretion. The investor's realization of the change in stock price is equally discretionary because the stocks are highly liquid (i.e., they can be sold at their market price at any time, with the seller receiving the proceeds in cash within three business days).

SUMMARY

Net cash flow is the measure of economic income that most financial analysts prefer to use today when using the cost of capital for valuation or project selection. If valuing cash flows to equity, the discount rate should be the cost of equity capital. If valuing cash flows to debt, the discount rate should be the cost of the debt capital. If valuing cash flows available for all invested capital, the discount rate should be the weighted average cost of capital (WACC).

There are two good reasons why financial analysts lean toward using net cash flow as the preferred measure of economic income when using cost of capital for valuation or project selection:

1. *Conceptually*, it is the amount of discretionary money available to be distributed without disrupting the projected ongoing operations of the enterprise.
2. *Empirically*, it is the economic income measure for which the best historical data is available for estimating cost of equity capital.

Net cash flows should be measured as the mathematical expected value of the probability-weighted distribution of expected outcomes for each projected period of returns, not the most likely value. In Chapter 5, we define risk by uncertainty of possible outcomes, a definition intended to encompass the entire range of possible returns for each future period.

Note

1. *Stocks, Bonds, Bills and Inflation, Valuation Edition 2002 Yearbook* (Chicago: Ibbotson Associates, 2002), 13. Used with permission. All rights reserved.

Chapter 4

Discounting versus Capitalizing

Capitalization Formula
Example: Valuing a Preferred Stock
Functional Relationship between Discount Rate and Capitalization Rate
Major Difference between Discounting and Capitalizing
Gordon Growth Model
Combining Discounting and Capitalizing (Two-stage Model)
Equivalency of Discounting and Capitalizing Models
Midyear Convention
  Midyear Discounting Convention
  Midyear Capitalization Convention
  Midyear Convention in the Two-stage Model
Converting from After-tax Rate to Pretax Rate
  Converting After-tax Capitalization Rate to Pretax Capitalization Rate
  Converting After-tax Rate to Pretax Discount Rate
Summary

In the context of cost of capital applications, there is a very clear distinction between a discount rate and a capitalization rate. The first two chapters explained that the cost of capital is used as a discount rate to discount a stream of future returns to a present value. This process is called *discounting*.

In discounting, we project *all* expected returns (cash flows or other measure of economic income) from the subject investment to the respective class or classes of capital over the life of the investment. Thus, the percentage return that we call the discount rate represents the total compound rate of return that an investor in that class of investment expects to achieve over the life of the investment.

There is a related process for estimating present value, which we call *capitalizing*. In capitalizing, instead of projecting all future returns on the investment to the respective class(es) of capital, we focus on the return of just one single period, usually the return expected in the first year immediately following the valuation date. We then divide that single number by a divisor called the *capitalization rate*. This process is called *capitalizing*.

As will be seen, the process of capitalizing is really just a shorthand form of discounting, and the capitalization rate is actually a derivative of the discount rate. That is, the capitalization rate, as used in the income approach to valuation or project selection, is formed by derivation from the discount rate. (This differs from the market approach to valuation, where capitalization rates for various economic income measures are observed directly in the marketplace.)

CAPITALIZATION FORMULA

Putting this concept into a formula, we have:

Formula 4.1

$$PV = \frac{NCF_1}{c}$$

where:

PV = Present value
NCF_1 = Net cash flow expected in the first period immediately following the valuation date
c = Capitalization rate

EXAMPLE: VALUING A PREFERRED STOCK

A simple example of applying Formula 4.1 uses a preferred stock for which a risk rating has been estimated. Let us make five assumptions:

1. The preferred stock pays dividends of $5 per share per year.
2. The preferred stock is issued in perpetuity and is not callable.
3. It pays dividends once a year, at the end of the year. (This, of course, is a simplifying assumption. Some privately owned preferred stocks pay only annually, but most publicly traded preferred stocks pay dividends quarterly.)
4. As of the valuation date, the market yield for preferred stocks of the same risk grade as the subject preferred stock is 10%. (We also must assume comparable rights, such as voting, liquidation preference, redemption, conversion, participation, cumulative dividends, etc.)
5. There is no prospect of liquidation.

Note that the par value of the preferred stock is irrelevant, since the stock is issued in perpetuity and there is no prospect of a liquidation. The entire cash flow an investor can expect to receive over the life of the investment (perpetuity in this case) is the $5 per-year per-share dividend.

Substituting numbers derived from the preceding assumptions into Formula 4.1 produces:

Formula 4.2

$$PV = \frac{\$5.00}{0.10}$$
$$= \$50.00$$

In this example, the estimated fair market value of the subject preferred stock is \$50 per share. That is the amount a willing buyer would expect to pay and a willing seller would expect to receive (before considering any transaction costs).

FUNCTIONAL RELATIONSHIP BETWEEN DISCOUNT RATE AND CAPITALIZATION RATE

The preceding example presented the simplest possible scenario in which to apply the cost of capital through the capitalization method: a fixed cash flow stream in perpetuity. This is the one unique situation in which the discount rate (cost of capital) equals the capitalization rate. The discount rate equals the capitalization rate because no growth or decline in the investor's cash flow is expected. But most real-world investments are not quite that simple.

In the case of an investment in common stock, a partnership interest, or a capital budgeting project in an operating company, investors often are expecting some level of growth over time in the cash flows available to pay dividends or partnership withdrawals. Even if unit volume is expected to remain constant (i.e., no real growth), investors still might expect cash flows to grow at a rate approximating expected inflation. If the expected annually compounded rate of growth is stable and sustainable over a long period of time, then the discount rate (cost of capital) can be converted to a capitalization rate.

As stated earlier, the capitalization rate is a function of the discount rate. This obviously raises the question: What is the functional relationship between the discount rate and the capitalization rate?

Assuming stable long-term growth in the cash flows available to the investment being valued, the capitalization rate equals the discount rate minus the expected long-term growth rate. In a formula, this functional relationship can be stated as:

Formula 4.3

$$c = k - g$$

where:

c = Capitalization rate
k = Discount rate (cost of capital) for the subject investment
g = Expected long-term sustainable growth rate in the cash flow available to the subject investment

The critical assumption in this formula is that the growth in the return available to the capital is relatively constant over the long term (technically in perpetuity).

Caveat: Note carefully the phrase "return available to the capital." This does not include growth in overall company cash flows that are dependent on future capital investment. A common error is to use a rate of growth that could not be achieved without additional capital investment. The only growth that counts is in returns to the *existing* capital, or the capital investment being evaluated.

Now we know two essential things about using the cost of capital to estimate present value using the capitalization method, assuming relatively stable long-term growth in the return available to the investor:

1. Present value equals the next period's expected cash flow divided by the capitalization rate.
2. The capitalization rate is the discount rate (cost of capital) less the sustainable expected long-term rate of growth in the cash flow. (Technically, sustainable growth in this context means in perpetuity. However, after 15 or 20 years, the remaining rate of growth has minimal impact on the present value, due to very small present value factors.)

We can combine these two relationships into a single formula as:

Formula 4.4

$$PV = \frac{NCF_1}{k - g}$$

where:

PV = Present value
NCF_1 = Net cash flow expected in period 1, the period immediately following the valuation date
k = Discount rate (cost of capital)
g = Expected long-term sustainable growth rate in net cash flow to investor

A simple example of substituting numbers into Formula 4.4 is an equity investment with a constant expected growth in net cash flow. Let us make three assumptions:

1. The net cash flow in period 1 is expected to be \$100.
2. The cost of capital (i.e., the market-required total return or the discount rate) for this investment is estimated to be 13%.
3. The sustainable rate of growth in net cash flow from year 1 to perpetuity is expected to be 3%.

Substituting numbers from the preceding assumptions into Formula 4.4 gives us:

Formula 4.5

$$\begin{aligned} PV &= \frac{\$100}{0.13 - 0.03} \\ &= \frac{\$100}{0.10} \\ &= \underline{\underline{\$1,000}} \end{aligned}$$

In this example, the estimated fair market value of the investment is \$1,000. That is the amount a willing buyer would expect to pay and a willing seller would expect to receive (before considering any transaction costs).

MAJOR DIFFERENCE BETWEEN DISCOUNTING AND CAPITALIZING

From the foregoing, we can now deduce the following critical insight: *The difference between discounting and capitalizing is in how we reflect changes over time in expected future cash flows.*

In *discounting*: Each future increment of return is estimated specifically and put in the numerator.

In *capitalizing*: Estimates of changes in future returns are lumped into one annually compounded growth rate, which is then subtracted from the discount rate in the denominator.

If we assume that there really is a constant compounded growth rate in net cash flow to the investor in perpetuity, then it is a mathematical truism that the discounting method and the capitalizing method will produce identical values. (See the section in this chapter titled "Equivalency of Discounting and Capitalizing Models" for an illustration of how this truism works.)

GORDON GROWTH MODEL

One frequently encountered minor modification to Formulas 4.4 and 4.5 is to use as the "base period" the period just completed prior to the valuation date, instead of next period's estimate. The assumption is that cash flows will grow evenly in perpetuity from the period immediately preceding the valuation date. This scenario is stated in a formula known as the Gordon Growth Model:

Formula 4.6

$$PV = \frac{NCF_0(1+g)}{k-g}$$

where:

PV = Present value

NCF_0 = Net cash flow in period 0, the period immediately preceding the valuation date

k = Discount rate (cost of capital)

g = Expected long-term sustainable growth rate in net cash flow to investor

Note that for this model to make economic sense, NCF_0 must represent a normalized amount of cash flow from the investment for the previous year, from which a steady rate of growth is expected to proceed. Therefore, NCF_0 need not be the actual cash flow for period 0 but may be the result of certain normalization adjustments, such as elimination of the effect of one or more nonrecurring factors.

In fact, if NCF_0 *is* the actual net cash flow for period 0, the valuation analyst must take reasonable steps to be satisfied that NCF_0 is indeed the most reasonable base from which to start the expected growth embedded in the growth rate. Furthermore, the valuation report should state the steps taken and the assumptions made in concluding that last year's actual results are the most realistic base for expected growth. Mechanistic acceptance of recent results as representative of future expectations is one of the most common errors in implementing the capitalization method of valuation.

For a simple example of using numbers in Formula 4.6, accept all assumptions in the previous example, with the exception that the \$100 net cash flow expected in period 1 is instead the normalized base cash flow for period 0. (The \$100 is for the period just ended, rather than the expectation for the period just starting.) Substituting the numbers with these assumptions into Formula 4.6 produces:

Formula 4.7

$$\begin{aligned} PV &= \frac{\$100(1+0.03)}{0.13-0.03} \\ &= \frac{\$103}{0.10} \\ &= \underline{\underline{\$1,030}} \end{aligned}$$

In this example, the estimated fair market value of the investment is \$1,030. That is the amount a willing buyer would expect to pay and a willing seller would expect to receive (before considering any transaction costs).

Note that the relationship between this and the previous example is simple and straightforward. We backed up the receipt of the \$100 by one period, and the value of the investment was higher by 3%, the growth rate. In a constant growth model, assuming that all of the available cash flows are distributed, the value of the investment grows at the same rate as the rate of growth of the cash flows. The reason is because, in defining net cash flow (as we did in the previous chapter), we have already subtracted the amount of reinvestment necessary to support the projected growth.

The investor in the above example thus earns a *total* rate of return of 13%, comprised of 10% current return (the capitalization rate) plus 3% annually compounded growth in the value of the investment.

COMBINING DISCOUNTING AND CAPITALIZING (TWO-STAGE MODEL)

For many investments, even given an accurate estimate of the cost of capital, there are practical problems with either a pure discounting or a pure capitalizing method of valuation.

Problem with discounting: There are few equity investments for which returns for each specific incremental period can be projected with accuracy many years into the future.

Problem with capitalizing: For most equity investments, it is not reasonable to expect a constant growth rate in perpetuity from either the year preceding or the year following the valuation date.

This dilemma is typically dealt with by combining the discounting method and the capitalizing method into a *two-stage model*. The idea is to project discrete cash flows for some number of periods into the future and then to project a steady growth model starting at the end of the discrete projection period. Each period's discrete cash flow is discounted to a present value, and the capitalized value of the projected cash flows following the end of the discrete projection period is also discounted back to a present value. The sum of the present values is the total present value. The capitalized value of the projected cash flows following the discrete projection period is called the *terminal value* or *residual value*.

The preceding narrative explanation of a two-stage model is summarized in seven steps:

1. Decide on a reasonable length of time for which discrete projections can be made.
2. Estimate specific amounts of expected cash flow for each of the discrete projection periods.
3. Estimate a long-term sustainable rate of growth in cash flows from the end of the discrete projection period forward.
4. Use the Gordon Growth Model (Formulas 4.6 and 4.7) to estimate value as of the end of the discrete projection period.
5. Discount each of the increments of cash flow back to a present value at the discount rate (cost of capital) for the number of periods until it is received.
6. Discount the terminal value (estimated in step 4) back to a present value for the number of periods in the discrete projection period (the same number of periods as the last increment of cash flow).
7. Sum the value derived from steps 5 and 6.

These steps can be summarized in the next formula:

Formula 4.8

$$PV = \frac{NCF_1}{(1+k)} + \frac{NCF_2}{(1+k)^2} + \cdots + \frac{NCF_n}{(1+k)^n} \frac{\dfrac{NCF_n(1+g)}{k-g}}{(1+k)^n}$$

where:

$NCF_1 \ldots NCF_n$ = Net cash flow expected in each of the periods 1 through n, n being the last period of the discrete cash flow projections

k = Discount rate (cost of capital)
g = Expected long-term sustainable growth rate in net cash flow, starting with the last period of the discrete projections as the base year

The discrete projection period in the two-stage model is typically between five and 10 years. However, for simplicity in applying Formula 4.8, we will just use a three-year discrete projection period. Let us make three assumptions:

1. Expected net cash flows for years 1, 2, and 3 are \$100, \$120, and \$140, respectively.
2. Beyond year 3, cash flow is expected to grow fairly evenly at a rate of about 5% in perpetuity.
3. The cost of capital for this investment is estimated to be 12%.

Substituting numbers derived from these assumptions into Formula 4.8 produces:

Formula 4.9

$$PV = \frac{\$100}{(1+0.12)} + \frac{\$120}{(1+0.12)^2} + \frac{\$140}{(1+0.12)^3} + \frac{\frac{\$140(1+0.05)}{0.12-0.05}}{(1+0.12)^3}$$

$$= \frac{\$100}{1.12} + \frac{\$120}{1.2544} + \frac{\$140}{1.4049} + \frac{\frac{\$147}{0.07}}{1.4049}$$

$$= \$89.30 + \$95.66 + \$99.65 + \frac{\$2,100}{1.4049}$$

$$= \$89.30 + \$95.66 + \$99.65 + \$1,494.77$$

$$= \underline{\underline{\$1,779.38}}$$

Thus, the estimated fair market value of this investment is \$1,779.38. This is the amount a willing buyer would expect to pay and a willing seller would expect to receive (before considering any transaction costs).

A common error is to discount the terminal value for $n + 1$ periods instead of n periods. The assumption we have made is that the nth period cash flow is received at the end of the nth period, and the terminal value is the amount for which we estimate we could sell the investment as of the end of the nth period. The end of one period and the beginning of the next period are the same moment in time, so they must be discounted for the same number of periods.

Note that, in the preceding example, the terminal value represents 84% of the total present value (\$1,494.77 ÷ \$1,779.38 = 0.84). The analyst should always keep in mind two relationships when using cost of capital in a two-stage model for valuation:

1. The shorter the projection period, the greater the impact of the terminal value on the total present value.

2. The closer the estimated growth rate is to the cost of capital, the more sensitive the model is to changes in assumptions regarding the growth rate. (This is true for the straight capitalization model as well as the two-stage model.) Of course, if the growth rate exceeds the cost of capital, the model implodes and is useless.

In some cases, the terminal value may not be a perpetuity model. For example, one might assume liquidation at that point, and the terminal value could be a salvage value.

EQUIVALENCY OF DISCOUNTING AND CAPITALIZING MODELS

As stated earlier, if all assumptions are met, the discounting and capitalizing methods of using the cost of capital will produce identical estimates of present value. Let us test this on the example used in Formula 4.5. Recall that we assumed cash flow in period 1 of \$100, growing in perpetuity at 3%. The cost of capital (the discount rate) was 13%, so we subtracted the growth rate of 3% to get a capitalization rate of 10%. Capitalizing the \$100 (period 1 expected cash flow) at 10% gave us an estimated present value of \$1,000 (\$100 ÷ 0.10 = \$1,000).

Let us take these same assumptions and put them into a discounting model. For simplicity, we will only use three periods for the discrete projection period, but it would not make any difference how many discrete projection periods we used.

Formula 4.10

$$
\begin{aligned}
PV &= \frac{\$100}{(1+0.13)} + \frac{\$100(1+0.03)}{(1+0.13)^2} + \frac{\$100(1.03)^2}{(1+0.13)^3} + \frac{\dfrac{\$100(1.03)^3}{0.13-0.03}}{(1.13)^3} \\
&= \frac{\$100}{1.13} + \frac{\$103}{1.2769} + \frac{\$106.09}{1.4429} + \frac{\dfrac{\$109.27}{0.10}}{1.4429} \\
&= \$88.50 + \$80.66 + \$73.53 + \frac{\$1092.73}{1.4429} \\
&= \$88.50 + \$80.66 + \$73.53 + \$757.31 \\
&= \underline{\underline{\$1,000}}
\end{aligned}
$$

This example, showing the equivalency of using cost of capital in either the discounting or the capitalizing model, when all assumptions are met, demonstrates the point that capitalizing is really just a shorthand form of discounting. When using a capitalizing model, the analyst should consider whether the answer would work out the same if it were expanded to a full discounting model. If not, it may be propitious to review and possibly adjust certain assumptions. If the discounting and capitalization models produce different answers using the same cost of capital and the same inputs, there may be some kind of internal inconsistency.

MIDYEAR CONVENTION

In all of our examples, we have assumed that cash flows are received at the end of each year. Even if a company realizes cash flows throughout the year, payouts to the investors may be made only at the end of the year when the managers have seen the results of the entire year and have an idea about next year's projections.

For some companies or investments, however, it may be more reasonable to assume that the cash flows are distributed more or less evenly throughout the year. To accommodate this latter assumption, we can modify our formulas for what we call the *midyear convention*. The midyear convention always results in a higher value, because the discount or capitalization rate remains the same, and the assumption is that the investor does not have to wait quite as long to receive returns.

Midyear Discounting Convention

We can make a simple modification to Formula 2.1 (discounting) to what we call the *midyear discounting convention*. We merely subtract a half year from the exponent in the denominator of the equation.

Formula 2.1, the discounting equation, now becomes:

Formula 4.11

$$PV = \frac{NCF_1}{(1+k)^{0.5}} + \frac{NCF_2}{(1+k)^{1.5}} + \cdots + \frac{NCF_n}{(1+k)^{n-0.5}}$$

Midyear Capitalization Convention

Similarly, we can make a modification to the capitalization formula to reflect the receipt of cash flows throughout the year. The modification to Formula 4.4, the capitalization equation, is handled by accelerating the returns by a half year in the numerator[1]:

Formula 4.12

$$PV = \frac{NCF_1(1+k)^{0.5}}{k-g}$$

Midyear Convention in the Two-stage Model

Combining discrete period discounting and capitalized terminal value into a two-stage model as shown in Formula 4.8, the midyear convention two-stage equation becomes:

Formula 4.13

$$PV = \frac{NCF_1}{(1+k)^{0.5}} + \frac{NCF_2}{(1+k)^{1.5}} + \cdots + \frac{NCF_n}{(1+k)^{n-0.5}} + \frac{\dfrac{NCF_n(1+g)(1+k)^{0.5}}{k-g}}{(1+k)^n}$$

Using the same assumptions as in Formula 4.9 (where the value was \$1,779.38) produces:

Formula 4.14

$$PV = \frac{\$100}{(1+0.12)^{0.5}} + \frac{\$120}{(1+0.12)^{1.5}} + \frac{\$140}{(1+0.12)^{2.5}} + \frac{\frac{\$140(1+0.05)(1+.12)^{0.5}}{0.12-0.05}}{(1+0.12)^{3}}$$

$$= \frac{\$100}{1.0583} + \frac{\$120}{1.1853} + \frac{\$140}{1.3275} + \frac{\frac{\$155.5270}{0.07}}{1.4049}$$

$$= \$94.49 + \$101.24 + \$105.46 + \frac{\$2,221.81}{1.4049}$$

$$= \$94.49 + \$101.24 + \$105.46 + \$1,581.47$$

$$= \underline{\underline{\$1,882.66}}$$

In this case, using the midyear convention increased the value by \$103.28 (\$1,882.66 – \$1,779.38 = \$103.28) or 5.8% (\$103.28 ÷ \$1,779.38 = 0.058).

An alternative version of the terminal value factor in the two-stage model actually is equivalent to that used in the preceding formula.

Instead of using the modified capitalization equation in the numerator of the terminal value factor, the normal terminal value capitalization equation is used, and the terminal value is discounted by $n - 0.5$ years instead of n years.

This equation reads as:

Formula 4.15

$$PV = \frac{NCF_1}{(1+k)^{0.5}} + \frac{NCF_2}{(1+k)^{1.5}} + \cdots + \frac{NCF_n}{(1+k)^{n-0.5}} + \frac{\frac{NCF_n(1+g)}{k-g}}{(1+k)^{n-0.5}}$$

Using the same numbers as in Formula 4.14, this works out to:

$$PV = \frac{\$100}{(1+0.12)^{0.5}} + \frac{\$120}{(1+0.12)^{1.5}} + \frac{\$140}{(1+0.12)^{2.5}} + \frac{\frac{\$140(1+0.05)}{0.12-0.05}}{(1+0.12)^{2.5}}$$

$$= \frac{\$100}{1.0583} + \frac{\$120}{1.1853} + \frac{\$140}{1.3275} + \frac{\frac{\$147}{0.07}}{1.3275}$$

$$= \$94.49 + \$101.24 + \$105.46 + \frac{\$2,100}{1.3275}$$

$$= \$94.49 + \$101.24 + \$105.46 + \$1,581.92$$

$$= \underline{\underline{\$1,883.11}}$$

(The difference is a matter of rounding.)

CONVERTING FROM AFTER-TAX RATE TO PRETAX RATE

We have emphasized that the cash flows that we are capitalizing are *after taxes*. We can convert after-tax capitalization rates to pretax capitalization rates, and even to pretax discount rates, *provided that we assume zero or constant growth*.

Converting After-tax Capitalization Rate to Pretax Capitalization Rate

To convert an after-tax capitalization rate to a pretax capitalization rate, the formula is:

Formula 4.16

$$c(_{pt}) = \frac{c}{1-t}$$

where:

c = Capitalization rate (on after-tax cash flows)
$c(_{pt})$ = Capitalization rate on pretax cash flows
t = Tax rate

Assuming a tax rate of 30%, substituting in Formula 4.16, we have:

Formula 4.17

$$\frac{0.10}{1-0.30} = \frac{0.10}{0.70} = 14.3\%$$

But the above is *not* a discount rate, unless the assumption is that there will be no growth.

Converting After-tax Rate to Pretax Discount Rate

Suppose that we arrived at the above capitalization rate of 10% by starting with a discount rate of 15% and subtracting an estimated sustainable growth rate of 5% (0.15 – 0.05 = 0.10). We are not going to apply a discount rate of 14.3% to pretax cash flows! To get a discount rate applicable to pretax cash flows, we have to add the growth rate to the pretax capitalization rate. In this case, we have 14.3% + 5% = a 19.3% discount rate for pretax cash flows. The proof for this is shown in Appendix G.[2]

SUMMARY

This chapter has shown the mechanics of *discounting* and *capitalizing* and has defined the difference between a *discount rate* and a *capitalization rate*.

It has shown that capitalizing is merely a short-form version of discounting. The essential difference between the discounting method and the capitalizing method is how changes in expected cash flows over time are reflected in the respective formulas. All things being equal, the discounting method and the capitalizing method will yield identical results. However, the validity of the capitalizing method in the income approach to valuation depends on the assumption that the difference between the discount rate and the capitalization rate represents a long-term average rate of growth in the income variable being capitalized.

Because many companies are likely to expect near-term changes in levels of their returns that are not expected to be representative of longer-term expectations, many analysts use a combination of discounting and capitalizing for valuation. To accomplish this, they implement five steps:

1. Project discrete amounts of return for some period of years until the company is expected to reach a stabilized level from which relatively constant growth may be expected to proceed.
2. Use the Gordon Growth Model to estimate a "terminal value" as of the end of the discrete projection period.
3. Discount each discrete projected cash flow to a present value at the cost of capital for the number of periods until it is expected to be received.
4. Discount the terminal value to a present value at the cost of capital for the number of periods in the discrete projection period (the *beginning* of the assumed stable growth period).
5. Add the values from steps 3 and 4.

Most discounting and capitalization formulas reflect the implicit assumption that investors will realize their cash flows at the end of each year. If it is assumed that investors will receive cash flows more or less evenly throughout the year, the formulas can be modified by the midyear convention.

Notes

1. Proof of the accuracy of this method was presented in Todd A. Kaltman, "Capitalization Using a Mid-Year Convention," *Business Valuation Review* (December 1995): 178–182.
2. This was first demonstrated in Mary Ann Lerch, "Pretax/Aftertax Conversion Formula for Capitalization Rates and Cash Flow Discount Rates," *Business Valuation Review* (March 1990): 18–22.

Chapter 5

Relationship between Risk and the Cost of Capital

Defining Risk
Types of Risk
  Maturity Risk
  Systematic Risk
  Unsystematic Risk
How Risk Impacts the Cost of Capital
Cost of Equity Capital
Cost of Conventional Debt and Preferred Equity Capital
Cost of Overall Invested Capital
Summary

The cost of capital for any given investment is a combination of two basic factors[1]:

1. A *risk-free rate*. By "risk-free rate" we mean a rate of return that is available in the market on an investment that is free of default risk, usually the yield to maturity on a U.S. government security.
2. A *premium for risk*. An expected amount of return over and above the risk-free rate to compensate the investor for accepting risk.

Quantifying the amount by which risk affects the cost of capital for any particular company or investment is arguably one of the most difficult analyses in the field of corporate finance, including valuation and capital budgeting.

DEFINING RISK

Probably the most widely accepted definition of risk in the context of business valuation is *the degree of uncertainty as to the realization of expected future economic income*.[2] This means uncertainty as to both the amounts and the timing of expected income. Note that the definition implies as the reference point *expected returns*. By expected returns, in a technical sense, we mean the expected value (mean average) of the probability distribution of possible returns for each forecast period. This concept was explained in Chapter 3 in the discussion of net cash flow. The point to understand here is that the uncertainty encompasses the full distribution of possible returns for each period both above and below the expected value.

Inasmuch as uncertainty is within the mind of each individual investor, we cannot measure the risk directly. Consequently, participants in the financial markets have developed ways of measuring factors that investors normally would consider in their effort to incorporate risk into their required rate of return.

TYPES OF RISK

Although risk arises from many sources, this chapter addresses risk in the economic sense, as used in the conventional methods of estimating cost of capital. In this context, capital market theory divides risk into three components:

1. *Maturity risk* (also called *horizon risk* or *interest rate risk*)
2. *Systematic risk* (also called *market risk*)
3. *Unsystematic risk* (sometimes called *company risk*, *specific risk*, or *residual risk*)

Maturity Risk

Maturity risk (also called horizon risk or interest rate risk) is the risk that the value of the investment may increase or decrease because of changes in the general level of interest rates. The longer the term of an investment, the greater the maturity risk. For example, market prices of long-term bonds fluctuate much more in response to changes in levels of interest rates than do short-term bonds or notes. When we refer to the yields of U.S. government bonds as *riskless rates*, we mean that we regard them as free from the prospect of default, but we recognize that they do incorporate maturity risk: The longer the maturity, the greater the susceptibility to change in market price in response to changes in market rates of interest. In regard to interest rates, much of the uncertainty derives from the uncertainty of future inflation levels.

Systematic Risk

Systematic risk (also called market risk) is the uncertainty of future returns because of the sensitivity of the return on a subject investment to movements in returns for the investment market as a whole. Although this is a broad conceptual definition, in practical application the investment market as a whole is generally limited to the U.S. equity markets and is typically measured by returns on either the New York Stock Exchange (NYSE) Composite Index or the Standard & Poor's (S&P) 500 Index.

Some theoreticians say that the only risk the capital markets reward with an expected premium rate of return is systematic risk, because *unsystematic risk* can be eliminated by holding a well-diversified portfolio of investments. Although this may be true for active publicly traded securities, it generally is not practical to hold a portfolio of closely held companies that is diversified enough to eliminate all risk except that of the market itself.

As we cover the chapters on the various methods of estimating the cost of capital, we will see that systematic risk is a factor specifically measured for a particular company or industry in some methods, but not at all or not necessarily in others. Systematic risk is taken into consideration in the Capital Asset Pricing Model (CAPM), which is the subject of Chapter 9. It is commonly measured by a factor called *beta*, which attempts to measure the sensitivity of the returns realized by a company or an industry to movements in returns of "the market," usually defined as either the NYSE Composite Index or the S&P 500 Index.

Unsystematic Risk

Unsystematic risk (also called specific risk or residual risk) is the uncertainty of expected returns arising from factors other than the market itself. These factors typically include characteristics of the industry and the individual company. In international investing, they also can include characteristics of a particular country.

Much of the unsystematic risk of an investment may be captured in the *size premium*, which is the subject of Chapter 11. Fully capturing unsystematic risk in the discount rate requires analysis of the company, in comparison with other companies, which is also discussed in Chapter 11. However, while the size premium captures many risk factors, the analyst must be careful to capture all the risk factors and at the same time avoid double-counting.

HOW RISK IMPACTS THE COST OF CAPITAL

As noted earlier, the cost of capital (the expected rate of return that the market requires to attract money to the subject investment) has two components[3]:

1. A riskless rate
2. A risk premium

As the market's perception of the degree of risk of an investment increases, the rate of return that the market requires (the discount rate) increases. The higher the market's required rate of return, the lower the present value of the investment.

Risk is the ultimate concern to investors. The riskless rate compensates investors for renting out their money, so to speak; that is, for delaying consumption over some future time period and receiving back dollars with less purchasing power. This component of the cost of capital is readily observable in the marketplace and generally differs from one investment to another only to the extent of the time horizon (maturity) selected for measurement of the riskless rate.

The risk premium, however, is due to the uncertainty of expected returns. It is much harder to estimate and also varies widely from one prospective capital investment to another. We could say that the market abhors uncertainty and consequently demands a high price (in terms of required rate of return or cost of capital) to accept uncertainty. Since uncertainty as to timing and amounts of future receipts is greatest for equity investors, the high risk forces equity as a class to have the highest cost of capital. The risk premium varies greatly from one company or project to another, but for most smaller companies the risk premium component of the cost of capital is greater than the riskless rate component.

COST OF EQUITY CAPITAL

When using either the build-up method (Chapter 8) or the Capital Asset Pricing Model (CAPM) (Chapter 9), we estimate one or more components of a risk premium and add the total risk premium to the riskless rate in order to estimate the cost of equity capital.

When using publicly traded stock data to estimate the cost of equity capital—the discounted cash flow (DCF) method (Chapter 12)—we get a total cost of equity capital without any explicit breakdown regarding how much of it is attributable to a riskless rate and how much is attributable to the risk premium.

COST OF CONVENTIONAL DEBT AND PREFERRED EQUITY CAPITAL

The cost of debt and preferred stock capital generally depends on risk factors identified by fixed income rating services, such as Standard & Poor's (now a division of McGraw-Hill) and Moody's (now published by Mergent's, Inc.). The rates of return for securities with comparable risk factors generally can be observed in the market.[4]

COST OF OVERALL INVESTED CAPITAL

The cost of total invested capital is a blending of the costs of each component, called the *weighted average cost of capital* (WACC). Chapter 6 discusses each component in the capital structure, and Chapter 7 addresses the weighted average cost of capital.

SUMMARY

The cost of capital is a function of the market's risk-free rate plus a premium for the risk associated with the investment. *Risk* is the degree of uncertainty regarding the realization of the expected returns from the investment.

Most analysts choose to use a long-term government security as a proxy for the "risk-free" rate. It is assumed that this rate is free of default risk, but it is recognized that it includes interest rate risk; that is, the market value of the principal will change with changes in the general level of interest rates.

In an economic sense, the market distinguishes between *systematic risk* and *unsystematic risk*. Systematic risk is the sensitivity of returns on the subject investment to returns on the overall market. Unsystematic risk is unique to the subject company or industry as opposed to the market as a whole.

Risk impacts the cost of each of the components of capital: debt, senior equity, and common equity. Because risk has an impact on each capital component, it also has an impact on the weighted average cost of capital.

As risk increases, the cost of capital increases, and value decreases. Because risk cannot be observed directly in the market, it must be estimated. The impact of risk on the cost of capital is at once one of the most essential and one of the most difficult analyses in corporate finance and investments.

Notes

1. A third factor is liquidity, but that is usually treated as a separate adjustment, as discussed in Chapter 16.
2. Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. (New York: McGraw-Hill, 2000), 44.
3. As noted in note 1, a third element—lack of marketability or liquidity—may be embedded in the discount rate, but more often it is treated as a separate adjustment to value. This is covered in Chapter 16.
4. This text deals primarily with the cost of equity and the weighted average cost of capital. For detailed discussions on cost of debt and cost of preferred equity, see Chapters 23 and 24 respectively, in Pratt, Reilly, and Schweihs, *Valuing a Business*.

Chapter 6

Cost Components of a Company's Capital Structure

Debt
- Tax Effect Lowers Cost of Debt
- Personal Guarantees

Preferred Equity
Convertible Debt or Preferred Stock
Common Stock or Partnership Interests
Summary

The capital structure of many companies includes two or more components, each of which has its own cost of capital. Such companies may be said to have a complex capital structure. The major components commonly found are:

- Debt
- Preferred stock
- Common stock or partnership interests

Similarly, a project being considered in a capital budgeting decision may be financed by multiple components of capital.

In a complex capital structure, each of these general components may have subcomponents, and each subcomponent may have a different cost of capital. In addition, there may be hybrid or special securities, such as convertible debt or preferred stock, warrants, options, or leases.

Ultimately, a company's or project's overall cost of capital is a result of the blending of the individual costs of each of these components. This chapter briefly discusses each of the capital structure components, and Chapter 7 shows the process of blending them into a company's or project's overall cost of capital, which is called the weighted average cost of capital (WACC).

Estimation of the costs of conventional fixed-income components of the capital structure, that is, straight debt and preferred stock, is relatively straightforward, because costs of capital for securities of comparable risk usually are directly observable in the market and the company's actual embedded cost is often at or very close to current market rates. Although there can be many controversies surrounding costs of fixed income capital, especially if unusual provisions exist, we discuss these components only briefly here. The rest of this book deals primarily with the critically important but highly elusive and controversial issue of the cost of equity.

DEBT

Conceptually, only long-term liabilities are included in a capital structure. However, many closely held companies, especially smaller ones, use what is technically short-term interest-bearing debt as if it were long-term debt. In these cases, it becomes a matter of the analyst's judgment whether to reclassify the short-term debt as long-term debt and include it in the capital structure for the purpose of estimating the company's overall cost of capital (weighted average cost of capital).

Usually the cost of debt is equivalent to the company's interest expense (after tax effect) and is readily ascertainable from the footnotes to the company's financial statements (if the company has either audited or reviewed statements or compiled statements with footnote information). If the rate the company is paying is not a current market rate (e.g., long-term debt issued at a time when market rates were significantly different), then the analyst should estimate what a current market rate would be for that component of the company's capital structure.

Standard & Poor's (now a division of McGraw-Hill) publishes debt rating criteria along with the *Standard & Poor's Bond Guide*. The analyst can see where the investment would fit within the bond rating system, then check the financial press to find the yields for the estimated rating.[1] The analyst should consider that smaller companies may have higher costs of debt than larger companies. Also, smaller companies may not be able to borrow as high a proportion of their capital structure as larger companies. Some companies have more than one class of debt, each with its own cost of debt capital.

The relevant market "yield" is either the *yield to maturity* or the *yield-to-call date*. Either of these is the total return the debt holder expects to receive over the life of the debt instrument, including current yield and any appreciation or depreciation from the market price, to the redemption of the debt at either its maturity or call date, if callable. If the stated interest rate is above current market rates, the bond would be expected to sell at a premium, and the yield-to-call date usually would be the appropriate yield, because it probably would be in the issuer's best interest to call it (redeem it) as soon as possible and refinance it at a lower interest cost. If the stated interest rate is below current market rates, then it usually would not be attractive to the company to call it, and the yield to maturity would be the most appropriate rate.

Tax Effect Lowers Cost of Debt

Because interest expense on debt is a tax-deductible expense to a company, the net cost of debt to the company is the interest paid less the tax savings resulting from the deductible interest payment. This cost of debt can be expressed by the next formula:

Formula 6.1

$$k_d = k_{d(pt)}(1 - t)$$

where:

$k_{d(pt)}$ = Rate of interest on debt
k_d = Discount rate for debt (the company's after-tax cost of debt capital)
t = Tax rate (expressed as a percentage of pretax income)

For decision-making purposes, most corporate finance theoreticians recommend using the marginal tax rate (the rate of tax paid on the last incremental dollar of taxable income) if that differs from the company's effective tax rate.[2] That makes sense, since the marginal rate will be the cost incurred as a result of the investment. However, the focus should be on the marginal rate over the life of the investment, if that is different from the marginal cost incurred initially.

Personal Guarantees

When estimating the cost of private company debt, the analyst should ascertain whether the debt is secured by personal guarantees. If so, this is an additional cost of debt that is not reflected directly in the financial statements (or, in some cases, might not even be disclosed). Such guarantees would justify an upward adjustment in the company's cost of debt. The author is not aware of any published studies to help quantify this factor. Therefore, it becomes a subjective adjustment on the part of the analyst.[3]

In the late 1990s, insurance companies offered guarantees on seller financing. That is, when a company was sold with some percentage of the price as a down payment and the buyer gave the seller a promissory note for the balance (a common procedure in the sale of small businesses and professional practices), the insurance company would guarantee the note to the seller. The required down payment was at least 30% of the purchase price, and the premium was about 3% of the face value of the note. So perhaps 3% is as good a guide as can be found for adding to the cost of debt to reflect personal guarantees.

PREFERRED EQUITY

If the capital structure includes preferred equity, the yield rate can be used as the cost of that component. If the dividend is at or close to the current market rate for preferred stocks with comparable features and risk, then the stated rate can be a proxy for market yield. If the rate is not close to a current market yield rate, then the analyst should estimate what a current market yield rate would be for that component of the company's capital structure.

Standard & Poor's (a division of McGraw-Hill) publishes preferred stock rating criteria along with the *Standard & Poor's Stock Guide*. The analyst can see where the company's preferred stock would fit within the preferred stock rating system, then check the financial press to find the yields for preferred stocks with similar features and estimated rating.[4] The analyst must adjust for any differences in features often found in privately issued preferred equity, such as special voting or liquidation rights. If the preferred stock is callable, the same analysis (of the market rate of dividend compared to the dividend relative to call price as discussed with respect to debt) applies to the preferred stock.

CONVERTIBLE DEBT OR PREFERRED STOCK

Convertible debt or convertible preferred stock is essentially two securities combined into one: a straight debt or preferred stock element plus a warrant. The cost of capital for the convertible instrument is the sum of the costs of these two elements.

A *warrant* is a long-term call option issued by a company on a specific class of its own common equity, usually at a fixed price. Understanding convertibles is easiest if they are analyzed first as debt or nonconvertible preferred stocks and then as warrants (long-term call options).[5]

COMMON STOCK OR PARTNERSHIP INTERESTS

Part II of this book is devoted to estimating the cost of common equity. Unlike yields to maturity on debt or yields on preferred stock, the cost of common equity for specific companies or risk categories cannot be directly observed in the market.

The cost of equity capital is the expected rate of return needed to induce investors to place funds in a particular equity investment. As with the returns on bonds or preferred stock, the returns on common equity have two components:

1. Dividends or distributions
2. Changes in market value (capital gains or losses)

Because the cost of capital is a forward-looking concept, and because these expectations regarding amounts of return cannot be directly observed, they must be estimated from current and past market evidence. Analysts primarily use two methods of estimating the cost of equity capital from market data, each with variations:

1. Single-factor or multifactor approaches:
   a. Build-up models (Chapter 8)
   b. Capital Asset Pricing Model (CAPM) (Chapters 9 and 10)
2. Discounted cash flow (DCF) approach (Chapter 12)
   a. Single-stage DCF model
   b. Multistage DCF models

Exhibit 6.1 Capital Structure Components

Short-term notes	Not technically part of the capital structure, but may be included in many cases, especially if being used as if long term (e.g., officer loans)
Long-term debt	YES
Capital leases	Normally YES
Preferred stock	YES
Common stock Additional paid-in capital Retained earnings	YES—all part of common equity
Off-balance sheet options or warrants	Normally YES
Operating leases	Increasingly common

Another multifactor method gaining acceptance in some circles, primarily oriented to larger companies, is called arbitrage pricing theory (Chapter 14). Each of these methods of estimating the cost of equity capital is described in detail in its respective subsequent chapter.

SUMMARY

The typical components of a company's capital structure are summarized in Exhibit 6.1. In addition to the straight debt, preferred equity, and common equity shown, some companies have hybrid securities, such as convertible debt or preferred stock and options or warrants.

The next chapter explains how to combine the costs of each of these components to derive a company's overall cost of capital, called the weighted average cost of capital. Whereas this chapter has addressed briefly the cost of each component, the rest of the book focuses primarily on the many ways to estimate the cost of equity capital.

Notes

1. A detailed description of estimating the cost of debt is found in Chapter 23, "Valuing Debt Securities" in Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. (New York: McGraw-Hill, 2000), 517–529.
2. See, for example, Richard A. Brealey and Stewart C. Myers, *Principles of Corporate Finance*, 6th ed. (Boston: Irwin McGraw-Hill, 2000), 543.
3. If any reader can shed light on quantifying the cost of personal guarantees as a part of the cost of the company's debt capital, please contact the author at the address shown in the preface, and it will be published in *Shannon Pratt's Business Valuation Update®*.

4. A detailed description of estimating the cost of preferred stock is found in Chapter 24, "Valuing Preferred Stock," in Pratt, Reilly, and Schweihs, *Valuing a Business*, 531–552.
5. Valuing warrants and options is beyond the scope of this book. For an extensive treatment of this topic, see Chapters 20 to 22 in Brealey and Myers, *Principles of Corporate Finance*.

Chapter 7

Weighted Average Cost of Capital

When to Use Weighted Average Cost of Capital
Weighted Average Cost of Capital Formula
Computing WACC for a Public Company
Computing WACC for a Private Company
Should an Actual or a Hypothetical Capital Structure Be Used?
Summary

In the last chapter we identified components of a company's capital structure. Here we blend their costs together to estimate the company's overall cost of capital. In other words, we want to estimate the weighted cost for all of the company's invested capital or the capital to be committed to a specific project.

WHEN TO USE WEIGHTED AVERAGE COST OF CAPITAL

The most obvious instance in which to use weighted average cost of capital (WACC) is when the objective is to value the entire capital structure of a company. An example would be when considering an acquisition and the buyer expects to pay off all equity and debt holders and refinance the whole company in a different way that better suits the buyer. Sometimes WACC is also used even when the objective is ultimately to value only the equity. One would value the entire capital structure and then subtract the market value of the debt to estimate the value of the equity. This procedure frequently is used in highly leveraged situations.

Weighted average cost of capital is especially appropriate for project selection in capital budgeting. The proportions of debt and equity that could be available to finance various projects might differ according to the project (e.g., asset-intensive projects may be financed with more debt), and the cost of capital should be based on the specific investment.

This introduces the idea that we have to compute or estimate the weight (percentage of the total) for each component of the capital structure. The critical point is that *the relative weightings of debt and equity or other capital components are based on the market values of each component, not on the book values.*

WEIGHTED AVERAGE COST OF CAPITAL FORMULA

As noted in the discussion of debt in Chapter 6, the weighted average cost of capital is based on the cost of each component net of any corporate-level tax effect of that component. In the return to the debt component, interest is a tax-deductible expense to a corporate taxpayer. Whatever taxes *are* paid are an actual cash expense to the company, and the returns available to equity holders are *after* the payment of corporate-level taxes.

Because we are interested in cash flows after entity-level taxes, literature and practitioners sometime refer to the WACC as an "after-tax WACC." The basic formula for computing the after-tax WACC for an entity with three capital structure components is:

Formula 7.1

$$WACC = (k_e \times W_e) + (k_p \times W_p) + (k_{d(pt)}[1 - t] \times W_d)$$

where:

$WACC$ = Weighted average cost of capital
k_e = Cost of common equity capital
W_e = Percentage of common equity in the capital structure, at market value
k_p = Cost of preferred equity
W_p = Percentage of preferred equity in the capital structure, at market value
$k_{d(pt)}$ = Cost of debt (pretax)
t = Tax rate
W_d = Percentage of debt in the capital structure, at market value

COMPUTING WACC FOR A PUBLIC COMPANY

For active publicly traded securities, one can compute the weights for each capital component by multiplying the amount of each component outstanding by the market price of each and then computing the percentage that each component represents of the total market value. The five steps for this procedure are:

1. Identify the number of shares or units of each component of the capital structure.
2. Determine the market price per unit of each component of the capital structure as of the valuation date.
3. Multiply the number of units of each component by the market price per unit. This gives the total market value for each capital structure component.
4. Sum the total market values of each component, from step 3. This gives the market value of invested capital (MVIC).

5. Divide the total market value of each component (from step 3) by the total MVIC (from step 4). This gives the percentage weight to be accorded to each component of the capital structure.

To illustrate the process of computing weights for each capital structure component, let us make these assumptions for American Brainstorming Company (ABC):

- 5 million shares of common stock issued and outstanding
- Closing common stock price per share: $8.00
- 1 million shares of preferred stock issued and outstanding
- Closing preferred stock price per share: $20.00
- $10 million face value of bonds issued and outstanding
- Closing bond price: 90 (This means 90% of face value. Because bonds usually have $1,000 face value, this would be $900 per bond.)

From the preceding information, the capital structure weights can be computed:

Component	No. of Shares (or $ of face value)	Price (or % of face value)	Component Total	Weight
Common stock	5,000,000	$ 8.00	$40,000,000	58%
Preferred stock	1,000,000	$20.00	$20,000,000	29%
Bonds	$10,000,000	0.90	$ 9,000,000	13%
Market value of invested capital			$69,000,000	100%

We still need four more pieces of information before we can compute the weighted average cost of capital:

1. *Cost of common equity.* Because we have not yet provided any information on how to estimate cost of common equity (the subject of Part II), we will *assume* that ABC's cost of common equity is 20%.
2. *Cost of preferred equity.* The cumulative, nonparticipating dividend on the preferred stock is $2.50 per share per year. Since its market price is $20, the cost of preferred equity is 12.5% ($2.50 ÷ $20.00 = 0.125).
3. *Cost of debt* (before tax effect). The bonds pay a 9% interest rate on their face value, or $90 per bond per year. Therefore, the current yield is 10% ($90 ÷ $900 = 0.10). However, remember that the cost of debt is the *yield to maturity*, not the current yield. We make the simplifying assumptions that the bonds mature three years from the valuation date and that the interest is paid only at the end of each year. This problem is very much like that addressed in Formulas 2.1 and 2.2, except that we know the present value (*PV*), but we have to solve for the cost of debt

capital ($k_{d(pt)}$) before tax effect. Putting it in the same form as Formulas 2.1 and 2.2 would look like:

Formula 7.2

$$\$900 = \frac{\$90}{(1+k_{d(pt)})} + \frac{\$90}{(1+k_{d(pt)})^2} + \frac{\$90}{(1+k_{d(pt)})^3} + \frac{\$1,000}{(1+k_{d(pt)})^3}$$

Instead of showing each step to solve for the independent variable as $k_{d(pt)}$, we will simply compute it on our financial calculator and find that $k_{d(pt)} \cong 13\%$. (Some readers may find it surprising that the example shows the pretax cost of the debt a half point [0.5%] higher than the cost of preferred stock, which is in a lower position of claims on the balance sheet. This sometimes happens when the preferred stock is attractive for taxable corporations to hold, because only a small portion of the dividends paid are taxable income to the receiving corporation.)

4. *Tax rate.* The combined federal and state income tax rate for ABC is 40%.

Now we are prepared to substitute all of these numbers into Formula 7.1 to compute a weighted average cost of capital for ABC:

Formula 7.3

$$\begin{aligned} WACC &= (0.20 \times 0.58) + (0.125 \times 0.29) + (0.13\ [1 - 0.40] \times 0.13) \\ &= 0.116 + 0.036 + (0.078 \times 0.13) \\ &= 0.116 + 0.036 + 0.010 \\ &= 0.162 \text{ or } 16.2\% \end{aligned}$$

Many people prefer to set up this formula in tabular form:

Component	Cost		Weight		Weighted Cost
Common stock	0.20	×	0.58	=	0.116
Preferred stock	0.125	×	0.29	=	0.036
Debt (after tax)	0.078	×	0.13	=	0.010
	Weighted average cost of capital				0.162

COMPUTING WACC FOR A PRIVATE COMPANY

In computing WACC for a closely held company, project, or proposed project, one important additional problem exists: Because there is no market for the securities, we have to *estimate* market values in order to compute the capital structure weightings. As we will see, estimating the weightings for each component of the capital structure becomes an *iterative process* for companies intending or assumed to operate with current levels of debt. Fortunately, computers perform this exercise very quickly. (To "iterate" means to repeat. An "iterative process" is a repetitious one. In this case, we estimate market value weights because the actual market values are unknown. We may reestimate weights several times until the computed market value weights come fairly close to the weights used in estimating the WACC.)

The eight steps in the iterative process for estimating capital structure component weights for a closely held company can be summarized in this way:

1. Estimate the market value of senior securities (debt and preferred equity), and hold that dollar amount fixed throughout the process.
2. Make a first estimate of the market value weights of the senior securities and the common equity. (Generally, the farther above book value the equity market value is expected to be, the greater the first estimate of the equity percentage compared with its percentage at book value.)
3. Using the first approximation weights, make a first approximation computation of the WACC, using Formula 7.1.
4. Project (a) the net cash flows available to all invested capital, and (b) the projected growth rate necessary for either a discounting valuation model (Formula 2.1) or a capitalizing valuation model (Formula 4.4).
5. Using the first approximation WACC from step 3 and the projected cash flows from step 4, compute a first approximation market value of invested capital.
6. Subtract from the MVIC from step 4 the value of the senior securities from step 1. This gives the first approximation value of the common equity.
7. Compute the capital structure weights using the equity value from step 6.
8. Repeat the process, starting with step 3, until the computed market value weights come reasonably close to the weights used in computing the WACC.

For simplicity, we will demonstrate this process using only a two-component capital structure, common equity and debt. To further simplify, we will use the capitalization model. (The iterative process works just as well with a discounting model, but a few more figures are involved.)

We will carry out the example based on these six assumptions for the Donald E. Frump company (DEF):

1. The *balance sheet* shows *book values* as follows:

Long-term debt	\$400,000	(40%)
Common equity	\$600,000	(60%)

2. *Interest rate* on the debt is 10%, and that approximates DEF's current cost of borrowing.
3. DEF's *cost of equity* has been estimated to be 25% (with the simplifying assumption that cost of equity is unaffected by differing levels of debt).
4. DEF's *tax rate* is 40%.

5. NCF_{f1} = \$250,000 (*estimated net cash flow* to all invested capital for the 12 months immediately following the valuation date).
6. Regarding *growth*, NCF_f (net cash flow available to all invested capital) is expected to grow fairly evenly following the first year at 5% per year.

If we start with the balance sheet book values as a first approximation of capital structure weightings, putting the assumed DEF balance sheet numbers into Formula 7.1, the first approximation of the capital structure weightings is:

Formula 7.4

$$\begin{aligned} WACC &= (0.25 \times 0.60) + (0.10[1 - 0.40] \times 0.40) \\ &= 0.15 + (0.06 \times 0.40) \\ &= 0.15 + 0.024 \\ &= 0.174 \end{aligned}$$

This implies an overall cost of capital (WACC) of 17.4%.

The next step in the iteration process is to compute the market value of all the invested capital at this WACC. Substituting numbers from the preceding information in the basic constant growth capitalization formula (Formula 4.4), we get:

Formula 7.5

$$\begin{aligned} PV_f &= \frac{\$250,000}{0.174 - 0.05} \\ &= \frac{\$250,000}{0.124} \\ &= \$2,016,129 \end{aligned}$$

Subtracting the debt of \$400,000 implies a market value of equity of \$1,616,129 (\$2,016,129 – \$400,000 = \$1,616,129). That is not even close to the book value of equity of \$600,000. In fact, on this basis, the proportions of the market values of the components of the capital structure would be:

Component	Value	Weight
Common stock	\$1,616,129	80%
Debt	400,000	20%
Market value of invested capital	\$2,016,129	100%

This certainly sends us back to the drawing board, because our first approximation was 60/40, and this calculation produced a significantly different (80/20) result. This time let us try these weights:

Common stock	75%
Debt	25%

Substituting these weights in the formula for WACC produces:

Formula 7.6

$$
\begin{aligned}
WACC &= (0.25 \times 0.75) + (0.10[1 - 0.40] \times 0.25) \\
&= 0.1875 + (0.06 \times 0.25) \\
&= 0.1875 + 0.015 \\
&= 0.2025
\end{aligned}
$$

This implies an overall cost of capital (WACC) of 20.25%, significantly higher than the 17.4% in our first approximation.

Taking the next step, substituting this new estimate of WACC in the constant growth capitalization formula, we get:

$$
\begin{aligned}
PV_f &= \frac{\$250,000}{0.2025 - 0.05} \\
&= \frac{\$250,000}{0.1525} \\
&= \$1,639,344
\end{aligned}
$$

Subtracting the debt of $400,000 implies a market value of equity of $1,239,344 ($1,639,344 – $400,000 = $1,239,344). On this basis, the proportions of the market values of the components of the capital structure are:

Component	Value	Weight
Common stock	$1,239,344	75.6%
Debt	400,000	24.4%
Market value of invested capital	$1,639,344	100.0%

This result is close enough for most applications. After all, computing WACC is not an exact science. A WACC of 20.25% is much more reasonable for this company than our first approximation of 17.4%. But it could be made more precise with further iterations.

The point is that the first approximation of the capital structure weighting led to a 30% overvaluation of DEF's stock. This fact certainly demonstrates the importance of using capital structure component weightings at market value, not at book value, to estimate a company's WACC. The iterative process can develop a good estimate of the WACC and therefore a sound and defensible estimate of the value of the overall capital, whether the valuation is using the WACC as a discount rate in the discounting method or as a base rate from which to subtract growth when applying the capitalization method.

For detail about implementing the iterative process to develop WACC using CAPM, see Appendix E.

SHOULD AN ACTUAL OR A HYPOTHETICAL CAPITAL STRUCTURE BE USED?

If a company or an interest in a company is to be valued as it is, assuming the capital structure will remain intact, then the amount of debt in the company's actual capital structure should be used. If a minority interest is to be valued by a procedure involving (first) valuing overall capital and (then) subtracting debt, the company's actual amount of debt in its capital structure may be appropriate. The reason is because it would be beyond the power of a minority stockholder to change the capital structure.

If a controlling interest is to be valued and the standard of value is *fair market value*, an argument can be made that an industry-average capital structure should be used, because a control buyer would have the power to change the capital structure and the industry average could represent the most likely result. However, it would be important to understand how the industry-average capital structure is derived and whether it is reasonable to expect the subject company to achieve it, given current conditions of the company itself and of the financial market. If the "industry average" capital structure is comprised of public companies and the subject is private, the subject may not be able to achieve the public company average, because public companies often have greater access to lower-cost senior capital than do private companies.

If a controlling interest is to be valued under the standard of *investment value* (value to a particular buyer or seller rather than the hypothetical buyer or seller assumed under the fair market value standard), then the buyer's or owner's actual or desired capital structure could be used.

Note that when using an industry-average capital structure, it must be at market value, not book value. Most composite industry statistics sources (e.g., *RMA Annual Statement Studies* and all the various services based on federal income tax return data) report balance sheet figures and ratios at book value. Industry-average capital structures at market value can be computed using data from selected guideline public companies in the industry or from sources such as Ibbotson Associates' *Cost of Capital Yearbook* (see Data Resources in Appendix C), remembering the caveat against assuming that private companies can achieve public company capital structures.

SUMMARY

We have outlined the process of computing a WACC for both public and private companies and for proposed capital projects. Because the weights of the capital structure components must be at market value, and because private company stocks do not have market values, the process of computing the WACC for a private company is an iterative one, starting with approximations of market value weights of capital structure components.

Under some circumstances (e.g., a minority interest valuation), a company's actual (currently existing) capital structure may be used to estimate the WACC. If a controlling interest valuation is sought where it is reasonable to alter the company's capital structure, a hypothetical capital structure may be used to estimate the WACC.

There is much controversy about the potential impact that altering the capital structure has on the WACC.

Part II

Estimating the Cost of Equity Capital

Chapter 8

Build-up Models

Formula for the Equity Cost of Capital Build-up Model
Risk-free Rate
Risk-free Rate Represented by U.S. Treasury Securities
Components of the Risk-free Rate
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Selecting the Best Risk-free Maturity
Equity Risk Premium
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Arithmetic or Geometric Mean Historical Average Equity Risk Premium?
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Estimating Equity Risk Premia by the DCF Method
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Example of a Build-up Model
Summary

Previous chapters discussed the cost of capital in terms of its two major components, a *risk-free rate* and a *risk premium*. This chapter examines these components in general, dividing the equity risk premium into its three principal subcomponents.

Accordingly, the typical "build-up model" for estimating the cost of common equity capital consists of two components:

1. A "risk-free" rate
2. A premium for risk, including any or all of the following subcomponents:
   a. A general equity risk premium
   b. A size premium
   c. A company-specific risk premium

In international investing, there may also be a country-specific risk premium, reflecting uncertainties owing to economic and political instability in the particular country.

With respect to using the weighted average cost of capital (WACC) in other countries, one should consider the risk-free rate in the foreign country (e.g., interest rate on that country's government debt) and the risk premium in that country as measured by stock market returns in excess of return on the foreign government bonds. The risk-free rate and foreign equity risk premium are likely to incorporate the foreign country-specific risk premium, including any currency-related risk. In some countries there might be a risk discount compared with the United States.

It is probably a mistake to use the U.S. risk-free rate in determining foreign country cost of capital. If a foreign country-specific risk premium is not available, which often is the case, especially with smaller countries, one can use the U.S. risk premium applied to the foreign risk-free rate. (Bloomberg is an excellent source of foreign country government bond rates.) Other adjustments may be necessary to account for regulatory or other economic differences.

FORMULA FOR THE EQUITY COST OF CAPITAL BUILD-UP MODEL

Stating the preceding concept in a formula, the equity cost of capital build-up model is:

Formula 8.1

$$E(R_i) = R_f + RP_m + RP_s + RP_u$$

where:

$E(R_i)$ = Expected (market required) rate of return on security i
R_f = Rate of return available on a risk-free security as of the valuation date
RP_m = General equity risk premium for the "market"
RP_s = Risk premium for small size
RP_u = Risk premium attributable to the specific company or to the industry (the u stands for unsystematic risk, as defined in Chapter 5)

After discussing how to develop each of these four components, we will substitute some numbers into the formula to reach an estimated cost of equity capital for a sample company.

An additional possible component, industry risk, is discussed in a later section in this chapter.

RISK-FREE RATE

The general notion of a "risk-free rate" is the return available as of the valuation date on a security that the market generally regards as free of the risk of default.

Risk-free Rate Represented by U.S. Treasury Securities

In the build-up model (as well as in other models), analysts typically use the yield to maturity on U.S. government securities, as of the effective valuation date, as the risk-free rate. They generally choose U.S. Treasury obligations of one of the following maturities:

- 30 days
- 5 years
- 20 years

Sources for yields to maturity for maturities of any length as of any valuation date can be found in the daily financial press. (It usually is not possible to find yields to match the exact length of maturity. One should just choose the closest maturity available.)

Components of the Risk-free Rate

The so-called risk-free rate reflects three components:

1. *Rental rate:* A real return for lending the funds over the investment period, thus forgoing consumption for which the funds otherwise could be used
2. *Inflation:* The expected rate of inflation over the term of the risk-free investment
3. *Maturity risk or investment rate risk:* As discussed in Chapter 5, the risk that the principal's market value will rise or fall during the period to maturity as a function of changes in the general level of interest rates

All three of these economic factors are embedded in the yield to maturity for any given maturity length. However, it is not possible to observe the market consensus about how much of the yield for any given maturity is attributable to each of these factors.

Very importantly, note that this basic risk-free rate *includes inflation*. Therefore, when this rate is used to estimate a cost of capital to discount expected future cash flows, those future cash flows also should reflect the effect of inflation. In the economic sense of nominal versus real dollars, we are building a cost of capital in nominal terms, and it should be used to discount expected returns that also are expressed in nominal terms.

Why Only Three Specific Maturities?

The risk-free rate typically is chosen from one of only three specific maturities because the build-up model incorporates a general equity risk premium often based on historical data developed by Ibbotson Associates.

- Ibbotson data provide short-term, intermediate-term, and long-term equity risk premium series, based on data corresponding to the aforementioned three maturities.
- Twenty years is the longest maturity because the Ibbotson data go all the way back to 1926, and 20 years was the longest U.S. Treasury obligation issued during the earlier years of that time period.

Selecting the Best Risk-free Maturity

The consensus of financial analysts today is to use the 20-year U.S. Treasury yield to maturity as of the effective date of valuation because:

- It most closely matches the often-assumed perpetual lifetime horizon of an equity investment.
- The longest-term yields to maturity fluctuate considerably less than short-term rates and thus are less likely to introduce unwarranted short-term distortions into the actual cost of capital.
- People generally are willing to recognize and accept that the maturity risk is embedded in this base, or otherwise risk-free, rate.
- It matches the longest-term bond over which the equity risk premium is measured in the Ibbotson Associates data series.

Many analysts use a 30-year yield, but as a practical matter it usually does not differ greatly from the 20-year yield.

Sometimes analysts select a five-year rate to match the perceived investment horizon for the subject equity investment. The 30-day rate is the purest risk-free base rate because it contains virtually no maturity risk. If inflation is high, it does reflect the inflation component, but it contains little compensation for inflation uncertainty.

To obtain a 20-year yield, most analysts go to the financial press (e.g., *The Wall Street Journal*, *The New York Times*) as of the valuation date and find the yield on a bond originally issued for 30 years with approximately 20 years left to maturity. The St. Louis branch of the Federal Reserve Bank also tracks 20-year yields. The direct link to its Web site is: *http://www.stls.frg.org/fred/data/irates/gs20*.

EQUITY RISK PREMIUM

On an equity investment, the return on investment that the investor will (or has the opportunity to) realize usually has two components:

1. Distributions during the holding period (e.g., dividends or withdrawals)
2. The capital gain or loss in the value of the investment (For an active public security, it is considered part of the return whether or not the investor chooses to realize it, because the investor has that choice at any time.)

Obviously, these expected amounts of returns on equities are much less certain (or more risky) than the interest and maturity payments on U.S. Treasury obligations. This difference in riskiness is well documented by much higher standard deviations (year-to-year volatility) in returns on the stock market, compared with the standard deviation of year-to-year returns on U.S. Treasury obligations.

To accept this greater risk, investors demand higher expected returns for investing in equities than for investing in U.S. Treasury obligations. This differential in expected return on the broad stock market over U.S. Treasury obligations (sometimes referred to as the *excess return*, but not to be confused with the *excess earnings method*) is called the *equity risk premium*.

In practice, a common method of estimating this expected equity risk premium is to use historical data. It is common to compute it as the average excess return (broad stock market over U.S. Treasuries) over some historical period of time. Because cost of capital is a forward-looking concept, a key implied assumption when using historical data is that the amount of excess return that investors expect for investing in stocks (over the amount expected from Treasuries) for their future time horizon is approximately equal to the excess returns that have actually been achieved in the broad stock market in the historical period for which the equity risk premium was computed.

Ibbotson now calculates the arithmetic average equity risk premium on two measures of the broad market:

1. Standard & Poor's (S&P) 500 Index
2. New York Stock Exchange (NYSE) Composite Index

In the *2002 SBBI Valuation Edition*, these equity risk premia were 7.4% and 7.1%, respectively. The *2002 Valuation Edition* says nothing about which is better to use under what circumstances. In the following examples, we will use the S&P 500, because that is what Ibbotson uses in most of its examples.

Ibbotson Associates Is the Primary Source of Historical Risk Premium Data

Ibbotson Associates publishes historical risk premium data in its annually updated *Stocks, Bonds, Bills and Inflation (SBBI)*, *Classic Edition* and *Valuation Edition*. These publications are described in some detail in Chapter 13 on using Ibbotson data, and ordering information is included in Appendix C. These sources are a core part of any corporate finance data library, especially for a practitioner of business valuations.

Arithmetic or Geometric Mean Historical Average Equity Risk Premium?

Ibbotson publishes both an arithmetic and a geometric mean equity risk premium series. In the arithmetic mean series, the procedure is to add up all the excess returns over the periods and divide by the number of periods.

The formula for the familiar arithmetic mean is:

Formula 8.2

$$\bar{x} = \frac{\sum_{1}^{n} R_i}{n}$$

where:

$\bar{x}$ = Mean average
R_i = Return for the ith period (The returns measured for each period are actually excess returns, i.e., the difference between the equity market return and the Treasury obligation income return for the period.)
n = Number of observation periods

In the geometric mean series, the procedure is to add 1 to the excess return for each period, multiply these all together, take the root of the number of periods, and subtract 1 at the end. The geometric mean result is the *annually compounded* rate of excess return.

The formula for the less familiar geometric mean is:

Formula 8.3

$$G = \left[\prod_{1}^{n} (1 + R_i)\right]^{\frac{1}{n}} - 1$$

Sometimes also written as:

$$G = \sqrt[n]{\prod_{1}^{n} (1 + R_i)} - 1$$

where:

G = Geometric average
R_i = Return for the ith period (The returns measured for each period are actually excess returns, i.e., the difference between the equity market return and the Treasury obligation income return for the period.)
n = Number of observation periods

Mathematically, the geometric mean is always lower than the arithmetic mean unless all observations are equal, in which case the arithmetic and geometric means are equal.

Ibbotson's position is that, for valuation purposes, the historical equity risk premium should be the arithmetic mean applied to the expected value of the probability distribution of the expected return for each period. This is discussed further in Chapter 13 on using Ibbotson data.

The Ibbotson position that the long-term arithmetic average equity risk premium is the best proxy for today's equity risk premium is widely accepted. For example, this view is supported by one of the leading corporate finance texts, Brealey and Myers' *Principles of Corporate Finance*, where the authors state, "If the cost of capital is estimated from historic returns or risk premiums, use arithmetic averages, not compound rates of return."[1] However, this view is not universally held. For a contrary view (i.e., that the true market risk premium lies somewhere between the arithmetic and geometric averages), see Tom Copeland, Tim Koller, and Jack Murrin's *Valuation: Measuring and Managing the Value of Companies*.[2] There have also been articles on this controversy in *Business Valuation Review*, the quarterly journal of the Business Valuation Committee of the American Society of Appraisers.[3] The arithmetic mean is technically correct when applied to the expected value of the cash flows. However, because a long time period is needed to develop a statistically valid historical risk premium, a short historical period could produce a high or low indicated cost of capital relative to any specific date's current market conditions.

Over What Historical Time Period Should the Equity Risk Premium Be Calculated?

Regarding the historical time period over which equity risk should be calculated, Ibbotson offers two observations:

1. Reasons to focus on recent history:
   a. The recent past may be most relevant to an investor.
   b. Return patterns may change over time.
   c. The longer period includes "major events" (e.g., World War I, World War II, the Depression) that have not repeated in some time.
2. Reasons to focus on long-term history:
   a. Long-term historical returns have shown surprising stability.
   b. Short-term observations may lead to illogical forecasts.
   c. Focusing on the recent past ignores dramatic historical events and their impact on market returns. We do not know what major events lie ahead.
   d. Law of large numbers: More observations lead to a more accurate estimate.

Exhibit 8.1 shows the arithmetic average equity risk premiums over various time periods as calculated by Ibbotson.

Note that as the length of the measurement period shortens, the standard error of the estimate widens substantially. This is one reason why Ibbotson advocates using the full historical period covered by the data, 1926 through the present, as discussed more fully in Chapter 13.

Exhibit 8.1 Stock Market Return and Equity Risk Premium over Time (1926–1998)

Period Length	Period Dates	Large Company Stock Arithmetic Mean Total Return	Long-Horizon Equity Risk Premium
73 years	1926–1998	13.20%	8.00%
70 years	1929–1998	12.40%	7.10%
60 years	1939–1998	13.70%	8.10%
50 years	1949–1998	14.80%	8.50%
40 years	1959–1998	13.10%	5.90%
30 years	1969–1998	13.90%	5.70%
20 years	1979–1998	18.50%	9.70%
15 years	1984–1998	18.60%	10.50%
10 years	1989–1998	20.00%	12.80%
5 years	1994–1998	24.80%	18.20%

Source: Ibbotson Associates, *Cost of Capital Workshop* (1999), chapter on "Equity Risk Premium," p. 14, copyright © 1999. Used with permission. All rights reserved.

Estimating Equity Risk Premia by the Discounted Cash Flow Method

An alternative to using the historical average equity risk premium data to estimate the current equity risk premium is the discounted cash flow (DCF) method. The DCF method uses market prices and analysts' growth estimates as of the effective date for individual companies and industries to estimate the market's implied expected rate of return.

The general idea of the DCF method is quite simple: Rearrange a capitalization model (e.g., the Gordon Growth Model) or a discounting model to make the present value (the market price of the stock) a known quantity and solve the equation for k, the implied cost of the equity capital. The difference between k (the implied cost of equity capital for the company or industry) and R_f (the risk-free rate as of the same time) is the implied equity risk premium.

The DCF method is widely used among investment bankers and portfolio managers. During the 1990s and up to the present time, the method consistently yielded lower equity risk premiums than the historical average equity risk premium method.

Data to implement the DCF method are included in Ibbotson's *Cost of Capital Yearbook* as well as in other sources included in Appendix C. Chapter 12 is devoted to further implementation of the DCF method.

SMALL STOCK PREMIUM

Recent studies have provided strong evidence that the degree of risk and corresponding cost of capital increase with the decreasing size of the company. The studies show that this addition to the equity risk premium is over and above the amount that would be warranted solely for the companies' systematic risk. The next two chapters explain the Capital Asset Pricing Model (CAPM) and the proper use of beta. Chapter 11 discusses the results of research on this phenomenon as well as the sources.

COMPANY-SPECIFIC RISK PREMIUM

To the extent that the subject company's risk characteristics are greater or less than the typical risk characteristics of the companies from which the equity risk premium and the size premium were drawn, a further adjustment may be necessary to estimate the cost of capital for the specific company. Such adjustment may be based on (but not necessarily limited to) analysis of five factors:

1. Size smaller than the smallest size premium group
2. Industry risk
3. Volatility of returns
4. Leverage
5. Other company-specific factors

Size Smaller Than the Smallest Size Premium Group

As will be seen in Exhibit 11.4, the smallest size group for which we have specific size premium data averages $30 million in market value of equity, $37 million in sales, and so forth. If the subject company is somewhat smaller than this cutoff, most observers believe that a further size premium adjustment is warranted, but there have not yet been adequate empirical studies to quantify this amount. Accordingly, a conservative approach may be appropriate, perhaps adding a point or two to the discount rate for a significantly smaller company and leaving any greater adjustments to be attributed to other specifically identifiable risk factors.

In the first edition of this book, I said that I planned to research this issue using the *Pratt's Stats*™ private company transaction database. The study is reported in Chapter 11 on the size effect. Indeed, the results show that the cost of capital is much greater for smaller companies.

Industry Risk

The industry in which the company operates may have more or less risk than the average of other companies in the same size category. This differential is very hard to quantify in the build-up model. However, if the company is obviously in a very low-risk industry (e.g., water distribution) or a very high-risk industry (e.g., airlines), a point or two adjustment, either downward or upward, for this factor may be warranted.

Ibbotson Associates now publishes industry risk adjustment factors (see Chapter 13), but the industry definitions are quite broad. We understand that the definitions will continue to grow and be refined down to more discrete Standard Industrial Classification (SIC) sorts than what we see so far.[4] It is too early to tell how well they will stand up to testing in the market.

