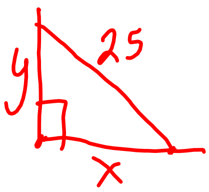
Name________________________________ Date_________________________
IB AP Calculus 2

Related Rates: Compare different rates of different quantities + arrive at a relationship between them.

1. Matt is standing at the top of a 25 ft. ladder that is leaning against a wall. Brian is at the foot of the ladder holding so that the ladder will not slip. Suddenly, Brian hears the ice cream truck, decides that he is hungry and runs away. Unfortunately for Matt, the bottom of the ladder starts to slip away from the wall at the rate of 3 ft/sec. How fast is the top of the ladder sliding down when the bottom of the ladder is 15 ft. from the wall?

deriv.



Step 1: Begin by defining the variable starting with t. (Include a diagram)

t = time in seconds

x = the number of feet between the wall + the foot of the ladder

y = the number of feet between the top of the ladder + the ground.

Step 2: Write down any numerical facts about x and y and their derivatives with respect to t. This is the information that we "know".

$\frac{dx}{dt} = 3$ ft/sec

Step 3: Write down what we wish to find. This is the "what".

Want $\frac{dy}{dt}$ when $x = 15$

Step 4: Write an equation to relate x and y. (Refer to your diagram)

$$x^2+y^2=25^2$$

Step 5: Differentiate both sides of the equation with respect to t.

$$\frac{2x\frac{dx}{dt}}{2}+\frac{2y\frac{dy}{dt}}{2}=\frac{0}{2}$$

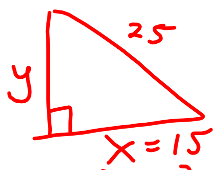
$$x\frac{dx}{dt}+y\frac{dy}{dt}=0$$

Step 6: Substitute your "know" and solve.

$$(15)(3)+(20)\frac{dy}{dt}=0$$

$$45+20\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=-2.25\text{ ft/sec}$$



$$15^2+y^2=25^2$$

$$y^2=400$$

$$y=20$$

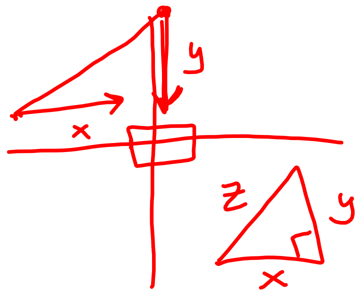
Step 7: Write a concluding sentence about your solution including direction and any other information that was asked.

When the bottom of the ladder is 15 ft from the wall the top of the ladder is sliding down at a rate of 2.25 ft/sec.

1

3. Two cars, one going due east at the rate of 90 km/hr and the other going due south at the rate of 60 km/hr, are traveling toward the intersection of two roads. At what rate are the cars approaching each other at the instant when the first car is .2 km and the second car is .15 km from the intersection?

Step 1: Define the variables and draw a diagram.



t = time in hours

x = distance from car1 to intersection (km)

y = distance from car 2 to intersection (km)

z = distance between the 2 cars

Step 2: Numerical facts about their variables and their derivatives; "know".

$\frac{dx}{dt} = -90$ km/hr $\frac{dy}{dt} = -60$ km/hr

Step 3: Write down what we wish to find; "what".

want $\frac{dz}{dt}$ when $x = .2$ km $y = .15$ km

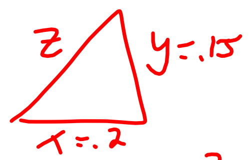
Step 4: Write an equation.

$$x^2 + y^2 = z^2$$

Step 5: Differentiate both sides of the equation with respect to t.

$$\frac{2x\frac{dx}{dt}}{2} + \frac{2y\frac{dy}{dt}}{2} = \frac{2z\frac{dz}{dt}}{2}$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = z\frac{dz}{dt}$$



$(.2)^2 + (.15)^2 = z^2$

$.0625 = z^2$

$z = .25$

Step 6: Substitute and solve.

$$(.2)(-90) + (.15)(-60) = (.25)\frac{dz}{dt}$$

$$-27 = .25\frac{dz}{dt}$$

$$\frac{dz}{dt} = -108 \text{ km/hr}$$

Step 7: Write a conclusion.

The cars are approaching each other at a rate of -108 km/hr when $x = .2$ & $y = .15$.

Page 150 #9, 40

9) Find the derivative: $r = 5\theta^2 \sec\theta$

$$\frac{dr}{d\theta} = 10\theta\sec\theta + 5\theta^2\sec\theta\tan\theta$$

40) Find the second derivative: $y = \csc x$

$$\frac{d^2y}{dx^2} = \cot^2 x\csc x + \csc^3 x$$

Page 170 Section 4.2 Exercises #1, 27

1) Find dy/dx: $x^2y + xy^2 = 6$

$$\frac{dy}{dx} = -\frac{2xy + y^2}{2xy + x^2}$$

$2x + 2y\frac{dy}{dx} = 0$

27) Use implicit to find dy/dx and the second derivative: $x^2 + y^2 = 1$ $\quad \frac{dy}{dx} = -\frac{2x}{2y}$

$\frac{dy}{dx} = -\frac{x}{y} = -\frac{x}{y}$ Quotient Rule

$\frac{d^2y}{dx^2} = \frac{(y)(-1) - (-x)\left(\frac{dy}{dx}\right)}{y^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2} \cdot \frac{y}{y}$

$= \frac{-y^2 - x^2}{y^3} = \frac{-(y^2+x^2)}{y^3}$

$\frac{d^2y}{dx^2} = -\frac{(x^2+y^2)}{y^3} = -\frac{1}{y^3} = -\frac{1}{y^3}$

Page 257 Section 5.6 Exercises #1, 3, 11

1) The radius r and the area A of a circle are related by the equation $A = \pi r^2$
Write an equation that relates dA/dt to dr/dt.

$$\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$$

3) The radius r, height h, and volume V of a right circular cylinder are related by the equation $V = \pi r^2 h$

a) How is dV/dt related to dh/dt if r is constant? $\frac{dV}{dt} = \pi r^2\frac{dh}{dt}$

b) How is dV/dt related to dr/dt if h is constant? $\frac{dV}{dt} = 2\pi rh\frac{dr}{dt}$

c) How is dV/dt related to dr/dt and dh/dt if neither r nor h is constant?

$$\frac{dV}{dt} = \pi r^2\frac{dh}{dt} + 2\pi rh\frac{dr}{dt}$$

11) A spherical balloon is inflated with helium at the rate of 100 pi ft³/min. Know $\frac{dV}{dt} = 100\pi$

a) How fast is the balloon's radius increasing at the instant the radius is 5 ft?

