Course: Math 3201 (Academic)
Book: Nelson's Principles of Mathematics 12

Assessment Overview:

Tests and Quizzes 20%
Assignments (including worksheets, projects, and journals)........................ 15%
Midterm Exam 15%
Final Exam.. 50%

Course Overview:

Math 3201 is broken down into four units and further broken into 9 chapters:

Chapter 1: Set Theory
Chapter 2: Counting Methods
Chapter 3: Probability
Chapter 4: Rational Expressions and Equations
Chapter 5: Polynomial Functions
Chapter 6: Exponential Functions
Chapter 7: Logarithmic Functions
Chapter 8: Sinusoidal Functions
Chapter 10: Financial Mathematics: Borrowing Money

Tentative Testing Timeline

The following is a tentative timeline for test dates. Please note that these dates are not exact and are subject to change. They are intended to give you a relative date to prepare for upcoming assessment.

Chapter 1: Set Theory	Sept 24th
Chapter 2: Counting Methods	Oct 23rd
Chapter 3: Probability	Nov 20th
Chapter 4: Rational Expressions and Equations	Dec 13th
Chapter 5: Polynomial Functions	Jan 17th
Chapter 6: Exponential Functions	Feb 26th
Chapter 7: Logarithmic Functions	Mar 23rd
Chapter 8: Sinusoidal Functions	Apr 28th
Chapter 9: Financial Mathematics: Borrowing Money	May 20th

Midterm: Your midterm exam will most likely occur in one of the two the last weeks of January.

Chapter 1: Set Theory

Section 1.1: Types of Sets and Set Notation

Terminology:

- **Set:**

  A collection of distinguishable objects.
  Ex. The set of whole numbers is W = {0, 1, 2, 3, ...}

- **Element:**

  An object in a set.
  Ex. 3 is an element in the set of whole numbers.

- **Universal Set:**

  A set of all the elements under consideration for a particular context (also called the sample space).
  Ex. The universal set of digits for a number is D = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

- **Subset:**

  A set whose elements all belong to another set.
  Ex. The set of odd digits O = {1, 3, 5, 7, 9} is a subset of D, the set of digits for a number.

  In Set Notation, this relationship, in which one set is a subset of another, is written as:

  $$O \subset D$$

- **Complement:**

  All elements of a universal set that do not belong to a subset of it.
  Ex. Given the odd digits O = {1, 3, 5, 7, 9}, than its complement would be the even digits {0, 2, 4, 6, 8}. Since the even digits are the complement of the odd digits, it would be denoted using the same symbol but with a prime. So the even digits will be denoted as O'.

- **Empty Set:**

  A set with no elements.
  Ex. A set of odd numbers that are divisible by two is the empty set.

  The empty set is denoted by { } or ∅.

- **Disjoint:**

  Two or more sets having no elements in common are referred to as being disjoint.
  Ex. The set of even numbers and odd numbers are disjoint.

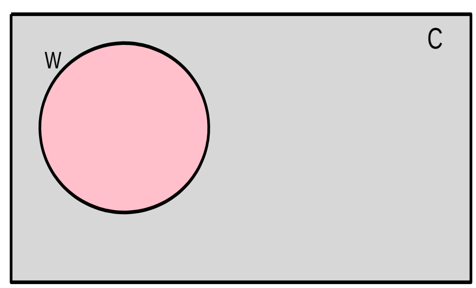
Investigation 1.1: Investigate the Math

Using the provinces and territories in Canada, answer the following questions:

1. List the *elements* of the *universal set (C)* of Canadian provinces and territories (abbreviations are acceptable).

2. One *subset* of *C* is the set of Western provinces and territories, *W*. Write *W* in set notation.

3. The Venn Diagram to the right represents the universal set, *C*. The circle in the diagram represents the subset, *W*.

   The *complement* of *W* is *W'*.
   i) Describe what *W'* contains

   ii) Write *W'* in set notation

   iii) Explain what *W'* represents in the Venn Diagram.

4. Jasmine wrote the set of Eastern provinces as:
   E = {NL, PEI, NS, NB, QC, ON}
   is *E* equal to *W'*? Explain.

5. List *T*, the set of territories in Canada. Is *T* a subset of *C*? Is it a subset of W, or a subset of W'? Explain using your Venn Diagram.