As noted earlier, Ibbotson's *SBBI Valuation Edition 2002 Yearbook* presents an expanded alternative build-up model that includes a separate variable for the industry risk premium. This model is shown in Formula 8.4:

Formula 8.4

$$E(R_i) = R_f + RP_m + RP_s +/- \boxed{\boldsymbol{RP}} + R_{pu}$$

where:

$E(R_i)$ = Expected rate of return
R_f = Risk-free rate of return
RP_m = Equity risk premium (market risk)
RP_s = Size premium
$\boldsymbol{RP_i}$ **= Industry risk premium**
RP_u = Company-specific risk premium (unsystematic risk)

Volatility of Returns

High volatility of returns (usually measured by the standard deviation of historical returns over some period) is another risk factor. However, without comparable data for the average of the other companies in the size category and/or industry, it is not possible to make a quantified comparison. If the analyst perceives that the subject company returns are either unusually stable or unusually volatile compared with others in the size category and/or industry, some adjustment for this factor may be warranted.

Leverage

Leverage is clearly a factor that can be compared between the subject company and its size peers. Exhibit 11.4 gives both the market value of equity and the market value of invested capital for each size category.

For example, the smallest size category averages $30 million in market value of equity and $41 million in market value of invested capital, or a capital structure of roughly 25% debt and 75% equity, at market value. Size breakdowns of other size measures show generally similar capital structures. If the subject company's capital structure significantly departs from this average, some upward or downward adjustment to the cost of equity relative to the average company in the size category would seem warranted. For example, highly leveraged companies should have higher equity costs of capital compared with companies with lower debt levels, all else being equal. Of course, a decrease in the required equity return might be warranted if the subject's capital structure has little or no debt.

Other Company-specific Factors

Other factors specific to a particular company that affect risk could include, for example:

- Concentration of customer base
- Key person dependence
- Key supplier dependence
- Abnormal present or pending competition
- Pending regulatory changes
- Pending lawsuits
- A wide variety of other possible specific factors

Because the size premium tends to reflect some factors of this type, the analyst should adjust further only for specific items that are truly unique to the subject company. Unfortunately, despite the widespread use by analysts and appraisers of a company-specific risk premium in a build-up (or CAPM) model, I am not aware of any academic research on the subject, and it remains in the realm of the analyst's judgment.

EXAMPLE OF A BUILD-UP MODEL

Now that we have discussed the factors in the build-up model, we can substitute some numbers into Formula 8.1. We start with the following four assumptions about Shannon's Bull Market (SBM), a regional steakhouse chain with excellent food and friendly service:

1. *Risk-free rate.* We will use the 20-year Treasury bond, for which the yield to maturity at the valuation date was 6.5%.
2. *Equity risk premium.* We will use the Ibbotson Associates' arithmetic average equity risk premium. The *SBBI Valuation Edition 2002 Yearbook* shows that to be 7.4%.
3. *Size premium.* The *SBBI Valuation Edition 2002 Yearbook* shows that the size premium for the tenth decile—smallest 10% of New York Stock Exchange (NYSE) stocks with American Stock Exchange (AMEX) and Nasdaq Stock Market (Nasdaq) stocks included—over and above the return estimated by CAPM is 5.33%.[5]
4. *Company-specific risk premium.* SBM is considerably smaller than the average of the smallest 10% of NYSE stocks, and our analyst perceives that the restaurant industry is riskier than the average for those companies. Although the assessment is somewhat subjective, our analyst recommends adding a company-specific risk factor of 3.0% because of risk factors identified as unique to this company.
5. Possibly, an *industry adjustment factor.*

Substituting the preceding information in Formula 8.1, we have the following:

Formula 8.5

$$E(R_i) = 6.5 + 7.4 + 5.3 + 3.0$$
$$= 19.2$$

So the estimated cost of capital for SBM is approximately 19.2%.

Some analysts prefer to present these calculations in tabular form, such as the following:

Build-up Cost of Equity Capital for SBM

Risk-free rate	6.5%
Equity risk premium	7.4%
Size premium	5.3%
Company-specific risk premium	3.0%
SBM cost of equity capital	19.2%

If we were using the Capital Asset Pricing Model (CAPM) (the subject of the next chapter), a portion of the size premium and probably all of the industry portion of the specific risk premium would be captured in the "beta" factor, which is the difference between CAPM and the straight build-up model. Of course, if these build-up model figures were presented in a formal valuation report, each of the numbers in the calculation would be footnoted as to its source, and each would be supported by a narrative explanation.

SUMMARY

The build-up model for estimating the cost of equity capital has four components:

1. A risk-free rate
2. A general equity risk premium
3. A size premium
4. A company-specific risk adjustment (which can be either positive or negative, depending on the risk comparisons between the subject company and others from which the size premium was derived)
5. Possibly, an industry adjustment factor

These factors are summarized schematically in Exhibit 8.2. In a sense, the build-up method is a version of the Capital Asset Pricing Model, the subject of the next chapter, without specifically incorporating systematic risk.

Exhibit 8.2 Summary of Development of Equity Discount Rate

Risk-free rate*	20-year, 5-year, or 30-day Treasury yield as of valuation date
+ Equity risk premium	Long-, intermediate-, or short-horizon equity risk premium (corresponding to risk-free yield above)**
+ Size premium	Small stock premium
± Specific risk	Specific risk difference in subject company relative to companies from which above data are drawn

*The "risk-free" actually has one element of risk: *maturity risk* (sometimes called *interest rate risk* or *horizon risk*)—the risk that the value of the bond will fluctuate with changes in the general level of interest rates.

**Corresponding to Ibbotson's historical risk premium studies, as found in the *Stocks, Bonds, Bills and Inflation, Classic Edition* and *Valuation Edition* yearbooks.

Notes

1. Richard A. Brealey and Stewart C. Myers, *Principles of Corporate Finance*, 6th ed. (Boston: Irwin McGraw-Hill, 2000), 157.
2. Tom Copeland, Tim Koller, and Jack Murrin, *Valuation: Measuring and Managing the Value of Companies*, 3rd ed. (New York: John Wiley & Sons, Inc., 2000), 218–221. For an additional supporting view that the discount rate should be somewhere between the arithmetic and geometric averages, see J. Michael Julius, "Market Returns in Rolling Multi-Year Holding Periods: An Alternative Interpretation of the Ibbotson Data," *Business Valuation Review* (June 1996): 57–63.
3. Brian Becker and Ian Gray, "Does a Small Firm Effect Exist when Using the CAPM? Not Since 1980 and Not when Using Geometric Mean of Historical Returns," *Business Valuation Review* (September 1999): 104–107; Brian C. Becker and Ian Gray, "Using Average Historical Data for Risk Premium Estimates: Arithmetic Mean, Geometric Mean, or Something Else?" *Business Valuation Review* (December 1998): 136–140; Allyn A. Joyce, "Why the Expected Rate of Return is a Geometric Mean," *Business Valuation Review* (March 1996): 17–19; George G. Cassiere, "Geometric Mean Return Premium Versus the Arithmetic Mean Return Premium—Expanding on the SBBI 1995 Yearbook Examples," *Business Valuation Review* (March 1996): 20–23; Paul D. Kaplan, "Why the Expected Rate of Return is an Arithmetic Mean," *Business Valuation Review* (September 1995): 126–129; Allyn A. Joyce, "Arithmetic Mean vs. Geometric Mean: THE Issue in Rate of Return Derivation," *Business Valuation Review* (June 1995): 62–68.
4. The July 2002 issue of *Shannon Pratt's Business Valuation Update®* features an interview on this subject with Michael W. Barad, senior editor of the *SBBI Valuation Edition 2002 Yearbook*.
5. Ibbotson Associates recommends using the size premium (return in excess of CAPM) analysis for both the build-up and CAPM cost of equity estimates. This data can be seen in the *SBBI Valuation Edition 2002 Yearbook* at page 125. See Chapter 13 for more information on Ibbotson's size premium methodology.

Chapter 9

Capital Asset Pricing Model[1]

Concept of Systematic Risk
Background of the Capital Asset Pricing Model
Systematic and Unsystematic Risk
Using Beta to Estimate Expected Rate of Return
Expanding CAPM to Incorporate Size Premium and Specific Risk
  Firm Size Phenomenon
  Company-specific Risk Factor
Expanded CAPM Cost of Capital Formula
Assumptions Underlying the Capital Asset Pricing Model
Recent Research on the Equity Risk Premium
Summary

CONCEPT OF SYSTEMATIC RISK

For more than 30 years financial theorists generally have favored the notion that using the Capital Asset Pricing Model (CAPM) is the preferred method to estimate the cost of equity capital. In spite of many criticisms, it is still one of the most widely used models for estimating the cost of equity capital, especially for larger companies.

The only difference between the CAPM and the build-up model presented in the preceding chapter is the introduction of *systematic risk* as a modifier to the general equity risk premium. Systematic risk is measured by a factor called *beta*. Beta measures the sensitivity of excess total returns (total returns over the risk-free rate returns) on any individual security or portfolio of securities to the total excess returns on some measure of the market, such as the New York Stock Exchange (NYSE) Composite Index or Standard & Poor's (S&P) 500 Index.

The next chapter discusses the specific measurements of beta. Note at this point, however, that beta is based on total returns, which have two components:

1. Dividends
2. Change in market price

Because privately held companies have no market price, their betas cannot be measured directly. Thus, to use the CAPM to estimate the cost of capital for a private company, it is necessary to estimate a proxy beta for that company. This usually is accomplished by using an average beta for the industry group or by selecting specific guideline public companies and using some composite, such as the average or median, of their betas.

CAPM is simply a mechanism to estimate the cost of equity capital, and in the end, all other things being equal, the cost of capital for any given company is the same whether you arrive at it by CAPM or by the build-up model. CAPM, however, requires public companies from which to develop betas. For some industries, especially those characterized by many small companies, public companies on which to base an estimate of beta simply do not exist.

BACKGROUND OF THE CAPITAL ASSET PRICING MODEL

The Capital Asset Pricing Model is part of a larger body of economic theory known as *capital market theory* (CMT). CMT also includes *security analysis* and *portfolio management theory*, a normative theory that describes how investors *should* behave in selecting common stocks for their portfolios, under a given set of assumptions. In contrast, the CAPM is a *positive* theory, meaning it describes the market relationships that *will* result if investors behave in the manner prescribed by portfolio theory.

The CAPM is a conceptual cornerstone of modern capital market theory. Its relevance to business valuations and capital budgeting is that businesses, business interests, and business investments are a subset of the investment opportunities available in the total capital market; thus, the determination of the prices of businesses theoretically should be subject to the same economic forces and relationships that determine the prices of other investment assets.

SYSTEMATIC AND UNSYSTEMATIC RISK

In Chapter 5 we defined risk conceptually as the degree of uncertainty regarding the realization of expected future economic income. Capital market theory divides risk into two components (other than maturity risk): systematic risk and unsystematic risk. Stated in nontechnical terms, *systematic risk* is the uncertainty of future returns owing to the sensitivity of the return on the subject investment to movements in the returns for a composite measure of marketable investments. *Unsystematic risk* is a function of the characteristics of the industry, the individual company, and the type of investment interest. To the extent that the industry as a whole is sensitive to market movements, that portion of the industry's risk would be captured in beta, the measure of systematic risk. Company-specific characteristics may include, for example, management's ability to weather changing economic conditions, relations between labor and management, the possibility of strikes, the success or failure of a particular marketing program, or any other factor specific to the company. Total risk depends on both systematic and unsystematic factors.

A fundamental assumption of the CAPM is that the risk premium portion of a security's expected return is a function of that security's systematic risk. This is because capital market theory assumes that investors hold, or have the ability to hold, common stocks in large, well-diversified portfolios. Under that assumption, investors will not require compensation (i.e., a higher return) for the unsystematic risk because they can easily diversify it away. Therefore, the only risk pertinent to a study of capital asset pricing theory is systematic risk. As one well-known corporate finance text puts it: "The crucial distinction between diversifiable and nondiversifiable risks . . . is the main idea underlying the capital asset pricing model."[2]

USING BETA TO ESTIMATE EXPECTED RATE OF RETURN

The CAPM leads to the conclusion that the equity risk premium (the required excess rate of return for a security over and above the risk-free rate) is a linear function of the security's beta. This linear function is described in this univariate linear regression formula:

Formula 9.1

$$E(R_i) = R_f + B(RP_m)$$

where:

$E(R_i)$ = Expected return (cost of capital) for an individual security
R_f = Rate of return available on a risk-free security (as of the valuation date)
B = Beta
RP_m = Equity risk premium for the market as a whole (or, by definition, the equity risk premium for a security with a beta of 1.0)

The preceding linear relationship is shown schematically in Exhibit 9.1, which presents the security market line (SML).

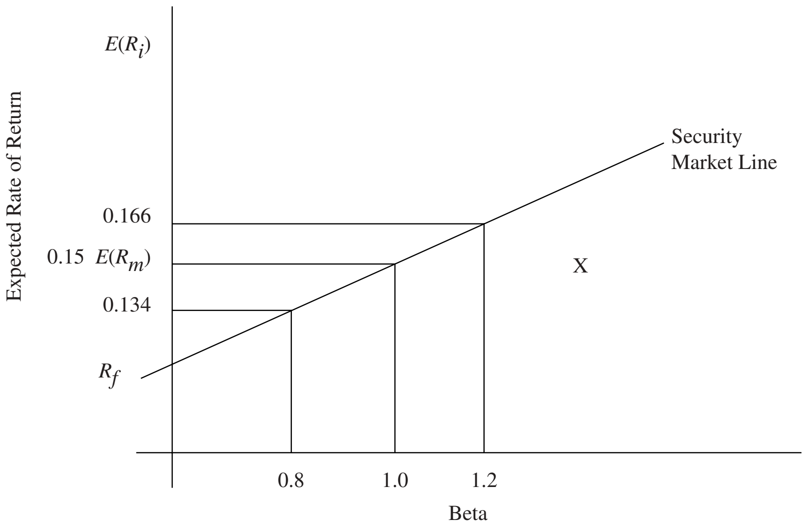
According to capital asset pricing theory, if the combination of an analyst's expected rate of return on a given security and its risk, as measured by beta, places it below the security market line, such as security X in Exhibit 9.1, the analyst would consider that security (e.g., common stock) mispriced. It would be mispriced in the sense that the analyst's expected return on that security is less than it would be if the security were correctly priced, assuming fully efficient capital markets.

To put the security in equilibrium according to that analyst's expectations, the price of the security must decline, allowing the rate of return to increase until it is just sufficient to compensate the investor for bearing the security's risk. In theory, all common stocks in the market, in equilibrium, adjust in price until the consensus expected rate of return on each is sufficient to compensate investors for holding them. In that situation the systematic risk/expected rate of return characteristics of all those securities will place them on the security market line.

As Exhibit 9.1 shows, the beta for the market as a whole is 1.0. Therefore, from a numerical standpoint, the beta has the following interpretations:

Beta > 1.0	When market rates of return move up or down, the rates of return for the subject tend to move in the same direction and with greater magnitude. For example, for a stock with no dividend, if the market is up 10%, the price of a stock with a beta of 1.2 would be expected to be up 12%. If the market is down 10%, the price of the same stock would be expected to be down 12%. Many high-tech companies are good examples of stocks with high betas.
Beta = 1.0	Fluctuations in rates of return for the subject tend to equal fluctuations in rates of return for the market.
Beta < 1.0	When market rates of return move up or down, rates of return for the subject tend to move up or down, but to a lesser extent. For example, for a stock with no dividend, if the market is up 10%, the price of a stock with a beta

Exhibit 9.1 Security Market Line



$E(R_i)$ = Expected return for the individual security

$E(R_m)$ = Expected return on the market

R_f = Risk-free rate available as of the valuation date

In a market in perfect equilibrium, all securities would fall on the security market line. The security X is mispriced, with a return less than it would be on the security market line.

Source: Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies,* 4th ed. (New York: McGraw-Hill, 2000), 167. Reprinted with permission.

	of .8 would be expected to be up 8%. The classic example of a low-beta stock would be a utility that has not diversified into riskier activities.
Negative beta (rare)	Rates of return for the subject tend to move in the opposite direction from changes in rates of return for the market. Stocks with negative betas are rare. A few gold-mining companies have had negative betas in the past.

To illustrate, using the preceding formula as part of the process of estimating a company's cost of equity capital, consider stocks of average size, publicly traded companies *i*, *j*, and *k*, with betas of 0.8, 1.0, and 1.2, respectively; a risk-free rate in the market of 7% (0.07) at the valuation date; and a market equity risk premium of 8% (0.08).

For company *i*, which is less sensitive to market movements than the average company, we can substitute in Formula 9.1 as follows:

Formula 9.2

$$\begin{aligned} E(R_i) &= 0.07 + 0.8(0.08) \\ &= 0.07 + 0.064 \\ &= 0.134 \end{aligned}$$

Thus, the cost of equity capital for company *i* is estimated to be 13.4% because it is less risky, in terms of systematic risk, than the average stock on the market.

For company *j*, which has average sensitivity to market movements, we can substitute in Formula 9.1 as follows:

Formula 9.3

$$\begin{aligned} E(R_j) &= 0.07 + 1.0(0.08) \\ &= 0.07 + 0.08 \\ &= 0.15 \end{aligned}$$

So the cost of equity capital for company *j* is estimated to be 15%, the estimated cost of capital for the average stock, because its systematic risk is equal to the average of the market as a whole.

For company *k*, which has greater-than-average sensitivity to market movements, we can substitute in Formula 9.1 as follows:

Formula 9.4

$$\begin{aligned} E(R_k) &= 0.07 + 1.2(0.08) \\ &= 0.07 + 0.096 \\ &= 0.166 \end{aligned}$$

Thus, the cost of equity capital for company *k* is estimated to be 16.6% because it is riskier, in terms of systematic risk, than the average stock on the market.

Note that in the preceding *pure* formulation of the CAPM, the required rate of return is composed of only two factors:

1. The risk-free rate
2. The market's general equity risk premium, as modified by the beta for the subject security

EXPANDING CAPM TO INCORPORATE SIZE PREMIUM AND SPECIFIC RISK

Firm Size Phenomenon

Many empirical studies performed since CAPM was originally developed have indicated that the realized total returns on smaller companies have been substantially greater over a long period of time than the original formulation of the CAPM (as given in Formula 9.1) would have predicted. Ibbotson Associates comments on this phenomenon:

> One of the most remarkable discoveries of modern finance is that of a relationship between firm size and return. The relationship cuts across the entire size spectrum but is most evident among smaller companies, which have higher returns on average than larger ones.[3]
>
> The firm size phenomenon is remarkable in several ways. First, the greater risk of small stocks does not, in the context of the capital asset pricing model (CAPM), fully account for their higher returns over the long term. In the CAPM, only systematic or beta risk is rewarded; small company stocks have had returns in excess of those implied by their betas. Second, the calendar annual return differences between small and large companies are serially correlated. This suggests that past annual returns may be of some value in predicting future annual returns. Such serial correlation, or autocorrelation, is practically unknown in the market for large stocks and in most other equity markets but is evident in the size premia.[4]

The size effect is the subject of Chapter 11.

Company-specific Risk Factor

The notion that the only component of risk that investors care about is systematic risk is based on the assumption that all unsystematic risk can be eliminated by holding a perfectly diversified portfolio of risky assets that will, by definition, have a beta of 1.0.

Without addressing the validity of that assumption for the public markets (for the purpose of this book), it is obviously not feasible for investors in privately held companies to hold such a perfectly diversified portfolio that would eliminate all unsystematic risk. Therefore, for the cost of capital for privately held companies, even when using the CAPM, we have to consider whether there may be other risk elements that neither the beta factor (systematic risk) nor the size premium fully accounts for. If so, an adjustment to the discount rate for unsystematic risk would be appropriate.

Just as in the build-up model, the "specific risk" factor could be negative if the analyst concluded that the subject company was less risky than the average of the other companies from which the proxy estimates for the other elements of the cost of equity capital were drawn. For example, a company could have a well-protected, above-average price for its products as a result of a strong trademark, resulting in significantly less earnings volatility than experienced by its competitors.

EXPANDED CAPM COST OF CAPITAL FORMULA

If we expand CAPM to also reflect the size effect and specific risk, we can expand the cost of equity capital formula to add these two factors:

Formula 9.5

$$E(R_i) = R_f + B(RP_m) + RP_s + RP_u$$

where:

$E(R_i)$ = Expected rate of return on security i
R_f = Rate of return available on a risk-free security as of the valuation date
RP_m = General equity risk premium for the market
RP_s = Risk premium for small size
RP_u = Risk premium attributable to the specific company (u stands for unsystematic risk)
B = Beta

Note that the *only* difference between this formula and the build-up model formula (Formula 8.1) is the addition of the beta coefficient. The value of the company-specific risk premium, however, is likely to differ from that used in the build-up model, because some portion of the company-specific risk may have been captured in beta. Also, in the build-up model, an industry adjustment factor may be incorporated. To put some numbers into Formula 9.5, we will make five assumptions about Lazard, Hazard, and Zipp (LHZ), a fictional investment banking firm with publicly traded stock:

1. *Risk-free rate.* As of the valuation date, the yield to maturity on 20-year U.S. government bonds is 7.0%.
2. *Beta.* The LHZ beta is 1.3.
3. *Equity risk premium.* The general equity risk premium is 8.0%.

4. *Size premium.* The size premium for this size firm in excess of the risk captured in CAPM through beta is 3.3%. (We will assume here that this is on the borderline between Ibbotson's ninth and tenth size deciles.)
5. *Company-specific risk factor.* Because of special risk factors, the analyst has estimated that there should be an additional specific risk factor of 1.0%.

Substituting this information in Formula 9.5, we have the following:

Formula 9.6

$$\begin{aligned} E(R_i) &= 7.0 + 1.3(8.0) + 3.3 + 1.0 \\ &= 7.0 + 10.4 + 3.3 + 1.0 \\ &= 21.7 \end{aligned}$$

Thus, the estimated cost of equity capital for LHZ is 21.7%.

Some analysts prefer to present the preceding calculations in tabular form:

CAPM Cost of Equity Capital for LHZ

Risk-free rate		7.0%
Equity risk premium:		
General equity risk premium	8.0	
Beta	× 1.3	
		10.4
Small stock size premium		3.3
Specific risk premium		1.0
LHZ cost of equity capital		21.7%

Of course, if this information were presented in a formal valuation report, each of the numbers would be footnoted as to its source, and each would be supported by narrative explanation.

ASSUMPTIONS UNDERLYING THE CAPITAL ASSET PRICING MODEL

Eight assumptions underlie the CAPM:

1. Investors are risk averse.
2. Rational investors seek to hold efficient portfolios, that is, portfolios that are fully diversified.
3. All investors have identical investment time horizons (i.e., expected holding periods).

4. All investors have identical expectations about such variables as expected rates of return and how capitalization rates are generated.
5. There are no transaction costs.
6. There are no investment-related taxes. (However, there may be corporate income taxes.)
7. The rate received from lending money is the same as the cost of borrowing money.
8. The market has perfect divisibility and liquidity (i.e., investors can readily buy or sell any desired fractional interest).

Obviously, the extent to which these assumptions are or are not met in the real world will have a bearing on the application of the CAPM for the valuation of closely held businesses, business interests, or investment projects. For example, while the perfect divisibility and liquidity assumption approximates reality for publicly traded stocks, the same is not true for privately held companies. This is one reason why the company-specific, nonsystematic risk factor may be rewarded in expected returns for closely held companies, even if it is not for public companies.

The CAPM, like most economic models, offers a theoretical framework for how certain relationships would exist subject to certain assumptions. Although not all assumptions are met in the real world, the CAPM provides a reasonable economic model for estimation of the cost of capital. Other models are discussed in later chapters.

RECENT RESEARCH ON THE EQUITY RISK PREMIUM

As noted in the preface, much research has been conducted in the last few years on the critical subject of measurements of the equity risk premium. At the Ibbotson Associates 1996 Cost of Capital and Equity Risk Premium Conference, Roger Ibbotson asserted, "The best estimate of the equity risk premium is the long-term arithmetic mean—[which is] 7.4%, with about 2.5% standard error of the estimate." Burton Malkiel, a nationally known professor of economics at Princeton University, replied, "Maybe 1% to 2%; it sure as hell isn't 5%."[5]

A recent working paper by Roger Ibbotson and Peng Chen concludes, "The equity risk premium is estimated to be 3.97% and 5.90% on an arithmetic basis."[6] If you accept Copeland *et al's* conclusion that the equity risk premium is somewhere between the arithmetic and geometric averages, that would imply about 5%, which I find is approximately the consensus at mid-2002.

In an article published in 2002, Fama/French used the DCF model (the subject of Chapter 12) applied to both dividends and earnings. They concluded, "For the half century, from 1951 to 2000, . . . , the equity risk premium estimates from the dividend and earnings growth models, 2.55 percent and 4.32 percent, are far below the estimate from the average return, 7.43 percent. We argue that the dividend and earnings growth estimates of the equity premium for 1951 to 2000 are closer to the true expected value. . . . [O]ur main message is that the unconditional expected equity premium of the last 50 years is probably far below the realized premium."[7]

Other recent research is cited in the Bibliography.

Exhibit 9.2 Capital Asset Pricing Model Method of Estimating Equity Discount Rate

Risk-free rate*	20-year, 5-year, or 30-day Treasury yield as of valuation date	
+ Equity risk premium**	Long-, intermediate-, or short-horizon equity risk premium (corresponding to risk-free yield above)	In CAPM, multiply the general equity risk premium by beta.
+ Size premium	Small stock premium	
± Specific risk	Specific risk difference in subject company relative to companies from which above data are drawn	

*The "risk-free" rate actually has one element of risk: *maturity risk* (sometimes called *interest risk* or *horizon risk*)—the risk that the value of the bond will fluctuate with changes in the general level of interest rates.

**This assumes that the equity risk premium will be based on Ibbotson's historical data. The equity risk premium could also be estimated from DCF model data, as discussed in Chapter 12.

SUMMARY

The Capital Asset Pricing Model (CAPM) expands on the build-up model by introducing the beta coefficient, an estimate of *systematic risk*, the sensitivity of returns for the subject to returns for the market. The CAPM has several underlying assumptions, which may be met to a greater or lesser extent for the market as a whole or for any particular company or investment.

Exhibit 9.2 is a schematic summary of using the CAPM to estimate the cost of equity capital.

Notes

1. Chapter 9 draws heavily on Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. (New York: McGraw-Hill, 2000).
2. Richard A. Brealey and Stewart C. Myers, *Principles of Corporate Finance*, 6th ed. (Boston: Irwin McGraw-Hill, 2000), 1007.
3. Ibbotson Associates, *Stocks, Bonds, Bills and Inflation, Valuation Edition 2002 Yearbook* (Chicago: Ibbotson Associates, 2002), 117.
4. Ibid. at 122.
5. "Evidence Suggests Equity Risk Premium Lower Than Conventional Wisdom Thinks," Special Report in *Shannon Pratt's Business Valuation Update®* (July 1996): 1, 4–5, at 1.
6. Roger G. Ibbotson and Peng Chen, "Stock Market Returns in the Long Run: Participating in the Real Economy," *Yale ICF Working Paper No. 00-44,* March 2002; can be downloaded without charge from the Social Science Research Network Electronic Paper Collection at *http://papers.ssrn.com/abstract=274150*.
7. Eugene F. Fama and Kenneth R. French, "The Equity Premium," *The Journal of Finance* (April 2002): 637–659.

Chapter 10

Proper Use of Betas

Estimation of Beta
Differences in Estimation of Beta
- Length of the Measurement Period
- Frequency of Data Measurement
- Choice of Market Index
- Choice of Risk-free Rate

Levered and Unlevered Betas
Modified Betas: Shrunk and Lagged
- Adjusted Beta Incorporates Industry Norm (Shrunk Beta)
- "Sum Beta" Incorporates Lag Effect

Summary

As discussed in the previous chapter, the beta is used as a modifier to the equity risk premium in the context of the Capital Asset Pricing Model (CAPM). The purpose of this chapter is to explore some widely used variations in the construction and application of betas.

ESTIMATION OF BETA

Systematic risk is measured in CAPM by a factor called *beta*. Beta is a function of the relationship between the return on an individual security and the return on the market as measured by a broad market index such as the Standard & Poor's (S&P) 500 Index.

Beta often is measured by comparing the excess return on an individual security relative to the excess return on the market index. By *excess return*, we mean the total return (which includes both dividends and capital gains and losses) over and above the return available on a risk-free investment (e.g., U.S. Treasuries).

Theorists prefer to measure beta on the basis of excess returns rather than total returns. However, a comparison of measurements by the two choices show that, as a practical matter, it makes little difference. Ibbotson Associates uses excess returns in all its computations.

A common method of calculating beta is to compute the slope of the best-fit line between the (excess) return on the individual security and the (excess) return on the market. An example of this is shown in Exhibit 10.1.

Exhibit 10.1 Example of One Common Method for the Calculation of Beta

Month End, t[a]	Return on Security A[b]	Return on S&P Index[c]	Calculated Covariance[d]	Calculated Variance[e]
1/89	0.041	0.069	0.00211	0.00325
2/89	(0.007)	(0.029)	0.00045	0.00168
3/89	0.052	0.021	0.00043	0.00008
•				
•				
•				
10/98	0.113	0.077	0.00709	0.00423
11/98	0.033	0.057	0.00131	0.00203
12/98	(0.016)	0.055	(0.00086)	0.00185
Sum	0.500	1.488	0.21060	0.26240
Average	0.004	0.012	0.00176[f]	0.00219[g]

$$\text{Beta} = \frac{\text{Covariance (Security A, S\&P Index)}}{\text{Variance of S \& P Index}} = \frac{0.00176}{0.00219} = 0.80$$

a. 10 years or 120 months.

b. Returns based on end-of-month prices and dividend payments (versus quarterly or annually).

c. Returns based on end-of-month S&P Index.

d. Values in this column are calculated as:

(Observed return on Security A – Average return on Security A) × (Observed return on S&P Index – Average return on S&P Index)

0.00211 = [(0.041 – 0.004) × (0.069 – 0.012)]

e. Values in this column are calculated as:

(Observed return on S&P Index – Average return on S&P Index)^2 0.00325 = (0.069 – 0.012)^2

f. The average of this column is the covariance between Security A and the S&P Index.

g. The average of this column is the variance of return on the S&P Index.

Source: Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. (New York: McGraw-Hill, 2000), 165. Reprinted with permission.

Because we cannot compute a beta directly for a privately held company, as noted in Chapter 9, we go to reference sources to obtain betas for guideline public companies or industries to determine a proxy beta for our private company. Details on these sources can be found in Appendix C.

DIFFERENCES IN ESTIMATION OF BETA

Be aware that significant differences exist between betas for the same stock published by different financial reporting services. One of the implications of this fact is that betas for guideline companies used in a valuation should all come from the same source. If all betas for guideline companies are not available from a single source, the best solution is probably to use the source providing betas for the greatest number of guideline companies and not use betas given for the others. Otherwise an "apples and oranges" mixture will result.

Differences in the beta measurement derive from choices within four variables:

1. The length of the total time period over which the returns are measured
2. The periodicity (frequency) of measurement within that time period
3. The choice of an index to use as a market proxy
4. The risk-free rate above which the excess returns should be measured

In addition to how these four variables are treated, adjustments can be made to recognize the beta's tendency to adjust toward 1.0. These adjustments are discussed later in the chapter.

Length of the Measurement Period

Most services that calculate beta use a two- to five-year measurement period, with five years being the most common. The Ibbotson Associates *Beta Book* uses 60 months for most stocks but includes a beta based on as few as 36 months if data are available for only this length of time.

Frequency of Data Measurement

Data may be measured daily, weekly, monthly, quarterly, or annually. Monthly is the most common frequency, although Value Line uses five years of weekly data.

Choice of Market Index

The market index used in calculating beta could be any of the following or, in some cases, another index:

- Standard & Poor's (S&P) 500 Index
- New York Stock Exchange (NYSE) Composite Index
- NYSE and American Stock Exchange (AMEX)
- NYSE, AMEX, and over-the-counter (OTC)
- Value Line Index

For an index to be representative of the market, it must be market-capitalization weighted. That is, the weight for each company in the index is determined by the market value of its equity. The sizes of the companies in the S&P 500 Index are so great that the index comprises about 70% of the total capitalization of all of the stock constituting the combined indexes listed here. Furthermore, the broader market indexes shown above correlate almost perfectly with the S&P 500 Index.

As a result, it does not make a great deal of difference which index is used. Ibbotson uses the NYSE in its calculations for the *Cost of Capital Yearbook* and the *Beta Book*.

Choice of Risk-free Rate

To avoid the maturity risk (interest rate risk) inherent in long-term bonds, the risk-free rate used to compute excess returns generally is either the Treasury-bill rate or the income return only from Treasury bonds. Ibbotson uses the 30-day Treasury bill in its calculations for the *Cost of Capital Yearbook* and the *Beta Book*.

A list of sources for betas is included in Appendix C.

LEVERED AND UNLEVERED BETAS

Published betas for publicly traded stocks typically reflect the capital structure of each respective company. These betas are sometimes referred to as *levered betas*, betas reflecting the leverage in the company's capital structure. If the leverage of the company subject to appraisal differs significantly from the leverage of the guideline companies selected for analysis, or if the debt levels of the guideline companies differ significantly from one another, it may be desirable to remove the effect that leverage has on the betas before using them as a proxy to estimate the beta of the subject.

When the firm's beta is measured based on observed historical total returns (as most betas are), its measurement necessarily includes volatility related to the company's financial risk. In particular, the equity of companies with higher levels of debt is riskier than the equity of companies with less leverage (all else being equal). In other words, levered betas incorporate two risk factors that bear on systematic risk: business risk and financial (or capital structure) risk.

This adjustment for leverage differences is performed by first computing unlevered betas for the guideline companies. An *unlevered beta* is the beta a company would have if it had no debt. The second step is to decide where the subject company's risk would fall on an unlevered basis relative to the guideline companies. The third and final step is to relever the beta for the subject company on the basis of one or more assumed capital structures. The result will be a market-derived beta specifically adjusted for the degree of financial leverage of the subject company.

To summarize, the three steps are:

1. Compute an unlevered beta for each of the guideline companies.
2. Decide where the risk would fall for the subject company relative to the guideline companies, assuming all had 100% equity capital structures.
3. Relever the beta for the subject company based on one or more assumed capital structures.

The formulas and an example of this process are shown in Exhibit 10.2. Of course, this leverage adjustment procedure takes as given all the assumptions of the Capital Asset Pricing Model.

With respect to levered versus unlevered betas, the capital structure of companies often can change significantly over the measurement period of the beta. For example, a beta could be measured during a five-year period in which, for the majority of time, a company was unleveraged. If at the end of the five-year measurement period the company has become highly leveraged, the levered betas computed would incorporate very little leverage. Yet in unlevering the beta, the analyst would incorporate the current level of high leverage. Thus the unlevered beta could be highly underestimated. The reverse effect applies for a company that deleverages during the beta measurement period. There is no specific method of correcting for this other than accounting for capital structure changes when unlevering the beta. A reasonable approach might be to determine the average leverage for the company during the beta measurement period rather than the leverage at the end of the measurement period.

Capital structures for both the guideline companies and the subject companies are assumed to be based on market values in this process. If the relevered beta is used to estimate the market value of a company on a controlling basis, and if it is anticipated that the actual capital structure will be adjusted to the proportions of debt and equity in the assumed capital structure, then only one assumed capital structure is necessary. However, if the amount of debt in the subject capital structure will *not* be adjusted, an iterative process may be required. The initial assumed capital structure for the subject will influence the cost of equity, which will, in turn, influence the relative proportions of debt and equity at market value. It may be necessary to try several assumed capital structures until one of them produces an estimate of equity value that actually results in the assumed capital structure.

This process of unlevering and relevering betas to an assumed capital structure is based on the assumption that the subject business interest has the ability to change the capital structure of the subject company. In the case of the valuation of a minority ownership interest, for example, the subject business interest may not have that ability.[1]

Exhibit 10.2 Computing Unlevered and Relevered Betas

The following is the formula for computing an unlevered beta (a beta assuming 100% equity in the capital structure):

$$B_u = \frac{B_L}{1 + (1 - t)W_d \,/\, W_e}$$

where:

B_u = Beta unlevered
B_L = Beta levered
t = Tax rate for the company
W_d = Percent debt in the capital structure
W_e = Percent equity in the capital structure

Example

Assume that for guideline company A:
Levered (published) beta: 1.2
Tax rate: 0.40
Capital structure: 30% debt, 70% equity

$$\begin{aligned} B_u &= \frac{1.2}{1 + (1 - 0.40)0.30 / 0.70} \\ &= \frac{1.2}{1 + 0.60(0.429)} \\ &= \frac{1.2}{1.257} \\ &= 0.95 \end{aligned}$$

Assume you made the previous calculation for all the guideline companies, the average unlevered beta was 0.90, and you believe the riskiness of your subject company, on an unlevered basis, is about equal to the average for the guideline companies. The next step is to relever the beta for your subject company based on its tax rate and one or more assumed capital structures. The formula to adjust an unlevered beta to a levered beta is:

$$B_L = B_u(1 + (1 - t)W_d/W_e)$$

where the definitions of the variables are the same as in the formula for computing unlevered betas.

(continued)

Exhibit 10.2 (Continued)

Example

Assume for the subject company:

Unlevered beta: 0.90

Tax rate: 0.30

Capital structure: 60% debt, 40% equity

$$\begin{aligned} B_L &= 0.90(1 + (1 - 0.30)0.60/0.40) \\ &= 0.90(1 + 0.70(1.5)) \\ &= 0.90(2.05) \\ &= 1.85 \end{aligned}$$

Source: Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. (New York: McGraw-Hill, 2000), 169. Reprinted with permission.

Appendix E discusses the use of the iterative process using CAPM for estimating the cost of equity, including the calculation of a relevered beta.

MODIFIED BETAS: SHRUNK AND LAGGED

Several research studies have provided significant support for two interesting hypotheses regarding betas:

1. *Tendency toward industry average.* Over time, a company's beta tends toward its industry's average beta. The higher the standard error in the regression used to calculate the beta, the greater the tendency to move toward the industry average.
2. *The lag effect.* For all but the largest companies, the prices of individual stocks tend to react in part to movements in the overall market with a lag. The smaller the company, the greater the lag in the price reaction. Recognizing these phenomena, former Ibbotson Associates vice president and economist Paul D. Kaplan, himself a participant in some of the relevant studies, introduced new methodologies in the first 1997 *Beta Book* to reflect this latest research, but Ibbotson dropped it starting with the Second 2001 Edition.

Adjusted Beta Incorporates Industry Norm (Shrunk Beta)

The adjusted beta is computed by a rather sophisticated technique called *Vasicek Shrinkage*.[2] The general idea is that betas with the highest statistical standard errors are adjusted toward the industry average more than are betas with lower standard errors. Because high-beta stocks also tend to have the highest standard errors in their betas, they tend to be subject to the most adjustment toward their industry average.

"Sum Beta" Incorporates Lag Effect

Because of the lag in all but the largest companies' sensitivity to movements in the overall market, traditional betas tend to understate systematic risk. As the First 2001 Edition of the *Beta Book* explains it, "Because of nonsynchronous price reactions, the traditional betas estimated by ordinary least squares are biased down for all but the largest companies."[3] As noted above, however, the Second 2001 Edition discontinued presentation of sum betas.

The research suggests that this understatement of systematic risk by the traditional beta measurements accounts in part, but certainly not wholly, for the fact that small stocks achieve excess returns over their apparent Capital Asset Pricing Model-required returns (where the market equity risk premium is adjusted for beta).

A *sum beta* consists of a multiple regression of a stock's current month's excess returns over the 30-day Treasury-bill rate on the market's current month's excess returns and on the market's previous month's excess returns, and then a summing of the coefficients. This helps to capture more fully the lagged effect of comovement in a company's returns with returns on the market (systematic risk).[4]

Exhibit 10.3 is an excerpt from Ibbotson's Second 2001 Edition *Beta Book* (which is published twice annually). Note that it includes a traditional least squares regression beta and the Fama-French three-factor models.

Chapter 13 on using Ibbotson data shows an entire sample page from the Second 2001 Edition *Beta Book*.

The *2002 SBBI Valuation Edition* has a table (Table 7-7) titled "Long-term Return in Excess of CAPM for Decile Portfolios of the NYSE/AMEX/NASDAQ, with Sum Beta."[5] The returns in excess of CAPM are much lower than for the ordinary least squares (OLS) betas, reflecting the superiority of sum betas over OLS betas. Graph 7-4 on the same page shows how much closer the portfolios track the Security Market Line, except for the tenth decile.[6] If sum betas are used, the size premium in excess of CAPM is greatly reduced.

Sum betas for individual stocks can be calculated using an Excel spreadsheet and 61 months of performance data, which is available from several sources, such as COMPUSTAT. Thus, even though the sum betas have been removed from the *Beta Book*, some analysts prefer to calculate their own sum betas for a peer group of public companies (which they use as a proxy for the beta of their subject private company in the context of CAPM), and thus make a smaller adjustment for the size effect. The theory is that this corrects for the larger size effect that is principally due to a misspecification of beta when using traditional OLS betas for the smaller companies.

SUMMARY

A *beta* is a measure of the sensitivity of the movement in returns on a particular stock to movements in returns on some measure of the market. As such, beta measures systematic risk. In cost of capital estimation, beta is used as a modifier to the general equity risk premium in using the Capital Asset Pricing Model.

Exhibit 10.3 Excerpt from Second 2001 Edition *Beta Book*

		CAPM: Ordinary Least Squares							Fama-French Three-Factor Model						
		Levered					Unlevered								
Ticker	Company	Raw Beta	t-Stat	R-Sqr	Pr Grp Beta	Ibbotson Beta	Raw Beta	Ibbotson Beta	FF Beta	FF t-Stat	SMB Prem	SMB t-Stat	HML Prem	HML t-Stat	FF R-Sqr
OTL	OCTEL CORP*	0.44	1.11	0.03	0.66	0.46	0.29	0.22	0.79	1.93	1.58	3.29	3.45	9.63	0.16
OCLR	OCULAR SCIENCES INC*	0.56	1.20	0.03	0.79	0.60	0.56	0.59	1.02	2.12	1.75	3.07	4.59	10.39	0.15
OCN	OCWEN FINANCIAL CORP*	0.85	2.20	0.08	0.97	0.86	0.69	0.63	1.18	2.86	1.03	2.06	3.39	8.48	0.15
ODETA	ODETICS INC -CL A	1.22	1.92	0.06	1.77	1.34	0.85	0.77	0.61	1.03	3.18	4.40	–6.54	–11.20	0.33
ODWA	ODWALLA INC	–0.13	–0.32	0.00	0.58	–0.05	–0.13	–0.05	–0.13	–0.31	2.81	5.32	–0.24	–0.57	0.11
ODP	OFFICE DEPOT INC	1.30	3.26	0.16	0.92	1.26	1.09	1.01	1.46	3.52	3.46	6.87	1.39	3.43	0.26
OMX	OFFICEMAX INC	1.11	3.03	0.14	0.93	1.09	0.88	0.76	1.45	3.98	4.18	9.47	3.21	9.02	0.31
OLOG	OFFSHORE LOGISTICS	0.97	2.49	0.10	0.99	0.97	0.71	0.61	0.98	2.28	–1.07	–2.04	0.22	0.53	0.11
OGE	OGE ENERGY CORP	0.02	0.14	0.00	0.12	0.02	0.01	0.01	0.18	1.29	–0.45	–2.57	1.73	12.42	0.25
OGLE	OGLEBAY NORTON CO	0.49	1.73	0.05	0.58	0.50	0.28	0.13	0.80	2.80	3.02	8.66	2.99	10.64	0.22
OCAS	OHIO CASUALTY CORP	0.60	1.88	0.06	0.75	0.61	0.50	0.48	0.72	2.02	0.23	0.53	1.17	3.36	0.07
OVBC	OHIO VALLEY BANC CORP	0.08	0.55	0.01	0.97	0.10	0.06	0.06	0.13	0.76	0.14	0.72	0.45	2.79	0.01
ODC	OIL DRI CORP AMERICA	0.26	1.06	0.02	1.02	0.30	0.18	0.15	0.57	2.21	1.32	4.20	3.11	12.25	0.15
OLGR	OILGEAR CO	0.06	0.18	0.00	1.60	0.19	0.04	0.08	0.41	1.11	2.54	5.69	3.36	9.35	0.10
ODFL	OLD DOMINION FREIGHT	0.00	0.01	0.00	0.66	0.03	0.00	0.01	0.12	0.47	1.93	6.12	1.08	4.26	0.10
OLDB	OLD NATIONAL BANCORP	0.56	3.22	0.15	0.97	0.57	0.38	0.29	0.59	3.10	–0.51	–2.21	0.36	1.93	0.18
ORI	OLD REPUBLIC INTL CORP	0.52	2.16	0.07	0.75	0.53	0.49	0.49	0.65	2.54	–0.48	–1.52	1.45	5.77	0.14
OSBC	OLD SECOND BANCORP INC/IL	0.19	1.30	0.03	0.97	0.21	0.17	0.18	0.35	2.23	0.81	4.24	1.56	10.15	0.12
OLN	OLIN CORP	0.69	2.10	0.07	0.90	0.71	0.56	0.54	0.85	2.33	1.12	2.53	1.50	4.22	0.09
OLY	OLYMPIC CASCADE FINL	1.23	1.92	0.06	1.84	1.37	1.11	1.21	1.17	1.69	2.46	2.91	–0.76	–1.12	0.10

*Company with less than 60 months' data (minimum 36 months).

Source: Ibbotson Associates, *Beta Book*, Second 2001 Edition (Chicago: Ibbotson Associates, 2001), 3.

There are many variations on the way betas are measured by different sources of published betas. Thus, a beta for a stock computed by one source may be very different from a beta computed for the same stock by another source.

Modern research is attempting to improve betas. Two such improvements implemented are the "shrunk beta," which blends the individual stock beta with the industry beta, and the "lagged beta," also called the "sum beta," which blends the beta for the stock and the market during a concurrent time period with a beta regressed on the market's previous period returns. These two adjustments both help to reduce "outliers," thus perhaps making the betas based on observed historical data a little more representative of future expectations. The size premium in excess of CAPM is much lower using sum betas.

Notes

1. This section has assumed beta of debt is zero. Actually, debt tends to have betas of about 0.2 to 0.3, slightly alleviating the unlevered/levered beta differential.
2. The formula, used in the Ibbotson *Beta Book*, was first suggested by Oldrich A. Vasicek, "A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Prices," *Journal of Finance* (1973). The company beta and the peer group (industry) beta are weighted. The greater the statistical confidence in the company beta, the greater the weight on the company beta relative to the peer group beta.
3. Ibbotson Associates, *Beta Book,* Second 2001 Edition (Chicago: Ibbotson Associates, 2001), vi.
4. The sum beta estimates conform with the expectation that betas are higher for lower capitalization stocks. Research also shows that sum betas are positively related to subsequent realized returns over a long period of time; see Roger G. Ibbotson, Paul D. Kaplan, and James D. Peterson, "Estimates of Small-Stock Betas Are Much Too Low," *Journal of Portfolio Management* (summer 1997).
5. Ibbotson Associates, *Stocks, Bonds, Bills and Inflation, Valuation Edition 2002 Yearbook* (Chicago: Ibbotson Associates, 2002), 129.
6. Ibid.

Chapter 11

Size Effect

Ibbotson Associates Studies

Standard & Poor's Corporate Value Consulting Studies (formerly the PricewaterhouseCoopers Studies)

Twenty-five Size Categories, Eight Measures of Size

Use of Measures of Risk from Company Financial Statements

Extension of Data to Smaller Size Categories: Results from the *Pratt's Stats*™ Database

Summary

In the chapters on the build-up and the Capital Asset Pricing Model (CAPM) cost of equity estimation models, we have made reference to the "size effect," the general idea that smaller size is associated with higher risk and, therefore, higher cost of capital. To help quantify the size effect in terms of its impact on cost of equity capital, this chapter presents empirical data from two independent sets of studies:

1. Ibbotson Associates Studies
2. Standard & Poor's Corporate Value Consulting Studies (formerly the PricewaterhouseCoopers studies)

Both of these sets of studies use rate of return data developed at the University of Chicago Center for Research in Security Prices (CRSP).

In addition, this chapter presents a comparative valuation multiple study of small companies from data on *Pratt's Stats*™, a database of private company sales.

IBBOTSON ASSOCIATES STUDIES

For many years, Ibbotson Associates has broken down New York Stock Exchange (NYSE) stock returns into quintiles by size, as measured by the aggregate market value of the common equity. Recently Ibbotson has further refined the breakdowns into decile groups. The excess returns over the basic general equity risk premium increase dramatically with decreasing size, as shown in Exhibit 11.1. This excess return is especially noticeable for the smallest 10% of the companies. Exhibit 11.2 shows the market capitalization by value of company equity of the largest company in each of the respective decile groups as of September 30, 2000.

Exhibit 11.1 Long-term Returns in Excess of CAPM Estimation for Decile Portfolios of the NYSE/AMEX/NASDAQ, with Tenth Decile Split 1926–2000

	Beta	Arithmetic Mean Return	Realized Return in Excess of Riskless Rate	Estimated Return in Excess of Riskless Rate	Size Premium (Return in Excess of CAPM)
1-Largest	0.91	12.06%	6.84%	7.03%	–0.20%
2	1.04	13.58%	8.36%	8.05%	0.31%
3	1.09	14.16%	8.93%	8.47%	0.47%
4	1.13	14.60%	9.38%	8.75%	0.62%
5	1.16	15.18%	9.95%	9.03%	0.93%
6	1.18	15.48%	10.26%	9.18%	1.08%
7	1.24	15.68%	10.46%	9.58%	0.88%
8	1.28	16.60%	11.38%	9.91%	1.47%
9	1.34	17.39%	12.17%	10.43%	1.74%
10a	1.43	19.11%	13.89%	11.10%	2.78%
10b-Smallest	1.41	24.56%	19.33%	10.91%	8.42%
Mid-Cap, 3–5	1.12	14.46%	9.23%	8.65%	0.58%
Low-Cap, 6–8	1.22	15.75%	10.52%	9.45%	1.07%
Micro-Cap, 9–10	1.36	18.41%	13.18%	10.56%	2.62%

*Betas are estimated from monthly portfolio total returns in excess of the 30-day U.S. Treasury bill total return versus the S&P 500 total returns in excess of the 30-day U.S. Treasury bill, January1926–December 2000.

**Historical riskless rate is measured by the 75-year arithmetic mean income return component of 20-year government bonds (5.22 percent).

†Calculated in the context of the CAPM by multiplying the equity risk premium by beta. The equity risk premium is estimated by the arithmetic mean total return of the S&P 500 (12.98 percent) minus the arithmetic mean income return component of 20-year government bonds (5.22 percent) from 1926–2000.

Source: Stocks, Bonds, Bills and Inflation® Valuation Edition 2001 Yearbook (Chicago: Ibbotson Associates, 2001), p. 123,

More recently, the tenth decile has been further broken down into 10a and 10b, 10a being the top half of the tenth decile and 10b being the bottom half of the tenth decile. Exhibits 11.1 and 11.2 show the dramatic difference between the smallest 5% of companies and the next smallest 5%.[1]

From 1926 through 1981, Ibbotson's "small stock" group was composed of stocks making up the fifth quintile (i.e., ninth and tenth deciles) of the NYSE, ranked by capitalization (price times number of shares outstanding). From 1982 forward, the small stock return series is the total return achieved by the Dimensional Fund Advisors (DFA) Small Company 9/10 (for ninth and tenth deciles) Fund. The Fund is a market-value–weighted index of the ninth and tenth deciles of the NYSE, plus stock listed on

Exhibit 11.2 Size-Decile Portfolios of the NYSE/AMEX/NASDAQ, Largest Company and Its Market Capitalization by Decile September 30, 2000

Decile	Market Capitalization of Largest Company (in thousands)	Company Name
1-Largest	$524,351,578	General Electric Co.
2	10,343,765	National City Corp.
3	4,143,902	Reader's Digest Association Inc.
4	2,177,448	Engelhard Corp.
5	1,327,582	Price Communications Corp.
6	840,000	Student Loan Corp.
7	537,693	APAC Customer Services Inc.
8	333,442	IHOP Corp. New
9	192,598	SCPIE Holdings Inc.
10-Smallest	84,521	Fibermark Inc.

Source: Center for Research in Security Prices, University of Chicago. *Stocks, Bonds, Bills, and Inflation® Valuation Edition 2001 Yearbook* (Chicago: Ibbotson Associates, 2001), p. 109,

the American Stock Exchange (AMEX) and over-the-counter (OTC) with the same or less capitalization than the upper bound of the NYSE ninth decile.

The Ibbotson data in the *Stocks, Bonds, Bills, and Inflation (SBBI) Valuation Edition Yearbook* show, for all size categories, both total realized returns in excess of the riskless rate and the size effect over and above CAPM (the latter having already accounted for beta, which tends to be higher for smaller stocks), so the data can be used either with a straight build-up model or with a CAPM model. They also show the average arithmetic mean return for each size category and the arithmetic average return on the Standard & Poor's (S&P) 500 Index, so the "small stock premium" could be derived by subtracting the difference between the two (a procedure that Ibbotson used to suggest). If using the data with a CAPM model, one would use the size effect over the CAPM-indicated equity risk premium, in which the beta would have captured some of the size effect.

In the build-up model, the applicable procedure is less clear-cut. Ibbotson now recommends starting with the return in excess of CAPM for *both* the build-up and CAPM models.

For the CAPM, there would be no further adjustment (except for a possible company-specific adjustment), because beta would presumably reflect any industry effects. In the build-up model, Ibbotson recommends starting with the return in excess of CAPM and then adding (or subtracting) an industry adjustment (which Ibbotson's *SBBI Valuation Edition Yearbook* presents for about 300 SIC codes).

However, not all practitioners have endorsed the latest procedure that Ibbotson recommends for the build-up model. Typical of the dissenting opinions is the following from Michael Mattson, a reviewer of this book and former Ibbotson employee:

> I am not in agreement with Ibbotson's contention that the only size premium to use is the one that is "beta adjusted." The problem in the build-up approach is that we have no place for a beta, so the aspect of size that is captured by a higher beta—an additional 0.4 over the market beta of 1.0 for 10th decile stocks—is not captured anywhere. Using the full size premium, as opposed to the beta-adjusted one, assumes that the small company being valued has similar risk characteristics to the average 10th decile company—this may not be such a bad assumption for many of the companies we value. Assuming that 10th decile companies are not in riskier industries than companies in the other size groupings, then their higher beta is due primarily to their size and the size effect is in both the beta *and* the premium over the CAPM line.
>
> I'm not persuaded that the industry premium would be anything other than another factor in the buildup model. The final problem with using just beta-adjusted size premium in the build-up model is that, absent an alpha, you would never come up with a discount rate as high as the historical average rate of return on small stocks. Is there any reason to believe that the differential between small and large stocks has actually shrunken over the last 75 years? Historically the difference between companies in the decile 1 and those in 10b have been 13.1%, yet the difference in their beta-adjusted size premia is only 9.4%. Why should our expectations be 3.7% lower than what we have seen historically, assuming we have no industry differences? I don't think there [has] been either a change in the way the data are calculated or any significant conceptual change that should cause us to change [our] thinking on this matter. I would be happy to be persuaded otherwise on this issue.

Further discussion of the use of the Ibbotson small stock data is included in Chapter 13 on using Ibbotson Associates data.

STANDARD & POOR'S CORPORATE VALUE CONSULTING STUDIES (FORMERLY THE PRICEWATERHOUSECOOPERS STUDIES)

Roger Grabowski and David King, formerly of PricewaterhouseCoopers (PwC), now with Standard & Poor's Corporate Value Consulting (S&P CVC), extended the study of the small stock phenomenon to encompass considerable additional detail. Their initial results were reported in the first edition of this book, and they have expanded their research significantly since then.

Twenty-five Size Categories, Eight Measures of Size

First, Grabowski and King broke down the New York Stock Exchange stocks into 25 size categories, plus a high-financial-risk category. This was done for eight different measures of size:

1. Market value of equity
2. Book value of equity

3. Five-year average net income
4. Market value of invested capital
5. Total assets (*Note:* Total assets replaced book value of invested capital as a measure of size in their original study.)
6. Five-year average earnings before interest, taxes, depreciation, and amortization (EBITDA)
7. Sales
8. Number of employees

The universe of stocks considered were those on both CRSP and Standard & Poor's COMPUSTAT database. The starting date of 1963 was selected because the COMPUSTAT database was established in 1963.

For each year since 1963, they filtered the universe to exclude:

- American Depository Receipts (ADRs)
- Nonoperating holding companies
- All financial companies (SIC code 6000 series)
- Companies lacking five years of publicly traded price history
- Companies with sales below $1 million in any of the previous five fiscal years
- Companies with a negative five-year EBITDA

They also excluded from the universe of stocks to be placed in portfolios based on size, those considered "high financial risk," and placed them in a separate portfolio. The high-financial-risk portfolio included companies with any one of these characteristics:

- Companies identified by COMPUSTAT as in bankruptcy or liquidation
- Companies with five-year average net income available to common stockholders less than zero (either in absolute terms or as a percentage of the book value of common equity)
- Companies with five-year average operating income after depreciation less than zero (either in absolute terms or as a percentage of net sales)
- Companies with negative book value of equity at any of the previous five fiscal year-ends
- Companies with debt-to-total capital of more than 80% (with debt measured in book value terms and total capital measured as book value of debt plus market value of equity)

For each year, they formed portfolios by sorting all of the companies in the base set that traded on the NYSE. The size cutoffs (or "breakpoints") were chosen so as to divide the NYSE companies evenly into 25 groups. Once the breakpoints were chosen, companies from the AMEX (available after 1962) and companies quoted on the Nasdaq Stock Market (Nasdaq) (available after 1972) were added to these portfolios.

Since NASDAQ and AMEX companies are generally small relative to NYSE companies, their addition to the data set produces portfolios that are more heavily populated with "small cap" stocks.

The portfolios were rebalanced annually. That is, the companies were reranked and sorted at the beginning of each year. Portfolio rates of return were calculated using an equal-weighted average of the companies in the portfolio, as opposed to market-value weighting.

A recent article by Tyler Shumway provided evidence that the CRSP database omits delisting returns for a large number of companies.[2] These returns are missing for the month in which a company is delisted from an exchange. Shumway collected data for a large number of companies that had been delisted for performance reasons (e.g., bankruptcy or insufficient capital). He found that investors incurred an average loss of about 30% after delisting. He further showed that delisting for nonperformance reasons (e.g., mergers or changes of exchange) tended to have a neutral impact in the month that the delisting occurred.

Grabowski and King have incorporated Shumway's evidence into their rate-of-return calculations. In calculating rates of return, they have imputed a 30% loss in the month of delisting in all cases in which CRSP identified the reason for delisting as performance related and also in all cases in which the reason for delisting was unknown.

To estimate equity risk premiums, they first calculated an average rate of return for each portfolio over our sample period. Then they subtracted the average income return earned on long-term Treasury bonds over the same period (using Ibbotson's *SBBI* data) to arrive at an average equity premium.

A summary of the results is presented in Exhibit 11.3.

The S&P CVC data cover the years 1963 (the first year of Compustat data) through the present, as compared with 1926 through the present for the Ibbotson data. Two results of the S&P CVC studies seem strikingly significant:

1. In spite of the different time period, the average results are very close to the Ibbotson results.
2. The results are significantly similar for all eight measures of company size.

Although the market value of common equity has both the highest degree of statistical significance and the steepest slope when regressing average returns against size, all size measures show a high degree of statistical significance. This is quite convenient in the context of valuing private companies, since it enables the analyst to start with a known size measure rather than an estimated market value of equity, which is the value being sought.

Exhibit 11.3 shows both the actual premium for each size group and the smoothed premium. The smoothed premium is based on regression analysis. In most parts of the size range, the smoothed premium is probably most appropriate to use.

Note, however, that a pronounced jump exists in the premium in the smallest 4% of companies. This fact is of interest to many business valuators, since this jump occurs in a size category in which, as a practical matter, many more valuation assignments are performed. For seven of the eight size measures, the actual premium for the

Exhibit 11.3 Premiums over Long-term Riskless Rate

Historical Equity Risk Premiums: Averages Since 1963 — Summary Schedule (1 of 3)
Data for Year Ending December 31, 1999

	Market Value of Equity			Book Value of Equity			5-Year Average Net Income			Market Value of Invested Capital		
Portfolio Rank by Size	Average ($mils.)	Arithmetic Average Premium	Smoothed Average Premium	Average ($mils.)	Arithmetic Average Premium	Smoothed Average Premium	Average ($mils.)	Arithmetic Average Premium	Smoothed Average Premium	Average ($mils.)	Arithmetic Average Premium	Smoothed Average Premium
1	91,406	7.0%	3.2%	12,736	6.8%	4.6%	2,472	6.9%	4.4%	106,439	6.4%	3.3%
2	20,934	5.2%	5.0%	5,591	5.3%	5.6%	685	6.3%	5.9%	26,269	5.0%	5.1%
3	12,062	4.3%	5.7%	3,637	6.9%	6.2%	447	6.1%	6.4%	16,486	5.7%	5.6%
4	8,758	6.1%	6.1%	2,715	5.3%	6.5%	322	5.6%	6.8%	11,328	5.3%	6.1%
5	6,022	5.1%	6.5%	1,980	6.8%	6.9%	236	6.0%	7.1%	8,795	5.1%	6.4%
6	4,730	6.7%	6.8%	1,578	7.8%	7.2%	173	8.9%	7.5%	6,329	8.1%	6.9%
7	3,744	6.6%	7.1%	1,428	8.2%	7.3%	132	7.1%	7.8%	5,271	7.4%	7.1%
8	3,172	8.1%	7.3%	1,132	6.3%	7.6%	121	7.5%	7.9%	4,280	6.4%	7.3%
9	2,385	5.8%	7.7%	936	7.4%	7.9%	102	7.7%	8.1%	3,638	7.1%	7.6%
10	2,137	7.2%	7.8%	779	6.9%	8.1%	84	7.0%	8.4%	2,841	7.4%	7.9%
11	1,744	8.6%	8.1%	692	8.9%	8.3%	73	9.1%	8.5%	2,371	5.9%	8.1%
12	1,358	8.4%	8.4%	611	8.8%	8.4%	60	8.5%	8.8%	2,008	7.7%	8.3%
13	1,241	7.1%	8.5%	517	7.2%	8.6%	49	8.6%	9.0%	1,639	9.0%	8.6%
14	972	9.2%	8.8%	421	9.3%	8.9%	44	7.7%	9.1%	1,449	8.6%	8.7%
15	812	8.9%	9.0%	365	8.8%	9.1%	38	10.2%	9.3%	1,207	9.8%	8.9%
16	684	8.3%	9.2%	333	9.7%	9.2%	31	8.4%	9.5%	983	9.3%	9.2%
17	567	8.6%	9.4%	282	8.4%	9.4%	26	9.7%	9.7%	867	9.0%	9.4%
18	490	10.0%	9.6%	242	7.9%	9.6%	21	9.5%	10.0%	691	8.4%	9.7%
19	401	8.3%	9.9%	212	8.8%	9.7%	18	9.9%	10.1%	568	8.3%	9.9%
20	344	9.9%	10.1%	177	10.4%	10.0%	16	10.3%	10.3%	469	10.4%	10.1%
21	277	10.8%	10.3%	147	9.9%	10.2%	13	11.0%	10.5%	399	10.5%	10.3%
22	211	10.2%	10.7%	127	11.4%	10.4%	11	11.8%	10.7%	333	11.1%	10.6%
23	163	11.4%	11.0%	100	11.3%	10.7%	8	12.0%	11.1%	242	11.2%	11.0%
24	104	12.5%	11.5%	72	11.1%	11.1%	6	11.5%	11.5%	161	11.5%	11.5%
25	39	15.7%	12.8%	30	14.0%	12.2%	2	13.8%	12.7%	61	15.5%	12.7%
Constant			0.1729			0.1648			0.1356			0.1792
Slope			–0.0285			–0.0290			–0.0271			–0.0291

Summary Schedule (2 of 3)

Portfolio Rank by Size	Total Assets			5-Year Average EBITDA			Sales			Number of Employees		
	Average ($mils.)	Arithmetic Average Premium	Smoothed Average Premium	Average ($mils.)	Arithmetic Average Premium	Smoothed Average Premium	Average ($mils.)	Arithmetic Average Premium	Smoothed Average Premium	Average	Arithmetic Average Premium	Smoothed Average Premium
1	47,082	6.1%	4.5%	6,574	7.3%	4.9%	38,477	7.2%	5.9%	179,912	6.4%	6.0%
2	17,733	5.2%	5.7%	2,104	5.3%	6.2%	14,953	5.9%	6.8%	73,564	6.7%	6.8%
3	11,333	6.9%	6.2%	1,461	6.7%	6.6%	9,964	7.4%	7.1%	45,168	7.0%	7.2%
4	8,369	6.0%	6.6%	1,030	6.8%	7.0%	7,019	8.7%	7.5%	35,209	8.7%	7.5%
5	6,029	8.0%	7.0%	770	7.5%	7.3%	5,409	8.6%	7.7%	27,682	8.6%	7.7%
6	4,904	8.4%	7.2%	580	7.7%	7.6%	4,299	8.1%	7.9%	21,476	8.2%	7.9%
7	4,111	7.0%	7.4%	482	7.6%	7.8%	3,666	7.4%	8.1%	16,456	7.4%	8.1%
8	3,170	6.4%	7.7%	393	6.8%	8.0%	2,708	8.0%	8.4%	14,012	7.5%	8.3%
9	2,676	7.5%	7.9%	307	7.4%	8.3%	2,357	6.0%	8.5%	10,915	9.4%	8.5%
10	2,172	7.6%	8.2%	266	7.5%	8.5%	1,992	8.1%	8.6%	9,510	8.3%	8.6%
11	1,820	8.5%	8.4%	229	8.6%	8.6%	1,573	8.6%	8.9%	8,211	9.2%	8.7%
12	1,436	6.9%	8.7%	201	9.3%	8.8%	1,472	10.8%	8.9%	6,888	8.8%	8.9%
13	1,367	7.3%	8.7%	163	10.5%	9.0%	1,249	10.2%	9.1%	6,337	8.6%	9.0%
14	1,128	10.8%	9.0%	139	7.8%	9.2%	1,137	8.7%	9.2%	5,163	7.9%	9.1%
15	960	9.1%	9.2%	115	8.0%	9.4%	948	8.6%	9.3%	4,485	10.4%	9.3%
16	825	9.5%	9.3%	101	10.4%	9.6%	787	8.1%	9.5%	3,949	9.2%	9.4%
17	678	10.2%	9.6%	83	9.9%	9.8%	699	10.7%	9.6%	3,325	8.4%	9.5%
18	579	9.5%	9.8%	72	9.5%	10.0%	592	8.6%	9.8%	2,861	8.7%	9.7%
19	461	10.1%	10.0%	60	10.6%	10.1%	506	9.6%	9.9%	2,351	7.8%	9.8%
20	394	10.3%	10.2%	49	9.2%	10.4%	420	9.3%	10.1%	1,997	11.2%	10.0%
21	347	8.2%	10.4%	42	9.8%	10.5%	344	9.3%	10.3%	1,539	8.8%	10.2%
22	276	9.9%	10.6%	35	11.3%	10.8%	278	9.8%	10.5%	1,210	11.3%	10.4%
23	218	11.3%	10.9%	27	12.3%	11.1%	226	12.2%	10.7%	897	10.6%	10.7%
24	157	12.6%	11.3%	20	12.0%	11.4%	165	11.0%	11.0%	589	11.7%	11.0%
25	63	14.0%	12.4%	8	13.4%	12.5%	68	13.8%	11.8%	230	13.4%	11.9%
Constant			0.1732			0.1475			0.1573			0.1663
Slope			–0.0273			–0.0258			–0.0215			–0.0202

(continued)

Exhibit 11.3 (Continued)

Historical Equity Risk Premiums: Averages Since 1963 — Summary Schedule (3 of 3)
Data for Year Ending December 31, 1999

	Operating Income Margin			CV(Operating Income Margin)			CV(ROE)		
Portfolio Rank by Size	Average	Arithmetic Average Premium	Smoothed Average Premium	Average	Arithmetic Average Premium	Smoothed Average Premium	Average	Arithmetic Average Premium	Smoothed Average Premium
1	35.5%	6.8%	5.9%	164.4%	12.9%	13.1%	500.3%	11.8%	12.4%
2	28.6%	5.0%	6.6%	78.9%	10.8%	12.0%	181.5%	12.0%	11.3%
3	25.1%	6.0%	6.9%	58.2%	11.6%	11.5%	111.5%	11.7%	10.8%
4	22.1%	7.0%	7.3%	46.3%	10.8%	11.2%	82.7%	9.0%	10.5%
5	19.3%	8.0%	7.7%	38.7%	11.6%	10.9%	66.2%	9.6%	10.3%
6	17.3%	8.1%	8.0%	33.5%	9.7%	10.7%	55.2%	10.4%	10.1%
7	15.6%	8.4%	8.3%	29.8%	9.6%	10.5%	47.2%	10.2%	10.0%
8	14.2%	8.0%	8.6%	26.7%	12.7%	10.3%	41.2%	9.9%	9.8%
9	13.1%	9.3%	8.8%	24.0%	8.8%	10.2%	36.3%	10.1%	9.7%
10	12.2%	8.1%	9.0%	21.7%	11.1%	10.0%	32.1%	10.5%	9.6%
11	11.4%	9.0%	9.2%	19.7%	10.4%	9.9%	28.7%	10.7%	9.5%
12	10.7%	8.5%	9.4%	17.9%	9.2%	9.7%	25.7%	10.0%	9.3%
13	10.0%	10.6%	9.6%	16.2%	9.9%	9.6%	23.1%	9.8%	9.2%
14	9.5%	8.6%	9.8%	14.8%	10.8%	9.4%	20.8%	9.4%	9.1%
15	8.9%	9.9%	10.0%	13.5%	11.5%	9.3%	18.6%	8.8%	9.0%
16	8.3%	10.9%	10.1%	12.3%	7.4%	9.2%	16.6%	7.6%	8.9%
17	7.8%	12.4%	10.3%	11.1%	8.8%	9.0%	14.8%	7.4%	8.8%
18	7.3%	11.6%	10.5%	10.0%	9.2%	8.9%	13.0%	9.2%	8.6%
19	6.8%	11.1%	10.8%	9.0%	8.8%	8.7%	11.5%	8.0%	8.5%
20	6.2%	12.6%	11.0%	8.0%	8.6%	8.5%	10.1%	8.1%	8.4%
21	5.6%	12.6%	11.3%	7.0%	8.8%	8.3%	8.8%	8.1%	8.2%
22	5.0%	10.6%	11.7%	6.0%	7.6%	8.1%	7.5%	7.5%	8.1%
23	4.2%	12.7%	12.1%	5.0%	6.1%	7.8%	6.2%	6.8%	7.9%
24	3.3%	12.9%	12.8%	3.8%	7.4%	7.4%	4.7%	8.8%	7.6%
25	2.0%	11.9%	14.4%	2.4%	6.7%	6.7%	2.9%	7.5%	7.1%
Constant			0.0291			0.1232			0.1073
Slope			–0.0671			0.0347			0.0235

Source: Standard & Poor's Corporate Value Consulting (S&P CVC) Risk Premium Report 2000 (formerly the *PricewaterhouseCoopers Risk Premium Study 2000)*, 18–20. Updates to the *S&P CVC Risk Premium Report* are available at *http://valuation.ibbotson.com*.

smallest group was greater than the "smoothed" premium, generally by a considerable margin.

As a point of reference, the arithmetic risk premium from Ibbotson *SBBI* data, calculated since 1926, comes to 7.60% for large stocks (average of first and second deciles) and 18.41% for small stocks (ninth and tenth deciles combined), which would lead one to conclude that the data are quite compatible.

Note that the S&P CVC data shown in Exhibit 11.3 do not show small stock returns in excess of CAPM. Therefore, the data as shown in the exhibit are suitable to use with a straight build-up model but not with a CAPM model. The returns shown in the exhibit include the general equity risk premium. Therefore, analysts can approximate an equity cost of capital reflecting various size measures (before any company-specific adjustment) by adding the appropriate equity risk premium from the exhibit (or the current version at Ibbotson's Cost of Capital Center) to the 20-year Treasury-bond rate.

