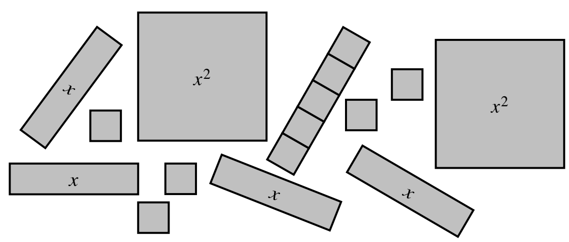
CPM EDUCATIONAL PROGRAM

SAMPLE LESSON: ALGEBRA TILES FOR FACTORING AND MORE
HIGH SCHOOL CONTENT

ALGEBRA TILES (MODELS)

Algebra Tiles are models that can be used to represent abstract concepts. This packet works with only one variable. You may either have tiles that are a blue color on one side, or a purple color on one side; both can be used to represent the variable x.

For students to take full advantage of the algebra tiles, your attitude is critical. The tiles provide students with the opportunity to "see" abstract algebraic expressions and equations with variables. They are not mere "math toys" or a diversion; rather, they will greatly enhance the learning of algebra for the majority of your students. Even if you are hesitant to use the tiles, presenting them to students in a positive manner is crucial for maximization of students' learning.



The activities and problems provided in this handout are written for use with students and are excerpts from CPM's middle school and high school courses, *Core Connections Course 3, Algebra,* and *Algebra 2*. The ability of your students to collaborate within teams and their previous exposure to algebra tiles will determine how much time these problems will take. We expect that students will be working in teams on these problems, and have access to algebra tiles. Pairs or teams of four are ideal. Let them work through the problems while you circulate and question students to check for understanding and to move their thinking forward. We strongly urge you to work all the problems before you assign them to your students. By doing this, you will be able to anticipate problems and prepare advancing questions. Also, there are several instances where the problem asks students to share their results with the class. By working the problems ahead of time, you will be aware of these problems and be prepared to orchestrate the sharing. These problems begin with a brief review of the tiles names (if tile are new to you and your students, you may want to begin with the algebra tiles lesson for middle school) quickly moves through expressions, solving equations, and ends with completing the square. Note: some problems mention a Learning Log. This is a notebook that students can use to write notes in and to reflect on specific topics. Any notebook will work. To learn more about these other resources see cpm.org.

Whenever you are using manipulatives, remember that it is important to have the students **Build it! Draw it! Write it!** This will help them transition from the concrete to the abstract.

EXPLORING VARIABLES AND EXPRESSIONS

In Algebra and in future mathematics courses, you will work with unknown quantities that can be represented using variables. To help you answer some important questions, we will introduce manipulatives called "algebra tiles" so you can consider questions such as *"What is a variable?"* and *"How can we use it?"*

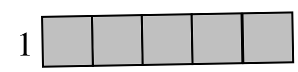
1. AREAS OF ALGEBRA TILES

   Remove one of each shape from the bag and put it on your desk. Trace around each shape on your paper. Look at the different sides of the shapes.

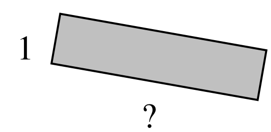
   a. With your team, discuss which shapes have the same side lengths and which ones have different side lengths. Be prepared to share your ideas with the class. On your traced drawings, color-code or somehow represent lengths that are the same.

   b. Each type of tile is named for its area. In this course, the smallest square will have a side length of 1 unit, so its area is 1 square unit. Thus, this tile will be called "one" or the "unit tile." Can you use the unit tile to determine the side lengths of the other rectangles? Why or why not?

   c. If the side lengths of a tile can be measured exactly, then the area of the tile can be calculated by multiplying these two lengths together. The area is measured in square units. For example, the tile at right measures 1 unit by 5 units, so it has an area of 5 square units.

   The next tile at right has one side length that is exactly one unit long. If you cannot give a numerical value to the other side length, what can you call it?

   d. If the unknown length is called "x," label the side lengths of each of the four algebra tiles you traced. What is each area? Use it to name each tile. Be sure to include the name of the type of units it represents.

2. JUMBLED PILES

   Your teacher will show you a jumbled pile of algebra tiles and will challenge you to write a name for the collection. What is the best description for the collection of tiles? Is your description the best possible?

   We use the area of algebra tiles to name the tiles, but we can calculate the perimeter of tiles. What is perimeter and how do you calculate it?

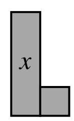
   There are several ways to calculate perimeter, and different ways to "see" perimeter. Sometimes, with complex shapes, a convenient shortcut can help you determine the perimeter more quickly. Be sure to share any insight into calculating perimeter with your teammates and with the whole class as you work through the next problems. Soon you will be able to determine the perimeter of complex shapes formed with a collection of tiles.

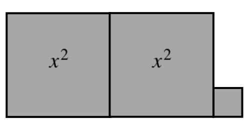
After your teacher has given your team a set of algebra tiles, separate one of each shape and review its name (area). Then determine the *perimeter* of each tile. Decide with your team how to write a simplified expression that represents the perimeter of each tile. Be prepared to share the perimeters with the class.

3. For each of the shapes formed by algebra tiles below:

    - Use tiles to build the shape.
    - Sketch and label the shape on your paper. Then write an expression that represents the perimeter.
    - Simplify your perimeter expression as much as possible. This process of writing the expression more simply by collecting together the parts of the expression that are the same is called **combining like terms**.

a. 

b. 

c. 

4. Calculate the perimeter of the shapes in problem 3 if the length of each x-tile is 3 units.

  MORE MATH FOR MORE PEOPLE

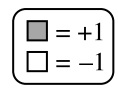
Now you will look at algebraic expressions and see how they can be interpreted using an Expression Mat. To achieve this goal, you first need to understand the different meanings of the "minus" symbol, which is found in expressions such as $5-2$, $-x$, and $-(-3)$.

5. LEARNING LOG

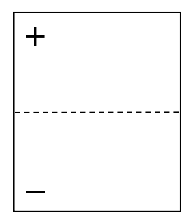
What does " – " mean? List as many ways as you can to describe this symbol and discuss how these descriptions differ from one another. Share your ideas with the class and record the different uses in your Learning Log. Title this entry "Meanings of Minus" and include today's date.

