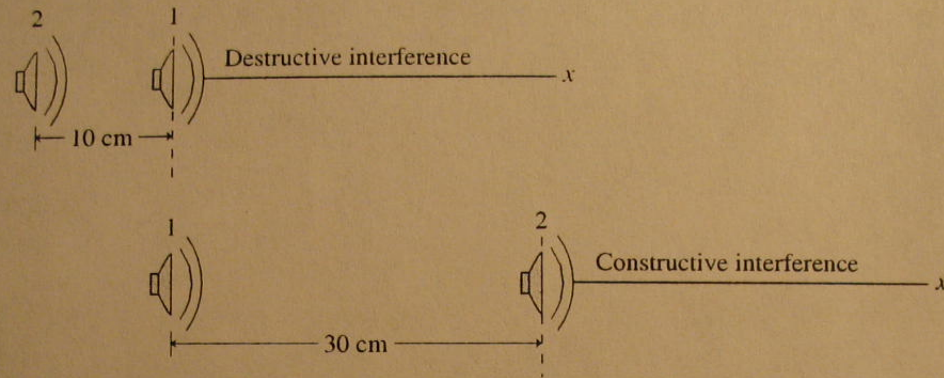
58. Two loudspeakers emit sound waves of the same frequency along the x-axis. The amplitude of each wave is a. The sound intensity is minimum when speaker 2 is 10 cm behind speaker 1. The intensity increases as speaker 2 is moved forward and first reaches maximum, with amplitude $2a$, when it is 30 cm in front of speaker 1. What is

a. The wavelength of the sound?
b. The phase difference between the two loudspeakers?
c. The amplitude of the sound if the speakers are placed side by side?

21.58. **Model:** Constructive or destructive interference occurs according to the phases of the two waves.

Visualize:



Solve: **(a)** To go from destructive to constructive interference requires moving the speaker $\Delta x = \frac{1}{2}\lambda$, equivalent to a phase change of π rad. Since $\Delta x = 40$ cm, we find $\lambda = 80$ cm.

(b) Destructive interference at $\Delta x = 10$ cm requires

$$2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0 = 2\pi\left(\frac{10\text{ cm}}{80\text{ cm}}\right)\text{rad} + \Delta\phi_0 = \pi\text{ rad} \Rightarrow \Delta\phi_0 = \frac{3\pi}{4}\text{ rad}$$

(c) When side by side, with $\Delta x = 0$, the phase difference is $\Delta\phi = \Delta\phi_0 = 3\pi/4$ rad. The amplitude of the superposition of the two waves is

$$a = \left|2a\cos\left(\frac{\Delta\phi}{2}\right)\right| = \left|2a\cos\frac{3\pi}{8}\right| = 0.765a$$

$a\cos(kx - \omega t) + a\cos(kx - \omega$

$= a\cos(\theta) + a\cos(\theta + \Delta\phi)$

$= a\cos(\theta + \frac{\Delta\phi}{2} - \frac{\Delta\phi}{2}) + a\cos(\theta + \frac{\Delta\phi}{2}$

21.59. **Model:** The amplitude is determined by the interference of the two waves.

Course PHYSICS260

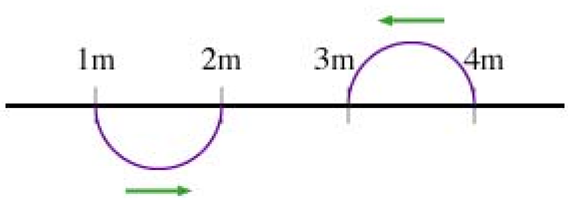
Assignment 4

Due at 11:00pm on Wednesday, February 27, 2008

A Simple Introduction to Interference

Description: Interference is discussed for pulses on strings and then for sinusoidal waves.
Learning Goal: To understand the basic principles underlying interference.
One of the most important properties of waves is the *principle of superposition*. The principle of superposition for waves states that when two waves occupy the same point, their effect on the medium adds algebraically. So, if two waves would individually have the effect "+1" on a specific point in the medium, then when they are both at that point the effect on the medium is "+2." If a third wave with effect "-2" happens also to be at that point, then the total effect on the medium is zero. This idea of waves adding their effects, or canceling each other's effects, is the source of *interference*.