Using similar methodology, Grabowski and King computed "premiums over CAPM." (Recall that beta captures some, but not all, of the size premium.) These can be used with the Capital Asset Pricing Model. Summary results of this study through 1999 are shown as Exhibit 11.4.

Use of Measures of Risk from Company Financial Statements[3]

Grabowski and King also examined measures of risk derived from company financial statements:

- Operating margin (the lower the operating margin, the greater the risk)
- Coefficient of variation in operating margin (the greater the coefficient of variation, the greater the risk)
- Coefficient of variation in return on equity (the greater the coefficient of variation, the greater the risk)

Coefficient of variation is the standard deviation divided by the mean. It measures volatility relative to the average value of the variable under consideration. This normalizes for differences in the magnitude of the subject variables.

Grabowski and King report that "the three fundamental measures of risk are at least as closely correlated with size as is the equity risk premium."[4]

They constructed portfolios of companies ranked by the above risk measures in the same manner as by company size. The summary results are shown in Exhibit 11.5.

EXTENSION OF DATA TO SMALLER SIZE CATEGORIES: RESULTS FROM THE *PRATT'S STATS*™ DATABASE

The size effect studies presented in this chapter are based entirely on transactions in the public stock markets. The smallest 4% of the stocks in the S&P CVC study

Exhibit 11.4 Premiums over CAPM

Historical Equity Risk Premiums: Averages Since 1963 — Summary Schedule (1 of 2)
Data for Year Ending December 31, 1999

	Market Value of Equity			Book Value of Equity			5-Year Average Net Income			Market Value of Invested Capital		
Portfolio Rank by Size	Average ($mils.)	Premium Over CAPM	Smoothed Premium over CAPM	Average ($mils.)	Premium Over CAPM	Smoothed Premium over CAPM	Average ($mils.)	Premium Over CAPM	Smoothed Premium over CAPM	Average ($mils.)	Premium Over CAPM	Smoothed Premium over CAPM
1	91,406	1.1%	–2.0%	12,736	0.8%	–0.5%	2,472	0.8%	–0.6%	106,439	0.6%	–1.8%
2	20,934	–0.9%	–0.8%	5,591	–0.4%	0.1%	685	0.8%	0.2%	26,269	–0.9%	–0.7%
3	12,062	–1.4%	–0.4%	3,637	0.8%	0.4%	447	0.6%	0.5%	16,486	–0.5%	–0.3%
4	8,758	0.0%	–0.1%	2,715	–0.3%	0.6%	322	–0.1%	0.7%	11,328	–0.9%	0.0%
5	6,022	–1.3%	0.2%	1,980	1.0%	0.8%	236	–0.2%	1.0%	8,795	0.0%	0.2%
6	4,730	0.3%	0.4%	1,578	1.7%	1.0%	173	2.3%	1.2%	6,329	1.5%	0.4%
7	3,744	1.0%	0.6%	1,428	1.7%	1.1%	132	1.0%	1.4%	5,271	1.0%	0.6%
8	3,172	1.7%	0.7%	1,132	0.4%	1.2%	121	0.8%	1.4%	4,280	–0.1%	0.7%
9	2,385	–0.8%	0.9%	936	0.6%	1.4%	102	1.7%	1.5%	3,638	0.6%	0.9%
10	2,137	–0.1%	1.0%	779	0.3%	1.5%	84	0.3%	1.7%	2,841	0.8%	1.1%
11	1,744	1.5%	1.2%	692	2.3%	1.6%	73	2.7%	1.8%	2,371	–1.1%	1.2%
12	1,358	1.6%	1.4%	611	2.2%	1.7%	60	1.8%	1.9%	2,008	0.9%	1.4%
13	1,241	0.4%	1.5%	517	0.8%	1.8%	49	1.8%	2.0%	1,639	2.1%	1.5%
14	972	2.7%	1.6%	421	2.5%	1.9%	44	1.1%	2.1%	1,449	1.4%	1.6%
15	812	1.6%	1.8%	365	1.8%	2.0%	38	2.6%	2.2%	1,207	2.4%	1.8%
16	684	1.0%	1.9%	333	2.5%	2.1%	31	1.1%	2.3%	983	2.4%	1.9%
17	567	1.1%	2.1%	282	1.5%	2.2%	26	2.2%	2.4%	867	1.8%	2.0%
18	490	3.0%	2.2%	242	1.2%	2.3%	21	2.0%	2.6%	691	1.0%	2.2%
19	401	0.3%	2.4%	212	1.2%	2.4%	18	2.2%	2.7%	568	0.8%	2.4%
20	344	2.1%	2.5%	177	3.6%	2.6%	16	2.7%	2.8%	469	2.7%	2.5%
21	277	3.3%	2.7%	147	2.3%	2.7%	13	3.6%	2.9%	399	2.7%	2.6%
22	211	2.4%	2.9%	127	4.2%	2.8%	11	4.4%	3.0%	333	3.5%	2.8%
23	163	3.4%	3.1%	100	3.3%	3.0%	8	3.9%	3.2%	242	2.4%	3.0%
24	104	4.2%	3.5%	72	2.9%	3.2%	6	3.5%	3.5%	161	3.6%	3.4%
25	39	7.0%	4.3%	30	4.8%	3.8%	2	4.8%	4.2%	61	6.8%	4.1%
Constant			0.0723			0.0631			0.0466			0.0742
Slope			–0.0187			–0.0167			–0.0156			–0.0184

Portfolio Rank by Size	Total Assets Average ($mils.)	Total Assets Premium Over CAPM	Total Assets Smoothed Premium over CAPM	5-Year Average EBITDA Average ($mils.)	5-Year Average EBITDA Premium Over CAPM	5-Year Average EBITDA Smoothed Premium over CAPM	Sales Average ($mils.)	Sales Premium Over CAPM	Sales Smoothed Premium over CAPM	Number of Employees Average ($mils.)	Number of Employees Premium Over CAPM	Number of Employees Smoothed Premium over CAPM
1	47,082	0.1%	–0.7%	6,574	1.0%	–0.3%	38,477	0.8%	0.1%	179,912	–0.9%	–0.3%
2	17,733	–0.6%	0.0%	2,104	–0.1%	0.5%	14,953	–0.6%	0.6%	73,564	0.1%	0.3%
3	11,333	1.0%	0.3%	1,461	1.1%	0.7%	9,964	1.3%	0.9%	45,168	1.0%	0.6%
4	8,369	0.0%	0.5%	1,030	0.7%	0.9%	7,019	2.3%	1.0%	35,209	2.2%	0.8%
5	6,029	2.2%	0.8%	770	1.5%	1.1%	5,409	2.6%	1.2%	27,682	2.2%	0.9%
6	4.904	1.7%	0.9%	580	1.8%	1.3%	4,299	1.5%	1.3%	21,476	1.5%	1.1%
7	4,111	0.7%	1.0%	482	0.9%	1.4%	3,666	1.2%	1.4%	16,456	0.6%	1.3%
8	3,170	0.0%	1.2%	393	0.5%	1.5%	2,708	1.5%	1.6%	14,012	1.2%	1.4%
9	2,676	0.9%	1.3%	307	0.9%	1.7%	2,357	–1.1%	1.7%	10,915	2.3%	1.5%
10	2,172	0.8%	1.5%	266	1.3%	1.8%	1,992	1.4%	1.7%	9,510	1.1%	1.6%
11	1,820	1.6%	1.6%	229	2.0%	1.9%	1,573	1.1%	1.9%	8,211	1.9%	1.7%
12	1,436	0.2%	1.8%	201	2.3%	2.0%	1,472	3.6%	1.9%	6,888	1.7%	1.8%
13	1,367	1.1%	1.8%	163	3.4%	2.1%	1,249	3.6%	2.0%	6,337	1.5%	1.9%
14	1,128	3.6%	1.9%	139	1.0%	2.2%	1,137	1.8%	2.1%	5,163	0.9%	2.0%
15	960	2.7%	2.0%	115	1.0%	2.3%	948	1.8%	2.2%	4,485	3.5%	2.1%
16	825	2.4%	2.2%	101	3.2%	2.4%	787	0.8%	2.3%	3,949	1.8%	2.2%
17	678	2.7%	2.3%	83	3.0%	2.5%	699	3.3%	2.3%	3,325	1.2%	2.3%
18	579	2.4%	2.4%	72	2.1%	2.6%	592	1.4%	2.4%	2,861	1.5%	2.4%
19	461	2.9%	2.6%	60	3.0%	2.7%	506	2.1%	2.5%	2,351	0.4%	2.5%
20	394	2.2%	2.7%	49	1.7%	2.9%	420	2.2%	2.6%	1,997	3.8%	2.6%
21	347	0.8%	2.8%	42	2.5%	3.0%	344	2.2%	2.7%	1,539	1.5%	2.8%
22	276	2.2%	2.9%	35	4.0%	3.1%	278	2.2%	2.8%	1,210	3.9%	3.0%
23	218	3.6%	3.1%	27	4.0%	3.3%	226	3.8%	3.0%	897	3.2%	3.2%
24	157	3.8%	3.3%	20	3.9%	3.4%	165	3.2%	3.1%	589	4.2%	3.4%
25	63	5.2%	4.0%	8	4.4%	4.1%	68	5.1%	3.6%	230	4.8%	4.0%
Constant			0.0687			0.0537			0.0597			0.0754
Slope			–0.0162			–0.0147			–0.0128			–0.0149

Source: Standard & Poor's Corporate Value Consulting (S&P CVC) Risk Premium Report 2000 (formerly the *PricewaterhouseCoopers Risk Premium Study 2000*), 21–22. Updates to the *S&P CVC Risk Premium Report* are available at *http://valuation.ibbotson.com*.

Exhibit 11.5 Companies Ranked by Measure of Risk

Companies Ranked by Operating Margin
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 1999

Portfolio Rank by Size	Median Operating Margin	Log of Median Op Margin	Number as of 1999	Beta Annual Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Equity Risk Premium	*Smoothed Average Equity Risk Premium*	Average Debt/ MVIC
1	35.5%	–0.45	84	0.88	17.53%	13.02%	14.30%	6.80%	*5.93%*	32.30%
2	28.6%	–0.54	69	0.82	14.93%	11.48%	12.47%	4.97%	*6.56%*	36.74%
3	25.1%	–0.60	58	0.88	15.33%	12.37%	13.46%	5.96%	*6.94%*	33.44%
4	22.1%	–0.66	76	0.97	17.22%	13.21%	14.54%	7.04%	*7.31%*	26.68%
5	19.3%	–0.71	70	1.01	18.33%	14.00%	15.47%	7.97%	*7.70%*	22.57%
6	17.3%	–0.76	83	1.03	18.63%	14.04%	15.57%	8.07%	*8.03%*	19.39%
7	15.6%	–0.81	73	1.07	19.71%	14.25%	15.91%	8.41%	*8.32%*	19.40%
8	14.2%	–0.85	72	1.06	19.88%	13.73%	15.53%	8.03%	*8.59%*	20.25%
9	13.1%	–0.88	66	1.15	20.92%	15.00%	16.82%	9.32%	*8.83%*	21.34%
10	12.2%	–0.91	61	1.13	20.97%	13.66%	15.57%	8.07%	*9.04%*	21.87%
11	11.4%	–0.94	71	1.23	23.24%	14.15%	16.45%	8.95%	*9.23%*	22.89%
12	10.7%	–0.97	69	1.18	23.45%	13.63%	15.96%	8.46%	*9.42%*	23.44%
13	10.0%	–1.00	78	1.20	23.62%	15.69%	18.10%	10.60%	*9.61%*	24.00%
14	9.5%	–1.02	89	1.18	22.82%	13.88%	16.12%	8.62%	*9.78%*	23.92%
15	8.9%	–1.05	80	1.20	23.70%	15.04%	17.44%	9.94%	*9.96%*	25.95%
16	8.3%	–1.08	85	1.23	24.51%	15.72%	18.36%	10.86%	*10.15%*	27.59%
17	7.8%	–1.11	98	1.23	25.11%	17.32%	19.91%	12.41%	*10.34%*	28.30%
18	7.3%	–1.14	92	1.32	27.08%	16.11%	19.14%	11.64%	*10.54%*	28.88%
19	6.8%	–1.17	97	1.15	23.98%	16.13%	18.56%	11.06%	*10.76%*	29.97%
20	6.2%	–1.21	98	1.28	27.28%	17.04%	20.09%	12.59%	*11.00%*	32.24%
21	5.6%	–1.25	101	1.26	26.21%	17.23%	20.05%	12.55%	*11.29%*	32.27%
22	5.0%	–1.30	84	1.27	26.33%	15.14%	18.08%	10.58%	*11.66%*	32.81%
23	4.2%	–1.37	135	1.19	26.02%	17.41%	20.20%	12.70%	*12.14%*	34.87%
24	3.3%	–1.48	103	1.29	29.77%	16.80%	20.35%	12.85%	*12.83%*	34.80%
25	2.0%	–1.71	169	1.27	28.86%	16.08%	19.38%	11.88%	*14.38%*	34.19%
High financial risk			644	1.50	37.27%	14.97%	20.24%	12.74%		49.68%
Large Stocks (Ibbotson SBBI data)						12.81%	13.92%	6.42%		
Small Stocks (Ibbotson SBBI data)						15.36%	18.09%	10.59%		
Long-term Treasury Income (Ibboston SBBI data)						7.48%	7.50%			

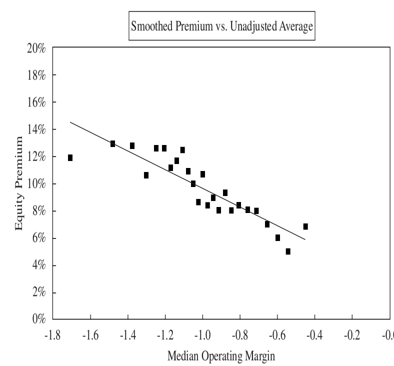
Equity Risk Premium Study: Data through December 31, 1999

Data Smoothing with Regression Analysis
Dependent Variable: Average Premium
Independent Variable: Log of Median Operating Margin

Regression Output:

Constant		2.907%
Std Err of Y Est		1.086%
R Squared		78%
No. of Observations		25
Degrees of Freedom		23
X Coefficient(s)	–6.714%	
Std Err of Coef.	0.741%	
t-Statistic	–9.06	

*Smoothed Premium = 2.907% – 6.714% * Log(Operating Margin)*



Companies Ranked by CV (Operating Margin)
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 1999

Portfolio Rank by Size	Median CV(OpInc%)	Log of Median CV(OpInc%)	Number as of 1999	Beta Annual Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	Arithmetic Equity Risk Premium	*Smoothed Average Equity Risk Premium*	Average Debt/ MVIC
1	164.4%	0.22	220	1.39	33.56%	16.11%	20.44%	12.94%	*13.07%*	28.63%
2	78.9%	–0.10	127	1.22	27.65%	15.14%	18.34%	10.84%	*11.96%*	30.93%
3	58.2%	–0.24	99	1.39	32.22%	15.20%	19.11%	11.61%	*11.50%*	31.67%
4	46.3%	–0.33	112	1.25	27.00%	15.21%	18.28%	10.78%	*11.16%*	30.89%
5	38.7%	–0.41	95	1.33	26.87%	16.12%	19.13%	11.63%	*10.89%*	29.44%
6	33.5%	–0.47	97	1.27	24.59%	14.54%	17.17%	9.67%	*10.67%*	29.02%
7	29.8%	–0.53	98	1.20	25.06%	14.42%	17.10%	9.60%	*10.49%*	29.64%
8	26.7%	–0.57	94	1.22	25.12%	17.60%	20.24%	12.74%	*10.33%*	28.72%
9	24.0%	–0.62	77	1.20	23.66%	13.84%	16.25%	8.75%	*10.17%*	28.15%
10	21.7%	–0.66	89	1.21	23.68%	16.22%	18.57%	11.07%	*10.01%*	28.69%
11	19.7%	–0.71	68	1.13	22.05%	15.83%	17.89%	10.39%	*9.87%*	27.82%
12	17.9%	–0.75	78	1.11	21.72%	14.63%	16.66%	9.16%	*9.73%*	27.13%
13	16.2%	–0.79	66	1.14	21.85%	15.38%	17.44%	9.94%	*9.58%*	28.56%
14	14.8%	–0.83	67	1.16	22.61%	16.22%	18.34%	10.84%	*9.44%*	27.21%
15	13.5%	–0.87	79	1.17	22.15%	16.88%	18.96%	11.46%	*9.30%*	26.39%
16	12.3%	–0.91	87	1.00	18.59%	13.37%	14.85%	7.35%	*9.16%*	26.67%
17	11.1%	–0.95	80	1.03	19.19%	14.67%	16.29%	8.79%	*9.01%*	26.02%
18	10.0%	–1.00	64	1.06	19.76%	15.01%	16.73%	9.23%	*8.86%*	26.61%
19	9.0%	–1.04	64	1.03	18.96%	14.69%	16.27%	8.77%	*8.69%*	25.76%
20	8.0%	–1.10	81	0.99	17.94%	14.66%	16.08%	8.58%	*8.52%*	25.20%
21	7.0%	–1.15	73	0.92	16.86%	14.99%	16.26%	8.76%	*8.31%*	25.84%
22	6.0%	–1.22	61	0.99	17.76%	13.74%	15.12%	7.62%	*8.08%*	26.55%
23	5.0%	–1.30	65	0.87	15.62%	12.44%	13.57%	6.07%	*7.79%*	27.25%
24	3.8%	–1.42	61	0.90	16.33%	13.68%	14.86%	7.36%	*7.40%*	26.47%
25	2.4%	–1.63	59	0.90	16.54%	12.98%	14.18%	6.68%	*6.67%*	24.67%
High financial risk			643	1.51	37.39%	15.04%	20.33%	12.83%		49.67%
Large Stocks (Ibbotson SBBI data)						12.81%	13.92%	6.42%		
Small Stocks (Ibbotson SBBI data)						15.36%	18.09%	10.59%		
Long-term Treasury Income (Ibbotson SBBI data)						7.48%	7.50%			

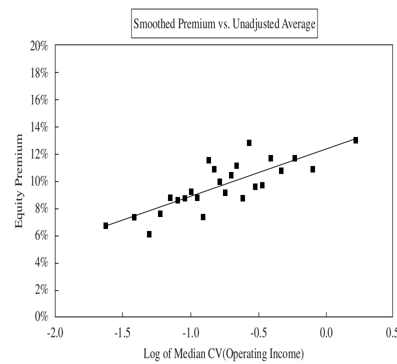
Equity Risk Premium Study: Data through December 31, 1999

Data Smoothing with Regression Analysis
Dependent Variable: Average Premium
Independent Variable: Log of Median CV (Operating Margin)

Regression Output:

Constant		12.319%
Std Err of Y Est		1.081%
R Squared		66%
No. of Observations		25
Degrees of Freedom		23
X Coefficient(s)	3.470%	
Std Err of Coef.	0.519%	
t-Statistic	–6.68	

*Smoothed Premium = 12.319% + 3.470% * Log(CV Op. Margin)*



(continued)

Exhibit 11.5 (Continued)

Companies Ranked by CV (ROE)
Historical Equity Risk Premium: Average Since 1963
Data for Year Ending December 31, 1999

Portfolio Rank by Size	Median CV(ROE)	Log of Median CV(ROE)	Number as of 1999	Beta Annual Since '63	Standard Deviation of Returns	Geometric Average Return	Arithmetic Average Return	*Smoothed* Arithmetic Equity Risk Premium	*Average Equity Risk Premium*	Average Debt/ MVIC
1	500.3%	0.70	102	1.43	32.70%	15.10%	19.25%	11.75%	*12.37%*	37.98%
2	181.5%	0.26	106	1.37	31.39%	15.45%	19.46%	11.96%	*11.34%*	35.60%
3	111.5%	0.05	122	1.28	27.91%	15.93%	19.17%	11.67%	*10.84%*	34.12%
4	82.7%	–0.08	116	1.30	26.88%	13.45%	16.50%	9.00%	*10.53%*	33.42%
5	66.2%	–0.18	99	1.24	24.22%	14.51%	17.08%	9.58%	*10.31%*	30.72%
6	55.2%	–0.26	92	1.22	25.62%	15.10%	17.87%	10.37%	*10.12%*	30.43%
7	47.2%	–0.33	99	1.16	24.17%	15.14%	17.73%	10.23%	*9.96%*	30.05%
8	41.2%	–0.38	89	1.13	22.89%	15.17%	17.42%	9.92%	*9.82%*	28.39%
9	36.3%	–0.44	89	1.20	23.40%	15.21%	17.59%	10.09%	*9.69%*	27.86%
10	32.1%	–0.49	81	1.16	23.40%	15.73%	18.00%	10.50%	*9.57%*	27.74%
11	28.7%	–0.54	78	1.21	23.12%	15.93%	18.23%	10.73%	*9.45%*	26.90%
12	25.7%	–0.59	80	1.12	21.35%	15.61%	17.49%	9.99%	*9.34%*	26.26%
13	23.1%	–0.64	88	1.07	22.18%	15.30%	17.27%	9.77%	*9.23%*	26.32%
14	20.8%	–0.68	85	1.07	20.91%	15.04%	16.95%	9.45%	*9.12%*	26.39%
15	18.6%	–0.73	85	1.10	20.71%	14.48%	16.31%	8.81%	*9.01%*	25.62%
16	16.6%	–0.78	94	1.05	20.16%	13.32%	15.11%	7.61%	*8.90%*	26.11%
17	14.8%	–0.83	78	1.07	19.87%	13.18%	14.91%	7.41%	*8.77%*	26.81%
18	13.0%	–0.89	75	0.97	17.96%	15.30%	16.71%	9.21%	*8.65%*	26.08%
19	11.5%	–0.94	90	1.07	19.29%	13.85%	15.46%	7.96%	*8.52%*	26.73%
20	10.1%	–1.00	66	0.97	17.34%	14.21%	15.58%	8.08%	*8.38%*	27.72%
21	8.8%	–1.06	80	0.95	18.56%	14.12%	15.58%	8.08%	*8.24%*	26.65%
22	7.5%	–1.13	69	0.87	16.17%	13.93%	15.05%	7.55%	*8.08%*	27.82%
23	6.2%	–1.21	82	0.90	17.11%	13.03%	14.28%	6.78%	*7.88%*	27.91%
24	4.7%	–1.33	63	0.91	16.83%	15.04%	16.32%	8.82%	*7.61%*	25.53%
25	2.9%	–1.54	56	0.91	16.83%	13.79%	15.03%	7.53%	*7.09%*	23.73%
High financial risk			640	1.47	36.63%	14.96%	20.07%	12.57%		49.64%
Large Stocks (Ibbotson SBBI data)						12.81%	13.92%	6.42%		
Small Stocks (Ibbotson SBBI data)						15.36%	18.09%	10.59%		
Long-term Treasury Income (Ibbotson SBBI data)						7.48%	7.50%			

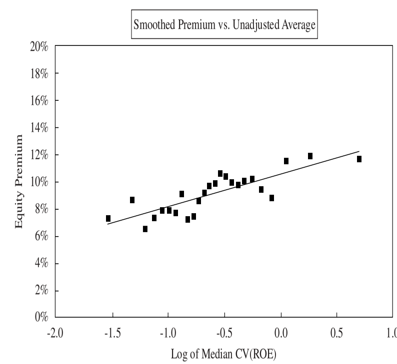
Equity Risk Premium Study: Data through December 31, 1999

Data Smoothing with Regression Analysis
Dependent Variable: Average Premium
Independent Variable: Log of Median CV (ROE)

Regression Output:

Constant		10.729%
Std Err of Y Est		0.818%
R Squared		69%
No. of Observations		25
Degrees of Freedom		23
X Coefficient(s)	2.353%	
Std Err of Coef.	0.326%	
t-Statistic	7.22	

*Smoothed Premium = 10.729% + 2.353% * Log(CV ROE)*



Source: Standard & Poor's Corporate Value Consulting (S&P CVC) Risk Premium Report 2000 (formerly the *PricewaterhouseCoopers Risk Premium Study 2000)*, 47–49. Updates to the *S&P CVC Risk Premium Report* are available at *http://valuation.ibbotson.com*.

Exhibit 11.6 *Pratt's Stats*™ Median Values by SIC Code
Data range 01/21/91–01/23/02

Pratt's Stats™ Median Value for SIC Code 1000–1999

Deal Price	Deal Price/ EBITDA	Number of Transactions	Deal Price/ Net Sales	Number of Transactions
< $1M	3.60	37	0.35	55
$1M–$10M	5.12	43	0.57	52
$10M–$50M	6.38	47	0.88	52

Pratt's Stats™ Median Value for SIC Code 2000–2999

Deal Price	Deal Price/ EBITDA	Number of Transactions	Deal Price/ Net Sales	Number of Transactions
< $1M	4.61	46	0.59	73
$1M–$10M	6.71	55	0.66	93
$10M–$50M	7.81	72	0.93	103

Pratt's Stats™ Median Value for SIC Code 3000–3999

Deal Price	Deal Price/ EBITDA	Number of Transactions	Deal Price/ Net Sales	Number of Transactions
< $1M	5.57	47	0.49	80
$1M–$10M	6.19	141	0.70	252
$10M–$50M	7.74	152	0.98	238

Pratt's Stats™ Median Value for SIC Code 4000–4999

Deal Price	Deal Price/ EBITDA	Number of Transactions	Deal Price/ Net Sales	Number of Transactions
< $1M	3.11	31	0.37	46
$1M–$10M	8.80	68	0.70	105
$10M–$50M	10.54	66	1.60	104

(continued)

Exhibit 11.6 (Continued)

Pratt's Stats™ Median Value for SIC Code 5000–5999

Deal Price	Deal Price/ EBITDA	Number of Transactions	Deal Price/ Net Sales	Number of Transactions
< $1M	3.59	296	0.36	483
$1M–$10M	7.26	147	0.55	213
$10M–$50M	8.43	124	0.45	173

Pratt's Stats™ Median Value for SIC Code 6000–6999

Deal Price	Deal Price/ EBITDA	Number of Transactions	Deal Price/ Net Sales	Number of Transactions
< $1M	3.57	12	0.47	18
$1M–$10M	6.88	34	0.68	43
$10M–$50M	9.91	40	2.52	51

Pratt's Stats™ Median Value for SIC Code 7000–7999

Deal Price	Deal Price/ EBITDA	Number of Transactions	Deal Price/ Net Sales	Number of Transactions
< $1M	3.65	184	0.51	303
$1M–$10M	10.29	204	1.40	372
$10M–$50M	12.09	170	2.29	310

Pratt's Stats™ Median Value for SIC Code 8000–8999

Deal Price	Deal Price/ EBITDA	Number of Transactions	Deal Price/ Net Sales	Number of Transactions
< $1M	4.46	134	0.59	244
$1M–$10M	6.35	123	0.85	189
$10M–$50M	11.02	62	1.81	124

Source: Pratt's Stats™, Business Valuation Resources, LLC. Available at *www.BVMarketData.com*sm.

have an average market value of $30 million. There are literally hundreds of thousands of companies smaller than this for which cost of capital estimation is relevant, both for valuation and for potential project assessment purposes.

Where the data leave off seems to raise the question: Is it valid to extrapolate these results beyond the observed population to infer even higher costs of capital for smaller companies? From purely a statistician's viewpoint, the answer would be no. We cannot know with certainty whether the population beyond the observed range would continue the trend. But most corporate finance practitioners and academicians with whom the author has discussed this question, as well as most business brokers and merger and acquisition intermediaries, conclude that the answer is yes.

In the first edition of this book, I said "One of our goals over the next year or two is to study this issue further, which will be facilitated by the new database being developed, *Pratt's Stats*, covering business sale transactions all the way from $100,000 to $100 million in value." *Pratt's Stats*™ is now the official database of the International Business Brokers Association, distributed by Business Valuation Resources, LLC, online at *www.BVMarketData.com*sm. It covers business sales with a market value of invested capital from under $100,000 to $1 billion.

Most of the companies in Ibbotson's 10b and S&P CVC's 25th size category have market capitalizations between $10 million and $50 million. In order to test the hypothesis that it would be valid to extrapolate the trend to companies smaller than $10 million, we calculated median transaction value multiples (market value of invested capital [MVIC]/EBITDA and MVIC/sales) for eight broad industry groups for transactions with $10 million to $50 million MVIC, $1 million to $10 million, and under $1 million. The results are shown in Exhibit 11.6.

As shown, for every broad industry group, median price EBITDA multiples were lower for the $1 million to $10 million group than for the $10 million to $50 million group, and lower yet for the under $1 million group. Similar results, although less consistent for a few industries, were observed for the deal price/sales multiples. I believe that this provides further strong evidence that the size effect does indeed exist and that it is valid to extrapolate it for companies with market values of equity below $10 million.

SUMMARY

Three independent sets of empirical studies provide strong support for the proposition that cost of capital tends to increase with decreasing company size. Users of cost of capital data should make themselves aware of updates of these and possibly other similar studies to incorporate the latest current size effect data in cost of capital estimates, whether using build-up models, CAPM, or other cost of equity models. The data currently available provide empirical evidence to help quantify the cost of capital for smaller companies, and the subject is attracting considerable new research interest.

Notes

1. *Stocks, Bonds, Bills & Inflation, Valuation Edition 2001 Yearbook* (Chicago: Ibbotson Associates, 2001), 123.
2. Tyler Shumway, "The Delisting Bias in CRSP Data," *Journal of Finance* (March 1997).
3. See "New Studies Quantifying Size Premiums Offer Strong Cost of Capital Support," *Shannon Pratt's Business Valuation Update®* (August 1997): 1–3; see also Shannon Pratt, "New Measures of Risk That Really Work," *Shannon Pratt's Business Valuation Update®* (December 1999): 1–4; and see Roger Grabowski and David King, "The Size Effect and Equity Returns," Appendix 5A in Jay Fishman et al., *Guide to Business Valuations*, 12th ed. (Forth Worth, TX: Practitioners Publishing Company, 2001), 5–75 et seq.
4. PricewaterhouseCoopers *Risk Premium Study 2000*, p. 12 (now the Standard & Poor's Corporate Value Consulting Risk Premium Reports).

Chapter 12

DCF Method of Estimating Cost of Capital

Theory of the DCF Method
Mechanics of the DCF Method
Single-stage DCF Model
Multistage DCF Models
Sources of Information
Summary

As discussed earlier in this book, there are several ways to estimate the cost of equity capital. Up to this point, all of the methods have had one thing in common: They begin with a risk-free rate and add one or many factors, based on the risks of the investment. The discounted cash flow (DCF) method is completely different.

THEORY OF THE DCF METHOD

At least in theory, the DCF method is more direct and simpler than the build-up model or the Capital Asset Pricing Model. The important assumption of the DCF method is that the public company's current stock price embodies the market's expectation of the rate of return that will be realized by investing in that stock.

In other words, the assumption is that the current stock price is actually the sum of the present values of the expected future returns to the investors (dividends and stock price change). The implied assumption is that the current stock price is equal to the expected future returns discounted to a present value at a discount rate that represents the equity cost of capital for that company.

The theory of the DCF method to estimate cost of capital is to use the DCF formula for computing present value backward. That is, since the present value (i.e., the current stock price) is known, the calculations are reconfigured to solve for k_e, the cost of equity capital.

The relationship between the DCF method of valuing a business and the DCF method of estimating cost of capital is the matter of which are the known and unknown variables. In using the DCF method to value a company, division, or project, the cost of capital already has been estimated and is given as a known rate in the formula to estimate the present value. In using the DCF method to estimate the cost of capital, the present value (market price of the stock) is known and placed into the formula to then solve for the discount rate (cost of capital).

Two main types of models are used to implement the DCF method as it is applied to estimating cost of capital. The first, and most popular, is the *single-stage model*. The second, and most accurate in most instances, is the *multistage model*. Although these models can be used to estimate the weighted average cost of capital, they typically are used to calculate the expected equity rate of return. The discussion that follows is based on equity rates of return only.

MECHANICS OF THE DCF METHOD

All methods for estimating the cost of capital derive all or part of the expected rates of return from current capital market data. With the exception of possible adjustments for private companies, the DCF method derives all of the implied expected return from current market data used in conjunction with analysts' growth expectations.

Although the models used are much different, some of the steps undertaken in estimating the cost of equity capital of a privately held company are the same as those used in the other methods. In particular, the DCF method of estimating cost of capital can be *directly* applied only to publicly traded companies (the current stock price is the essential ingredient here); therefore, for private companies, a set of guideline companies (i.e., those similar to the subject) must be identified. Alternatively, an industry average for companies in the subject's industry may be used as the starting point.

For public companies, the cost of equity estimated by the DCF method represents the *entire* cost of equity. That is, it encompasses in a single number all the factors considered in the build-up and Capital Asset Pricing Model (CAPM) methods: the risk-free rate, the equity risk premium, the beta, the size effect, and any company-specific factors.

To apply the cost of equity capital developed from public companies to a private company, the characteristics of the public companies must be compared with characteristics of the subject private company. Such comparisons could lead to adjustments for size and/or company-specific risk factors to get from the cost of equity estimate for the public companies to an estimate for a particular private company.

SINGLE-STAGE DCF MODEL

The single-stage DCF model is based on a rewrite (an algebraic manipulation) of a constant growth model, such as the Gordon Growth Model, presented earlier as Formula 4.6 and repeated here:

Formula 12.1

$$PV = \frac{NCF_0(1+g)}{k-g}$$

where:

PV = Present value
NCF_0 = Net cash flow in period 0, the period immediately preceding the valuation date
k = Discount rate (cost of capital)
g = Expected long-term sustainable growth rate in net cash flow to investor

When the present value (i.e., the market price) is known, but the discount rate (i.e., the cost of capital) is unknown, Formula 12.1 can be rearranged to solve for the cost of capital:

Formula 12.2

$$k = \frac{NCF_0(1+g)}{PV}$$

where the variables have the same definitions as in Formula 12.1.

In public companies, the net cash flow that the investor actually receives is the dividend. We can substitute some numbers into Formula 12.2 and thus illustrate estimating the cost of equity capital for Alpha Utilities, Inc. (AUI), an electric, gas, and water utility conglomerate, by making these three assumptions:

1. *Dividend.* AUI's dividend for the latest 12 months was $3.00 per share.
2. *Growth.* Analysts' consensus estimate is that the long-term growth in AUI's dividend will be 5%.
3. *Present value.* AUI's current stock price is $36.00 per share.

Substituting this information into Formula 12.2, we have:

Formula 12.3

$$\begin{aligned} k &= \frac{\$3.00(1+0.05)}{\$36.00} + 0.05 \\ &= \frac{\$3.15}{\$36.00} + 0.05 \\ &= 8.8 + 0.05 \\ &= 13.8 \end{aligned}$$

Thus, according to this computation, AUI's cost of equity capital is estimated to be 13.8% (8.8% dividend yield plus 5.0% expected stock price increase).

The preceding is the formulation used in Ibbotson Associates' *Cost of Capital Yearbook* "Analysts Single-Stage Discounted Cash Flow" cost of equity capital estimate. Ibbotson's source of growth estimates is the I/B/E/S database (now Thomson Financial) of consensus long-term growth rate estimates. A number of other sources of growth estimates are included in Appendix C.

This single-stage DCF model often is used in utility rate hearings to estimate a utility's cost of equity.[1]

Like the capitalization "shortcut" version of the discounting model used for valuation, the single-stage DCF model for estimating cost of capital is deceptively simple.

In utility settings, the dividend yield is assumed to be an appropriate estimate of the first input, cash flow yield. This is reasonable, because publicly traded utilities typically pay dividends, and these dividends represent a high percentage of available cash flows. In cases where the utility's dividend yield is abnormally high or low, a "normal" dividend yield is used. It is difficult, however, to use dividend yields with all publicly traded companies.

For many companies, dividend payments may have little to do with earnings or cash flows. A large number of companies do not pay dividends or pay only a token amount. In these cases, theoretically, the growth component, g, will be larger than that of an otherwise similar company that pays higher dividends. In practice, properly adjusting for this lack of dividends is extremely difficult.

One way to avoid the dividend issue is to define cash flows more broadly. Instead of considering only the cash flows investors actually receive (dividends), the analyst might define net cash flows as those amounts that could be paid to equity investors without impeding a company's future growth. As noted in Chapter 3, net cash flow is usually defined as:

	Net income (after tax)
+	Noncash charges (e.g., depreciation, amortization, deferred revenue, deferred taxes)
–	Capital expenditures*
–	Additions to net working capital*
±	Changes in long-term debt (add cash from borrowing, subtract repayments)*
=	Net cash flow to equity

*Only amounts necessary to support projected operations

Of course, these cash flows are not those paid to investors, but, presumably, investors ultimately will realize the benefit of these amounts through higher future dividends, a special dividend, or, more likely, stock price appreciation. Some analysts assume that over the very long run, net (after-tax) income should be quite close to cash flows. Therefore, they assume that net income can be used as a proxy for net cash flow. This assumption should be questioned on a case-by-case basis. For a growing company, capital expenditure and working capital requirements may make the assumed equivalence of net income and net cash flow so remote as to be irrelevant.

The other, and perhaps more problematic, input is the expected growth rate. An important characteristic of the growth rate is that it is the *perpetual* annual growth rate.

Future growth rates do not have to be the same for every year; however, the "average" rate should be equal to this perpetual rate. For example, if a company is expected to grow at 10% per year for the next four years and 3% per year thereafter, then the average growth rate into perpetuity could be estimated as about 5%. On the other hand, if the company is expected to grow by 10% per year for the next 20 years and 3% per year thereafter, the average growth rate is probably closer to 9%. However, this would be an extreme case. It is theoretically impossible for the sustainable perpetual growth rate for a company to significantly exceed the growth rate in the economy. Anything over a 6–7% perpetual growth rate should be questioned carefully.

A common approach to deriving a perpetual growth rate is to obtain stock analysts' estimates of earnings growth rates. The advantage of using these growth estimates is that they are prepared by people who follow these companies on an ongoing basis. These professional stock analysts develop a great deal more insight on these companies than a casual investor or valuation analyst not specializing in the industry is likely to achieve.

There are, however, three caveats when using this information:

1. These earnings growth estimates typically are for only the next two to five years; they are not perpetual. Therefore, any use of these forecasts in a single-stage DCF model must be tempered with a longer-term forecast.
2. Most published analysts' estimates come from "sell-side" stock analysts who work for firms that are in business to sell stocks. Thus, although their earnings forecasts fall within the range of "reasonable" possibilities, they may be on the high end of the range.
3. Usually these estimates are obtained from firms that provide consensus earnings forecasts; that is, they aggregate forecasts from a number of analysts and report certain summary statistics (mean, median, etc.) on these forecasts. For a small publicly traded firm, there may be only one or even no analyst following the company. The potential for forecasting errors is greater when the forecasts are obtained from a very small number of analysts. These services typically report the number of analysts who have provided earnings estimates, which should be considered in determining how much reliance to place on forecasts of this type.

Many of the problems inherent in using the single-stage model to estimate cost of capital are addressed by using a multistage model.

MULTISTAGE DCF MODELS

Multistage models come closer to reversing the discounting process than do single-stage models that simply reverse the capitalization process. Multistage models do not go to the extent of incorporating specific expected return amounts for specific years, but they do incorporate different growth rates for different expected growth stages, most often three stages.

Multistage models have one main advantage over single-stage models in that using more than one growth rate reduces reliance on a single such rate. Furthermore, it is unnecessary to compute a blended growth rate.

The main disadvantage of a multistage model is its computational complexity relative to the single-stage model. Unlike the single-stage model, the multistage model must be solved iteratively.

It also differs from the single-stage model in that there is no single form of the multistage model. Two main factors determine the form of the model:

1. The number of growth stages—usually either two or three
2. The length of each stage—usually between three and five years

In a three-stage model, the discounting formula that must be reversed to solve for k, the cost of capital, looks like this:

Formula 12.4

$$PV = \sum_{n=1}^{5} \frac{\left[NCF_0(1+g_1)^n\right]}{(1+k)^n} + \sum_{n=6}^{10} \frac{\left[NCF_5(1+g_2)^{n-5}\right]}{(1+k)^n} + \frac{\dfrac{NCF_{10}(1+g_3)}{k-g_3}}{(1+k)^{10}}$$

where:

NCF_0	= Net cash flow (or dividend) in the immediately preceding year
NCF_5	= Expected net cash flow (or dividend) in the fifth year
NCF_{10}	= Expected net cash flow (or dividend) in the tenth year
g_1, g_2, and g_3	= Expected growth rates in *NCF* (or dividends) through each of stages 1, 2, and 3, respectively
k	= Cost of capital (discount rate)

The above "stages" can be done in three-year increments or in increments of any number of years. Also, the length of the second stage can differ from the length of the first stage.

As noted earlier, this equation must be solved iteratively for k. Fortunately, many spreadsheet software packages, such as Excel, can perform this calculation.

Ibbotson, for example, in its *Cost of Capital Yearbook*, uses two five-year stages and then a growth rate applicable to earnings over all future years following the first 10 years. In the first and second stages it uses estimated cash flows instead of dividends. Ibbotson defines cash flows for this purpose as net income plus noncash charges less capital expenditures. This definition comes close to our definition of net cash flow to equity, except that it does not subtract additions to working capital or adjust for changes in outstanding debt principal. Ibbotson's third-stage (long-term) growth rate is the expected long-term inflation forecast plus the historical gross domestic product (GDP) growth rate.

SOURCES OF INFORMATION

To perform the DCF cost of capital analysis rather than use data compiled by one of the services, a variety of inputs is necessary, including company-specific data, industry outlook data, and long-term macroeconomic forecasts.

Company data can be obtained from Securities and Exchange Commission (SEC) filings or services such as Standard & Poor's (a division of McGraw-Hill), Moody's (published by Mergent, Inc.), and Value Line Publishing, Inc. Analysts' estimates can be compiled from individual analysts' reports or from one of the three earnings consensus reporting services: Thomson Financial (formerly First Call and I/B/E/S), Multex-Ace, and Zack's Investment Research, Inc.

There are a great number of different industry forecasts. For some industries, excellent material is available from industry trade associations, although they tend to focus primarily on revenues rather than on cash flows. There is also a wide variety of macroeconomic forecast information. Appendix C lists details on many sources providing data in all these categories. A more comprehensive compilation of the industry forecasts is the *Business Valuation Data, Publications & Internet Directory*, published annually by Business Valuation Resources, LLC, (888) BUS-VALU [(888) 287-8258], *www.BVResources.com*.

SUMMARY

The DCF method of cost of capital estimation attempts to use current public stock price information to estimate implied costs of equity capital. Single-stage models use a Gordon Growth Model type of formula, with the present value (i.e., the stock price) known, solving for k, the cost of capital. Multistage models use two or more growth estimates for different future periods. As with the CAPM, applying the method to privately held companies involves using public companies in a similar industry group to develop a proxy starting point, with modifications for differences in the characteristics between the public guideline companies and the subject company.

Analysts can obtain DCF-based cost of capital estimates for public companies and industries from several services that compile them or can build their own estimates from scratch.

The author is working on a research paper showing examples of applying the DCF method of estimating the cost of capital using several different sources of growth estimates. Readers may obtain a complimentary copy of this research paper when it is completed by contacting the author at the address shown in the preface.

Note

1. For a concise discussion of the use of this model for utility rate-setting, see Richard A. Brealey and Stewart C. Myers, *Principles of Corporate Finance*, 6th ed. (Boston: Irwin McGraw-Hill, 2000), 67–69.

Chapter 13

Using Ibbotson Associates Cost of Capital Data

Michael W. Barad and Tara McDowell

Stocks, Bonds, Bills and Inflation
- Cost of Equity Models
- Equity Risk Premium
- Firm Size Premium
- Industry Premium
- Ibbotson Data and Taxes
- Ibbotson Data: Minority or Controlling Interest?

Cost of Capital Yearbook
- Organization of Data
- Cost of Equity Models
- Capital Asset Pricing Model
- Fama-French Three-factor Model
- Discounted Cash Flow Models

Ibbotson *Beta Book*
- Beta Estimation Methodologies
- Levered and Unlevered Betas
- Ibbotson (Adjusted) Betas
- Peer Group and Industry Betas

Cost of Capital Center

Notation for This Chapter: The notation used in this chapter is that used in the Ibbotson data sources discussed herein and may differ slightly from the notation used elsewhere in this book. There are, however, no conceptual discrepancies between equations in this chapter and similar equations elsewhere in this book.

Ibbotson Associates is a financial software, data, consulting, and training firm headquartered in Chicago, Illinois. Established in 1977 by Roger Ibbotson, Ibbotson Associates is a leading provider of financial information to business valuation analysts, corporate finance professionals, and investment analysts. Ibbotson produces four publications that valuation and corporate finance professionals at all levels have found useful in the estimation of the cost of capital for companies of various industries and sizes.

The firm's benchmark publication is the *Stocks, Bonds, Bills and Inflation (SBBI) Yearbook*. Now commonly referred to as the *Classic Edition*, the *SBBI Yearbook* is based on Roger Ibbotson and Rex Sinquefield's original 1976 study of long-term market analysis. This publication has become a staple in the field of finance and is updated annually by Ibbotson Associates. The *SBBI Yearbook* is a leading source for historical market data, including data on the equity risk premium and firm size premium, market commentary, and other historical analysis on the capital markets.

While the *SBBI Classic Edition Yearbook* served as the foundation for discount rate, capitalization rate, and cost of capital estimates for many years, the introduction of the *Stocks, Bonds, Bills, and Inflation Valuation Edition Yearbook* has become the new standard for such valuation data. The *SBBI Valuation Edition Yearbook* was first introduced in 1999 to address the growing data needs of the valuation profession. The *Valuation Edition* was authored by Michael Annin and Dominic Falaschetti and expanded greatly on the cost of capital data already presented in the *Classic Edition* by drawing primarily from the cost of capital workshops then held by Ibbotson Associates. The *Valuation Edition*, updated annually by Ibbotson Associates, not only presents data along with examples on how to use it, but also addresses topical issues and controversies each year with alternative calculation methods and new studies as they become available. The *Valuation Edition* focuses primarily on the equity risk premium, size premium, industry premium, beta, and other issues related to the cost of capital.

Ibbotson Associates' *Cost of Capital Yearbook* is another publication geared directly to business valuation and corporate finance decisions. Ibbotson has published the *Cost of Capital Yearbook*, formerly referred to as *Cost of Capital Quarterly (CCQ)*, since 1994. The *Cost of Capital Yearbook* is an industry analysis publication that presents cost of capital and other financial information useful in business valuation and corporate finance. Over 300 industries organized by Standard Industrial Classification (SIC) code are presented in the *Cost of Capital Yearbook*. While the yearbook is published annually with data through March, quarterly updates are available.

The *Beta Book* rounds out Ibbotson's library of publications. Published semiannually since 1995, the *Beta Book* provides beta and three-factor model information on more than 5,000 companies. The *Beta Book* allows practitioners to select a company-specific measure of risk directly applicable to a publicly traded company, or to construct a custom peer group for analysis of a private company. This chapter discusses Ibbotson Associates publications in detail.

STOCKS, BONDS, BILLS AND INFLATION

Ibbotson's *Stocks, Bonds, Bills and Inflation (SBBI) Classic Edition Yearbook* and *Valuation Edition Yearbook* are some of the most commonly cited references in valuation reports. The *Classic Edition*, as it is now known, is based on the original study of long-term capital market performance done by Roger Ibbotson and Rex Sinquefield in 1976. The *Classic Edition* was first published in 1983 to satisfy the demand for a comprehensive historical data set along with corresponding analysis on the capital markets. The *Valuation Edition* was first published in 1999 to expand on the cost of capital concepts and data covered in both the *Classic Edition* and Ibbotson's cost of capital workshops. Both publications are updated annually by Ibbotson Associates.

While both yearbooks provide discussions and estimates of the equity risk premium and firm size premium, the *Valuation Edition* focuses primarily on the science of valuation and has much greater coverage on topics related to the cost of capital. The *Valuation Edition* discusses current issues and controversies related to cost of capital and includes any of Ibbotson's advances in the field of cost of capital analysis. Some examples of such coverage presented only in the *Valuation Edition* would be the inclusion of industry premia for use in the build-up method and alternative measures of firm size premia, including a further breakout of the smallest companies (the tenth decile) into even smaller divisions (10a and 10b). While the *Classic Edition* still provides much useful information to the valuation industry in the form of capital market analysis and discussions on the performance of the economy, the *Valuation Edition* is critical to anyone performing cost of capital analysis using the income approach and has become the industry standard for such data. For this reason, it is the *Valuation Edition* that serves as the basis for the following discussion.

Cost of Equity Models

The cost of equity capital is equal to the expected rate of return for a firm's equity. There are several widely used models for estimating cost of equity for a firm, the two most common being the build-up method and the Capital Asset Pricing Model (CAPM) (see Exhibit 13.1). Other methods such as the Fama-French three-factor model and the discounted cash flow model will be discussed later. Both the build-up method and the CAPM are very similar, with the major exception being the use of beta.

The risk-free rate, equity risk premium, and firm size premium are components shared by both the build-up method and the CAPM. These models should provide very similar if not identical results if implemented correctly. Let us take a private company as an example. For a private company, there is no market data to derive a beta. Therefore, to use the CAPM the analyst must use a beta from comparable companies. If we assume that the beta used in this example is an industry beta for the subject company's industry, then both the build-up and CAPM models have a provision for including industry risk. Theoretically, the industry premium in the build-up model and the industry beta used in the CAPM should lead both models toward identical results.

Equity Risk Premium

The expected equity risk premium is defined as the additional return investors expect to receive to compensate for the additional risk associated with investing in equities as opposed to riskless assets. The equity risk premium is a critical component of many of the cost of equity models, including the build-up method, CAPM, and Fama-French three-factor model.

While the equity risk premium (ERP) has many uses throughout the field of finance, for the purpose of business valuation it should be a forward-looking measure

Exhibit 13.1 Build-up versus CAPM Cost of Equity Models

Build-up Model		Capital Asset Pricing Model (CAPM)	
	Risk-free Rate		Risk-free Rate
+	Equity Risk Premium	+	(Equity Risk Premium × Beta)
+	Firm Size Premium	+	Firm Size Premium
+/–	Industry Premium ?		Cost of Equity
	Cost of Equity		

of what investors can expect. Unfortunately, a forward-looking measure of the ERP is not directly observable in the market. The most common way of capturing expectations on the ERP is to measure the historical relationship of stocks to bonds. It is in measuring this historical relationship that such choices as benchmark selection, the appropriate range of data, and using arithmetic versus geometric averages become important decisions.

Ibbotson Associates measures the ERP by calculating the arithmetic average total return on the Standard & Poor's (S&P) 500 Index over the arithmetic average income return on the appropriate horizon Treasury security. Ibbotson provides ERP estimates for the short-, intermediate-, and long-term horizons. Since most companies do not have a defined life span and are valued as going concerns, the long-term discount rate typically is most appropriate for business valuation purposes. The appropriate horizon should be a function of the investment, not the investor.

To determine the long-horizon equity risk premium, Ibbotson calculates the arithmetic average total return on the S&P 500 less the arithmetic average income return on long-term Treasury bonds using annual data from 1926 to the present. Price return (and ultimately total return) for a bond is sensitive to changes in interest rates and can lead to gains or losses. For the purpose of calculating an ERP, income return from a bond better represents the truly riskless portion of the bond's return.

Choosing the components that comprise the equity risk premium is one of the first critical decisions. Ibbotson has chosen the S&P 500 to represent the stock market and a 20-year bond to represent the riskless asset. For stock market representation, other common benchmarks are the New York Stock Exchange (NYSE) Composite Index and NYSE 1-2 index (largest 20% of stocks by market capitalization traded on the NYSE). While the Dow Jones Industrial Average is a common investment reference for the market, it is too narrow for ERP calculation. Ibbotson presents ERP estimates using both the S&P 500 and the NYSE 1-2 market benchmarks. However, the S&P 500 is most used throughout Ibbotson publications because it represents a large sample of companies across a large sample of industries. As of December 31, 1993, 88 separate industry groups were included in the index, and the industry composition of the index has not changed since. The S&P 500 is also one of the most widely accepted market benchmarks. All things considered, Ibbotson believes that the S&P 500 is a good measure of the equity market as a whole.

Ibbotson uses a 20-year bond to represent the income return of a riskless asset for computing the long-horizon equity risk premium. The Treasury does not currently issue

20-year bonds, however. Both 10-year and 30-year bonds are possible benchmark options, but Ibbotson chooses to use bonds on the market with approximately 20 years to maturity to keep the basis of the time series consistent across time. The Treasury did not issue 30-year or 10-year bonds as far back as 1926, and as of the date of this publication the Treasury had stopped issuing 30-year bonds altogether. Ibbotson believes that the best option for a consistent time series of long-term bond data extending back to 1926 is the 20-year bond.

The *SBBI* equity risk premium covers the time range from 1926 to present. The original data source for the raw data series comprising the ERP is the Center for Research in Security Prices (CRSP) at the University of Chicago. The CRSP chose 1926 as a starting date for its data series because this is when good-quality financial data became available. This period was chosen also because it includes one full business cycle before the 1929 market crash. While Roger Ibbotson and other researchers have published data back to the early nineteenth century, these data are not of the same quality as the data that begin in 1926. In basic terms, these are the main reasons why Ibbotson uses a range back to 1926 in equity risk premium calculations.

The period from 1926 to present is most relevant because it includes a number of different economic scenarios. Some practitioners argue for a shorter historical period, such as the last 20 or 30 years. This argument is based on the assumption that investors only factor in a more recent economic climate into their expectations and that "unusual" economic events prior to recent times are not likely to repeat in the future. All periods contain unusual events, some of which took place most recently. For example, the inflation of the late 1970s and early 1980s, the October 1987 market crash, the collapse of the Soviet Union, the development of the European Economic Community, and the September 2001 terrorist attack on the United States are but a few of the "unusual" events that occurred in recent times. While we do not expect these events to occur again in the future, they are representative of the type of events that can occur unexpectedly and have massive effects on the economy. Focusing on a shorter historical date range would magnify the effect of the most recent unusual events. Using a longer range of data places less emphasis on each event and better captures long-term performance. By including market data measured over the entire set of economic scenarios available, Ibbotson believes that the resulting computations can better anticipate similar events in the future.

The equity risk premium presented by Ibbotson Associates is an arithmetic average risk premium, as opposed to a geometric average risk premium. Ibbotson believes that arithmetic averages are appropriate for use in discounting future cash flows. A geometric average is better for reporting past performance since it represents the compound average return. Mathematically, the arithmetic mean assumes that the cash flow being discounted each period is the expected value of the probability distribution of possible outcomes for that period.

Arithmetic averages better incorporate the volatility in a data series. Take bonds and stocks as an example. Bonds have lower volatility than stocks, on average. When comparing the arithmetic mean with the geometric mean for each asset class, the difference between arithmetic and geometric will be greater for the asset class with higher volatility. The arithmetic mean captures the volatility of a time series. Since the ERP is a volatile data set and we are using it in a forward-looking capacity, the arithmetic average will better capture the uncertainty associated with the ERP. In general terms, arithmetic averages are better forward-looking point estimates, and geometric averages are better for historical analysis of a defined data range.

Numerous alternatives to using pure historical data for calculating the equity risk premium bear discussion. A few of these methods for calculating ERP are:

- The use of survey results
- The exponential weighting of historical periods
- The supply-side perspective

The first of these alternatives takes the approach that a survey of ERP expectations from the appropriate people will yield more useful information. Typically, the survey is conducted on academics, money managers, or other professionals deemed to have an educated idea on the direction of the market. The difficulty in relying on this method stems from the subjective nature of the "opinions" submitted by the participants, along with a bias for participants to form estimates based heavily on the current economic condition.

For those who struggle with the appropriate time period to use in their historical ERP calculation, exponential weighting offers a solution. Using an exponential weighting scheme to average historical data allows for more importance to be placed on current data (compared with an equal weighting scheme). Of course, this assumes that an unbiased reason exists for assuming that the future will bring with it an economic climate more similar to that of recent periods.

The last alternative to calculating the ERP is the supply-side model. This method estimates what the economy can supply going forward, as opposed to its actual historical performance. In general, research has shown that the supply-side estimate is lower than the historical estimate, indicating that the market cannot supply the type of long-term growth that it has demonstrated to date.

All of the alternatives to estimating ERP have advantages and disadvantages. For cost of capital analysis, it is important that all of the cost-of-equity components are developed so they can work together in a model. If using a size premium, industry premium, or any other addition to the cost of equity, it is imperative that all of the components be on the same basis. For example, the time period and weighting scheme of the ERP and size premium should be identical. Similarly, both components should be from either a historical perspective or a supply-side perspective. Consistency is a critical attribute in cost of capital analysis.

Firm Size Premium

The relationship between firm size and return is one of the most remarkable discoveries of modern finance. This relationship cuts across the entire size spectrum but is most evident among smaller companies. While many studies have examined the size effect, Ibbotson Associates is the most cited source of size premium data as published in both the *SBBI Classic Edition Yearbook* and the *SBBI Valuation Edition Yearbook*.

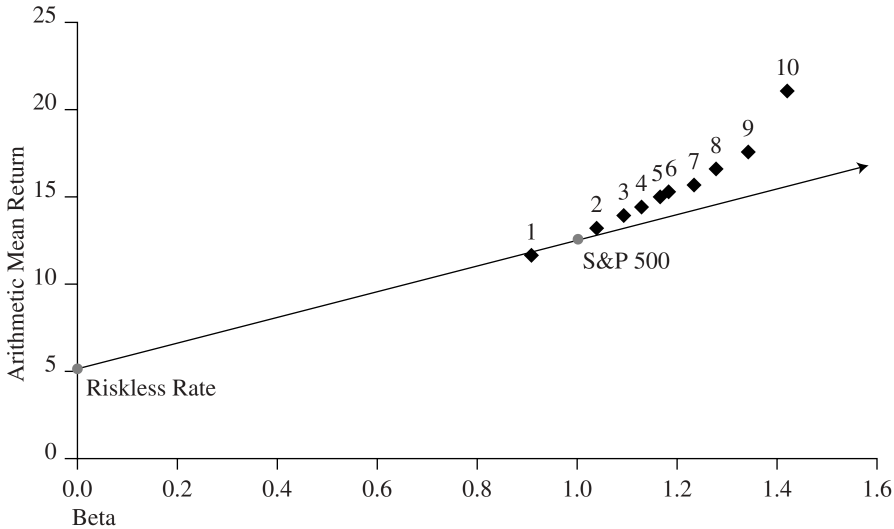
What is the firm size premium? Historically, small stocks have shown greater risk and greater return than their larger capitalization counterparts. This makes perfect sense since investors will demand higher return to compensate for increased risk. If small stocks did not provide a higher return to compensate for this risk, there would be no demand to invest in them. To capture the additional return exhibited by smaller stocks, we calculate a size premium that can be used as an addition to either the CAPM or build-up model of estimating cost of equity.

The first point worth noting is the evolution of Ibbotson's size premium and the difference between what is presented in the *Classic Edition* and *Valuation Edition* yearbooks. Through the 1994 yearbook, the *Classic Edition* simply presented what Ibbotson calls the small stock premium. The small stock premium was measured as the simple difference between Ibbotson's Small Company Stock series and the S&P 500 total returns. The Small Company Stock series is a representation of publicly traded micro-cap stocks. Beginning in 1995, the *SBBI Classic Edition* presented a chapter on firm size that analyzed the small stock effect across all 10 deciles of the stock market. At this point the CRSP data for companies traded on the NYSE was used in the firm size analysis. At the same time that data was introduced for all 10 deciles, a new method for measuring the size effect was introduced. This new method was calculated as the return in excess of what CAPM predicts given the beta for a decile, otherwise known as a beta-adjusted size premium. In 1999 Ibbotson Associates introduced the first *SBBI Valuation Edition Yearbook*. The *Valuation Edition* was created to address current topics relating primarily to cost of capital analysis and to present advances made in this field. While a firm size chapter exists in both yearbooks, the *Valuation Edition* has much more extensive coverage and analysis on the topic of firm size. The remainder of this section discusses some of the coverage offered in the *Valuation Edition*.

The *Valuation Edition Yearbook* has continued to present advances in firm size analysis since its inception in 1999. Beginning with the 2001 edition, the yearbook revised all size premium calculations to include the population of stocks traded on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and Nasdaq Stock Market (Nasdaq). The AMEX and Nasdaq securities were added to the analysis to capture the performance of the many small stocks traded on these exchanges. The NYSE is used to create the breakpoints that define the deciles, to which AMEX and Nasdaq securities of similar size are then added. Also in the 2001 edition, size premia were added for deciles 10a and 10b. This breakout of the tenth decile into two components allows for further analysis of the smallest companies. Additional statistics relating to 10a and 10b were presented beginning in the 2002 yearbook.

Exhibit 13.2 is a graph from the *SBBI Valuation Edition 2001 Yearbook* showing the actual returns achieved by the 10 deciles and the security market line on which the CAPM would predict the portfolios would fall. If the CAPM were functioning properly, all of the decile portfolios would fall directly on the line indicated on the graph. Instead, most of the deciles fall above the security market line, indicating that the CAPM underreports cost of equity for all but the largest companies. The vertical space between a decile and the security market line is the graphical representation of the size premium. This premium must be added back into the cost of equity to fully explain the returns of all but the largest companies.

Exhibit 13.2 Security Market Line versus Size-Decile Portfolios of the NYSE/AMEX/NASDAQ (1926–2000)



Source: Stocks, Bonds, Bills, and Inflation® Valuation Edition 2001 Yearbook, © 2001 Ibbotson Associates, Inc. Used with permission. All rights reserved.

Exhibit 13.3 is a table from the *SBBI Valuation Edition 2001 Yearbook* detailing the calculation of the size premium for each decile. In addition to the 10 deciles, this table also presents size premia for the mid-, low-, and micro-cap size groupings for consolidation and generalization purposes. The first column of data next to the decile names represents the beta for each decile measured against the S&P 500 market benchmark. The next two columns show the actual historical returns and returns in excess of the riskless rate for each decile. The second-to-last column represents the return predicted by CAPM in excess of the riskless rate. This is calculated as beta multiplied by the equity risk premium. The last column shows the size premia, which is the difference between the actual returns (minus the riskless rate) and the returns predicted by CAPM (minus the riskless rate). As companies get smaller, their beta and CAPM-predicted return increase; however, beta does not fully explain the full returns of these smaller companies, and a size premium must be added to complete the model.

Should these beta-adjusted size premia be used in both the CAPM and the build-up models for cost of equity analysis? Yes. The size premia calculated in *SBBI* are constructed within the context of CAPM. It is clear that this type of size premium can be used as an addition to the CAPM cost of equity. More debate surrounds the use of this data for addition to the build-up model. Some practitioners argue that the non–beta-adjusted, or simple excess return, method, should be used instead. Ibbotson believes that the beta-adjusted size premia constructed within the context of CAPM are appropriate for build-up use.

Exhibit 13.3 Long-term Returns in Excess of CAPM Estimation for Decile Portfolios of the NYSE/AMEX/NASDAQ (1926–2000)

Decile	Beta*	Arithmetic Mean Return	Realized Return in Excess of Riskless Rate**	Estimated Return in Excess of Riskless Rate***	Size Premium (Return in Excess of CAPM)
1-Largest	0.91	12.06%	6.84%	7.03%	–0.20%
2	1.04	13.58%	8.36%	8.05%	0.31%
3	1.09	14.16%	8.93%	8.47%	0.47%
4	1.13	14.60%	9.38%	8.75%	0.62%
5	1.16	15.18%	9.95%	9.03%	0.93%
6	1.18	15.48%	10.26%	9.18%	1.08%
7	1.24	15.68%	10.46%	9.58%	0.88%
8	1.28	16.60%	11.38%	9.91%	1.47%
9	1.34	17.39%	12.17%	10.43%	1.74%
10-Smallest	1.42	20.90%	15.67%	11.05%	4.63%
Mid-Cap, 3–5	1.12	14.46%	9.23%	8.65%	0.58%
Low-Cap, 6–8	1.22	15.75%	10.52%	9.45%	1.07%
Micro-Cap, 9–10	1.36	18.41%	13.18%	10.56%	2.62%

*Betas are estimated from monthly portfolio total returns in excess of the 30-day U.S. Treasury bill total return versus the S&P 500 total returns in excess of the 30-day U.S. Treasury bill, January 1926-December 2000.

**Historical riskless rate is measured by the 75-year arithmetic mean income return component of 20-year government bonds (5.22 percent)

***Calculated in the context of the CAPM by multiplying the long-horizon equity risk premium by beta. The equity risk premium is estimated by the annual arithmetic mean total return of the S&P 500 (12.98 percent) minus the annual arithmetic mean income return component of 20-year government bonds (5.22 percent) from 1926–2000.

Note that all data have been rounded for presentation purposes and any calculation discrepancies are due to rounding.

Source: Stocks, Bonds, Bills, and Inflation® Valuation Edition 2001 Yearbook, © 2001 Ibbotson Associates, Inc. Used with permission. All rights reserved.

In the CAPM, an adjustment for different types of risk (i.e., industry risk) are included in the beta measure. In the build-up model, beta is absent and these additional risk factors must be added directly to the model in the form of risk premia. Many practitioners add a company-specific risk premium to the build-up model. The beta-adjusted size premium should be used in the build-up model because it makes no assumptions about the risk of the company by isolating the return due solely to size. The return due to beta (risk) for each decile has been removed from the actual returns to leave a size premium that is absent of risk assumptions. As mentioned, this risk is accounted for in the CAPM through the use of beta. In the build-up model, the risk that has been removed from the size premium calculation may be added back in through the use of other risk premia.

This method for isolating size only in the size premium and accounting for other risk factors by using industry premia, company-specific premia, and the like presents a very clean model for estimating cost of equity. It also reduces the likelihood of double-counting risk factors. If one were to use a simple excess return small stock premium, one would be assuming that the beta of the subject company is the same as the beta of all small companies in the index. This contradicts the nature of most analysis to add or subtract risk premia to the model and assumes that there is no adjustment for industry risk. By using an excess return size premium such as this along with something like an industry premium, there is a high likelihood of double-counting risk factors. For the reasons presented here, Ibbotson believes that the beta-adjusted size premia are appropriate for application to the CAPM and build-up models.

In addition to the size premia presented in Exhibits 13.2 and 13.3, Ibbotson publishes a variety of variations and additions to the analysis in its *SBBI Valuation Edition Yearbook*. Some of the additional information is meant to demonstrate that the size premium still exists even when altering the data in a number of ways suggested by critics, while other data add options for cost of equity analysis. Ibbotson repeats the size premium graph and table:

- Using the NYSE market benchmark instead of the S&P 500
- Calculated with sum betas instead of raw ordinary least squares (OLS) betas
- Calculated with annual betas instead of monthly
- With the tenth decile split into 10a and 10b

All of the variations on the firm size analysis support the addition of a size premium for the smallest stocks.

A common question among practitioners is whether to use the micro-cap decile, tenth decile, or 10a/10b split for analysis of the smallest companies. For example, say that we are analyzing a very small company whose market capitalization (or equivalent) would place it into the 10b size grouping. A company of this size also fits into the tenth decile and micro-cap aggregation of the ninth and tenth deciles. Which one should be used? Ibbotson feels that this is up to the practitioner to determine, but suggests consistency across valuation assignments.

Industry Premium

Both the CAPM and build-up models should take the characteristics of the subject company's industry into account when determining cost of equity. The CAPM has the ability to incorporate industry risk into the beta measure. For the build-up model, valuation practitioners often add an industry premium or incorporate industry risk into a company-specific premium. Prior to 2000, the formation of industry premia for use in the build-up model was not quantitative in nature. Since 2000, however, Ibbotson Associates has published industry premia for use in the build-up model in its *SBBI Valuation Edition Yearbook*. In its first publication, Ibbotson presented data on over 60 industries organized by two-digit Standard Industrial Classification

(SIC) code. Since 2001, Ibbotson has expanded coverage to include nearly 300 industries down to the three-digit SIC code level. This introduction of quantifiable objective industry data can now account for the industry risk that was once left up to the practitioner to measure.

The method Ibbotson uses in formation of industry premia relies on the full-information beta estimation process outlined later in the *Beta Book* section. The full-information beta methodology uses data from all companies that participate in an industry to determine the risk characteristics of that industry. The approach provides a risk index for each industry that can be used to compare the risk level of the industry with that of the market as a whole. The industry risk premium methodology uses this equation:

$$IRP_i = (RI_i \times ERP) - ERP$$

where:

IRP_i = The expected industry risk premium for industry i, or the amount by which investors expect the future return of the industry to exceed that of the market as a whole
RI_i = The risk index (full-information beta) for industry i
ERP = The expected equity risk premium

For an industry with a risk index of 1.0 (the same as that of the market), the expected industry risk premium would be 0. For those industries with a risk index greater than 1.0, the industry premium will be positive; for those with a risk index less than 1.0, it will be negative. The industry risk premium can in fact be a negative number that actually must be subtracted from the cost of equity. This makes perfect sense, since just as many industries should have less risk than the market as those that have more risk. Also remember that the beta-adjusted size premium is more appropriate than the simple excess returns size premium to use in conjunction with an industry premium. The systematic risk removed from a beta-adjusted size premium can be replaced by the risk included in the industry premium as a better measure of systematic risk.

Ibbotson Data and Taxes

All of the risk premium statistics presented in any Ibbotson Associates publication are derived from market returns earned by an investor. An investor receives dividends and realizes price appreciation after the corporation has paid its taxes. Therefore, the underlying data used in these risk premia calculations represent returns after corporate taxes but before personal taxes.

When performing discounted cash flow analysis, it is important that both the discount rate and the cash flows be on the same tax basis. Since most valuation settings rely on after-tax cash flows, the use of an after-tax discount rate is appropriate in most cases. However, there are some instances (usually because of regulations or statutes) in which it is necessary to calculate a pretax value. Should a pretax cost of capital be required, there is no easy way to accurately modify the underlying market returns to a pretax basis. This modification would require estimating pretax returns for all the publicly traded companies that comprise the market benchmark. Although not completely accurate, the easiest way to convert an after-tax discount rate to a pretax basis is to divide the after-tax rate by (1 minus the tax rate). This will gross up the discount rate to an estimated pretax basis.

The tax rate selected for use in this method can have a substantial effect on the results. While the tendency is to use the top marginal tax rate, each case should be analyzed to determine whether this is appropriate. Many companies do not always pay the top marginal tax rate.

Determining value for an S corporation can be even trickier. An S corporation is a form of corporation with less than 75 shareholders that enjoys the benefits of incorporation but is taxed similarly to a partnership. For an S corporation all taxes are passed through to the individual level (no corporate taxes). There has been much controversy regarding whether to tax-adjust data derived from publicly traded companies for application in discounting S corporations. Ibbotson's opinion is that in many cases it would make perfect sense to tax-adjust either the cash flow stream or the discount rate to put them on the same tax basis. However, the valuation practitioner should evaluate each case individually to determine what adjustments, if any, should be taken. Should an adjustment to the discount rate be warranted, there may be methods other than the approach described above that would be more appropriate.

Ibbotson Data: Minority or Controlling Interest?

Ibbotson Associates uses publicly traded company data in its risk premium calculations, but is a minority discount implicit in these data? This is an important issue for the valuation practitioner because applying a minority discount or control premium can have a material impact on the ultimate value derived in the appraisal.

The Ibbotson long-horizon equity risk premium is derived from returns on the S&P 500. The Ibbotson size premia data are calculated from returns of stocks on the NYSE, AMEX, and Nasdaq. All of these indexes include a preponderance of minority-held companies.

Because most of the companies underlying Ibbotson risk premium data are minority held, some valuation practitioners assume that the risk premia represent minority returns and therefore have an implicit minority discount. Ibbotson does not believe this is entirely correct, however. The returns generated by the S&P 500, NYSE, and the like represent returns to equity holders. While most of these companies are in fact minority held, there is no evidence that higher rates of return could be earned if all of these companies were acquired by majority shareholders. The Ibbotson risk premia represent expected premiums that holders of securities of a similar nature can expect to achieve, on average, into the future. There is no distinction between minority and controlling owners.

The discount rate is meant to represent the underlying risk of a particular industry or line of business. There may be instances in which a majority shareholder can acquire a company and improve its cash flow, but that would not necessarily have an impact on the general risk level of the cash flows generated by that company.

In applying the income approach to valuation, adjustments for minority or controlling interest value should be made to the projected cash flows of the subject company instead of to the discount rate. Adjusting the expected cash flows better measures the potential impact a controlling party may have while not overstating or understating the actual risk associated with a particular line of business.