1 ft/min

want $\frac{dr}{dt}$ when $r = 5$

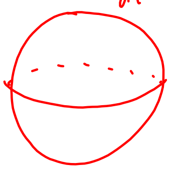
$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$

$100\pi = 4\pi(5)^2\boxed{\frac{dr}{dt}}$

$100\pi = 100\pi\frac{dr}{dt}$



b) How fast is the surface area increasing at that instant?

40 pi ft²/min

$r = 5$

$\frac{dr}{dt} = 1$

$SA = 4\pi r^2$

$\frac{dSA}{dt} = 8\pi r\frac{dr}{dt}$

$= 8\pi(5)(1) = 40\pi$ ft²/min

2. Water is flowing at the rate of 2 m³/min into a tank in the form of an inverted cone having an altitude of 16 m and a radius of 4 m. How fast is the water level rising when the water is 5 m deep?

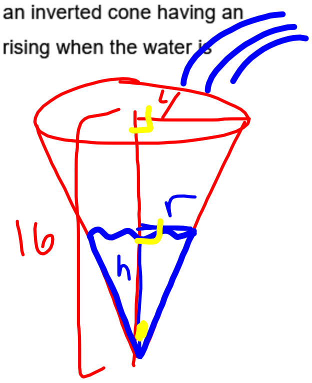
Step 1: Define the variables and draw a diagram.

t = time in minutes

h = height of water in meters

r = radius of water in meters

V = Volume of water (m^3)



"Know" Step 2: Numerical facts about their variables and their derivatives; "know".

$\frac{dV}{dt} = 2$

$\frac{16}{h} = \frac{4}{r}$

"Want" Step 3: Write down what we wish to find; "want".

$\boxed{\frac{dh}{dt}}$ when $h = 5$

$16r = 4h$

$r = \frac{h}{4}$

Step 4: Write an equation.

Product rule

$V = \left(\frac{1}{3}\pi r^2\right)(h)$

Step 5: ~~Differentiate~~ both sides of the equation with respect to t.

$\frac{dV}{dt} = \left(\frac{1}{3}\pi r^2\right)\left(\frac{dh}{dt}\right) + (h)\left(\frac{1}{3}\pi\cdot 2r\frac{dr}{dt}\right)$

Step 6: Substitute and solve.

$2 = \left(\frac{1}{3}\pi r^2\right)\boxed{\left(\frac{dh}{dt}\right)} + (5)\left(\frac{2}{3}\pi r\frac{dr}{dt}\right)$

Step 4 Redo: Write an equation.

$V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{h^2}{16}\right)h$

$V = \frac{\pi}{48}h^3$

$\frac{dV}{dt} = \frac{\pi}{\cancel{48}16}\cdot\cancel{3}h^2\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{16}\cdot h^2\frac{dh}{dt}$

$2 = \frac{\pi}{16}(5)^2\frac{dh}{dt}$

$2 = \frac{25\pi}{16}\frac{dh}{dt}$

$\frac{16}{25\pi}\cdot 2 = \frac{dh}{dt}$

$\boxed{+\frac{32}{25\pi}\text{ m/min} = \frac{dh}{dt}}$

Step 5 Redo: Differentiate both sides of the equation with respect to t.

Step 6: Substitute and solve.

Step 7: Write a conclusion.

5.

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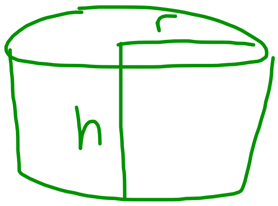
Question 3

Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5 centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?



"Know" $\frac{dV}{dt} = 2000\,cm^3/min$

"want" $\frac{dh}{dt}$ when

$\frac{dr}{dt} = 2.5$ cm/min

$r = 100$cm

$h = 0.5$cm

$V = \pi r^2 h$

$$\frac{dV}{dt} = (\pi r^2)\left(\frac{dh}{dt}\right) + (h)\left(\pi \cdot 2r\frac{dr}{dt}\right)$$

$$2000 = (\pi(100)^2)\boxed{\left(\frac{dh}{dt}\right)} + \left(\frac{1}{\cancel{2}}\right)\left(\pi \cdot \cancel{2}\,(\cancel{100}\ 50)\left(\frac{5}{\cancel{2}}\right)\right)$$

$$2000 = 10{,}000\pi\boxed{\frac{dh}{dt}} + 250\pi$$

$$\frac{2000 - 250\pi}{10{,}000\pi}\ \text{cm/min} = \frac{dh}{dt}$$

(a) When $r = 100$ cm and $h = 0.5$ cm, $\frac{dV}{dt} = 2000\ \text{cm}^3/\text{min}$ and $\frac{dr}{dt} = 2.5$ cm/min.

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$$

$$2000 = 2\pi(100)(2.5)(0.5) + \pi(100)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = 0.038 \text{ or } 0.039 \text{ cm/min}$$

$4 : \begin{cases} 1 : \frac{dV}{dt} = 2000 \text{ and } \frac{dr}{dt} = 2.5 \\ 2 : \text{expression for } \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

$$\frac{dh}{dt} = 0.038$$

$$0.039$$

6.

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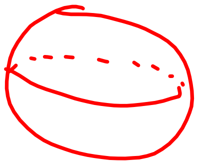
Question 5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

Want $\frac{dV}{dt}$ when $t=5$, $r = 30$ ft



$V = \frac{4}{3}\pi r^3$

$\boxed{\frac{dV}{dt}} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2 \cdot \frac{dr}{dt} = 4\pi(30)^2 r'(5)$

$4\pi(30)^2(2) = \boxed{8\pi(30)^2}$

$= 7200\pi \text{ ft}^3/\text{min}$

(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$

$\left.\frac{dV}{dt}\right|_{t=5} = 4\pi(30)^2 2 = \boxed{7200\pi \text{ ft}^3/\text{min}}$

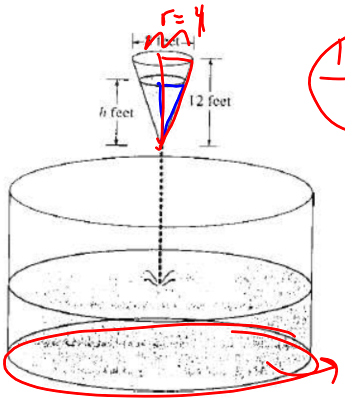
$3 : \begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

7.