6. USING AN EXPRESSION MAT

So far, your work with algebra tiles has involved only positive values. Today you will look at how you can use algebra tiles to represent "minus." Below are several tiles with their associated values. Note that the shaded tiles are positive and the un-shaded tiles are negative. The diagram at right will appear throughout the text as a reminder.

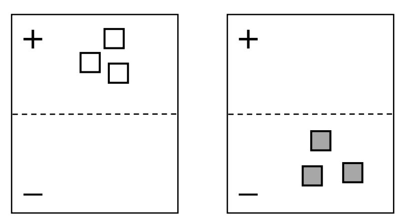


"Minus" can also be represented with a new tool called an **Expression Mat**, shown at right. An Expression Mat is an organizing tool that will be used to represent expressions. Notice that there is a positive region at the top and a negative (or "opposite") region at the bottom.



Using the Expression Mat, the value –3 can be shown in several ways, two of which are shown at right.

Note that in these examples, the diagram on the left side uses negative tiles in the "+" region, while the diagram on the right side uses positive tiles in the "–" region.



Value: –3 Value: –3

a. Build two different representations for $-2x$ using an Expression Mat.

b. Similarly, build $3x-(-4)$. How many different ways can you build $3x-(-4)$?

7. As you solved problem 6 did you see all of the different ways to represent "minus" that you listed in problem 5? Discuss how you could use an Expression Mat to represent the different meanings for minus discussed in class.

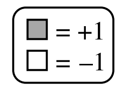
8. BUILDING EXPRESSIONS

Use the Expression Mat to create each of the following expressions with algebra tiles. Create at least two different representations for each expression. Sketch each representation on your paper. Be prepared to share your different representations with the class.

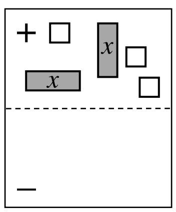
a. $-3x+4$ b. $-(x-2)$ c. $-x-3$ d. $5x-(3-2x)$

9. In the last problem, you represented algebraic expressions with algebra tiles. In this problem, you will need to reverse your thinking to write an expression from a diagram of algebra tiles.

Working with a partner, write algebraic expressions for each representation below. Start by building each problem using your algebra tiles.



a.



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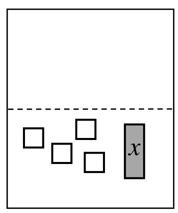

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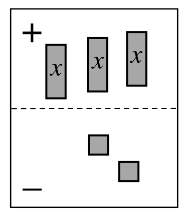
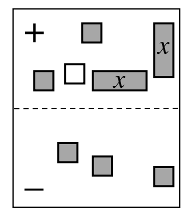
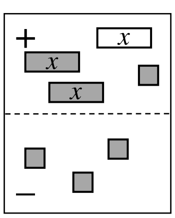


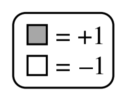
d.



10. How can you represent zero with tiles on an Expression Mat? With your team, try to create at least two different ways to do this (and more if you can). Be ready to share your ideas with the class.

11. Gretchen used seven algebra tiles to build the expression shown below.





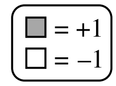
a. Build this collection of tiles on your own Expression Mat and write its value.

b. Represent this same value three different ways, each time using a *different number* of tiles. Be ready to share your representations with the class.

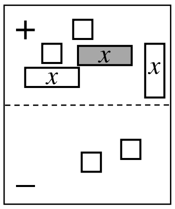
12. Build each expression below so that your representation does not match those of your teammates. Once your team is convinced that together you have created four different, valid representations, sketch your representation on your paper and be ready to share your answer with the class.

a. $-3x+5+x$ b. $-(-2x+1)$ c. $2x-(x-4)$

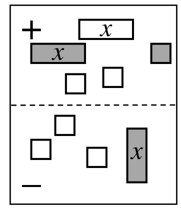
13. Write the algebraic expression shown on each Expression Mat below. Build the model and then simplify the expression by removing as many tiles as you can *without changing the value* of the expression. Finally, write the simplified algebraic expression.



a.



b.



14. For each expression below:

- Use an Expression Mat to build the expression.
- Create a different way to represent the same expression using tiles.

a. $7x - 3$

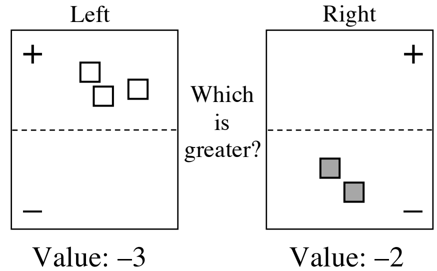
b. $-(-2x + 6) + 3x$

USING ALGEBRA TILES TO SIMPLIFY ALGEBRAIC EXPRESSIONS

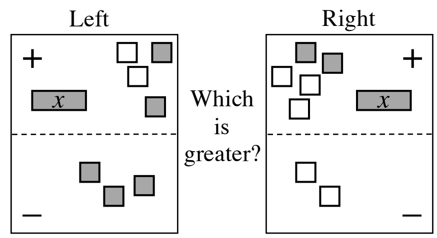
Which is greater: 58 or 62? That question might seem easy, because the numbers are easily compared. However, if you are asked which is greater, $2x+8-x-3$ or $6+x+1$, the answer is not so obvious! Now, you and your teammates will investigate how to compare two algebraic expressions and decide whether one is greater.

15. COMPARING EXPRESSIONS

Two expressions can be represented at the same time using an **Expression Comparison Mat**. The Expression Comparison Mat puts two Expression Mats side-by-side so you can compare them and see which one is greater. For example, in the picture at right, the expression on the left represents –3, while the expression on the right represents –2. Since $-2>-3$, the expression on the right is greater.



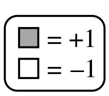
Build the Expression Comparison Mat shown at right. Write an expression representing each side of the Expression Mat.



a. Can you simplify each of the expressions so that fewer tiles are used? Develop a method to simplify both sides of the Expression Comparison Mats. Why does it work? Be prepared to justify your method to the class.

b. Which side of the Expression Comparison Mat do you think is greater (has the largest value)? Agree on an answer as a team. Make sure each person in your team is ready to justify your conclusion to the class.