COST OF CAPITAL YEARBOOK

The *Cost of Capital Yearbook* is a comprehensive source of industry-level financial data. The yearbook presents statistics critical in applying the income and market approaches to business valuation. Cost of equity, cost of capital, capital structure ratios, growth rates, industry multiples, and other useful financial data are presented on over 300 industries. For each statistic, Ibbotson presents the median and average estimate, along with estimates for the smaller and larger companies in the industry. This book is an excellent resource for industry analysis and is a necessary tool for obtaining comparable market data applicable to privately held company valuation. The *Cost of Capital Yearbook* is published annually containing data calculated through March, with quarterly supplements available throughout the year.

Organization of Data

Those industries included in the yearbook are organized by SIC code ranging from one digit (the most general) to four digits (the most specific). Companies appearing in a four-digit SIC code also are included in three-, two-, and single-digit classifications. For example, the company OMNI Rail Products, Inc., is located in industry 3069 as well as in industries 306, 30, and 3. In this way, companies of a similar type are classified together in a systematic manner. Only those industries that contain five or more companies are listed in the *Cost of Capital Yearbook*.

The primary source of company data for the *Cost of Capital Yearbook* is provided by Standard & Poor's COMPUSTAT database. COMPUSTAT provides data for more than 10,000 companies, but the yearbook includes within a particular industry only those companies that meet the criteria of a rigorous sorting and screening process. This process attempts to provide the purest industry statistics possible by excluding companies that have incomplete data or contain other characteristics that would misrepresent a given industry's financial statistics.

A company may be excluded for any of the following reasons:

- A company does not have sales for the most recent fiscal year, or no stock price has been reported for the most recent month.
- Company sales are less than $100,000 for the most recent fiscal year, or market value does not exceed $10,000.

- A company has not reported financial results for each of the last three fiscal years or month-end stock prices for the last 24 months.
- A company has less than 75% of sales in a single SIC code.

The following example illustrates the selection process based on the sales of a company in a particular industry. As shown in the accompanying table, the total sales for fictitious companies A and B are distributed between more than one SIC code.

Company A		**Company B**	
SIC	$Sales	SIC	$Sales
3443	730	2531	5,308
3533	649	3085	968
3534	1,709	3691	774
3559	993	3822	2,960
	$4,081		$10,010

Neither company A nor company B would be included at the four-digit level, because sales to any one SIC code are not greater than 75%. However, at the two-digit level, industry 35 represents 82% of company A's sales. Thus company A would be found in industries 35 and 3. The analysis for company B reveals that industry 2 represents 53% of sales and industry 3 represents 47% of sales. Company B's sales do not meet the 75% sales criteria for any of the industries in which it participates, and thus it would be excluded from the yearbook.

Currently the yearbook includes statistics on over 300 industries for help in performing discounted cash flow analysis. For each industry, a comprehensive set of financial parameters (levels of profitability, capitalization requirements, capital structure, and risk) are displayed. In addition to these financial statistics, and unique to the *Cost of Capital Yearbook*, are multiple cost of equity and weighted average cost of capital (WACC) measures.

Cost of Equity Models

The yearbook calculates cost of equity and average cost of capital estimates based on five separate models for each industry. These models include the ordinary least squares (OLS) Capital Asset Pricing Model, variations of the discounted cash flow (DCF) model, and the Fama-French three-factor model. The results of each model can be seen in the bottom section of Exhibit 13.4.

Capital Asset Pricing Model

Most practitioners are familiar with the Capital Asset Pricing Model for calculating the cost of equity developed by William Sharpe and John Linter. The principal assumption behind the model is that a direct linear relationship exists between the

Exhibit 13.4 Sample Page from the 2001 *Cost of Capital Yearbook*

STATISTICS FOR SIC CODE 275

Commercial Printing

This Industry Comprises 12 Companies

Industry Description

Establishments primarily engaged in commercial printing by the lithographic process and in gravure printing.

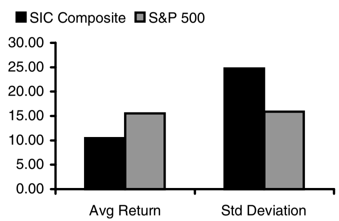
Sales (million$)

Total	9,425
Average	785.4
Three Largest Companies	
DONNELLEY (R R) & SONS CO	5,764.3
BANTA CORP	1,537.7
BOWNE & CO INC	1,010.8
Three Smallest Companies	
GEOGRAPHICS INC	27.3
LASER MASTER INTL INC	17.5
DIMENSIONAL VISIONS GRP LTD	1.0

Total Capital (million$)

Total	6,061
Average	505.1
Three Largest Companies	
DONNELLEY (R R) & SONS CO	4,161.8
BANTA CORP	830.4
CONSOLIDATED GRAPHICS INC	429.1
Three Smallest Companies	
SUCCESSORIES INC	18.5
LASER MASTER INTL INC	11.5
DIMENSIONAL VISIONS GRP LTD	1.1

SIC vs. S&P 500 for Last 10 Years (%)



Number of Companies & Total Capital (billion$)

S&P Debt Rating	Large Cap	Mid Cap	Low Cap	Micro Cap	Totals	
AAA, AA, A	0	1	0	0	1	(companies)
	0.0	4.2	0.0	0.0	4.2	(capital)
BBB	0	0	0	0	0	
	0.0	0.0	0.0	0.0	0.0	
BB, B, CCC, CC, D	0	0	0	0	0	
	0.0	0.0	0.0	0.0	0.0	
Not Rated	0	0	2	9	11	
	0.0	0.0	1.2	0.7	1.9	
Totals	0	1	2	9	12	
	0.0	4.2	1.2	0.7	6.1	

Annualized Statistics for Last 10 Years (%)

	Avg Return	Std Deviation
S&P 500	15.50	15.86
SIC Composite	10.46	24.72
Large Composite	8.85	25.29
Small Composite	20.75	102.82

Compound Annual Equity Return (%)

	5 Years	10 Years
75th Percentile	6.02	6.51
Median	-1.26	4.74
25th Percentile	-29.89	2.85
SIC Composite	3.61	6.12
Large Composite	0.71	6.03
Small Composite	-10.16	NMF

Sales, Income & Market Capitalization (billion$)

	Sales	Operating Income	Net Income	Equity Capital	Debt Capital
Current Yr.	9.4	1.3	0.4	4.4	1.6
Last Yr.	8.2	1.3	0.4	3.9	1.7
2 Yrs. Ago	7.7	1.2	0.4	6.3	1.4
3 Yrs. Ago	7.1	1.1	0.3	8.7	1.5
4 Yrs. Ago	8.5	1.0	-0.1	7.0	1.7

Growth Over Last 5 Years (%)

	Net Sales	Operating Income	Net Income
Median	17.97	6.07	-2.26
SIC Composite	2.54	1.45	1.86
Large Composite	0.99	-0.94	-1.62
Small Composite	19.08	9.58	-35.20

Capital Structure Ratios (%)

	Debt/Total Capital Latest	Debt/Total Capital 5-Year Avg	Debt/MV Equity Latest	Debt/MV Equity 5-Year Avg
Median	28.56	24.17	39.98	31.87
SIC Composite	22.22	19.67	28.57	24.49
Large Composite	23.24	19.58	30.27	24.35
Small Composite	46.81	43.68	88.01	77.55

Distribution of Sales & Total Capital (million$)

	Distribution of Sales Latest	Distribution of Sales 5-Year Avg	Total Capital Latest	Total Capital 5-Year Avg
90th Percentile	1,485.0	1228.2	790.3	847.7
75th Percentile	721.4	401.7	419.8	535.1
Median	98.8	83.7	44.9	63.5
25th Percentile	46.5	44.0	18.5	30.4
10th Percentile	18.5	15.1	12.2	14.6

Margins (%)

	Operating Margin Latest	Operating Margin 5-Year Avg	Net Margin Latest	Net Margin 5-Year Avg	Asset Turnover Latest	Asset Turnover 5-Year Avg	Return on Inv. Cap. Latest	Return on Inv. Cap. 5-Year Avg	Return on Assets Latest	Return on Assets 5-Year Avg	Return on Equity Latest	Return on Equity 5-Year Avg
Median	10.04	8.85	3.01	2.71	150.73	138.57	5.56	5.40	4.13	3.97	9.08	5.31
SIC Composite	13.64	14.34	4.40	3.58	150.83	135.88	8.09	6.87	6.63	4.86	9.01	4.57
Large Composite	13.44	14.51	4.25	3.57	155.14	136.09	9.28	7.08	6.60	4.86	8.39	4.67
Small Composite	6.57	-2.64	-0.50	-13.48	114.22	95.77	-1.88	-15.78	-0.57	-12.91	-0.76	-22.75

Equity Valuation Ratios (Multiples)

	Price/Earnings Latest	Price/Earnings 5-Year Avg	Market/Book Latest	Market/Book 5-Year Avg	Price/Sales Latest	Price/Sales 5-Year Avg	Price/Cash Flow Latest	Price/Cash Flow 5-Year Avg	Price/Operating Income Latest	Price/Operating Income 5-Year Avg	Dividend Yield (% of Price) Latest	Dividend Yield (% of Price) 5-Year Avg
Median	11.02	18.84	1.27	2.08	0.36	0.68	9.47	18.01	3.27	4.52	0.00	0.00
SIC Composite	11.10	16.47	1.60	2.39	0.49	0.78	8.66	19.69	3.58	5.46	3.20	2.12
Large Composite	11.91	15.72	1.82	2.40	0.51	0.77	9.49	19.02	3.77	5.28	3.41	2.35
Small Composite	NMF	NMF	0.70	1.92	0.66	0.59	11.26	NMF	9.99	NMF	0.00	0.00

Growth Rates (%) / **Cost of Equity Capital** (%)

	Growth Rates: Analysts' Estimate	CAPM	CAPM + Size Prem	3-Factor Fama-French	Discounted Cash Flow 1-Stage	Discounted Cash Flow 3-Stage
Median	11.63	9.79	11.32	13.96	11.63	14.40
SIC Composite	11.63	10.03	11.10	15.93	11.87	17.30
Large Composite	11.44	9.93	10.51	15.97	11.98	17.20
Small Composite	11.63	10.16	12.78	14.56	11.63	6.50

Weighted Average Cost of Capital (%) / **Levered Betas** / **Unlevered Betas**

	CAPM	CAPM + Size Prem	3-Factor Fama-French	Discounted Cash Flow 1-Stage	Discounted Cash Flow 3-Stage	Levered Raw Beta	Levered Adjusted Beta	Unlevered Adjusted Beta
Median	9.80	11.62	13.27	11.81	13.11	0.55	0.54	0.27
SIC Composite	9.77	10.57	14.19	11.15	15.22	0.61	0.57	0.46
Large Composite	9.66	10.10	14.26	11.22	15.20	0.61	0.56	0.47
Small Composite	11.02	12.08	12.80	11.61	9.53	0.95	0.59	0.26

risk of an asset relative to the market and the return that can be expected from that asset. The CAPM model determines the cost of equity for any company as equal to the riskless rate plus an amount proportionate to its systematic risk. The formula is:

$$k_i = r_f + (b_i \times ERP)$$

where:

k_i = The cost of equity
r_f = The riskless yield
b_i = Beta of company i
ERP = Expected equity risk premium

The regression for each company is run over 60 months of total return data. All CAPM models in the yearbook use the yield on a 20-year Treasury bond for the riskless rate and the long-horizon equity risk premium for the equity risk premium. Values for both the expected riskless rate and the expected equity risk premium (ERP) can be found in the *SBBI Valuation Edition Yearbook*. The yearbook also provides industry betas that can be used to make modifications to the current CAPM assumptions.

The *Cost of Capital Yearbook* displays two CAPM-based cost of equity estimates with identical OLS-adjusted betas; however, the second CAPM model incorporates a size premium. The size premium is included to account for the additional return provided by small companies over large companies but not captured by the standard CAPM model (see the section on firm size premium for more information). For a particular industry, the size premium is determined from the individual company size premia, which are based on the equity capitalization of each company. (A size premium is added only to mid-cap, low-cap, and micro-cap companies.) For composites, the size premium is an equity-capitalization-weighted average of the size premia of the companies included in the industry.

When determining a cost of equity for an industry, the CAPM adjusted for size best represents the entire industry. The size adjustment for an industry dominated by large companies will be less than for an industry comprised mostly of small- or mid-cap companies. When valuing individual companies, however, it would be most appropriate to use the CAPM model that is not adjusted for size and instead add a size premium relative to the equity capitalization of the subject company. Full size premium analysis at the firm level is presented in the *SBBI Valuation Edition Yearbook*.

Fama-French Three-factor Model

The Fama-French three-factor model is a multiple linear regression model developed by Eugene Fama and Kenneth French. The model is estimated by running a time series multiple regression for each company. The dependent variable is the company's monthly excess returns over Treasury-bill returns. The independent variables are:

- The monthly excess return on the market over Treasury bills
- *SMB* (small minus big): The difference between the monthly return on small-cap stocks and large-cap stocks

- *HML* (high minus low): The difference between monthly returns on high book-to-market stocks and low book-to-market stocks

The Fama-French three-factor model is shown in the next equation:

$$k_i = r_f + (b_i \times ERP) + (s_i \times SMBP) + (h_i \times HMLP)$$

where:

k_i	= Cost of equity capital for company *i*
r_f	= The risk-free rate
b_i	= Market coefficient in the Fama-French regression
ERP	= The expected equity risk premium
s_i	= Small-minus-big coefficient in the Fama-French regression
SMBP	= The expected small-minus-big risk premium, estimated as the difference between the historical average annual returns on the small-cap and large-cap portfolios
h_i	= High-minus-low coefficient in the Fama-French regression
HMLP	= The expected high-minus-low risk premium, estimated as the difference between the historical average annual returns on the high book-to-market and low book-to-market portfolios

The Fama-French three-factor model attempts to improve on the single-variable CAPM model by incorporating additional market variables to explain a company's expected return more adequately. These variables include the size of the company and its book-to-market ratio (capturing the size effect and the financial distress of the firm), in addition to the single-market variable of the CAPM. Because this model incorporates more information into the cost of equity estimate than the typical CAPM, estimates on average tend to be higher.

Discounted Cash Flow Models

The discounted cash flow model (DCF), or income approach, was developed by John Burr Williams and elaborated by Myron J. Gordan and Eli Shapiro. The idea behind the DCF model is that the present value of a company can be estimated by discounting its dividends or expected cash flows using the firm's appropriate discount rate. Therefore, the set of inputs needed to determine a company's present value include the value of future cash flows, the rate at which these cash flows will grow, and the discount rate that will equate future cash flows to their present value. Alternatively, given these inputs, we can use the DCF model to solve for the discount rate or the cost of equity.

Assuming that the present value of a company and its projected cash flows are known, there still exists the difficulty of obtaining a cash flow growth forecast, since cash flows tend not to grow at a constant rate forever. In this case, the yearbook uses the expected earning growth rates from the I/B/E/S database of consensus long-term growth rate estimates.

The one-stage and three-stage discounted cash flow models presented in the *Cost of Capital Yearbook* are both rooted in the Gordon Growth Model, which is:

$$P_i = D / (k_i - g_i)$$

where:

P_i = The price per share for company i
D_i = The dividend per share for company i at the end of year 1
k_i = The discount rate for company i
g_i = The dividend growth rate for company i

To solve for the cost of equity capital, the formula is rewritten as:

$$k_i = \{[D_i \times (1 + g)]/P_i\} + g_i$$

The single-stage growth model describes the cost of equity capital for a company that has a constant expected cash flow growth rate projected indefinitely into the future. One drawback of the single-stage model is that if a company pays no dividends, its cost of equity is equivalent to its growth rate. Given such shortcomings, the practitioner must recognize the limitations of the model on its own and view the results as a point of comparison with other models in estimating the cost of equity.

It is probably unrealistic to assume that a firm's cash flows will grow at a constant rate in perpetuity. To improve the predictability of the DCF cost of equity model, a more realistic assumption might be that the growth rate of cash flows changes over time. The three-stage DCF model estimates the cost of equity employing three different growth rate estimates at different future time periods. The *Cost of Capital Yearbook* calculates the three-stage DCF cost of equity for each industry assuming that cash flows will grow at the analyst's company-specific rates for the first five years, at an industry-average growth rate for the next five years, and finally at a growth rate for the entire economy (expected real growth rate plus an inflation estimate) for all other time periods. The results of each model are shown in the bottom section of Exhibit 13.4.

IBBOTSON *BETA BOOK*

The Ibbotson *Beta Book*, published semiannually, includes company-specific betas for more than 5,000 companies. Company betas are calculated for the ordinary least squares (OLS) CAPM and Fama-French model with regression factors. A sample page from the *Beta Book* can be seen in Exhibit 13.5.

The *Beta Book* does not provide cost of equity estimates for individual companies; however, it does provide practitioners with the statistics necessary for calculating cost of equity under both the Capital Asset Pricing Model and the three-factor model described by Fama and French.

Those companies included in the *Beta Book* are selected from the COMPUSTAT database based on three criteria:

1. A company must have at least 36 months of return data available.
2. It must have sales greater than $100,000 in the most recent year.
3. It must have a market capitalization greater than $10,000 for the most recent month.

Beta Estimation Methodologies

For the CAPM model, the Ibbotson *Beta Book* provides several equity beta statistics including traditional unlevered and levered ordinary least squares beta estimates, peer group betas, and Ibbotson adjusted betas. For the Fama-French model, the beta factor in addition to size and value factors identified in the model are provided.

All CAPM regressions in the *Beta Book* are run over a 60-month period, using the S&P 500 total returns as a proxy for the market returns and the yield on a 30-day Treasury bill as a proxy for the risk-free asset. If a company has less than 60 months of historical returns, then betas are calculated using its available return data with a minimum of 36 months being acceptable.

Please refer to the "*Cost of Capital Yearbook*" section of this chapter for detailed information about the CAPM and Fama-French models.

Levered and Unlevered Betas

The OLS CAPM results for each company include both levered and unlevered betas. Whereas the levered beta incorporates both the business and financing risks undertaken by the company and borne by the equity shareholders, the unlevered beta excludes the risk implicit in the financial structure of a company and only reflects its business risk. This allows a practitioner to make adjustments to the capital structure inherent in a levered beta by relevering a company's unlevered beta with a debt structure more similar to the subject company being valued. The unlevered beta is also useful when it may be difficult to make comparisons within an industry for a company that has much higher leverage than its peers. In this case, because the unlevered beta reflects only business risk and not financial risk, the practitioner could relever the company's beta with that of its industry to make a more comparable peer group analysis.

Ibbotson (Adjusted) Betas

The *Beta Book* includes an adjusted or forward-looking beta for each of the CAPM models. The adjustment incorporates the theory that a company's beta tends to revert toward its industry's average beta over time. The adjusted beta is calculated using the

Exhibit 13.5 Sample Page from the *Beta Book* Second 2001 Edition

Ibbotson Associates' Beta Book

		CAPM: Ordinary Least Squares				
		Levered				
Ticker	Company	Raw Beta	t-Stat	R-Sqr	Pr Grp Beta	Ibbotson Beta
OTL	OCTEL CORP*	**0.44**	1.11	0.03	0.66	**0.46**
OCLR	OCULAR SCIENCES INC*	**0.56**	1.20	0.03	0.79	**0.60**
OCN	OCWEN FINANCIAL CORP*	**0.85**	2.20	0.08	0.97	**0.86**
ODETA	ODETICS INC -CL A	**1.22**	1.92	0.06	1.77	**1.34**
ODWA	ODWALLA INC	**-0.13**	-0.32	0.00	0.58	**-0.05**
ODP	OFFICE DEPOT INC	**1.30**	3.26	0.16	0.92	**1.26**
OMX	OFFICEMAX INC	**1.11**	3.03	0.14	0.93	**1.09**
OLOG	OFFSHORE LOGISTICS	**0.97**	2.49	0.10	0.99	**0.97**
OGE	OGE ENERGY CORP	**0.02**	0.14	0.00	0.12	**0.02**
OGLE	OGLEBAY NORTON CO	**0.49**	1.73	0.05	0.58	**0.50**
OCAS	OHIO CASUALTY CORP	**0.60**	1.88	0.06	0.75	**0.61**
OVBC	OHIO VALLEY BANC CORP	**0.08**	0.55	0.01	0.97	**0.10**
ODC	OIL DRI CORP AMERICA	**0.26**	1.06	0.02	1.02	**0.30**
OLGR	OILGEAR CO	**0.06**	0.18	0.00	1.60	**0.19**
ODFL	OLD DOMINION FREIGHT	**0.00**	0.01	0.00	0.66	**0.03**
OLDB	OLD NATIONAL BANCORP	**0.56**	3.22	0.15	0.97	**0.57**
ORI	OLD REPUBLIC INTL CORP	**0.52**	2.16	0.07	0.75	**0.53**
OSBC	OLD SECOND BANCORP INC/IL	**0.19**	1.30	0.03	0.97	**0.21**
OLN	OLIN CORP	**0.69**	2.10	0.07	0.90	**0.71**
OLY	OLYMPIC CASCADE FINL	**1.23**	1.92	0.06	1.84	**1.37**
ZEUS	OLYMPIC STEEL INC	**0.24**	0.57	0.01	0.79	**0.30**
OMG	OM GROUP INC	**0.62**	2.90	0.13	0.66	**0.62**
OMEF	OMEGA FINL CORP	**0.36**	2.28	0.08	0.97	**0.38**
OHI	OMEGA HEALTHCARE INVS INC	**0.75**	2.06	0.07	0.30	**0.71**
OME	OMEGA PROTEIN CORP*	**0.60**	0.99	0.03	0.58	**0.60**
OMM	OMI CORP*	**0.65**	0.86	0.02	0.81	**0.70**
OMNI	OMNI ENERGY SERVICES CORP*	**-0.59**	-0.65	0.01	0.87	**-0.03**
3ZONE	OMNI NUTRACEUTICALS INC	**-0.07**	-0.07	0.00	0.65	**0.24**
3ORXR	OMNI RAIL PRODUCTS INC	**-0.19**	-0.25	0.00	0.83	**0.12**
OUSCD	OMNI USA INC	**0.99**	1.50	0.04	1.41	**1.10**
OCR	OMNICARE INC	**0.81**	1.78	0.05	0.90	**0.82**
OMC	OMNICOM GROUP	**1.00**	5.50	0.34	1.74	**1.01**
OMTL	OMTOOL LTD*	**2.32**	2.63	0.13	1.74	**2.11**
ASGN	ON ASSIGNMENT INC	**0.66**	1.92	0.06	1.74	**0.74**
ONCO	ON COMMAND CORP*	**0.64**	1.61	0.04	1.40	**0.72**
3ONST	ON STAGE ENTERTAINMENT INC*	**0.31**	0.19	0.00	0.75	**0.59**
ONTC	ON TECHNOLOGY CORP	**2.46**	2.71	0.11	1.74	**2.19**
ONSS	ON-SITE SOURCING INC*	**0.04**	0.07	0.00	1.61	**0.38**
OLP	ONE LIBERTY PROPERTIES INC	**0.10**	0.92	0.01	0.30	**0.10**
ONPR	ONE PRICE CLOTHING STORES	**0.10**	0.19	0.00	1.10	**0.28**
3ONCL	ONECLASS SYNERGY CORP	**-3.33**	-1.47	0.04	0.79	**-0.07**
OCQ	ONEIDA LTD	**0.57**	1.57	0.04	0.55	**0.57**
OKE	ONEOK INC	**0.42**	1.83	0.05	0.30	**0.41**
3OGAM	ONLINE GAMING SYS LTD	**1.56**	1.94	0.06	1.74	**1.62**
3ONSE	ONSITE ENERGY CORP -CL A	**-1.25**	-1.06	0.02	0.66	**-0.29**

OCTEL CORP*

Source: Beta Book Second 2001 Edition, © 2001 Ibbotson Associates, Inc. Used with permission.

Exhibit 13.5 (Continued)

Second 2001 Edition
Data Through June 2001

Unlevered Raw Beta	Unlevered Ibbotson Beta	FF Beta	FF t-Stat	SMB Prem	SMB t-Stat	HML Prem	HML t-Stat	FF R-Sqr
0.29	**0.22**	**0.79**	1.93	**1.58**	3.29	**3.45**	9.63	0.16
0.56	**0.59**	**1.02**	2.12	**1.75**	3.07	**4.59**	10.39	0.15
0.69	**0.63**	**1.18**	2.86	**1.03**	2.06	**3.39**	8.48	0.15
0.85	**0.77**	**0.61**	1.03	**3.18**	4.40	**-6.54**	-11.20	0.33
-0.13	**-0.05**	**-0.13**	-0.31	**2.81**	5.32	**-0.24**	-0.57	0.11
1.09	**1.01**	**1.46**	3.52	**3.46**	6.87	**1.39**	3.43	0.26
0.88	**0.76**	**1.45**	3.98	**4.18**	9.47	**3.21**	9.02	0.31
0.71	**0.61**	**0.98**	2.28	**-1.07**	-2.04	**0.22**	0.53	0.11
0.01	**0.01**	**0.18**	1.29	**-0.45**	-2.57	**1.73**	12.42	0.25
0.28	**0.13**	**0.80**	2.80	**3.02**	8.66	**2.99**	10.64	0.22
0.50	**0.48**	**0.72**	2.02	**0.23**	0.53	**1.17**	3.36	0.07
0.06	**0.06**	**0.13**	0.76	**0.14**	0.72	**0.45**	2.79	0.01
0.18	**0.15**	**0.57**	2.21	**1.32**	4.20	**3.11**	12.25	0.15
0.04	**0.08**	**0.41**	1.11	**2.54**	5.69	**3.36**	9.35	0.10
0.00	**0.01**	**0.12**	0.47	**1.93**	6.12	**1.08**	4.26	0.10
0.38	**0.29**	**0.59**	3.10	**-0.51**	-2.21	**0.36**	1.93	0.18
0.49	**0.49**	**0.65**	2.54	**-0.48**	-1.52	**1.45**	5.77	0.14
0.17	**0.18**	**0.35**	2.23	**0.81**	4.24	**1.56**	10.15	0.12
0.56	**0.54**	**0.85**	2.33	**1.12**	2.53	**1.50**	4.22	0.09
1.11	**1.21**	**1.17**	1.69	**2.46**	2.91	**-0.76**	-1.12	0.10
0.14	**0.11**	**0.69**	1.60	**3.48**	6.66	**4.42**	10.50	0.14
0.47	**0.43**	**0.80**	3.50	**0.48**	1.73	**1.86**	8.32	0.19
0.31	**0.31**	**0.45**	2.57	**0.44**	2.07	**0.86**	4.99	0.11
0.39	**0.06**	**1.25**	3.34	**2.36**	5.22	**4.94**	13.52	0.22
0.48	**0.45**	**0.90**	1.39	**2.36**	3.11	**3.17**	5.61	0.08
0.44	**0.38**	**0.46**	0.61	**4.48**	5.05	**-1.08**	-1.64	0.20
-0.42	**-0.02**	**-0.27**	-0.27	**0.51**	0.42	**3.20**	3.58	0.03
-0.05	**0.10**	**-0.47**	-0.47	**7.39**	6.14	**-4.81**	-4.95	0.21
-0.12	**0.01**	**-0.94**	-1.16	**-1.77**	-1.80	**-7.65**	-9.66	0.10
0.67	**0.38**	**1.10**	1.51	**2.88**	3.26	**0.89**	1.25	0.07
0.62	**0.58**	**1.20**	2.50	**0.22**	0.38	**4.09**	8.71	0.14
0.92	**0.93**	**0.90**	4.53	**-0.31**	-1.29	**-0.95**	-4.86	0.36
2.31	**2.10**	**2.20**	2.40	**4.76**	4.40	**-1.29**	-1.55	0.23
0.66	**0.74**	**0.67**	1.77	**0.85**	1.85	**0.03**	0.09	0.07
0.45	**0.33**	**0.90**	2.11	**2.77**	5.39	**2.34**	5.67	0.12
0.19	**0.24**	**-0.42**	-0.24	**0.52**	0.26	**-7.34**	-4.68	0.05
2.32	**2.05**	**3.05**	3.31	**9.43**	8.43	**5.32**	5.90	0.26
0.03	**0.20**	**-0.27**	-0.47	**4.31**	6.10	**-4.00**	-7.05	0.26
0.06	**0.03**	**0.23**	2.10	**1.22**	9.12	**1.25**	11.57	0.21
0.06	**0.05**	**0.15**	0.26	**1.56**	2.14	**0.38**	0.65	0.02
-2.50	**-0.04**	**-2.13**	-0.86	**6.22**	2.06	**11.94**	4.90	0.06
0.38	**0.30**	**1.01**	2.70	**2.22**	4.88	**4.43**	12.10	0.17
0.25	**0.14**	**0.66**	2.80	**-0.07**	-0.23	**2.49**	10.86	0.20
1.05	**0.54**	**2.56**	3.16	**7.75**	7.87	**9.82**	12.35	0.23
-1.08	**-0.24**	**-1.09**	-0.84	**0.96**	0.61	**1.55**	1.21	0.02

ONSITE ENERGY CORP -CL A

Vasicek Shrinkage Technique, which takes the statistically weighted average of the company beta and the industry beta. The formula is:

$$\text{Adjusted Beta} = (1 - \text{weight}) \times \text{peer group beta} + \text{weight} \times \text{company beta}$$

where,

$$\text{Weight} = (\text{cross-sectional standard error})^2/[(\text{cross-sectional standard error})^2 + (\text{time series beta standard error})^2]$$

The Vasicek adjustment focuses on the statistical significance of the beta estimate, using the standard error to appropriately weight the company and industry beta. If the historical company beta displays low statistical significance, the higher standard error will result in the company's having a lower weighting than the industry, and vice versa. The greater the statistical confidence of the company's regression beta, the closer that weight is to 1.0.

The motivation behind adjusting a beta is to calculate a forward-looking estimate from a historical beta. An advantage of the Vasicek technique is that the adjustment does not need to be made toward the market as a whole but can be made toward an industry or a peer group. Also, if a company's beta estimate seems statistically unreliable, confidence in the prospective beta can be increased by assigning a greater weight to its industry or peer group.

Peer Group and Industry Betas

In addition to individual company raw and adjusted betas, the Ibbotson *Beta Book* also provides peer group betas. Peer group betas are calculated by taking the sales-weighted average of the betas for each industry in which a company has sales. These betas can be useful for comparison purposes or in place of a company beta displaying poor regression statistics.

The *Beta Book* includes OLS betas for industries defined by two-digit SIC codes. Previous versions of the publication constructed each industry beta using the market-capitalization-weighted average of the betas of all companies in the industry. Only companies with 75% of their sales in the industry as defined by the primary SIC codes of its business segments were included. However, this "pure play" approach tends to exclude many domestic conglomerates that are major participants in the industry.

In order to calculate an industry beta using data from all companies participating in a particular industry, the *Beta Book* uses the Full Information Beta procedure developed by Paul Kaplan and James Peterson. In this procedure, a single market-capitalization-weighted cross-sectional regression is performed using individual company betas as the dependent variable and the percentage of exposure or sales to the industry as the independent variables. The resulting regression coefficients for each industry are the estimates of the pure-play industry betas for which data from every company in a particular industry has been incorporated.

COST OF CAPITAL CENTER

The Cost of Capital Center at *www.ibbotson.com* is a Web site devoted to business valuation issues. As the name suggests, this site is the center of all things cost of capital. The site offers analysis on more than 300 industries, 5,000 companies, and 145 countries.

Ibbotson Associates uses the Cost of Capital Center as an outlet for much of its valuation-related data. Visitors to the site can purchase data from the *Cost of Capital Yearbook* and the *Beta Book* on a per-usage basis. Anyone interested in international cost of capital analysis will find the collection of international reports at the site critical in calculations. The Cost of Capital Center is segmented into four main product groups:

1. Industry Analysis
2. Company Analysis
3. Global Analysis
4. Risk Premia

Industry analysis available at the Cost of Capital Center is taken directly from the *Cost of Capital Yearbook* published by Ibbotson Associates. Industries are organized by SIC code and are available for immediate download at the site. Ibbotson presents industry analysis on more than 300 industries and updates the data at the Web site quarterly. Included in the analysis are multiple measures of cost of equity, WACC, beta, capital structure ratios, growth rates, industry multiples, and other important financial statistics. (See the section on the *Cost of Capital Yearbook* for more information.)

Company analysis presented on the Web site includes individual company betas and tax rates. Over 5,000 company betas are taken directly from Ibbotson's *Beta Book* and sold at the site on a per-company basis. For each company, multiple measures of beta are displayed, including levered and unlevered betas, betas adjusted toward their peer group, and statistics necessary to calculate Fama-French three-factor cost of equity. (See the section on the *Beta Book* for more information.) In addition to beta analysis at the company level, Ibbotson also presents tax rate estimates. Research has shown that using the top marginal tax rate in cost of capital calculations may be overstating the effect of taxes. Ibbotson provides tax rate estimates on more than 5,000 companies that can be used in discounting projected future cash flows. Tax rate estimates based on the most recent fiscal year and the five-year average are presented for each company. Company betas are updated on the Web site quarterly, while company tax rates are updated annually (usually in the fall).

The remaining sections of the Web site present data reports for global analysis and risk premia analysis. The global analysis section provides equity risk premia and cost of equity estimates for a variety of countries. The *International Equity Risk Premia Report* provides ERP estimates on 16 different developed countries. The analysis can be customized to view the ERP for any time period covered for each country. When available, data are presented for both the long- and short-horizon ERP, in both

U.S. dollars and local currency. Most of the ERP estimates use a historical date range that extends back to 1970. Using a longer string of data, Ibbotson is also able to provide a *Canadian Risk Premia over Time Report* and a *United Kingdom Risk Premia over Time Report*.

Some of the most popular reports available on the Web site are the *International Cost of Capital Report* and the *International Cost of Capital Perspectives Report*. The first presents cost of equity estimates on approximately 145 countries from the perspective of U.S. investors. This report offers estimates using up to five different cost of equity models for each country, based on data availability. The "perspectives" report also covers 145 countries but from the perspective of international investors. This report utilizes only one model for cost of equity estimation (the country risk rating model) and presents the estimates from the perspective of investors in Australia, Canada, France, Germany, Japan, and the United Kingdom. Examples of the data offered in both reports can be seen in Exhibits 13.6 and 13.7. All international reports are updated annually on the Web site.

In addition to the international reports available at the Cost of Capital Center, a number of U.S.-based risk premium reports are presented: specifically, the *Risk Premia over Time Report* and the *Standard & Poor's Corporate Value Consulting Risk Premium Report* (formerly the *PricewaterhouseCoopers Risk Premium Study*). The *Risk Premia over Time Report* provides equity and size premia over all historical time periods dating back to 1926. The analysis can be customized by choosing the beginning and ending date for risk premia estimation. The report contains long-, intermediate- and short-term equity risk premia and mid-, low- and micro-cap size premia. All of the content in this report is from the *SBBI Valuation Edition Yearbook*, primarily from Appendix A. The *Standard & Poor's Corporate Value Consulting Risk Premium Report* (formerly the *PricewaterhouseCoopers Risk Premium Study*) is written by Roger Grabowski and David King, currently with Standard & Poor's Corporate Value Consulting, formerly of PricewaterhouseCoopers. Companies are divided into 25 different size groups based on:

- Market value of equity
- Book value of equity
- Five-year average net income
- Market value of invested capital
- Total assets
- Five-year average EBITDA
- Sales
- Number of employees

Exhibit 13.6 *International Cost of Capital Report*

	International Cost of Capital Models						
	Country Credit Rating						
Country	Log Model	Linear Model	Country Spread	International CAPM Model	Globally Nested CAPM	Relative Standard Deviation Model	Available Models
	Thru 3-01		Thru 12-00	Thru 12-00	Thru 12-00	Thru 12-00	
Afghanistan	47.61	30.53					2
Albania	35.20	28.31					2
Algeria	24.87	24.19					2
Angola	39.16	29.24					2
Argentina	20.34	21.16	19.92	16.22	30.49	49.39	6
Australia	12.19	12.50		13.09		18.14	4
Austria	11.36	11.31		11.90		16.15	4

Source: International Cost of Capital Report 2001,

Exhibit 13.7 *International Cost of Capital Perspectives Report*

	International Cost of Capital Country Credit Rating Model from International Perspectives											
	Australia Country Credit Rating		Canada Country Credit Rating		France Country Credit Rating		Germany Country Credit Rating		Japan Country Credit Rating		U.K. Country Credit Rating	
Country	Log Model	Linear Model	Log Model	Linear Model	Log Model	Linear Model	Log Model	Linear Model	Log Model	Linear Model	Log Model	Linear Model
	Thru 3-01		Thru 3-01		Thru 3-01		Thru 3-01		Thru 3-01		Thru 3-01	
Afghanistan	48.22	32.67	48.62	31.72	44.77	31.16	44.95	30.70	40.82	26.28	44.76	30.26
Albania	37.12	30.73	36.37	29.53	34.79	29.35	34.37	28.76	30.13	24.34	34.27	28.39
Algeria	27.87	27.12	26.17	25.47	26.47	25.99	25.56	25.14	21.23	20.73	25.54	24.92
Angola	40.66	31.55	40.28	30.45	37.97	30.11	37.75	29.57	33.54	25.15	37.62	29.18
Argentina	23.81	24.46	21.68	22.48	22.82	23.52	21.69	22.49	17.32	18.08	21.70	22.38
Australia	16.52	16.88	13.63	13.96	16.25	16.46	14.73	14.90	10.29	10.51	14.81	15.10
Austria	15.78	15.84	12.82	12.78	15.59	15.49	14.03	13.86	9.58	9.47	14.11	14.10

Source: International Cost of Capital Perspectives Report 2001,

Chapter 14

Arbitrage Pricing Model

Explanation of the APT Model
APM Formula
Summary

The concept of the *arbitrage pricing theory* (APT) was introduced by academicians in 1976.[1] However, it was not until 1988 that data in a commercially usable form became generally available to permit the application of the theory to the estimation of required rates of return in day-to-day practice. Despite the theory's longevity, it still is not widely used by practitioners today.

EXPLANATION OF THE APT MODEL

As noted in Chapter 9, the Capital Asset Pricing Model (CAPM) is a univariate model; that is, CAPM recognizes only one risk factor—systematic risk relative to a market index. In a sense, APT is a multivariate extension of the CAPM. APT recognizes a variety of risk factors that may bear pervasively on an investment's required rate of return, one of which may be a CAPM-type "market" or "market timing" risk. It may be argued that the CAPM and APT are not mutually exclusive, nor is one of greater or lesser scope than the other. It also can be argued that the CAPM beta implicitly reflects the information included separately in each of the APT "factors." However, in spite of its more limited use, most academicians consider the arbitrage pricing model (APM) richer in its information content and explanatory and predictive power.

Whereas the nature of the CAPM is a single regression, the nature of the APM is a multiple regression. In the APM, the cost of capital for an investment varies according to that investment's sensitivity to each of several different risk factors.

The model itself does not specify what the risk factors are. Most formulations of the APM consider only risk factors of a pervasive macroeconomic nature, such as:

- *Yield spread.* The differential between risky and less risky bonds as a measure of investors' consensus confidence in economic prosperity
- *Interest rate risk.* Measured by the difference between long-term and short-term yields

- *Business outlook risk.* Measured by changes in forecasts for economic variables such as gross national product (GNP)
- *Inflation risk.* Measured by changes in inflation forecasts

The beta used in the CAPM may or may not be one of the risk factors included in any particular practitioner's version of the APM. In some versions, more industry-specific factors may be included, such as changes in oil prices. Exhibit 14.1 explains one version of APT risk factors.

APM FORMULA

The econometric estimation of the arbitrage pricing model with multiple risk factors yields this formula:

Formula 14.1

$$E(R_i) = R_f + (B_{i1}K_1) + (B_{i2}K_2) + \dots + (B_{in}K_n)$$

where:

$E(R_i)$	= Expected rate of return on the subject security
R_f	= Rate of return on a risk-free security
$K_1 \dots K_n$	= Risk premium associated with factor K for the average asset in the market
$B_{i1} \dots B_{in}$	= Sensitivity of security i to each risk factor relative to the market average sensitivity to that factor

Roger Ibbotson and Gary Brinson make the following observations regarding APT:

> In theory, a specific asset has some number of units of each risk; those units are each multiplied by the appropriate risk premium. Thus, APT shows that the equilibrium expected return is the risk-free rate plus the sum of a series of risk premiums. APT is more realistic than CAPM because investors can consider other characteristics besides the beta of assets as they select their investment portfolios.[2]

Research has shown that the cost of equity capital as estimated by the APM tends to be higher for some industries (e.g., oil) and lower for others (e.g., certain utility groups) than the cost of equity capital using the CAPM. Early research also suggests that the multivariate APT model explains expected rates of return better than does the univariate CAPM.[3]

So, if the APM is more powerful than the CAPM, why is the APM not used more? For one thing, the variables are not specified. Also, there is no universal consensus about which variables are likely to have the greatest efficacy. Furthermore, the APM is complicated in that coefficients for several factors, rather than just one factor, must be worked out for each company for each specific time it is going to be applied.

Exhibit 14.1 Explanation of APT Risk Factors

Confidence Risk

Confidence Risk is the unanticipated changes in investors' willingness to undertake relatively risky investments. It is measured as the difference between the rate of return on relatively risky corporate bonds and the rate of return on government bonds, both with 20-year maturities, adjusted so that the mean of the difference is zero over a long historical sample period. In any month when the return on corporate bonds exceeds the return on government bonds by more than the long-run average, this measure of Confidence Risk is positive. The intuition is that a positive return difference reflects increased investor confidence because *the required yield on risky corporate bonds has fallen relative to safe government bonds*. Stocks that are positively exposed to the risk then will rise in price. (Most equities *do* have a positive exposure to Confidence Risk, and small stocks generally have greater exposure than large stocks.)

Time Horizon Risk

Time Horizon Risk is the unanticipated changes in investors' desired time to payouts. It is measured as the difference between the return on 20-year government bonds and 30-day Treasury bills, again adjusted to be mean zero over a long historical sample period. A positive realization of Time Horizon Risk means that the price of long-term bonds has risen relative to the 30-day Treasury bill price. This is a signal that investors require a lower compensation for holding investments with relatively longer times to payouts. The price of stocks that are positively exposed to Time Horizon Risk will rise to appropriately decrease their yields. (Growth stocks benefit more than income stocks when this occurs.)

Inflation Risk

Inflation Risk is a combination of the unexpected components of short- and long-run inflation rates. Expected future inflation rates are computed at the beginning of each period from available information: historical inflation rates, interest rates, and other economic variables that influence inflation. For any month, Inflation Risk is the unexpected surprise that is computed at the end of the month, i.e., it is the difference between the actual inflation for that month and what had been expected at the beginning of the month. Since most stocks have negative exposures to inflation risk, a positive inflation surprise causes a negative contribution to return, whereas a negative inflation surprise (a deflation shock) contributes positively toward return.

Industries whose products tend to be "luxuries" are most sensitive to Inflation Risk. Consumer demand for "luxuries" plummets when real income is eroded through inflation, thus depressing profits for industries such as retailers, services, eating places, hotels and motels, and toys. In contrast, industries least sensitive to Inflation Risk tend to sell "necessities," the demands for which are relatively insensitive to declines in real income. Examples include foods, cosmetics, tire and rubber goods, and shoes. Also companies that have large asset holdings such as real estate or oil reserves may benefit from increased inflation.

Business Cycle Risk

Business Cycle Risk represents unanticipated changes in the level of real business activity. The expected values of a business activity index are computed both at the beginning and end of the month, using only information available at those times. Then, Business Cycle Risk is calculated as the difference between the end-of-month value and the beginning-of-month value. A positive realization of Business Cycle Risk indicates that the expected growth rate of the economy,

(Continued)

Exhibit 14.1 (Continued)

measured in constant dollars, has increased. Under such circumstances firms that are more positively exposed to business cycle risk—for example, firms such as retail stores that do well when business activity increases as the economy recovers from a recession—will outperform those such as utility companies that do not respond much to increased levels in business activity.

Market Timing Risk

Market Timing Risk is computed as that part of the S&P 500 total return that is not explained by the first four macroeconomic risks and an intercept term. Many people find it useful to think of the APT as a generalization of the CAPM, and by including this Market Timing factor, the CAPM becomes a special case: If the risk exposures to all of the first four macroeconomic factors were exactly zero, then Market Timing Risk would be proportional to the S&P 500 total return. Under these extremely unlikely conditions, a stock's exposure to Market Timing Risk would be equal to its CAPM beta. Almost all stocks have a positive exposure to Market Timing Risk, and hence positive Market Timing surprises increase returns, and vice versa.

A natural question, then, is: "Do Confidence Risk, Time Horizon Risk, Inflation Risk, and Business Cycle Risk help to explain stock returns better than I could do with just the S&P 500?" This question has been answered using rigorous statistical tests, and the answer is very clearly that they do.

Source: Presented in a talk based on a paper, "A Practitioner's Guide to Arbitrage Pricing Theory," by Edwin Burmeister, Richard Roll, and Stephen A. Ross, written for the Research Foundation of the Institute of Chartered Financial Analysts, 1994. The exhibit is drawn from Notes for "Controlling Risks Using Arbitrage Pricing Techniques," by Edwin Burmeister. Reprinted with permission.

Although The Alcar Group no longer provides APT cost of equity capital estimates, due to low demand for the inputs, William Roper, a vice president of Alcar, had the following to say about APT theory:

> The practical application of APT theory is difficult because it is very hard to determine a valid statistical relationship between an individual company's returns and the macroeconomic factors. There is simply too much "noise" in these relationships. In Exhibit 14.2, the relationship of an individual company is first measured against a portfolio of companies, then the relationship of the portfolio of companies to the macroeconomic factor is measured. However, while this two-step approach tries to relate the individual company to the macroeconomic factors, the relationship still tends to have too much noise.
>
> Therefore, Alcar's formulation was to take a *peer group* of companies relative to the portfolios shown in exhibit 14.2, and combine this with the relationship of the portfolios to the macroeconomic factors. This approach does tend to yield a better (statistically significant?) relationship for the peer group to the macroeconomic factors but the ability to extend this relationship and apply it to an individual company in the peer group remains problematic.

BIRR Portfolio Analysis, Inc., is still a source of information for inputs to the APM. Contact information is given in Appendix C.

SUMMARY

The arbitrage pricing model is a multivariate model for estimating the cost of equity capital. The risk factor variables are not specified, but most formulations use macroeconomic factors that may impact different companies' rates of return to different degrees. The beta in the CAPM may or may not be one of the factors.

Partly because of lack of consensus on the specific factors and the complexity of the model, it has not enjoyed wide usage. Moreover, the macroeconomic factors used in current applications of APT may have a considerably less significant systematic

Exhibit 14.2 APT and CAPM Cost of Equity Capital Estimates Example

APT Estimated Cost of Equity for Air Prods & Chems Inc.

	Beta	× Risk Premium	= Contribution
Risk-Free Rate	N/A	N/A	10.00%
+ CAPM Equity Premium	1.09	3.00%	3.26%
+ Large Capitalization	(0.58)	(0.23)%	0.13%
+ Small Capitalization	0.36	3.30%	1.19%
+ High Cash Flow/Price	0.00	2.53%	0.01%
+ Low Cash Flow/Price	0.43	(3.76)%	(1.64)%
APT Cost of Equity			12.95%
Business Risk (Unlevered K_e)			12.41%
+ Financial Risk			0.54%
APT Cost of Equity			12.95%
Risk-Free Rate			10.00%
+ Short-Term Inflation			(0.40)%
+ Long-Term Inflation			0.24%
+ Interest Yield Term			0.20%
+ Default Risk			2.47%
+ Monthly Production			0.43%
APT Cost of Equity			12.95%

APT uses additional portfolios to overcome limitations of CAPM

Three ways to look at equity risk

APT techniques link the cost of equity to underlying economic factors. This business is quite sensitive to "default risk," or investor confidence in the economy (the spread between high- and low-grade bonds)

Selected Yield Term:	20 years
Selected Market Risk Premium:	Alcar Forecast
Debt/Equity Ratio:	33.69%
R-Squared	43%
Specific Risk:	5%

(Continued)

Exhibit 14.2 (Continued)

Estimated Cost of Equity for Air Prods & Chems Inc.

CAPM/APT Reconciliation

Economic Factors

Portfolio Factors	Short-term Infl.	Long-term Infl.	Interest Yield Term	Default Risk	Monthly Prodn.	TOTAL	Memo: Average Risk Premia
Risk-free Rate						10.00%	
CAPM Eq. Prem.	–0.13%	0.47%	0.49%	1.84%	0.58%	3.26%	3.00%
CAPM K_e						13.26%	
Large Cap	0.02%	0.02%	0.02%	0.06%	0.02%	0.13%	–0.23%
Small Cap	0.11%	0.10%	0.08%	0.73%	0.16%	1.19%	3.30%
High CF/Price	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	2.53%
Low CF/Price	–0.40%	–0.36%	–0.39%	–0.17%	–0.33%	–1.64%	–3.76%
Difference	–0.27%	–0.24%	–0.28%	0.63%	–0.15%	–0.31%	
APT K_e	–0.40%	0.24%	0.20%	2.47%	0.43%	12.95%	
Memo: Avg. Risk Premia	–0.25%	0.31%	0.19%	3.99%	0.60%		

"Small capitalization," or small stock, premium is due to this company's higher sensitivity to default risk.

This Low Cash Flow–to–Price effect indicates stability and lower risk which more than offsets the small cap effect, resulting in a lower estimated cost of equity under APT than CAPM.

Selected Yield Term:	20 years
Selected Market Risk Premium:	Alcar Forecast
Debt/Equity Ratio:	33.69%
APT R-squared improvement:	7%

Source: Exhibit Highlights from *APT!*™: Alcar's Financial Policy Information Service. Reprinted with permission.

impact on the cost of capital for smaller companies or on individual divisional or project decisions than for large national companies.

Notes

1. Stephen A. Ross, "The Arbitrage Theory of Capital Asset Pricing," *Journal of Economic Theory* (December 1976): 241–260; and Stephen A. Ross, "Return, Risk, and Arbitrage," in *Risk and Return in Finance*, Irwin I. Friend and I. Bisksler, eds. (Cambridge, MA: Ballinger, 1977), 189–218. See also Stephen A. Ross, Randolph W. Westerfield, and Jeffrey F. Jaffe, *Corporate Finance*, 3d ed. (Burr Ridge, IL: McGraw-Hill, 1993), 315–337.
2. Roger G. Ibbotson and Gary P. Brinson, *Investment Markets* (New York: McGraw-Hill, 1987), 32. For a more extensive discussion of APT, see Frank K. Reilly, *Investment Analysis and Portfolio Management*, 4th ed. (Fort Worth, TX: The Dryden Press, 1994), 288–291.
3. Tom Copeland, Tim Koller, and Jack Murrin, *Valuation: Measuring and Managing the Value of Companies*, 3d ed. (New York: John Wiley & Sons, Inc., 2000), 226–228.

Part III

Other Topics Related to Cost of Capital

Chapter 15

Minority versus Control Implications of Cost of Capital Data

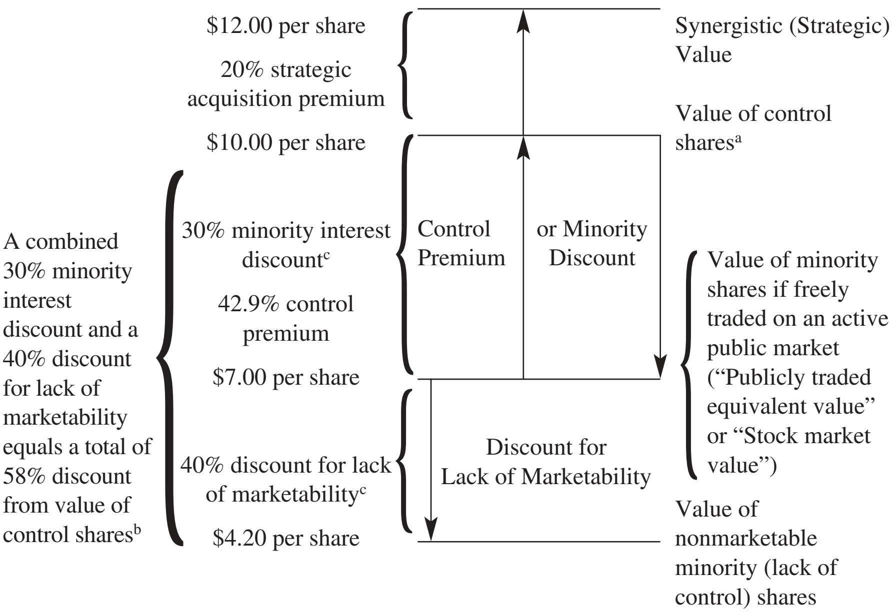
Minority versus Control Has Little or No Impact on Cost of Capital
Company Efficiency versus Shareholder Exploitation
Impact of the Standard of Value
Under What Circumstances Should a Control Premium Be Applied?
  Projected Income May Not Reflect What a Control Owner Would Achieve
  Investment Value Reflecting Synergies
  Factors Affecting a Control Premium a Financial Buyer Might Pay
A Tale of Two Markets
Many Takeovers at Less Than Public Trading Price
Summary

There is much confusion about whether the results of applying cost of capital data, as discussed in this book, in the context of a company or stock valuation produces a minority value or a control value. The difference between the per-share value of a share that represents control and the per-share value of a share that represents a minority interest can be quite significant. (See, e.g., the traditional "levels of value" chart, Exhibit 15.1.) As with many such questions in economics and finance, the answer is: *It depends*.

More than anything else, when the cost of capital is used in the context of valuation, the question of whether the result of discounting or capitalizing represents a minority or a control value depends primarily on *the nature of the cash flows being discounted or capitalized rather than on the discount or capitalization rate*.

In some cases, the answer to this question may hinge on the definition of value sought, for example, fair market value (the value to a *hypothetical* buyer and/or seller) or investment value (the value to a *particular* buyer and/or seller).[1]

Exhibit 15.1 "Levels of Value" in Terms of Characteristics of Ownership



Notes:

[a]Control shares in a privately held company may also be subject to some discount for lack of marketability, but usually not nearly as much as minority shares.

[b]Minority and marketability discounts normally are multiplicative rather than additive. That is, they are taken in sequence:

$	10.00	Control Value
–	3.00	Less: Minority interest discount (.30 × $10.00)
$	7.00	Marketable minority value
–	2.80	Less lack of Marketability discount (.40 × $7.00)
$	4.20	Per share value of non-marketable minority shares

[c]Note that neither the minority/control nor the marketability issue are "all or nothing" matters. Each covers a spectrum of degrees as discussed in the accompanying text.

Source: Jay E. Fishman, Shannon P. Pratt, J. Clifford Griffith, and Mark Wells, *Guide to Business Valuations*, 12th ed. (Fort Worth, TX: Practitioners Publishing Company, 2002), 8–15, copyright © 2002. All rights reserved. Reprinted with permission of Practitioner's Publishing Company. Copies of this *Guide* can be ordered by calling PPC at (800) 323-8724 or log on to *www.ppcnet.com.*

MINORITY VERSUS CONTROL HAS LITTLE OR NO IMPACT ON COST OF CAPITAL

Regardless of which of the major approaches is used to estimate cost of capital (e.g., build-up model, Capital Asset Pricing Model, discounted cash flow model, or arbitrage pricing model), the information is derived from publicly traded stocks. Because these public market transactions represent minority ownership interests, some analysts think that the cost of capital should be adjusted downward in valuing a controlling ownership interest.

This generally is not true!

Recall that the discounting method of valuation and the capitalization method of valuation have two basic elements in common:

1. A numerator consisting of an amount or amounts of expected economic income
2. A denominator consisting of a rate of return at which the economic income is discounted or capitalized

Almost all the difference in the control value versus the minority value in the income approach to valuation is found in the numerator—the expected economic income available to the investor—rather than in the denominator—the discount or capitalization rate.

As Roger Ibbotson has succinctly stated the case:

> When you are purchasing a company you are acquiring the ability to potentially control future cash flows. To acquire this option to exercise control, you must pay a premium. Holding all else constant, it should not impact the discount rate.[2]

Generally speaking, investors will not accept a lower expected rate of return for purchase of a controlling interest than for purchase of a minority interest. Control buyers pay premiums because they expect to do something to increase the cash flows, not because they are willing to accept a lower expected rate of return. What they may do to increase cash flows can range from eliminating inactive relatives from the payroll to drastically increasing prices for products or services of both acquirer and target as a result of absorbing a direct competitor.

Of course, many a public stock has taken a huge tumble in its market price as a result of an acquisition. This usually is because the acquisition failed to achieve the expected increase in cash flows for the target and/or the acquirer. If the market perceives that the returns a company is likely to achieve will fall short of the market's required rate of return, then the market simply adjusts the stock price downward until the expected returns *do* meet the market's required rate of return.

The "control premium" in *Mergerstat/Shannon Pratt's Control Premium Study*™ includes *both* the "control premium" and the "synergistic" or "strategic" premium, as shown in Exhibit 15.1.

COMPANY EFFICIENCY VERSUS SHAREHOLDER EXPLOITATION

Benefits available to a minority shareholder are a function of two distinct factors:

1. Efficiency at the overall company level
2. Differential benefits between control stockholders and minority stockholders (e.g., cash flow that could be available for dividends used for extra compensation to controlling owners)

Controlling stockholders enjoy a number of prerogatives of control, which can have an impact on both of the aforementioned factors as they affect minority stockholders. Some of the more common prerogatives of control include the ability to:

- Appoint or change operational management.
- Appoint or change members of the board of directors.
- Determine management compensation and perquisites.
- Set operational and strategic policy and change the course of the business.
- Acquire, lease, or liquidate business assets, including plant, property, and equipment.
- Select suppliers, vendors, and subcontractors with whom to do business and award contracts.
- Negotiate and consummate mergers and acquisitions.
- Liquidate, dissolve, sell out, or recapitalize the company.
- Sell or acquire Treasury shares.
- Register the company's equity securities for an initial or secondary public offering.
- Register the company's debt securities for an initial or secondary public offering.
- Declare and pay cash and/or stock dividends.
- Change the articles of incorporation or bylaws.
- Set one's own compensation (and perquisites) and the compensation (and perquisites) of related-party employees.
- Select joint venturers and enter into joint venture and partnership agreements.
- Decide what products and/or services to offer and how to price those products/services.
- Decide what markets and locations to serve, to enter into, and to discontinue serving.
- Decide which customer categories to market to and which not to market to.
- Enter into inbound and outbound license or sharing agreements regarding intellectual properties.
- Block any or all of the above actions.[3]

It is apparent that exercise of some of these prerogatives may have an impact on the total cash flows available to the firm, and others will affect the relative benefits ultimately realized by control shareholders versus minority shareholders. It should also be apparent that in projecting expected cash flows, the amounts available to a control owner may not be the same as those available to a minority owner. For example, in many companies control owners set their own compensation (often reflecting their own perceived genius) rather than having an independent compensation committee. In any case, whether a result of company efficiency or differential shareholder benefits, it is the expected cash flows to the investor that drive the value to the investor, not differences in cost of capital between minority investors and control investors.

Enron and its ilk notwithstanding, the exploitation of minority shareholders is far less prevalent in public companies than in private companies, at least in larger public companies. If company cash flows are already maximized and the returns are already distributed pro rata to all shareholders, then there may be no difference between a control value and a minority value.

IMPACT OF THE STANDARD OF VALUE

Some analysts suggest that the appropriate cost of capital for an acquisition should be that of the acquirer rather than the target. This position departs from the standard of *fair market value* (the price at which the property would change hands between *hypothetical* buyers and sellers, with no special motivations) and introduces an element of *investment value* (the value to a *particular* buyer or seller). If the standard of value is fair market value, then the principle that "The cost of capital is a function of the investment, not the investor" clearly applies.

The idea of fair market value is that there is a consensus value that, in economic terms, "clears the market." If the cost of capital is a function of the investment, not the investor, then it is conceivable that the risk perceived by one investor may depart from the market consensus regarding the investment's risk. If the estimated cost of capital for a particular investment for a particular investor is driven by a view that departs from the market consensus, then we are moving away from the standard of fair market value (a consensus value) and toward investment value (value to some particular investor, driven by that investor's unique perceptions or circumstances).

UNDER WHAT CIRCUMSTANCES SHOULD A CONTROL PREMIUM BE APPLIED?

We have made the case that the cost of capital is the same or nearly the same for controlling interests as for minority interests. But we know that acquisitions are made at prices reflecting a control price premium over public market minority share trading prices.[4] So if we are valuing a controlling interest by the discounting or capitalizing method, and if we are using the cost of capital that we have estimated, under what circumstances should we add a control premium?

Projected Income May Not Reflect What a Control Owner Would Achieve

We said earlier that the control premium would be reflected as a result of the increased cash flows that a control owner would expect to achieve. If such control cash flows have been either discounted or capitalized, then little or no further control premium should be applied. However, if the projected cash flows used do *not* reflect a control owner's expectations, then a control premium may be warranted.

Investment Value Reflecting Synergies

If a buyer may achieve strategic or synergistic benefits by an acquisition, then that buyer may be willing to pay a control premium. For example, if the target is a direct competitor, then the buyer may benefit by raising *his or her own* prices as well as those of the target. Remember, however, that a price premium reflecting benefits to such a *particular* buyer brings in the element of investment value as opposed to a pure fair market value.

In the context of this thinking, the "levels of value" chart (Exhibit 15.1) added another level at the top since the first edition of this book was published, "synergistic value," reflecting a potential premium over control value on a stand-alone basis.

Factors Affecting a Control Premium a Financial Buyer Might Pay

Financial buyers sometimes pay control premiums, even if they do not have any opportunities for synergistic benefits, albeit typically much lower premiums than those paid by synergistic buyers. For example, one control prerogative that control owners can implement that minority owners cannot is to register a public offering. Other control prerogatives would be to sell interests to employees or to others, to repurchase outstanding minority interests, or to recapitalize. Some will pay a premium to "call the shots." Some may perceive psychological advantages to control of certain companies.

A TALE OF TWO MARKETS

In my opinion, a strong case could be made that when the share price of a public company is wildly out of line with its intrinsic value (to a financial buyer, and even possibly to a normal strategic buyer), then perhaps the public market value is meaningless and should be disregarded entirely.

In 2001 Mark Lee, a well-known business valuation analyst for over 25 years, went public with his views on the issue. Lee observed:

> The stock market is a market for minority interests in common stock. The principal buyers and sellers are individuals, mutual funds, and financial institutions. The market is highly liquid, individual investment horizons may be short, and risk tolerances can be

> greater than in illiquid markets. Financing is often readily available from banks and brokers at short-term money rates. Investors are generally passive. Individual investments are usually purchased as part of diversified portfolios, which leads to greater tolerance to risk.
>
> The [mergers and acquisitions] market is a market for whole companies. The principal buyers and sellers are controlling shareholders, corporations, and [leveraged buyout] houses. The market is not liquid; as a result, individual investment horizons tend to be longer. Risk tolerances in the short term tend to be lower than in a liquid market. Transactions are financed using long-term debt from banks, insurance companies, mezzanine funds, equity of large corporations, and private equity funds. [Mergers and acquisitions] investors take an active role in managing their companies.
>
> The relationship of the two markets is not linear [as shown in the single bar of the levels of value chart]. [This linearity] presupposes that acquisition premiums apply in all situations; and acquisition premiums are roughly the same amount generally or in each industry.
>
> The relationship of the two markets is better shown as the two overlapping forms as shown in [Exhibit 15.2]. . . .
>
> Clearly, the existence of an acquisition premium and its magnitude is a "facts and circumstances" test for each individual valuation.[5]

It is obvious that one must be extremely cautious about applying a control premium to public market values to determine a control level of value. Conversely, if guideline stocks are trading at or near control value in a given case, valuation of a minority interest by applying a discount for lack of control from the guideline indicators (in addition to a lack of marketability discount) might be supported since the minority owner lacks the control prerogative of taking the company public or registering his or her stock in an offering.

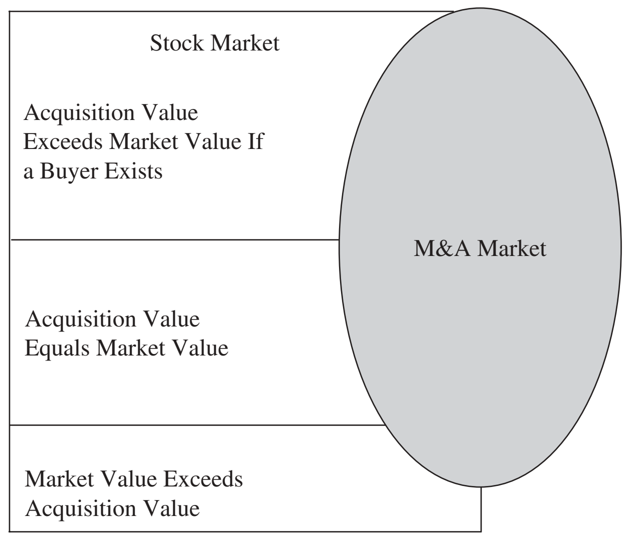
MANY TAKEOVERS AT LESS THAN PUBLIC TRADING PRICE

Many people are surprised to learn just how many takeovers occur at prices below the public trading price. In 2001, Business Valuation Resources assumed distribution of the *Mergerstat Control Premium Study*™, renamed it *Mergerstat/Shannon Pratt's Control Premium Study*™, and put it online so that people could access and analyze it electronically. Utilizing this, Exhibit 15.3 is a summary of the takeovers of public companies from 1998 through 2001.

In the 16 quarters analyzed, over 16% of the takeovers were at prices below their recent public market trading prices! Also as shown in Exhibit 15.3, including the "negative premiums" in the medians and averages has a dramatic impact.

The database is searchable by Standard Industrial Classification (SIC) code, North American Industry Classification System (NAICS) code, time frame, and size of company (by several measurements). Before naively applying an "average" control premium or implied minority discount, I would recommend searching the database to select those transactions that are truly relevant to the subject at hand.[6]

Exhibit 15.2 Schematic Relationship of Stock Market and M&A Market



1. The oval in the chart above is the M&A market. The box is the stock market. (The sizes of the two are not proportionate.)
2. If a potential acquirer believes that it can create sufficient added economic benefits, the acquisition value of the company will exceed its market value. The additional economic benefit can pay for the cost of the acquisition premium. These are the transactions reported in the *Control Premium Study* and similar publications.
3. Most publicly traded companies are not taken over in a given year. Generally, there is no market available that can create benefits large enough to justify payment of the premium required for the acquisition of these companies in view of other alternatives.

   If there is no M&A market available to sell a company at a premium to its stock market value, then there is little or no acquisition premium, much less a "theoretical" premium based on an average of acquisitions of dissimilar companies.
4. In emerging industries, such as the Internet in 1998 and 1999, the value of the common stock of a corporation as a whole often is worth less than the aggregate market value of common stock trading as minority interests. While the new industry is viewed as very attractive for investment, individual corporations are perceived as too risky. As a result, individual and institutional investors will pay more for minority interests as part of a diversified industry portfolio than individual acquirers will pay for the entire company.
5. Similarly, many companies spin off units or sell them in an IPO rather than sell the units in the M&A market because a higher price can be obtained in the market than in an M&A transaction.

Source: Adapted from Mark Lee of Sutter Securities, Incorporated, "Control Premiums and Minority Discounts: the Need for Specific Economic Analysis." *Shannon Pratt's Business Valuation Update®* (August 2001):1–5. Reprinted in Shannon P. Pratt, *Business Valuation Discounts and Premiums* (New York: John Wiley & Sons, Inc., 2001), 41, copyright © 2001. Used with permission.

Exhibit 15.3 *Mergerstat/Shannon Pratt's Control Premium Study*™ Takerovers from 1998 to 2001
ALL Foreign and Domestic Transactions

Control Premiums by Quarter, with and without Negative Premiums and TIC/EBITDA

	Mean Premium	Mean Premium	Median Premium	Median Premium	Total	Total	Percent	Median
	w/o Negatives	w/Negatives	w/o Negatives	w/Negatives	Count	Negatives	Negative	TIC/EBITDA[1]
1Q1998	35%	30%	32%	29%	93	10	11%	14.90
2Q1998	35%	32%	30%	26%	79	5	6%	10.10
3Q1998	27%	16%	19%	15%	93	22	24%	11.85
4Q1998	43%	27%	33%	21%	141	34	24%	10.72
1Q1999	49%	42%	33%	29%	138	14	10%	9.43
2Q1999	49%	41%	33%	27%	150	21	14%	9.86
3Q1999	42%	38%	31%	30%	176	13	7%	10.12
4Q1999	46%	39%	34%	29%	214	25	12%	10.78
1Q2000	55%	48%	36%	32%	165	16	10%	11.92
2Q2000	50%	33%	38%	28%	268	56	21%	10.45
3Q2000	47%	35%	37%	31%	244	39	16%	10.14
4Q2000	45%	30%	36%	23%	248	54	22%	8.88
1Q2001	53%	37%	37%	27%	228	43	19%	7.75
2Q2001	58%	42%	42%	33%	203	38	19%	7.59
3Q2001	49%	30%	35%	23%	223	53	24%	8.35
4Q2001	48%	26%	30%	22%	186	49	26%	8.11
				Total	2,849	492	17%	

(Continued)

Exhibit 15.3 (Continued)
ONLY Domestic Transactions

Control Premiums by Quarter, with and without Negative Premiums and TIC/EBITDA

	Mean Premium	Mean Premium	Median Premium	Median Premium	Total	Total	Percent	Median
	w/o Negatives	w/Negatives	w/o Negatives	w/Negatives	Count	Negatives	Negative	TIC/EBITDA[1]
1Q1998	35%	29%	33%	29%	75	9	12%	15.10
2Q1998	35%	33%	31%	27%	59	3	5%	10.10
3Q1998	27%	10%	17%	11%	58	20	34%	12.30
4Q1998	42%	25%	31%	16%	105	29	28%	11.50
1Q1999	52%	45%	33%	29%	109	12	11%	11.03
2Q1999	52%	44%	36%	29%	115	14	12%	10.36
3Q1999	45%	40%	32%	30%	141	10	7%	10.18
4Q1999	45%	39%	32%	28%	168	19	11%	11.07
1Q2000	59%	52%	34%	32%	103	10	10%	13.01
2Q2000	52%	30%	43%	29%	139	37	27%	10.84
3Q2000	51%	41%	42%	37%	137	17	12%	11.15
4Q2000	50%	34%	45%	32%	148	30	20%	9.18
1Q2001	56%	38%	35%	26%	121	24	20%	8.75
2Q2001	64%	50%	51%	38%	107	17	16%	8.86
3Q2001	51%	36%	39%	32%	112	18	16%	11.62
4Q2001	54%	37%	33%	27%	106	22	21%	9.36
				Total	1,803	291	16%	

	Mean Premium	Mean Premium	Median Premium	Median Premium	Total	Total	Percent	Median
	w/o Negatives	w/Negatives	w/o Negatives	w/Negatives	Count	Negatives	Negative	TIC/EBITDA[1]
1Q1998	37%	32%	29%	29%	18	1	6%	NMF
2Q1998	37%	31%	28%	25%	20	2	10%	NMF
3Q1998	27%	25%	23%	19%	35	2	6%	8.75
4Q1998	45%	36%	35%	33%	36	5	14%	8.68
1Q1999	35%	31%	24%	24%	29	2	7%	6.56
2Q1999	37%	30%	25%	23%	35	7	20%	NMF
3Q1999	32%	29%	26%	21%	35	3	9%	11.33
4Q1999	48%	38%	38%	34%	46	6	13%	9.86
1Q2000	47%	40%	37%	34%	62	6	10%	10.74
2Q2000	47%	37%	32%	27%	129	19	15%	10.17
3Q2000	42%	28%	32%	26%	107	22	21%	9.10
4Q2000	38%	25%	24%	17%	100	24	24%	7.86
1Q2001	50%	36%	37%	28%	107	19	18%	6.74
2Q2001	51%	33%	36%	27%	96	21	22%	6.48
3Q2001	45%	24%	31%	18%	111	35	32%	7.56
4Q2001	38%	13%	25%	15%	80	27	34%	6.81
				Total	1,046	201	19%	

[1]Excludes negative TIC/EBITDA. Includes companies with negative Control Premiums.

TIC = Target Invested Capital = Target company's implied total invested capital based on the sum of implied market value of equity (MVE) plus the face value of total interest bearing debt and the book value of preferred stock outstanding prior to the announcement date. Target MVE is based on the purchase price per share times total shares outstanding reported in the period prior to the transaction's announcement date. Market value of equity is reported in millions of dollars and rounded.