1995 AB5/BC3

Answer

:



$\frac{12}{h} = \frac{4}{r}$

$12r = 4h$

$r = \frac{4h}{12} = \frac{h}{3}$

$A = 400\pi\ ft^2$

As shown in the figure above, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ feet per minute. (The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

$\frac{dh}{dt} = h-12$

(a) Write an expression for the volume of water in the conical tank as a function of h.

(b) At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate units of measure.

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

(a) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 \cdot h = \frac{1}{3}\cdot\frac{\pi}{1}\cdot\frac{h^2}{9}\cdot\frac{h}{1} = \boxed{\frac{\pi h^3}{27}}$

(b) Cone want $\frac{dV}{dt}$ when $h=3$

$\frac{dh}{dt} = h-12 = 3-12 = -9$

$V = \frac{\pi}{27}h^3$

$\boxed{\frac{dV}{dt}} = \frac{\pi}{27}\cdot 3h^2 \cdot \frac{dh}{dt} = \frac{\pi}{\cancel{27}}\cdot\cancel{3(3)^2}(-9) = -9\pi\ ft^3/min$

(c) Want $\frac{dy}{dt}$ when $h=3$

$\frac{dV}{dt}_{\text{Cylinder}} = +9\pi$ entering

$V = \pi r^2 h$

$V_{\text{Cylinder}} = (\text{Area circle base})\, h$

$V = 400\pi\, y$

$\frac{dV}{dt} = 400\pi \boxed{\frac{dy}{dt}}$

$\frac{9\pi}{400\pi} = \frac{400\pi}{400\pi}\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{9}{400}\ ft/min$

1995 AB5/BC3
Solution

(a) $\frac{r}{h}=\frac{4}{12}=\frac{1}{3}\quad r=\frac{1}{3}h$

$V=\frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h=\frac{\pi h^3}{27}$

(b) $\frac{dV}{dt}=\frac{\pi h^2}{9}\frac{dh}{dt}$

$=\frac{\pi h^2}{9}(h-12)=-9\pi$

V is decreasing at 9π ft^3/min

(c) Let W = volume of water in cylindrical tank

$W=400\pi y;\quad \frac{dW}{dt}=400\pi\frac{dy}{dt}$

$400\pi\frac{dy}{dt}=9\pi$

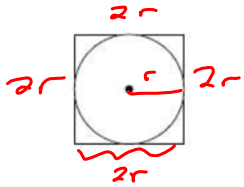
y is increasing at $\frac{9}{400}$ ft/min

☆

8.

Answer

1994 AB 5-BC 2



Know $\frac{dC}{dt} = +6 \text{ in/sec}$

A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$)

(a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

$P =$ $\frac{dP}{dt}$

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

when $A = 25\pi = \pi r^2$

want $\frac{dA}{dt}$

$25 = r^2$

$r = 5$

(a) $P = 8r$

$$\boxed{\frac{dP}{dt}} = 8\frac{dr}{dt}$$

$$\frac{dC}{dt} = 6$$

$$C = 2\pi r$$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

$$\frac{6}{2\pi} = \frac{2\pi}{2\pi}\frac{dr}{dt}$$

$$\frac{3}{\pi} = \frac{dr}{dt}$$

know

$$\frac{dP}{dt} = \frac{8}{1}\left(\frac{3}{\pi}\right) = \frac{24}{\pi} \text{ in/sec}$$

(b) Area = Area Square − Area circle

$$A = (2r)^2 - \pi r^2$$

$$A = 4r^2 - \pi r^2$$

$$\boxed{\frac{dA}{dt}} = 4 \cdot 2r\frac{dr}{dt} - \pi \cdot 2r\frac{dr}{dt}$$

$$= 8(5)\left(\frac{3}{\pi}\right) - \cancel{\pi} \cdot 2(5)\left(\frac{3}{\cancel{\pi}}\right)$$

$$\boxed{\frac{120}{\pi} - 30 \text{ in}^2/\text{sec}}$$

$\boxed{r = 5}$

$$\frac{dr}{dt} = \frac{3}{\pi}$$

1994 AB 5-BC 2

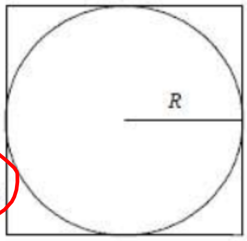
(a) $P = 8R$

$\frac{dP}{dt} = 8\frac{dR}{dt}$

$6 = \frac{dC}{dt} = 2\pi\frac{dR}{dt}$

$\frac{dR}{dt} = \frac{3}{\pi}$; $\frac{dP}{dt} = \frac{24}{\pi}$ inches/second

≈ 7.639 inches/second



(b) Area $= 4R^2 - \pi R^2$

$\frac{d(\text{Area})}{dt} = 8R\frac{dR}{dt} - 2\pi R\frac{dR}{dt}$

$= (4-\pi)2R\frac{dR}{dt}$

Area of circle $= 25\pi = \pi R^2$

$R = 5$

$\frac{d(\text{Area})}{dt} = \frac{120}{\pi} - 30$ inches^2/second

$= (4-\pi)\frac{30}{\pi}$ inches^2/second

≈ 8.197 inches^2/second

Answer

Know $\frac{dx}{dt} = +5\text{ ft/sec}$

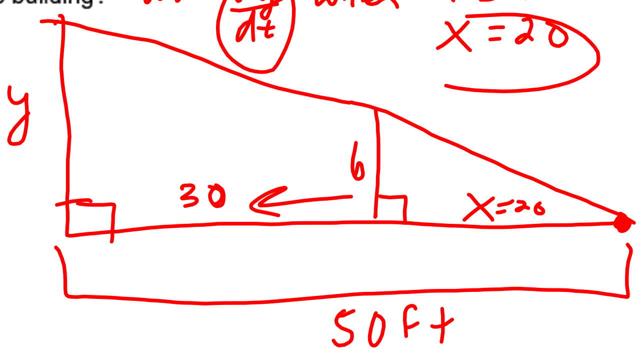
10. A man 6 feet tall is walking toward a building at the rate of 5 ft/sec. If there is a light on the ground 50 feet from the building, how fast is the man's shadow on the building growing shorter when he his 30 ft from the building?