First, consider two wave pulses on a string, approaching each other.



Assume that each moves with speed 1 meter per second. The figure shows the string at time $t = 0$. The effect of each wave pulse on the string (which is the medium for these wave pulses) is to displace it up or down. The pulses have the same shape, except for their orientation. Assume that each pulse displaces the string a maximum of 0.5 meters, and that the scale on the *x* axis is in meters.

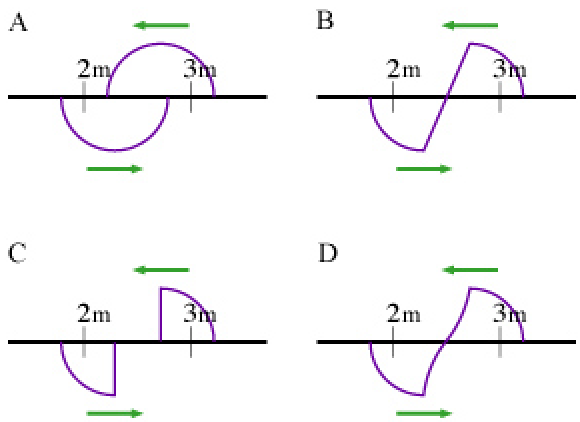
Part A

At time $t = 1\,\mathrm{s}$, what will be the displacement Δy at point $x = 2.5\,\mathrm{m}$?
Express your answer in meters, to two significant figures.

ANSWER: Δy = 0 m

Part B

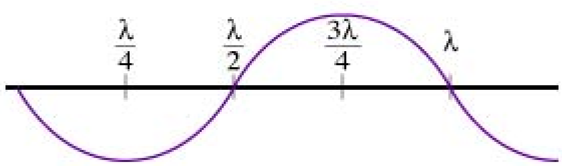
Choose the picture that most closely represents what the rope will actually look like at time $t = 0.75\,\mathrm{s}$.



ANSWER:

- ☐ A
- ☐ B
- ☐ C
- ☒ D

The same process of superposition is at work when we talk about continuous waves instead of wave pulses. Consider a sinusoidal wave as in the figure.



Part C

How far Δx to the left would the original sinusoidal wave have to be shifted to give a wave that would completely cancel the original? The variable λ in the picture denotes the wavelength of the wave.

Express your answer in terms of λ.

ANSWER: Δx = $\frac{1}{2}\lambda$

Part D

In talking about interference, particularly with light, you will most likely speak in terms of phase differences, as well as wavelength differences. In the mathematical description of a sine wave, the phase corresponds to the argument of the sine function. For example, in the function $y = A\sin(kx)$, the value of kx at a particular point is the phase of the wave at that point. Recall that in radians a full cycle (or a full circle) corresponds to 2π radians. How many radians would the shift of half a wavelength from the previous part correspond to?

Express your answer in terms of π.

ANSWER: phase difference = π radians

Part E

The phase difference of π radians that you found in the previous part provides a criterion for destructive interference. What phase difference corresponds to completely *constructive* interference (i.e., the original wave and the shifted wave coincide at all points)?

Express your answer as a number in the interval $[-\pi, \pi]$.