Sources: Mergerstat Control Premium Study™, 1998–2000, inclusive. Copyright © 1998–2000. Mergerstat, LP.

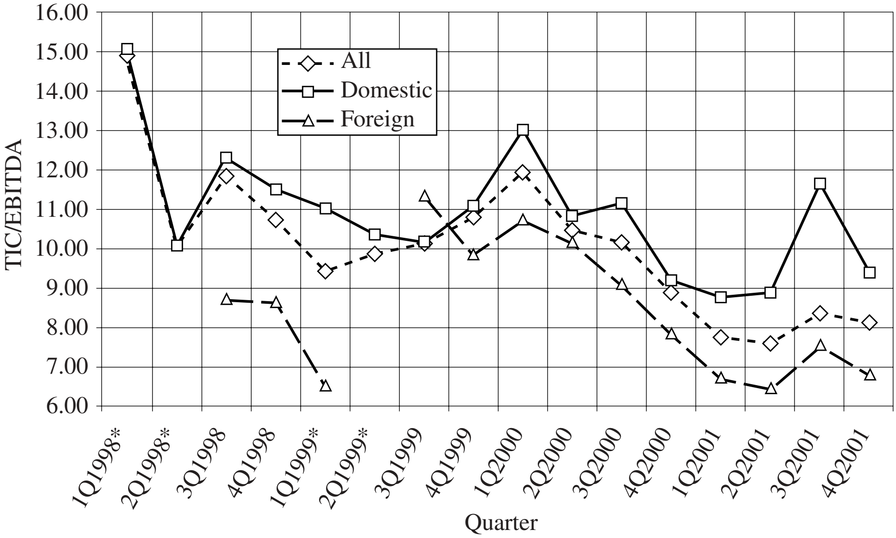
Mergerstat/Shannon Pratt's Control Premium Study™, 2001, inclusive. Copyright © 2001. Mergerstat, LP.

Available at *www.BVMarketData.com*℠. See Shannon Pratt and Doug Twitchell, "Valuation Multiples and Control Premiums Down in 4th Quarter 2001," Market Data Corner in *Shannon Pratt's Business Valuation Update*® (May 2002): 10–11.

(Continued)

Exhibit 15.3 (Continued)

Mergerstat/Shannon Pratt's Control Premium Study™
Median TIC/EBITDA by Quarter



*Insufficient data to determine median TIC/EBITDA for foreign transactions (i.e., less than 12 data points).

Mergerstat/Shannon Pratt's Control Premium Study™
Median Control Premium w/Negatives Included

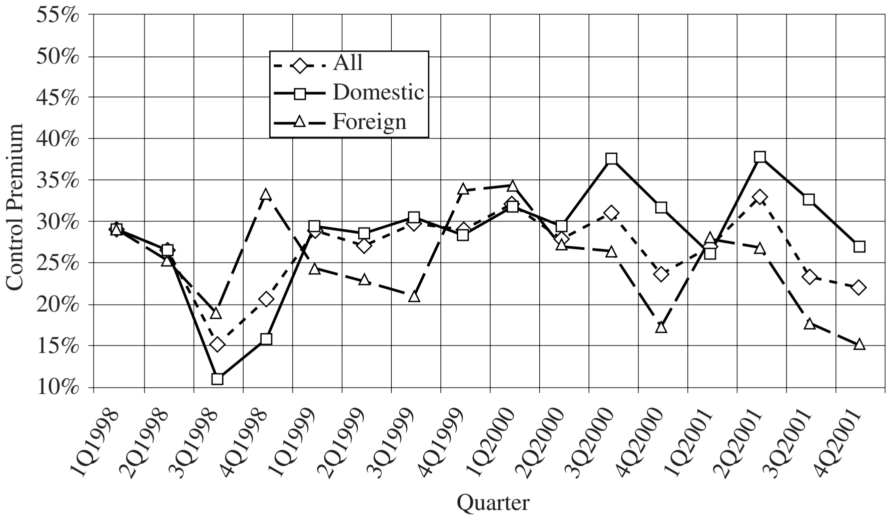
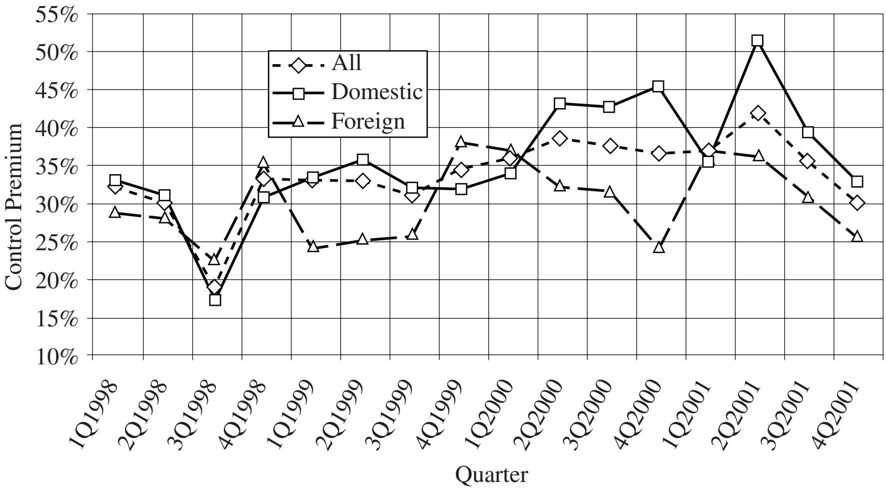


Exhibit 15.3 (Continued)

Mergerstat/Shannon Pratt's Control Premium Study™
Median Control Premiums w/Negatives Excluded



Sources: Mergerstat Control Premium Study™, 1998–2000, inclusive. Copyright © 1998–2000, Mergerstat, LP. *Mergerstat/Shannon Pratt's Control Premium Study*™, 2001, inclusive. Copyright © 2001, Mergerstat, LP.

SUMMARY

Generally, the cost of capital is the same for minority interest investments as for controlling interest investments. Investors typically do not reduce their required rate of return because they are buying a controlling interest rather than a minority interest.

Therefore, although empirical data used to estimate the cost of capital are drawn almost entirely from the public stock market (which represents transactions in minority shares), the cost of capital thus estimated is applicable to either control or minority investments. Premiums above current market trading prices often are paid to acquire controlling interests. However, these premiums are paid because of anticipated increases in the cash flows available to the controlling investors, not because of a lower cost of capital.

Increased cash flows to control buyers may come as a result of improved operating efficiency, synergies with a buying company, or redistribution of already available cash flows to new control owners.

The standard of value conceivably could affect the cost of capital. *Fair market value* assumes a market consensus cost of capital, whereas an *investment value* standard may reflect a cost of capital driven by a particular investor's perceptions or circumstances, which may depart from the market consensus.

The merger and acquisition market is a distinct market from the public stock market. While most takeovers occur at prices above the stock's public market price prior to the acquisition, a significant number occur at prices below the public market trading price. Careful research should be undertaken before applying a "control premium" or "implied minority discount" to a private company valuation to determine whether such a premium or discount is warranted.

Notes

1. For a detailed discussion of definitions of various standards of value in commonly encountered legal contexts, see "Standards of Value" in Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. (New York: McGraw-Hill, 2000), 28–32.
2. *Ibbotson Associates Cost of Capital Workshop* (Chicago: Ibbotson Associates, 1998), Chapter 1, p. 12.
3. Pratt, Reilly, and Schweihs, *Valuing a Business*, 347–348, 365–366 (McGraw-Hill): Used with permission.
4. See, for example, *Mergerstat/Shannon Pratt's Control Premium Study*™ (Los Angeles: Mergerstat, LP), distributed exclusively by Business Valuation Resources, LLC, Portland, Oregon, available online, updated quarterly, at *www.BVMarketData.com*℠.
5. Mark Lee, "Control Premiums and Minority Discounts: The Need for Specific Economic Analysis," *Shannon Pratt's Business Valuation Update*® (August 2001): 1–5. Used with permission.
6. See, for example, Daniel L. McConaughy, "Negative Takeover Premia and Stock Price Levels in Internet Stocks," *Valuation Strategies* (March/April 2002): 20–29.

Chapter 16

Handling the Discount for Lack of Marketability

Discrete Percentage Discount for Lack of Marketability
Minority Ownership Interests
Controlling Ownership Interests
Building the Discount for Lack of Marketability into the Discount Rate
Venture Capitalists' Required Rates of Return
Quantifying the Marketability Factor in the Discount Rate
Summary

As noted earlier, whether the cost of capital estimation is based partly on historical market data or entirely on current market data, the data represent publicly traded stock transactions in the highly liquid U.S. public stock markets. Investors in companies without an established trading market for their stock place a high premium on liquidity or, conversely, demand a high discount for lack of liquidity, compared with companies with an established trading market for their stock.

Having estimated required rates of return from market data for publicly traded stocks, there are two ways to adjust for the lack of liquidity for closely held stock:

1. After estimating a value as if publicly traded, subtract a percentage discount for lack of marketability.
2. Build the lack of marketability factor into the discount rate by adding some number of percentage points into the discount or capitalization rate, developed from any of the models discussed in earlier chapters of this book.

DISCRETE PERCENTAGE DISCOUNT FOR LACK OF MARKETABILITY[1]

The most common way to handle the lack of marketability issue is by a percentage deduction from the value indicated after discounting or capitalizing expected cash flows at a rate derived from public market data.

Minority Ownership Interests

Many empirical studies have provided extensive transaction data to help quantify the amount of such a discount in the case of minority interest transactions. The studies consistently show a central tendency for discounts for lack of marketability for minority interests to be 30% to 50% from the value as if they were publicly traded. However, it is noteworthy that there are many transactions above and below this range.

For any given valuation developed by the income approach, it is tempting to simply take the average from the studies and apply that as a discount for lack of marketability. However, with the broad ranges around the measures of central tendency, like the minority discount/control premium issue, it is more accurate (and more convincing to a court) to select from the available databases those transactions with characteristics closest to the subject company to estimate the discount for lack of marketability.

Fortunately, since the first edition of this book, two databases have been put online to enable researchers to do this. One is a restricted stock study and the other is a pre–initial public offering (IPO) study.

Restricted Stock Studies

A "restricted stock" is a stock of a public company that is identical in all respects to the stock that trades publicly, except that it is restricted from trading on the public market. It could be, for example, stock issued in an acquisition, stock issued in a financing, or stock of insiders not registered in a public offering. Such stock is, however, eligible for block transactions with institutional and other qualified investors.

The essence of restricted stock studies is to compare the price at which a restricted stock transaction takes place with the public market price on the same day. The percentage difference is a proxy for the discount for lack of marketability.

A summary of the restricted stock studies published to date is shown as Exhibit 16.1. The Securities and Exchange Commission (SEC) loosened the restrictions in 1990, and the average discount went down from the mid-30s to the mid-20s. In 1997 the SEC reduced the required holding period under Rule 144 from two years to one year, and the average discount was 13% in the only post-1997 study as of press time. The reduction in discounts for restricted stocks should not be interpreted to indicate a reduction in the discount for lack of marketability for minority interests in closely held companies, but merely as a reaction to the loosening of restrictions.

The most detailed study to date is *The FMV Restricted Stock Study*™ by FMV Opinions, Inc. It covers 243 transactions from 1980 through 1997 (the year the SEC cut the required holding period from two years to one year), with 53 data items for each transaction. A sample transaction report from the FMV study is shown as Exhibit 16.2. It was placed online in 2001 and is fully searchable at *www.BVMarketData.com*℠.

When an analyst is trying to quantify a discount for lack of marketability for a particular company, he or she can search the database to find companies with comparable characteristics to use for guidance. Such characteristics might include com-

Exhibit 16.1 Summary of Restricted Stock Transaction Studies

Time Period	Study	Number of Transactions	Average Discount	Time Period[6]
1/66–6/69	SEC Institutional Investor	398	25.8%[1]	A
1/68–12/70	Milton Gelman	89	33.0%	A
1/68–12/72	Robert Trout	60	33.5%	A
1/68–12/72[2]	Robert Moroney	148	35.6%	A
1/69–12/73	Michael Maher	33	35.4%	A
10/78–6/82	Standard Research Consultants	28	45.0%[3]	A
1/81–12/88	William Silber	69	33.8%	A
1/79–4/92	FMV Opinions, Inc.[7]	>100	23.0%	B
1/80–12/96	Management Planning, Inc.	53	27.1%	B
1/91–12/95	Bruce Johnson	70	20.0%	C
1/96–4/97	Columbia Financial Advisors	23	21.0%[4]	C
5/97–12/98	Columbia Financial Advisors	15	13.0%[5]	D

[1]The average was 32.6% for OTC companies not required to file reports with the Securities and Exchange Commission.

[2]The exact ending month is not specified.

[3]Median.

[4]Median was 14.0.

[5]Median was 9.0.

[6]A Pre-1900 (before SEC loosened reporting requirements)
B Straddles 1990 (some before, some after, but more after 1997)
C Post-1990 but pre-1997
D Post-1997 (after SEC reduced required holding period under Rule 144 from two years to one year)

[7]FMV Opinions has updated its study through 1997, and it now includes 243 transactions and is available online at *www.BVMarketData.com.*

Source: Adapted from Shannon P. Pratt, *Business Valuation Discounts and Premiums* (New York: John Wiley & Sons, Inc., 2001), 81, copyright © 2001. Used with permission.

pany size (measured by either sales or assets), size of the block (as a percentage of the total outstanding), earnings (or deficit), and so on.

Pre–Initial Public Offering Studies

When a company goes public for the first time, it is required by the SEC to disclose in its prospectus all the transactions in its stock for the previous three years. A comparison of the prices of those transactions with the initial public offering (IPO) price is the essence of the "pre-IPO studies."

The percentage below the IPO price at which the transactions took place (adjusted for changes in company fundamentals) is a proxy for the discount for lack of marketability. For a long time, only two such series of studies existed, the Willamette

Exhibit 16.2 FMV Restricted Stock Study™ Transaction Report

Company	
SIC	2022 Natural, Processed, and Imitation Cheese
NAICS	—No description—
Name	Galaxy Foods Company
Company Description	
City	
State	
Country	
Ticker	GALX
Exchange	OTC

Transaction Data		**Financial Data ($000's)**	
Transaction Date	10/1/93	Market Value	8,970.66
Registration Rights	N	Book Value	875.00
Discount	59.18%	MTB Ratio	10.25
Offering Price	$1.25	Intangible Assets	0.00
Prior Month High	$2.88	Total Assets	6,275.00
Prior Month Low	$1.38	Debt	2,375.00
Prior Month High-Low Average	$2.13	Total Revenues	5,311.00
Prior Month Volume	280,900	Depreciation Expense	306.00
Shares Placed to Volume Ratio	1.6x	Interest Expense	185.00
Shares Outstanding	4,221.487	Pretax Income	–2,620.00
Shares Placed	440,000	Net Income	–2,620.00
Placement Amount	550,000	Prior Year Dividend ($)	0.00
Shares Placed/Shares After	0.09	EBITDA	–2,129.00
Transaction Month High	$4.00	Operating Profit Margin	–0.46
Transaction Month Low	$2.13	Net Profit Margin	–0.49
Transaction Month High-Low Average	$3.06		
Transaction Month Close	$3.75		
Transaction Month Volume	447,300		

N/A = Not Available

Source: Available at *www.BVMarketData.com.*

Management Associates studies and the Emory studies (formerly the Baird & Co. studies). Both indicated average discounts in the mid-40s.

A summary of the Willamette studies is shown as Exhibit 16.3. Because the Willamette studies incorporate transactions back three years from the IPO, discounts are computed in terms of relative price/earnings ratios instead of absolute dollar comparisons.

A summary of the Emory studies is shown as Exhibit 16.4. Because the Emory studies collect transactions only five months prior to the IPO, discounts are computed on unadjusted prices.

Finally, the broadest pre-IPO study was instituted by Valuation Advisors in 2000, called the *Valuation Advisors Lack of Marketability Discount Study*™. While the

Exhibit 16.3 Summary of Discounts for Private Transaction P/E Multiples Compared to Public Offering P/E Multiples Adjusted for Changes in Industry P/E Multiples

Time Period	Number of Companies Analyzed	Number of Transactions Analyzed	Standard Mean Discount	Trimmed Mean Discount*	Median Discount	Standard Deviation
1975–78	17	31	34.0%	43.4%	52.5%	58.6%
1979	9	17	55.6%	56.8%	62.7%	30.2%
1980–82	58	113	48.0%	51.9%	56.5%	29.8%
1983	85	214	50.1%	55.2%	60.7%	34.7%
1984	20	33	43.2%	52.9%	73.1%	63.9%
1985	18	25	41.3%	47.3%	42.6%	43.5%
1986	47	74	38.5%	44.7%	47.4%	44.2%
1987	25	40	36.9%	44.9%	43.8%	49.9%
1988	13	19	41.5%	42.5%	51.8%	29.5%
1989	9	19	47.3%	46.9%	50.3%	18.6%
1990	17	23	30.5%	33.0%	48.5%	42.7%
1991	27	34	24.2%	28.9%	31.8%	37.7%
1992	36	75	41.9%	47.0%	51.7%	42.6%
1993	51	110	46.9%	49.9%	53.3%	33.9%
1994	31	48	31.9%	38.4%	42.0%	49.6%
1995	42	66	32.2%	47.4%	58.7%	76.4%
1996	17	22	31.5%	34.5%	44.3%	45.4%
1997	34	44	28.4%	30.5%	35.2%	46.7%

*Excludes the highest and lowest deciles of indicated discounts.

Source: © Willamette Management Associates, *www.willamette.com*. Used with permission.

Willamette and Emory studies set limiting criteria for transactions considered, Valuation Advisors recorded *every* transaction for two years prior to the IPO date. They recorded 14 data points for each transaction:

- Standard Industrial Classification (SIC) code
- North American Industry Classification System (NAICS) code
- Company name
- Company description
- Sales
- EBIT (earnings before interest and taxes)
- Assets
- IPO price
- IPO date
- Transaction price
- Transaction date

Exhibit 16.4 Value of Marketability as Illustrated in Initial Public Offerings of Common Stock

Study	Number of IPO Prospectuses Reviewed	Number of Qualifying Transactions	Discounts %	
			Mean	Median
*1997–2000	92	53	54	54
1995–1997	732	91	43	42
1994–1995	318	46	45	45
1992–1993	433	54	45	44
1990–1992	266	35	42	40
1989–1990	157	23	45	40
1987–1989	98	27	45	45
1985–1986	130	21	43	43
1980–1981	97	13	60	66
All 9 Studies	2333	363	47%	44%

*1997–2000 study was for dot-com companies—not comparable to other studies (see text).

Source: John D. Emory. "The Value of Marketability as Illustrated in Initial Public Offerings of Common Stock (Eighth in a Series) November 1995 through April 1997," *Business Valuation Review*, vol. 16, no. 3 (September 1997): 125, published by the Business Valuation Committee of the American Society of Appraisers; John D. Emory, Sr., F.R. Dengel, III, and John D. Emory, Jr., "The Value of Marketability as Illustrated in Dot.Com IPOs: May 1997–March 2000, *Shannon Pratt's Business Valuation Update*, vol. 6, no. 7 (July 2000): 1–2. © Emory Business Valuation, LLC. Reprinted with permission.

- Whether transaction was
  - Common stock
  - Option
  - Convertible preferred stock
- Percentage discount from IPO price
- Industry description

Transactions may be searched online on any of these above fields at *www.BVMarketData.com*sm.

Results of the Valuation Advisors study for the year 2000 are presented in Exhibit 16.5. As can be seen clearly from the exhibit, the discounts diminish rapidly as they approach the IPO date.

Three sample transaction reports from the Valuation Advisors study are shown as Exhibit 16.6. Since these are all recent initial public offerings, if additional data regarding a transaction are desired, they can be readily obtained from public company filings.

The pre-IPO studies are the only ones that actually represent transactions in private company stock (although the parties in most cases probably had hopes for a public offering), and thus come closest to a proxy for a discount for lack of marketability for

Exhibit 16.5 Results of Valuation Advisors Study for 2000

Table 1: Complete Study Results

Time of transaction before IPO	1–90 Days	91–180 Days	181–270 Days	271–365 Days	1–2 Yrs.
Number of transactions	123	165	105	86	134
Average discount	31.50%	43.58%	56.47%	64.39%	71.61%
Average one-year discount	47.07%	–	–	–	–

Table 2: Narrowed Discount Range

Time of transaction before IPO	1–90 Days	91–180 Days	181–270 Days	271–365 Days	1–2 Yrs.
Number of transactions	99	146	94	73	106
Average discount	40.60%	49.29%	59.16%	65.95%	66.85%
Average one-year discount	52.40%	–	–	–	–

Table 2.1: Narrowed Discount Range—Excluding CPS Transactions

Time of transaction before IPO	1–90 Days	91–180 Days	181–270 Days	271–365 Days	1–2 Yrs.
Number of transactions	82	95	53	49	50
Average discount	39.56%	47.64%	56.98%	63.17%	63.54%
Average one-year discount	49.76%	–	–	–	–

Table 3: CPS Transactions Only

Time of transaction before IPO	1–90 Days	91–180 Days	181–270 Days	271–365 Days	1–2 Yrs.
Number of transactions	24	56	43	30	73
Average discount	31.83%	47.78%	57.51%	73.00%	75.45%
Average one-year discount	52.96%	–	–	–	–

Source: Brian K. Pearson, "2000 Marketability Discounts as Reflected in Initial Public Offerings," *Shannon Pratt's Business Valuation Update®* (September 2001): 1–7, tables at 4. Used with permission.

Exhibit 16.6 Valuation Advisors' Lack of Marketability Study™ Transaction Report (1 of 3)

Company

Company	Plumtree Software
Product, Service, or Business	Corporate Portal Software
SIC	7371 Computer Programming Services
NAICS	5112 Software Publishers

Transaction Data		**Financial Data**	
Pre-IPO Timeframe	6 mth(s)	Net Sales	$81,473,000
Transaction Date	12/6/01	Percent Discount	1.882%
Transaction Price per Share	$8.34	Total Assets	$54,268,000
CPS, S, or O	O	Operating Income	($7,132,000)
IPO Date	6/4/02	Operating Profit Margin	–8.754%
IPO Price per Share	$8.50		

N/A = Not Available

(Continued)

Exhibit 16.6 (Continued) (2 of 3)

Company	
Company	PDF Solutions, Inc.
Product, Service, or Business	Performance Enhancers for Semiconductors
SIC	7371 Computer Programming Services
NAICS	334413 Semiconductor and Related Device Manufacturing

Transaction Data		**Financial Data**	
Pre-IPO Timeframe	0 mth(s)	Net Sales	$2,013,500
Transaction Date	7/15/01	Percent Discount	8.333%
Transaction Price per Share	$11.00	Total Assets	$15,034,000
CPS, S, or O	O	Operating Income	($9,081,000)
IPO Date	7/28/01	Operating Profit Margin	–451.006%
IPO Price per Share	$12.00		

(3 of 3)

Company	
Company	Signature Eyeware
Product, Service, or Business	Eyeglass Frames
SIC	3827 Optical Instruments and Lenses
NAICS	333314 Optical Instrument and Lens Manufacturing

Transaction Data		**Financial Data**	
Pre-IPO Timeframe	16 mth(s)	Net Sales	$28,280,086
Transaction Date	5/15/96	Percent Discount	72.200%
Transaction Price per Share	$2.78	Total Assets	$10,293,057
CPS, S, or O	S	Operating Income	$2,012,569
IPO Date	9/11/97	Operating Profit Margin	7.117%
IPO Price per Share	$10.00		

Source: Copyright © 2001. *Valuation Advisors' Lack of Marketability Discount Study*™. All rights reserved. Available at *www.BVMarketData.com*. Used with permission.

stock of a privately held company. Nevertheless, there are some challenges to the use of these studies, and the appraiser should be aware of the challenges when relying on them to support the discount.[2]

Controlling Ownership Interests

The case for discounts for lack of marketability for controlling interest transactions is not as clear. A controlling interest holder cannot merely call a stockbroker, execute a transaction in seconds, and have cash in hand within three business days. It may take months to prepare a controlling interest for sale, incurring significant legal, accounting, and management time costs in the process. Furthermore, compared with public companies, most private companies have much less ready access to the capital markets to raise additional equity and/or debt capital.

Despite these limitations, some analysts would say that discounts for lack of marketability are not applicable to controlling interests. These same analysts may, however, recognize the realities of these factors under the rubric of "liquidity." Courts frequently have recognized discounts for lack of marketability for controlling stock interests held in estates. Discounts for lack of liquidity (marketability) for controlling ownership interests, when appropriate (such as those recognized in the U.S. Tax Court), are often in the range of 10% to 25%,[3] not as great as for minority ownership interests.

BUILDING THE DISCOUNT FOR LACK OF MARKETABILITY INTO THE DISCOUNT RATE

Venture Capitalists' Required Rates of Return

Venture capitalists typically say that they look for expected rates of return between 30% and 50% on their portfolios, which means higher rates on very risky start-ups. One reason why these rates are so high is the illiquidity of the companies and securities in which venture capitalists invest, even though they virtually always have an exit strategy in place if everything works out as projected. Unfortunately, there are no data available to indicate how much of their required rate of return is for illiquidity. (If any reader can shed light on this issue, please contact the author at the phone number or address shown in the preface.)

Quantifying the Marketability Factor in the Discount Rate

In building the lack of marketability factor into the discount rate, determining how many percentage points to add to the discount rate is difficult and quite subjective. Z. Christopher Mercer, chief executive officer of Mercer Capital, has compiled a list of factors to consider, generally adding somewhere between zero and four percentage points for each factor considered important in the particular case. This list is shown in Exhibit 16.7. In most of his examples, Mercer considers about four to six factors, and his cumulative adjustments tend to run between one and six percentage points.[4] The applicable cash flow for Mercer's "Quantitative Marketability Discount Model" discount rate are those expected to be available to the minority investor.

Of course, no empirical basis exists for assigning numbers in a matrix like this, and doing so may give the valuation report a sense of spurious accuracy. When using a table like this, the analyst might want to include a disclaimer to the effect that the numbers are presented not for precise quantification, but merely to show the thinking of the analyst.

Exhibit 16.7 Estimating Cost of Capital, Including Illiquidity Factor

		Range of Returns	
Components of the Required Holding Period Return		Lower	Higher
Base equity discount rate (adjusted Capital Asset Pricing Model)			
Current yield-to-maturity composite long-term Treasuries		0.0%	0.0%
+ Adjusted Ibbotson large stock premium	0.0%		
× Applicable beta statistic	× 1.0		
= Beta-adjusted large stock premium		0.0%	0.0%
+ Adjusted Ibbotson small stock premium		0.0%	0.0%
= Base equity discount rate		0.0%	0.0%
Investment-Specific Risk Premiums—Factors to Consider			
Uncertainties related to length of expected holding period			
General illiquidity of the investment			
Lack of expected interim cash flow			
Uncertainties related to expected interim cash flow			
Potential for adverse cash flow from tax pass-through entity			
Uncertainties related to potential for favorable exit from investment			
General unattractiveness of the investment			
Lack of diversification of assets			
Unattractive asset mix			
Unlikely candidate for merger/sale/acquisition/initial public offering			
Uncertainties related to buy-sell agreement			
Small shareholder base			
Adjustment for large size of the entity			
Large size of the investment limits market			
Other			
Range of specific risk premiums for the investment		0.0%	0.0%
Initial range of required returns		0.0%	0.0%
Concluded range of required holding period returns		0.0%	0.0%

Source: Z. Christopher Mercer, *Quantifying Marketability Discounts* (Memphis: Peabody Publishing, LP, 1997), 323. Reprinted with permission.

SUMMARY

Investors, especially in the United States, cherish liquidity and abhor illiquidity. Because our empirical data to estimate cost of equity capital all come from the public stock market, the comparative lack of liquidity must be addressed when using cost of capital data to estimate value for privately held interests or companies.

There are two ways to handle the liquidity difference between public company interests and private company interests:

1. *Use a discrete percentage discount for lack of marketability.* Value the interest as if it were publicly traded, and then subtract a discount for lack of marketability

from the estimated publicly traded equivalent value. Two sets of studies to provide quantitative guidance for this are:

   1. Restricted stock studies
   2. Pre–initial public offering (IPO) studies

2. *Adjust the discount rate.* Add percentage points to the discount rate used to discount the expected cash flows to present value.

The notes provide reference sources for assistance in implementing either one of these methods.

Notes

1. For a comprehensive discussion of discounts for lack of marketability, see Chapters 3 through 11 in Shannon P. Pratt, *Business Valuation Discounts and Premiums* (New York: John Wiley & Sons, Inc., 2001), 45–211; see also Chapter 17, "Discounts for Illiquidity and Lack of Marketability," in Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. (New York: McGraw-Hill, 2000), 391–423.
2. See, for example, Shannon Pratt, "Lack of Marketability Discounts Suffer More Controversial Attacks," editor's column in *Shannon Pratt's Business Valuation Update®* (February 2002): 1–3; see also Mukesh Bajaj, "Dr. Bajaj Responds to Dr. Pratt's February 2002 Editorial," reader/editor exchange in *Shannon Pratt's Business Valuation Update®* (March 2002): 12–14.
3. See reference in note 1; see also Bradley A. Fowler, "How Do You Handle It?" part of a special report, "ASA Conference Offers Valuable Insights," *Shannon Pratt's Business Valuation Update®* (July 1997): 1–2.
4. Z. Christopher Mercer, *Quantifying Marketability Discounts: Developing and Supporting Marketability Discounts in the Appraisal of Closely Held Business Interests* (Memphis: Peabody Publishing, LP, 1997). This book also comprehensively covers empirical studies of discounts for lack of marketability conducted over the 30 years up until its publication.

Chapter 17

How Cost of Capital Relates to the Excess Earnings Method of Valuation

Basic "Excess Earnings" Valuation Method
- Conceptual Basis for the Method
- Steps in Applying the Excess Earnings Method
- Example of the Excess Earnings Method

Cost of Capital Reasonableness Check
- Computing the Weighted Average Excess Earnings Capitalization Rate
- Estimating a Build-up Model Capitalization Rate
- Discussion of the Example

Vagaries of the Excess Earnings Method

Summary

The excess earnings method of valuation was originally created for the purpose of valuing intangible assets, specifically intangible value in the nature of goodwill. It was devised to determine how much the U.S. government would compensate brewers and distillers for the economic loss of their goodwill as a result of Prohibition.

This valuation method has since been embodied in Revenue Ruling 68-609, included here as Exhibit 17.1. Although it was originally designed to value only intangible assets, it is widely used (and misused) today in the valuation of small businesses and professional practices.

This chapter has a single point: An estimate of the cost of capital, developed by methods discussed in this book, can be used as a test of the reasonableness of the assumptions and results achieved by using the excess earnings method. This test can be applied either by a person preparing an excess earnings method valuation or by someone reviewing an excess earnings method valuation prepared by someone else.

This chapter gives only enough of the skeletal basics of the excess earnings method to allow the reader to understand how to apply the reasonableness test proposed herein. Most basic texts on business valuation contain a full chapter, or a major portion of a chapter, on the excess earnings method.[1]

Exhibit 17.1 Revenue Ruling 68-609

The "formula" approach may be used in determining the fair market value of intangible assets of a business only if there is no better basis available for making the determination; A.R.M. 34, A.R.M. 68, O.D. 937, and Revenue Ruling 65-192 superseded.

SECTION 1001.—DETERMINATION OF AMOUNT OF AND RECOGNITION OF GAIN OR LOSS

26 CFR 1.1001-1: Computation of gain or loss. Rev. Rul. 68-609[1]
(Also Section 167; 1.1679(a)-3.)

The purpose of this Revenue Ruling is to update and restate, under the current statute and regulations, the currently outstanding portions of A.R.M. 34, C.B. 2, 31 (1920), A.R.M. 68, C.B. 3, 43 (1920), and O.D. 937, C.B. 4, 43 (1921).

The question presented is whether the "formula" approach, the capitalization of earnings in excess of a fair rate of return on net tangible assets, may be used to determine the fair market value of the intangible assets of a business.

The "formula" approach may be stated as follows:

> A percentage return on the average annual value of the tangible assets used in a business is determined using a period of years (preferably not less than five) immediately prior to the valuation date. The amount of the percentage return on tangible assets, thus determined, is deducted from the average earnings of the business for such period and the remainder, if any, is considered to be the amount of the average annual earnings from the intangible assets of the business for the period. This amount (considered as the average annual earnings from intangibles), capitalized at a percentage of, say, 15 to 20 percent, is the value of the intangible assets of the business determined under the "formula" approach.

The percentage of return on the average annual value of the tangible assets used should be the percentage prevailing in the industry involved at the date of valuation, or (when the industry percentage is not available) a percentage of 8 to 10 percent may be used.

The 8 percent rate of return and the 15 percent rate of capitalization are applied to tangibles and intangibles, respectively, of businesses with a small risk factor and stable and regular earnings: the 10 percent rate of return and 20 percent rate of capitalization are applied to businesses in which the hazards of business are relatively high.

The above rates are used as examples and are not appropriate in all cases. In applying the "formula" approach, the average earnings period and the capitalization rates are dependent upon the facts pertinent thereto in each case.

The past earnings to which the formula is applied should fairly reflect the probable future earnings. Ordinarily, the period should not be less than five years, and abnormal years, whether above or below the average, should be eliminated. If the business is a sole proprietorship or partnership, there should be deducted from the earnings of the business a reasonable amount of services performed by the owner or partners engaged in the business. See *Lloyd B. Sanderson Estate v. Commissioner*, 42 F. 2d 160 (1930). Further, only the tangible assets entering into net worth, including accounts and bills receivable in excess of accounts and bills payable, are used for determining earnings on the tangible assets. Factors that influence the capitalization rate include (1) the nature of the business, (2) the risk involved, and (3) the stability or irregularity of earnings.

The "formula" approach should not be used if there is better evidence available from which the value of intangibles can be determined. If the assets of a going business are sold upon the basis of a rate of capitalization that can be substantiated as being realistic, though it is not within the range of figures indicated here as the ones ordinarily to be adopted, the same rate of capitalization should be used in determining the value of intangibles.

Accordingly, the "formula" approach may be used for determining the fair market value of intangibles of a business only if there is no better basis therefor available.

See also Revenue Ruling 59-60, C.B. 1959-1, 237, as modified by Revenue Ruling 65-193, C.B. 1965-2, 370, which sets forth the proper approach to use in the valuation of closely held corporate stocks for estate and gift tax purposes. The general approach, methods, and factors, outlined in Revenue Ruling 59-60, as modified, are equally applicable to valuations of corporate stocks for income and other tax purposes as well as for estate and gift tax purposes. They apply also to problems involving the determination of the fair market value of business interests of any type, including partnerships and proprietorships, and of intangible assets for all tax purposes.

A.R.M. 34, A.R.M. 68, and O.D. 937 are superseded, since the positions set forth therein are restated to the extent applicable under current law in this Revenue Ruling 65-192, C.B. 1965-2, 259, which contained restatements of A.R.M. 34 and A.R.M. 68, is also superseded.

[1]Prepared pursuant to Rev. Proc. 67-6, C.B. 1967-1, 576.

Source: Rev. Rul. 68-609, 1968-2, C.B. 327.

BASIC "EXCESS EARNINGS" VALUATION METHOD

The excess earnings method is a form of a capitalization method requiring separate estimation of *two* capitalization rates. The reason for the two rates is that the income stream being capitalized is divided into two parts:

1. *Income attributable to tangible assets.* Less risky, lower required rate of return
2. *Income attributable to intangible assets.* More risky, generally higher required rate of return

The rule is simply that the weighted average of the two rates based on asset components (weighted at market values, of course) should approximately equal capitalization rates based on capital structure components estimated by methods discussed in this book.

Conceptual Basis for the Method

The *Guide to Business Valuations* explains the concept of the excess earnings method:

> The model for the excess earnings method computes the company's equity value based on the *appraised* value of tangible assets, plus an additional amount for intangible assets. A company's tangible assets should provide a current return to the owner. Since there are risks associated with owning the company's assets, the rate of return on those assets should be commensurate with the risks involved. That rate of return should be either the prevailing rate of return required to attract capital to that industry or an appropriate rate above the risk-free rate. Any returns produced by the company above the rate on tangible assets are considered to arise from intangible assets. Accordingly, the weighted average capitalization rate for tangible assets and intangible assets should be equivalent to the capitalization rate for the entire company.[2]

Steps in Applying the Excess Earnings Method

The *Guide to Business Valuations* lists the steps required in implementing the excess earnings method:

Step 1. Obtain the company's financial statements. Apply the necessary GAAP (generally accepted accounting principles) and normalization adjustments (including adjustments for nonoperating assets). Recompute federal and state income taxes, if necessary, based on normalized pretax earnings.

Step 2. Determine the value of the company's net tangible assets.

Step 3. Determine a reasonable rate of return (as of the valuation date) on the appraised value of the company's net tangible assets.

Step 4. Multiply the reasonable rate of return (Step 3) times the company's net tangible asset value (Step 2). That amount is the "reasonable return" on those assets.

Step 5. Subtract the calculated reasonable return (Step 4) from normalized net earnings (Step 1). That difference is the company's "excess earnings."

Step 6. Determine an appropriate capitalization rate (as of the valuation date) for the company's excess earnings, which are assumed to be attributable to goodwill or other intangible assets.

Step 7. Capitalize the excess earnings (divide excess earnings by the capitalization rate).

Step 8. Add the amount computed in Step 7 and the value of the net tangible assets (Step 2).

Step 9. Perform "sanity checks" to determine the reasonableness of the value determined in Steps 1 through 8.

Step 10. Determine an appropriate value for any excess or nonoperating assets that were adjusted for in Step 1. If applicable, add the value of those assets to the value determined in Step 8. If asset shortages were identified in Step 1, determine if the value estimate should be reduced to reflect the value of such shortages. If the normalized income statement was adjusted for identified asset shortages, it is not necessary to further reduce the value estimate.

Step 11. Determine if the value of the company computed in Step 8 should be adjusted for a minority interest discount, discount for lack of marketability, or other discounts.[3]

A very good "sanity check," as referred to in Step 9, is the cost of capital reasonableness check outlined in this chapter.

Example of the Excess Earnings Method

For an example, we will use a company with 100% equity in its capital structure. This simplifies the example, although it works just as well in valuing overall invested capital. As a practical matter, though, a majority of the companies to which this method is applied have no long-term debt. We are reviewing an excess earnings method valuation prepared by Sam Shoveler for a company owner's wife in a divorce proceeding.

The company is Kenny's Landscaping Mob (KLM), a sole proprietorship with several years of history in a residential area primarily populated by employees of lumber, plywood, and papermill companies. Mill shutdowns are frequent, impacting KLM's business, and there has even been talk of permanent closures. Kenny, now 45, supervises a high-turnover workforce whose members are paid a small fraction of the hourly rate that Kenny charges his clients for the gardening and landscaping work performed. There is substantial client turnover, but Kenny advertises heavily and finds new customers, at least when the mills are fully operating.

To illustrate a simple valuation of KLM by the excess earnings method, we will make five assumptions:

1. An appraiser accredited by the American Society of Appraisers in Machinery and Equipment Appraisal and with experience in landscaping and gardening equipment has appraised KLM's tangible assets on a value-in-use basis at $200,000.
2. Shoveler has determined that a reasonable rate of return on the company's net tangible assets is 8%.
3. Shoveler has also determined that an appropriate capitalization rate for the company's excess earnings is 20%.
4. Normalized net cash flow for KLM, after reasonable compensation to Kenny, is $50,000 per year.
5. Because Kenny is always scrambling for both customers and workers, and because the community's industrial base is flat at best, growth in net cash flow is expected to be only at the rate of inflation, estimated at 3%.

Shoveler's summary of the excess earnings method of valuation of KLM is:

Tangible asset value		$200,000
Net cash flow	$50,000	
Required return on tangible assets: $0.08 \times \$200,000$	16,000	
Return attributable to intangible assets	$34,000	
Intangible asset value (capitalized excess earnings)		
$\$34,000 \div 0.20$		170,000
Total value of KLM		$370,000

COST OF CAPITAL REASONABLENESS CHECK

The cost of capital reasonableness check is a fairly simple two-step process:

Step 1. Estimate a reasonable capitalization rate for the subject company by one or more of the cost of capital estimation methods discussed in this book.

Step 2. Compute the weighted average capitalization rate (the weighted average of the returns on tangible assets and excess earnings, the latter representing the return on intangible assets) implied in the excess earnings valuation, and compare it with the capitalization rate estimate in Step 1.

If the results of Step 1 and Step 2 are close, this implies passing marks on one test of the reasonableness of the rates used in the excess earnings method. (The analyst should recognize, of course, that the weighted average of the excess earnings method rates could have been close to the overall capitalization rate by accident, and that different tangible asset values or cash flows could still produce an unreasonable result.)

Computing the Weighted Average Excess Earnings Capitalization Rate

Returning to the excess earnings method valuation, we see that the company's estimated value of $370,000 was composed of $200,000 tangible asset value and $170,000 intangible asset value. Computing the relative weights of these asset values, we have:

Tangible assets	$200,000 ÷ $370,000 = 54.1%
Intangible assets	$170,000 ÷ $370,000 = 45.9
	= 100.0%

Weighting the required rates of return on the tangible and intangible asset value components gives us:

Tangible asset value	0.541 × 0.08 = 0.043
Intangible asset value	0.459 × 0.20 = 0.092
Weighted asset-based capitalization rate	= 0.135

Estimating a Build-up Model Capitalization Rate

For a very small company like KLM, the build-up method, presented in Chapter 8, is usually the best method for estimating an equity capitalization rate. To implement the build-up method, we will make four assumptions:

1. *Risk-free rate.* At the valuation date, 7.0%
2. *Equity risk premium.* Arithmetic average from *SBBI Valuation Edition Yearbook*, 7.4%
3. *Size premium.* Ibbotson's tenth decile size premium from *SBBI Valuation Edition Yearbook,* 5.33%
4. *Company-specific risk premium.* The company is tiny compared with Ibbotson's tenth-decile New York Stock Exchange stocks. The company has high specific risk because of lack of stability of customer base and economic vulnerability of its customer base due to conditions in the industry on which it is dependent. In addition, there is a key person issue: How easily could Kenny be replaced? Although this decision is quite subjective, it seems conservative to add a specific risk factor of 5.0%.

Adding up the pertinent factors gives us this discount rate:

Risk-free rate	7.00%
Equity risk premium	7.40
Size premium	5.33
Company-specific premium	5.00
Estimated KLM cost of equity (discount rate)	24.73%

We then subtract the estimated growth rate from the discount rate to get the estimated capitalization rate:

Estimated Capitalization Rate for KLM Cash Flows

Discount rate	24.73%
– Estimated long-term growth rate	3.00
= Estimated capitalization rate	21.73%

A 13.5% asset-based capitalization rate certainly is significantly different from a 21.73% capitalization rate based on capital structure components. If we divided the \$50,000 cash flow by the 13.5% asset-based capitalization rate, we would, of course, get the excess earnings method value of \$370,000 (\$50,000 ÷ 0.135 ≅ \$370,000).

If we divided the \$50,000 by the 21.73% build-up method capitalization rate, we would get an indicated value of \$230,097 (\$50,000 ÷ 0.2173 ≅ \$230,097).

Which do you believe?

Discussion of the Example

Considering the risks involved, it is unlikely that anyone would pay \$170,000 for the blue sky (a colloquial term used loosely to refer to intangible assets) in KLM. And it can be argued that the 21.73% capitalization rate is an already conservative capitalization rate itself.

Obviously, the capitalization rates for tangible assets and excess earnings used by Mr. Shoveler in his excess earnings exercise are considerably too low. Note that the asset appraisal assumes value *in use*. This is the value to an operating business, not a liquidation value. There appears to be plenty of risk associated with these tangible assets. Accordingly, one point over the risk-free rate is not nearly an adequate risk premium. Capitalizing excess earnings at 20% implies that a buyer will pay for five years of expected excess earnings (1 ÷ 0.20 = 5). Typically, buyers will pay that implied multiple only for a very stable customer base. The KLM customer base certainly is not stable.

Obviously, this is an extreme example. Its purpose is merely to illustrate the mechanics of using cost of capital as a reasonableness check on an excess earnings method valuation. However, there have been worse abuses. There are even people who think that the 30-day U.S. Treasury-bill rate is a satisfactory return rate of the tangible assets employed in a business. Watch out for such abuses!

VAGARIES OF THE EXCESS EARNINGS METHOD

Revenue Ruling 68-609 is not very specific on many points, such as how income is defined. Definitions of income other than net cash flow may require some adjustment to the capitalization rate, as we illustrated its development.

The many vagaries of the excess earnings method have been explored at great length and are beyond the scope of this book.[4] The purpose here is simply to show the mechanics of demonstrating whether the weighted capitalization rate implied in an excess earnings valuation is within a reasonable range.

SUMMARY

The excess earnings method uses two capitalization rates:

1. A required return on tangible assets
2. A rate at which to capitalize "excess earnings," returns over and above amounts necessary to support the tangible assets in a business

The position presented in this chapter is that the weighted average of these two rates (weighted at market value, of course) should be approximately equal to a company's capitalization rate (developed as discussed in this book). This chapter has illustrated the mechanics of how to make such a comparison.

To explore the many vagaries of implementing the excess earnings valuation method, readers are encouraged to avail themselves of the references listed in the notes.

Notes

1. See, for example, Jay E. Fishman, Shannon P. Pratt, J. Clifford Griffith, and Mark Wells, *Guide to Business Valuations*, 12th ed. (Fort Worth, TX: Practitioners Publishing Company, 2001, updated annually in May), 7-28–7-40; Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing Small Businesses and Professional Practices*, 3d ed. (New York: McGraw-Hill, 1998), Chapter 23, "The Capitalized Excess Earnings Method"; Shannon P. Pratt, Robert F. Reilly, and Robert P. Schweihs, *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. (New York: McGraw-Hill, 2000), Chapter 13, "The Capitalized Excess Earnings Method."
2. Fishman, Pratt, Griffith, and Wells, *Guide to Business Valuations*, 7-28. Copyright © 2002 by Practitioners Publishing Company. All rights reserved. Reproduced with permission from *Guide to Business Valuations, Twelfth Edition* (January 2002). For product information, phone 800-323-8724 or log onto *www.ppcnet.com*.
3. Ibid. at 7-28 to 7-29.
4. See the preceding references plus "Practitioners Disagree Strongly on Excess Earnings Methodology," *Shannon Pratt's Business Valuation Update®* (April 1997): 1–3. These references provide guidance on estimating rates of return on tangible and intangible assets, among other things.

Chapter 18

Common Errors in Estimation and Use of Cost of Capital

Confusing Discount Rates with Capitalization Rates
Using the Firm's Cost of Capital to Evaluate a More or Less Risky Acquisition or Project
Mistaking Historical Rates of Return for Expected Rates of Return
Mismatching the Discount Rate with the Economic Income Measure
- Using a Safe Rate to Discount or Capitalize a Risky Return
- Applying a Discount Rate in Real Terms to an Economic Income Projection in Nominal (Current) Terms
- Applying Costs of Capital Derived from After-tax Returns to Pretax Returns
- Subtracting a Short-term Supergrowth Rate from the Discount Rate to Get a Capitalization Rate
- Applying a Discount Rate Applicable to Net Cash Flow to Net Income

Performing an Excess Earnings Method Valuation That Results in an Unrealistic Cost of Capital
Projecting Growth beyond That Which the Capital Being Valued Will Support
Internally Inconsistent Capital Structure Projection
Assumptions That Produce a Standard of Value Other Than That Specified in the Valuation Engagement
Incorrect or Inadequately Supported Data in Estimating the Cost of Equity
Summary

The purpose of this chapter is to call attention to some of the errors frequently encountered in estimation and applications of the cost of capital. We point out these errors partly so that readers will not fall into the same traps themselves when estimating or using cost of capital. Another reason is to help readers readily identify such errors when reviewing the work of others and have an understanding about how the errors should be corrected.

CONFUSING DISCOUNT RATES WITH CAPITALIZATION RATES

The *discount rate* is the cost of capital, and it applies to *all* prospective economic income. This includes all distributions and realized or readily realizable capital appreciation. The *capitalization rate* is a divisor applied to some particular economic income (e.g., earnings, cash flow, etc., for the latest 12 months, upcoming 12 months, or some other period). Only when the expected level of economic income is constant in perpetuity are these two rates equal, other than by sheer coincidence.

Nevertheless, some analysts fall into the trap of using the discount rate (i.e., cost of capital) as a capitalization rate. The opposite is also seen from time to time: the use of a capitalization rate to discount prospective cash flows or other expected economic income to a present value.

The relationship between discount rates and capitalization rates is the subject of Chapter 4.

USING THE FIRM'S COST OF CAPITAL TO EVALUATE A MORE OR LESS RISKY ACQUISITION OR PROJECT

We have emphasized throughout this book that the cost of capital is market driven and that it is a function of the investment, not the investor.

If an acquirer uses its own cost of capital to set an acquisition price by discounting the expected cash flows of a riskier acquiree, then the result will be some increase in the risk of the acquiring company after the acquisition. This will result in an increase in the company's overall risk and cost of capital, to which the market would be expected to respond by reducing the stock price. Decreases in acquirers' stock prices as a result of acquisitions are very common phenomena, although it is not possible to sort out the extent to which this is a result of perceived overestimation of future cash flows or a market adjustment to the company's cost of capital.

The same principle applies to internal capital budgeting and project selection as to acquisitions. If the project under consideration is more or less risky than the activities of the company as a whole, then the expected cash flows from the project should be evaluated by a correspondingly higher or lower discount rate. In deciding among competing potential projects, an analyst should be certain to appropriately reflect the risk of each project in the discount rate applied to the respective project.

MISTAKING HISTORICAL RATES OF RETURN FOR EXPECTED RATES OF RETURN

Remember, cost of capital is a forward-looking concept. Cost of capital is the *expected* rate of return that the market requires to induce investment in a subject security.

Historical returns rarely are what investors expect in the future. Even if one were to extrapolate historical returns into the future, one would be wrong most of the time.

The analyst should use historical returns only for guidance about what to expect in the future. If extrapolating historical returns into the future, the analyst should have some basis for the assumption that the future will resemble the past on an extrapolated basis. This assumption should be articulated in the analyst's report. Rarely are conditions in the economy, the industry, and the specific company comparable to what they were in the past. The common error is for the analyst to naively assume that the future will fall somewhere near the line of extrapolation of the past. If this is, indeed, the assumption, the analyst should demonstrate that he or she has done the analysis and concluded that this will be true or that, at least, it is the best estimate of the future.

A related error is to take the recent average historical rates of return that have been achieved for an industry, often from a source of industry composite statistics such as the Risk Management Association's (RMA) (formerly Robert Morris Associates) *Annual Statement Studies*, and to assume that this average is the expected return required to attract investment in that industry. The returns actually achieved for a particular industry in recent past years may be well above or below the level of expected return required to attract capital to the industry and certainly do not represent a reliable indicator of the cost of capital. Furthermore, returns shown in sources such as the RMA *Annual Statement Studies* are based on book values, whereas the relevant measure is return on market values. (The preceding discussion is not intended to totally eliminate *all* consideration of historical returns, such as a long-term average equity risk premium.)

MISMATCHING THE DISCOUNT RATE WITH THE ECONOMIC INCOME MEASURE

The most common type of error in application of the income approach to valuation is to use a discount or capitalization rate that is not appropriate for the definition of economic income being discounted or capitalized. This general category of error has almost infinite variations. Those discussed in the next paragraphs are only a few.

Using a Safe Rate to Discount or Capitalize a Risky Return

Although not the most common version of the mismatching error, the use of a safe rate to discount or capitalize a risky return certainly is one of the most egregious. On occasion, analysts erroneously discount a highly risky series of projected economic income by the U.S. Treasury-bill rate!

Applying a Discount Rate in Real Terms to an Economic Income Projection in Nominal (Current) Terms

In discounting or capitalizing, some analysts erroneously subtract the anticipated inflation rate from the discount rate and then apply the adjusted discount rate to an economic income projection that includes inflation (and vice versa). It is noteworthy that all the Ibbotson data are presented in nominal terms—that is, they include inflation. The most common way of performing the income approach to valuation in the United States and in other mature economies is to express the cash flows in nominal terms (including the effect of inflation) and use a nominal discount rate. In countries with hyperinflation, it is more common to express expected cash flows in real terms and to use a discount rate not including expected inflation.

Applying Costs of Capital Derived from After-tax Returns to Pretax Returns

Whether costs of capital are estimated by the build-up model, the Capital Asset Pricing Model (CAPM), or the discounted cash flow (DCF) method, in all cases they are returns realized *after* the payment of corporate-level income taxes. If the entity being valued is subject to entity-level income taxes, then it is inappropriate to apply the cost of capital estimated by those methods to pretax return flows.

As we go to press, three controversial U.S. Tax Court cases have been issued that many analysts believe are guilty of this error.[1] The cases all involved S corporations, and the Tax Court applied discount rates developed on an after-tax basis to S corporation earnings, which are pretax, on the theory that the corporation itself did not pay the taxes on the earnings. But *someone* had to pay the taxes. All S corporation status does is pass through the tax liability to the owners and thus avoid double taxation. But S corporation status does not avoid, or even defer, the obligation to pay the taxes on the earnings.

Most public companies have a rather low payout ratio of dividends to earnings, if they even pay dividends at all. But in the case of S corporations, the earnings are all taxed to the owners, regardless of whether the earnings are actually distributed. We will undoubtedly hear more about this issue in the months and years to come.

Subtracting a Short-term Supergrowth Rate from the Discount Rate to Get a Capitalization Rate

Converting a discount rate to a capitalization rate involves subtracting an estimate of the *long-term sustainable growth rate*. Many companies expect high short-term growth that will tend to dampen over time. If the high short-term growth rate is subtracted from the discount rate, the proper capitalization rate will be understated, resulting in overvaluation. In such circumstances, a two-stage or three-stage DCF valuation model usually will produce a more valid valuation than a straight capitalization model.

Applying a Discount Rate Applicable to Net Cash Flow to Net Income

One of the reviewers of this book said that this is the most common error he sees in practice.

Ibbotson data produces a discount rate that is applicable to net cash flow, which usually is lower than net income. In such cases, the discount rate developed for net cash flow (or the capitalization rate derived by subtracting growth from the discount rate) would result in overvaluation if applied to net income.

If a consistent relationship exists between net cash flow and net income, then it is possible to adjust the discount rate by the proportion of net income to net cash flow. In a capitalization model, this is a reasonable approximation. However, very few companies have an adequately consistent relationship between net cash flow and net income to make this any more than an approximation.

PERFORMING AN EXCESS EARNINGS METHOD VALUATION THAT RESULTS IN AN UNREALISTIC COST OF CAPITAL

One very useful application of the cost of capital analysis is to do a sanity check on the reality of a valuation performed by the excess earnings method, as discussed in the previous chapter.

In the excess earnings method, two capitalization rates are estimated:

1. A capitalization rate for tangible assets
2. A capitalization rate for excess earnings (return over and above the amount required to support the company's tangible assets)

The excess earnings method derives its capitalization rates by very different methods from those discussed earlier in this book. It is based on required returns to categories of assets rather than on required returns to categories of capital. Nevertheless, at the end of the day, the value as estimated by the excess earnings method should reflect a capitalization rate very similar to that which would be derived if we developed a discount rate by any of the cost of capital estimation methods presented in this book and subtracted a reasonable estimate of long-term sustainable growth.

The following is an example of such a sanity check:

Sanity Check. Is the overall equity capitalization rate approximately equal to what you would expect using a build-up capitalization rate?

1. Analysis of overall equity cap rate using excess earnings method:	
Net cash flow to equity	$270
Divided by: indicated equity value	$1,205
Equals implied cap rate on equity:	22.4%
($270 ÷ $1,205 = 22.4%)	
2. Build-up cap rate:	
20-year government bond rate	7.0%
Small stock equity risk premium (combined general equity premium and small stock premium)	15.8%
Specific risk premium for subject	5.0%

Total required rate of return (discount rate)	27.8%
Less: expected sustainable growth rate	4.0%
Equals cap rate applicable to net cash flow:	23.8%

According to the sanity check, the results of the excess earnings method seem reasonable. If we capitalize the $270 net cash flow to equity at 23.8%, we would have an indicated value of $1,134, compared with $1,205 achieved by the excess earnings method. This is a reasonable range of difference. If the results were significantly different, we would reexamine all our calculations and assumptions.

In the above example, we dealt only with a capitalization rate for equity, because most of the data sources used to perpetrate this error show only returns to equity rather than returns to total capital. The excess earnings method, however, is used more often to value controlling interests than minority interests. Consequently, the return to total capital, as measured by the weighted average cost of capital (WACC), is relevant, and thus the capitalization rate for overall invested capital also should be considered in the reasonableness test.

This use of cost of capital as a reasonableness check for an excess earnings method valuation was the subject of Chapter 17. That example demonstrated a significant overvaluation by the excess earnings method.

PROJECTING GROWTH BEYOND THAT WHICH THE CAPITAL BEING VALUED WILL SUPPORT

As businesses expand, they typically need additional working capital and capital expenditures to support the increased level of operations. One of the many advantages of using net cash flow as the prospective economic income measure is that it forces the analyst to explicitly consider these needs. Nevertheless, they often are underestimated.

When cost of capital is used for valuation, it values only the investment as of the valuation date. The calculation of net cash flow allows for reinvestment for capital expenditures and additions to working capital necessary to support projected operations. However, if the projections being discounted will *not* be totally supported by the capital expenditure and working capital allowances in the net cash flow projections, and additional investment will be required to achieve those projected results, then the *existing* investment will be overvalued.

Many analysts do not make adequate deductions for capital expenditures and additions to working capital. Analysts should be sure that these items bear a reasonable relationship to revenue, especially to revenue growth. A good idea is to project balance sheets as well as income statements. This helps to show potential asset deficiencies.

These items also should be examined when calculating the terminal value. If growth is assumed when calculating the terminal value, then capital expenditures following the discrete projection period should necessarily exceed depreciation following the discrete projection period. Many analysts assume capital expenditures to equal depreciation when estimating the terminal value, which results in overestimation of expected net cash flow and overvaluation, where growth is expected.[2]

INTERNALLY INCONSISTENT CAPITAL STRUCTURE PROJECTION

Methods using weighted average cost of capital and betas adjusted for leverage require projections about the subject company's capital structure. These projected capital structures are on the basis of market value. Analysts often assume a capital structure in the process of estimating a market value of equity, and the resulting estimated market value of equity makes the capital structure, at the estimated market value, different from that which was assumed.

In such cases, the projected capital structure has to be adjusted and the process iterated until the estimated market value of equity results in a capital structure consistent with that which is projected in estimating the cost of capital.

Even worse, of course, is to not even estimate a market value capital structure but simply to use book value. If the company is earning good returns, then the market value of equity is likely to exceed book value. This is true not only for the subject company but also for peer companies that may be used to estimate an industry-average capital structure. If the market value of equity is understated, then the assumed proportion of low-cost debt in the capital structure will be too high. This will result in an understatement of the WACC and an overstatement of value.

Chapter 7 (WACC) addresses estimating capital structure by the iterative process, and Appendix E illustrates using the iterative process in the context of CAPM.

ASSUMPTIONS THAT PRODUCE A STANDARD OF VALUE OTHER THAN THAT SPECIFIED IN THE VALUATION ENGAGEMENT

A common error is to project a capital structure other than the company's actual capital structure (thereby deriving a weighted average cost of capital different from the company's actual WACC) when the standard of value is fair market value on a minority basis. If an acquirer were to use its own WACC, then the implied result would be *investment value* to that acquirer instead of fair market value. Moreover, if the equity ownership interest is a minority interest, the holder could not force a change in capital structure.

Another enormous assumption that would lead to investment value rather than fair market value would be to inject the benefits of synergies into the projected cash flow stream. This would produce the value to a particular buyer who could take advantage of the synergies, rather than a hypothetical buyer.

INCORRECT OR INADEQUATELY SUPPORTED DATA IN ESTIMATING THE COST OF EQUITY

The first mistake sometimes made in this category is to mismatch the risk-free rate with the equity risk premium. Ibbotson has equity premiums series that attach to 30-day Treasury-bill maturities, five-year Treasury-note maturities, and 20-year

Treasury-bond maturities. The equity risk premium should be selected to match one of those maturities.

Ibbotson's *Stocks, Bonds, Bills and Inflation Valuation Edition Yearbook* specifies the range for each of the size premia, and these vary from year to year. The size is measured in terms of market value of common equity capital. The size premium decreases as the market value increases. Mismatching the company size to the company size premium can either understate or overstate the equity risk premium and therefore understate or overstate value.

Another common error is inadequate support for the company-specific risk premium. Within the CAPM, a portion of the company-specific risk may be captured in beta. Certainly major portions of the company-specific risk are captured in the size premium. The remaining company-specific premium is totally a matter of the analyst's judgment. It should be based on quantitative and/or qualitative analysis, which should be detailed in the report. The company-specific risk premium should be as small as possible—sometimes we see 10 percentage points in the company-specific risk premium, and that, depending on the company and the industry, is normally too much. Ten points would push the discount rate close to a venture capital or start-up company rate. In any case, the company-specific risk premium should be supported with a very strong narrative.

SUMMARY

Cost of capital is one of the most critical components in valuation, capital budgeting, and other financial decision making. There are many ways to err in both estimating the cost of capital and applying it in practice. The errors regularly seen in actual practice are prolific. Some of the major areas that require careful consideration include:

- Properly distinguishing between discount rates and capitalization rates
- Making sure that the data supporting discount and capitalization rates represent *expected* returns (current market-required returns), not past returns that do not represent future expectations
- Making sure that the discount or capitalization rate used *matches* the definition of expected returns being discounted or capitalized
- Making sure that the implied weighted capitalization rate used in an excess earning valuation procedure is reasonably close to capitalization rates developed by the cost of capital estimation methods discussed in this book
- Being careful that projected returns being discounted or capitalized can be achieved without having to dilute the existing capital with additional outside capital
- Being sure that capital structure assumptions fully reflect the *market values* of the capital structure components
- Being sure that valuation results are estimated consistently with the definition of value specified in the valuation assignment
- Making sure that the cost of equity is calculated correctly and supported adequately

Avoid all of these traps and you get a gold star! More important, your company and your clients will be well served, and your cost of capital work should withstand rigorous scrutiny.

Notes

1. *Gross v. Commissioner*, T.C. Memo 1999-254, 78 T.C.M. (CCH) 201 (U.S. Tax Ct. 1999), *affirmed*, 272 F.3d 333 (6th Cir. 2001); *Estate of Heck v. Commissioner*, T.C. Memo 2002-34, 83 T.C.M. (CCH) 1181 (U.S. Tax Ct. 2002); *Estate of Adams v. Commissioner*, T.C. Memo 2002-80 (U.S. Tax Ct. 2002).
2. For a good discussion of this common error, see Gilbert E. Matthews, "Cap X = Depreciation Is Unrealistic Assumption for Most Terminal Values," *Shannon Pratt's Business Valuation Update*® (March 2002): 1–3.

Chapter 19

Cost of Capital in the Courts

Cost of Capital in Shareholder Disputes
Cost of Capital in the Tax Court
Cost of Capital in Family Law
Cost of Capital in Bankruptcy Reorganizations
  Setting Interest Rates
  Valuation of Stock by the Income Approach
Cost of Capital Included in Damages
Cost of Capital in Utility Rate-setting
Taxicab Lease Rates
Summary

Cost of capital is getting ever-increasing attention in the courts within many contexts. These include valuations for many judicial purposes, allowed rates of return as a component in rate-setting, and other applications.

This chapter touches briefly on many of the contexts in which many millions of dollars can hinge significantly on the court's determination of the relevant cost of capital. For each context, we have cited one or more cases that are typical of contemporary court deliberations on the subject.

COST OF CAPITAL IN SHAREHOLDER DISPUTES

Delaware traditionally has been the case law trendsetter on shareholder disputes. A landmark Delaware Supreme Court case in 1983 reversed a lower court case because it did not consider future earnings projections. The court made the point that a determination of fair value (the statutory standard of value in Delaware, as well as in most other states, for dissenting stockholder actions) "must include proof of value by any techniques or methods which are generally considered acceptable in the financial community."[1]

Since that time, Delaware courts have increasingly embraced the discounted cash flow (DCF) method of valuation. In a 1997 case, the Delaware Chancery Court characterized the DCF method as "increasingly the model of choice for valuations in this Court."[2]

In a typical case before the Delaware Supreme Court, both experts used the DCF method as well as other methods. The court rejected the other methods and focused on the DCF method. The court accepted one expert's Capital Asset Pricing Model (CAPM) for the cost of equity capital, with minor modifications to the beta ("[T]he beta employed shall be based on the average beta of [the subject company's] comparable companies.") The court made some minor adjustments to the cash flow projections because "it was apparent by the date of the merger [the effective valuation date] that [the subject company] would have a very difficult time meeting this projection." Both experts used market multiples for their terminal values. The court accepted the multiple of the expert who had "convincingly demonstrated the appropriateness of [his] selection of [comparable companies]."[3]

However, in another case before the Delaware Court of Chancery, both experts used the DCF method, with one using a 12% discount rate and the other an 18% discount rate. The court determined that the difference between the experts' discount rates was "attributable primarily to their different estimates of [the subject company's] cost of equity capital, and their different assumptions of the company specific risks confronting [the subject company] at the time of the merger." The court disagreed with certain of the other assumptions applied by both of the parties' experts. The court of chancery ultimately concluded that it could not rely on the DCF valuation opinion of either party's expert.

The Delaware Supreme Court affirmed, stating: "Similarly, by recognizing the discounted cash flow model as one proper valuation technique, the Court of Chancery was not required to use that methodology to make its own independent valuation calculation by either adapting or blending the factual assumptions of the parties' experts. The ultimate selection of a valuation framework is within the Court of Chancery's discretion."[4]

COST OF CAPITAL IN THE TAX COURT

Both the Internal Revenue Service (IRS) and the U.S. Tax Court traditionally have leaned more toward the market approach than the income approach. The reason is partly because of language in Revenue Ruling 59-60, written before the development of modern capital market theory, which evolved in the 1960s. The market approach has also been favored partly because of concern about possible manipulation of both cash flow forecasts and discount rates in the DCF method. Nonetheless, as the DCF method has achieved greater utilization in the professional financial community, it has also achieved greater acceptance in the Tax Court.

In a 1985 case the IRS challenged income tax returns reflecting deductions for a company's contributions to the employee stock ownership trust at $61.35 per share, based on the fair market value estimated by an independent appraisal firm. The IRS asserted that the value was between $5.36 and $8.00 per share. The independent appraisal heavily emphasized earning power and dividend-paying capacity, whereas the IRS stressed net asset value (book value was $7.05 per share).

The court agreed with the emphasis on earning power and dividend-paying capacity. The court was somewhat concerned "that the appraisal took into account a 20-year earnings projection" but thought it was "not unreasonable in light of past earnings increases." The court concluded that the only reasonable appraisal presented to it was the one at $61.35 per share.[5]

Yet the Tax Court has *rejected* the DCF method in cases where it believed that the model used by the expert was far too sensitive to minor changes in assumptions, such as the discount rate and/or the growth rate.[6]

The Tax Court also has been known to reach its conclusion by giving partial weight to a DCF method and partial weight to a market approach method. For example, one case gave 70% weight to the market method and 30% weight to the DCF method.[7]

One Tax Court judge, obviously very knowledgeable about CAPM, rejected testimony offered in the context of the DCF but gave the following critique of the testimony for the guidance of other appraisers:

> *Beta of 1.00 too low.* Beta, a measure of systematic risk, is a function of the relationship between return on an individual security and the return on the market as a whole. . . . However, because the betas for small corporations tend to be larger than the betas for larger corporations, it may be difficult to find suitable comparables when valuing a small, closely held corporation. . . . [T]here are substantial differences in size and operations between [the subject company] and the banks on the VL bank list [*Value Line Investment Survey* (4th ed., Apr. 9, 1993)]; we do not believe that their betas are representative of the greater business risks faced by [the subject company].
>
> We do not believe that an investment in [the subject company], a small, single-location bank, whose earnings were susceptible to impending interest rate mismatches and sluggish local economic conditions, presents the same systematic risk as an investment in an index fund holding shares in 500 of the largest corporations in the United States. . . .[8]
>
> *Failure to add small stock premium.* Although [the witness] cited Ibbotson as his source for equity risk premium, in his initial report he ignored a crucial aspect of the Ibbotson approach to constructing a cost of capital—the small stock premium. In his rebuttal report, [the witness] unsuccessfully tried to persuade us that the small stock premium is not supported by financial theory, characterizing the risk associated with a firm's size as unsystematic risk, for which the market does not compensate. The relationship between firm size and return is well known. Size is not an unsystematic risk factor and cannot be eliminated through diversification. "On average, small companies have higher returns than large ones." Ibbotson at 125. . . . [I]t has been found that the greater risk of small stocks is not fully reflected by CAPM, in that actual returns may exceed those expected based on beta. . . . Consequently, when calculating a cost of capital under CAPM on a small stock . . . , it is appropriate to add a small stock premium to the equity risk premium, to reflect the greater risk associated with an investment in a small stock in comparison to the large stocks from which the equity-risk premium is calculated. Based on [the subject company's] size, a microcapitalization equity size premium of 3.6 percent should have been added. See Ibbotson at 161. . . .[9]
>
> *Failure to account for unsystematic risk.* [B]ecause CAPM assumes that an investor holding a diversified portfolio will encounter only systematic risk, the only type of risk for which an investor can be compensated is systematic or market risk, which represents the sensitivity of the future returns from a given asset to the movements of the market as a whole [citing Brealey & Myers, *Principles of Corporate Finance* 137–138, 143–144 (4th ed. 1991); Pratt et al., *Valuing a Business* 166 (3rd ed. 1996)]. . . .

> [The witness] followed the principles of CAPM and did not make any provision for [the subject company's] unsystematic risk, based on the assumption that such risk was diversifiable. . . . [R]espondent and [the witness] have overlooked the difficulties in diversifying an investment in a block of stock they argued is worth approximately $8.94 million. Construction of a diversified portfolio that will eliminate most unsystematic risk requires from 10 to 20 securities of similar value. See Brealey & Myers, supra at 137–139. Thus, proper diversification of an investment in the [the subject company] shares owned by petitioner, as valued by respondent, would require a total capital investment of at least $89 million. We do not think the hypothetical buyer should be limited only to a person or entity that has the means to invest $89 million in Peoples and a portfolio of nine other securities. . . .[10]

In another case, a witness added a small stock premium applicable to companies much smaller than the subject company. For this reason, the court rejected that witness's cost of equity developed by the CAPM model and accepted the cost of equity capital developed by the opposing witness, also by the CAPM model.[11]

The court rejected the DCF model for a small company in another case because it did not believe that CAPM and the WACC were "the proper analytical tools to value a small, closely held corporation with little possibility of going public."[12]

Another case rejected the conclusion of a witness because he failed to apply a small stock premium and used too low a beta:

> Respondent relies on an article by Bajaj & Hakala, "Valuation for Smaller Capitalization Companies," published in Financial Valuation: Business and Business Interests, ch. 12A (Hanan & Sheeler ed. 1998), for the proposition that there is no small-stock premium. We find [petitioner's expert's] analysis to be more persuasive."
>
> [Respondent's expert] testified that it is appropriate to use the Ibbotson Associates data from the 1978–92 period rather than from the 1926–92 period because small stocks did not consistently outperform large stocks during the 1980's and 1990's. We give little weight to [respondent's expert's] analysis. [He] appeared to selectively use data that favored his conclusion. He did not consistently use Ibbotson Associates data from the 1978–92 period; he relied on data from the 1978–92 period to support his theory that there is no small-stock premium but used an equity risk premium of 7.3 percent from the 1926–92 data (rather than the equity risk premium of 10.9 percent from the 1978–92 period). If he had used data consistently, he would have derived a small stock premium of 5.2 percent and an equity risk premium of 7.3 percent using the 1926–92 data, rather than a small-stock premium of 2.8 percent and an equity risk premium of 10.9 percent using the 1978–92 data.
>
> We conclude that [petitioner's expert] appropriately applied a small-stock premium in valuing the [subject company's] stock.[13]

In the same case, another issue in the income approach was the petitioner's expert's build-up method versus the respondent's expert's CAPM method. Respondent's expert lowered his equity discount rate by multiplying it by a beta of 0.7. Judge Colvin stated the following:

> We disagree with [respondent's expert's] use of a .7 beta because [the subject company] was a small, regional company, had customer concentrations, faced litigation and envi-

> ronmental claims, had inadequate insurance, was not publicly traded, and had never paid a dividend. A beta . . . can only be correctly estimated on the basis of the betas of comparable publicly traded companies. . . . [Respondent's expert] stated that he selected the beta based on a review of comparable companies. However, he did not identify these comparable companies or otherwise give any reason for his use of a .7 beta. We believe [his] use of a .7 beta improperly increased his estimate of the value of the [subject company's] stock.[14]

As we go to press, three cases involving S corporations recently were decided. Since S corporation earnings are pretax, many appraisers valuing S corporation stock either tax-affect the earnings (apply hypothetical taxes as if they were a C corporation) or increase the discount or capitalization rate by dividing the after-tax rate by 1 minus the effective tax rate to derive a rate applicable to pretax earnings.[15]

The first such case was *Gross v. Commissioner*.[16] One expert tax-affected the S corporation earnings and the other did not. The Tax Court accepted the procedure of not tax-affecting the earnings. The case was appealed to the Sixth Circuit Court of Appeals, where the Tax Court decision was upheld by a two-to-one vote of the deciding judges. The minority judge wrote a lengthy dissenting opinion.[17]

The next case was *Estate of Heck v. Commissioner*.[18] In this case neither the expert for the estate nor the expert for the IRS tax-affected the earnings or adjusted the after-tax discount rate. Since both agreed, it was not an issue in the case, and the after-tax discount rate applied to the S corporation's pretax earnings was allowed to stand. The expert for the taxpayer in that case was the same expert who testified for the IRS in the *Gross* case.