want $\frac{dy}{dt}$ when ~~x=30~~ $x = 20$

$\frac{-75}{20} = -\frac{15}{4}$ ft/sec

y = height of shadow on the building

x = distance between man & light



Similar Δ's $\quad \frac{50}{x} = \frac{y}{6}$

$xy = 300 \longrightarrow \frac{(20)y}{20} = \frac{300}{20}$

$y = 15$

Product Rule

$(x)\boxed{\left(\frac{dy}{dt}\right)} + (y)\left(\frac{dx}{dt}\right) = 0$

$(20)\boxed{\frac{dy}{dt}} + (15)(5) = 0$

$20\frac{dy}{dt} + 75 = 0$

$\frac{dy}{dt} = \frac{-75}{20} = -\frac{15}{4}$ ft/sec

Homework

Page 257-258 #9, 16

9) The length l of a rectangle is decreasing at the rate of 2 cm/sec while the width w is increasing at the rate of 2 cm/sec. When l = 12 cm and w = 5 cm, find the rates of change of:

a) the area 14 cm^2/sec

b) the perimeter 0 cm/sec

c) the length of the diagonal of the rectangle
-14/13 cm/sec

d) Which of these quantities are decreasing and which are increasing?

Area increasing, derivative is positive

Perimeter is not changing, derivative is 0

Diagonal length is decreasing, derivative is neg.

16) Sand falls from a conveyer belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eigths of the base diameter. How fast are the

(a) height 90/(256pi) m/min

(b) and radius changing 120/(256pi) m/min

when the pile is 4 m high?

Page 258-260 #19, 42

19) A 13 foot ladder is leaning against a house when its base starts to slide away. By the time the base is 12ft from the house, the base is moving at the rate of 5 ft/sec.

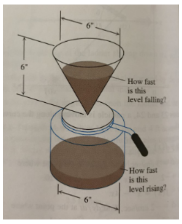
(a) How fast is the top of the ladder sliding down the wall at that moment? 12 ft/sec

(b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing at that moment? -119/2 ft^2/sec

(c) At what rate is the angle between the ladder and the ground changing at that moment?

-1radian/sec

42) Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of 10 in^3/min.

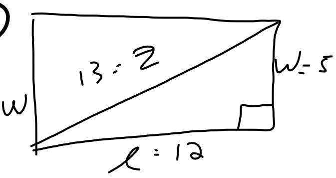


(a) How fast is the level in the pot rising when the coffee in the cone is 5 in. deep?

10/(9pi) in/min

(b) How fast is the level in the cone falling at that moment? 8/(5pi) in/min

(c)

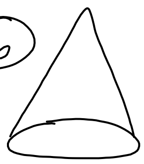


$w^2 + l^2 = z^2$

$2w\frac{dw}{dt} + 2l\frac{dl}{dt} = 2z\frac{dz}{dt}$

$(5)(2) + (12)(-2) = (13)\frac{dz}{dt}$

(16)



know

$\frac{dV}{dt} = 10$

$h = \frac{3}{8}D$

want $\frac{dh}{dt}$

$\frac{dr}{dt}$

when $h = 4$

$\boxed{V = \frac{1}{3}\pi r^2 h}$

$h = \frac{3}{\cancel{8}\,4}(\cancel{2}r) \rightarrow h = \frac{3}{4}r$

$h = \frac{3}{4}r$

$r = \frac{4}{3}h$

$\frac{dh}{dt} = \frac{3}{4}\frac{dr}{dt}$

$\frac{90}{256\pi} = \frac{3}{4}\frac{dr}{dt}$

$\frac{4}{\cancel{3}} \cdot \frac{\overset{30}{\cancel{90}}}{256\pi} = \frac{dr}{dt}$

$\frac{120}{256\pi}$ m/min $= \frac{dr}{dt}$

$V = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{16}{9}h^2\right)h$

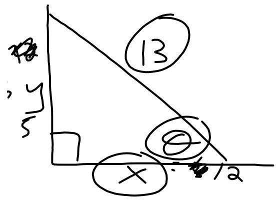
$V = \frac{16\pi}{27}h^3$

$\frac{dV}{dt} = \frac{16\pi}{\cancel{27}\,9}\cancel{3}h^2\frac{dh}{dt}$

$10 = \frac{16\pi}{9}(4)^2\frac{dh}{dt}$

$10 = \frac{256\pi}{9}\frac{dh}{dt}$

$\boxed{\frac{90}{256\pi} \text{ m/min} = \frac{dh}{dt}}$



$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \left(\frac{1}{2}x\right)\frac{dy}{dt} + y\left(\frac{1}{2}\frac{dx}{dt}\right)$

$\cos\theta = \frac{x}{13} = \frac{1}{13}\cdot x$

$-\sin\theta \boxed{\frac{d\theta}{dt}} = \frac{1}{13}\frac{dx}{dt}$

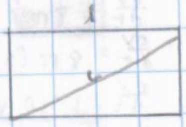
$-\frac{5}{13}\frac{d\theta}{dt} = \frac{1}{13}\cdot 5$

$\boxed{\frac{d\theta}{dt} = -1 \text{ radian/sec}}$

Textbook HW

pg 257-258

(9)



c = length diagonal
t = time in sec
l = length in cm
w = width in cm
p = perimeter in m
a = area in m^2

$\frac{dl}{dt} = -2$ cm/sec
$\frac{dw}{dt} = 2$ cm/sec

a) $a = lw$

$\frac{da}{dt} = \frac{dl}{dt} \cdot w + \frac{dw}{dt} \cdot l$

$\frac{da}{dt} = (-2\text{ cm/sec})(5) + (2\text{ cm/sec})(12)$

$\frac{da}{dt} = -10 + 24$

$\boxed{\frac{da}{dt} = 14\ \text{cm}^2/\text{sec}}$

b) $p = l + l + w + w$

$\frac{dp}{dt} = \frac{dl}{dt} + \frac{dl}{dt} + \frac{dw}{dt} + \frac{dw}{dt}$

$\frac{dp}{dt} = -2 - 2 + 2 + 2$

$\boxed{\frac{dp}{dt} = 0\ \text{cm/sec}}$

c) $c = \sqrt{l^2 + w^2}$

$c = (l^2 + w^2)^{\frac{1}{2}}$

$\frac{dc}{dt} = \frac{1}{2}(l^2 + w^2)^{-\frac{1}{2}} \cdot \left(2l\frac{dl}{dt} + 2w\frac{dw}{dt}\right)$

$\frac{dc}{dt} = \frac{1}{2}(12^2 + 5^2)^{-\frac{1}{2}}\left(2(12)(-2) + 2(5)(2)\right)$