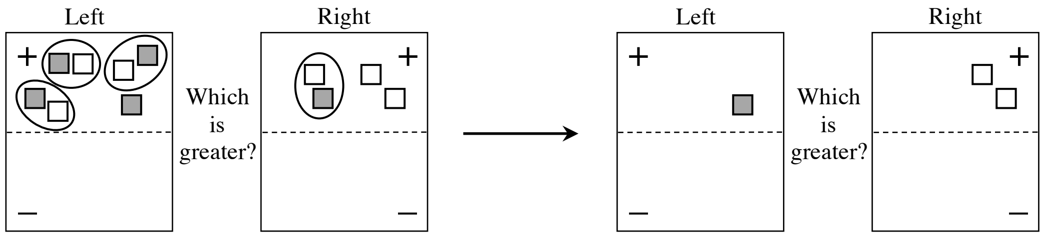
16. As Karl simplified some algebraic expressions, he recorded his work on the diagrams below.



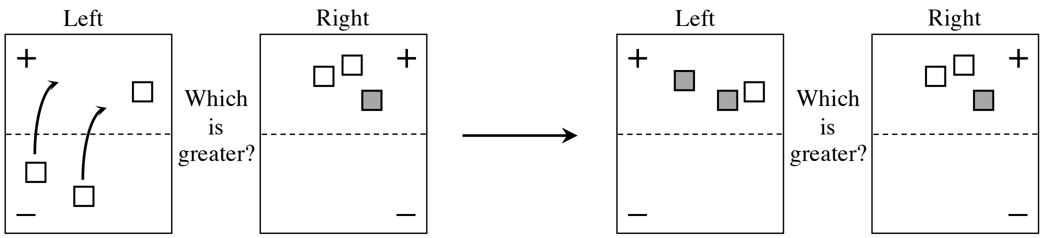
- Explain in writing what he did to each Expression Comparison Mat on the left to get the Expression Comparison Mat on the right.
- If necessary, simplify further to determine which Expression Mat is greater. How can you tell if your final answer is correct?

(problem continues on next page)

a.



b.



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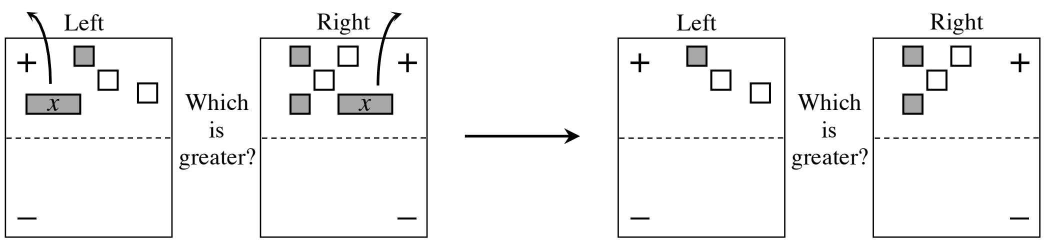
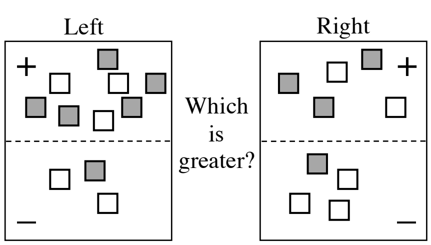
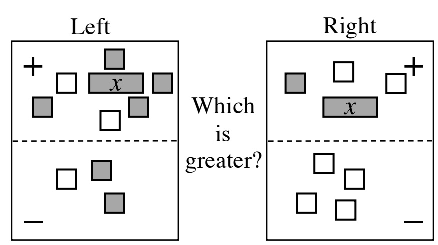


17. Use Karl's "legal" simplification moves to determine which side of each Expression Comparison Mat below is greater. Record each of your "legal" moves on your paper by drawing on it the way Karl did in problem 16. After each expression is simplified, state which side is greater (has the largest value.) Be prepared to share your process and reasoning with the class.

a.



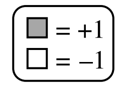
b.



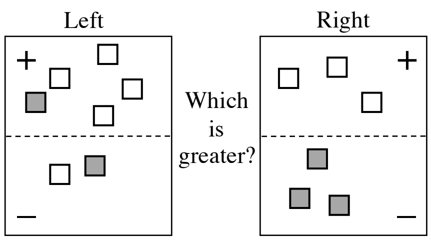
Can you always tell whether one algebraic expression is greater than another? With the next few problems you will compare the values of two expressions, practicing the different simplification strategies you have learned so far.

18. WHICH IS GREATER?

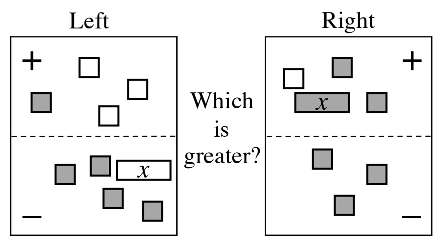
Write an algebraic expression for each side of the Expression Comparison Mats given below. Use the "legal" simplification moves to determine which expression on the Expression Comparison Mat is greater.



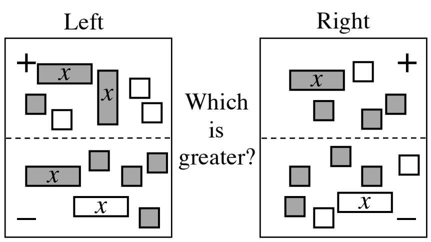
a.



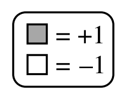
b.



c.

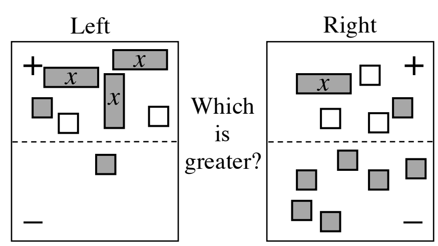


19. Build the Expression Comparison Mat shown below with algebra tiles.



a. Simplify the expressions using the "legal" moves that you have developed.

b. Can you tell which expression is greater? Explain in a few sentences on your paper. Be prepared to share your conclusion with the class.

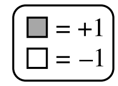


20. Use algebra tiles to build the expressions below on an Expression Comparison Mat. Use "legal" simplification moves to determine which expression is greater, if possible. If it is not possible to tell which expression is greater, explain why.