ANSWER: phase difference = 0 radians

Part F

Since sinusoidal waves are cyclical, a particular phase difference between two waves is identical to that phase difference plus a cycle. For example, if two waves have a phase difference of $\frac{\pi}{4}$, the interference effects would be the same as if the two waves had a phase difference of $\frac{\pi}{4} + 2\pi$. The complete criterion for constructive interference between two waves is therefore written as follows:

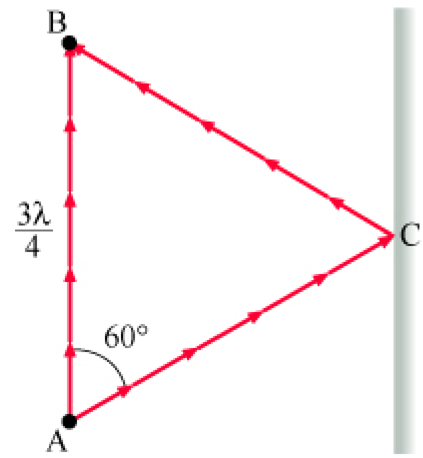
$$\text{phase difference} = 0 + 2\pi n \qquad \text{for any integer } n$$

Write the full criterion for *destructive* interference between two waves.

Express your answer in terms of π and n.

ANSWER: phase difference = $\pi + 2\pi n$ for any integer n

The phase for a plane wave is a somewhat complicated expression that depends on both position and time. For most interference problems, you will work at a specific time and with coherent light sources, so that only geometric considerations are relevant. Consider two light rays propagating from point A to point B in the figure, which are $\frac{3\lambda}{4}$ apart. One ray follows a straight path, and the other travels at a $60°$ angle to that path and then reflects off a plane surface to point B. Both rays have wavelength



λ.

Part G

Find the phase difference between these two rays at point B.

Part G.1 Find the difference in distance

Find the difference in length between the direct path and the reflected path. You can use the fact that triangle ABC is an equilateral triangle.

Express your answer in terms of λ.

ANSWER: path length difference = $\frac{3\lambda}{4}$

Now that you have the difference in path length, convert that to radians. Recall that every cycle of 2π radians is equivalent to one wavelength.

Express your answer in terms of π.

ANSWER: phase difference = $\frac{3\pi}{2}$ radians

Part H

Suppose that the reflected ray receives an extra half-cycle phase shift when it reflects. What is the new phase shift at point B?

Hint H.1 How many radians in a half cycle?

Since 2π radians corresponds to a full cycle, a half cycle must correspond to π radians.

Express your answer in terms of π.

ANSWER:

phase difference = $\frac{5\pi}{2}$ $\frac{\pi}{2}$ radians

Whenever light reflects from a transparent interface, moving from lower index of refraction to higher index of refraction, it gets an extra half cycle phase difference. Being able to accurately find the phase differences between waves at various points will be useful in both interference and diffraction problems.

Normal Modes and Resonance Frequencies

Description: Multiple choice questions about the definition and origin of normal modes. Then compute the frequency and wavelength of the first three normal modes in a string.
Learning Goal: To understand the concept of normal modes of oscillation and to derive some properties of normal modes of waves on a string.

A *normal mode* of a closed system is an oscillation of the system in which all parts oscillate at a single frequency. In general there are an infinite number of such modes, each one with a distinctive frequency f_i and associated pattern of oscillation.

Consider an example of a system with normal modes: a string of length L held fixed at both ends, located at $x = 0$ and $x = L$. Assume that waves on this string propagate with speed v. The string extends in the *x* direction, and the waves are transverse with displacement along the *y* direction.

In this problem, you will investigate the shape of the normal modes and then their frequency.

The normal modes of this system are products of trigonometric functions. (For linear systems, the time dependance of a normal mode is always sinusoidal, but the spatial dependence need not be.) Specifically, for this system a normal mode is described by

$$y_i(x,t) = A_i \sin\left(2\pi\frac{x}{\lambda_i}\right)\sin(2\pi f_i t) .$$

Part A

The string described in the problem introduction is oscillating in one of its normal modes. Which of the following statements about the wave in the string is correct?

Hint A.1 Normal mode constraints

The key constraint with normal modes is that there are two spatial boundary conditions, $y_i(0,t)=0$ and $y_i(L,t)=0$, which correspond to the string being fixed at its two ends.