The last case before we went to press was *Estate of Adams v. Commissioner*.[19] In the *Adams* case, one appraiser adjusted the capitalization rate (derived from Ibbotson after-tax discount rates) to apply to pretax earnings and the other did not. The Tax Court accepted the nonadjusted rate, resulting in an outcry from the professional business appraisal community. We have yet to see whether the *Adams* case will be appealed.

It is my opinion that these cases, if allowed to stand unchallenged and cited as precedent, represent bad case law and a misinterpretation of fair market value. Taxes on S corporation earnings have to be paid. Denying tax-affecting of the earnings limits the pool of potential buyers to nontaxable entities. I believe that we will see some modification of these results in appeals of these cases or in future cases.

COST OF CAPITAL IN FAMILY LAW

Cost of capital is getting increasing attention in family law courts as those courts are becoming more receptive to the DCF method of valuation of closely held businesses for marital property divisions.

In a 1996 Ohio case, for example, the trial judge stated from the bench that the DCF method had never been used before within her family law jurisdiction. She was willing to consider it, based on testimony that members of the professional financial community would be likely to use the method in valuing a company of the particular type at issue in the case. Ultimately the court not only accepted the method but used the value indicated by the DCF method as its final conclusion of value. The case was appealed and upheld.[20] Other courts have similarly followed suit.[21]

Shannon Pratt's Business Valuation Update® consistently reports family law cases that recognize the DCF method.

COST OF CAPITAL IN BANKRUPTCY REORGANIZATIONS

Cost of capital arises in bankruptcy proceedings in at least two contexts:

1. Setting interest rates
2. Valuing companies or interests in companies by the income approach

Setting Interest Rates

The concept of the cost of capital as described in this book is recognized for the purpose of setting interest rates in the case of a bankruptcy reorganization. For example, the Seventh Circuit Court of Appeals rejected a trustee's notion that the interest rate a creditor should receive should be the Treasury-bill rate.

The court stated that the creditor was entitled to "indubitable equivalence of its property interest, which means a stream of payments including interest that adds up to the present value of its claim. . . . [T]he creditor must get the market rate of interest . . . for loans for equivalent duration and risk."

The court then added: "To say that the lender is limited to its 'cost of capital' . . . is therefore to say that the lender is entitled to the market rate of interest, for that is what its cost of capital is: the price it must pay to its own lenders, plus the costs of making and administering loans, plus reserves for bad debts (that is, the anticipated rate of nonpayment)."[22]

Another court of appeals case in a different circuit stated the same concept. In considering a reorganization plan involving interest payments, it stated that the issue is whether the interest rate provides the plaintiffs with the "present value" of their claims. The court explained: " 'Present value' is a market rate concept, determined by the use of an interest rate which fairly compensates the creditor for not receiving the amount of its secured claim upon confirmation of the debtor's plan."

The same court added, "[A]n entity forced to delay payment that it is entitled to receive is, in effect, extending a loan." The court further stated: "[T]he purpose . . . is . . . 'to put the secured creditor in an economic position equivalent to the one it would have occupied had it received the allowed secured amount immediately.' . . . [T]he appropriate rate of interest is 'that which the secured creditor would charge, at the effective data of the plan, for a loan similar in character, amount and duration to the credit which the creditor would be required to extend under the plan'."[23]

The preceding case quoted the Third Circuit's case of first impression on this issue, a frequently-quoted case. In effect, the case rejected use of the prime lending rate and required a market rate for similar credits:

> Some courts, including the bankruptcy court in the cases we are here reviewing, have suggested a "cost of funds" theory which would determine present value by looking to the creditor and the market in which the creditor borrows capital.
>
> There is more involved, however, than the mere cost of funds. . . .
>
> It is only by acknowledging the coerced loan aspects of a cramdown and by compensating the secured creditor at the rate it would voluntarily accept for a loan of similar character,* amount and duration that the creditor can be placed in the same position he would have been in but for the cramdown.
>
> We hold that the bankruptcy and district courts erred in utilizing the prime rate to determine whether the proposed plan, as required by [the bankruptcy code], provided for payments to the creditor having a present value equal to the value of its allowed secured claim. The appropriate interest rate for this purpose is the rate of interest currently being charged by the creditor in the regular course of its business for loans similar in character, amount and duration to the loan being coerced in the cramdown.[24]
>
> ---
>
> * By a loan of "similar character" we mean a loan that the creditor regularly extends to other debtors who are not in bankruptcy but who are otherwise similarly situated to the debtor who is the recipient of the loan coerced by the Chapter 13 proceeding and who are seeking the same kind of credit (e.g., auto loan, home equity loan, etc.).

The Fifth Circuit reached a similar conclusion in a case decided September 8, 1997.[25]

A common issue is setting appropriate interest rates to establish present value in "cram down" situations. In these situations, the bankruptcy court, in effect, forces the creditor to accept a new loan for the property rather than allowing the creditor to repossess the secured property.

In a consolidated 1998 case,[26] the debtors' Chapter 13 plan proposed to provide a 9% interest rate on the balances due the creditors on automobiles in the debtors' possession. Both creditors maintained that a 9% rate was inadequate to provide them with an amount equal to the present value of their claims as required by the bankruptcy code. Both creditors argued that the appropriate capitalization rate should be based on the rate of interest they currently charge for similar loans in the region. According to the creditors, the appropriate rate should be 20.19% in one case and 21% in the other.

The court determined that any analysis of this issue necessarily begins with the Tenth Circuit opinion of *In re Hardzog*.[27] The Tenth Circuit held that "in the absence of special circumstances, such as the market rate being higher than the contract rate, bankruptcy courts should use the current market rate of interest used for similar loans in the region."

The debtors argued that the "current market rate of interest" should be some variation on the "prime lending rate" or "prevailing market rate" as reflected, for example, in *The Wall Street Journal*. The creditors argued that the correct interpretation of "current market rate of interest," as used in *Hardzog*, means the rate charged by the particular or objecting creditor for similar loans in the region in the recent past.

Citing *Hardzog*, the court determined that "similar" loans means a loan that the creditor regularly extends to other borrowers who are not in bankruptcy but who are otherwise similarly situated to the debtor. Thus, the court ordered that the appropriate rates for the debtors should be 20.1% and 21%, respectively.

Another case that followed shortly thereafter articulated the same conclusion[28]:

The remaining matter for the Court to decide is the appropriate cram down discount, or interest rate n1 for payment for Household on the secured portion of its claim under [the bankruptcy code]. Debtors' Chapter 13 plan proposed to pay an interest rate of 10% on the secured portion of Household's claim, and Household objected to this interest rate as being insufficient. The testimony of the witness from Household indicated that based on the Debtors' credit history, Household's current interest rate would be in a range between 18.95% and 24.95%. Debtors' contract rate of interest is 22.95%. . . .

n1 For the sake of clarity, the Court will refer to "interest rate" though "discount rate" or "capitalization rate" may be more appropriate terms.

[The code] requires that a secured creditor receive payments that provide the present value of the secured amount of its claim. In determining the appropriate interest rate, this Court must begin its analysis *with Memphis Bank & Trust Co. v. Whitman, 692 F.2d 427 (6th Cir. 1982)*. In Memphis Bank, the Sixth Circuit stated that it would be inappropriate to arbitrarily establish an interest rate for cram down under [the code], but instead "bankruptcy courts should use the current market rate of interest used for similar loans in the region." *Memphis Bank, 692 F.2d at 431*. In making this determination, the Court must "assess interest on the secured claim for the present value of the collateral . . . in order not to dilute the value of that claim through delay in payment. In effect, the law requires the creditor to make a new loan in the amount of the value of the collateral rather than repossess it, and the creditor is entitled to interest on his loan." *Memphis Bank, 692 F.2d at 429*.

The Sixth Circuit subsequently ruled that "the most equitable rate to establish in this type of situation is the prevailing market rate of interest on similar types of secured loans at the time of allowance of the creditor's claim and the confirmation of the plan in bankruptcy with a maximum limitation on such rate to be the underlying contract rate of interest." *In re Colegrove, 771 F.2d 119, 123 (6th Cir. 1985)*. The use of the "current market rate" of interest for cram down under . . . [the bankruptcy code], was also adopted in *U.S. v. Arnold, 878 F.2d 925 (6th Cir. 1989)*. The Arnold court noted that the contract interest rate maximum set forth in Colegrove was applicable only for fully secured creditors who were not required to write-down any portion of a loan. *Arnold, 878 F.2d at 929, 930. . . .*

In light of Memphis Bank, Arnold and Colegrove, interest rates offered to low risk borrowers are not necessarily appropriate for cram down. . . . Other circuits have adopted a "coerced loan" approach, consistent with Memphis Bank, for determining appropriate cram down interest rate. . . . See *General Motors Acceptance Corp. v. Jones, 999 F.2d 63 (3d Cir. 1993); In re Hardzog, 901 F.2d 858 (10th Cir. 1990)*. The various approaches to determining the appropriate cram down interest rate were analyzed in a recent article, David G. Epstein, Don't Go and Do Something Rash About Cram Down Interest Rates, *49 Ala. L.Rev. 435 (1998):*

> Accordingly, in applying "value, as of the effective date of the plan," bankruptcy courts should endeavor to leave the secured creditor as well off as if it had been paid the amount of its secured claim in cash. The cram down interest rate should reflect what the secured creditor would have earned had it taken that cash and reinvested it in loans with terms comparable to the terms proposed by the debtors' plan and with risks comparable to the risks presented by the debtor's nonpayment. . . .

This court finds that evidence of the relevant creditor's recent loan rate for similar loans within the region will be instrumental for setting the proper cram down interest rate. . . .

In this case, Debtors presented no testimony or evidence concerning the market rate of interest that was available for borrowers with similar credit histories, but simply proposed an arbitrary interest rate of 10% for cram down. . . . Absolutely no support was provided for this rate, and the Court cannot accept such an arbitrary rate in light of Memphis Bank. The only evidence requiring an adjustment of the contract rate of interest, to accurately reflect the current market interest rate for similar loans, came from Household's witness who testified that the current market rate of interest for similar loans ranged from 18.95% to 24.95%. The Court believes that the reduced risk associated with Debtors' payments to the Chapter 13 trustee, and discharge of debts on completion of their Chapter 13 plan supports the use of the interest rate at the low end of Household's "range," and will set 18.95% as the interest rate to be paid on the secured portion of Household's claim under § 1325(a)(5)(B).

The above case contains references to many other cases espousing this position.

Valuation of Stock by the Income Approach

One case found flaws in the application of the discounted cash flow method[29]:

In discussing [the expert's] application of the discounted cash flow method, the court found several flaws in [his] report and testimony:

- He failed to include information regarding additional working capital or capital expenditures that would be required for QSA to expand its business as he projected,
- He projected earnings without management input,
- He failed to account for Mahoney's salary as president and CEO of QSA, and
- He assumed that QSA would terminate a distributorship contract, and further assumed an estimated cost savings associated with this termination.[30]

The court ultimately rejected the above expert witness's valuation and accepted the valuation of the opposing expert witness.

The issue in another interesting adversary proceeding[31] was whether the transfer of stock of Danbury Surgical Center, Inc. (DSC) and Bridgeport Surgical Center, Inc. (BSC) by debtors Googel and Sisti to the defendants Steinberg and Simons were fraudulent transfers under the bankruptcy code.

The transfer agreement recited the consideration as $100,000 for all of the stock. There was conflicting testimony regarding whether the release of debtors' liability on the centers' financing was part of the consideration and whether additional consideration was promised to be paid "after the debtors' financial troubles were over."

The trustee's expert used the discounted net cash flow method to value the stock, projected net cash flows for three years, used the industry average capital structure of 24% debt and 76% equity, computed a discount rate of 17.5%, and determined that DSC's business enterprise value was $7,124,000. From this value he subtracted interest-bearing debt of $3,798,000, applied a 20% discount for lack of marketability, and applied an additional 5% discount for restrictions on voting rights. Based on Googel and Sisti's ownership percentages of the total stock of DSC, the trustee's expert concluded that each debtor's ownership interest was worth $349,100. He further tested the validity of his valuation by comparing the results with the value of publicly traded surgical centers and with comparable centers that were traded within a reasonable period before the subject transfer.

Defendants provided expert valuation testimony but did not independently appraise the value of the DSC stock. Instead, the expert provided a critical review of the trustee's expert's appraisal. Defendant's expert's criticisms fell into three main areas. He testified that:

- [Trustee's expert] failed to fully consider the impact of the Connecticut economy, which was in a recession;
- The discount rate and weighted average cost of capital [he] used did not account for the "unsystematic risks" associated with valuing future cash flows of a small, privately owned, highly leveraged business; and
- [His] selection of a 20% discount for lack of marketability was too low—based upon a number of discount studies a 35% discount for lack of marketability was appropriate.[32]

Defendant's expert recommended changes in the calculations and concluded that each debtor's ownership interest in DSC was worth $126,000.

The court concluded that the trustee had not proven his claim of actual fraud because the only evidence that additional consideration for the stock would be paid at a later date was the testimony of Googel. (It is of some interest that Googel testified by videotape due to his incarceration in federal prison for wire fraud, bank fraud, and impeding administration of IRS laws.)

Regarding the trustee's constructive fraud claim, the defendants conceded that debtors were insolvent at the time of the transfer. Therefore, the trustee needed to prove that debtors did not receive reasonably equivalent value for their stock. Based on the expert valuation testimony, the court concluded that the debtors indeed did not receive reasonably equivalent value for the DSC stock and the trustee therefore could void the transfer as fraudulent.

The court generally credited the testimony of the trustee's expert but adjusted his valuation conclusion by applying the 35% lack of marketability discount suggested by defendant's expert.

COST OF CAPITAL INCLUDED IN DAMAGES

In a District of Columbia (D.C.) case, the plaintiff appealed the district court's decision to award the cost of its lost capital, calculated on the basis of the total amount of damages.

The D.C. Court of Appeals ruled:

> It remains unclear whether prejudgment interest [interest from the time of the tort to the date of court judgment] is available in a negligence action. Nonetheless . . . [i]n addition

> to finding Williams negligent, the district court found that Straight had breached its contract with Smoot. Further, because Williams agreed to indemnify Straight in full, the court did not err by including the cost of capital in the damage award assessed against Williams—whether or not District of Columbia law allows a cost of capital award in a negligence action.

The court of appeals did, however, require an adjustment in the district court's calculation of the time period for which portions of the cost of capital were awarded. The district court had awarded cost of capital on the full amount of the damages through the last day of the trial. The court of appeals noted, however, that portions of the damages, such as increased insurance premiums and legal fees, were incurred over an extended period of time, and ordered a recalculation based on time of actual incurrence of damages.[33]

COST OF CAPITAL IN UTILITY RATE-SETTING

Many providers of essential services are subject to federal or state regulation in respect to the rates they can charge for their services. In setting rates, it is virtually universally recognized that one of the costs the service provider is entitled to recover is its cost of capital. This generally is interpreted to mean its weighted average cost of capital, as discussed in Chapter 7.

This principle was articulated by the U.S. Supreme Court more than 50 years ago:

> The Supreme Court has stated that a just and reasonable rate should be "sufficient to assure confidence in the financial integrity of the enterprise, so as to maintain its credit and to attract capital"; the rate should also be "commensurate with returns on investments in other enterprises having corresponding risks."[34]

The next quote from a case appealing a Federal Communications Commission (FCC) rate order is typical:

> The FCC relied on "classic" DCF methodology, which assumes that the price of a share of stock is equal to the present value of the cash flows the stock will generate. J. Bonbright et al., *Principles of Public Utility Rates* 318 (2d ed. 1988).* These cash flows are in the form of dividends.** Because a dollar available now is worth more than a dollar available only later, the future cash flows must be reduced by a rate that reflects investors' opportunity costs, i.e., their required rate of return or discount rate, Id. Assuming that this discount rate and the growth rate of dividends both remain constant, one calculates the price of the stock using the following formula: $P = D/(r - g)$, where P is the current price of the stock, D is the total dividend in the first year, r is the rate of return, and g is the expected annual growth of dividends. Id.; see also A. Kolbe et al., *The Cost of Capital: Estimating the Rate of Return for Public Utilities* 53–54 (1984). Since regulatory commissions are interested in the rate of return, they rearrange the equation to solve for r: $r = D/P + g$.

* The DCF method "has become the most popular technique of estimating the cost of equity, and it is generally accepted by most commissions. Virtually all cost of capital witnesses use this method, and most of them consider it their primary technique." Id. at 317–18.

** Cash flows also result from the ultimate sale of the stock. R. Brealey and S. Myers, *Principles of Corporate Finance* 49 (4th ed. 1991). However, the theory is that the next investor will be willing to buy the stock at a price based on his estimate of future dividends and the price at which he will be able to sell; so too the third investor and the fourth, ad infinitum. The most basic form of the DCF model therefore assumes that the stock is held forever. Bonbright et al., supra at 318. For a detailed explanation, together with the mathematics, see 1.[35]

Similarly, a case challenging a Federal Energy Regulatory Commission (FERC) rate order supports the use of the Gordon Growth Model version of DCF analysis for estimating the cost of equity. Courts reviewing rate decisions generally will not require the use of one method rather than another to estimate the cost of equity capital but will accept methods in common use in the financial community. The next excerpt is typical of approval of regulatory methodology:

> In fact, the Commission appears quite wedded to DCF analysis and to efficient market theory as its theoretical mainstay. In Montaup Electric Co., 38 FERC ¶ 61,252 (1987), for example, the Commission adopted the DCF methodology over risk premium analysis for a period of rapidly declining interest rates and reasoned that "a market-oriented analysis such as a DCF analysis accounts for all risk factors perceived by investors." Id. at 61,866. At about the same time as this court's first remand, it published its third annual "generic Determination of Rate of Return on Common Equity for Public Utilities," in which it defended the use of the DCF methodology against attacks based on criticism of the Efficient Market Hypothesis. Order No. 461, III FERC Regulations Preambles ¶ 30,722 (1987). It declared enthusiastically, "The concept of an efficient market is astonishingly simple and remarkably well supported by the facts."[36]

TAXICAB LEASE RATES

The City of Chicago Commissioner of Consumer Services retained a consulting firm to recommend the rate of return on invested capital that Yellow Cab Company should be allowed to include in its maximum allowable lease rate for taxicabs. The commissioner adopted the consultant's recommended rate of 14%, and Yellow Cab appealed to the U.S. District Court.

According to the court, "Based on extensive research and the concept of a weighted average cost of capital, which resulted in a 12 percent rate of return assigned to the debt and a 20 percent rate of return given to the equity, [the consultant] determined that the maximum lease rates should afford a 14 percent rate of return."

Yellow Cab's actual cost of debt at the time was 7.25%, so the cost of capital was based on the risks and costs of the lease transaction, not on Yellow's debt cost. In granting summary judgment for the defendants, the court stated, "Yellow Cab did not show that the risk of the taxicab industry in Chicago entitled it to a rate of return exceeding 20 percent."[37]

SUMMARY

In matters such as valuation and allowed rates of return or interest rates, courts attempt to reflect the realities of financial decision making as practiced in the contemporary financial environment. This includes the attempt to embrace modern capital market theory in reaching determinations of the appropriate cost of capital in many contexts.

These contexts include but are not limited to:

- Shareholders disputes
- Gift and estate tax valuations
- Marital property valuations
- Bankruptcy reorganizations
- Damage awards
- Rate-setting
- Lease rates

Courts are moving away from arbitrary cost of capital decisions and relying heavily on expert witnesses who use current market data in conjunction with the cost of capital estimation methods discussed in this book.

Notes

1. *Weinberger v. UOP, Inc.*, 457 A.2d 701 (Del. 1983).
2. *Grimes v. Vitalink Comm. Corp.*, 1997 Del. Ch. LEXIS 124 (Del. Ch. 1997) (Oct. 1997 *BVU*).
3. *M.P.M. Enter., Inc. v. Gilbert*, 731 A.2d 790 (Del. 1999).
4. *M.G. Bancorp., Inc. v. LeBeau*, 737 A.2d 513 (Del. 1999).
5. *Las Vegas Dodge, Inc. v. United States*, 1985 U.S. Dist. LEXIS 21577 (D. Nev. 1985).
6. *Morton v. Commissioner*, T.C. Memo 1997-166, 73 T.C.M. (CCH) 2520 (U.S. Tax Ct. 1997) (June 1997 *BVU*).
7. *Estate of Freeman v. Commissioner*, T.C. Memo 1996-372, 72 T.C.M. (CCH) 373 (U.S. Tax Ct. 1996) (Oct. 1996 *BVU*).
8. *Estate of Hendrickson v. Commissioner*, T.C. Memo 1999-278, 78 T.C.M. (CCH) 322 (U.S. Tax Ct. 1999) (Oct. 1999 *J&L*) (Oct. 1999 *BVU*).
9. *Id.*
10. *Id.*
11. *Gross v. Commissioner*, T.C. Memo 1999-254, 78 T.C.M. (CCH) 201 (U.S. Tax Ct. 1999) (Sept. 1999 *J&L*) (Sept. 1999 *BVU*), *affirmed*, 272 F.3d 333 (6th Cir. 2001) (Jan. 2002 *BVU*).
12. *Estate of Maggos v. Commissioner*, T.C. Memo 2000-129, 79 T.C.M. (CCH) 1861 (U.S. Tax Ct. 2000) (July 2000 *BVU*).
13. *Estate of Klauss v. Commissioner*, T.C. Memo 2000-191, (U.S. Tax Ct. 2000) (July 2000 *BVU*) (July 2000 *J&L*).
14. *Id.*

15. Tax-affecting S corporation earnings is advocated by the *IRS Valuation Training for Appeals Officers Coursebook* (Chicago: CCH Incorporated, 1998), 7–12.
16. *Gross v. Commissioner*, T.C. Memo 1999-254, 78 T.C.M. (CCH) 201 (U.S. Tax Ct. 1999) (Sept. 1999 *J&L*) (Sept. 1999 *BVU*).
17. *Gross v. Commissioner*, 272 F.3d 333 (6th Cir. 2001). For an analysis of this discussion, see George Hawkins, "A Gross Result in the *Gross* Case Calls into Question Circumstances in Which Tax Affecting Is Valid," *Shannon Pratt's Business Valuation Update®* (Jan. 2002): 5–7 (Jan. 2002 *BVU*).
18. *Estate of Heck v. Commissioner*, T.C. Memo 2002-34, 83 T.C.M. (CCH) 1181 (U.S. Tax Ct. 2002) (March 2002 *BVU*).
19. *Estate of Adams v. Commissioner*, T.C. Memo 2002-80 (U.S. Tax Ct. 2002) (May 2002 *BVU*).
20. *Sergi v. Sergi*, 1996 Ohio App. LEXIS 3241 (Ohio Ct. App. 1996), *appeal denied*, 673 N.E.2d 147 (Ohio 1996) (Sept. 1996 *BVU*).
21. *Guiffre v. Baker*, 1996 Ohio App. LEXIS 3673 (Ohio Ct. App. 1996) (Nov. 1996 *BVU*).
22. *Koopmans v. Farm Credit Serv.*, 102 F.3d 874 (7th Cir. 1996).
23. *Rankin v. DeSarno*, 89 F.3d 1123 (3d Cir. 1996), *certiorari denied*, 519 U.S. 1108 (U.S. Sup. Ct. 1997).
24. *General Motors Acceptance Corp. v. Jones*, 999 F.2d 63 (3d Cir. 1993).
25. *Green Tree Fin. Serv. Corp. v. Smithwick*, 121 F.3d 211 (5th Cir. 1997), *rehearing denied*, 57 F.3d 1190 (D.C. Cir. 1991).
26. *In re Oglesby and Jones*, 221 B.R. 515 (1998).
27. *In re Hardzog*, 901 F.2d 858 (10th Cir. 1990).
28. *In re Glueck*, 223 B.R. 514 (1998).
29. "Bankruptcy Court Determines Fair Market Value Under Pennsylvania Judgment," *Judges & Lawyers Business Valuation Update*™ (May 2000): 7.
30. *In re Mahoney*, 251 B.R. 748 (2000) (May 2000 *J&L*) (June 2000 *BVU*).
31. *In re Colonial Realty Co.* 202 B.R. 185 (Bankr. D. Conn. 1996).
32. "Stock Recipients Liable for Fraudulent Transfer Underpayment," *BVU* (Jan. 1999): 10.
33. *Williams Enter., Inc. v. The Sherman R. Smoot Co.*, 938 F.2d 230 (D.C. Cir. 1991), *rehearing denied*, 57 F.3d 1190 (D.C. Cir. 1991).
34. *Federal Power Comm'n v. Hope Natural Gas Co.*, 320 U.S. 591 (U.S. Sup. Ct. 1944), quoted in *Illinois Bell Tel. Co. v. Federal Comm. Comm'n*, 988 F.2d 1254 (D.C. Cir. 1993).
35. *Id.*
36. *Tennessee Gas Pipeline Co. v. Federal Energy Regulatory Comm'n*, 926 F.2d 1206 (D.C. Cir. 1991).
37. *Yellow Cab Company v. City of Chicago*, 938 F.Supp. 500, U.S. Dist. N.D. Ill. (September 10, 1996).

*Months in parentheses after case citations refer to the issue in which the case was abstracted, either in *Shannon Pratt's Business Valuation Update®* (*BVU*), or in *Judges & Lawyer's Business Valuation Update*™ (*J&L*) (now merged with *BVU*), as noted above. The full texts of most cases abstracted in either *BVU* or *J&L* are available online at *www.BVLibrary.com*.

Chapter 20

Cost of Capital in Ad Valorem Taxation

Carl R.E. Hoemke

Introduction to Ad Valorem Taxation
Some Examples of Law That Promulgates the Definition of Income to Discount
General Categories of Legislative Constraints Where Adjustments to the Cost of Capital Are Necessary
Cost of Capital in a Constant, Perpetual Cash Flow Scenario
Different Types of Adjustments
  Multiplicative Income Adjustments
  Linear Income Adjustments: Constant Perpetual Income
  Linear Income Adjustments with a Growth Variant
  Random Income Variant
  Multiplicative Value Adjustments
Other Adjustments to the Cost of Capital
  Ex Post and *Ex Ante* Risk Premia
  Size Premium
  Industry Long-term Capitalization
Summary

Most people think of ad valorem as applicable only to real estate, but it applies in many business valuation contexts as well. For example, there is a great deal of work for business appraisers (in which I have participated) among railroads, where the company as a whole is valued and that value allocated among the states through which the railroad runs. The most commonly used valuation method is the discounted cash flow (DCF) method, and a point or two difference in the discount rate can make millions of dollars of difference in the resultant ad valorem taxes. Also, in some states, intangible assets are not subject to ad valorem taxation.

Where value is assessed starting with the company as a whole, companies sometimes save millions of dollars in taxes by subtracting the value contributed by intangible assets. Also, I have undertaken assignments to quantify economic obsolescence for an industrial plant, a task for which business appraisers may be more qualified than most real estate appraisers.

—Shannon Pratt

INTRODUCTION TO AD VALOREM TAXATION

Ad valorem taxation is a process in which government entities assess a tax or levy on the value of property. Of the three basic types of taxes—the tax on wealth, the tax on income, and the tax on transactions, or excise taxes—the tax on wealth or property is the oldest and provides the revenue foundation for local governments. This tax is assessed per value, thus the Latin term *ad valorem*. Property value is determined by the taxing jurisdiction, and a rate or levy is applied to the value for assessment. Proceeds are collected and used to fund services to the population of the taxing entity. These services may include police and fire protection, school funding, road funding, governmental administration, and others.

Many states have constitutional and statutory provisions that establish terms to define the value to be used by the assessor for tax purposes. The terms *full cash value*, *actual cash value*, *fair cash value*, *fair value in exchange*, *value in exchange*, and *true and fair value* are among those most commonly used. The courts have consistently interpreted these terms as indicating the same kind of value, which is value in exchange or value in the marketplace, normally termed *market value*. Therefore, we may consider that the basis of assessment is market value and that the assessor's task is to estimate the market value of property. For this reason, there is a great need to estimate accurately the value of property so that it can be fairly applied within its context.

The process of valuation has been described as an art form rather than a science. The appraiser's final conclusion must be made by judgment to determine value. The assessment of value therefore is limited by its tendency to be somewhat subjective; thus governmental authorities have attempted to make the process of assessment more objective through policymaking. Some of the policies that have been created to simplify and standardize the application of variables to value have served only to increase the complexity in the way cost of capital should be measured. This chapter identifies the adjustments to the cost of capital as applied in the income approach. The increased complexity derives from the fact that standardization forces the analyst to deviate from preferred measurements of income.

There are three approaches to value:

1. Income
2. Cost
3. Market

Properties that are not frequently sold in the markets but have income streams are the best candidates for the income approach to value. The income approach is a useful, although sensitive, appraisal tool. It is useful because, for most types of property, it is the most valid approach to value; it is sensitive, and therefore must be used with care, because any small original error will be compounded.

One of the parameters of concern in the income approach is the cost of capital. A basic assumption of the income approach is that people purchase property for the income it will yield. A different way of stating the assumption is that the value of the property depends on the income it produces. In valuing income-producing property, the valuation of an income stream is a function of the level of income being measured. The level of income corresponds to ownership rights to the cash flows.

The preferred measure of return is net cash flow.[1] There is certainly much discussion and many theories on this measure, as previous chapters identify. Using this return requires that the income to discount is the net cash flow income (see Chapter 3 for definition). Both creditors and shareholders expect to be compensated for the opportunity cost of investing their funds in one particular business instead of in others with equivalent risk. In ad valorem taxation, the level of income to discount depends on the limits placed on it by statutory requirements. Because of legislative constraints on the definition of income, adjustments must be made to the net cash flow return to render it comparable with the measure of income to discount.

SOME EXAMPLES OF LAW THAT PROMULGATES THE DEFINITION OF INCOME TO DISCOUNT

In many instances, limits or standards have promulgated the measure of income. For instance, if a governing body limits the definition of income to be discounted to the earnings before depreciation, interest, and taxes (*EBDIT*), then the analyst must recognize these barriers and make opposing adjustments in the cost of capital to compensate for the recognition of taxes and capital expenditures. Another example: If statutory requirements or practices require the subtraction of book depreciation from *EBDIT*, then changes in the rate are necessary to match that specific level of income compared with net cash flow measurements. Statutory requirements can thus dramatically alter the cost of capital and leave the analyst with many adjustments to the standard net cash flow cost of capital. It is essential that the analyst fully understand the cost of capital and its basis, then make any adjustments that make the cost of capital comparable with the income being measured.

GENERAL CATEGORIES OF LEGISLATIVE CONSTRAINTS WHERE ADJUSTMENTS TO THE COST OF CAPITAL ARE NECESSARY

A good rule to use in making these adjustments is that any change made to the return on net cash flow ultimately should result in a value that would have been arrived at by measuring the present value of the net cash flow under no constraints. These are a few situations in which adjustments to the cost of capital may be necessary:

- Cost of capital is used as a capitalization rate.
- Earnings are used as a proxy for free cash flow.
- Before-tax cash flow is used as the proxy for income.
- Adjustments are made for flotation costs.
- Adjustments are made for book value capital structure.

- Adjustments are made in the cost of capital for unaccounted capital expenditures in the cash flow.

COST OF CAPITAL IN A CONSTANT, PERPETUAL CASH FLOW SCENARIO

In its simplest form, the cost of capital is a function of a numerator income and a denominator value. It expresses the relationship of income to value.

The preferred level of income to discount is net cash flow. Net cash flow is the income available for the equity investor after all obligations to other forms of capital have been satisfied. The discount rate applicable to this level of income is the rate that is the subject of this book. If the expected net cash flows are constant and perpetual, then the discount rate (k) is simply the constant annual net cash flow (NCF) divided by the present value (PV) of all future net cash flows.

Formula 20.1

For a constant and perpetual income (NCF):

$$k = \frac{NCF}{PV}$$

Under these assumptions, any changes to the numerator will result in a direct and proportional change in k.

DIFFERENT TYPES OF ADJUSTMENTS

This section addresses how the changes in k influence the rate and how we may adjust accordingly. The discussion covers three areas:

1. Income adjustments
2. Changes in relation to a percentage of value
3. Handling a combination of income and value adjustments

Each of these areas is elaborated in regard to both multiplicative and linear adjustments. A *multiplicative adjustment* is the application of a factor or percentage to the numerator and/or denominator. Similarly, a *linear adjustment* is one in which a variable is added or subtracted to the numerator and/or denominator. Within the discussions of multiplicative and linear adjustments are analyses of the effects of changing variables.

Multiplicative Income Adjustments

The primary adjustment associated with a multiplicative income adjustment is the effect of income taxes on the discount rate. Before-tax income is multiplied by an effective tax rate to determine the after-tax income. The simplifying assumption here is that tax-deductible items and other noncash items do not materially affect the multiplicative relationship with before-tax income. For instance, "after-tax income is equal to 60% of before-tax income" is a multiplicative proposition. This is synonymous with "before-tax income is reduced by a 40% tax rate." The tax rate in this case is not necessarily the actual income tax rate on the taxable income. This rate takes into account the comparison of after-tax income with before-tax income, which is not necessarily taxable income.

Accounting for Income Tax within the Cost of Capital

If the numerator in Formula 20.1 is changed to before-tax net cash flows, then the numerator is increased by the annual amount of income tax. If the before-tax income is NCF_{pt} and after-tax income is NCF, then the tax rate (t) on before-tax income is 1 – (NCF/NCF_{pt}). Therefore:

Formula 20.2

$$k = \frac{NCF}{PV} \text{ or } \frac{NCF_{pt} \times (1-t)}{PV}$$
$$k = \frac{NCF_{pt}}{PV} \text{ or } \frac{NCF \div (1-t)}{PV}$$

Let us assume that annual net cash flow (NCF) is \$84, the effective income tax rate on before-tax income is 40%, and the net present value (NPV) is equal to \$1,000. If k were converted to a before-tax rate (k_{pt}), then the numerator would change from \$84 to \$140, or \$84 ÷ (1 – 40%). The cost of capital increases by the annual tax divided by the present value, or 56 ÷ 1000, or 5.6%.[2]

Formula 20.3

$$k = \frac{\$84}{\$1,000} = 8.40\%$$
$$k_{pt} = \frac{\$84 \div (1-40\%)}{\$1,000} = 14.00\%$$

Accounting for Income Tax within the Individual Components of the WACC

Alternatively, income tax adjustments can be measured directly in the weighted average cost of capital (WACC) under the assumption of constant and perpetual net cash flows.

The After-tax Measurement of WACC. The weighted cost of each security is expressed as:

Formula 20.4

$$WACC = (k_e \times W_e) + (k_p \times W_p) + (k_d \times W_d)$$

Each component is expressed in after-tax terms.

After-tax Cost of Equity (k_e). Unless otherwise stated, the cost of equity (k_e) generally is assumed to be the discount rate applicable to net cash flow available to common equity. Therefore, because this rate is in after-tax terms, no adjustment is necessary.

After-tax Cost of Preferred Stock (k_p). Preferred stock is a hybrid of debt and equity. Under certain provisions, preferred stock dividends are tax-deductible, and under other circumstances they are not. The analyst must determine the preferred stock's taxability. If it has tax-deductible components, then its cost should be treated like debt; otherwise, it requires no adjustment.

After-tax Cost of Debt (k_d). Comparatively, the cost of debt (k_d) should be expressed in after-tax terms. The rate of interest on debt is measured by the yield-to-maturity. Because interest on debt is tax-deductible, the net cost of debt to the company is reduced by the tax rate. Therefore, the yield rate must be adjusted to convert it to an after-tax debt cost. This adjustment is the before-tax rate of debt multiplied by 1 minus the marginal tax rate ($1 - t$). The marginal tax rate is the rate that equates to the incremental tax associated with the inclusion of the additional profit to the purchasing company. Typically, this rate is assumed to be the top corporate tax rate levied by the federal government. If the property's income is subject to state income tax, then that rate must also be included in the tax rate. Because state taxes typically are deductible from federal taxes, the state taxes must be tax-affected before adding them to the total marginal rate.

Formula 20.5

$$k_d = k_{d(pt)} \times (1 - t)$$

Therefore:

if:	$k_{d(pt)}$	= 8%
	t	= 40% (combined federal 35% and state 5%)
then:	k_d	= 4.8%

Therefore:

if:	k_e	= 12%
	k_d	= 4.80%

$$\begin{array}{rll} & W_e & = 0.50 \\ & W_d & = 0.50 \\ \text{then:} & WACC & = 8.40\% \end{array}$$

Although it is true that this is the after-tax cost of debt, one must be sure that the correct level of income is measured in utilizing this rate. A common error in discounting net cash flow to invested capital is that the interest expense is added back at its face amount and not measured net of the tax deduction resulting from interest as a tax-deductible expense. If before-tax interest is added back to the net cash flow to invested capital, then the before-tax cost of debt must be used to discount the income.

For instance, if earnings before interest and taxes (*EBIT*) equals the pretax income, and interest is tax-deductible, then *EBIT* less interest equals the taxable income (*EBT*). An income tax rate (t) multiplied by *EBT* is the income tax (T). Subtracting T from *EBT* equals net income. If the desired income to discount is the net operating income (*EBI*), then an amount of interest must be added to the net income. Algebraically, the equation is:

$$(EBIT - I)\,(1 - t) = NI$$

Alternatively:

Formula 20.6

$$EBIT\,(1 - t) - I\,(1 - t) = NI$$

If the desired income is earnings before interest but after-tax (*EBI*), then the equation can be rewritten as:

Formula 20.7

$$EBI = NI + I\,(1 - t)$$

Therefore, the tax-affected interest is correctly added back to net income to result in the desired income.

Special Case After-tax Cost of Capital (WACC). In many appraisals of companies subject to regulatory accounting, the level of income measured is the net operating income. In regulatory filing, the net operating income is measured by subtracting the income tax from the operating income. This is mathematically equivalent to adding back the full interest expense (before-tax impacted) to the net income. Thus, in cases where this occurs, the before-tax cost of debt is the correct measure; otherwise, the taxes must be adjusted to remove the effect of the interest's being tax-deductible.

In the special case after-tax cost of capital, the cost of each security is appropriately expressed as:

Formula 20.8

$$WACC = (k_e \times W_e) + (k_{d(pt)} \times W_d)$$

Therefore:

$$\begin{aligned} \text{if:}\quad & k_e && = 12\% \\ & k_{d(pt)} && = 8\% \\ & W_e && = 0.50 \\ & W_d && = 0.50 \\ \text{then:}\quad & WACC && = 10\% \end{aligned}$$

The Before-tax Measurement of k: Constant Level Income

Before-tax Cost of Capital ($WACC_{pt}$). The cost of each security is appropriately expressed as:

Formula 20.9

$$WACC_{pt} = (k_{e(pt)} \times W_e) + (k_{d(pt)} \times W_d)$$

Before-tax Cost of Debt ($k_{d(pt)}$). Because the cost of debt (k_d) is calculated in before-tax terms as stated earlier, no adjustment is necessary.

Before-tax Cost of Equity ($k_{e(pt)}$). The cost of equity (k_e) must be adjusted because it is measured in after-tax terms. With the assumption that the cash flows are constant and perpetual, the adjustment is simply the cost of equity k_e divided by 1 minus the tax rate (note that by using these assumptions the capitalization rate is equal to the cost of capital):

Tautology:

Formula 20.10

$$c_{e(pt)} = \frac{c_e}{(1-t)}$$

Under the given assumptions that cash flows are constant and perpetual, the capitalization rate is equal to the discount rate. The before-tax cost of equity can be expressed as:

Formula 20.11

$$k_{e(pt)} = \frac{k_e}{(1-t)}$$

Therefore:

if:	k_e	$= 12\%$
	t	$= 40\%$
then:	$k_{e(pt)}$	$= 20\%$

Therefore:

if:	$k_{e(pt)}$	$= 20\%$
	$k_{d(pt)}$	$= 8\%$
	W_e	$= 0.50$
	W_d	$= 0.50$
then:	$WACC_{pt}$	$= 14\%$

The Impact of Taxes on the Equity Cost of Capital: The Growth Variant. As identified earlier, the calculations presented here are for determining a before-tax cost of capital under constant cash flow assumptions. For calculation of the before-tax cost of capital where a constant growth rate is present, the rate can be determined by expanding on the preceding tautology. The before-tax equity capitalization rate ($c_{e(pt)}$) is equal to the after-tax equity capitalization rate (c_e) divided by 1 minus the effective tax rate on the income to be capitalized:

Tautology:

Formula 20.12

$$c_{e(pt)} = \frac{c_e}{(1-t)}$$

Therefore, since:

$c_{e(pt)} = k_{e(pt)} - g_{e(pt)}$
$c = k_e - g_e$

where:

g_e = Growth rate in after-tax equity income
$g_{e(pt)}$ = Growth rate in before-tax equity income

The tautology can be rewritten to account for growth:

Formula 20.13

$$k_{e(pt)} - g_{e(pt)} = \frac{k_e - g_e}{(1-t)}$$

Then rearranging and solving for $k_{e(pt)}$:

Formula 20.14

$$k_{e(pt)} = \frac{k_e - g_e}{(1-t)} + g_{e(pt)}$$

This calculation requires an analyst to calculate the growth rate on after-tax income and before-tax income. Differences in the two growth parameters can arise from several circumstances. One circumstance in particular is the effect of accelerated tax depreciation. Normally, the effective tax rate on before-tax income is smaller in the initial years and grows with each year until it reaches a constant level. This difference can serve to reduce growth on after-tax cash flows, because it allows for a higher after-tax income in the near-term years and a lower income in the later years, offsetting some of the normal inflationary changes in the before-tax income. Otherwise, growth patterns in the two levels of income typically are not widely divergent. When comparing the changes in growth with a simple nongrowth model, two observations can be made:

1. Growth tends to reduce the impact of taxes on the before-tax cost of capital.
2. Conversely, decline increases the cost of capital.

For example, assume the growth rates of pretax and after-tax cash flows are equal:

If $k = 15\%$, $t = 40\%$, and
$g = 10\%$, then $k_{e(pt)} = 18.33\%$
$g = 0\%$, then $k_{e(pt)} = 25.00\%$
$g = -10\%$, then $k_{e(pt)} = 31.67\%$

Linear Income Adjustments: Constant Perpetual Income

The primary adjustment associated with a linear income adjustment originates with the differences between nonoperating cash outflows and noncash items—specifically, capital expenditures, depreciation, and accrual versus cash accounting. This adjustment is necessary in capitalizing a level of income that includes any items that should or should not be included in net cash flow. Some assessor practices traditionally have excluded adjustments for depreciation, other noncash items, and capital expenditures. In many cases the income identified as the income to capitalize or discount is the earnings before interest (*EBI*), otherwise known as net operating income (*NOI*). Consider Formula 20.1 again:

$$k = \frac{NCF}{PV}$$

Again, the same assumptions must hold true (constant and perpetual income). Thus, if a constant *e* (any adjustment to *NCF* required by the assessor) is added to the numerator to represent a linear adjustment, then it too must be added to k to maintain the integrity of the calculation. Where k_q is the adjusted discount rate:

Formula 20.15

$$k_q = k + \frac{e}{PV} = \frac{NCF + e}{PV}$$

Notice how the addition of a constant e affects the left side of the equation, compared with Formula 20.1. The constant's relation to present value (PV) linearly changes the rate. In most cases, constant e represents the difference between book depreciation and capital expenditures. In other words, if annual depreciation is equal to 4% of the value and capital expenditures are equal to 5% of the value, then the discount rate increases by 1%. This is consistent with real experiences of deficient book depreciation compared with capital expenditures. Inflation, deflation, or incorrect estimates of the service life of an asset can cause these differences.

In other cases assessors require adding back depreciation to *NOI* or *EBI*. To further complicate the equation, capital expenditures are not subtracted from the cash flow, resulting in the necessity of a large e adjustment. The e in this scenario is comparable to what is called a recapture rate. Some analysts add back the annual straight-line depreciation rate. For example, if a project has a 20-year life with no salvage value, it depreciates 5% per year. Ideally, the rate in this instance should be equal to the ratio of capital expenditures to value.

Linear Income Adjustments with a Growth Variant

Inflation or deflation can account for differences in capital expenditures and depreciation. The preceding formula accounts for a constant and perpetual difference. This constant difference is also paired with a constant net cash flow. A variant to this equation is the influence of growth. Refer again to Formula 20.1:

$$k = \frac{NCF}{PV}$$

As identified, k represents a rate that is true for a constant and perpetual income stream. Alternatively, if the rate of income is growing at a constant rate into perpetuity, then this equation can be altered to account for the constant growth component:

Formula 20.16

$$c_q = k - g = \frac{NCF}{PV}$$

and, $\quad k_q = c_q + g$

thus, $\quad k_q = k$

where:

c_q = Capitalization rate
k = Unadjusted discount rate
k_q = Adjusted discount rate
g = Constant growth in NCF

Adding a linear parameter to the numerator is allowed only if that parameter is growing at the same rate as the income. The formula is restated to account for this constant where k_q is the adjusted discount rate:

Formula 20.17

$$c_q = k - g + \frac{e}{PV} = \frac{NCF + e}{PV}$$

and $$k_q = c_q + g$$

thus, $$k_q = k + \frac{e}{PV}$$

where:

e = Linear parameter
g = Growth in both NCF and e

Otherwise, if the growth rate of NCF and e differ, then the equation becomes much more complex. Essentially, the problem is to solve for the discount rate necessary to discount the two separate income streams (NCF and e) sufficiently to equal the present value of NCF discounted with the return on net cash flow rate. To represent this formula, let the adjusted discount rate be equal to k_q. The equation can be demonstrated as:

Formula 20.18

$$\frac{NCF}{k_q - g} + \frac{e}{k_q - g_e} = \frac{NCF}{k - g}$$

where:

k_q = Adjusted discount rate for the inclusion of e
g_e = Growth in e
g = Growth rate in NCF

Solving for k_q mathematically is difficult because the equation is in a polynomial form. The easiest way to solve for k_q is to use a solver function in a spreadsheet.

Random Income Variant

Because both net cash flow and adjusted income are subject to the same parameters, both are subject to a variety of variables, which sometimes makes it difficult to fit the streams into rigid constant formulas. As noted in earlier sections, the formula constrains the analyst with measurements available only in a sterile environment. It requires constant levels of growth and perpetuity incomes. For situations that fall outside this environment, no quick-and-dirty estimate can be performed. The only way to determine the effect of the adjustment on the discount rate when the subject falls outside the constraints is to use internal rate of return (*IRR*) calculations.

The *internal rate of return* is the discount rate necessary to convert future income into the investment. The tools available to calculate the cost of capital on an after-tax basis have been widely established; therefore, it is easier to simply discount the preferred level of income (net cash flow). Once the present value of the net cash flow is calculated, the result can be used for determination of the discount rate. This discount rate would then be the appropriate rate to use in the alternative income streams. By using methods of determining the internal rate of return, one can specifically measure the effect of a desired variable on the cost of capital.

Therefore, let us observe the given income streams on the investment proposed earlier:

Year	Net Cash Flow	Before-tax Income
0	($1,000)	($1,000)
1	77	133
2	81	137
3	85	141
4	89	145
5	1,081	1,137

Observations:

1. Year 0 is the investment or the present value of cash flows.
2. Year 5 is a reversion value plus Year 5's income.
3. The difference between each net cash flow and before-tax income is $56. This difference is the annual tax associated with each respective cash flow.
4. The net present value in the net cash flow is determined by using an 8.23% discount rate.
5. The internal rate of return for the before-tax income, using $1,000 as the investment, is 13.82%.
6. The incremental effect to the cost of capital for adding back the income tax is 5.58%.

This process will return a rate equivalent to the before-tax discount rate. This is the desired method of calculating the true effect of taxes on the discount rate. Several things are occurring here that lead to a result on a before-tax basis. Generally, the reason for calculating the IRR is that inconsistent growth rates between net cash flow and before-tax income are difficult to model in an easy-to-understand formula. Unfortunately, the downside to this process is that it is more complex and a little more difficult to explain.

Multiplicative Value Adjustments

Ad Valorem Tax Addback

The most common multiplicative value adjustment in ad valorem assessment is the addback of ad valorem taxes. Many assessors want to remove the historical bias resulting from prior valuations. Therefore, they may prefer to account for property tax within the discount rate. They do so by adding back to the discount rate the percent relationship of tax to market value. This adjustment is most similar to the linear adjustment in income. The difference is that the adjustment is a direct function of value. In other words, if the value increases, the adjustment increases directly with the value, and vice versa. This can be demonstrated by the next formula:

Formula 20.19

$$k_q = k + (o \times PV) \div PV = \frac{1 \div (o \times PV)}{PV}$$

thus, $$k_q = k + o$$

where:

o = Percent of tax to value

And with the addition of a growth component (g), the formula expands to:

Formula 20.20

$$k_q - g = k - g + o$$

thus, $$k_q = k + o$$

The same formula can be used for any adjustment that is equal to a percentage of value. This holds true even in random changes in value. The only caveat is that the percent relationship to value must remain constant. This adjustment is quite powerful and easy to demonstrate, which is likely the reason for its popularity.

Flotation Costs

Another type of multiplicative value adjustment is flotation costs. Flotation costs occur when new issues of stock or debt are sold to the public. The firm usually incurs several kinds of flotation or transaction costs, which reduce the actual proceeds received by the firm. Some of these are direct out-of-pocket outlays, such as fees paid to underwriters, legal expenses, and prospectus preparation costs. Because of this reduction in proceeds, the firm's required returns on these proceeds equate to a higher return to compensate for the additional costs. Flotation costs can be accounted for either by amortizing the cost, thus reducing the cash flow to discount, or by incorporating the cost into the cost of capital. Because flotation costs are not typically applied to operating cash flow, one must incorporate them into the cost of capital.

The cost of flotation is a function of size and risk. The larger the issuance, the lower the cost as a percentage of the issuance price. Flotation costs are the greatest for equity issuance and the least for debt issuance. Preferred stock flotation costs tend to be somewhere in between. The next table shows examples of the relation of flotation cost to size of an issuance of stock that occurred during 1996 and 1997.

Company	Total Issuance	Total Flotation
Excite	39,100,000	9.46%
Team Rental	52,000,000	6.76%
Amazon	54,000,000	8.57%
IXC	89,600,000	8.67%
General Cigar	108,000,000	8.28%
Ciena	115,000,000	7.96%
Capstar	166,500,000	7.68%
General Cable	354,900,000	5.94%
Sabre	545,400,000	5.77%
Hartford Life	649,750,000	6.50%

OTHER ADJUSTMENTS TO THE COST OF CAPITAL

In the property tax arena, traditional techniques are king. Any new approaches are met with skepticism, because the results of many new techniques tend to lower the market value of the project and, thus, the taxes. This is true despite the validity of such approaches. The next paragraphs identify four "newer" techniques introduced in the ad valorem arena in the 1990s.

Ex Post and *Ex Ante* Risk Premia

The expected equity risk premium is unobservable in the market and must be estimated. For both the Capital Asset Pricing Model (CAPM) and the build-up method, *ex post* and *ex ante* risk premia are used to obtain estimates for the cost of equity.

An *ex post* risk premium is based on the assumption that historical returns are the best predictor of future returns. It is calculated by subtracting the long-term arithmetic average of the income return on long-term government bonds for the CAPM or long-term corporate bonds for the measurement of the build-up model. Each is measured from the long-term arithmetic average stock market return measured over the same period. The duration of the bond must be the same as that used to estimate the equity risk premium (RP_m).

The *ex ante* risk premium is forward looking. The Gordon Growth Model is applied to determine the resulting risk premium. The premium is determined by first estimating the cost of equity for the proxy market. The proxy market is a market large enough to remove the effects of nondiversification. Typically, the Standard & Poor's (S&P) 500 Index or the NYSE Composite Index is used as this proxy. The data necessary for this analysis are more abundant in the S&P 500 because analysts follow these stocks more than any other large grouping of an index, and the size of the index is sufficient for this measurement.

The first parameter to estimate is the expected growth on the dividends of the market. Dividend growth typically is not measured, because it is a function of management decision on capitalization of the firm. The more retention of capital, the greater the growth in dividends; whereas the lower the retention rate, the lower the growth in dividends. If the retention rate remains the same in relation to the net income, then the growth in earnings per share is the best proxy for the growth in dividends.

The first step in deriving the *ex ante* risk premium is to use a single-stage discounted cash flow analysis (otherwise known as the Gordon Growth Model) to calculate the cost of equity for the market proxy (i.e., the S&P 500). The cost of equity is calculated by using the most recent consensus long-term growth rates for each firm in the S&P 500 and adding it to the dividend growth yield.

The dividend yield for the S&P 500 should be an estimate for Year 1's dividend (D_1). D_1 can be estimated by multiplying the S&P 500's current weighted average dividend yield (D_0) by 1 plus its weighted average long-term earnings growth rate. By adding the weighted average long-term growth rate to the dividend yield at the end of Year 1, the cost of equity is estimated. If, for example, the long-term growth rate is equal to 10% and the current dividend yield is 4%, then the cost of equity is (4% × 1.1) + 10%, or 14.40%. This can also be described by the formula:

Formula 20.21

$$k_{e500} = DY \times (1 + g) + g$$

where:

DY = Dividend yield
g = Long-term growth rate
k_{e500} = Cost of equity for the S&P 500

The second step is to calculate the risk premium of the S&P 500 (RP_{500}). For the CAPM, the *ex ante* risk premium is calculated by subtracting the risk-free rate (R_f) from the cost of equity for the S&P 500. For the build-up method, the *ex ante* risk premium is calculated by subtracting the weighted average bond yield for the S&P 500 from the cost of equity for the S&P 500.

$$RP_{500} = k_{e500} - R_f$$

Size Premium

Many analysts recommend including a size premium in the cost of capital. They support the argument for including this premium with the fact that small companies have historically earned returns greater than those explained by the beta-times-risk-premium alone. In other words, although betas for small companies tend to be greater than betas for large companies, they still do not account for *all* of the risks faced by investors in small companies. This premium is added directly to the results obtained using the CAPM. The size premium was discussed earlier in this book and is applicable in the property tax arena.

Industry Long-term Capitalization

Traditionally, property tax assessors have used a book value capital structure as the appropriate measure of the employment of capital. This was the result of regulatory influence in economic analysis. Regulators define the allowed earnings as a return on the original investment. A utility company would receive a return on its investment and a return of its investment. Depreciation serves as a return *of* the investment, and net book value (otherwise termed *rate base*) is identified as the basis for the return *on* the investment. Therefore, the utility company would be limited to a return on the net book value, not on the fair market value, of the assets. With regulation of telecommunication and electric utility companies coming to an end (as seen in state and federal deregulation advances), investors are looking at the returns on the market value of assets out of regulation (i.e., fair market value). Therefore, the cost of capital would be subject to a market-weighted capital structure.

SUMMARY

The use of the cost of capital in ad valorem taxation must be done with care. Because of various legislative caveats and exceptions, analysts must examine the impacts of different methods and determine whether they affect the cost of capital. With the tools given in this chapter, this approach can ease the burden and the complexity of the adjustments to the cost of capital.

Notes

1. Net cash flow is defined differently in different jurisdictions for ad valorem purposes and may not always correspond to the definition of net cash flow used consistently throughout this book.
2. Some states do not subtract income taxes in defining net cash flow.

Chapter 21

Capital Budgeting and Feasibility Studies

Invest for Returns above Cost of Capital
DCF Is Best Corporate Decision Model
Focus on Net Cash Flow
Adjusted Present Value Analysis
Use Target Cost of Capital over Life of Project
Summary

Good cost of capital estimation is essential to sound capital budgeting and feasibility analysis decisions.

INVEST FOR RETURNS ABOVE COST OF CAPITAL

When addressing capital budgeting and feasibility analysis decisions, popular phrases in contemporary corporate finance literature are *shareholder value added* (SVA) and *economic value added* (EVA). The essence of the way to add value is to invest funds in a project that will earn a higher rate of return than its cost of capital (see Chapter 22).

As Brealey and Myers say in their capital budgeting chapter of their classic text on *Principles of Corporate Finance*, "Accept any project that more than compensates for the *project's beta*."[1]

In the case of selection among multiple potential projects competing for limited funds, analysts recommend investing in those with the highest *net present value* (NPV). Net present value is estimated by discounting the expected cash outflows and expected cash inflows from the project by the project's cost of capital.

Note two important points in the last sentence:

1. *Cash flow* is the preferred measure of economic income.
2. *Project's cost of capital* is the preferred focus, as opposed to the company's cost of capital.

"[T]he company cost of capital rule can also get a firm into trouble if the new projects are more or less risky than its existing business. Each project should be evaluated on its *own* opportunity cost of capital."[2]

DCF IS BEST CORPORATE DECISION MODEL

At a seminar on Frontiers in Corporate Valuation, Tom Copeland, coauthor of *Valuation: Measuring and Managing the Value of Companies*, compared the use of ratios, formulas, and discounted cash flow (DCF) analysis for purposes of corporate decision making. In evaluating the three approaches, he noted, "The most important criterion for comparing approaches is that they result in good decisions, because the model value is close to the equilibrium market value."[3]

Copeland asks these questions:

- How well do the model values match market values?
- Is the model logical?
- Is the approach easy to understand and use?
- Does the approach easily lend itself to a wide variety of decision-making applications?

Copeland's list of pros and cons for each of the three approaches to corporate decision making is shown in Exhibit 21.1. Copeland concludes unequivocally that the DCF approach is superior.

FOCUS ON NET CASH FLOW

Copeland notes that the DCF approach captures all elements of value. He also states:

> Managers who are interested in maximizing share value should use discounted cash flow analysis to make decisions, not earnings per share. . . . The market is not fooled by cosmetic earnings increases; only earnings increases that are associated with improved long-term cash flows will increase share prices. The evidence that the market focuses on cash flows can be grouped into four areas, studies showing that:
>
> - Accounting earnings is not very well correlated with share prices
> - Earnings "window dressing" does not improve share prices
> - The market evaluates management decisions based on their expected long-term cash flow impact, not the short-term earnings impact
> - There are many decisions where cash flows and earnings per share give opposing results.[4]

ADJUSTED PRESENT VALUE ANALYSIS

Generally speaking, most contemporary corporate finance literature and seminars advocate discounting expected cash inflows and outflows at a weighted average cost of capital (WACC). The characteristics of a project, either risk or special financing

Exhibit 21.1 Pros and Cons of Approaches to Corporate Decision Making

Ratios are the oldest form of valuation methodology because they are easy to use. They provide a direct, simple link between easy-to-observe variables like earnings and market prices.

Pros and Cons of Ratios

Pros	Cons
• Easy to use • Based on comparables	• Difficult to find exact comparables • Heavily dependent on accounting standards • No logic that leads back to a fundamental understanding (e.g., should earnings in a P/E ratio be normalized) • P/E ratio does not focus on balance sheet, and market/book ratio does not focus on income statement • Generally low correlation with actual market values • Not particularly useful for day-to-day operating decisions

Formulas are also fairly simple to use, but are crude tools because their simplicity requires that they make strong (often unrealistic) assumptions.

Pros and Cons of Formulas

Pros	Cons
• Easy to use • Logic does tie back to fundamentals (e.g., cash flows to the owner)	• Make strong implicit assumptions (e.g., constant growth forever) • Depend strongly on a point estimate of cash flows or earnings • Require modest amounts of training regarding the underlying math • Not useful for day-to-day operating decisions

Discounted cash flows is best for decision-making but is more complex than the alternatives.

Pros and Cons of DCF

Pros	Cons
• Clear logical link to the underlying fundamentals • Matches actual market values quite well • Lends itself to a wide variety of decision-making applications • Not dependent on changes in accounting principles, depends only on actual cash flow	• Complex, requires training

Source: Tom Copeland, Seminar on Frontiers in Corporate Valuation, New York University Leonard N. Stern School of Business, November 6–7, 1997. Reprinted with permission.

opportunities unique to the project, may cause the WACC for the project to differ from the company's overall WACC.

However, a variation on DCF analysis, dubbed *adjusted present value* (APV), advocates taking DCF analysis for project selection in a different direction. Instead of an overall project WACC, the APV approach estimates a *base-case value* by unbundling the components of value and analyzing each separately. APV starts with a base-case value, discounting all cash flows from the project as if they were financed by equity. It then adds or subtracts increments or decrements of value from all financing side effects. The list includes:

- Interest tax shields
- Cost of financial distress
- Subsidies
- Hedges
- Issue costs
- Other costs[5]

Timothy A. Leuhrman, author of "Using APV: A Better Tool for Valuing Operations," claims "the particular version of DCF that has been accepted as the standard over the past 20 years—using the weighted average cost of capital (WACC) as the discount rate—is now obsolete. . . . *Adjusted present value* (APV) is especially versatile and reliable, and will replace WACC as the DCF methodology of choice among generalists."[6]

USE TARGET COST OF CAPITAL OVER LIFE OF PROJECT

In any case, while the consensus advocates focusing on the cost of capital of the project rather than on the overall company cost of capital to the extent that they differ, the focus should encompass the life of the project rather than any temporary effects. For example, if the project requires an abnormal level of debt financing that would temporarily change the company's capital structure, the WACC should reflect the company's *target* capital structure rather than the abnormal structure when the investment initially is made.

SUMMARY

This is a short chapter, because the essential principles of using cost of capital for capital budgeting and project selection are essentially the same as for other applications already discussed.

The general consensus is:

- Discounted cash flow is the best model for corporate finance decisions.
- Focus on net cash flow as the economic income variable of choice.

- Each project should be analyzed in light of its own cost of capital characteristics, rather than automatically using the company's overall cost of capital.
- The cost of capital used should be the target cost of capital over the life of the project.
- New variations of cost of capital applications are constantly being developed.

Notes

1. Richard A. Brealey and Stewart C. Myers, *Principles of Corporate Finance*, 6th ed. (Boston: Irwin McGraw-Hill, 2000), 222.
2. Ibid, 221.
3. Tom Copeland, Seminar on Frontiers in Corporate Valuation, New York University, Leonard N. Stern School of Business, November 6–7, 1997.
4. Ibid.
5. Timothy A. Luehrman, "Using APV: A Better Tool for Valuing Operations," *Harvard Business Review* (May–June 1997): 145.
6. Ibid., 145–153.

Chapter 22

Central Role of Cost of Capital in Economic Value Added[1]

Joel M. Stern, G. Bennett Stewart, III, and Donald H. Chew, Jr.

EVA Financial Management System
EVA and the Corporate Reward System
EVA Bonus Plan: Simulating Ownership
Leveraged Stock Options: Making Ownership Real
Summary

An economic value added (EVA)–based performance measurement system makes the cost of capital explicit. In its simplest form, EVA is net operating profit after taxes less a charge for the capital employed to produce those profits. The capital charge is the required, or minimum, rate of return necessary to compensate all the firm's investors, debt holders, and shareholders for the risk of the investment.

EVA is charged for capital at a rate that compensates investors for bearing the firm's explicit business risk. The assessment of business risk is based on the Capital Asset Pricing Model (CAPM), which allows for a specific, market-based evaluation of risk for a company and its individual business units using the concept of "beta." In addition, the tax benefit of debt financing is factored into the cost of capital, but in such a way as to avoid the distortions that arise from mixing operating and financing decisions. To compute EVA, the operating profit for the company and for each of the units is charged for capital at a rate that blends the after-tax cost of debt and equity in the *target* proportions each would plan to employ rather than the actual mix each actually uses year-by-year. Moreover, operating leases are capitalized and considered a form of debt capital for this purpose. As a result, new investment opportunities are neither penalized nor subsidized by the specific forms of financing employed.

To illustrate, a company with a 10% cost of capital that earns a 20% return on $100 million of net operating assets has an EVA of $10 million. This says the company is earning $10 million more in profit than is required to cover all costs, including the opportunity cost of tying up scarce capital on the balance sheet. In this sense, EVA combines operating efficiency and balance sheet management into one measure that can be understood by operating people.

For operating heads and top management alike, EVA holds out three principal ways of increasing shareholder value:

1. Increase the return derived from the assets already tied up in the business. Run the income statement more efficiently without investing any more capital on the balance sheet.
2. Invest additional capital and aggressively build the business so long as the return earned exceeds the cost of that new capital. (Targets based on rates of return such as return on earnings (ROE) or return on investment (ROI), incidentally, can actually discourage this objective when divisions are earning well above their cost of capital, because taking on some EVA-increasing projects will lower their average return.)
3. Stop investing in, and find ways to release capital from, activities that earn substandard returns. This means everything from turning working capital faster and speeding up cycle times to consolidating operations and selling assets worth more to others.

Besides making the cost of capital explicit, the EVA performance measure also can be designed to encourage tax-minimizing accounting choices and to incorporate a number of other adjustments intended to eliminate distortions of economic performance introduced by conventional accounting measures like earnings or ROE. For example, one notable shortcoming of GAAP (generally accepted accounting principles) accounting stems from its insistence that many corporate outlays with longer-term payoffs (like research and development [R&D] or training) be fully expensed rather than capitalized and amortized over an appropriate period. While well suited to creditors' concerns about liquidation values, such accounting conservatism can make financial statements unreliable as guides to going-concern values. More important, to the extent GAAP's conservatism is built into a company's performance measurement and compensation system, it can unduly shorten managers' planning horizon.

In setting up EVA systems, we sometimes advise companies to capitalize portions of their R&D, marketing, training, and even restructuring costs. In cases of other "strategic" investments with deferred payoffs, we have also developed a procedure for keeping such capital "off the books" (for internal evaluation purposes) and then gradually readmitting it into the manager's internal capital account to reflect the expected payoffs over time. As these examples are meant to suggest, EVA can be used to encourage a more farsighted corporate investment policy than traditional financial measures based upon GAAP accounting principles.

In defining and refining its EVA measure, Stern Stewart & Co. has identified over 120 shortcomings in conventional GAAP accounting. In addition to GAAP's inability to handle R&D and other corporate investments, we have addressed performance measurement problems associated with standard accounting treatments of:

- Inventory costing and valuation
- Depreciation
- Revenue recognition
- The writing-off of bad debts
- Mandated investments in safety and environmental compliance
- Pension and postretirement medical expense
- Valuation of contingent liabilities and hedges
- Transfer pricing and overhead allocations
- Captive finance and insurance companies
- Joint ventures and start-ups
- Special issues of taxation, inflation, and currency translation

For most of these accounting issues, we have crafted a series of cases to illustrate the performance measurement problem and devised a variety of practical methods to modify reported accounting results in order to improve the accuracy with which EVA measures real economic income.

Of course, no one company is likely to trigger all 120 measurement issues. In most cases, we find it necessary to address only some 15 to 25 key issues in detail—and as few as five or 10 key adjustments actually are made in practice. We recommend that adjustments to the definition of EVA be made only in those cases that pass four tests:

1. Is it likely to have a material impact on EVA?
2. Can the managers influence the outcome?
3. Can the operating people readily grasp it?
4. Is the required information relatively easy to track or derive?

For any one company, then, the definition of EVA that is implemented is highly customized with the aim of striking a practical balance between simplicity and precision.

To make the measure more user-friendly, we have also developed a management tool called "EVA Drivers" that enables management to trace EVA through the income statement and balance sheet to key operating and strategic levers available to them in managing their business. This framework has proven to be quite useful in focusing management's attention, diagnosing performance problems, benchmarking with peers, and enhancing planning. More generally, it has helped people up and down the line to appreciate the role they have to play in improving value. It also can help guard against an excessive preoccupation with improving individual operational metrics to the detriment of overall performance. For example, a drive to increase productivity—or, say, a single-minded obsession with winning the Malcolm Baldrige Award—could lead to unwarranted capital spending or to shifts in product mix that result in less EVA and value, not more. In the end, management must be held accountable for delivering value, not improving metrics.[2]

EVA FINANCIAL MANAGEMENT SYSTEM

The real success of business today depends not on having a well-thought-out, far-reaching strategy but rather on reengineering a company's business systems to respond more effectively to the new business environment of continuous change. Our contention at Stern Stewart is that just as this information revolution has created a need for business process reengineering, it also has precipitated a need to reengineer the corporate *financial management system*.

What do we mean by a financial management system? A financial management system consists of all those financial policies, procedures, methods, and measures that guide a company's operations and its strategy. It has to do with how companies address such questions as:

- What are our overall corporate financial goals and how do we communicate them, both within the company and to the investment community?
- How do we evaluate business plans when they come up for review?
- How do we allocate resources—everything from the purchase of an individual piece of equipment, to the acquisition of an entire company, to opportunities for downsizing and restructuring?
- How do we evaluate ongoing operating performance?
- Last but not least, how do we pay our people, what is our corporate reward system?

Many companies these days have ended up with a needlessly complicated and, in many respects, hopelessly obsolete financial management system. For example, most companies use discounted cash flow analysis for capital budgeting evaluations. But when it comes to other purposes, such as setting goals and communicating with investors, the same companies tend to reach for accounting proxies—measures like earnings, earnings per share (EPS), EPS growth, profit margins, ROE, and the like. To the extent this is true, it means there is already a "disconnect" between the cash flow–based capital budget and accounting-based corporate goals. To make matters worse, the bonuses for operating people tend to be structured around achieving some annually negotiated profit figure.

This widespread corporate practice of using different financial measures for different corporate functions creates inconsistency, and thus considerable confusion, in the management process. And, given all the different, often conflicting, measures of performance, it is understandable that corporate operating people tend to throw up their hands and say, "So, what are you really trying to get me to do here? What is the real financial mission of our company?"

With EVA, all principal facets of the financial management process are tied to just one measure, making the overall system far easier to administer and understand.

That is, although the process of coming up with the right definition of EVA for any given firm is often complicated and time-consuming, the measure itself, once established, becomes the focal point of a simpler, more integrated overall financial management system—one that can serve to unite all the varied interests and functions within a corporation.

Why is it so important to have only one measure? As we noted earlier, the natural inclination of operating managers in large public companies is to get their hands on more capital to spend and grow the empire. This tendency in turn leads to an overtly political internal competition for capital—one in which different performance measures are used to gain approval for pet projects. And because of this tendency toward empire-building, top management typically feels compelled to intervene excessively—not in day-to-day decision making but in capital spending decisions. Why? Because they do not trust the financial management system to guide their operating managers to make the right decisions. There is no real accountability built into the system, and there is no real incentive for operating heads to choose only those investment projects that will increase value.

EVA is the internal measure that management can decentralize throughout the company and use as the basis for a completely integrated financial management system. It allows all key management decisions to be clearly modeled, monitored, communicated, and rewarded according to how much value they add to shareholders' investment. Whether reviewing a capital budgeting project, valuing an acquisition, considering strategic plan alternatives, assessing performance, or determining bonuses, the goal of increasing EVA over time offers a clear financial mission for management and a means of improving accountability and incentives. In this sense, it offers a new model of internal corporate governance.

EVA AND THE CORPORATE REWARD SYSTEM

Incentive compensation is the anchor of the EVA financial management system. The term "incentive compensation" is not quite right, however, for in practice too much emphasis gets placed on the word "compensation" and not enough on the word "incentive." The proper objective is to make managers behave as if they were owners. Owners manage with a sense of urgency in the short term but pursue a vision for the long term. They welcome change rather than resisting it. Above all else, they personally identify with the successes and the failures of the enterprise.