6. Explain why you can represent the set of Canadian provinces south of Mexico as the *empty set?*

7. Consider *C, W, W',* and *T*. List the pairs of disjoint sets. Is there more than one pair of sets that are disjoint?

8. Complete the Venn diagram by listing the elements of each subset in the appropriate circle.

Sorting Numbers Using Set Notation and a Venn Diagram

Example 1:

(a) Indicate the multiples of 5 and 10, from 1 to 50, using set notation. List any subsets.

(b) Represent the sets and subsets in a Venn diagram.

Example 2:

(a) Indicate the multiples of 4 and 12, from 1 to 60 inclusive, using set notation. List any subsets.

(b) Represent the sets and subsets in a Venn diagram

NOTATION:

The phase "from 1 to 5" means "from 1 to 5 inclusive."
In set notation, the number of elements of the set X is $n(X)$.

For example, if the set X is defined as the set of numbers from 1 to 5:
$X = \{1, 2, 3, 4, 5\}$
$n(X) = 5$

Terminology:

- **Finite Set:**

  A set with a countable number of elements.
  Ex: The set of even natural numbers less than 10, $E = \{2, 4, 6, 8\}$, is finite.

- **Infinite Set:**

  A set with an infinite number of elements.
  Ex. The set of natural numbers, $N = \{1, 2, 3, 4, 5, \ldots\}$, is infinite.

Describing the Relationship(s) Between Sets

Example 1: Alden and Connie rescue homeless animals and advertise in the local newspaper to find homes for the animals. They are setting up a web page to help them advertise the animals that are available. They currently have dogs, cats, rabbits, ferrets, parrots, lovebirds, macaws, iguanas, and snakes.

(a) Design a way to organize the animals on the webpage. Represent your organization using a Venn Diagram.

(b) Name any disjoint sets.

(c) Show which sets are subsets of one another using set notation.

(d) Alden said that the set of fur-bearing animals could form one subset. Name another set of animals that is equal to this subset.

Example 2: Jake runs a produce stand. He offers oranges, apples, peas, corn, pears, carrots, pineapples, potatoes, beans, and bananas.

(a) Design a way to organize the produce available. Represent your organization using a Venn Diagram.

(b) Name any disjoint sets.

(c) Show which sets are subsets of one another using set notation.

(d) Let U represent the produce that are usually eaten without pealing. Is this set a subset of any others? Is it disjoint to any other set?

Solving a Problem Using a Venn Diagram

Example 1: Bilyana recorded the possible subs that can occur when you roll two four-sided dice in an outcome table.

(a) Display the following sets in one Venn diagram:
- Rolls that produce a sum less than 5
- Rolls that produce a sum greater than 5

(b) Record the number of elements in each set.

(c) Determine a formula for the number of ways that a sum less than or greater than 5 can occur. Verify your formula.

Example 2: Two six-sided dice are rolled.

(a) Display the following sets in one Venn diagram:
- Rolls that produce a sum less than 6
- Rolls that produce a sum greater than 6

(b) Record the number of elements in each set.

(c) Determine a formula for the number of ways that a sum less than or greater than 6 can occur. Verify your formula.

Using Set Notation to Determine the Size of a Set

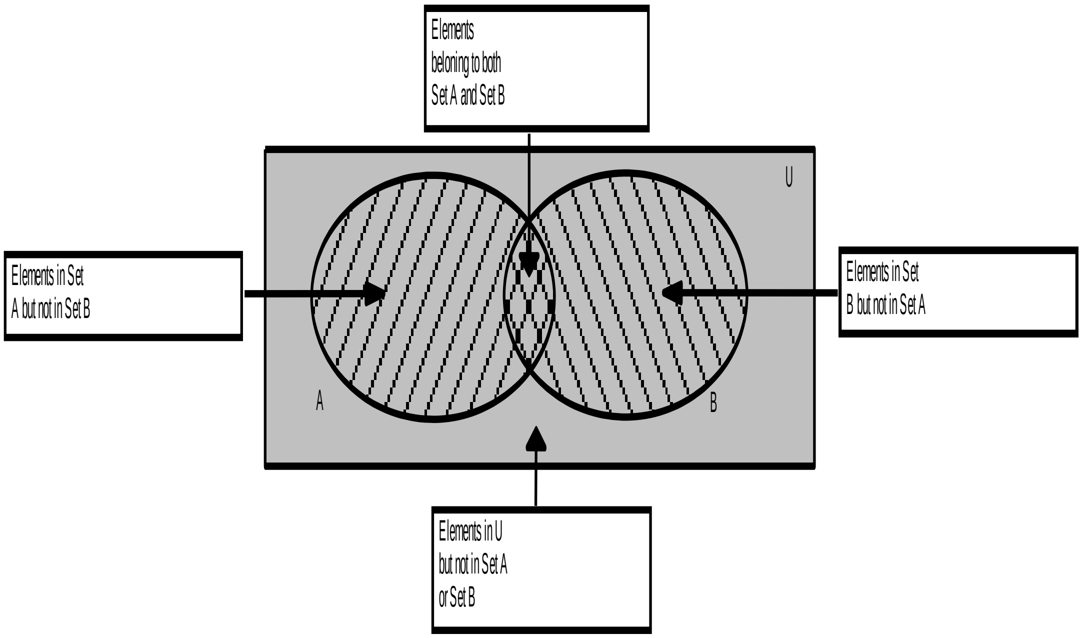
Example 1: In the universal set U [the set of natural numbers to 150], determine the value of n(P) if n(P') = 78.