ANSWER:

- ◻ The wave is traveling in the +*x* direction.
- ◻ The wave is traveling in the -*x* direction.
- ◻ The wave will satisfy the given boundary conditions for any arbitrary wavelength λ_i.
- ⊙ The wavelength λ_i can have only certain specific values if the boundary conditions are to be satisfied.
- ◻ The wave does not satisfy the boundary condition $y_i(0,t)=0$.

Part B

Which of the following statements are true?

ANSWER:

- ⊙ The system can resonate at only certain resonance frequencies f_i and the wavelength λ_i must be such that $y_i(0,t)=y_i(L,t)=0$.
- ◻ A_i must be chosen so that the wave fits exactly on the string.
- ◻ Any one of A_i or λ_i or f_i can be chosen to make the solution a normal mode.

The key factor producing the normal modes is that there are two spatial boundary conditions, $y_i(0,t)=0$ and $y_i(L,t)=0$, that are satisfied only for particular values of λ_i.

Part C

Find the three longest wavelengths (call them λ_1, λ_2, and λ_3) that "fit" on the string, that is, those that satisfy the boundary conditions at $x=0$ and $x=L$. These longest wavelengths have the lowest frequencies.

Hint C.1 How to approach the problem

The nodes of the wave occur where

$$\sin\left(2\pi\frac{x}{\lambda_i}\right)=0$$.

This equation is trivially satisfied at one end of the string (with $x=0$), since $\sin(0)=0$.

The three largest wavelengths that satisfy this equation at the other end of the string (with $x = L$) are given by $2\pi L/\lambda_i = z_i$, where the z_i are the three *smallest*, *nonzero* values of z that satisfy the equation $\sin(z) = 0$.

Part C.2 **Values of z that satisfy $\sin(z) = 0$**

The spatial part of the normal mode solution is a sine wave. Find the three smallest (nonzero) values of z (call them z_1, z_2, and z_3) that satisfy $\sin(z) = 0$.
Express the three nonzero values of z as multiples of π. List them in increasing order, separated by commas.

ANSWER:

z_1, z_2, z_3 =

1 2 3

π 2 3

1 2π 3

π 2π 3

1 2 3π

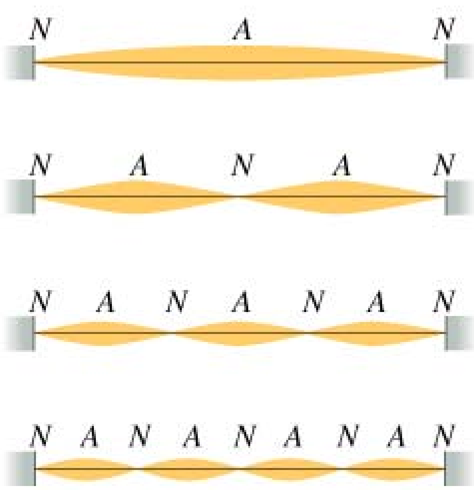
π 2 3π

1 2π 3π

π 2π 3π

Hint C.3 Picture of the normal modes

Consider the lowest four modes of the string as shown in the figure.



The letter N is written over each of the *nodes* defined as places where the string does not move. (Note that there are nodes in addition to those at the end of the string.) The letter A is written over the *antinodes*, which are where the oscillation amplitude is maximum.

Express the three wavelengths in terms of L. List them in *decreasing* order of length, separated by commas.

ANSWER: λ_1, λ_2, λ_3 = $2L$ L $\frac{2}{3}L$

The procedure described here contains the same mathematics that leads to *quantization* in quantum mechanics.

Part D

The frequency of each normal mode depends on the spatial part of the wave function, which is characterized by its wavelength λ_i.

Find the frequency f_i of the *i*th normal mode.