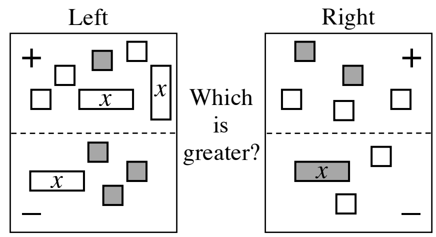
a. Which is greater: $3x-(2-x)+1$ or $-5+4x+4$?

b. Which is greater: $2x^2-2x+6-(-3x)$ or $-(3-2x^2)+5+2x$?

21. RECORDING YOUR WORK

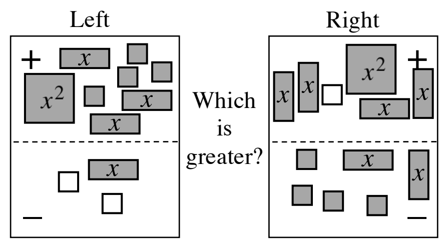


Although using algebra tiles can make some things easier because you can "see" and "touch" the math, it can be difficult to remember what you did to solve a problem unless you take good notes.

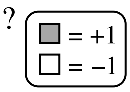


Use the simplification strategies you have learned to determine which expression on the Expression Comparison Mat at right is greater. Record each step in a way that others can understand and follow. Also record the simplified expression that remains after each move. This will be a written record of how you solved this problem. Discuss with your team the best way to record your moves.

22. While Athena was comparing the expressions shown at right, she was called out of the classroom. When her teammates needed help, they looked at her paper and saw the work shown below. Unfortunately, she had forgotten to explain her simplification steps.



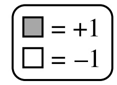
Can you help them figure out what Athena did to get each new set of expressions?



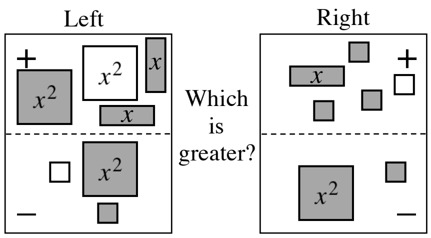
Left Expression	**Right Expression**	**Explanation**
$3x+4-x-(-2)+x^2$	$-1+x^2+4x-(4+2x)$	Original expressions
$3x+4-x-(-2)$	$-1+4x-(4+2x)$	
$3x+4-x+2$	$-1+4x-4-2x$	
$2x+6$	$2x-5$	
6	-5	
Because $6 > -5$, the left side is greater.		

  MORE MATH FOR MORE PEOPLE

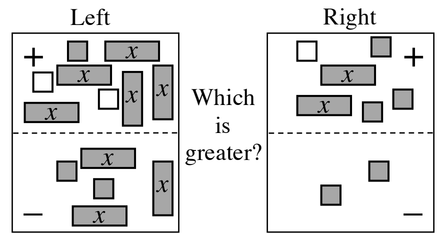
23. For each pair of expressions below, determine which is greater, carefully recording your steps as you go. If you cannot tell which expression is greater, state, "Not enough information." Make sure that you record your result after each type of simplification. For example, if you flip all of the tiles from the "–" region to the "+" region, record the resulting expression and indicate what you did using either words or symbols. Be ready to share your work with the class.



a.



b.



c. Which is greater: $5-(2x-4)-2$ or $-x-(1+x)+4$? Why?

d. Which is greater: $9-4x-7+x$ or $-2x-(x-2)$? Why?

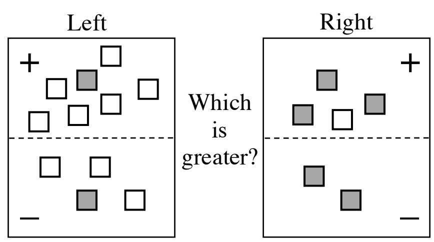
USING ALGEBRA TILES TO SOLVE FOR *X*

Can you always tell whether one algebraic expression is greater than another? In this section, you will continue to practice the different simplification strategies you have learned so far to compare two expressions and see which one is greater. Sometimes when you are comparing you do not have enough information about the expressions, but you can learn even more about x when both sides of an equation are equal.

24. WHICH IS GREATER?

Build each expression represented below with the tiles provided by your teacher. Use "legal" simplification moves to determine which expression is greater, if possible. If it is not possible to determine which expression is greater, explain why it is impossible. Be sure to record your work on your paper.

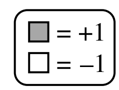
a.



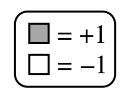
b. Which is greater:

$x+1-(1-2x)$ or

$3+x-1-(x-4)$?

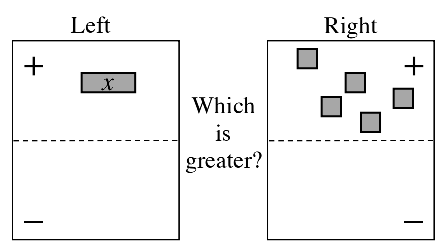


25. WHAT IF BOTH SIDES ARE EQUAL?



If the number 5 is compared to the number 7, then it is clear that 7 is greater. However, what if you compare x with 7? In this case, x could be smaller, larger, *or equal to* 7.

Examine the Expression Comparison Mat below.



a. If the left expression is smaller than the right expression, what does that tell you about the value of x?

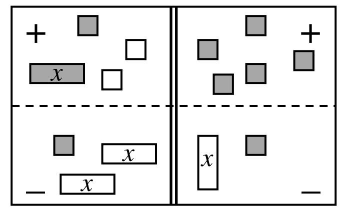
b. If the left expression is greater than the right expression, what does that tell you about the value of x?

c. What if the left expression is equal to the right expression? What does x have to be for the two expressions to be equal?

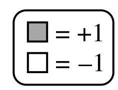
26. SOLVING FOR *X*

In later courses, you will learn more about situations like parts (a) and (b) in the preceding problem, called "inequalities." For now, to learn more about x, assume that the left expression and the right expression are equal. The two expressions will be brought together on one mat to create an **Equation Mat**, as shown in the figure below. The double line down the center of an Equation Mat represents the word "equals." It is a wall that separates the left side of an equation from the right side.

a. Obtain an Equation Mat from your teacher. Build the equation represented by the Equation Mat at right using algebra tiles. Simplify as much as possible and then solve for x. Be sure to record your work.