Example 2: Given that n(D) = 37 and n(D') = 59, Determine n(S), the universal set.

Practice Questions:
2,3,4,5,6,7,8,9,10,**17** pg 15-18 (**Good Question**)

Section 1.2: Exploring Relationships Between Sets

Areas of a Venn Diagram



- Sets that are not disjoint, share common elements.
- Each area of a Venn Diagram represents something different.
- When two non-disjoint sets are represented in a Venn diagram, you can count the elements in both sets by counting the elements in each region of the diagram just once.

IMPORTANT NOTES ABOUT VENN DIAGRAMS:

- Each element in a universal set appears only once in a Venn diagram.
- If an element occurs in more than one set, it is placed in the area of the Venn diagram where the sets overlap.

Example 1: Consider the following sets:

$U = \{2,4,6,8,12,16,18,20,24,28,30,32,36,40\}$
$A = \{6,12,18,24,30,36\}$
$B = \{4,8,12,16,20,24,28,32,36,40\}$

(a) Illustrate these sets using a Venn Diagram.
(b) Determine the number of elements in:

i) Set A

ii) Set A but not Set B

iii) Set B

iv) Set B but not Set A

v) Set A **and** Set B

vi) Set A **or** Set B

vii) Set A’

NOTE:

When we are determining the number of elements in a given situation:

AND – Means that you only count the elements that are shared between the given sets

OR – Means that you count every element that belongs to both independent sets and the elements they share. Remember that you only count each element once, be careful not to count the shared elements twice!!

Example 2: There are 62 grads planning on attending Safe Grad this year. Each grad had to decide whether they wished to go bowling, watch a movie, both, or neither. The final tally indicated that 36 wished to go bowling, 28 wanted to watch a movie, and 5 didn't want to partake in either activity.

(a) Determine:

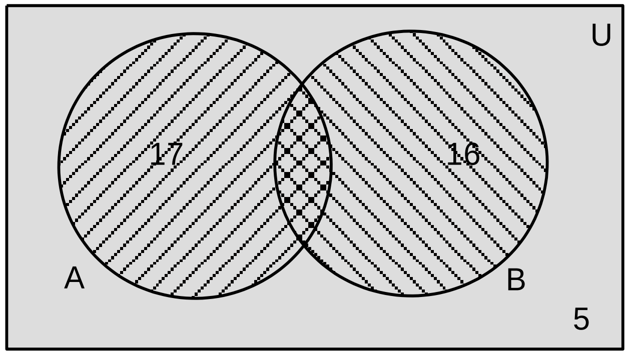
i) How many grads plan to bowl and watch a movie?

ii) How many grads plan to bowl only?

iii) How many grads plan to watch a movie only?

(b) Draw a Venn diagram with the data that you determined in part (a)

Example 3: Nick drew the Venn diagram incorrectly. There are 34 items in the universal set, U, and 5 items that are not in Set A or Set B.