Hint D.1 Propagation speed for standing waves

Your expression will involve v, the speed of propagation of a wave on the string. Of course, the normal modes are standing waves and do not travel along the string the way that traveling waves do. Nevertheless, the speed of wave propagation is a physical property that has a well-defined value that happens to appear in the relationship between frequency and wavelength of normal modes.

Hint D.2 Use what you know about traveling waves

The relationship between the wavelength and the frequency for standing waves is the same as that for traveling waves and involves the speed of propagation v.

Express f_i in terms of its particular wavelength λ_i and the speed of propagation of the wave v.

ANSWER: f_i = $\frac{v}{\lambda_i}$

The frequencies f_i are the *only* frequencies at which the system can oscillate. If the string is excited at one of these *resonance frequencies* it will respond by oscillating in the pattern given by $y_i(x,t)$, that is, with wavelength λ_i associated with the f_i at which it is excited. In quantum mechanics these frequencies are called the *eigenfrequencies*, which are equal to the energy of that mode divided by Planck's constant h. In SI units, Planck's constant has the value $h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s}$.

Part E

Find the three lowest normal mode frequencies f_1, f_2, and f_3.

Express the frequencies in terms of L, v, and any constants. List them in *increasing* order, separated by commas.

ANSWER: $f_1, f_2, f_3 = \frac{\frac{1}{2}v}{L}, \frac{v}{L}, \frac{\frac{3}{2}v}{L}$

Note that, for the string, these frequencies are multiples of the lowest frequency. For this reason the lowest frequency is called the *fundamental* and the higher frequencies are called *harmonics* of the fundamental. When other physical approximations (for example, the stiffness of the string) are not valid, the normal mode frequencies are not exactly harmonic, and they are called *partials*. In an acoustic piano, the highest audible normal frequencies for a given string can be a significant fraction of a semitone sharper than a simple integer multiple of the fundamental. Consequently, the fundamental frequencies of the lower notes are deliberately tuned a bit flat so that their higher partials are closer in frequency to the higher notes.

Thin Film (Oil Slick)

Description: This problem explores thin film interference for both transmission and reflection.

A scientist notices that an oil slick floating on water when viewed from above has many different rainbow colors reflecting off the surface. She aims a spectrometer at a particular spot and measures the wavelength to be 750 nm(in air). The index of refraction of water is 1.33.

Part A

The index of refraction of the oil is 1.20. What is the minimum thickness t of the oil slick at that spot?

Hint A.1 Thin-film interference

In thin films, there are interference effects because light reflects off the two different surfaces of the film. In this problem, the scientist observes the light that reflects off the air-oil interface and off the oil-water interface. Think about the phase difference created between these two rays. The phase difference will arise from differences in path length, as well as differences that are introduced by certain types of reflection.

Recall that if the phase difference between two waves is 2π(a full wavelength) then the waves interfere constructively, whereas if the phase difference is π(half of a wavelength) the waves interfere destructively.

Hint A.2 Path-length phase difference

The light that reflects off the oil-water interface has to pass through the oil slick, where it will have a different wavelength. The total "extra" distance it travels is twice the thickness of the slick (since the light first moves toward the oil-water interface, and then reflects back out into the air).

Hint A.3 Phase shift due to reflections

Recall that when light reflects off a surface with a higher index of refraction, it gains an extra phase shift of πradians, which corresponds to a shift of half of a wavelength. What used to be a maximum is now a minimum! Be careful, though; if two beams each reflect off a surface with a higher index of refraction, they will both get a half-wavelength shift, canceling out that effect.

Express your answer in nanometers to three significant figures.

$$\lambda_{air} n_{air} = \lambda_{oil} n_{oil}$$

$$\lambda_{oil} = \frac{n_{air}}{n_{oil}} \lambda_{air} = \frac{750nm}{1.20} = 625nm$$

$2t = n\lambda_{oil}$, where n is an integer.

When n=1, t has minimum value

$$t = \frac{\lambda_{oil}}{2} = 313nm$$

ANSWER: t = 313 nm

Part B

Suppose the oil had an index of refraction of 1.50. What would the minimum thickness t be now?