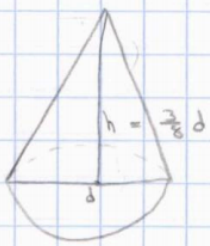
$\frac{dc}{dt} = \frac{1}{2}(169)^{-\frac{1}{2}}(-28)$

$\frac{dc}{dt} = \frac{1}{2} \cdot \frac{1}{13} \cdot (-28)$

$\boxed{\frac{dc}{dt} = \frac{-14}{13}\ \text{cm/sec}}$

d) a - increasing, $\frac{da}{dt}$ is pos
p - not changing $\frac{dp}{dt} = 0$
c - decreasing, $\frac{dc}{dt}$ is neg

(16)



t = time in min
h = height in m
d = diameter in m
r = radius in m
v = volume in m^3

$h = \frac{3}{8}d$

$h = \frac{3}{8}(2r)$

$r = \frac{8h}{6}$

$r = \frac{4h}{3}$

$\frac{dv}{dt} = 10\ m^3/\text{min}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{4h}{3}\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{16}{9}h^2\right)h$

$V = \frac{16}{27}\pi h^3$

$\frac{dv}{dt} = \frac{16}{27}\pi \cdot 3h^2\frac{dh}{dt}$

$\frac{dv}{dt} = \frac{48}{27}\pi h^2\frac{dh}{dt}$

$10 = \frac{48}{27}\pi(4)^2\frac{dh}{dt}$

$10 = \frac{768}{27}\pi\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{270}{768\pi}$

$\frac{dh}{dt} = \frac{45}{128\pi}$ m/min

$\frac{dh}{dt} = \frac{4500}{128\pi}$ cm/min

$\boxed{\frac{dh}{dt} = \frac{1125}{32\pi}\ \text{cm/min}}$

$r = \frac{4}{3}h$

$\frac{dr}{dt} = \frac{4}{3}\frac{dh}{dt}$

$\frac{dr}{dt} = \frac{4}{3}\left(\frac{1125}{32\pi}\right)$

$\frac{dr}{dt} = \frac{4500}{96\pi}$

$\boxed{\frac{dr}{dt} = \frac{375}{8\pi}\ \text{cm/min}}$

Pg 258-260

19a) $y = \sqrt{-x^2 + 13^2}$

$\frac{dy}{dt} = \frac{1}{2}(-x^2+13^2)^{-\frac{1}{2}} \cdot -2x\,\frac{dx}{dt}$

$\frac{dy}{dt} = \frac{1}{2}(-12^2+13^2)^{-\frac{1}{2}} \cdot -2(-12)(5)$

$\frac{dy}{dt} = \frac{1}{2} \cdot \frac{1}{5} \cdot 120$

$\frac{dy}{dt} = \frac{120}{10}$

$\boxed{\frac{dy}{dt} = 12 \text{ ft/sec}}$

19b) Area $= \frac{1}{2}bh$

$A = \frac{1}{2}xy$

$\frac{dA}{dt} = \frac{1}{2}x\,\frac{dy}{dt} + \frac{1}{2}y\,\frac{dx}{dt}$

$\frac{dA}{dt} = \frac{1}{2}(12)(-12) + \frac{1}{2}(5)(5)$

$\frac{dA}{dt} = -72 + 12.5$

$\boxed{\frac{dA}{dt} = -59.5 \text{ ft}^2\text{/sec}}$

$y = \sqrt{-x^2+13^2}$

$y = \sqrt{-12^2+13^2}$

$y = 5$

19c) $\sin\theta = \frac{y}{13}$

$\cos\theta\,\frac{d\theta}{t} = \frac{1}{13} \cdot \frac{dy}{dt}$

$\frac{d\theta}{t} = \frac{\frac{1}{13} \cdot \frac{dy}{dt}}{\cos\theta}$

$\frac{d\theta}{t} = \frac{\frac{1}{13} \cdot -12}{\frac{12}{13}}$

$\boxed{\frac{d\theta}{t} = -1 \text{ rad/sec}}$

42a) let h = height of coffee in pot

v = volume of pot

r = radius of pot

$V = \pi r^2 h$

$V = \pi(3)^2 h$

$\frac{dV}{dt} = 9\pi\,\frac{dh}{dt}$

$10 = 9\pi\,\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{10}{9\pi} \text{ in/min}}$

42b) $V = \frac{1}{3}\pi r^2 h$

$\frac{6}{h} = \frac{3}{r}$

$6r = 3h$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2\,\frac{dh}{dt}$

$-10 = \frac{\pi}{12} \cdot 3(5)^2\,\frac{dh}{dt}$

$-10 = \frac{75\pi}{12}\,\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-120}{75\pi} \text{ in/min}}$

pg 257-258

9

a) $14\ cm^2/sec$

b) 0 cm/sec

c) $-\frac{14}{13}$ cm/sec

d)

a) "know" $\frac{d\ell}{dt} = -2$ cm/sec

$\frac{dw}{dt} = +2$ cm/sec

"want" $\frac{dA}{dt}$ when $\ell = 12$ $w = 5$

$A = \ell \cdot w$

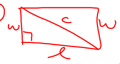
$\boxed{\frac{dA}{dt}} = \ell\frac{dw}{dt} + w\frac{d\ell}{dt}$

$= (12)(2) + (5)(-2)$

$= 24 - 10$

$\boxed{\frac{dA}{dt} = 14\ cm^2/sec}$

c)



$w^2 + \ell^2 = c^2$

$\cancel{2}w\frac{dw}{dt} + \cancel{2}\ell\frac{d\ell}{dt} = \cancel{2}c\frac{dc}{dt}$

$(5)(2) + (12)(-2) = (13)\frac{dc}{dt}$

$10 - 24 = 13\frac{dc}{dt}$

$\frac{-14}{13} = \frac{dc}{dt}$

16 Hourglass

know $\frac{dV}{dt} = 10\ m^3/min$

$\rightarrow h = \frac{3}{8}d = \frac{3}{8}(2r) = \frac{3}{4}r$

a) $\frac{dh}{dt}$ h = 4m

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h$

$V = \frac{16}{27}\pi h^3$

$\frac{dV}{dt} = \frac{16}{\cancel{27}_9}\pi \cdot \cancel{3}h^2\frac{dh}{dt}$

$10 = \frac{16}{9}\pi(4)^2\frac{dh}{dt}$

$10 = \frac{256\pi}{9}\frac{dh}{dt}$

$\frac{90}{256\pi}$ m/min $= \frac{dh}{dt}$

b) $\frac{dr}{dt}$ h = 4

$\frac{3}{4}r = 4$

$r = \frac{16}{3}$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{\cancel{3}}\pi r^2\left(\frac{\cancel{3}}{4}r\right)$