(a) Determine:

i) *n(A and B)*

ii) *n(A or B)*

iii) *n(A only)*

iv) *n(B only)*

(b) Redraw Nick's Venn Diagram with the data determined in (a)

Practice Problems
1-5 pg 20-21

Section 1.3: Intersection and Union of Two sets

Terminology:

- **Intersection:**

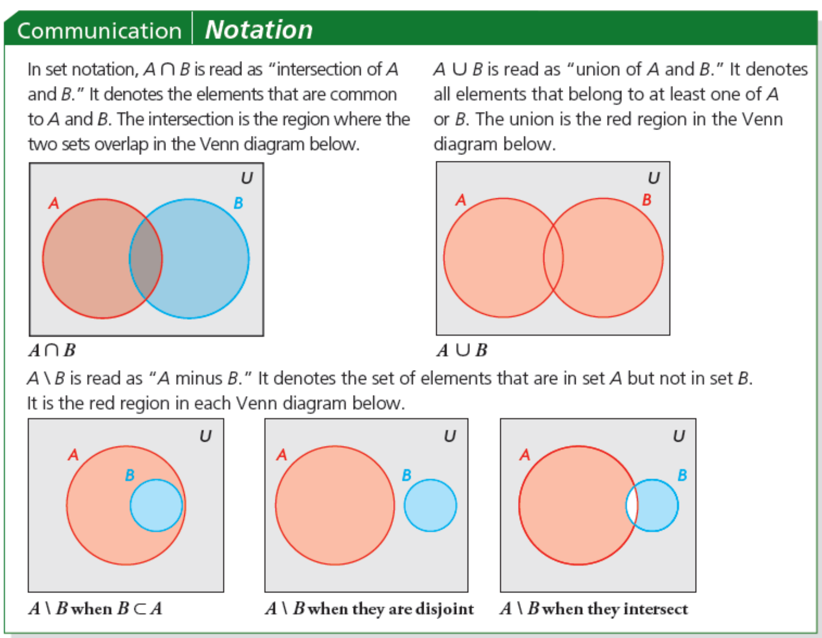
  The set of elements that are common to two or more sets.
  In set notation, $A \cap B$ denotes the intersection of sets A and B.
  Ex. If $A = \{1,2,3\}$ and $B = \{3,4,5\}$, then $A \cap B = \{3\}$.
  NOTE: This is the notation that corresponds to the word "AND"

- **Union:**

  The set of all the elements in two or more sets.
  In set notation, $A \cup B$ denotes the union of sets A and B.
  Ex. If $A = \{1,2,3\}$ and $B = \{3,4,5\}$, then $A \cup B = \{1,2,3,4,5\}$.
  NOTE: This is the notation that corresponds to the word "OR"

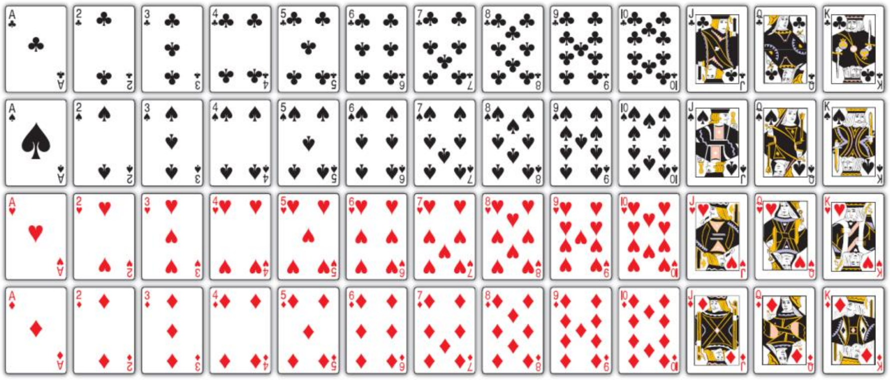
- **Minus:**

  The set of elements that are part of one set but not part of another.
  In set notation, $A \backslash B$ denotes the elements of set A minus the elements of set B.
  Ex. If $A = \{1,2,3\}$ and $B = \{3,4,5\}$, then $A \backslash B = \{1,2\}$.
  Ex. If $A = \{1,2,3\}$ and $B = \{3,4,5\}$, then $B \backslash A = \{4,5\}$.



Determining the Union and Intersection of Disjoint Sets

If you draw a card at random from a standard deck of cards, you will draw a card from one of four suits: Clubs (C), Spades(S), Hearts (H), or Diamonds (D).



(a) Describe sets C, S, H, D, and the universal set U for this situation.

(b) Determine n(C), n(S), n(H), n(D), and n(U)

(c) Describe the union of S and H. Determine n($S \cup H$).

(d) Describe the intersection of S and H. Determine n($S \cap H$).

(e) Determine whether sets S and H are disjoint.