Hint B.1 Phase shift due to reflections

Keep in mind that when light reflects off a surface with a higher index of refraction, it gains an extra shift of half of a wavelength. What used to be a maximum is now a minimum! Be careful, though; if two beams reflect, they will both get a half-wavelength shift, canceling out that effect. Also, reflection off a surface with a lower index of refraction yields no phase shift.

Express your answer in nanometers to three significant figures.

$$\lambda_{oil} = \frac{n_{air}}{n_{oil}} \lambda_{air} = \frac{750nm}{1.50} = 500nm$$

$2t = (n + 1/2)\lambda_{oil}$ where n is an integer.

When n=0, t has minimum value

$$t = \frac{\lambda_{oil}}{4} = 125nm$$

ANSWER: t = 125 nm

Part C

Now assume that the oil had a thickness of 200 nm and an index of refraction of 1.5. A diver swimming underneath the oil slick is looking at the same spot as the scientist with the spectromenter. What is the longest wavelength λ_{water} of the light *in water* that is transmitted most easily to the diver?

Hint C.1 How to approach the problem

For transmission of light, the same rules hold as before, only now one beam travels straight through the oil slick and into the water, while the other beam reflects twice (once off the oil-water interface and once again off the oil-air interface) before being finally transmitted to the water.

Part C.2 Determine the wavelength of light in air

Find the wavelength of the required light in air.
Express your answer numerically in nanometers.
The wave most easily transmitted into the water is the one most hard to reflect out of the water, i.e., find out the wave that has destructive effect with its reflected wave.

Also note that index of refraction for oil is1.5, water is 1.33, so it gains an extra shift of half of a wavelength while reflecting on the air-water interface.

$2t = k\lambda_{oil} = kn_{air}\lambda_{air} / n_{oil} = k\lambda_{air} / 1.50$, where n is an integer.
The longest wavelength in air is for k=1.
λ_{air} =1.50*$2t = 600nm$

ANSWER: λ_{air} = 600 nm

Hint C.3 Relationship between wavelength and index of refraction

There is a simple relationship between the wavelength λ_1 of light in one medium (with one index of refraction n_1) and the wavelength λ_2 in another medium (with a different index of refraction n_2):

$$n_1\lambda_1 = n_2\lambda_2.$$

Express your answer in nanometers to three significant figures.

ANSWER: λ_{water} = 451 nm

This problem can also be approached by finding the wavelength with the minimum reflection. Conservation of energy ensures that maximum transmission and minimum reflection occur at the same time (i.e., if the energy did not reflect, then it must have been transmitted to conserve energy), so finding the wavelength of minimum reflection must give the same answer as finding the wavelength of maximum transmission. In some cases, working the problem one way may be substantially easier, so you should keep both approaches in mind.

Two Loudspeakers in an Open Field

Description: Determine if constructive interference occurs at a certain point and then find the shortest distance you need to walk in order to experience destructive interference.
Imagine you are in an open field where two loudspeakers are set up and connected to

the same amplifier so that they emit sound waves in phase at 688 Hz. Take the speed of sound in air to be 344 $\mathrm{m/s}$.

Part A

If you are 3.00 m from speaker *A* directly to your right and 3.50 m from speaker *B* directly to your left, will the sound that you hear be louder than the sound you would hear if only one speaker were in use?

Hint A.1 How to approach the problem

The perceived loudness depends on the amplitude of the sound wave detected by your ear. When two sound waves arrive at the same region of space they overlap, and interference occurs. The resulting wave has an amplitude that can vary depending on how the two waves interfere. If destructive interference occurs, the total wave amplitude is zero and no sound is perceived; if constructive interference occurs, the total wave amplitude is twice the amplitude of a single wave, and sound is perceived as louder than what it would be if only one wave reached your ear.