$V = \frac{1}{4}\pi r^3$

$\frac{dV}{dt} = \frac{1}{4}\pi \cdot 3r^2\frac{dr}{dt}$

$10 = \frac{1}{4}\pi \cdot 3\left(\frac{16}{3}\right)^2\frac{dr}{dt}$

$10 = \frac{1}{4}\pi \cdot \cancel{3} \cdot \frac{256}{\cancel{9}_3}\frac{dr}{dt}$

$10 = \frac{256\pi}{12}\frac{dr}{dt}$

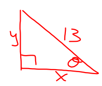
$\frac{120}{256\pi} = \frac{dr}{dt}$

19

a) 12 ft/sec

b) $-\frac{119}{2}\ ft^2/sec$

c) -1 radian/sec



b) $\frac{dA}{dt}$ when $x = 12$

$\frac{dx}{dt} = 5$ ft/sec

$A = \left(\frac{1}{2}x\right)(y)$

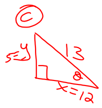
$\frac{dA}{dt} = \left(\frac{1}{2}x\right)\left(\frac{dy}{dt}\right) + (y)\left(\frac{1}{2}\frac{dx}{dt}\right)$

$\frac{dA}{dt} = \left(\frac{1}{2}\right)(12)(-12) + (5)\left(\frac{1}{2}\right)(5)$

$= \frac{-144}{2} + \frac{25}{2}$

$= -\frac{119}{2}$

c)



$\sin\theta = \frac{y}{13} = \frac{1}{13}y$

$\cos\theta \cdot \boxed{\frac{d\theta}{dt}} = \frac{1}{13}\left(\frac{dy}{dt}\right)$

$\left(\frac{12}{13}\right)\frac{d\theta}{dt} = \frac{1}{13}(-12)$

$\frac{12}{13}\frac{d\theta}{dt} = \frac{-12}{13}$

$\frac{d\theta}{dt} = -1$ radian/sec

pg 260

42

a $\frac{10}{9\pi} \approx 0.354$

b $\frac{8}{5\pi} \approx 0.509$

$\frac{dV}{dt} = -10 \text{ in}^3/\text{min}$

$\frac{6}{3} = \frac{h}{r}$

$h = \frac{6r}{3} = 2r$

$r = \frac{1}{2}h$

$\frac{dV}{dt} = 10 \text{ in}^3/\text{min}$

a want $\boxed{\frac{dh}{dt} \text{ of cylinder}}$ when cone $h = 5$

$V = \pi r^2 h$

$V = \pi (3)^2 h$

$V = 9\pi h$

$\frac{dV}{dt} = 9\pi \boxed{\frac{dh}{dt}}$

$10 = 9\pi \frac{dh}{dt}$

$\boxed{\frac{10}{9\pi} = \frac{dh}{dt}}$

b $\frac{dh}{dt}$ cone when cone $h = 5$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$

$V = \frac{1}{12}\pi h^3$

$\frac{dV}{dt} = \frac{1}{\cancel{12}_4}\pi \cdot \cancel{3}h^2 \frac{dh}{dt}$

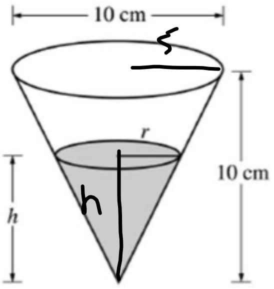
$-10 = \frac{1}{4}\pi (5)^2 \frac{dh}{dt}$

$-10 = \frac{25\pi}{4} \frac{dh}{dt}$

$\frac{-40}{25\pi} = \frac{dh}{dt}$

$-\frac{8}{5\pi}$ ✓

9.



$\frac{10}{h} = \frac{5}{r}$

know

$\frac{dh}{dt} = -\frac{3}{10}$

A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(Note: The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

a $V = \frac{1}{3}\pi r^2 h$

$\frac{10}{5} = \frac{5}{r}$

$r = \frac{5}{2}$

$V = \frac{1}{3}\pi \left(\frac{5}{2}\right)^2 (5)$

$V = \frac{125\pi}{12}$ cm^3

b

$\frac{10}{h} = \frac{5}{r}$

$r = \frac{h}{2}$

$V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h$

$V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{\cancel{12}_{4}} \cdot \cancel{3} h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{4}(5)^2\left(-\frac{3}{10}\right)$

$\frac{dV}{dt} = -\frac{75\pi}{40}$ cm^3/hr

(a) When $h = 5$, $r = \frac{5}{2}$; $V(5) = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 5 = \frac{125}{12}\pi \text{ cm}^3$	$1:\ V$ when $h = 5$
(b) $\frac{r}{h} = \frac{5}{10}$, so $r = \frac{1}{2}h$ $V = \frac{1}{3}\pi\left(\frac{1}{4}h^2\right)h = \frac{1}{12}\pi h^3;\ \frac{dV}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt}$ $\left.\frac{dV}{dt}\right\rvert_{h=5} = \frac{1}{4}\pi(25)\left(-\frac{3}{10}\right) = -\frac{15}{8}\pi \text{ cm}^3/\text{hr}$ OR $\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2\frac{dh}{dt} + 2rh\frac{dr}{dt}\right);\ \frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$ $\left.\frac{dV}{dt}\right\rvert_{h=5,r=\frac{5}{2}} = \frac{1}{3}\pi\left(\left(\frac{25}{4}\right)\left(-\frac{3}{10}\right) + 2\left(\frac{5}{2}\right)5\left(-\frac{3}{20}\right)\right)$ $= -\frac{15}{8}\pi \text{ cm}^3/\text{hr}$	5 $\begin{cases} 1:\ r = \frac{1}{2}h \text{ in (a) or (b)} \\ 1:\ \begin{cases} V \text{ as a function of one variable in (a) or (b)} \\ \text{OR} \\ \frac{dr}{dt} \end{cases} \\ 2:\ \frac{dV}{dt} \\ \quad < -2 > \text{ chain rule or product rule error} \\ 1:\ \text{evaluation at } h = 5 \end{cases}$
(c) $\frac{dV}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt} = -\frac{3}{40}\pi h^2$ $= -\frac{3}{40}\pi(2r)^2 = -\frac{3}{10}\pi r^2 = -\frac{3}{10}\cdot \text{area}$ The constant of proportionality is $-\frac{3}{10}$.	2 $\begin{cases} 1:\ \text{shows } \frac{dV}{dt} = k\cdot \text{area} \\ 1:\ \text{identifies constant of proportionality} \end{cases}$
units of cm^3 in (a) and cm^3/hr in (b)	**1 : correct units in (a) and (b)**

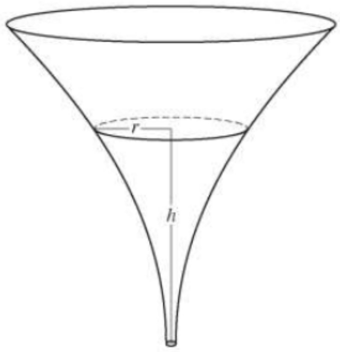
Skip

11. Two sides of a triangle have lengths 12 m and 15 m. The angle between them is increasing at a rate of 2 degrees per minute. How fast is the length of the third side increasing when the angle between the sides of fixed length is 60 degrees?