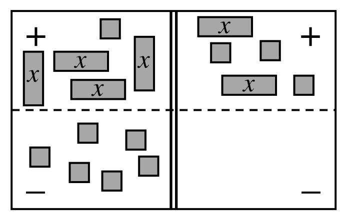


b. Build the equation $2x-5=-1+5x+2$ using your tiles by placing $2x-5$ on the left side and $-1+5x+2$ on the right side. Then use your simplification skills to simplify this equation as much as possible so that x is alone on one side of the equation. Use the fact that both sides are equal to solve for x. Record your work.



27. For this activity, share algebra tiles and an Equation Mat with your partner.

a. Start by setting up your Equation Mat as shown at right. Write the equation on your paper.



b. Next, solve the equation on your Equation Mat one step at a time. Every time you make a step, record your work in two ways:

- Record the step that was taken to get from the old equation to the new equation.
- Write a new equation that represents the tiles on the Equation Mat.

c. With your partner, create a way to check if your solution is correct.

28. WHAT IS A SOLUTION?

In the last problem you have found a solution to an algebraic equation. But what exactly is a solution? Answer each of these questions with your study team, but *do not use algebra tiles*. Be prepared to justify your answers!

a. Preston solved the equation $3x-2=8$ and got the solution $x=100$. Is he correct? How do you know?

b. Edwin solved the equation $2x+3-x=3x-5$ and got the solution $x=4$. Is he correct? How do you know?

c. With your partner, discuss what you think a solution to an equation is. Write down a description of what you and your partner agree on.

DISTRIBUTIVE PROPERTY EQUATIONS

Not all equations are as simple as the equations you have solved so far. However, many complicated-looking equations just need to be broken into simpler, familiar parts. Now you will use algebra tiles to work with situations that combine addition and multiplication. Then you will solve equations that contain complicated parts.

29. So far you have solved single-variable equations like $3x+7=-x-3$. Consider this change to that equation: $3(x+7)=-x-3$. What is different about the equations? How will the changes made to the original equation change the steps needed to solve the equation?

30. Use algebra tiles to build, draw, and simplify each expression.

    a. $3(x+4)$ b. $4(2x-1)$

    c. $2(x+5)+3$ d. $3(x-2)+5$

31. In a previous class, you used the **Distributive Property** to rewrite problems with parentheses similar to the ones above. Use the Distributive Property to fill in the blanks and simplify each expression below.

    a. $2(x+5)=2\cdot x+2\cdot 5=2x+$ __ b. $3(2x+1)=3\cdot 2x+3\cdot$ __ $=$ ______

    c. $-2(x+3)=-2\cdot$ _ $+-2\cdot$ _ $=$ ______ d. $-3(2x-5)=$ ________ $=$ _________

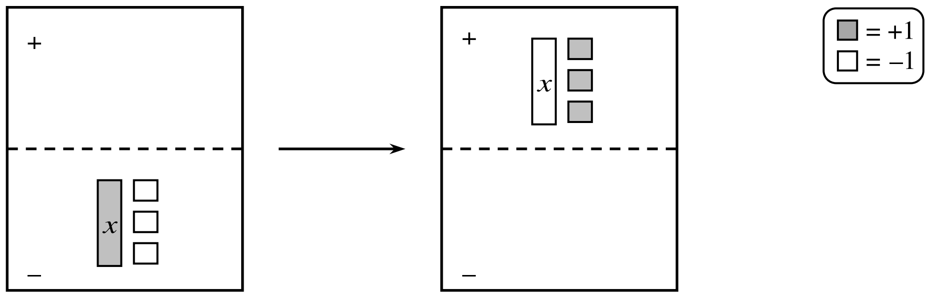
32. Now use what you learned in the previous three problems to solve for x in the equation $3(x+7)=-x-3$. Show your steps and check your answer. You may want to use algebra tiles and an Equation Mat to help you visualize the equation.

33. Solve each of the following equations for x. Show your steps and check your answers.

    a. $3x-2(5x+3)=14-2x$ b. $3(x+1)-8=14-2(3x-4)$

  MORE MATH FOR MORE PEOPLE

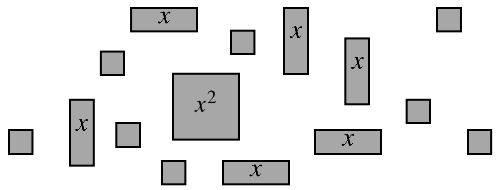
34. Earlier in this course, you learned that $-(x-3)$ was the same as $-x+3$, because $(x-3)$ in the "$-$" region could be "flipped" to $-x+3$ in the "+" region, as shown below.



Use what you have learned in this lesson to explain *algebraically* why "flipping" works. That is, why does $-(x-3)=-x+3$?

In the last few problems, you used tiles to represent algebraic equations. Today you will use algebra tiles again, but this time to represent expressions using multiplication.

35. Your teacher will put this group of tiles on the overhead:



a. Using your own tiles, arrange the same group of tiles into one large rectangle, with the x^2 tile in the lower left corner. On your paper, sketch what your rectangle looks like.

b. What are the dimensions (length and width) of the rectangle you made? Label your sketch with its dimensions, then write the area of the rectangle as a *product*, that is, length · width.

c. The area of a rectangle can also be written as the sum of the areas of all its parts. Write the area of the rectangle as the *sum* of its parts. Simplify your expression for the sum of the rectangle's parts.

d. Write an equation that shows that the area written as a product is equivalent to the area written as a sum.