Hint A.2 Constructive and destructive interference

Constructive interference occurs when the distances traveled by two sound waves differ by a integer number of wavelengths. If the difference in paths is equal to any half-integer number of wavelengths, destructive interference occurs.

Part A.3 Find the wavelength of the sound

What is the wavelength λ of the sound emitted by the loudspeakers?

Hint A.3.a Relationship between wavelength and frequency

In a periodic wave, the product of the wavelength λ and the frequency f is the speed at which the wave pattern travels; that is,

$$v = \lambda f.$$

Express your answer in meters.

ANSWER: $\lambda = \frac{344}{688}$ m

ANSWER:
- (●) yes
- ( ) no

Part B

What is the shortest distance d you need to walk forward to be at a point where you cannot hear the speakers?

Hint B.1 How to approach the problem

You will not be able to hear the speakers if you are at a point of destructive interference. At a point of destructive interference, the lengths of the paths traveled by the sound waves differ by a half-integer number of wavelengths. Therefore, you can find the shortest distance you need to walk in the forward direction by determining the difference in distance from the two speakers that corresponds to the smallest possible half-integer multiple of the wavelength. Then, figure out how far forward you need to walk to obtain this path-length difference.

Part B.2 Find the path-length difference at a point of destructive interference

If d_A is the distance between you and speaker *A* and d_B is the distance between you and speaker *B*, by how much does d_A differ from d_B if you are now at the closest possible point of destructive interference?

Hint B.2.a Condition for destructive interference

Destructive interference occurs if the difference in paths traveled by sound waves is equal to any half-integer number of wavelengths. Therefore, the closest possible point of destructive interference corresponds to a path-length difference of half a wavelength.

Express your answer in meters.

ANSWER: $d_A - d_B = \frac{1}{4}$ m

Now combine this result with the Pythagorean Theorem and solve for d_A.

Part B.3 Find your distance from speaker *A*

If initially you were 3.00 m from speaker *A* and then you walked forward the shortest possible distance needed to experience destructive interference, what is your new distance d_A from that same speaker?

Hint B.3.a Geometrical considerations

Geometrically, your initial distance from one speaker and the distance you walked north represent the legs of a right triangle, whose hypothenuse is your new distance from that same speaker. If you apply the Pythagorean Theorem twice, you can write an expression that links d_A to d_B. This equation, combined with the relation previously found by imposing the condition of destructive interference, allows you to find d_A.

$$\sqrt{3.5^2 + d^2} - \sqrt{3^2 + d^2} = 0.25$$

d=5.625m

Express your answer in meters to four significant figures.

ANSWER: $d_A = 6.375$ m

Express your answers in meters to three significant figures.

ANSWER: $d = 5.62$, 5.63 m

Problem 21.75

Description: A flutist assembles her flute in a room where the speed of sound is 340 m/s. When she plays the note A, it is in perfect tune with a 440 Hz tuning fork. After a few minutes, the air inside her flute has warmed to where the speed of sound is v_2.

Part A

How many beats per second will she hear if she now plays the note A as the tuning fork is sounded?

1. Flute frequency in air $f_1 = \frac{340}{\lambda} Hz = 440 Hz \quad \lambda = \frac{340}{440} m$

2. Flute frequency in air $f_2 = \frac{v_2}{\lambda} Hz$

Beats frequency $f_{beat} = f_2 - f_1 = \frac{v_2 - 340}{\lambda} = \frac{(v_2 - 340) \cdot 440}{340}$

ANSWER: $\frac{(v_2 - 340) \cdot 440}{340} + 0.05$ beats

Part B

How far does she need to extend the "tuning joint" of her flute to be in tune with the tuning fork?
Assuming after extending the tuning joint the wavelength of note A traveling in room2 is

$$\lambda' = \frac{v_2}{f_1} = \frac{v_2}{440} m$$

The original wavelength of note A is

$$\lambda = \frac{340}{440} m$$

$$2\Delta L = \Delta\lambda = \lambda' - \lambda = \left(\frac{v_2}{440} - \frac{340}{440}\right) m$$

$$\Delta L = \frac{\left(\frac{v_2}{440} - \frac{340}{440}\right)}{2} * 1000 mm$$