Then go to logs notebook

12.

2016 AP® CALCULUS BC FREE-RESPONSE QUESTIONS



5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height h, the radius of the funnel is given by $r = \frac{1}{20}(3 + h^2)$, where $0 \le h \le 10$. The units of r and h are inches.

~~(a)~~ Find the average value of the radius of the funnel.

~~(b)~~ Find the volume of the funnel.

(c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is $h = 3$ inches, the radius of the surface of the liquid is decreasing at a rate of $\frac{1}{5}$ inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?

When $h = 3$, $\frac{dr}{dt} = -\frac{1}{5}$

what is $\frac{dh}{dt}$

$r = \frac{1}{20}(3 + h^2)$

$3 + h^2$

$\frac{dr}{dt} = \frac{1}{\cancel{20}\ 10} \cdot \cancel{2} h \frac{dh}{dt}$

$-\frac{1}{5} = \frac{1}{10}(3)\frac{dh}{dt}$

$-\frac{10}{15} = -\frac{2}{3}$ in/sec $= \frac{dh}{dt}$

12.

(a) Average radius $= \frac{1}{10}\int_0^{10} \frac{1}{20}\left(3+h^2\right)\,dh = \frac{1}{200}\left[3h+\frac{h^3}{3}\right]_0^{10}$

$= \frac{1}{200}\left(\left(30+\frac{1000}{3}\right)-0\right) = \frac{109}{60}$ in

$3: \begin{cases} 1: \text{integral} \\ 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$

(b) Volume $= \pi\int_0^{10}\left(\left(\frac{1}{20}\right)\left(3+h^2\right)\right)^2 dh = \frac{\pi}{400}\int_0^{10}\left(9+6h^2+h^4\right)dh$

$= \frac{\pi}{400}\left[9h+2h^3+\frac{h^5}{5}\right]_0^{10}$

$= \frac{\pi}{400}\left(\left(90+2000+\frac{100000}{5}\right)-0\right) = \frac{2209\pi}{40}$ in^3

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$

(c) $\frac{dr}{dt} = \frac{1}{20}(2h)\frac{dh}{dt}$

$-\frac{1}{5} = \frac{3}{10}\frac{dh}{dt}$

$\frac{dh}{dt} = -\frac{1}{5}\cdot\frac{10}{3} = -\frac{2}{3}$ in/sec

$3: \begin{cases} 2: \text{chain rule} \\ 1: \text{answer} \end{cases}$

Homework MC

Page 14 #11 = E

$\frac{dr}{dt} = 0.3 = \frac{3}{10}$

11. The radius r of a sphere ~~is increasing~~ at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$ want $\frac{dV}{dt}$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

$\frac{100\pi}{4\pi} = \frac{4\pi r^2}{4\pi}$

$25 = r^2$

$r = 5$

$\frac{dV}{dt} = \cancel{\frac{4}{3}}\pi \cancel{3} r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(5)^2\left(\frac{3}{10}\right) = 10\cancel{0}\pi \cdot \frac{3}{\cancel{10}} = 30\pi$

Page 18 #28 = B

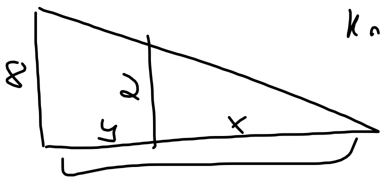
28. The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

Page 20 #34 = D

34. A person 2 meters tall walks directly away from a streetlight that is 8 meters above the ground. If the person is walking at a constant rate and the person's shadow is lengthening at the rate of $\frac{4}{9}$ meter per second, at what rate, in meters per second, is the person walking?

(A) $\frac{4}{27}$ (B) $\frac{4}{9}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$ (E) $\frac{16}{9}$



Know $\frac{dx}{dt} = +\frac{4}{9}$

$\frac{x}{2} = \frac{x+y}{8}$

$8x = 2x + 2y$

$6x = 2y$

$x = \frac{1}{3}y$

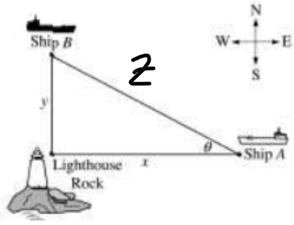
$\frac{dx}{dt} = \frac{1}{3}\frac{dy}{dt}$

$\frac{4}{9} = \frac{1}{3}\frac{dy}{dt}$

$3 \cdot \frac{4}{9} = \boxed{\frac{4}{3} = \frac{dy}{dt}}$

4. Homework AP FR Page 2 (page 3)

Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.



(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

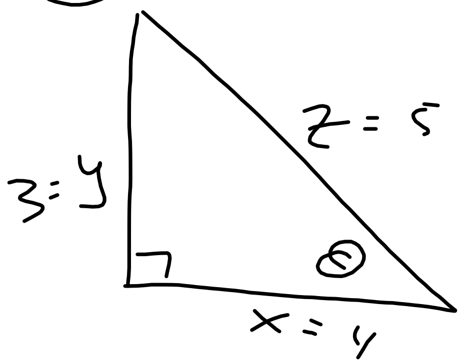
(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

know $\frac{dx}{dt} = -15$

$\frac{dy}{dt} = 10$

$\frac{dz}{dt} = -6$

c



$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$

$\tan\theta = \frac{y}{x}$

$\downarrow$

$$\underbrace{\sec^2\theta}\,\frac{d\theta}{dt} = \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}$$

$$\left(\frac{5}{4}\right)^2 \frac{d\theta}{dt} = \frac{(4)(10) - (3)(-15)}{4^2}$$

$$\frac{25}{16}\frac{d\theta}{dt} = \frac{40 + 45}{16}$$

$$\frac{d\theta}{dt} = \frac{85}{\cancel{16}} \cdot \frac{\cancel{16}}{25}$$

$$= \frac{85}{25} \text{ radians/hr}$$

(a) Distance $= \sqrt{3^2 + 4^2} = 5$ km

1 : answer

(b) $r^2 = x^2 + y^2$

$$2r\frac{dr}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

or explicitly:

$$r = \sqrt{x^2 + y^2}$$

$$\frac{dr}{dt} = \frac{1}{2\sqrt{x^2 + y^2}}\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$$