(f) Describe the complement of $S \cup H$

(g) A student thinks that $n(S) + n(H) = n(S \cup H)$. Is she correct? Explain.

Determining the Number of Elements in a Set Using a Formula

Ex1: The athletics department at a large high school offers sixteen different sports:

Badminton	Hockey	Tennis
Basketball	Lacrosse	Ultimate Frisbee
Cross-Country Running	Rugby	Volleyball
Curling	Cross-Country Skiing	Wrestling
Football	Soccer	
Golf	Softball	

Determine the number of sports that require the following types of equipment:

(a) A ball and an implement (such as a stick, club, or racket)

(b) Only a ball

(c) An implement but mot a ball

(d) Either a ball or an implement

(e) Neither a ball nor an implement

Ex2: The athletics department at a large high school offers seventeen different sports:

Badminton	Hockey	Tennis
Basketball	Lacrosse	Ultimate Frisbee
Cross-Country Running	Rugby	Volleyball
Curling	Cross-Country Skiing	Wrestling
Football	Soccer	Water Polo
Golf	Softball	

Determine the number of sports that require the following types of equipment:

(a) Special headgear and special footwear

(b) Only special headgear

(c) Special footwear but not special headgear

(d) Either special headgear or footwear

(e) Neither special headgear nor footwear

Formula: The Principle of Inclusion and Exclusion

The number of elements in the union of two sets is equal to the sum of the number of elements in each set subtract the number of elements in both sets.

In set notation, this is written as:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

This formula can be rearranged to solve for the intersection as well:

$$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

Determining the Number of Elements in a Set by Reasoning

Ex1: Jamaal surveyed 34 people at his gym. He learned that 16 people do weight training three times a week, 21 people do cardio training three times a week, and 6 people train fewer than three times a week. How can Jamaal interpret these results?
(hint: Venn Diagram)

Ex2: Jamaal surveyed 50 other gym members. Of these members, 9 train fewer than three times a week, 11 do only cardio training three times a week, and 16 do both cardio and weight training three times a week. Determine how many of these members do weight training three times per week.

Correcting Errors in Venn Diagrams

Morgan surveyed the 30 students in his mathematics class about their eating habits.

- 18 of these students eat breakfast.
- 5 of the 18 also eat a healthy lunch.
- 3 students do not eat breakfast and do not eat a healthy lunch.

How many students eat a healthy lunch?

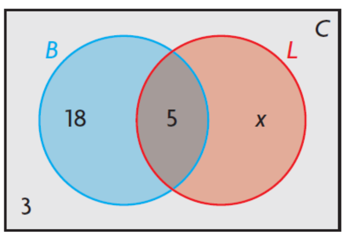
Erica solved this problem, as shown below, but made an error. What error did she make? Determine the correct solution.

Erica's Solution

Let C represent the universal set, the students in Morgan's math class. Let B represent those who eat breakfast, and let L represent those who eat a healthy lunch.

There are 30 students in total.

I drew a Venn diagram showing the number of elements in each region.



There are 18 students in set B. I put the 5 students who are in sets B and L in the overlap.

There are 3 students who do not belong in either set. This means that there are 30-3 or 27 people in sets A and B.

$$18 + 5 + x = 27$$
$$x = 4$$

I determined the total number of elements in set L.

$$n(L) = 5 + 4$$
$$n(L) = 9$$

Therefore, there are 9 students that eat a healthy lunch.

CORRECT SOLUTION

Ex2: Susan surveyed 34 students in her chemistry class.

- 14 students eat breakfast.
- 16 students eat a healthy lunch.
- 4 students eat breakfast and a healthy lunch.

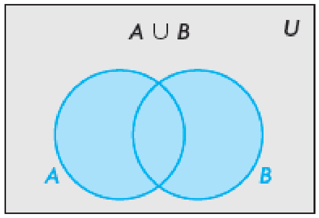
Since $14 + 16 + 4 = 34$, Susan concluded that everyone in her class eats either breakfast or a healthy lunch, or both.

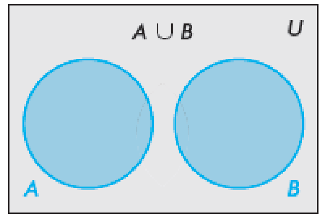
What error did Susan make How many students do not eat either meal?