ANSWER: $\frac{\frac{1000 \cdot 340\left(\frac{v_2}{340} - 1\right)}{440} \cdot 1}{2}$ mm

Problem 21.40

Description: A violinist places her finger so that the vibrating section of a mu string has a length of L, then she draws her bow across it. A listener nearby in a 20 degree(s) C room hears a note with a wavelength of lambda. (a) What is the tension in the string?

A violinist places her finger so that the vibrating section of a 1.30 g/m string has a length of 40.0 cm, then she draws her bow across it. A listener nearby in a $20^\circ C$ room hears a note with a wavelength of 50.0 cm.

Part A

What is the tension in the string?
The fundamental frequency of a vibrating string is

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Wave traveling in the room where the listener stays

$$f = \frac{v}{\lambda}$$

Therefore, $T = \mu(2Lf)^2 = \mu\left(2L\frac{v}{\lambda}\right)^2 = \mu\left(2L\frac{343}{\lambda}\right)^2$

ANSWER: $\mu\left(\frac{2L \cdot 343}{\lambda}\right)^2$ N

Problem 21.53

Description: A L_1-long wire with a linear density of mu passes across the open end of an L_2-long open-closed tube of air. If the wire, which is fixed at both ends, vibrates at its fundamental frequency, the sound wave it generates excites the second vibrational mode of the tube of air. What is the tension in the wire?

A 21.0 cm-long wire with a linear density of 10.0 g/m passes across the open end of an 84.0 cm-long open-closed tube of air. If the wire, which is fixed at both ends, vibrates at its fundamental frequency, the sound wave it generates excites the second vibrational mode of the tube of air. What is the tension in the wire?

Part A

Assume $v_{sound} = 340\ m/s$.
The fundamental frequency of the wire is

$$f = \frac{1}{2L_1}\sqrt{\frac{T}{\mu}}$$

The second vibration of the open-closed tube of air is

$$f_2 = \frac{3v}{4L_2} = f \quad f_2 = \frac{3v}{4L_2} = f$$

$$T = \mu(2L_1 f)^2 = \mu\left(2L_1 \frac{3 \cdot 340}{4L_2}\right)^2$$

ANSWER: $\left(\frac{3 \cdot 340}{4L_2}\right)^2 (2L_1)^2 \mu$ N

Problem 21.61

DescriptionTwo loudspeakers emit sound waves along the *x*-axis. A listener in front of both speakers hears a maximum sound intensity when speaker 2 is at the origin and speaker 1 is at $x =$0.480 m. If speaker 1 is slowly moved forward, the sound intensity decreases and then increases, reaching another maximum when speaker 1 is at $x =$ 0.890 m.

Part A

What is the frequency of the sound? Assume $v_{sound} = 340 \mathrm{m/s}$.
Assuming the listener is at position s on the x-axis.

$$s - x_1 = m\lambda$$

$$s - x_2 = (m-1)\lambda$$

$$x_2 - x_1 = \lambda = \frac{v}{f} = \frac{340m/s}{f}$$

$$f = \frac{340}{x_2 - x_1} Hz$$

ANSWER: $\frac{340}{x_2 - x_1}$ Hz

Part B

What is the phase difference between the speakers?
Consider while speaker1 is at x1.

$$\frac{2\pi}{\lambda} x_1 + \Delta\varphi = 2\pi$$

Phase difference $\varphi = 2\pi - \frac{2\pi}{\lambda} x_1 = 2\pi(1 - \frac{x_1}{x_2 - x_1}) rad$

ANSWER: $2 \cdot 3.14 \left(1 - \frac{x_1}{x_2 - x_1}\right)$ rad