Extending an ownership interest is also the best way to motivate managers in the information age. As the pace of change increases and the world becomes ever less predictable, line managers need more general as opposed to specific measures of performance to which they will be held accountable. They need more leeway to respond to changes in the environment. They need a broader and longer-range mandate to motivate and guide them. Maximizing shareholder value is the one goal that remains constant, even as the specific means to achieve it are subject to dramatic and unpredictable shifts.

Making managers into owners should not be undertaken as an "add-on" to current incentive compensation methods. Rather, it should replace them. In place of the traditional short-term bonus linked to budget and ordinary stock option grants, the EVA ownership plan employs two simple, distinct elements:

1. A cash bonus plan that simulates ownership
2. A leveraged stock option (LSO) plan that makes ownership real

EVA Bonus Plan: Simulating Ownership

The cash bonus plan simulates ownership primarily by tying bonuses to *improvements* in EVA over time. Paying for improvements in, rather than absolute levels of, EVA is designed mainly to solve the problem of "unequal endowments." This way managers of businesses with sharply negative EVA can be given a strong incentive to engineer a turnaround—and those managers of businesses already producing large positive EVA do not receive a windfall simply for showing up.

Besides leveling the playing field for managers inheriting different circumstances, bonuses tied to improvements in rather than levels of EVA are also "self-financing" in the following sense: To the extent that a company's current stock tends to reflect current levels of EVA, it is only *changes* in current levels of EVA that are likely to be correlated with changes in stock price.[3] And, to the extent the managers of a given company succeed in increasing a company's EVA and so earn higher bonus awards for themselves, those higher bonuses are more than paid for by the increase in shareholder value that tends to accompany increases in EVA.

As with a true ownership stake, EVA bonuses are not capped. They are potentially unlimited (on the downside as well as upside), depending entirely on managerial performance. But, to guard against the possibility of short-term "gaming" of the system, we have devised a "bonus bank" concept that works in this way: Annual bonus awards are not paid out in full but instead are banked forward and held "at risk," with full payout contingent on continued successful performance. Each year's bonus award is carried forward from the prior year and a fraction—for example, one-third—of that total is paid out, with the remainder banked into the next year.

Thus, in a good year, a manager is rewarded—much like a shareholder who receives cash dividends and capital appreciation—with an increase in both the cash bonus paid out and in the bonus bank carried forward. But in a poor year—again, much like a shareholder—the penalty is a shrunken cash distribution and a depletion in the bank balance that must be recouped before a full cash bonus distribution is again possible. Because the bonus paid in any one year is an accumulation of the bonuses earned over time, the distinction between a long-term and a short-term bonus plan becomes meaningless.

When combined with such a bonus bank system, EVA incentive plans tied to continuous improvement also help to break the counterproductive link between bonuses and budgets. EVA targets are automatically reset from one year to the next by formula, not annual negotiation. For example, if EVA should decline for whatever reason, management will suffer a reduced, possibly negative bonus in that year. In the following year, however, the minimal standard of performance for the next year's bonus will be set somewhat lower—again, by a preset formula. This automatic lowering of expectations is designed to help companies retain and motivate good managers through bad times by giving them a renewed opportunity to earn a decent bonus if they can reverse the company's fortune. At the same time, however, it avoids the problem—inherent in the stock option "repricing" practices of so many public companies—of rewarding managers handsomely when the stock drops sharply and then simply returns to current levels.

In combination with a bonus bank, then, the use of objective formulas to reset targets eliminates the problems of "sandbagging" on budgets and encourages collaborative, long-range planning. Instead of wasting time managing the expectations of their supervisors, managers are motivated to propose and execute aggressive business plans. Moreover, because it compensates the end of creating value rather than the means of getting there, the EVA bonus plan is entirely consistent with the movement to decentralize and empower.

In sum, the banking of bonuses tied to continuous improvements in EVA helps companies to smooth cyclical bumps and grinds, extends managers' time horizons, and encourages good performers accumulating equity in their bank accounts to stay and poor performers running up deficits to go. In so doing, the EVA bonus bank functions as both a long-term and short-term plan at one and the same time.

Leveraged Stock Options: Making Ownership Real

The annual EVA cash bonus is intended to simulate an owner's stake. In many cases, however, it is often valuable to supplement the bonus plan with actual stock ownership by management. Pursuit of that goal, however, runs headlong into this fundamental contradiction: How can managers with limited financial resources be made into significant owners without unfairly diluting the current shareholders? Showering them with stock options of restricted stock is apt to be quite expensive for the shareholders, notwithstanding the incentive for the managers. And asking the managers to buy lots of stock is apt to be excessively risky for them.

One approach we recommend to resolve this dilemma is to encourage (or require) managers to purchase common equity in the form of special leveraged stock options (LSOs). Unlike ordinary options, these are initially in-the-money and not at-the-money, are bought and not granted, and project the exercise price to rise at a rate that sets aside a minimal acceptable return for the shareholders before management participates.

Although managers' purchase of LSOs could be funded by them as a one-time investment, we typically recommend that managers be allowed to buy them only with a portion of their EVA bonuses. Besides providing even more deferred compensation, this practice helps ensure that only those managers who have added value in their own operations are allowed to participate in the success of the entire enterprise.

To illustrate how an LSO operates, consider a company with a current common share price of \$10. The initial exercise price on the LSO is set at a 10% discount from the current stock price, or $9, making the option worth $1 right out of the gate. But instead of just handing the LSOs to management, managers are required to purchase them for the $1 discount, and that money is put at risk. Another difference between LSOs and regular options is that the exercise price is projected to increase at a rate that approximates the cost of capital (less a discount for undiversifiable risk and illiquidity)—let's say 10% per annum. In this case, over a five-year period (and ignoring compounding for simplicity), the exercise price will rise 50% above the current $9 level to $13.50. In sum, management pays $1 today for an option to purchase the company's stock (currently worth $10) for $13.50 five years down the road.

Only if the company's equity value grows at a rate faster than the exercise price will management come out ahead. Indeed, if the exercise price rises at a rate equal to the cost of capital (less the dividend yield), then the LSOs will provide exactly the same incentives as an EVA bonus plan. It rewards management for generating a spread between the company's rate of return on capital and the cost of that capital (as reflected by the rate of increase in the exercise price) times the capital employed by management to purchase the shares.

Perhaps a better comparison, however, is between the incentives held out by LSOs and those provided by leveraged buyouts. LSOs can be seen as putting management in the position of participating in an LBO, but without requiring an actual LBO of the company. By virtue of their being purchased 10% in the money, LSOs effectively replicate the 90% debt and 10% equity that characterized the structure of the LBOs of the 1990s.

At bottom, then, LSOs (and LBOs, as we have seen) also boil down to EVA, to the idea that management should participate only in those returns in excess of a company's required rate of return. But while conceptually identical to an EVA bonus plan, LSOs are likely to be an even more powerful motivator because they amplify the risks and rewards for management. Any improvement in EVA that investors think will be sustained is capitalized into the value of the shares; for example, a company with a cost of capital of 10% that increases its EVA by $1 million will see its value appreciate by $10 million. For managers holding the LSOs, such capitalized increases in value are themselves further leveraged 10 to 1, thus creating $100 of added managerial wealth for each $1 improvement in EVA. This leveraging effect makes LSOs a potent way to get management to concentrate on building EVA over the long haul.

In sum, the EVA ownership plan replaces the traditional short-term bonus linked to budget and ordinary stock option grants with two components:

1. A cash bonus plan that simulates ownership
2. A leveraged stock option plan that confers actual ownership

The cash bonus plan simulates ownership by tying bonuses to sustained improvements in EVA over time, with a large portion of awarded bonuses held in escrow and subject to loss to ensure that improvements are permanent. The LSO plan corrects the deficiencies of normal stock option plans in two ways:

1. The leverage factor allows managers to purchase significantly more stock for a given amount of dollars (thus replicating an LBO's effect on ownership)
2. A steadily rising exercise price ensures that managers win only if shareholders do

SUMMARY

An EVA financial management system represents a way to institutionalize the running of a business in accordance with basic microeconomic and corporate finance principles. When properly implemented, it is a closed-loop system of decision making, accountability, and incentives—one that has the potential to make the entire organization and not just the chief executive officer responsible for the successes and failures of the enterprise. It can result in a self-regulated and self-motivated system of "internal" governance.

As a concept, EVA starts simple, but in practice it can be made as comprehensive as necessary to accommodate management's needs and preferences. EVA is most effective, however, when it is more than just a performance measure. At its best, EVA serves as the centerpiece of a completely integrated framework of financial management and incentive compensation.

The anchor of the EVA financial management system is a powerful incentive compensation plan that consists of two parts:

1. A cash bonus plan tied to continuous improvement in EVA, in which a significant portion of the awarded bonuses is carried forward in a "bonus bank" and held at risk
2. A leveraged stock option plan, in which managers use part of their cash bonus awards to make highly leveraged purchases of company stock

Such an EVA reward system holds out major benefits over more conventional compensation plans:

- Rewarding managers for continuous improvement in (rather than levels of) EVA means that new managers neither receive windfalls for inheriting already profitable divisions, nor are they penalized for stepping into turnaround situations.
- In contrast to compensation plans that continually revise performance criteria to provide "competitive" compensation levels *each year*, the EVA bonus plan has a "long-term memory" in the form of a bonus bank that ensures that only consistent, sustainable increases in value are rewarded.
- EVA bonuses are tied to a performance measure that is highly correlated with shareholder value, thus aligning managers' with shareholders' interests.
- The strength of the correlation between changes in EVA and in shareholder value also means that the EVA compensation system is effectively "self-financing";

that is, managers win big only when shareholders are winning—and managers are truly penalized when shareholders lose.

Proper internal governance is certainly no guarantee of success, and it is no substitute for leadership, entrepreneurism, and hustle. But an EVA financial management and incentive system can help. We like to say that EVA works like the proverbial Trojan horse: What is wheeled in appears to be an innocuous new financial management and incentive program, but what jumps out is a new culture that is right for times of rapid change and decentralized decision making. By increasing accountability, strengthening incentives, facilitating decentralized decision making, establishing a common language and integrated framework, and fostering a culture that prizes building value above all else, it significantly improves the chances of winning. That's all any shareholder can reasonably expect from governance in today's business environment of continuous change.

Notes

1. This chapter is adapted with permission from a portion of "The EVA® Financial Management System," *Journal of Applied Corporate Finance* (summer 1995): 32–36.
2. Nevertheless, our research suggests a remarkably strong correlation between a company's EVA performance, its shareholder value added (or MVA), and its standing in *Fortune's Most Admired* survey, a ranking based on an assessment of such criteria as customer responsiveness, innovation, time-to-market, and management quality. See Bennett Stewart, "EVA: Fact and Fantasy," *Journal of Applied Corporate Finance*, Vol. 7, No. 2 (summer 1994).
3. Our own research indicates that the changes in companies' EVAs over a five-year period account for nearly 50% of the changes in their market value added (MVAs), over that same time frame. (MVA, which is a measure of the shareholder value added by management, is roughly equal to the difference between the total market value and the book value of the firm's equity.) By comparison, growth in sales explained just 10% of the MVA changes, growth in earnings per share about 15% to 20%, and return on equity only 35%. For a description of this research, see Bennett Stewart, "Announcing the Stern Stewart Performance 1,000," *Journal of Applied Corporate Finance*, Vol. 3 No. 2 (summer 1990).

Appendixes

Appendix A

Bibliography

This bibliography is separated into two sections—books and articles. Appendix B references relevant courses and conferences, and Appendix C lists data resources for information on cost of capital.

BOOKS

Abrams, Jay B. *Quantitative Business Valuation—A Mathematical Approach for Today's Professionals*. New York: McGraw-Hill, 2001.

Adams, David P., et al. *Fundamentals of Business Valuation—Part I*. Lewisville, TX: American Institute of Certified Public Accountants, 2001.

ASA Business Valuation Standards and Portions of Uniform Standards of Professional Appraisal Practice. Herndon, VA: American Society of Appraisers, revised 1994.

Bierman, Harold, Jr., and Seymour Smidt. *The Capital Budgeting Decision: Economic Analysis of Investment Projects*, 8th ed. New York: Macmillan, 1992.

Brealey, Richard A., and Stewart C. Myers. *Principles of Corporate Finance*, 6th ed. Boston: Irwin McGraw-Hill, 2000.

Brown, Lawrence D., ed. *I/B/E/S Research Bibliography*, 5th ed. New York: I/B/E/S International, Inc., 1996.

Copeland, Tom, Tim Koller, and Jack Murrin. *Valuation: Measuring and Managing the Value of Companies*, 3rd ed. New York: John Wiley & Sons, Inc., 2000.

Cornell, Bradford. *Corporate Valuation: Tools for Effective Appraisal and Decision Making*. New York: McGraw-Hill, 1993.

———. *The Equity Risk Premium: The Long-Run Future of the Stock Market*. New York: John Wiley & Sons, Inc., 1999.

Damodaran, Aswath. *Damodaran on Valuation: Security Analysis for Investment and Corporate Finance*. New York: John Wiley & Sons, Inc., 1994.

———. *Investment Valuation*, 2nd ed. New York: John Wiley & Sons, Inc., 2002.

Ehrhardt, Michael C. *The Search for Value: Measuring the Company's Cost of Capital*. Boston: Harvard Business School Press, 1994.

Friend, Irwin I., and I. Bisksler, eds. *Risk and Return in Finance*. Cambridge, MA: Ballinger, 1977.

Fishman, Jay E., Shannon P. Pratt, J. Clifford Griffith, and Mark Wells. *Guide to Business Valuations*, 11th ed. Fort Worth, TX: Practitioners Publishing Co., 2001 (updated annually).

Hitchner, James R., *Financial Valuation: Applications and Models*. New York: John Wiley & Sons, Inc., forthcoming.

Hull, J. *Options, Futures and Other Derivative Securities*. Englewood Cliffs, NJ: Prentice-Hall, 1993.

Ibbotson Associates. *Beta Book*, First 2001 ed. Chicago: Ibbotson Associates, 2001.

———. *Cost of Capital Workshop*. Chicago: Ibbotson Associates, 1998.

———. *Cost of Capital Yearbook*. Chicago: Ibbotson Associates, 2002.

———. *Stocks, Bonds, Bills and Inflation Classic Edition 2002 Yearbook*. Chicago: Ibbotson Associates, 2002.

Ibbotson, Roger G., and Gary P. Brinson. *Global Investing: The Professional's Guide to the World Capital Markets*. New York: McGraw-Hill, 1993.

Ibbotson, Roger G., and Gary P. Brinson. *Investment Markets*. New York: McGraw-Hill, 1987.

IRS Training for Appeals Officers Coursebook. Chicago: CCH Incorporated, 1998.

Kaufman, Mike. *Handbook of Budgeting*, 4th ed. New York: John Wiley & Sons, Inc., 1999.

Mercer, Z. Christopher, *Quantifying Marketability Discounts—Developing and Supporting Marketability Discounts in the Appraisal of Closely Held Business Interests*, Memphis, TN: Peabody Publishing, LP, 1997.

———. *Valuing Financial Institutions*. Homewood, IL: Business One Irwin, 1992.

Pratt, Shannon P. *Business Valuation Discounts and Premiums*. New York: John Wiley & Sons, Inc., 2001.

Pratt, Shannon P., Robert F. Reilly, and Robert P. Schweihs. *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. New York: McGraw-Hill, 2000.

———. *Valuing Small Businesses and Professional Practices*, 3d ed. New York: McGraw-Hill, 1998.

Rachlin, Robert, and H. W. Allend Sweeny. *Handbook of Budgeting*, 3d ed. New York: John Wiley & Sons, Inc., 1993. See especially Chapter 11, "Determining the Cost of Capital," by Mike Kaufman.

Rapport, Alfred. *Creating Shareholder Value*, revised ed. New York: The Free Press, 1998.

Reilly, Frank K. *Investment Analysis and Portfolio Management*, 4th ed. Forth Worth, TX: Dryden Press, 1994.

Ross, Stephen A., Randolph W. Westerfield, and Jeffrey F. Jaffe. *Corporate Finance*, 4th ed. Burr Ridge, IL: McGraw-Hill, 1996.

Sharpe, William F., G. J. Alexander, and J.V. Bailey. *Investments*, 6th ed. Englewood Cliffs, NJ: Prentice-Hall, 1998.

Standard & Poor's Corporate Value Consulting. *Risk Premium Report* (formerly the PricewaterhouseCoopers *Risk Premium Study*).

Williams, John Burr. *The Theory of Investment Value*. Cambridge, MA: Harvard University Press, 1938.

ARTICLES

Abrams, Jay B. "A Breakthrough in Calculating Reliable Discount Rates." *ASA Valuation* (August 1994): 8–24.

———. "The Right Tools Can Simplify Company Valuation." *Accounting Today* (November 6, 1995): 14, 35.

Annin, Michael. "Fama-French and Small Company Cost of Equity Calculations." *Business Valuation Review* (March 1997): 3–13.

———. "Using Ibbotson Associates' Data to Develop Minority Discount Rates." *CPA Expert* (Winter 1997): 1–4.

Annin, Michael E., and Dominic A. Falaschetti. "Equity Risk Premium Still Produces Debate." *Valuation Strategies* (January/February 1998): 17–23, 44.

———. "Is There Still a Size Premium?" *CPA Expert* (Winter 1998): 7–9.

Ashton, D. J. "The Cost of Equity Capital and a Generalization of the Dividend Growth Model." *Accounting & Business Research* (Winter 1995): 3–17.

Asness, Clifford S. "Stocks versus Bonds: Explaining the Equity Risk Premium." *Financial Analysts Journal* (March/April 2000): 96–113.

"Bankruptcy Court Determines Fair Market Value Under Pennsylvania Judgment." *Judges & Lawyers Business Valuation Update* (May 2000): 7.

Baptiste, Laurent, Gregory Borges, and Gary Carr. "Utility Bond Ratings and the Cost of Capital." *Public Utilities Fortnightly* (October 27, 1988).

Barber, Gregory A. "Valuation of Pass-Through Entities." *Valuation Strategies* (March/April 2001): 4–11, 44.

Becker, Brian C., and Ian Gray. "Does a Small Firm Effect Exist When Using the CAPM? Not Since 1980 and Not When Using Geometric Means of Historical Returns." *Business Valuation Review* (September 1999): 104–107.

———. "Using Average Historical Data for Risk Premium Estimates: Arithmetic Mean, Geometric Mean, or Something Else?" *Business Valuation Review* (December 1998): 136–140.

Bendixen, Christian L. "Improved Estimation of Equity Risk Premiums." *Business Valuation Review* (March 1994): 22–32.

Berk, Jonathan B. "Does Size Really Matter?" *Financial Analysts Journal* (September–October 1997): 12–18.

Bishop, David M., and Frank C. Evans. "Avoiding a Common Error in Calculating the Weighted Average Cost of Capital." *CPA Expert* (Fall 1997): 4–6.

Black, Fischer. "Estimating Experted Return." *Financial Analysts Journal* (September–October 1993): 36–38.

Bishop, David M., and Frank C. Evans. "Avoiding a Common Error in Calculating the Weighted Average Cost of Capital." *CPA Expert* (Fall 1997): 4–6.

Bishop, David M., and John P. White. "Overview of the Multiple Period Discounting Method: Practical Considerations for Application in the Valuation of Businesses." *American Society of Appraisers International Conference 2001*. Pittsburgh, Pa. Available at *www.BVLibrary.com*.

Bolton, Steven, and Yan Wang. "The Impact of Management Depth on Valuation." *Business Valuation Review* (September 1997): 143–146.

Booth, Laurence. "The Capital Asset Pricing Model + Equity Risk Premiums and the Privately-Held Business." 1998 CICBV/ASA Joint Business Valuation Conference, September 24–25, 1998, Montreal, Canada.

Booth, Laurence, and Cleveland S. Patterson. "Estimating the Cost of Equity Capital of a Non-Traded Unique Canadian Entity: Reply." *Canadian Journal of Administrative Sciences* (June 1993): 122–133.

Botosan, Christine A. "Disclosure Level and the Cost of Equity Capital." *Accounting Review* (July 1997): 323–349.

Boudoukh, Jacob, Matthew Richardson, and Robert F. Whitelaw. "Nonlinearities in the Relation Between the Equity Risk Premium and the Term Structure." *Management Science* (March 1997): 371–385.

Budyak, James T. "Getting a Grip on Foreign Discount Rates." (January 2000). Available at *www.BVLibrary.com*.

———. "Establishing Discount Rates in a Global Environment." *ASA International Appraisal Conference*. Pittsburgh, Penn. July 2001.

Burmeister, Edwin, Richard Roll, and Stephen A. Ross. "A Practitioner's Guide to Arbitrage Pricing Theory." In *A Practitioner's Guide to Factor Models*. Charlottesville, VA: Research Foundation of the Institute of Chartered Financial Analysts, 1994.

Burns, Richard M. "Using DCF Analysis." *1999 Institute of Business Appraisers National Conference—Business Valuations*. Orlando, Fla. Available at *www.BVLibrary.com*.

Carlton, Willard T., and Josef Lakonishok. "Risk and Return on Equity: The Use and Misuse of Historical Estimates." *Financial Analysts Journal* (January–February 1985): 38–47.

Cassiere, George G. "Geometric Mean Return Premium Versus the Arithmetic Mean Return Premium—Expanding on the SBI 1995 Yearbook Examples." *Business Valuation Review* (March 1996): 20–23.

Chen, Nai-fi, Richard Roll, and Stephen A. Ross. "Economic Forces and the Stock Market: Testing the APT and Alternative Asset Pricing Theories." *UCLA Graduate School of Management Working Paper No. 20-83* (December 1983).

Clineball, John M., Douglas R. Kahl, and Jerry L. Stevens. "Time-Series Properties of the Equity Risk Premium." *Journal of Financial Research* (Spring 1994): 105–116.

Clubb, Colin D. B., and Paul Doran. "On the Weighted Average Cost of Capital with Personal Taxes." *Accounting & Business Research* (Winter 1992): 44–48.

Copeland, Tom. Seminar on Frontiers in Corporate Valuation. New York University Leonard N. Stern School of Business (November 6–7, 1997).

Cornell, Bradford. "Estimating the Cost of Equity Capital." *Proceedings of the 27th Annual Wichita Program: Appraisal for Ad Valorem Taxation*, August 3–7, 1997. Wichita, KS: Wichita State University, 1997.

Cummins, J. David, and Joan Lamm-Tennant. "Capital Structure and the Cost of Equity Capital in the Property-Liability Insurance Industry." *Insurance: Mathematics & Economics* (December 1994): 187–201.

Damodaran, Aswath. "Estimating Equity Risk Premiums." *Stern School of Business Working Paper No. FIN-99-021,* 1999. Can be downloaded without charge at *http://www.stern.nyu.edu/fin*.

Dempsey, Mike. "The Cost of Equity Capital at the Corporate and Investor Levels: Allowing a Rational Expectations Model with Personal Taxations." *Journal of Business Finance & Accounting* (December 1996): 1319–1331.

Dickerson, Gregg. "Estimating the Cost of Equity or They Ripped Out My CAPM and Stomped That Sucker Flat." *Proceedings of the 27th Annual Wichita Program: Appraisal for Ad Valorem Taxation*, August 3–7, 1997, Wichita, Kansas.

Dietrich, Mark O. "Tax Effecting Nontaxable Entities' Earnings Post-Gross." July 10, 2000. Available at *www.cpa.net/postgross.html*.

Dilbeck, Harold R. "A Constant-Dollar Discount Rate for Closely Held Businesses Based on Risk Premiums from Five-Year Holding Periods." *Business Valuation Review* (March 1994): 11–18.

Dobner, Michael. "Mid Year Discounting and Seasonality Factors." *Business Valuation Review* (March 2002): 16–18.

Downs, Thomas W., and Robert W. Ingram. "Beta, Size, Risk, and Return." *Journal of Financial Research* (Fall 2000): 245–260.

Durand, David. "Afterthoughts on a Controversy with MM, Plus New Thoughts on Growth and the Cost of Capital." *Financial Management* (Summer 1989): 12–18.

Duvall, Richard M. "Mid-year or End-of-year Discounting." *Business Valuation Review* (December 2000): 208–211.

———. "Capitalization of Earnings with Temporary Rapid Growth." *Business Valuation Review* (June 2001): 3–4.

Eccles, Robert G., Kersten L. Lanes, and Thomas C. Wilson. "Are You Paying Too Much for That Acquisition?" *Harvard Business Review* (July–August 1999): 136–146.

Evans, Frank C. "Practice Development/Business Valuation: Tips for the Valuator." *Journal of Accountancy* (March 2000): 5–42.

———. "Making Sense of Rates of Return and Multiples." *Business Valuation Review* (June 1999): 51–57.

Evans, Frank C., and Kelly L. Strimbu. "Debt and Equity Weightings in WACC." *CPA Expert* (Fall 1998): 4–5.

Fama, Eugene, and Kenneth French. "Common Risk Factors in the Returns on Stocks and Bonds." *University of Chicago Working Paper No. 360* (November 1992).

Fama, Eugene F., and Kenneth R. French. "Taxes, Financing Decisions, and Firm Value." *Journal of Finance* (June 1998): 819–843.

———. "The Equity Premium." *The Journal of Finance* (April 2002): 637–659.

Fowler, Bradley A. "How Do You Handle It?" part of a Special Report "ASA Conference Offers Valuable Insights." *Shannon Pratt's Business Valuation Update* (July 1997): 1–2.

Glazer, Jeffrey S. "The Capital Asset Pricing Model: Risk, Valuation, Judicial Interpretation and Market Bias." 50 *Bus. Law* 686 (1995).

Goldenberg, David H., and Ashok J. Robin. "The Arbitrage Pricing Theory and Cost-of-Capital Estimation: The Case of Electric Utilities." *Journal of Financial Research* (Fall 1991): 181–196.

———. "The Cross Section of Expected Stock Returns." *Journal of Finance* 47 (1992): 427–465.

Good, Walter R. "Yes, Virginia, There Is a Risk Premium, But . . ." *Financial Analysts Journal* (January–February 1994): 11–12.

Grabowski, Roger, and David King. "New Evidence on Size Effects and Rates of Return." *Business Valuation Review* (September 1996): 103–115.

———. "Size Effects and Equity Returns, An Update." *Business Valuation Review* (March 1997): 22–26.

———. "New Evidence on Equity Returns and Company Risk." Business Valuation Review (September 1999): 112–130.

———. "New Evidence on Equity Returns and Company Risk: A Revision." Business Valuation Review (March 2000): 32.

Graham, John R. "Debt and the Marginal Tax Rate." *Journal of Financial Economics* (1996): 41–73.

———. "Proxies for the Corporate Marginal Tax Rate." *Journal of Financial Economics* (1996): 187–221.

Graham, John R., Michael L. Lemmon, and James S. Schallheim. "Debt, Leases, Taxes, and the Endogeneity of Corporate Tax Status." *Journal of Finance* (February 1998): 131–162.

Greer, Willis R. "The Growth Rate Term in the Capitalization Model." *Business Valuation Review* (June 1996): 72–79.

Gullihur, James K. "The Gross Decision—Tax-effecting S Corporation Earnings." Pittsburgh, Pa. (October 2001). Available at *www.BVLibrary.com*.

Hanley, Frank J., and A. Gerald Harris. "Does Diversification Increase the Cost of Equity Capital?" *Public Utilities Fortnightly* (July 15, 1991): 26–30.

Hawkins, George B. "Valuation of S Corporations Using the Discounted Future Income Method—A Tax Court Valuation Bombshell." *Fair Value* (Fall 1999): 1–4.

———. "A Gross Result in the *Gross* Case Calls into Question Circumstances in Which Tax Affecting Is Valid." *Shannon Pratt's Business Valuation Update* (January 2002): 5–7.

———. "Critically Assessing a Business Valuation: Is the Capitalization Rate Used Reasonable?" *Fair Value* (Spring 1996): 1–6.

Honnold, Keith L. "The Link Between Discount Rates and Capitalization Rates: Revisited." *Appraisal Journal* (April 1990): 190–195.

Howton, Shelly W., and David R. Peterson. "A Cross-sectional Empirical Test of a Dual-state Multi-factor Pricing Model." *Financial Review* (August 1999): 47–64.

Ibbotson, Roger G. "Equity Risk Premium: Where We Stand Today." *Proceedings of the Equity Risk Premium Conference*, University of Chicago, June 6, 1996. Chicago: Ibbotson Associates, 1996.

Ibbotson, Roger G., and Peng Chen. "Stock Market Returns in the Long Run: Participating in the Real Economy." *Yale ICF Working Paper No. 00-44*, March 2002. Can be downloaded without charge at *http://papers.ssrn.com/abstract=274150*.

Ibbotson, Roger G., Paul D. Kaplan, and James D. Peterson. "Estimates of Small-Stock Betas Are Much Too Low." *Journal of Portfolio Management* (Summer 1997).

Ickert, Donald W. "Unsystematic Risk & Size Effects on Valuation." Presented at American Society of Appraisers 2001 International Appraisal Conference, Pittsburgh, PA (July 23–25, 2001): Available at *BVLibrary.com*.

———. "Unsystematic Risk & Size Effects on Valuations." (October 2001). Available at *www.BVLibrary.com*.

Indro, Daniel C., and Wayne Y. Lee. "Biases in Arithmetic and Geometric Averages as Estimates of Long-run Expected Returns and Risk Premia." *Financial Management* (Winter 1997): 81–90.

Johnson, Bruce A. "S Corporation Tax Treatment." *2001 ASA International Appraisal Conference*. Available at *www.BVLibrary.com*.

Jackson, Marcus. "The Gordon Growth Model and the Income Approach to Value." *Appraisal Journal* (January 1994): 124–128.

Joyce, Allyn A. "Arithmetic Mean vs. Geometric Mean: THE Issue in Rate of Return Derivation." *Business Valuation Review* (June 1995): 62–68.

———. "Why the Expected Rate of Return Is a Geometric Mean." *Business Valuation Review* (March 1996): 17–19.

Julius, J. Michael. "Market Returns in Rolling Multi-Year Holding Periods: An Alternative Interpretation of the Ibbotson Data." *Business Valuation Review* (June 1996): 57–71.

Kairys, Joseph P., Jr. "Predicting Sign Changes in the Equity Risk Premium Using Commercial Paper Rates." *Journal of Portfolio Management* (Fall 1993): 41–51.

Kaltman, Todd A. "Capitalization Using a Mid-Year Convention." *Business Valuation Review* (December 1995): 178–182.

Kaplan, Paul D. "Why the Expected Rate of Return Is an Arithmetic Mean." *Business Valuation Review* (September 1995): 126–129.

Kaplan, Paul D., and James D. Peterson. "Full-information Industry Betas." *Financial Management* (Summer 1998): 85–93.

Kaplan, Steven J., and Richard S. Ruback. "The Valuation of Cash Flow Forecasts: An Empirical Analysis." 50 *Journal of Finance* 1054, 1091 (1995).

Keck, Tom, Eric Levengood, and Al Longfield. "Using Discounted Cash Flow Analysis in an International Setting: A Survey of Issues in Modeling the Cost of Capital." *Journal of Applied Corporate Finance* (Fall 1998): 82–99.

Kenny, Thomas J. "Closely Held Corporation Valuation: Determining a Proper Discount Rate." *Business Valuation Review* (March 1992): 22–30.

Kholdy, Shady, and Ahmad Sohrabian. "Internal Finance and Corporate Investment." *Financial Review* (May 2001): 97–114.

Kihm, Steven G. "The Superiority of Spot Yields in Estimating Cost of Capital." *Public Utilities Fortnightly* (February 1, 1996): 42–45.

Kincheloe, Stephen C. "The Weighted Average Cost of Capital—The Correct Discount Rate." *Appraisal Journal* (January 1990): 88–95.

King, David W. "The Equity Risk Premium for Cost of Capital Studies: Alternatives to Ibbotson." *Business Valuation Review* (September 1994): 123–129.

———. “Recent Evidence on Discount Rates.” *Proceedings of the AICPA 1995 National Business Valuation Conference*, New Orleans, LA, December 1995, pp. 12-i–12-13. Available on *Business Valuation Update Online*.

Krueger, Mark K., and Charles M. Linke. “A Spanning Approach for Estimating Divisional Cost of Capital.” *Financial Management* (Spring 1994): 64–70.

Lally, Martin. “The Accuracy of CAPM Proxies for Estimating a Firm’s Cost of Equity.” *Accounting & Finance* (May 1995): 63–72.

Lamdin, Douglas J. “New Estimates of the Equity Risk Premium and Why Business Economists Need Them.” Department of Economics, University of Maryland, January 2002.

Lavely, Joe, and Frank Bacon. “Risk and Rate of Return for Electric Utilities.” *Public Utilities Fortnightly* (September 1, 1993): 18–20.

Leibowitz, Mark L., forum chair. “Equity Risk Premium Forum.” Association of Investment Management and Research, November 8, 2001.

Leibowitz, Martin L., “Spread-driven Dividend Discount Models.” *Financial Analysts Journal* (November/December 2000): 64–77.

Lerch, Mary Ann. “Pretax/Aftertax Conversion Formula for Capitalization Rates and Cash Flow Discount Rates.” *Business Valuation Review* (March 1990): 18–22.

———. “Are We Capitalizing the Right Measure of Cash Flow?” *Business Valuation Review* (September 2001): 32–34.

Leuhrman, Timothy A. “Using APV: A Better Tool for Valuing Operations.” *Harvard Business Review* (May–June 1997): 145.

Levy, Moshe, and Haim Levy. “The Danger of Assuming Homogeneous Expectations.” *Financial Analysts Journal* (May–June 1996): 65–70.

Lippitt, Jeffrey W., and Nicholas J. Mastriacchio. “Developing Capitalization Rates for Valuing a Business.” *The CPA Journal* (November 1995): 24–28.

Madden, Bartley J. “The CFROI Valuation Model.” *Journal of Investing* (Spring 1998): 31–44.

Mandron, Alix. “DCF Valuation Can, and Should, Be Improved.” Montreal, Canada. Available at *www.BVLibrary.com*.

Mard, Michael J., and James S. Rigby. “New Research to Estimate Cost of Capital.” *CPA Expert* (Fall 1995): 9–12.

Maris, Brian A., and Fayez A. Elayan. “Capital Structure and the Cost of Capital for Untaxed Firms: The Case of REITs.” *Journal of the American Real Estate & Urban Economics Association* (Spring 1990): 22–39.

Martin, Harold G., Jr., and Ronald L. Seigneur. “Cost of Capital.” Joint presentation made at the American Institute of Certified Public Accountants National Business Valuation Conference, Las Vegas, NV (December 4, 2001). Available at *www.BVLibrary.com*.

Mastriacchio, Nicholas J., and Jeffrey W. Lippitt. “A Comparison of the Earnings Capitalization and the Excess Earnings Models in the Valuation of Closely Held Businesses.” *Journal of Small Business Management* (January 1996): 1–12.

Matthews, Gilbert E. “Cap X = Depreciation Is Unrealistic Assumption for Most Terminal Values.” *Shannon Pratt’s Business Valuation Update* (March 2002): 1–3.

McConaughy, Daniel L. "Negative Takeover Premia and Stock Price Levels in Internet Stocks." *Valuation Strategies* (March–April 2002): 20–29.

Mercer, Z. Christopher. "The Adjusted Capital Asset Pricing Model for Developing Capitalization Rates." *Business Valuation Review* (December 1989): 147–156.

Meyer, James E., Patrick Fitzgerald, and Mostafa Moini. "Loss of Business Profits, Risk, and the Appropriate Discount Rate." *Journal of Legal Economics* (Winter 1994): 27–42.

Miller, Warren D. "Assessing Unsystematic Risk." *CPA Expert* (Summer 1999): 1–5.

———. "Assessing Unsystematic Risk: Part II—The Macroenvironment." *CPA Expert* (Winter 2000): 1–5.

Moyer, R. Charles, and Ajay Patel. "The Equity Market Risk Premium: A Critical Look at Alternative Ex Ante Estimates." *Proceedings of the Ibbotson Equity Risk Premium Conference*, University of Chicago, June 6, 1996. Chicago: Ibbotson Associates, 1996. Abstracted in *Shannon Pratt's Business Valuation Update* (August 1996): 1–2.

"New Studies Quantifying Size Premiums Offer Strong Cost of Capital Support." *Shannon Pratt's Business Valuation Update* (August 1997): 1, 3.

O'Brien, Thomas J. "The Global CAPM and a Firm's Cost of Capital in Different Currencies." *Journal of Applied Corporate Finance* (Fall 1999): 73–79.

Olsen, Robert A., and George H. Troughton. "Are Risk Premium Anomalies Caused by Ambiguity?" *Financial Analysts Journal* (March/April 2000): 24–31.

Paolo, Stanley B. S. "The Weighted Average Cost of Capital: A Caveat." *Engineering Economist* (Winter 1992): 178–183.

Paschall, Michael A., and George B. Hawkins. "Are Smaller Companies More Risky?" *Fair Value* (Fall 1999): 1, 5–12.

Pastor, Lubos, and Robert Stambaugh. "Cost of Equity Capital and Model Mispricing." *Journal of Finance* (February 1999): 67–114.

Patterson, Cleveland S. "The Cost of Equity Capital of a Non-Traded Unique Entity: A Canadian Study." *Canadian Journal of Administrative Sciences* (June 1993): 115–121.

Pratt, Shannon P. "Alternative Equity Risk Premium Measures Unstable; Lack Robust Predictive Power." *Shannon Pratt's Business Valuation Update* (August 1996): 1–2.

———. "Building Better Betas Is Ibbotson's Answer to Beta Controversy." *Shannon Pratt's Business Valuation Update* (August 1997): 1–2.

———. "Discount Rates Based on CAPM Don't Always Lead to Minority Value." *Shannon Pratt's Business Valuation Update* (March 2001): 1–3.

———. "Evidence Suggests Equity Risk Premium Lower Than Conventional Wisdom Thinks." *Shannon Pratt's Business Valuation Update* (July 1996): 1–5.

———. "New Measures of Risk That Really Work." *Shannon Pratt's Business Valuation Update* (December 1999): 1–4.

———. "Practitioners Disagree Strongly on Excess Earnings Methodology." *Shannon Pratt's Business Valuation Update* (April 1997): 1–3.

Raabe, William A., and Gerald E. Whittenburg. "Is the Capital Asset Pricing Model Appropriate in Tax Litigation?" *Valuation Strategies* (January/February 1998): 10–15, 36–37.

Reynolds, Kevin B. "Reconciling the Single Period Capitalization Method with the Discounted Future Earnings Method." *Business Appraisal Practice* (Spring 2000): 16–27.

Rigby, Jim, and Michael J. Mattson. "Capitalization and Discount Rates: Mathematically Related, but Conceptually Different." *CPA Expert* (Fall 1996): 1–3.

Roll, Richard, and Stephen A. Ross. "On the Cross-sectional Relation between Expected Returns and Betas." 49 *Journal of Finance* 101 (1994).

Ross, Stephen A. "The Arbitrage Theory of Capital Asset Pricing." *Journal of Economic Theory* (December 1976): 241–260.

Schramm, Ronald M., and Henry M. Wang. "Measuring the Cost of Capital in an International CAPM Framework." *Journal of Applied Corporate Finance* (Fall 1999): 63–72.

Scott, M. F. G. "The Cost of Equity Capital and the Risk Premium on Equities." *Applied Financial Economics* (March 1992): 21–32.

Sharpe, William F. "Factor Models, CAPMs, and the ABT." *Journal of Portfolio Management* (Fall 1984): 21–25.

———. "Capital Asset Prices With and Without Negative Holdings." Nobel Lecture, December 1990. Stockholm: The Nobel Foundation, Royal Swedish Academy of Sciences, 1990.

Shumway, Tyler. "The Delisting Bias in CRSP Data." *Journal of Finance* (March 1997).

Siegel, Jeremy, and Richard Thaler. "The Equity Risk Premium Puzzle." *Journal of Economic Perspectives*, vol. 11, no. 1 (Winter): 191–200.

Sliwoski, Leonard. "Capitalization Rates Developed Using the Ibbotson Associates Data: Should They Be Applied to Pretax or After Tax Earnings?" *Business Valuation Review* (March 1994): 8–10.

Sliwoski, Leonard J. "Risk Categorization, the Income Approach, and the Invested Capital Methodology (Part One)." *Business Appraisal Practice* (Winter 2000–2001): 10–21.

———. "Risk Categorization, the Income Approach, and the Invested Capital Methodology (Part Two)." *Business Appraisal Practice* (Spring/Summer 2001): 43–53.

Stanton, Thomas C. "The Market Model: An Alternative to the CAPM." *Business Valuation Review* (March 1999): 12–15. Available at *www.BVLibrary.com*.

———. "The Anatomy of Beta: Getting Down to Basics." *Business Appraisal Practice* (Fall 1999): 34–38. Available at *www.BVLibrary.com*.

"Stock Recipients Liable for Fraudulent Transfer Underpayment." *Shannon Pratt's Business Valuation Update* (January 1999): 10.

Stultz, René M. "Globalization, Corporate Finance, and the Cost of Capital." *Journal of Applied Corporate Finance* (Fall 1999): 8–25.

Suvas, Arto. "Cost of Equity Capital Redefined." *Quarterly Journal of Business & Economics* (Spring 1992): 53–71.

Swad, Randy. "Discount and Capitalization Rates in Business Valuations." *CPA Journal* (October 1994): 40–46.

Tarbell, Jeffrey S. "The Small Company Risk Premium: Does It Really Exist?" *American Society of Appraisers 18th Annual Advanced Business Valuation Conference*. New Orleans, La. October 29, 1999. Available at *www.BVLibrary.com*.

Thompson, Howard E., and Wing K. Wong. "On the Unavoidability of 'Unscientific' Judgment in Estimating the Cost of Capital." *Managerial & Decision Economics* (February 1991): 27–42.

Thompson, Samuel C., Jr. "A Lawyer's Guide to Modern Valuation Techniques in Mergers and Acquisitions." *The Journal of Corporation Law* (Spring 1996): 457–537.

———. "Demystifying the Use of Beta in the Determination of the Cost of Capital and an Illustration of Its Use in Lazard's Valuation of Conrail." *The Journal of Corporation Law* (Winter 2000): 241–306.

Trout, Robert R. "Mid-year Discounting Without Bias." *Business Valuation Review* (December 2001): 39–41.

Vander Linden, Eric. "Cost of Capital Derived from Ibbotson Data Equals Minority Value?" *Business Valuation Review* (December 1998): 123–127.

Vasicek, Oldrich A. "A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Prices." *Journal of Finance* (1973).

Welch, Ivo. "The Equity Premium Consensus Forecast Revisited." Cowles Foundation for Research in Economics at Yale University, *Cowles Foundation Discussion Paper No. 1325*, September 2001. Can be downloaded without charge at *http://papers.ssrn.com/abstract=285169*.

Yamaguchi, Katsunari. "Estimating the Equity Risk Premium from Downside Probability." *Journal of Portfolio Management* (Summer 1994): 17–27.

Young, David S. "Some Reflections on Accounting Adjustments and Economic Value Added." *Journal of Financial Statement Analysis* (Winter 1999): 7–19.

Appendix B

Courses and Conferences

Courses by Professional Associations
American Society of Appraisers
American Institute of Certified Public Accountants
Center for Advanced Valuation Studies
Institute of Business Appraisers
National Association of Certified Valuation Analysts
Schedules and Other Conferences

COURSES BY PROFESSIONAL ASSOCIATIONS

The American Society of Appraisers (ASA), the American Institute of Certified Public Accountants (AICPA), the Institute of Business Appraisers (IBA), and the National Association of Certified Valuation Analysts (NACVA) offer certain courses with substantial content on cost of capital.

American Society of Appraisers

The income approach to valuation, including estimation and use of cost of capital, is a major part of the ASA's BV202: "Introduction to BV, Part Two." This is a three-day course, plus a half-day exam, offered about a dozen times a year at various cities around the country. BV204 goes into the cost of capital in greater depth. For information call (800) ASA-VALU [272-8258].

American Institute of Certified Public Accountants

The AICPA's basic course "Fundamentals in Business Valuation (FBV) I," focuses primarily on the income approach, including estimation and use of cost of capital. The AICPA's advanced course "Advanced Cost of Capital Computations in a Complex World (ACCC)," is devoted to estimating cost of capital and applying it in business valuations. FBVI is a three-day course and ACCC is a one-day course. Both courses are offered frequently at many locations around the country. For information contact the nearest state CPA society or the AICPA/PDI at (888) 247-3277.

Center for Advanced Valuation Studies

The ASA, together with the Center for Advanced Valuation Studies (CAVS), has launched a series of advanced-level seminars, targeted toward seasoned professionals with a minimum of six to ten years experience in valuation or accounting, or expertise in areas such as finance, financial consulting, public accounting, or business.

The CAVS cost of capital seminar is two days, offered a couple of times per year. For more information, call (800) ASA-VALU [272-8258].

Institute of Business Appraisers

The IBA offers course 1008, "Discount and Capitalization Rates: Practical & Defensible Derivation," as part of its "Shannon Pratt/IBA Master Business Appraisal Series." This newly expanded course examines discount and capitalization rates in-depth for small and midsize businesses. This is a two-day course offered a couple of times a year. For information, call (800) 299-4130.

National Association of Certified Valuation Analysts

As part of a five-day intermediate-level course, NACVA offers two days of "Business Valuations: Fundamentals, Techniques and Theory (FT&T)." Some of the topics covered during this two-day course are selecting the proper income, estimating future earnings, selecting and developing proper capitalization rates, and distinguishing and converting pretax and after-tax rates. The course is offered several times a year in various cities around the country. For more information, call (800) 677-2009.

SCHEDULES AND OTHER CONFERENCES

Often various professional business appraisal organizations and other organizations that utilize the cost of capital sponsor sessions relating to cost of capital at conferences.

All new offerings are described as they are announced in the monthly "News Update" or "Cost of Capital Update" departments of *Shannon Pratt's Business Valuation Update*®. Conference schedules are included in the monthly "Calendar Update" section.

Appendix C

Data Resources

Ibbotson Associates Cost of Capital Data
Betas
Earnings Forecasts and Related Data
  Value Line Publishing, Inc.
  First Call and I/B/E/S Databases (Thomson Financial)
  Multex-ACE (Analysts' Consensus Estimates)
  Zacks Investment Research, Inc.
Arbitrage Pricing Model Data
  BIRR Portfolio Analysis, Inc.
Publicly Traded Stock Data
Larger Company Merger and Acquisition Transaction Sources
Private Company Sale Transaction Data
Partnership Transaction Data
Periodicals

IBBOTSON ASSOCIATES COST OF CAPITAL DATA

Ibbotson Associates. Attn: Product Sales, 225 N. Michigan Avenue, Suite 700, Chicago, IL 60601-7676; (800) 758-3557; (312) 616-1620; fax (312) 616 0404; *www.ibbotson.com.*

Stocks, Bonds, Bills and Inflation® Classic Edition Yearbook. Published annually in late March. Available in print. Provides historical data on U.S. asset classes. The data gives a comprehensive, historical view of the performance of capital markets dating back to 1926. Contains total returns and index values for large- and small-company stocks, long-term corporate bonds, long- and intermediate-term government bonds, Treasury bills, and inflation.

New enhancements include a chapter devoted to international equity investing and a summary of the study "Stock Market Returns in the Long Run: Participating in the Real Economy."

Stocks, Bonds, Bills and Inflation® Valuation Edition Yearbook. Published annually in late March. Available in print. Complete with real-world examples and useful graphs to illustrate the analyses, this edition will help readers make decisions in cost-of-capital estimates. Contains an overview and comparison of the build-up method, Capital Asset Pricing Model, Fama-French three-factor model, and the discounted cash flow approach.

New enhancements include a new chapter providing an overview of the most popular business valuation models and a study confirming that the "delisting bias" has little effect on the formation of size premia.

Cost of Capital Yearbook. Published annually in June plus three quarterly updates in print. Contains valuable information and data on over 300 different industries based on Standard Industrial Classification (SIC) code. Statistics include historical growth, equity returns, profitability measures, capital structure ratio, price/earnings ratio, market-to-book, price-to-sales, dividend and cash flow yield, levered and unlevered beta, projected discounted cash flow growth rates, and five measures of cost of equity capital. Data are presented for both the median and industry composite on all statistics. For industries containing more than 10 companies, statistics also are provided for large-company and small-company composites.

Beta Book. Published semiannually in February and August. Available in print. Includes statistics on more than 5,000 companies that are essential for calculating cost of equity with the Capital Asset Pricing Model and Fama-French (FF) three-factor model. Statistics include: standard beta, adjusted beta, peer group beta, levered and unlevered beta, FF beta, small-minus-big premium, high-minus-low premium, and FF R-squared.

Cost of Capital Center. Available online at *www.ibbotson.com*. *Cost of Capital Yearbook* data and company betas from the *Beta Book* are available on a per-use basis and can be searched by SIC code, industry description, or company name. The Center also offers a variety of international cost of capital reports.

Standard and Poor's Corporate Value Consulting Risk Premium Report (formerly the PricewaterhouseCoopers Risk Premium Study). Available in PDF format. Has been updated through year-end 2000. This revision includes eight added measures of size. Companies are divided into 25 different size groups based on the following entries: Market Value of Equity, Book Value of Equity, Five-Year Average Net Income, Market Value of Invested Capital, Total Assets, Five-Year Average EBITDA, Sales, and Number of Employees.

Risk Premia Over Time. Available in PDF format. Provides equity and size premia over all historical time periods. The report contains long-, intermediate-, and short-term equity risk premia and mid-, low-, and micro-cap size premia. In addition, beta calculations are provided for each of the three size groupings.

International Reports. Available in PDF format. Ibbotson offers a variety of reports covering equity risk premia and cost of equity estimates for numerous countries. These reports are available at Ibbotson's Cost of Capital Center.

BETAS

CompuServe, 5000 Arlington Centre Boulevard, P.O. Box 20212, Columbus, OH 43220; (800) 848-8990.

Ibbotson Associates, 225 North Michigan Avenue, Suite 700, Chicago, IL 60601-7676; (800) 215-2494—*Beta Book:* Individual company betas; *Cost of Capital Yearbook:* Industry betas. *www.ibbotson.com*.

Merrill Lynch, Merrill Lynch Capital Markets, Global Securities Research & Economics Group, 4 World Financial Center, North Tower, New York, NY 10080; (212) 449-1159.

Standard & Poor's, 55 Water Street, New York, NY 10041. Publications (212) 438-2000—Standard & Poor's *Compustat* and *Standard & Poor's Stock Reports*; *www.standardandpoors.com*, *www.compustat.com*.

Tradeline, 112 West Park Drive, Mt. Laurel, NJ 08054-1286; (800) 444-2515; *www.tradeline.com*.

Value Line Publishing, Inc., 220 East 42nd Street, 6th Floor, New York, NY 10017; (800) 634-3583; *www.valueline.com*.

EARNINGS FORECASTS AND RELATED DATA

The information offered by providers listed here goes well beyond just earnings forecasts, because the providers also compile varying but almost intimidatingly vast amounts of other data and opinions about stocks, industries, and markets that can be helpful in estimating cost of capital. Four of the services listed herein compile data from hundreds of brokerage house analysts and one, Value Line, develops its prognostications inhouse.

(The use of earnings forecasts in estimating cost of capital by the discounted cash flow method is the subject of Chapter 12.)

Value Line Publishing, Inc.

Value Line employs a staff of some 100 independent professional security analysts. Its basic *Value Line Investment Survey Standard Edition* covers 1,700 stocks, and the *Value Line Investment Survey—Plus Edition* covers 7,500 stocks in total.

In addition to historical financial data and betas, Value Line forecasts revenues, cash flow, earnings dividends, capital expenditures, book value, shares outstanding, income tax rates, net profit margins, capital structure ratios, returns on both total capital and equity, and a three- to five-year target price range for the stock.

For historical research, the *Value Line Investment Survey* is available on microfiche from 1980 and the *Expanded Edition* from March 1995. This organization has several other print services, including *The Value Line Mutual Fund Survey*, *The Value Line No-Load Fund Advisor*, *The Value Line OTC Special Situations Service*, *The Value Line Options Survey*, and *The Value Line Convertibles Survey*.

Value Line also has an array of electronic publications, starting with *Value Line Investment Survey for Windows*. The software includes 350 searchable data fields, more than 60 chart and graph options, and more than 100 screening options. In addition, there are several other electronic products.

The surveys are also available through CompuServe, and Value Line has an online bulletin board service updated weekly for subscribers.

There is a Value Line DataFile with fundamental data on more than 6,000 companies. It has annual data since 1955, quarterly since 1963, and full 10-Q data since

1985. It includes balance sheet and income data, risk measures, rates of return, and analytic ratios.

Value Line is at 220 East 42nd Street, 6th Floor, New York, NY 10017; (800) 634-3583 for print services; (800) 284-7607 for electronic services; fax (201) 939-9079; *www.valueline.com.*

First Call and I/B/E/S Databases (Thomson Financial)

First Call Real Time Earnings Estimates (RTEE) has more than 200 data items, including current and previous analysts' earnings estimates, operating data, expected reporting dates, footnotes, and the FIRST CALL consensus estimate. RTEE covers more than 17,500 companies, updated from more than 500 brokerage firms worldwide.

Other services include current research from 200 brokerage firms, more than 320,000 full-text research reports (including charts, graphs, color, and formatting), a Recommendations Database, and a Fundamentals Database on more than 7,000 companies updated weekly with balance sheet and income items, pricing and valuation data, and some financial ratios.

First Call has consensus earnings estimates updated weekly for about 6,500 companies via America Online and on the Web at *www.firstcall.com.*

All First Call products are available via flexible delivery options. They can be accessed through a dedicated First Call terminal, a local area network (LAN), via other third-party services, or through FIRST CALL On Call®, a dial-up method using a standard personal computer and modem. RTEE data are also available in various hard-copy reports and fax products.

I/B/E/S covers 6,000 U.S. companies and 12,000 companies in 47 additional countries. It provides earnings estimates, recommendations, stock charts, current summaries and history, and analyst directories. The U.S. estimates come from a little more than 3,000 analysts in about 230 firms. In addition to earnings per share (EPS), I/B/E/S forecasts include cash flow per share, dividends per share, and pretax profits.

I/B/E/S has a U.S. History database that covers more than 20 years of U.S. earnings estimates and results for more than 10,000 companies. Data items include annual EPS projections and actual results since January 1976, long-term (five-year) growth projections since 1981, and quarterly data since 1984. The database is combined with I/B/E/S Rewind Software compatible with Windows.

I/B/E/S data are distributed through nearly every major source of electronic financial information. Although they emphasize electronic distribution, some products including earnings estimates are also available in print versions.

This summary of I/B/E/S products has focused largely on domestic earnings forecast information, but I/B/E/S also has a staggering array of other financial information available, especially that of a global nature.

In addition, I/B/E/S has supported academic research on earnings estimates for many years and has published an annotated bibliography, edited by Lawrence D. Brown, with abstracts of more than 400 articles and reports on such research.

Thomson Financial is at 195 Broadway, New York, NY 10007; (800) 455-5844; (646) 822-2000; fax (646) 822-3230; *www.firstcall.com*.

Multex-ACE (Analysts' Consensus Estimates)

Multex-ACE (formerly Standard & Poor's) offers its Analysts' Consensus Estimates (ACE) through its Compustat distribution system. It provides EPS and five-year growth estimates for more than 5,200 of the 9,700 companies on the Compustat database. The EPS estimates for each company also include mean, median, high, low, and standard deviation. The estimates come from more than 2,300 analysts in more than 200 brokerage firms.

The data can be accessed through a PC Plus Windows-based software. The ACE file is available electronically for loading to the subscriber's computer system or through several COMPUSTAT® vendors distributing the file electronically.

Compustat is at 7400 South Alton Court, Englewood, CO 80112; (800) 523-4534.

Zacks Investment Research, Inc.

Zacks offers three quarterly print publications, *Zacks Earnings Forecaster*, *Zacks EPS Calendar*, and *Zacks Profit Guide*. They cover a total of just over 6,000 companies. Zacks material is also distributed electronically through several vendors.

Zacks is at 155 N. Wacker Drive, Chicago, IL 60606; (800) 767-3771; (312) 630-9880; fax (312) 630-0954; *www.zacks.com*.

ARBITRAGE PRICING MODEL DATA

Only one firm still offers data to implement the arbitrage pricing model (APT) cost of capital estimation.

BIRR Portfolio Analysis, Inc.

BIRR has a software system called *BIRR Risks and Returns Analyzer*®. BIRR is an acronym for Burmeister (Edwin), Ibbotson (Roger), Roll (Richard), and Ross (Stephen).

It provides APT multiregression factor inputs for companies and industries for five macroeconomic risk factors: confidence risk, time horizon risk, inflation risk, business cycle risk, and market timing risk. (Each of these factors is described in Exhibit 14.1 in the chapter on the arbitrage pricing model.)

BIRR is at 2200 West Main Street, Suite 210, Durham, NC 27705; (919) 687-7053, *www.birr.com*.

PUBLICLY TRADED STOCK DATA

Electronic Data Gathering, Analysis, and Retrieval (EDGAR) Service provides access to SEC filings for more than 15,000 companies through the Securities and Exchange Commission at *www.sec.gov*. There is no charge for access. Freeware that can be used to reformat EDGAR data to a more usable layout is available from these two sources as well as from Business Valuation Resources LLC.

Mergent, Inc. (formerly known as Moody's) provides a wide variety of publications on publicly traded companies, including *Moody's Industrial Manual*, *Moody's Bank & Financial Manual*, *Moody's OTC Industrial Manual*, *Moody's OTC Unlisted Manual*, *Moody's Public Utilities Manual*, *Moody's Transportation Manual*, *Moody's Company Data*, and *Moody's Industry Review*. These publications are available from 60 Madison Avenue, 6th Floor, New York, NY 10007; (800) 342-5647; (212) 413-7601; fax (212) 413-7777, *www.mergent.com*.

Standard & Poor's, a division of McGraw-Hill, provides a wide variety of publications, both print and electronic, on publicly traded companies, including *Compustat*, *Standard & Poor's Corporation Records*, *Standard & Poor's Industry Reports*, *Standard & Poor's Industry Survey*, *Standard & Poor's Stock Guide*, *Standard & Poor's Stock Reports*, *Standard & Poor's Earnings Guide*, *Standard & Poor's Analysts' Handbook*, and *Standard & Poor's Execucomp*. These are all available from Standard & Poor's at 55 Water Street, New York, NY 10041; (800) 221-5277; Publications (212) 438-2000; *www.standardandpoors.com*, *www.compustat.com*.

LARGER COMPANY MERGER AND ACQUISITION TRANSACTION SOURCES

Mergerstat Review, published annually in March, with quarterly update reports, tracks mergers and acquisitions involving U.S. companies, including privately held, publicly traded, and cross-border transactions. Also tracks unit divestitures, management buyouts, and certain asset sales. Includes industry analysis by size premium, and transaction multiples. Provides trend analysis by seller, type, deal size, and industry. Offers 25 years of summary merger and acquisition (M&A) statistics, including average premium and price/earnings ratio. Available from Mergerstat LP, 2150 Colorado Avenue, Suite 150, Santa Monica, CA 90404, (310) 315-3100; fax (310) 829-4855. *www.mergerstat.com*.

Mergerstat Online Transaction Roster 2001–2002 tracks mergers and acquisitions involving U.S. companies, including privately held, publicly traded, and cross-border transaction synopses, Standard Industrial Classification (SIC) codes, announce and close dates, and deal values for thousands of transactions. Deals are sorted by target industry group. Available from Mergerstat LP, 2150 Colorado Avenue, Suite 150, Santa Monica, CA 90404, (310) 315-3100; fax (310) 829-4855. *www.mergerstat.com*.

Mergers & Acquisitions, The Dealmaker's Journal, published monthly, covers the merger/acquisition/divestiture field with articles on techniques and merger methodology; case studies of recent noteworthy deals; detailed records and evaluation of business deals for each quarter accompanied by tables analyzing merger activities in the past quarter; interviews with key people in the field; news of current legislation and regulations affecting the industry. This publication is available from Securities Data Publishing, a division of Thomson Financial, 195 Broadway, New York, NY 10007; (800) 455-5844; (646) 822-2000; *www.sdponline.com.*

Mergerstat/Shannon Pratt's Control Premium Study™. Business Valuation Resources, LLC, 7412 S.W. Beaverton-Hillsdale Highway, Suite 106, Portland, OR 97225, 888-BUS-VALU [287-8258], fax (800) 846-2291, *www.BVResources.com.*, available online at BVMarketData.com. Published quarterly in print and updated quarterly online, this study is a tool to quantify minority discounts and control premiums, and to derive valuation multiples for companies sold. Subscribers may choose between a searchable, Web-based database or a binder of printed material. The Web-based version gives instant access to four years of back data. Data is searchable by industry, SIC code, control premium, keyword, business description, and financial data. Five valuation multiples are calculated, including price/book value, price/sales, and target invested capital/EBITDA. Currently contains approximately 2,500 transactions, with 46% of the deals having net sales of less than $100 million. Each quarterly print publication contains details on approximately 250 sold businesses.

Mergers & Acquisitions in Canada, published monthly, includes M&A data and analysis. Available from Crosbie & Company, Inc., One First Canadian Place, 9th Floor, Toronto, Ontario, Canada M5X 1A4; (416) 362-0020; fax (416) 362-3447; *www.crosbieco.com.*

Mergers & Acquisitions Sourcebook, published annually, is the most comprehensive source of M&A information (600 pages) available anywhere, with coverage of mergers and acquisitions, joint ventures, initial public offerings (IPOs), restructurings, and strategic minority stakes. Details on more than 3,000 transactions, including purchase price and three-year financial data on seller and buyer organized by seller's SIC code, special sections on leveraged buyouts (LBOs), buy-backs, and terminations; M&A and divestiture activity of company divisions, foreign M&A activity, and analysis of industry trends. This publication is available from NVST.COM, INC., 14450 NE 29th Place, Suite 108, Bellevue, WA 98007; (800) 843-9559; (425) 702-9733; fax (425) 702-9753; *www.nvst.com.*

Mergers & Acquisitions Quarterly is specifically designed to provide M&A sourcebook data on a quarterly basis. It offers purchase price data and ratios to seller's sales, earnings, and net worth data on more than 1,000 corporate growth transactions in all industries. In addition, it has detailed charts and graphs, book reviews, and M&A features not published elsewhere. This publication is available from NVST.COM, INC., 14450 NE 29th Place, Suite 108, Bellevue, WA 98007; (800) 843-9559; (425) 702-9733; fax (425) 702-9753; *www.nvst.com.*

The Weekly Corporate Growth Report, 50 issues per year, is a newsletter on corporate growth in the United States, with fast-breaking news of the M&A market, including acquisitions, mergers, divestitures, spinoffs, terminations, management buyouts, restructurings, and methods of increasing shareholder value. This publication is available from NVST.COM, INC., 14450 NE 29th Place, Suite 108, Bellevue, WA 98007; (800) 843-9559; (425) 702-9733; fax (425) 702-9753; *www.nvst.com*.

The Merger Yearbook, published annually in March, provides information on tens of thousands of announced and completed deals, including total purchase price, price paid per share, form of payment, division/unit purchased, parent company, acquiring company, type of transaction, SIC number and industry section of target company and acquirer, price to earnings ratio, plus dozens of charts covering transactions by dollar amount and industry. This publication is available from Securities Data Publishing, a division of Thomson Financial, 195 Broadway, New York, NY 10007; (800) 455-5844; (646) 822-2000; fax (646) 822-3230; *www.sdponline.com*.

PRIVATE COMPANY SALE TRANSACTION DATA

Several databases cover private company transactions.

BIZCOMPS®. Jack Sanders, Asset Business Appraisal, PO Box 711777, San Diego, CA 92171; (858) 457-0366; fax (619) 299-7171; *www.bizcomps.com*.

BIZCOMPS® 2001. Deals are sorted by industry and contain revenue and discretionary earnings multiples. Published annually in print in four editions: Western (2,350 transactions), Central (1,322 transactions), Eastern (2,150 transactions), National Industrial (753 larger transactions). Data for each sale includes SIC code, type of business, ask price, sale price, annual sales, seller's discretionary cash flow, percent down, terms, inventory, fixtures and equipment, rent percent of sales, general location, ratio of sale price to gross sales, and sale price to seller's discretionary cash flow. A variety of summaries and averages are presented for various subgroups of companies.

BIZCOMPS® 2001 On Disk. Windows-driven program cataloging over 5,000 transactions. Custom searches for any field in each record. Program also contains graphing and linear regression analysis.

BIZCOMPS® Online. Graphic now a feature. Registration available online through *www.BVMarketData.com* and *www.NVST.com*.

Done Deals Data. NVST.COM, INC., 14450 NE 29th Place, Suite 108, Bellevue, WA 98007; (800) 843-9559; (425) 702-9733; fax (425) 702-9753; *www.nvst.com*.

Done Deals Data. Available in CD-ROM (updated quarterly) and online (updated weekly). Provides midmarket transaction data, with approximately 80% of the selling companies over the past three years being privately owned. Reported transactions cover approximately 30 industries. Data includes sale price, terms, and ratios. Six different pricing multiples are included in graphical and statistical format. The user interface includes a Deal Navigator, which allows the user to search by SIC code, keywords, location, closing date, price, and by seller and buyer name. Transactions include five financial indicators that include assets, equity, revenue, net income, and cash from operations. Ratios corresponding to these fields are also provided. *Done Deals* currently has over 4,700 transactions, with over 840 from 2000, and up to 250 added each quarter.

DoneDeals Complete Transaction Handbook 2000–2001. Provides details for both public and private companies that sold for between $1 million and $100 million. Features include the contact information for buyer and seller, price, terms and sources of financing, key financials and ratios, four-digit SIC code, purchase price, description of company operations, and comparative charting.

***Pratt's Stats*™.** Business Valuation Resources, LLC, 7412 S.W. Beaverton-Hillsdale Highway, Suite 106, Portland, OR 97225; 888-BUS-VALU [(888) 287-8258]; fax (800) 846-2291; *www.BVResources.com*. Available online at *BVMarketData.com*. Updated monthly online and quarterly in print and on disk. Provides detailed information on the sales of privately and closely held businesses. Contains information on up to 70 different data fields for each sold business. The *Public Company Database*, also included with a subscription to *Pratt's Stats*, contains up to 53 data fields. The fields include income data, asset data, transaction details, and a general company description. The data can be searched by any number of criteria selected by the user, including SIC and NAICS classifications. Database currently contains over 4,000 private transactions and over 500 public transactions. The median revenue of *Pratt's Stats* is $4.3 million and $41.5 million for the *Public Company Database*.

IBA Market Data Base. Institute of Business Appraisers, PO Box 17410, Plantation, FL 33318; (954) 584-1144; fax (954) 584-1184; *www.instbusapp.org*. Published regularly with updates from brokers. Contains approximately 20,000 transactions submitted during the last 20 years in over 650 SIC codes. Database contains field such as annual sales, earnings, owner's compensation, sale price, location, the year and month of the transaction, and some other data for some transactions.

PARTNERSHIP TRANSACTION DATA

The Partnership Spectrum. Partnership Profiles, Inc., PO Box 7938, Dallas, TX 75209; (800) 634-4614; fax (817) 488-2726; *www.partnershipprofiles.com*. Published bimonthly, available in print. Tracks the partnership industry, especially focusing on but not limited to publicly traded real estate partnerships. The May/June issue is a compilation of empirical data concerning discounts from net asset value at which minority interests in real estate partnerships trade in the informal resale market. The other issues are valuable to the partnership industry; however, they are not predominantly data compilations. Detailed reports on approximately 90% of the limited partnerships included in the May/June issue are available. Information for each partnership includes specific property holdings, cash distribution history, debt levels, key operating statistics, and so on.

PERIODICALS

***Shannon Pratt's Business Valuation Update*™.** Business Valuation Resources, LLC, 7412 S.W. Beaverton-Hillsdale Highway, Suite 106, Portland, OR 97225; 888-BUS-VALU [(888) 287-8258]; fax (800) 846-2291; *www.BVResources.com*. Available in print and online. Monthly newsletter focusing on news, views, and resources, for business valuation professionals. Monthly features include guest articles and interviews, editor's columns, legal and court case update, cost of capital, data & publications update, market data corner, reader/editor exchange, news update, and calendar update.

Business Valuation Library Online Service. Business Valuation Resources, LLC, 7412 S.W. Beaverton-Hillsdale Highway, Suite 106, Portland, OR 97225; 888-BUS-VALU [(888) 287-8258]; fax (800) 846-2291; *www.BVLibrary.com.* This online database of information contains: the full text and abstracts of all important federal and state court cases involving business valuation; IRS materials relevant to business valuation; conference papers and other articles not published elsewhere; all articles featured in *Shannon Pratt's Business Valuation Update®* (*BVU*) since its inception in 1995; all articles printed in the *Judges & Lawyers Business Valuation Update*™ (now merged with *BVU*); restricted stock study papers; and more. All documents are keyword searchable.

Appendix D

Developing Cost of Capital (Capitalization Rates and Discount Rates) Using ValuSource PRO Software

Z. Christopher Mercer, ASA, CFA

Introduction
Cost of Capital, Discount Rates, and Capitalization Rates
Data for the (Adjusted) Capital Asset Pricing Model
Earnings Stream to Be Capitalized
Application of Marketability Discounts

INTRODUCTION

Wiley ValuSource PRO was introduced in its current form in late 1996. Shannon Pratt asked me, as its developer, to prepare a short appendix regarding the development of cost of capital estimates that could be helpful to users of the ValuSource PRO software. Although this appendix discusses cost of capital in the context of the ValuSource PRO software, it also may be of broader interest for appraisers generally. The discussion is framed in the context of the development of capitalization rates in the "Appraisal" section of the software package.

COST OF CAPITAL, DISCOUNT RATES, AND CAPITALIZATION RATES

The term "cost of capital" is foreign to some business appraisers and many users of valuation reports who may have entered the business appraisal field via a route other than traditional finance. As indicated in the introduction of this book, "The cost of capital estimate is the essential link that enables us to convert a stream of expected income into an estimate of present value."

The *Business Valuation Standards* of the American Society of Appraisers provides two similar definitions[1]:

Discount Rate. A rate of return used to convert a monetary sum, payable or receivable in the future, into present value.
Capitalization Rate. Any divisor (usually expressed as a percentage) that is used to convert income into value.

In the context of the typical business appraisal, appraisers are developing the cost of (equity) capital when they develop a capitalization rate (or factor) to *capitalize* an earnings estimate by converting an expected future stream of income into present value. As a result, the Wiley ValuSource PRO software provides a framework to develop discount rates and capitalization rates. That framework is based on the Capital Asset Pricing Model (CAPM) and, specifically, on what I have called the *Adjusted Capital Asset Pricing Model* (ACAPM).[2]

This book has discussed several sources of cost of capital data:

- Ibbotson Associates published the *Stocks, Bonds, Bills and Inflation* (*SBBI) Classic Edition* and *Valuation Edition Yearbooks* annually as well as the other publications, including the *Cost of Capital Yearbook* and the *Beta Book*.[3]
- Roger Grabowski and David King, now with Standard & Poor's Corporate Value Consulting, formerly of PricewaterhouseCoopers, also have done interesting work on the impact of size on historical rates of return in the public stock markets. This work has been published partially in *Business Valuation Review* and has been discussed in *Shannon Pratt's Business Valuation Update*®.[4]
- Others, like Michael Julius, have analyzed the Ibbotson historical data to address the question of whether the arithmetic mean, the geometric mean, or some other statistic should be used as the basis for equity premia.[5]

There is nothing magical about any of these studies. All are attempting to measure the historical returns generated in the public stock markets for differing groups of stocks. The *SBBI Yearbooks*, portions of the *Cost of Capital Yearbook*, and the Grabowski/King studies have focused on market returns and stratified the public markets by various measures of size (sales, market capitalization, etc.). The major portion of the *Cost of Capital Yearbook* focuses on stratifying the public markets by Standard Industrial Classification (SIC) codes.

Given the background of this book, we can focus briefly on CAPM to derive some guidance on how to develop capitalization rates (i.e., cost of capital) using the Wiley ValuSource PRO software.