36. Your teacher will assign several of the expressions below. For each expression, build a rectangle using all of the tiles, if possible. Sketch each rectangle, write its dimensions, and write an expression showing the equivalence of the area as a *sum* (like x^2+5x+6) and as a *product* (like $(x + 3)(x + 2)$). If it is not possible to build a rectangle, explain why not.

a. x^2+3x+2

b. $6x+15$

c. $2x^2+7x+6$

d. $2x^2+10x+12$

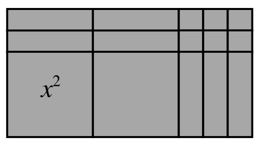
e. $2x^2+6x$

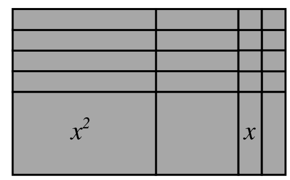
f. $3x^2+4x+1$

MULTIPLYING BINOMIALS AND THE DISTRIBUTIVE PROPERTY

In last two problems, you made rectangles with algebra tiles and found the dimensions of the rectangles. Starting with the area of a rectangle as a sum, you wrote the area as a product. Today you will reverse the process, starting with the product and determining its area as a sum.

37. For each of the following rectangles, write the dimensions (length and width) and write the area as the *product* of the dimensions and as the *sum* of the tiles. Remember to combine like terms whenever possible.

a. 

b. 

38. Your teacher will assign your team some of the expressions below. Use your algebra tiles to build rectangles with the given dimensions. Sketch each rectangle on your paper, label its dimensions, and write an equivalence statement for its area as a product and as a sum. Be prepared to share your solutions with the class.

a. $(x + 3)(2x + 1)$

b. $2x(x + 5)$

c. $(2x + 1)(2x + 1)$

d. $(2x + 5)(x + 2)$

e. $2(3x + 5)$

f. $(2x)(4x)$

39. With your team, examine the solutions you found for parts (b), and (e) of problem 38. This pattern is called the Distributive Property, which we were reminded of earlier. Multiply the following expressions without using your tiles and simplify. Be ready to share your process with the class.

a. $2x(x+5)$

b. $2(3x+5)$

USING GENERIC RECTANGLES TO MULTIPLY

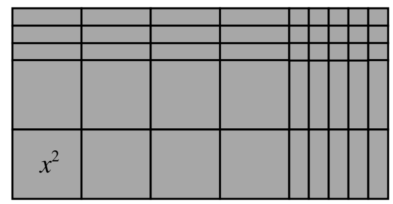
Today you will be introduced to a tool that will help you write the product of the dimensions of a rectangle. This will allow you to multiply expressions without tiles.

40. Use the Distributive Property to rewrite each product below.

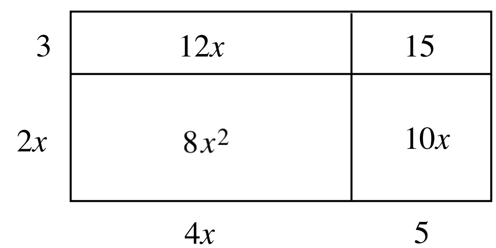
a. $6(-3x+2)$ b. $x^2(4x-2y)$

c. $5t(10-3t)$ d. $-4w(8-6k^2+y)$

41. Write the area as a *product* and as a *sum* for the rectangle shown at right.

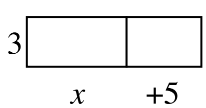


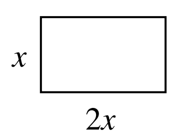
42. Now examine the following diagram. How is it similar to the set of tiles in problem 41? How is it different? Talk with your teammates and write down all of your observations.

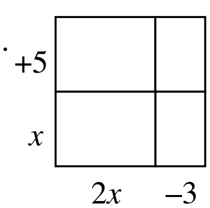


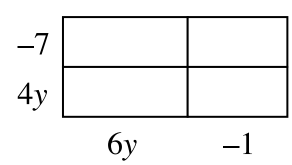
43. Diagrams like the one in problem 42 are referred to as **generic rectangles**. Generic rectangles allow you to use an area model to multiply expressions without using the algebra tiles. Using this model, you can multiply with values that are difficult to represent with tiles.

Draw each of the following generic rectangles on your paper. Then calculate the area of each part and write the area of the whole rectangle as a *product* and as a *sum*.

a.


b.


c.


d.


e. How did you calculate the area of the individual parts of each generic rectangle?

44. Multiply and simplify the following expressions using either a generic rectangle, the Distributive Property, and/or the tiles if you wish. For part (a), verify that your solution is correct by building a rectangle with algebra tiles.

a. $(x + 5)(3x + 2)$

b. $(2y - 5)(5y + 7)$

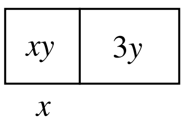
c. $3x(6x^2 - 11y)$

d. $(5w - 2p)(3w + p - 4)$

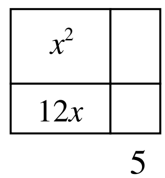
45. THE GENERIC RECTANGLE CHALLENGE

Copy each of the generic rectangles below and fill in the missing dimensions and areas. Then write the entire area as a product and as a sum. Be prepared to share your reasoning with the class.

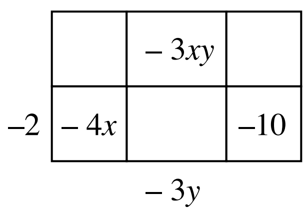
a.



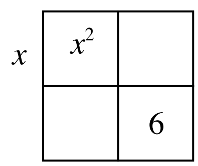
b.



c.

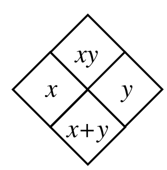


d.

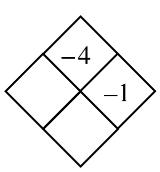


46. Diamond Problems

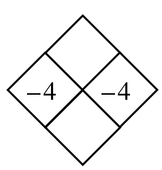
Copy and complete each of the Diamond Problems below. Fill in the missing parts of the Diamond by using the pattern shown at right.



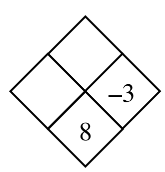
a.



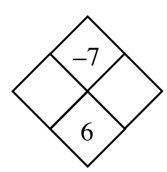
b.



c.



d.

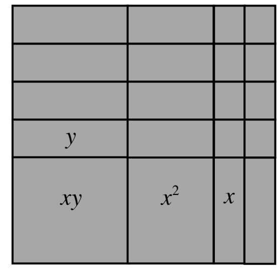


INTRODUCTION TO FACTORING QUADRATIC EXPRESSIONS

You have learned how to multiply algebraic expressions using algebra tiles and generic rectangles. This section will focus on reversing this process: How can you write a product when given a sum?