In Summary

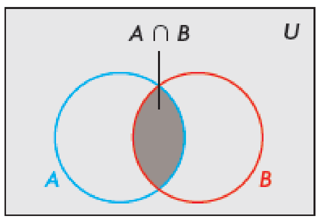
Key Ideas

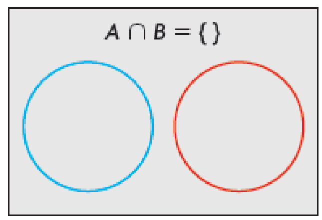
- The **union** of two or more sets, for example, $A \cup B$, consists of all the elements that are in at least one of the sets. It is represented by the entire region of these sets on a Venn diagram. It is indicated by the word "**or**."





- The **intersection** of two or more sets, for example, $A \cap B$, consists of all the elements that are common to these sets. It is represented by the region of overlap on a Venn diagram. It is indicated by the word "**and**."



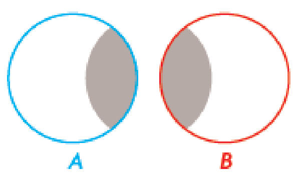


Need to Know

- If two sets, A and B, contain common elements, the number of elements in A or B, $n(A \cup B)$, is:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

This is called the Principle of Inclusion and Exclusion. To calculate $n(A \cup B)$, subtract the elements in the intersection so they are not counted twice, once in $n(A)$ and once in $n(B)$.



- If two sets, A and B are disjoint, they contain no common elements:

$$n(A \cap B) = 0 \text{ and}$$
$$n(A \cup B) = n(A) + n(B)$$

- Elements that are in set A but not in set B are expressed as $A \setminus B$. The number of elements in A or B, $n(A \cup B)$, can also be determined as follows:

$$n(A \cup B) = n(A \setminus B) + n(B \setminus A) + n(A \cap B)$$

Practice Questions:
1-5,7-15,16 pg 32-34

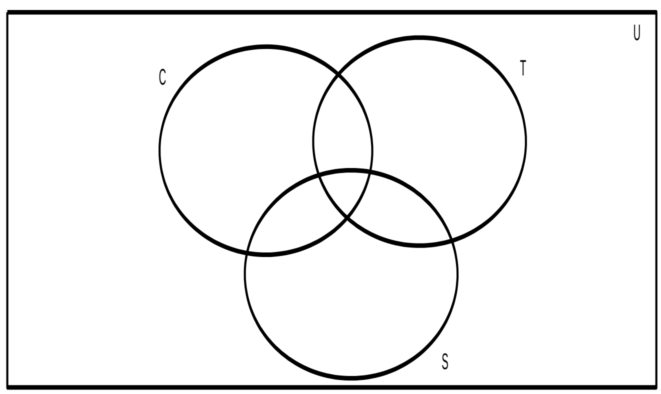
Section 1.4: Applications of Set Theory

Using Sets to Model and Solve Problems

Ex1: Rachel surveyed Grade 12 students about how they communicated with friends over the previous week.

- 66% called on a cellphone.
- 76% texted.
- 34% used social medial.
- 56% called on a cellphone and texted.
- 18% called on a cellphone and used social media.
- 19% texted and used social media.
- 12% used all three forms of communication.

(a) The Venn diagram below represents the following sets:
   - C = {Students who called on a cellphone}
   - T = {Students who texted}
   - S = {Students who used social media}



i) What does the universal set U represent in this situation?

ii) Record the percent of students who used all three forms of communication in the diagram.

(b) Determine the percent of students who texted and used a social networking site, but did not call on a cellphone. Update your diagram.

(c) Determine the percent of students who called on a cellphone and used a social networking site, but did not text. Determine the percent of students who called on a cellphone and texted, but did not use social media. Update your diagram.

(d) Determine the percentage of students who only called on a cellphone, only texted, or only used social media. Update your diagram.

(e) Determine the percent of students who use at least one of these three forms. Explain your answer.

(f) What percent of students who called on a cellphone or texted, but did not use social media. Express your results in set notation.

Formula: The Principle of Inclusion and Exclusion with Three Sets

The number of elements in the union of three sets is equal to the sum of the number of elements in each set subtract the number of elements shared between each set. Then we must add in the sum of the elements shared between all three sets.

In set notation, this is written as:

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$

Solving a Puzzle Using the Principle of Inclusion or Exclusion

Ex. Use the following clues to answer the questions below:

- 28 children have a dog, a cat, or a bird.
- 13 children have a dog.
- 13 children have a cat.
- 13 children have a bird.
- 4 children have only a dog and a cat.
- 3 children have only a dog and a bird.
- 2 children have only a cat and a bird.
- No child has more than one of each pet.