At $x = 4$, $y = 3$,

$$\frac{dr}{dt} = \frac{4(-15) + 3(10)}{5} = -6 \text{ km/hr}$$

4 $\begin{cases} 1 : \text{expression for distance} \\ 2 : \text{differentiation with respect to } t \\ \quad < -2 > \ \text{chain rule error} \\ 1 : \text{evaluation} \end{cases}$

(c) $\tan\theta = \dfrac{y}{x}$

$$\sec^2\theta\,\frac{d\theta}{dt} = \frac{\frac{dy}{dt}x - \frac{dx}{dt}y}{x^2}$$

At $x = 4$ and $y = 3$, $\sec\theta = \dfrac{5}{4}$

$$\frac{d\theta}{dt} = \frac{16}{25}\left(\frac{10(4) - (-15)(3)}{16}\right)$$

$$= \frac{85}{25} = \frac{17}{5} \text{ radians/hr}$$

4 $\begin{cases} 1 : \text{expression for } \theta \text{ in terms of } x \text{ and } y \\ 2 : \text{differentiation with respect to } t \\ \quad < -2 > \ \text{chain rule, quotient rule, or} \\ \qquad\qquad \text{transcendental function error} \\ \quad \text{note: } 0/2 \text{ if no trig or inverse trig} \\ \qquad \text{function} \\ 1 : \text{evaluation} \end{cases}$

Homework page 5 says page 9 on bottom

$16 + \cancel{12x^2} - \cancel{12x^2} + 12 = 1$ $\quad 28 = 1$

2. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.
   (a) Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.
   (b) Write an equation of each horizontal tangent line to the curve. $\frac{dy}{dx} = 0$
   (c) The line through the origin with slope -1 is tangent to the curve at point P. Find the x- and y-coordinates of point P.

(a) $\underbrace{6y^2\frac{dy}{dx}} + \underbrace{(6x^2)\frac{dy}{dx}} + (y)(12x) - 24x + \underbrace{6\frac{dy}{dx}} = 0$

$$\frac{dy}{dx} = \frac{24x - 12yx}{6y^2 + 6x^2 + 6} = \frac{\cancel{6}(4x - 2xy)}{\cancel{6}(x^2 + y^2 + 1)}$$

(b) $\frac{dy}{dx} = 0 = 4x - 2xy$

$2x(2 - y) = 0$

$x = 0 \quad y = 2$

Go back to the curve

$x = 0$	$y = 2$
$2y^3 + 6y = 1$	no solution
need calc.	
$y = .165$	
$(0, .165)$	
$m = 0$	
$y - .165 = 0(x - 0)$	
$y = .165$	

(c)

$(0,0)$, (x, y) } slope $= -1 = \frac{dy}{dx}$

$\frac{y - 0}{x - 0} = -1$

$\frac{y}{x} = -1$

$y = -x$

$$-1 = \frac{4x - 2xy}{x^2 + y^2 + 1}$$

$$-1 = \frac{4x - 2x(-x)}{x^2 + (-x)^2 + 1}$$

$$-1 = \frac{4x + 2x^2}{2x^2 + 1}$$

$-2x^2 - 1 = 4x + 2x^2$

$0 = 4x^2 + 4x + 1$

$0 = (2x + 1)(2x + 1)$

$x = -\frac{1}{2}$

$y = \frac{1}{2}$

(a) $6y^2\frac{dy}{dx} + 6x^2\frac{dy}{dx} + 12xy - 24x + 6\frac{dy}{dx} = 0$

$\frac{dy}{dx}(6y^2 + 6x^2 + 6) = 24x - 12xy$

$\frac{dy}{dx} = \frac{24x - 12xy}{6x^2 + 6y^2 + 6} = \frac{4x - 2xy}{x^2 + y^2 + 1}$

$$2\begin{cases} 1: & \text{implicit differentiation} \\ 1: & \text{verifies expression for } \frac{dy}{dx} \end{cases}$$

(b) $\frac{dy}{dx} = 0$

$4x - 2xy = 2x(2 - y) = 0$

$x = 0$ or $y = 2$

When $x = 0$, $2y^3 + 6y = 1$; $y = 0.165$

There is no point on the curve with y coordinate of 2.

$y = 0.165$ is the equation of the only horizontal tangent line.

$$4\begin{cases} 1: & \text{sets } \frac{dy}{dx} = 0 \\ 1: & \text{solves } \frac{dy}{dx} = 0 \\ 1: & \text{uses solutions for } x \text{ to find equations of horizontal tangent lines} \\ 1: & \text{verifies which solutions for } y \text{ yield equations of horizontal tangent lines} \end{cases}$$

Note: max 1/4 [1-0-0-0] if $dy/dx = 0$ is not of the form $g(x, y)/h(x, y) = 0$ with solutions for both x and y

c) $y = -x$ is equation of the line.

$2(-x)^3 + 6x^2(-x) - 12x^2 + 6(-x) = 1$

$-8x^3 - 12x^2 - 6x - 1 = 0$

$x = -1/2, \quad y = 1/2$

–or–

$\frac{dy}{dx} = -1$

$4x - 2xy = -x^2 - y^2 - 1$

$4x + 2x^2 = -x^2 - x^2 - 1$

$4x^2 + 4x + 1 = 0$

$x = -1/2, \quad y = 1/2$

$$3\begin{cases} 1: & y = -x \\ 1: & \text{substitutes } y = -x \text{ into equation of curve} \\ 1: & \text{solves for } x \text{ and } y \end{cases}$$

–or–

$$3\begin{cases} 1: & \text{sets } \frac{dy}{dx} = -1 \\ 1: & \text{substitutes } y = -x \text{ into } \frac{dy}{dx} \\ 1: & \text{solves for } x \text{ and } y \end{cases}$$

Note: max 2/3 [1-1-0] if importing incorrect derivative from part (a)