47. Review what you know about products and sums below.

    a. Write the area of the rectangle at right as a product and as a sum. Remember that the product represents the area found by multiplying the length by the width, while the sum is the result of adding the areas inside the rectangle.



    b. Use a generic rectangle to multiply $(6x - 1)(3x + 2)$. Write your solution as a sum.

48. The process of changing a sum to a product is called **factoring**. Can every expression be factored? That is, *does every sum have a product that can be represented with tiles?*

    Investigate this question by building rectangles with algebra tiles for the following expressions. For each one, write the area as a sum and as a product. If you cannot build a rectangle, be prepared to convince the class that no rectangle exists (and thus the expression cannot be factored).

    a. $2x^2 + 7x + 6$

    b. $6x^2 + 7x + 2$

    c. $x^2 + 4x + 1$

    d. $2xy + 6x + y^2 + 3y$

49. Work with your team to write the sum and the product for the following generic rectangles. Are there any special strategies you discovered that can help you determine the dimensions of the rectangle? Be sure to share these strategies with your teammates.

    a.

$2x$	5
$6x^2$	$15x$

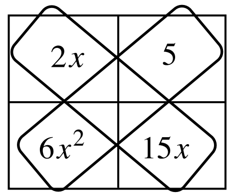
    b.

$-2y$	-6
$5xy$	$15x$

    c.

$-9x$	-12
$12x^2$	$16x$

50. While working on problem 49, Casey noticed a pattern with the diagonals of each generic rectangle. However, just before she shared her pattern with the rest of her team, she was called out of class! The drawing on her paper looked like the diagram below. Can you figure out what the two diagonals have in common?



51. Does Casey's pattern always work? Verify that her pattern works for all of the 2-by-2 generic rectangles in problem 49. Then describe Casey's pattern for the diagonals of a 2-by-2 generic rectangle in your Learning Log. Be sure to include an example. Title this entry "Diagonals of a Generic Rectangle" and include today's date.

FACTORING WITH GENERIC RECTANGLES

Since mathematics is often described as the study of patterns, it is not surprising that generic rectangles have many patterns. You saw one important pattern in problem 50. Now you will continue to use patterns while you develop a method to factor trinomial expressions.

52. Examine the generic rectangle shown at right.

$-35x$	14
$10x^2$	$-4x$

a. Review what you have learned by writing the area of the rectangle at right as a sum and as a product.

b. Does this generic rectangle fit Casey's pattern for diagonals? Demonstrate that the product of each diagonal is equal.

53. FACTORING QUADRATIC EXPRESSIONS

To develop a method for factoring without algebra tiles, first model how to factor with algebra tiles, and then look for connections within a generic rectangle.

a. Using algebra tiles, factor $2x^2+5x+3$; that is, use the tiles to build a rectangle, and then write its area as a product.

b. To factor with tiles (like you did in part (a)), you need to determine how to arrange the tiles to form a rectangle. Using a generic rectangle to factor requires a different process.

Miguel wants to use a generic rectangle to factor $3x^2+10x+8$. He knows that $3x^2$ and 8 go into the rectangle in the locations shown at right. Finish the rectangle by deciding how to place the ten *x*-terms. Then write the area as a product.

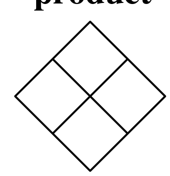
	8
$3x^2$	

c. Kelly wants to discover a shortcut to factor $2x^2+7x+6$. She knows that $2x^2$ and 6 go into the rectangle in the locations shown at right. She also remembers Casey's pattern for diagonals. Without actually factoring yet, what do you know about the missing two parts of the generic rectangle?

?	6
$2x^2$	?

d. To complete Kelly's generic rectangle, you need two *x*-terms that have a sum of $7x$ and a product of $12x^2$ Create and solve a Diamond Problem that represents this situation. Look back to problem 46 if you wish to review the pattern.

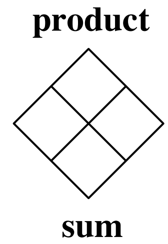
product



sum

e. Use your results from the Diamond Problem to complete the generic rectangle for $2x^2+7x+6$, and then write the area as a product of factors.

54. Factoring with a generic rectangle is especially convenient when algebra tiles are not available or when the number of necessary tiles becomes too large to manage. Using a Diamond Problem helps avoid guessing and checking, which can at times be challenging. Use the process from problem 53 to factor $6x^2+17x+12$. The questions below will guide your process.



a. When given a trinomial, such as $6x^2+17x+12$, what two parts of a generic rectangle can you quickly complete?

b. How can you set up a Diamond Problem to help factor a trinomial such as $6x^2+17x+12$? What goes on the top? What goes on the bottom?

c. Solve the Diamond Problem for $6x^2+17x+12$ and complete its generic rectangle.

d. Write the area of the rectangle as a product.

55. Use the process you developed in problem 53 to factor the following quadratics, if possible. If a quadratic cannot be factored, justify your conclusion.

a. $x^2+9x+18$

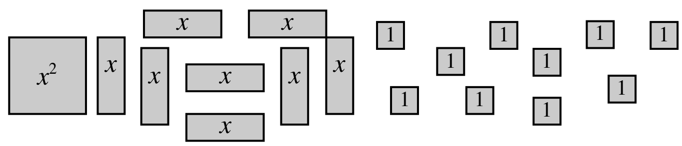
b. $4x^2+17x-15$

c. $4x^2-8x+3$

d. $3x^2+5x-3$

  MORE MATH FOR MORE PEOPLE

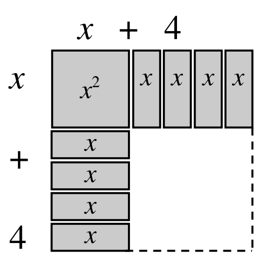
56. Jessica was at home struggling with her homework. She had missed class and could not remember how to **complete the square**. She was supposed to use the method to change $f(x)=x^2+8x+10$ to graphing form. Then her precocious younger sister, Anita, who was playing with algebra tiles, said, "*Hey, I bet I know what they mean*." Anita's algebra class had been using tiles to multiply and factor binomials. Anita explained, "$f(x)=x^2+8x+10$ *would look like this*,"



"*Yes*," said Jessica, "*I took Algebra 1 too, remember?*"

Anita continued, "*And you need to make it into a square!*"

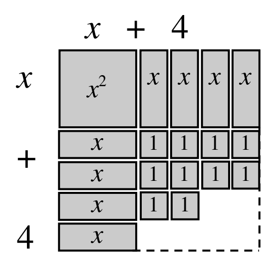
"*OK*," said Jessica, and she arranged her tiles as shown in the picture below.