(a) How many children have a cat, a dog, and a bird?

(b) How many children have only one pet?

Show internet search example to class at this point before continuing on…

Ex2 Shannon's high school starts a campaign to encourage students to use "green" transportation from travelling to and from school. At the end of the first semester, Shannon's class surveys the 750 students in the school to see if the campaign is working. They obtain the following results:

- 370 students use public transit.
- 100 students cycle and use public transit.
- 80 students walk and use public transit.
- 35 students walk and cycle.
- 20 students walk, cycle, and use public transit.
- 445 students cycle or use public transit.
- 265 students walk or cycle.

(a) How many students use green transportation for travelling to and from school?

(b) How many students use exactly one method of green transportation?

Ex: There are twenty five dogs at the dog show. Twelve of the dogs are black, eight of the dogs have short tails, and fifteen of the dogs have long hair. There is only one dog that is black with a short tail and long hair. Three of the dogs are black with short tails and do not have long hair. Two of the dogs have short tails and long hair but are not black. If all of the dogs in the kennel have at least one of the mentioned characteristics, how many dogs are black with long hair but do not have short tails?

Ex. Penny surveyed 115 teenagers regarding their favourite types of movies. She found the following:

- 49 like Horror Movies
- 60 like Science-Fiction Movies
- 45 like Comedy Movies
- 15 like both Horror and Comedy
- 19 like both Comedy and Science Fiction
- 8 like all three types of movies
- 4 like neither

Determine the number of teens that like only science fiction movies.

Ex. On a recent interest inventory to determine next years programming, the school asked 80 grade 11 students if they would like to do Art, Music, or Home Economics in their grade 12 year. The results found:

- 46 wanted to do Art
- 45 wanted to do Music
- 32 wanted to do Home Economics
- 15 wanted to do Home Economics and Art
- 21 wanted to do Home Economics and Music
- 9 wanted to do All three courses
- 3 wanted to do neither of these course

Determine the number of students who wish to take Art or Music.

Question Involving Set Theory

Remember from Grade 10 we used set theory for writing domain and range. For example $x|x \in (A \cup B)$ is read as x such that x is a member of A *union* B. This would mean that all values of x are those who belong to the union of A and B.

REMEMBER:
| - such that
∈ - is a member of/belongs
N – Natural Numbers
W – Whole Numbers
R – Real Numbers

Ex. If $E = \{2x|1 \leq x \leq 10, x \in N\}$ and $F = \{3x - 1|1 \leq x \leq 7, x \in N\}$, determine:

(a) $n(E \cap F)$

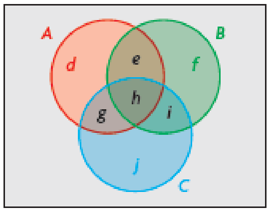
(b) $n(E \cup F)$

(c) Is it true that $E \cap F$ is the empty set? Explain your reasoning

In Summary

Key Ideas

- Set theory is useful for solving many types of problems, including Internet searches, database queries, data analyses, games, and puzzles.
- To represent three intersecting sets with a Venn diagram, use three intersecting circles. For example, in the following Venn diagram,



  - $A \cap B \cap C$ is represented by region h,
  - $A \cap B$ is represented by the union of regions e and h,
  - $A \cap C$ is represented by the union of regions g and h, and
  - $B \cap C$ is represented by the union of regions h and i.

  Each region of a Venn diagram contains elements that occur only in that particular region.
- You can use the Principle of Inclusion and Exclusion to determine the number of elements in the union of three sets:
  $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

Need to Know

- You can use concepts related to sets to search for websites on the Internet:
  - Put an exact phrase in quotation marks.
  - Connect words or phrases with "and" to search for sites that contain both. The word "and" represents the intersection of two or more sets.
  - Connect words or phrases with "or" to search for sites that contain either one or the other, or both. The word "or" represents the union of two or more sets.
- When solving a puzzle or problem, it is often useful to visualize the problem. First identify which sets are defined by the context. Then identify how the sets overlap. Finally, identify regions of the overlaps that are of interest in the puzzle or problem. It is often advisable to consider how much is known about each region, and use the information about the region that is most known to deduce information about regions that are less well known. A systematic approach will result in answers that are easier to verify.

Practice Problems
3,4,6,9 pg 51-53