DATA FOR THE (ADJUSTED) CAPITAL ASSET PRICING MODEL

A number of chapters have discussed the so-called build-up method for developing capitalization rates and CAPM. In my opinion, the basic build-up method is simply a variation of CAPM under the assumption that beta is equal to 1.0. In the absence of market evidence to the contrary, business appraisers sometimes assume that the appropriate assumption for beta is 1.0, or the expected volatility of the broader stock market, which forms the first building block of the "build-up" of an equity discount rate and reflects the long-run historical premium in returns of the broader market over long-term Treasuries. In years prior to 1994, this premium was referred to in the Ibbotson Associates *SBBI Yearbooks* as the *common stock premium*. Since then it has been renamed the *large company stock premium*. Too often, some appraisers and writers try to make an arbitrary distinction between the build-up method and the CAPM. But, clearly, the former is a special instance of the latter with beta equal to 1.0.

So first of all, users of ValuSource PRO have to be aware of this assumption each time a decision is made to use the build-up method.

Many business appraisers and other financial analysts have used the historical premium return analysis presented in each year's *SBBI Yearbook*. In recent years, that information has come from Table 2.1 in each *SBBI Yearbook*. Appraisers have typically used the current year's analysis (e.g., the *SBBI 2001 Yearbook*, which covers Ibbotson's analysis of historical return information from 1926 to 2000). Historical appraisals typically reference the cumulative premium data from the then-current *SBBI Yearbook*. The actual historical geometric and arithmetic mean returns for the cumulative periods are provided for large company stocks, small company stocks, and long-term government bonds, and the actual premiums are calculated:

- The large company stock premium returns in excess of long-term government bond returns
- The small company stock premium returns in excess of large stock returns
- The small company stock premium in excess of long-term government bond returns

The current numbers for the appropriate premia are often used by appraisers in building up discount rates. Users selecting the Capital Asset Pricing Model in ValuSource PRO find a screen providing the various components of a capitalization rate (or factor). An illustrative example is shown in Exhibit D.1. The figures for the arithmetic mean and the geometric mean returns come from the *SBBI 1997 Yearbook*, and the figures labeled "Julius Multi-Year Holding Period Analysis" are derived from the article referenced in note 5.

The CAPM components above are called ACAPM components for the Adjusted Capital Asset Pricing Model. I have referred to this model as the *adjusted* CAPM because the basic CAPM stops at the net cash flow or net earnings discount rate and, in the process, assumes that company-specific (nonsystematic) factors are "diversified away." The ACAPM incorporates company-specific risk factors.

Any user of ValuSource PRO should recognize from Exhibit D.1 that neither software nor any single publication will enable the appraiser to develop an appropriate net equity discount rate or capitalization rate without the exercise of considerable judgment and the review and understanding of numerous sources of direct or indirect market evidence. With all assumptions remaining the same in Exhibit D.1 except the selection of the arithmetic mean or geometric mean returns, a spread in implied base capitalization rates (CR) is developed, ranging from 12.0% to 17.0%. To put this in

Exhibit D.1 Calculating Build-up or CAPM Discount and Capitalization Rates

		Appraiser Decision Letters		
		A	B	C
Appraiser Decision Numbers	ACAPM Component	*SBBI 1997 Yearbook* Arithmetic Mean	*SBBI 1997 Yearbook* Geometric Mean	Julius Multi-Year Holding Period Analysis
1	**Risk-free rate of return**	6.5%	6.5%	6.5%
2	**Equity risk premium**	7.3%	5.6%	6.5%
3	× **Industry beta**	1.0	1.0	1.0
Calc	= Beta-adjusted common stock premium	7.3%	5.6%	6.5%
4	**+ Risk adjustment for size**	5.0%	1.9%	3.5%
Calc	= Base equity discount rate	18.8%	14.0%	16.5%
5	+ *Company-specific premium*	3.0%	3.0%	3.0%
6	+ *Cash flow to earnings conversion*	0.0%	0.0%	0.0%
Calc	= Net earnings discount rate *(cost of equity capital)*	21.8%	17.0%	19.5%
7	– *Sustainable growth*	5.0%	5.0%	5.0%
Calc	= Base capitalization rate for next year*	17.0%	12.0%	15.0%
Calc	Base capitalization rate for current year*	16.2%	11.4%	14.3%
Calc	Base capitalization factor*	5.88	8.33	6.67

*User options.

User selects desired factor.

Boldfaced items require market evidence and appraiser judgments.

Italicized items require specific appraiser judgments.

Calc = calculated by software.

Source: Data compiled from Michael J. Julius, "Market Returns in Rolling Multi-Year Holding Periods, An Alternative Interpretation of the Ibbotson Data," *Business Valuation Review* (June 1996): 57–71, published by the Business Valuation Committee of the American Society of Appraisers. Reprinted with permission.

perspective by converting these capitalization rates into price/earnings multiples (*P/E = 1/CR*), the arithmetic mean selection developed a net earnings multiple of 5.88× and the geometric mean selection developed a multiple of 8.33×, or some 42% greater. The use of the Julius multiyear holding period analysis produces a price/earnings multiple of 6.67×, which is higher than that developed using the arithmetic mean but closer to that result than to the multiple derived using the geometric mean.

My best advice to any appraiser, whether using ValuSource PRO or not, is to be very clear at each of the numbered decision points (noted in Exhibit D.1) about what market data are being used and why. Furthermore, appraisers should be clear about the assumptions made regarding the lettered decision points in Exhibit D.1 as well.

Appraisers referring to the *SBBI Yearbooks* will develop components for the common stock equity premium, the appropriate beta, if applicable, and the small stock premium. Those referring to the Grabowski/King analyses may have to calculate the implied size premium in relationship to the base equity premiums initially used. The point is that at numbered decision 4, the net size adjustment is developed by subtracting a total premium over Treasuries implied by Grabowski/King from the common stock premium used in the analysis. In addition, appraisers may make a judgmental adjustment for size in addition to any developed by Ibbotson, Grabowski/King, or anyone else—if their subject companies are substantially smaller than the public companies used as reference points.

In the ValuSource PRO software, size premia are best considered in the "Risk Adjustment for Size" line. Appraisers using other than a so-called standard small stock premium from Ibbotson should explain in their reports exactly how their size premia were developed.

Note that some appraisers have considered very small size as a company-specific risk factor. There is nothing conceptually wrong with this treatment; however, before doing so, they should be familiar with current research on size premia or run the possible risk of being viewed as arbitrary.

The *company-specific premium* is an integral part of the development of the cost of equity capital. A breakout of several possible factors to consider in developing this premium is provided in the software. There is no market evidence to help the appraiser deal with most of these factors, and appraiser judgment must be carefully exercised.[6]

ValuSource PRO provides a line called "Cash Flow to Earnings Conversion." Shannon Pratt has indicated that he believes that the CAPM (or ACAPM) discount rate is applicable to the net cash flow of a business enterprise. I have suggested that it may be applicable to the net income of the enterprise. In *Valuing Financial Institutions*, I prepared an analysis indicating a methodology for developing a conversion of a net cash flow discount rate to a net income discount rate, and suggested that for many private companies the differential might not be large.[7] This analysis was also turned into an article that was published in the *Business Valuation Review*.[8] Certainly in the very long run, the net cash flow of an enterprise will approximate its net income. In any event, appraisers should be clear in their own minds what they believe on this issue and why, and then develop their remaining judgments consistently from this vantage point.

At this point, we have conceptually developed a net cash flow or net earnings discount rate. This discount rate is the *equity cost of capital*. This discount rate would be applicable to projected net earnings in a discounted future earnings analysis (or, properly styled or adjusted, to the projected net cash flows in a discounted cash flow analysis).

However, many appraisals are prepared without specific projections. To develop a single-period capitalization rate, expected future earnings growth must be subtracted from the discount rate (for all the reasons explained earlier in this book).

EARNINGS STREAM TO BE CAPITALIZED

It should be fairly obvious that the discount rate or capitalization rate applied to any measure of earnings should be appropriately developed for that measure, whether net income, pretax income, debt-free pretax income, or another level of the income statement. The CAPM discount rate discussed here and elsewhere in this book is generally considered applicable to either the net income or the net cash flow of a business enterprise. In application in actual appraisals, however, a legitimate question can be raised: To what net income or cash flow does the discount rate apply?

There has been considerable discussion in recent years regarding whether discounted future earnings (DFE) or discounted cash flow (DCF) valuation methods develop minority interest or controlling interest indications of value. A detailed discussion of the concept of levels of value is beyond the scope of this appendix; however, the question deserves some treatment.[9]

The two major trains of thought are as follows:

1. Since the CAPM discount rate is applicable to the net income of a business enterprise, and since this discount rate is generally believed to develop value indications at the marketable minority interest level of value, the value indication from a discounted future cash flow or earnings valuation is a minority interest (marketable) conclusion. As a result, it would be proper to apply a control premium to this value indication if a controlling interest conclusion is called for in the appraisal.[10]
2. Since appraisers make so-called control adjustments in developing their projections for DFE or DCF methods, the income stream is said to be control-adjusted, and the resulting valuation indication is at the controlling interest level.[11]

According to the first argument, buyers of companies might appear to have different discount rates than hypothetical investors at the marketable minority interest level. According to the second argument, there is only one discount rate, and it is the same for appraisers at the marketable minority interest level and for acquirers at the controlling interest level.

According to the first argument, one would add an appropriate control premium to a DCF/DFE valuation method to arrive at a controlling interest level of value. According to the second argument, a control premium might not be appropriate.

As is often the case, the truth may lie somewhere in between. To begin to resolve the controversy, we can divide the so-called control adjustments into their two primary component parts:

1. *Normalizing Adjustments.* In developing capitalization rates using data from Ibbotson Associates, Grabowski/King, or any other source of market return information, there are implicit "market baskets" of publicly traded companies that constitute the basis of comparison with subject private companies. We know that the typical public company in a group is larger than many of the closely held businesses that appraisers value, and this size differential gives rise to premium-required returns. We also know, generally, that public companies must pay

competitive salaries to senior management or else run the risk of being penalized in their market capitalizations. Likewise, related party transactions, to the extent that they exist, must be conducted at arm's length, and nonworking members of the president's family are not normally found on the payroll of public companies. The point is that a significant portion of the control adjustments made in many appraisals are, in reality, adjustments to normalize the earnings of the subject company with the group of public companies with which it is implicitly being compared.

2. *Acquirer's Potential Economic (Control) Adjustments.* Logically, an acquirer would make the normalizing adjustments noted earlier in the context of an acquisition of a private company. Clearly, an owner is not going to be paid for the capitalized value of excess salary and then continue to receive that salary. However, acquirers look at acquisition prospects differently than do public market securities investors. Acquirers often have an opportunity to generate economic benefits from acquisitions that go beyond the normalizing adjustments noted earlier. For example, an acquirer in a similar business may be able to generate significant economies by stripping out general and administrative or selling expenses from the acquired entity. Alternatively, an acquirer may be able to generate economic benefits that are not readily visible on a private company's financial statements. For example, an acquirer may be willing to pay a premium for a business because of planned increased sales of existing products through the acquired company's sales force. These types of potential economic benefits (adjustments) may generate the willingness to pay an apparent control premium for a company that otherwise might not be immediately justified.

The example in Exhibit D.2 illustrates a delineation of potential valuation adjustments into those categorized as normalizing (Line 2) and those noted as economic (control) adjustments made by a potential acquirer of control (Line 3).

In most appraisals, the adjustments made normally fall into the category of *normalizing adjustments*. The analysis in Exhibit D.2 indicates that it is not at all inconsistent to suggest that the discount rates are the same for the potential buyer of a company as for the hypothetical willing buyer of a marketable minority interest. (See Line 7, where the same price/earnings multiple and, implicitly, discount rate, is applied to differing perceptions of a subject company's earnings.) This would suggest, however, that the economic benefits of control have not yet been factored into the appraisal process at the marketable minority interest level, and that a control premium may be necessary to reach a proper conclusion of value on a controlling interest basis (see Line 10, where the implied control premium is 20%).

In the alternative, the appraiser would estimate these economic benefits specifically and capitalize them to develop a controlling interest conclusion. In the example in Exhibit D.2, the control premium provides a vehicle to estimate the magnitude of the benefit of potential economic (control) adjustments and to reflect them in the appraisal.

For users of ValuSource PRO software, the message is clear. Be sure to understand what adjustments have been made in an appraisal. To the extent that the normalizing adjustments of an appraisal do not consider the potential economic benefits

Exhibit D.2 Calculating Indicated Values

Line	Item	As Reported	Appraiser's Normalizing Adjustments	Acquirer's Economic (Control) Adjustments
1	Reported pretax earnings	$1,000	$1,000	$1,000
2	+ Normalizing adjustment (owner compensation)	–	200	200
3	+ Acquirer's economic adjustments	–	–	240
4	= Adjusted pretax income	1,000	1,200	1,440
5	– Taxes @ assumed rate of 40%	(400)	(480)	(576)
6	= Adjusted net income	600	720	864
7	× Net income capitalization factor (l/cap rate=P/E)		6.67	6.67
8	= Indicated value		$4,802	$5,763
9	Indicated level of value		Marketable, minority	Control
10	Implied control premium over marketable minority value indication			20%
11	Apparent net multiple without economic adjustments			8

available to potential acquirers, a judgmental control premium may be appropriate. The software makes this option readily available. That control premium, however, should be justified by a separate analysis or discussion of the potential factors leading to the apparent additional value attributable to control relative to the initially derived discount rate.

APPLICATION OF MARKETABILITY DISCOUNTS

Earlier in this book it was suggested that the marketability discount can be considered a premium to the equity cost of capital. Conceptually, this is correct; however, such a consideration would make the implicit assumption that the cash flows from which the initial marketable minority interest value indication is derived are the same as those available to the prospective holder of nonmarketable minority interests of private companies, which is clearly not the case in many closely held businesses.

For this reason, among others, we have developed a Quantitative Marketability Discount Model (QMDM), which develops appropriate marketability discounts based on the facts and circumstances facing hypothetical willing buyers of a company's minority interests.[12] Unless the expected cash flows available to a hypothetical minority investor are the same as those that formed the basis for developing the marketable minority interest value indication (a very rare circumstance), it is preferable to develop a marketability discount analysis separate from the initial development of the equity cost of capital (i.e., the capitalization factor).

Wiley ValuSource offers the QMDM in a CD-ROM format. The resultant discounts from this CD-ROM product can be incorporated into ValuSource PRO (Versions 2001, 2000, and 1.06).[13]

Notes

1. *ASA Business Valuation Standards and Portions of Uniform Standards of Professional Appraisal Practice (USPAP)* (Herndon, VA: American Society of Appraisers, revised January 1994), 18.
2. Z. Christopher Mercer, "The Adjusted Capital Asset Pricing Model for Developing Capitalization Rates: An Extension of Previous 'Build-Up' Methodologies Based upon the Capital Asset Pricing Model," *Business Valuation Review* (December 1989): 147–156. Also see Chapter 13 in Z. Christopher Mercer, *Valuing Financial Institutions* (Burr Ridge, IL: McGraw-Hill, 1992), and Exhibit 8.2 in Z. Christopher Mercer, *Quantifying Marketability Discounts: Developing and Supporting Marketability Discounts in the Appraisal of Closely Held Business Interests*, revised reprint (Memphis, TN: Peabody Publishing, LP, 2001). *Quantifying Marketability Discounts* is available from the publisher at 800-769-0967.
3. *Stocks, Bonds, Bills and Inflation, Classic Edition* and *Valuation Edition*, published annually; *Cost of Capital Yearbook*, published annually with quarterly updates; and *Beta Book*, published semiannually. Chicago: Ibbotson Associates.
4. Roger Grabowski and David King, "Size Effects and Equity Returns, An Update," *Business Valuation Review* (March 1997): 22–26. Discussed in *Shannon Pratt's Business Valuation Update®* (August 1997): 1.
5. J. Michael Julius, "Market Returns in Rolling Multi-Year Holding Periods: An Alternative Interpretation of the Ibbotson Data," *Business Valuation Review* (June 1996): 57–71. There has been something of a controversy over whether the more appropriate average statistics from Ibbotson's *SBBI Yearbook* is the arithmetic mean or the geometric mean. At its simplest, the Julius analysis recognizes that the arithmetic mean of the Ibbotson return data from 1926 to 1997 is the arithmetic mean (average) of 71 annual returns. The annual returns are the geometric means of the annual observations. So the arithmetic mean advanced by Ibbotson Associates is the arithmetic mean of 71 annual (geometric) returns, reflecting 71 one-year holding periods. The geometric mean advanced by others is simply the compound growth rate in total return from 1926 to 1997, or the geometric mean return for the period, which represents a single, 71-year holding period. From a practical viewpoint, neither extreme makes logical sense (and I am oversimplifying complex logical arguments to be practical). The Julius analysis examines the arithmetic mean of geometric returns for multiyear holding periods that have occurred from 1926 to 1995 (in the cited article). The effect of this averaging process over many multiyear holding periods is to develop a series of average returns for more reasonable holding periods such as, say, five or 10 years. The result, incidentally, is effectively to split the difference between the arithmetic mean and the geometric mean as calculated by Ibbotson. We have used this analysis for years as a basis for determining the appropriate common stock and small stock premium return measures.
6. As the appraisal profession matures, various appraisers are creatively examining the public stock markets for guidance on fundamental issues like developing company-specific risk premiums. A recent article typifies these efforts: Steven Bolten and Yan Wang, "The Impact of Management Depth on Valuation," *Business Valuation Review* (September 1997): 143–146.

7. See Mercer, *Valuing Financial Institutions*, Exhibit 14.7, 262–266.
8. Z. Christopher Mercer, "Adjusted Capitalization Rates for the Difference Between Net Income and Net Free Cash Flow," *Business Valuation Review* (December 1992): 201–207.
9. See Mercer, *Quantifying Marketability Discounts*, Revised Reprint, Chapter 1.
10. See *Estate of Jung v. Commissioner* for a discussion of this argument, 101 T.C. 412 (U.S. Tax Ct. 1993).
11. See Chapter 13 of this book, written by Michael W. Barad and Tara McDowell of Ibbotson Associates, for elements of this argument.
12. See Mercer, *Quantifying Marketability Discount*, Revised Reprint, Chapter 8.
13. Z. Christopher Mercer, "Quantifying Marketability Discount Modeling," *Wiley ValuSource* (software), 1-800-825-8763.

Appendix E

Iterative Process Using CAPM to Calculate the Cost of Equity Component of the Weighted Average Cost of Capital[1]

Harold G. Martin, Jr., MBA, CPA/ABV, ASA, CFE

Overview
Capital Asset Pricing Model and Beta
Solution—The Iterative Process
    Iteration
Iterative Process Using a Financial Spreadsheet Model
    Iteration 1
    Iteration 2
    Iteration 3
    Iteration 4
Summary
Additional Reading

OVERVIEW

In Chapter 7 Dr. Pratt presents an iterative process for computing the weighted average cost of capital (WACC) for a closely held company. In determining the WACC, the market values of the capital structure components—that is, debt and equity—are required to determine the relative weights of each component. However, this sets up a "catch-22" scenario:

- Our objective is to determine the market value of equity for the closely held company based on some unknown WACC.
- To determine the WACC, we must solve for an unknown market value of equity.

Chapter 7 presents an estimation technique, the iterative process, that provides a method for circumventing this problem. This appendix expands on the technique and considers the additional complexities introduced to the iterative process when the Capital Asset Pricing Model (CAPM) is used to calculate the equity component of WACC. Further, it illustrates how to implement the iterative process using a financial spreadsheet model.

CAPITAL ASSET PRICING MODEL AND BETA

Chapter 9 presents an overview of CAPM. The mathematical model for the expanded CAPM is expressed in Formula E.1:

Formula E.1

$$E(Ri) = Rf + B(RPm) + RPs + RPu$$

where:

$E(Ri)$ = Expected return on an individual security
Rf = Rate of return available on a risk-free security as of the valuation date
B = Beta
RPm = Risk premium for equities ($B \times RPm$ equals systematic risk)
RPs = Risk premium for size
RPu = Risk premium for specific company (unsystematic risk)

In determining the CAPM, betas for the subject company's industry (as provided in such publications as Ibbotson's *Cost of Capital Yearbook*) or guideline public companies typically are used as proxies to estimate the beta for the closely held company. However, as noted in Chapter 10, the public company betas are "levered" betas; that is, the betas reflect the amount of debt in the public company's capital structure. If the amount of leverage for the public company differs materially from that of the subject company being valued *and* it is assumed that the amount of debt in the subject company capital structure will not be adjusted, then the public company beta should be adjusted to remove the effect of the leverage. This may be accomplished by "unlevering" the public company beta to remove the effect of the leverage and "relevering" the beta to reflect the subject company's leverage.

To unlever the public company beta, this formula is used:

Formula E.2

$$Bu_i = B_{Li}/[1 + (1 - t_i)Wd_i/We_i]$$

where:

Bu_i = Beta unlevered for industry (or guideline companies)
B_{Li} = Beta levered for industry (or guideline companies)

t_i = Federal and state income tax rate for industry (or guideline companies)
Wd_i = Weight of interest-bearing debt in capital structure at market for industry (or guideline companies)
We_i = Weight of common equity in capital structure at market for industry (or guideline companies)

Once the public company beta is unlevered, then the beta may be relevered for the subject company using this formula:

Formula E.3

$$B_L = Bu_i\,[1 + (1 - t)Wd/We]$$

where:

B_L = Beta relevered for subject company
Bu_i = Beta unlevered for industry (or guideline companies)
t = Federal and state income tax rate for subject company
Wd = Weight of interest-bearing debt in capital structure for subject company estimated using iterative approach
We = Weight of common equity in capital structure for subject company estimated using iterative approach

In relevering the unlevered beta, we have introduced a third unknown: *We need to know the market value of the subject company's equity in order to determine the relative weights to be assigned to the subject company's debt and equity for the purpose of relevering the beta.*

SOLUTION—THE ITERATIVE PROCESS

Each of the following three calculations depends on a single unknown value—the market value of the subject company's equity:

1. Subject company's relevered beta
2. WACC
3. Market value of equity

We can solve each of these calculations by using the iterative process to estimate the market value of equity. The following example illustrates this methodology.[2] For purposes of illustration, we have used a capital structure consisting of common equity and debt. Further, in applying the income approach, we have used the capitalization of economic income methodology instead of the discounted economic income methodology to simplify the calculations. Our example is based on seven assumptions:

1. Book value of long-term interest-bearing debt: \$400,000
2. Book value of common equity: \$600,000
3. Interest rate for debt: 10%
4. Tax rate (combined federal and state): 40%
5. Projected net cash flow to invested capital for year following valuation date: \$250,000
6. Estimated annual compounded long-term growth rate for net cash flow to invested capital: 5%
7. Cost of capital variables:

a. Risk-free rate:	6.28%
b. Equity risk premium:	8.10%
c. Beta—unlevered for industry:	1.12
d. Risk premium for size:	4.63%
e. Specific (unsystematic) risk:	2.00%

Iteration

Step 1. Inputs for Debt and Equity

For this iteration, the book values of the subject company's debt and equity will be used as proxies for the market values for purposes of calculating the weighting of the capital components of WACC:

Capital Component	Estimated Market Value	Percent of Capital[3]
Debt	\$400,000	0.40
Equity	600,000	0.60
Total	\$1,000,000	1.00

Step 2. Calculation of Relevered Beta for Subject Company

The next step in the iterative process is to estimate the beta for the subject company. This involves relevering the unlevered industry beta. To calculate the relevered beta for the subject company, we substitute the unlevered industry beta, the subject company tax rate, and the initial book values for debt and equity into Formula E.4:

Formula E.4

$$B_L = Bu_i[1 + (1 - t)Wd/We]$$
$$B_L = 1.12[1 + (1 - 0.4)400{,}000/600{,}000]$$
$$B_L = 1.57$$

Step 3. Estimation of Cost of Equity Using the Capital Asset Pricing Model

Next we calculate an estimate of cost of equity using CAPM. We substitute the known values for the CAPM variables as well as the relevered beta derived above, into Formula E.5:

Formula E.5

$$E(Ri) = Rf + B(RPm) + RPs + RPu$$
$$E(Ri) = 0.0628 + 1.57(0.0810) + 0.0463 + 0.02$$
$$E(Ri) = 0.2563$$

Step 4. Estimation of Weighted Average Cost of Capital

After calculating the initial estimate of the cost of equity, we next estimate the WACC. Using the same book values of the subject company's debt and equity as the weights and the cost of equity calculated using CAPM into Formula E.6, we calculate the WACC as:

Formula E.6

$$WACC = [ke \times We] + [kd(1 - t) \times Wd]$$
$$WACC = [0.2563 \times 0.6] + [0.1(1 - 0.4) \times 0.4]$$
$$WACC = 0.1778$$

As the WACC represents a discount rate for invested capital, we subtract the long-term growth rate of 5% to derive the capitalization rate of 12.78%.

Step 5. Capitalized Economic Income Method (Invested Capital Model)

Finally, we use Formula E.7 to estimate the market value of invested capital and subtract the value of the debt to derive the estimated value of equity:

Formula E.7

$$PV = NCF_1/c$$

where:

PV = Present value
NCF_1 = Net cash flow expected in the first period immediately following the valuation date
c = Capitalization rate

$$PV = NCF_1/c$$
$$PV = 250{,}000/0.1278$$
$$PV = 1{,}956{,}182$$

The market value of invested capital, $1,956,182, less the value of debt, $400,000, equals the estimated market value of equity, $1,556,182. However, this estimate of the market value of equity is materially different from the book value of $600,000 we used initially as a proxy and consequently results in very different market weights of debt and equity as indicated in the following chart:

	Estimated Market Value	Percent of Capital
Debt	$400,000	0.2045
Equity	1,556,182	0.7955
Total	$1,956,182	1.0000

Therefore, we must repeat the above calculations substituting the book value of equity used in step 1, $600,000, with the calculated value of equity, $1,556,182. We continue to recalculate the value of equity until the value of the equity input in step 1 (the value used to estimate the market weights in steps 2 and 4) equals the calculated equity in step 5.

ITERATIVE PROCESS USING A FINANCIAL SPREADSHEET MODEL

While we could repeat each of these calculations manually, the iterative process may be implemented more easily using a financial spreadsheet application and linking the cells containing the unknown weight of the equity value we are seeking to determine. The following illustration of a spreadsheet model is based on the previous example. This model is built in Microsoft Excel®, and all formulas are presented using Excel definitions. Note that the version of the model presented requires a user to manually input the estimated market value of equity for each iteration. This presentation is useful in illustrating how the iterative process is performed. However, advanced users of Excel may wish to consider using the "Solver," an Excel add-on tool, to calculate the value automatically.

Iteration 1

Worksheet 1.1 Inputs: Estimates of Debt and Equity

As previously discussed, the estimated market values of debt (C7) and equity (C8) for Iteration 1 are based on book values and serve as our initial inputs to the model. The relative weights of debt (D7) and equity (D8) are calculated.

A	B	C	D
3		Estimated	Percent
4		Market	of
5		Value	Capital
6			*w*
7	Long-term interest-bearing debt	400,000	0.4000
8	Equity	600,000	0.6000
9	Total capital	1,000,000	1.0000

Worksheet 1.2 Calculation of Relevered Beta for Subject Company

Worksheet 1.2 presents the calculation of the relevered beta. The estimated market weights of debt (C17) and equity (C18) are linked to the values in Worksheet 1.1 (D7 and D8, respectively). The tax rate (D17) and beta (E19) are manual inputs to the model. The relevered beta (F19) is calculated using Formula E.4.

A	B	C	D	E	F
13		Percent		Industry	Subject
14		of		(or guideline co.)	Company
15		Capital	Tax Rate	Unlevered Beta	Levered Beta
16		W	t	Bu_i	B_L
17	Debt	0.4000	0.40		
18	Equity	0.6000	N/A		
19	Total	1.0000		1.12	1.57

Cell formulas:

A	B	C	D	E	F
13		Percent		Industry	Subject
14		of		(or guideline co.)	Company
15		Capital	Tax Rate	Unlevered Beta	Levered Beta
16		W	t	Bu_i	B_L
17	Debt	=D7	0.4		
18	Equity	=D8	N/A		
19	Total	=SUM(C17:C18)		1.12	=ROUND((E19*(1+((1–D17)*C17)/C18)),2)

Worksheet 1.3 Estimation of Cost of Equity Using CAPM

Worksheet 1.3 presents the CAPM based on Formula E.5. All CAPM variables are manual inputs, except for beta (C26), which is linked to Worksheet 1.2 (F19). The cost of equity is calculated (D30).

A	B	C	D
23	Risk-free rate (*Rf*)		0.0628
24	Systematic risk		
25	Equity risk premium (*RPm*)	0.0810	
26	× Beta (*B*)	1.57	
27	Systematic risk		0.1272
28	Risk premium for size (*RPs*)		0.0463
29	Specific (unsystematic) risk (*RPu*)		0.0200
30	Cost of equity (*Ke*)		0.2563

Cell formulas:

A	B	C	D
23	Risk-free rate (*Rf*)		0.0628
24	Systematic risk		
25	Equity risk premium (*RPm*)	0.081	
26	× Beta (*B*)	=F19	
27	Systematic risk		=ROUND((C25*C26),4)
28	Risk premium for size (*RPs*)		0.0463
29	Specific (unsystematic) risk (*RPu*)		0.02
30	Cost of equity (*Ke*)		=SUM(D23:D29)

Worksheet 1.4 Estimation of WACC

Worksheet 1.4 calculates the WACC based on Formula E.6. As with Worksheet 1.2, the cells containing the estimated market weights of debt (C39) and equity (C40) are linked to the values in Worksheet 1.1 (D7 and D8, respectively). The cost of debt (D39) and tax rate (E39) are manual inputs and the tax-affected rate is calculated (F39). The cost of equity (D40) is linked to Worksheet 1.3 (D30). The weighted average cost of debt (G39) and equity (G40) are calculated and summed to derive the WACC (G41). The long-term growth rate (G45), a manual input, is deducted from the WACC (G44) to derive the capitalization rate (G46).

A	B	C	D	E	F	G
34		Percent	Cost of Capital			Weighted
35		of			Tax-affected	Average
36		Capital	Rate	Tax Rate	Rate	Cost
37		*W*	*k*	*t*	*k*[1–*t*]	WACC
38	**Calculation of WACC**					
39	Debt	0.4000	0.10	0.40	0.0600	0.0240
40	Equity	0.6000	0.2563	N/A	0.2563	0.1538
41	Total	1.0000				0.1778
42						
43	**Calculation of Capitalization rate**					
44	Discount rate for net cash flow					0.1778
45	Less long-term average growth rate					0.0500
46	Capitalization rate for net cash flow					0.1278

Cell formulas:

A	B	C	D	E	F	G
34		Percent	Cost of Capital			Weighted
35		of			Tax-affected	Average
36		Capital	Rate	Tax Rate	Rate	Cost
37		*W*	*k*	*t*	*k*[1–*t*]	WACC
38	**Calculation of WACC**					
39	Debt	=D7	0.1	0.4	=ROUND((D39*(1–E39)),4)	=ROUND((C39*F39),4)
40	Equity	=D8	=D30	N/A	=+D40	=ROUND((C40*F40),4)
41	Total	=SUM(C39:C40)				=ROUND(SUM(G39:G40),4)
42						
43	**Calculation of Capitalization rate**					
44	Discount rate for net cash flow					=ROUND(G41,4)
45	Less long-term average growth rate					0.05
46	Capitalization rate for net cash flow					=(G44–G45)

Worksheet 1.5 Capitalized Economic Income Method (Invested Capital Model)

Worksheet 1.5 presents the calculation of the value of invested capital using the capitalized economic income method based on Formula E.7, and also presents a calculation of the value of the equity. The net cash flow to invested capital (C50), a manual input, is multiplied by the capitalization rate (C51) linked to Worksheet 1.4 (G46) to derive the market value of invested capital (C52). From this amount, the market value of debt (C53), linked to Worksheet 1.1 (C7), is subtracted to derive the estimated market value of equity (C54).

A	B	C
50	Adjusted net cash flow to invested capital	250,000
51	Capitalization rate	0.1278
52	Indicated value of 100% of the business enterprise	1,956,182
53	Less interest-bearing debt	400,000
54	**Indicated value of a 100% Marketable Equity Interest**	1,556,182

Cell formulas:

A	B	C
50	Adjusted net cash flow to invested capital	250,000
51	Capitalization rate	=G46
52	Indicated value of 100% of the business enterprise	=ROUND(C50/C51,0)
53	Less interest-bearing debt	=C7
54	**Indicated value of a 100% Marketable Equity Interest**	=C52–C53

As the value of the calculated equity (C54), $1,556,182, is not equal to the initial estimate of equity input in Worksheet 1.1 (C8), $600,000, (the value used to estimate the market weights in Worksheets 1.3 and 1.4) the market value of equity must be estimated again and the calculations repeated.

Iteration 2

Worksheet 2.1 Inputs: Estimates of Debt and Equity

In the second iteration, the value of equity in Worksheet 2.1 (C8) is set equal to the value derived in Iteration 1, Worksheet 1.5 (C54). The relative weights of debt (D7) and equity (D8) are then recalculated. The model then automatically performs the calculations in Worksheets 2.2, 2.3, 2.4, and 2.5.

A	B	C	D
3		Estimated	Percent
4		Market	of
5		Value	Capital
6			*W*
7	Long-term interest-bearing debt	400,000	0.2045
8	Equity	1,556,182	0.7955
9	Total capital	1,956,182	1.0000

Worksheet 2.2 Calculation of Relevered Beta for Subject Company

A	B	C	D	E	F
13		Percent		Industry	Subject
14		of		(or guideline co.)	Company
15		Capital	Tax Rate	Unlevered Beta	Levered Beta
16		*W*	*t*	Bu_i	B_L
17	Debt	0.2045	0.40		
18	Equity	0.7955	N/A		
19	Total	1.0000		1.12	1.29

Worksheet 2.3 Estimation of Cost of Equity Using CAPM

A	B	C	D
23	Risk-free rate (*Rf*)		0.0628
24	Systematic risk		
25	Equity risk premium (*RPm*)	0.0810	
26	× Beta (*B*)	1.29	
27	Systematic risk		0.1045
28	Risk premium for size (*RPs*)		0.0463
29	Specific (unsystematic) risk (*RPu*)		0.0200
30	Cost of equity (*Ke*)		0.2336

Worksheet 2.4 Estimation of WACC

A	B	C	D	E	F	G
34		Percent	Cost of Capital			Weighted
35		of			Tax-affected	Average
36		Capital	Rate	Tax Rate	Rate	Cost
37		*W*	*k*	*t*	*k*[1–*t*]	WACC
38	**Calculation of WACC**					
39	Debt	0.2045	0.10	0.40	0.0600	0.0123
40	Equity	0.7955	0.2336	N/A	0.2336	0.1858
41	Total	1.0000				0.1981
42						
43	**Calculation of Capitalization rate**					
44	Discount rate for net cash flow					0.1981
45	Less long-term average growth rate					0.0500
46	Capitalization rate for net cash flow					0.1481

Worksheet 2.5 Capitalized Economic Income Method (Invested Capital Model)

A	B	C
50	Adjusted net cash flow to invested capital	250,000
51	Capitalization rate	0.1481
52	Indicated value of 100% of the business enterprise	1,688,049
53	Less interest-bearing debt	400,000
54	**Indicated value of a 100% Marketable Equity Interest**	1,288,049

As the value of the calculated equity (C54), \$1,288,049, is not equal to the estimate of equity input in Worksheet 2.1 (C8), \$1,556,182, the market value of equity must be estimated again and the calculations repeated.

Iteration 3

Worksheet 3.1 Inputs: Estimates of Debt and Equity

In Iteration 3, the value of equity in Worksheet 3.1 (C8) is set equal to the value derived in Iteration 2, Worksheet 2.5 (C54). The relative weights of debt (D7) and equity

(D8) are then recalculated. The model then automatically performs the calculations in Worksheets 3.2, 3.3, 3.4, and 3.5.

A	B	C	D
3		Estimated	Percent
4		Market	of
5		Value	Capital
6			*W*
7	Long-term interest-bearing debt	400,000	0.2370
8	Equity	1,288,049	0.7630
9	Total capital	1,688,049	1.0000

Worksheet 3.2 Calculation of Relevered Beta for Subject Company

A	B	C	D	E	F
13		Percent		Industry	Subject
14		of		(or guideline co.)	Company
15		Capital	Tax Rate	Unlevered Beta	Levered Beta
16		*W*	*t*	Bu_i	B_L
17	Debt	0.2370	0.40		
18	Equity	0.7630	N/A		
19	Total	1.0000		1.12	1.33

Worksheet 3.3 Estimation of Cost of Equity Using CAPM

A	B	C	D
23	Risk-free rate (*Rf*)		0.0628
24	Systematic risk		
25	Equity risk premium (*RPm*)	0.0810	
26	× Beta (*B*)	1.33	
27	Systematic risk		0.1077
28	Risk premium for size (*RPs*)		0.0463
29	Specific (unsystematic) risk (*RPu*)		0.0200
30	Cost of equity (*Ke*)		0.2368

Worksheet 3.4 Estimation of WACC

A	B	C	D	E	F	G
34		Percent	Cost of Capital			Weighted
35		of			Tax-affected	Average
36		Capital	Rate	Tax Rate	Rate	Cost
37		*W*	*k*	*t*	*k*[1–*t*]	WACC
38	**Calculation of WACC**					
39	Debt	0.2370	0.10	0.40	0.0600	0.0142
40	Equity	0.7630	0.2368	N/A	0.2368	0.1807
41	Total	1.0000				0.1949
42						
43	**Calculation of Capitalization rate**					
44	Discount rate for net cash flow					0.1949
45	Less long-term average growth rate					0.0500
46	Capitalization rate for net cash flow					0.1449

Worksheet 3.5 Capitalized Economic Income Method (Invested Capital Model)

A	B	C
50	Adjusted net cash flow to invested capital	250,000
51	Capitalization rate	0.1449
52	Indicated value of 100% of the business enterprise	1,725,328
53	Less interest-bearing debt	400,000
54	**Indicated value of a 100% Marketable Equity Interest**	1,325,328

As the value of the calculated equity (C54), $1,325,328, is not equal to the estimate of equity input in Worksheet 1 (C8), $1,288,049, the market value of equity must be estimated again and the calculations repeated.

Iteration 4

Worksheet 4.1 Inputs: Estimates of Debt and Equity

In the fourth iteration, the value of equity (C8) is set equal to the value derived in Iteration 3, Worksheet 3.5 (C54). The relative weights of debt (D7) and equity

(D8) are then recalculated. The model then automatically performs the calculations in Worksheets 4.2, 4.3, 4.4, and 4.5.

A	B	C	D
3		Estimated	Percent
4		Market	of
5		Value	Capital
6			*W*
7	Long-term interest-bearing debt	400,000	0.2318
8	Equity	1,325,328	0.7682
9	Total capital	1,725,328	1.0000

Worksheet 4.2 Calculation of Relevered Beta for Subject Company

A	B	C	D	E	F
13		Percent		Industry	Subject
14		of		(or guideline co.)	Company
15		Capital	Tax Rate	Unlevered Beta	Levered Beta
16		*W*	*t*	Bu_i	B_L
17	Debt	0.2318	0.40		
18	Equity	0.7682	N/A		
19	Total	1.0000		1.12	1.32

Worksheet 4.3 Estimation of Cost of Equity Using CAPM

A	B	C	D
23	Risk-free rate (*Rf*)		0.0628
24	Systematic risk		
25	Equity risk premium (*RPm*)	0.0810	
26	× Beta (*B*)	1.32	
27	Systematic risk		0.1069
28	Risk premium for size (*RPs*)		0.0463
29	Specific (unsystematic) risk (*RPu*)		0.0200
30	Cost of equity (*Ke*)		0.2360

Worksheet 4.4 Estimation of WACC

A	B	C	D	E	F	G
34		Percent	Cost of Capital			Weighted
35		of			Tax-affected	Average
36		Capital	Rate	Tax Rate	Rate	Cost
37		*W*	*k*	*t*	*k*[1–*t*]	WACC
38	**Calculation of WACC**					
39	Debt	0.2318	0.10	0.40	0.0600	0.0139
40	Equity	0.7682	0.2360	N/A	0.2360	0.1813
41	Total	1.0000				0.1952
42						
43	**Calculation of Capitalization rate**					
44	Discount rate for net cash flow					0.1952
45	Less long-term average growth rate					0.0500
46	Capitalization rate for net cash flow					0.1452

Worksheet 4.5 Capitalized Economic Income Method (Invested Capital Model)

A	B	C
50	Adjusted net cash flow to invested capital	250,000
51	Capitalization rate	0.1452
52	Indicated value of 100% of the business enterprise	1,721,763
53	Less interest-bearing debt	400,000
54	**Indicated value of a 100% Marketable Equity Interest**	1,321,763

As the value of the calculated equity (C54), $1,321,763, is approximately equal to the estimate of equity input in Worksheet 4.1 (C8), $1,325,328, we conclude that the market value of equity is approximately $1,320,000 in round numbers. We could continue the iterations until the two equity values equaled one another, but for purposes of illustration, the calculated value is considered sufficient.

SUMMARY

This appendix has expanded on the iterative process presented in Chapter 7 to consider the additional complexities when the CAPM is used to calculate the equity component of WACC. Further, it has provided an example illustrating the use of a financial spreadsheet model to perform the calculations required for the iterative process.

Notes

1. In developing the model presented here, I have received invaluable guidance from professional colleagues and resources such as the first edition of *Cost of Capital*. In particular, I wish to thank James R. Hitchner of Phillips Hitchner in Atlanta, Georgia, for sparking my initial interest in the invested capital valuation methodology, and Mark L. Zyla, also of Phillips Hitchner in Atlanta, Georgia, and Michael J. Mattson of The Financial Valuation Group in Chicago, Illinois, for their suggestions and critique of the model. Any errors relating to its application are solely my own. This model was first introduced in a presentation entitled "Cost of Capital" at the American Institute of Certified Public Accountants National Business Valuation Conference, December 4, 2001.
2. The example used in the illustration is based on a presentation entitled "Cost of Capital" presented at the American Institute of Certified Public Accountants National Business Valuation Conference, December 4, 2001. I have modified the example to mirror the example presented in Chapter 7, with the exception that I have expanded on it to illustrate the derivation of a relevered beta and the use of CAPM in deriving the cost of equity component of the WACC.
3. Percentages are expressed as decimal equivalents.

ADDITIONAL READING

Abrams, Jay B. *Quantitative Business Valuation: A Mathematical Approach for Today's Professionals*. New York: McGraw-Hill, 2001.

Adams, David P., et al. *Fundamentals of Business Valuation: Part I*. Lewisville, TX: American Institute of Certified Public Accountants, 2001.

Annin, Michael, and Dominic Falaschetti. *Stocks, Bonds, Bills, and Inflation Valuation Edition 2001 Yearbook*. Chicago: Ibbotson Associates, 2001.

Bishop, David M., and Frank C. Evans. "Avoiding a Common Error in Calculating the Weighted Average Cost of Capital." *CPA Expert*. New York: American Institute of Certified Public Accountants (fall 1997).

Evans, Frank C., and Kelly L. Strimbu. "Debt and Equity Weightings in WACC." *CPA Expert*. New York: American Institute of Certified Public Accountants (fall 1998).

Hitchner, James R., et al, *Financial Valuation: Application and Models*. New York: John Wiley & Sons, Inc., forthcoming.

Ibbotson Associates Cost of Capital 2001 Yearbook. Chicago: Ibbotson Associates, 2001.

Martin, Harold G., Jr. "Cost of Capital." Joint presentation made with Ronald L. Seigneur at the American Institute of Certified Public Accountants National Business Valuation Conference, Las Vegas, NV, December 4, 2001.

Pratt, Shannon P., et al. *Valuing a Business: The Analysis and Appraisal of Closely Held Companies*, 4th ed. New York: McGraw-Hill, 2000.

Appendix F

International Glossary of Business Valuation Terms

The second edition of the International Glossary of Business Valuation Terms is now out. It contains definitions of 38 additional terms (not in the original edition and denoted here by an asterisk). Definitions of only two terms are changes from the original edition, and the changes are only to clarify the wording, not changing the meaning. The glossary is a joint effort of the American Institute of Certified Public Accountants, the American Society of Appraisers, the Canadian Institute of Chartered Business Valuators, the National Association of Certified Valuation Analysts, and the Institute of Business Appraisers.

—Shannon Pratt

To enhance and sustain the quality of business valuations for the benefit of the profession and its clientele, the below identified societies and organizations have adopted the definitions for the terms included in this glossary.

The performance of business valuation services requires a high degree of skill and imposes upon the valuation professional a duty to communicate the valuation process and conclusion in a manner that is clear and not misleading. This duty is advanced through the use of terms whose meanings are clearly established and consistently applied throughout the profession.

If, in the opinion of the business valuation professional, one or more of these terms needs to be used in a manner that materially departs from the enclosed definitions, it is recommended that the term be defined as used within that valuation engagement.

This glossary has been developed to provide guidance to business valuation practitioners by further memorializing the body of knowledge that constitutes the competent and careful determination of value and, more particularly, the communication of how that value was determined.

Departure from this glossary is not intended to provide a basis for civil liability and should not be presumed to create evidence that any duty has been breached.

*__Adjusted Book Value Method__ A method within the asset approach whereby all assets and liabilities (including off-balance sheet, intangible, and contingent) are adjusted to their fair market values ***[NOTE: In Canada on a going concern basis.]***

*__Adjusted Net Asset Method__ *See* **Adjusted Book Value Method**.

Appraisal *See* **Valuation**.

Appraisal Approach *See* **Valuation Approach**.

Appraisal Date *See* **Valuation Date**.

Appraisal Method *See* **Valuation Method**.

Appraisal Procedure *See* **Valuation Procedure**.

***Arbitrage Pricing Theory** A multivariate model for estimating the cost of equity capital, which incorporates several systematic risk factors.

Asset (Asset-Based) Approach A general way of determining a value indication of a business, business ownership interest, or security using one or more methods based on the value of the assets net of liabilities.

Beta A measure of systematic risk of a stock; the tendency of a stock's price to correlate with changes in a specific index.

Blockage Discount An amount or percentage deducted from the current market price of a publicly traded stock to reflect the decrease in the per share value of a block of stock that is of a size that could not be sold in a reasonable period of time given normal trading volume.

***Book Value** *See* **Net Book Value**.

Business *See* **Business Enterprise**.

Business Enterprise A commercial, industrial, service, or investment entity (or a combination thereof) pursuing an economic activity.

***Business Risk** The degree of uncertainty of realizing expected future returns of the business resulting from factors other than financial leverage. *See* **Financial Risk**.

Business Valuation The act or process of determining the value of a business enterprise or ownership interest therein.

Capital Asset Pricing Model (CAPM) A model in which the cost of capital for any stock or portfolio of stocks equals a risk-free rate plus a risk premium that is proportionate to the systematic risk of the stock or portfolio.

Capitalization A conversion of a single period of economic benefits into value.

Capitalization Factor Any multiple or divisor used to convert anticipated economic benefits of a single period into value.

***Capitalization of Earnings Method** A method within the income approach whereby economic benefits for a representative single period are converted to value through division by a capitalization rate.

Capitalization Rate Any divisor (usually expressed as a percentage) used to convert anticipated economic benefits of a single period into value.

Capital Structure The composition of the invested capital of a business enterprise, the mix of debt and equity financing.

Cash Flow Cash that is generated over a period of time by an asset, group of assets, or business enterprise. It may be used in a general sense to encompass various levels of specifically defined cash flows. When the term is used, it should be supplemented by a qualifier (for example, "discretionary" or "operating") and a specific definition in the given valuation context.

***Common Size Statements** Financial statements in which each line is expressed as a percentage of the total. On the balance sheet, each line item is shown as a percentage of total assets, and on the income statement, each item is expressed as a percentage of sales.

Control The power to direct the management and policies of a business enterprise.

Control Premium An amount or a percentage by which the pro rata value of a controlling interest exceeds the pro rata value of a noncontrolling interest in a business enterprise, to reflect the power of control.

Cost Approach A general way of determining a value indication of an individual asset by quantifying the amount of money required to replace the future service capability of that asset.

Cost of Capital The expected rate of return that the market requires in order to attract funds to a particular investment.

*__Debt-Free__ ***We discourage the use of this term.*** *See* **Invested Capital**.

Discount for Lack of Control An amount or percentage deducted from the pro rata share of value of 100% of an equity interest in a business to reflect the absence of some or all of the powers of control.

*__Discount for Lack of Marketability__ An amount or percentage deducted from the value of an ownership interest to reflect the relative absence of marketability.

Discount for Lack of Voting Rights An amount or percentage deducted from the per share value of a minority interest voting share to reflect the absence of voting rights.

Discount Rate A rate of return used to convert a future monetary sum into present value.

*__Discounted Cash Flow Method__ A method within the income approach whereby the present value of future expected net cash flows is calculated using a discount rate.

*__Discounted Future Earnings Method__ A method within the income approach whereby the present value of future expected economic benefits is calculated using a discount rate.

*__Economic Benefits__ Inflows such as revenues, net income, net cash flows, etc.

Economic Life The period of time over which property may generate economic benefits.

Effective Date *See* **Valuation Date**.

Enterprise *See* **Business Enterprise**.

*__Equity__ The owner's interest in property after deduction of all liabilities.

Equity Net Cash Flows Those cash flows available to pay out to equity holders (in the form of dividends) after funding operations of the business enterprise, making necessary capital investments, and increasing or decreasing debt financing.

Equity Risk Premium A rate of return added to a risk-free rate to reflect the additional risk of equity instruments over risk-free instruments (a component of the cost of equity capital or equity discount rate).

Excess Earnings That amount of anticipated economic benefits that exceeds an appropriate rate of return on the value of a selected asset base (often net tangible assets) used to generate those anticipated economic benefits.

Excess Earnings Method A specific way of determining a value indication of a business, business ownership interest, or security determined as the sum of a) the value of the assets derived by capitalizing excess earnings and b) the value of the selected asset base. Also frequently used to value intangible assets. *See* **Excess Earnings**.

Fair Market Value The price, expressed in terms of cash equivalents, at which property would change hands between a hypothetical willing and able buyer and a hypothetical willing and able seller, acting at arms length in an open and unrestricted market, when neither is under compulsion to buy or sell and when both have reasonable knowledge of the relevant facts. ***[NOTE: In Canada, the term "price" should be replaced with the term "highest price."]***

*__Fairness Opinion__ An opinion as to whether or not the consideration in a transaction is fair from a financial point of view.

*__Financial Risk__ The degree of uncertainty of realizing expected future returns of the business resulting from financial leverage. *See* **Business Risk**.

Forced Liquidation Value Liquidation value, at which the asset or assets are sold as quickly as possible, such as at an auction.

*__Free Cash Flow__ ***We discourage the use of this term.*** *See* **Net Cash Flow**.

Going Concern An ongoing operating business enterprise.

Going Concern Value The value of a business enterprise that is expected to continue to operate into the future. The intangible elements of Going Concern Value result from factors such as having a trained work force, an operational plant, and the necessary licenses, systems, and procedures in place.

Goodwill That intangible asset arising as a result of name, reputation, customer loyalty, location, products, and similar factors not separately identified.

Goodwill Value The value attributable to goodwill.

*__Guideline Public Company Method__ A method within the market approach whereby market multiples are derived from market prices of stocks of companies that are engaged in the same or similar lines of business, and that are actively traded on a free and open market.

Income (Income-based) Approach A general way of determining a value indication of a business, business ownership interest, security, or intangible asset using one or more methods that convert anticipated economic benefits into a present single amount.

Intangible Assets Nonphysical assets such as franchises, trademarks, patents, copyrights, goodwill, equities, mineral rights, securities and contracts (as distinguished from physical assets) that grant rights and privileges, and have value for the owner.

*__Internal Rate of Return__ A discount rate at which the present value of the future cash flows of the investment equals the cost of the investment.

*__Intrinsic Value__ The value that an investor considers, on the basis of an evaluation of available facts, to be the "true" or "real" value that will become the market value when other investors reach the same conclusion. When the term applies to options, it is the difference between the exercise price or strike price of an option and the market value of the underlying security.

Invested Capital The sum of equity and debt in a business enterprise. Debt is typically (a) all interest-bearing debt or (b) long-term interest-bearing debt. When the term is used, it should be supplemented by a specific definition in the given valuation context.

Invested Capital Net Cash Flows Those cash flows available to pay out to equity holders (in the form of dividends) and debt investors (in the form of principal and interest) after funding operations of the business enterprise and making necessary capital investments.

Investment Risk The degree of uncertainty as to the realization of expected returns.

Investment Value The value to a particular investor based on individual investment requirements and expectations. ***[NOTE: In Canada, the term used is "Value to the Owner."]***

Key Person Discount An amount or percentage deducted from the value of an ownership interest to reflect the reduction in value resulting from the actual or potential loss of a key person in a business enterprise.

Levered Beta The beta reflecting a capital structure that includes debt.

*__Limited Appraisal__ The act or process of determining the value of a business, business ownership interest, security, or intangible asset with limitations in analyses, procedures, or scope.

Liquidity The ability to quickly convert property to cash or pay a liability.

Liquidation Value The net amount that would be realized if the business is terminated and the assets are sold piecemeal. Liquidation can be either "orderly" or "forced."

Majority Control The degree of control provided by a majority position.

Majority Interest An ownership interest greater than 50% of the voting interest in a business enterprise.

Market (Market-Based) Approach A general way of determining a value indication of a business, business ownership interest, security, or intangible asset by using one or more methods that compare the subject to similar businesses, business ownership interests, securities, or intangible assets that have been sold.

*__Market Capitalization of Equity__ The share price of a publicly traded stock multiplied by the number of shares outstanding.

*__Market Capitalization of Invested Capital__ The market capitalization of equity plus the market value of the debt component of invested capital.

*__Market Multiple__ The market value of a company's stock or invested capital divided by a company measure (such as economic benefits, number of customers).

Marketability The ability to quickly convert property to cash at minimal cost.

Marketability Discount *See* **Discount for Lack of Marketability**.

***Merger and Acquisition Method** A method within the market approach whereby pricing multiples are derived from transactions of significant interests in companies engaged in the same or similar lines of business.

***Midyear Discounting** A convention used in the Discounted Future Earnings Method that reflects economic benefits being generated at midyear, approximating the effect of economic benefits being generated evenly throughout the year.

Minority Discount A discount for lack of control applicable to a minority interest.

Minority Interest An ownership interest less than 50% of the voting interest in a business enterprise.

***Multiple** The inverse of the capitalization rate.

Net Book Value With respect to a business enterprise, the difference between total assets (net of accumulated depreciation, depletion, and amortization) and total liabilities as they appear on the balance sheet (synonymous with Shareholder's Equity). With respect to a specific asset, the capitalized cost less accumulated amortization or depreciation as it appears on the books of account of the business enterprise.

Net Cash Flows When the term is used, it should be supplemented by a qualifier. *See* **Equity Net Cash Flows** and **Invested Capital Net Cash Flows**.

***Net Present Value** The value, as of a specified date, of future cash inflows less all cash outflows (including the cost of investment) calculated using an appropriate discount rate.

Net Tangible Asset Value The value of the business enterprise's tangible assets (excluding excess assets and nonoperating assets) minus the value of its liabilities.

Nonoperating Assets Assets not necessary to ongoing operations of the business enterprise. ***[NOTE: In Canada, the term used is "Redundant Assets."]***

***Normalized Earnings** Economic benefits adjusted for nonrecurring, noneconomic, or other unusual items to eliminate anomalies and/or facilitate comparisons.

***Normalized Financial Statements** Financial statements adjusted for nonoperating assets and liabilities and/or for nonrecurring, noneconomic, or other unusual items to eliminate anomalies and/or facilitate comparisons.

Orderly Liquidation Value Liquidation value at which the asset or assets are sold over a reasonable period of time to maximize proceeds received.

Premise of Value An assumption regarding the most likely set of transactional circumstances that may be applicable to the subject valuation; e.g., going concern, liquidation.

***Present Value** The value, as of a specified date, of future economic benefits and/or proceeds from sale, calculated using an appropriate discount rate.

Portfolio Discount An amount or percentage deducted from the value of a business enterprise to reflect the fact that it owns dissimilar operations or assets that do not fit well together.

***Price/Earnings Multiple** The price of a share of stock divided by its earnings per share.

Rate of Return An amount of income (loss) and/or change in value realized or anticipated on an investment, expressed as a percentage of that investment.

Redundant Assets *See* **Nonoperating Assets**.

Report Date The date conclusions are transmitted to the client.

Replacement Cost New The current cost of a similar new property having the nearest equivalent utility to the property being valued.

Reproduction Cost New The current cost of an identical new property.

***Required Rate of Return** The minimum rate of return acceptable by investors before they will commit money to an investment at a given level of risk.

Residual Value The value as of the end of the discrete projection period in a discounted future earnings model.

***Return on Equity** The amount, expressed as a percentage, earned on a company's common equity for a given period.

***Return on Investment** *See* **Return on Invested Capital** and **Return on Equity**.

***Return on Invested Capital** The amount, expressed as a percentage, earned on a company's total capital for a given period.

Risk-Free Rate The rate of return available in the market on an investment free of default risk.

Risk Premium A rate of return added to a risk-free rate to reflect risk.

Rule of Thumb A mathematical formula developed from the relationship between price and certain variables based on experience, observation, hearsay, or a combination of these; usually industry specific.

Special Interest Purchasers Acquirers who believe they can enjoy post-acquisition economies of scale, synergies, or strategic advantages by combining the acquired business interest with their own.

Standard of Value The identification of the type of value being used in a specific engagement; e.g., fair market value, fair value, investment value.

Sustaining Capital Reinvestment The periodic capital outlay required to maintain operations at existing levels, net of the tax shield available from such outlays.

Systematic Risk The risk that is common to all risky securities and cannot be eliminated through diversification. The measure of systematic risk in stocks is the beta coefficient.

***Tangible Assets** Physical assets (such as cash, accounts receivable, inventory, property, plant and equipment, etc.).

Terminal Value *See* **Residual Value**.

***Transaction Method** *See* **Merger and Acquisition Method**.

Unlevered Beta The beta reflecting a capital structure without debt.

Unsystematic Risk The portion of total risk specific to an individual security that can be avoided through diversification.

Valuation The act or process of determining the value of a business, business ownership interest, security, or intangible asset.

Valuation Approach A general way of determining a value indication of a business, business ownership interest, security, or intangible asset using one or more valuation methods.

Valuation Date The specific point in time as of which the valuator's opinion of value applies (also referred to as "Effective Date" or "Appraisal Date").

Valuation Method Within approaches, a specific way to determine value.

Valuation Procedure The act, manner, and technique of performing the steps of an appraisal method.

Valuation Ratio A fraction in which a value or price serves as the numerator and financial, operating, or physical data serve as the denominator.

Value to the Owner ***[NOTE: In Canada, see Investment Value.]***

*__Voting Control__ *De jure* control of a business enterprise.

Weighted Average Cost of Capital (WACC) The cost of capital (discount rate) determined by the weighted average, at market value, of the cost of all financing sources in the business enterprise's capital structure.

Appendix G

Converting After-tax Discount Rates to Pretax Discount Rates[1]

In Chapter 4, "Discounting versus Capitalizing," we said that to convert an after-tax capitalization rate to a pretax capitalization rate, we divided the after-tax capitalization rate by 1 minus the tax rate. In a formula, this would be:

Formula G.1

$$c_{(pt)} = \frac{c}{1-t}$$

where:

c = capitalization rate (after tax)
$c_{(pt)}$ = capitalization rate (pretax)
t = tax rate

Assuming 10% after-tax capitalization rate an 30% taxes, this works out to:

Formula G.2

$$c_{(pt)} = \frac{0.10}{1-0.30} = \frac{0.10}{0.7} = .1429$$

So the pretax capitalization rate would be 14.29%.

But we said that this was *not* a discount rate. To convert a pretax capitalization rate to a discount rate, we have to add the estimated growth rate to the pretax capitalization rate. The formula for this would be:

Formula G.3

$$k_{(\mathrm{pt})} = c_{(pt)} + \mathrm{g}$$

where:

$k_{(pt)}$ = discount rate applicable to pretax cash flows
$c_{(pt)}$ = capitalization rate applicable to pretax cash flows
g = growth rate

If we estimated the 10% capitalization rate on after-tax cash flows by estimating a 15% discount rate less a 5% growth rate, substituting in Formula G.3, we have:

Formula G.4

$$k_{(pt)} = 14.29 + 5.00 = 19.29$$

So this gives us a 19.29% discount rate applicable to pretax cash flows.

If this is correct, discounting after-tax cash flows at 15% should derive the same answer as discounting pretax cash flows at 19.29%. To test this, we assumed for period 1 (the estimate for the pretax immediately following the valuation date), \$10,000 in pretax cash flow and a 30% tax rate, resulting in \$7,000 of after-tax cash flows. We tested the equivalency with a model consisting of two discrete forecast periods plus a terminal value.

First, discounting the after-tax cash flows at 15%, we have the following:

Formula G.5

$$\begin{array}{ccccccc}
 & \text{Period 1} & & \text{Period 2} & & \begin{array}{c}\text{Terminal value}\\ \text{(Period 3}\\ \text{and beyond)}\end{array} \\
\hline
 & \dfrac{\$7,000}{1.15} & + & \dfrac{\$7,000(1.05)}{(1.15)^2} & + & \dfrac{\dfrac{\$7,000(1.05)(1.05)}{0.15-0.05}}{(1.15)^2} \\
= & \dfrac{\$7,000}{1.15} & + & \dfrac{\$7,350}{1.3225} & + & \dfrac{\dfrac{\$7,350(1.05)}{0.10}}{1.3225} \\
= & \dfrac{\$7,000}{1.1929} & + & \dfrac{\$7,350}{1.4230} & + & \dfrac{\dfrac{\$7,717.50}{0.10}}{1.3225} \\
= & \dfrac{\$7,000}{1.15} & + & \dfrac{\$7,350}{1.3225} & + & \dfrac{\$77,175}{1.3225} \\
\cong & \$6,086.96 & + & \$5,557.66 & + & \$58,355.39 \\
\cong & \$70,000. & & & &
\end{array}$$

For discounting the pretax cash flows at 19.29%, we have the following:

Formula G.6

	Period 1		Period 2		Terminal value (Period 3 and beyond)
	$\frac{\$10,000}{1.1929}$	+	$\frac{\$10,000(1.05)}{(1.1929)^2}$	+	$\frac{\frac{\$10,000(1.05)(1.05)}{0.1929-0.05}}{(1.1929)^2}$
=	$\frac{\$10,000}{1.1929}$	+	$\frac{\$10,500}{1.4230}$	+	$\frac{\frac{\$10,000(1.05)}{0.1429}}{1.4230}$
=	$\frac{\$10,000}{1.1929}$	+	$\frac{\$10,500}{1.4230}$	+	$\frac{\frac{\$11,025}{0.1429}}{1.4230}$
=	$\frac{\$10,000}{1.1929}$	+	$\frac{\$10,500}{1.4230}$	+	$\frac{\$77,151.85}{1.4230}$
$\cong$	\$8,382.93	+	\$7,378.78	+	\$54,217.74
$\cong$	\$69,979.45				

(The difference between the above figure and $70,000 is due to rounding.)

To summarize, the steps in converting an after-tax discount rate to a pretax discount rate are as follows:

1. Convert the after-tax discount rate to an after-tax capitalization rate by subtracting the estimated growth rate.
2. Convert the after-tax capitalization rate to a pretax capitalization rate by dividing the after-tax capitalization rate by 1 minus the tax rate.
3. Convert the pretax capitalization rate to a pretax discount rate by adding the estimated growth rate to the pretax capitalization rate.

The strict validity of this conversion is subject to the following limiting assumptions:

1. The relationship between after-tax cash flows and pretax cash flows remains constant over time.
2. The growth rate is a long-term sustainable growth rate which remains constant over time.

Note

1. For an earlier exposition of this concept, see Mary Ann Lerch, "Pretax/Aftertax Conversion Formula for Capitalization Rates and Cash Flow Discount Rates," *Business Valuation Review* (March 1990):18–22.

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Multiple Choice:

1. For which of the following balance sheet components can cost of capital be measured?
   a. Common equity
   b. Preferred equity
   c. Long-term debt
   d. All of the above

2. Which of the following terms are properly used interchangeably?
   a. Discount rate, capitalization rate, and cost of capital
   b. Discount rate and capitalization rate, but not cost of capital
   c. Discount rate and cost of capital, but not capitalization rate
   d. Capitalization rate and cost of capital, but not discount rate

3. What is the measure of economic income preferred by most analysts for the income approach to valuation?
   a. Net income
   b. Net cash flow
   c. Pretax income
   d. Gross cash flow

4. Given the following:

Sales	$1,000
Cost of goods sold	600
Depreciation	50
Interest	30
Owners' compensation	100
Other expenses	200
Pretax income	220
Taxes	55
Net income	165
Capital expenditures	60
Additions to working capital	20

   What is the net cash flow to invested capital?
   a. $257.50
   b. $177.50

c. \$165.00
d. \$157.50

5. Given the same assumptions as in the previous question, what is the net cash flow to equity?
   a. 235
   b. 190
   c. 155
   d. 135

6. Given the following estimates of possible cash flows:

–10	10%
0	20%
10	40%
20	25%
30	5%

   What is the probability-weighted expected value of the cash flow?
   a. 10.5
   b. 10.0
   c. 9.5
   d. 8.5

7. All other things being equal, which is the correct statement of the relationship between the discounting and capitalizing methods?
   a. Discounting will always produce a higher value.
   b. Discounting will usually produce a higher value.
   c. Discounting and capitalizing will produce the same value.
   d. Discounting will produce a lower value.

8. What is the relationship between the value resulting from the midyear convention and the value resulting from the year-end convention?
   a. The midyear convention will always result in a higher value than the year-end convention.
   b. The midyear convention will always result in a lower value than the year-end convention.
   c. The midyear convention will always result in the same or higher value relative to the year-end convention.
   d. The midyear convention will always result in the same or lower value relative to the year-end convention.

9. The risk associated with the fact that as the general level of interest rates goes up, the value of a fixed income investment goes down, and vice versa, is called what?
   a. Maturity risk
   b. Systematic risk
   c. Unsystematic risk
   d. Operating risk

10. Based on the cost of insuring seller paper to the buyer in small business sales, what is the estimated added cost of debt where personal guarantees are required?
    a. 1%
    b. 3%
    c. 5%
    d. 7%

11. Which of the following statements is true with respect to estimating the cost of equity?
    a. The build-up method produces a discount rate in nominal terms, and the CAPM produces a discount rate in real terms.
    b. The CAPM produces a discount rate in nominal terms, and the build-up method produces a discount rate in real terms.
    c. Both the CAPM and the build-up method produce discount rates in nominal terms.
    d. Both the CAPM and build-up methods produce discount rates in real terms.

12. For estimation of the cost of equity capital, "size premium in excess of CAPM" is recommended by Ibbotson Associates for use with which of the following model or models?
    a. Build-up model
    b. Capital Asset Pricing Model
    c. Both a and b
    d. Neither a nor b

13. In the "build-up model" for estimating the equity risk premium:
    a. The implied value for beta is 1.0.
    b. The implied value for beta is < 1.0.
    c. The implied value for beta is > 1.0.
    d. None of the above.

14. Given the following:

Risk-free rate	7.0%
Equity risk premium	8.0%
Small stock size premium	4.0%
Company-specific risk premium	1.0%
Beta	1.10

Compute the cost of equity by the Capital Asset Pricing Model and select the correct answer:
   a. 21.2%
   b. 21.1%
   c. 20.8%
   d. 19.0%

15. Beta is a measure of what kind of risk?
   a. Maturity risk
   b. Systematic risk
   c. Unsystematic risk
   d. Operating risk

16. Which of the following is NOT an assumption of the Capital Asset Pricing Model?
   a. Investors do not reflect inflation risk in their investment decisions.
   b. There are no transaction costs.
   c. The rate received from lending money is the same as the cost of borrowing money.
   d. Investors are risk averse.

17. For use with the Capital Asset Pricing Model, Ibbotson's equity risk premium series correspond to which of the following maturities of U.S. Treasury securities?
   a. 30-day, 5-year, and 30-year
   b. 30-day, 10-year, and 20-year
   c. 1 year, 5-year, and 20-year
   d. 30-day, 5-year, and 20-year

18. Which of the following are differences in estimating beta among reporting services?
   a. Length of measurement period, frequency of observations, and choice of market index
   b. Length of measurement period and frequency of observations, but not choice of market index

   c. Length of measurement period and choice of market index, but not frequency of observations
   d. Frequency of observations and choice of market index, but not length of measurement period

19. Ibbotson Associates breaks down size of companies by which of the following criteria?
    a. Market value of common equity
    b. Market value of common and preferred equity
    c. Market value of invested capital
    d. Book value of invested capital

20. By which criteria were the *Pratt's Stats*™ companies broken down in the study of the size effect?
    a. Market value of common equity
    b. Market value of common and preferred equity
    c. Market value of invested capital
    d. Book value of invested capital

21. *Standard & Poor's Corporate Value Consulting Risk Premium Reports* (formerly the *PricewaterhouseCoopers Risk Premium Studies*) on the relationship of rate of return to size of company have divided the stocks on the New York Stock Exchange into how many size categories?
    a. 10
    b. 20
    c. 25
    d. 40

22. The method of estimating the cost of equity that uses present value and analysts' estimates as the inputs is called:
    a. The build-up model
    b. The Capital Asset Pricing Model
    c. The DCF model
    d. The arbitrage pricing model

23. Which of the following is a source of market value capital structures by industry?
    a. *Stocks, Bonds, Bills and Inflation Classic Edition Yearbook*
    b. *Stocks, Bonds, Bills and Inflation Valuation Edition Yearbook*
    c. *Cost of Capital Yearbook*
    d. Both b and c

24. A multivariate regression model for estimating the cost of equity capital is known as:
    a. The build-up model
    b. The Capital Asset Pricing Model
    c. The DCF model
    d. The arbitrage pricing model

25. According to the *Mergerstat/Shannon Pratt's Control Premium Study*™, what proportion of public company takeovers occurred at less than the previous public share trading price from 1998 through 2001?
    a. Less than 5%
    b. 5–10%
    c. 10–15%
    d. More than 15%

26. Which of the following is a correct statement about each of the discount for lack of marketability studies?
    a. The FMV Opinions study is a restricted stock study, and the Emory and Valuation Advisors studies are pre-IPO studies.
    b. The FMV Opinions and Emory studies are restricted stock studies, and the Valuation Advisors study is a pre-IPO study.
    c. The FMV Opinions and Valuation Advisors studies are restricted stock studies, and the Emory studies are pre-IPO studies.
    d. All of the above are pre-IPO studies.

27. Which of the following revenue rulings relates to the excess earnings method?
    a. 59–60
    b. 68–609
    c. 83–120
    d. 93–12

28. Computing a weighted average cost of capital using the buyer's capital structure rather than the subject company's capital structure results in which of the following standards of value?
    a. Fair market value
    b. Investment value
    c. Fair value
    d. Intrinsic value

29. Which of the following courts has (have) explicitly expressed a preference for the DCF method of valuation?
    a. The Delaware Court of Chancery
    b. The U.S. Tax Court
    c. Family law courts
    d. U.S. bankruptcy courts

30. Which of the following is a correct statement about capital budgeting and feasibility studies?
    a. The company's cost of capital over the life of the project should be used as a discount rate.
    b. The project's cost of capital should be used over the life of the project.
    c. The company's current cost of capital should be used.
    d. The project's initial cost of capital should be used.

True or False:

True or False	31. Cost of capital depends on the investor rather than the investment.
True or False	32. For use with Ibbotson's arithmetic mean risk premium, the economic income variable should be the expected value of the probability-weighted distribution rather than the most likely outcome.
True or False	33. Capitalizing can properly be regarded as a short-cut form of discounting.
True or False	34. The weighted average cost of capital is computed by taking the components of the capital structure at their relative book values.
True or False	35. In the context of estimating cost of capital, the "iterative process" refers to a technique for estimating weights of components in the capital structure.
True or False	36. With respect to sequential observations of rates of return, the arithmetic mean represents a simple average, while the geometric mean represents a compound average.
True or False	37. All other things being equal, smaller companies usually have higher costs of capital than larger companies.
True or False	38. In the context of estimating the cost of equity capital, the "discounted cash flow method" computes a present value of the subject investment.
True or False	39. Because Ibbotson data are all based on minority trading in public market stocks, the result of a discounted cash flow analysis using Ibbotson data produces minority value.
True or False	40. Net cash flow is used consistently as the economic income to discount in ad valorem valuation cases.

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