"*Oh*," said Jessica. "*So I just need 16 small unit tiles to fill in the corner*."

"*But you only have 10*," Anita reminded her.

"*Right, I only have ten*," Jessica replied. She put in the 10 small square tiles then drew the outline of the whole square and said:



"*Oh, I get it! The* ***complete square*** *is* $(x+4)^2$ *which is equal to* $x^2+4x+16$ *. But my original expression,* $x^2+8x+10$ *, has six fewer tiles than that, so what I have is* $(x+4)^2$ *, minus 6*."

"*Yes*," said Anita. "*You started with* $x^2+8x+10$ *, but now you can rewrite it as* $x^2+8x+10=(x+4)^2-6$ *.*"

Use your graphing calculator to show that $f(x)=x^2+8x+10$ and $f(x)=(x+4)^2-6$ are equivalent functions.

57. Help Jessica with a new problem. She needs to complete the square to write $y=x^2+4x+9$ in graphing form. Draw tiles to help her figure out how to make this expression into a square. Does she have too few or too many unit squares this time? Write her equation in graphing form.

58. How could you complete the square to change $f(x)=x^2+5x+2$ into graphing form? How would you split the five x-tiles into two equal parts?

Jessica decided to use force! She cut one tile in half, as shown below. Then she added her two unit tiles.

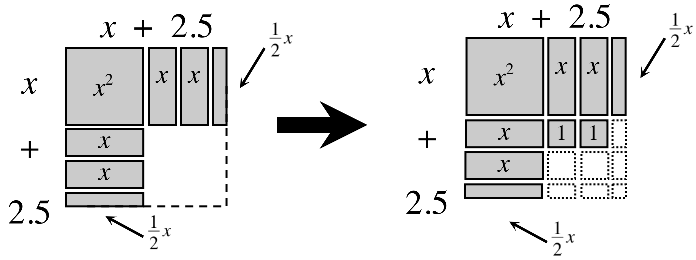


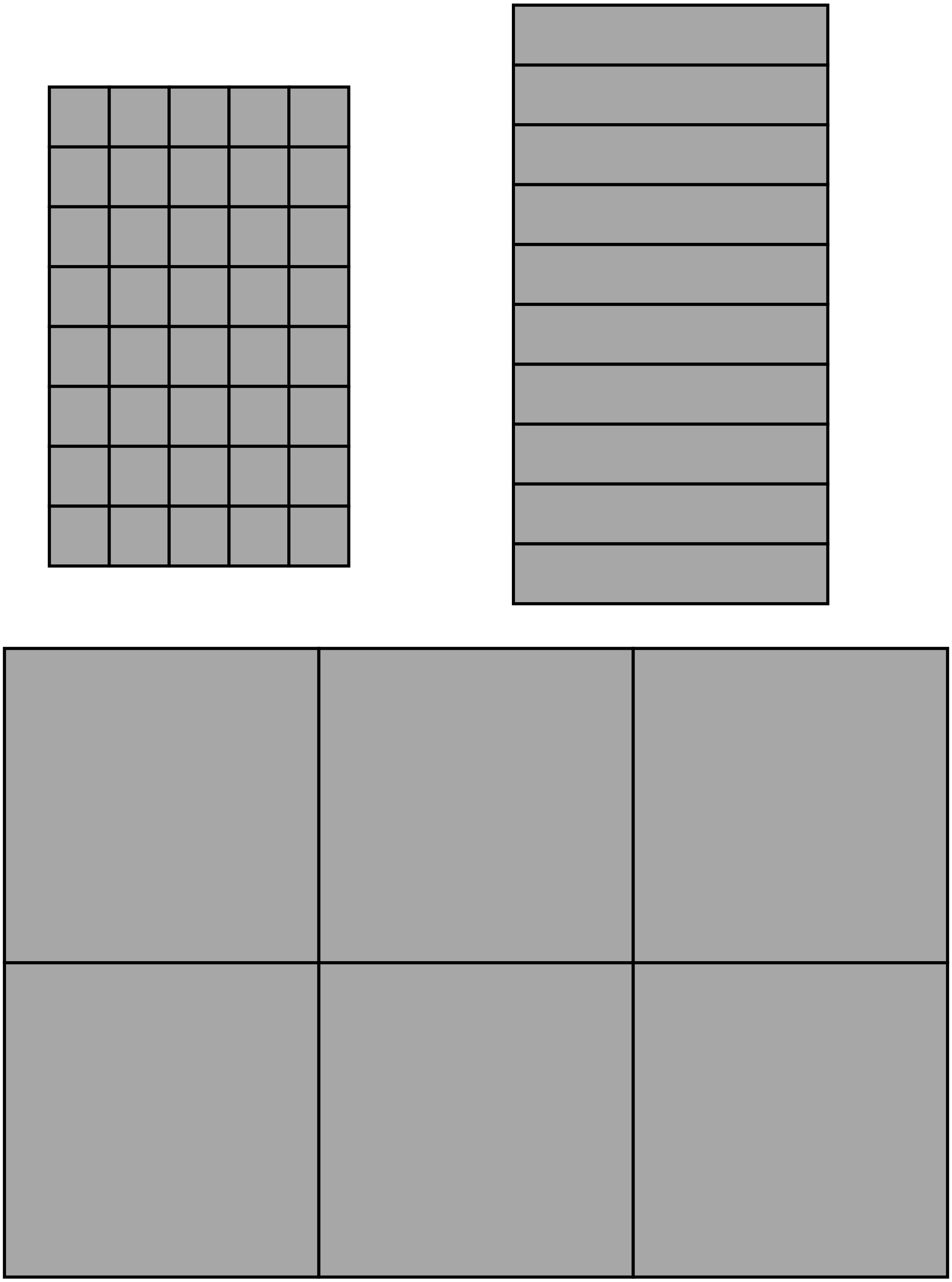
Figure A　　　Figure B

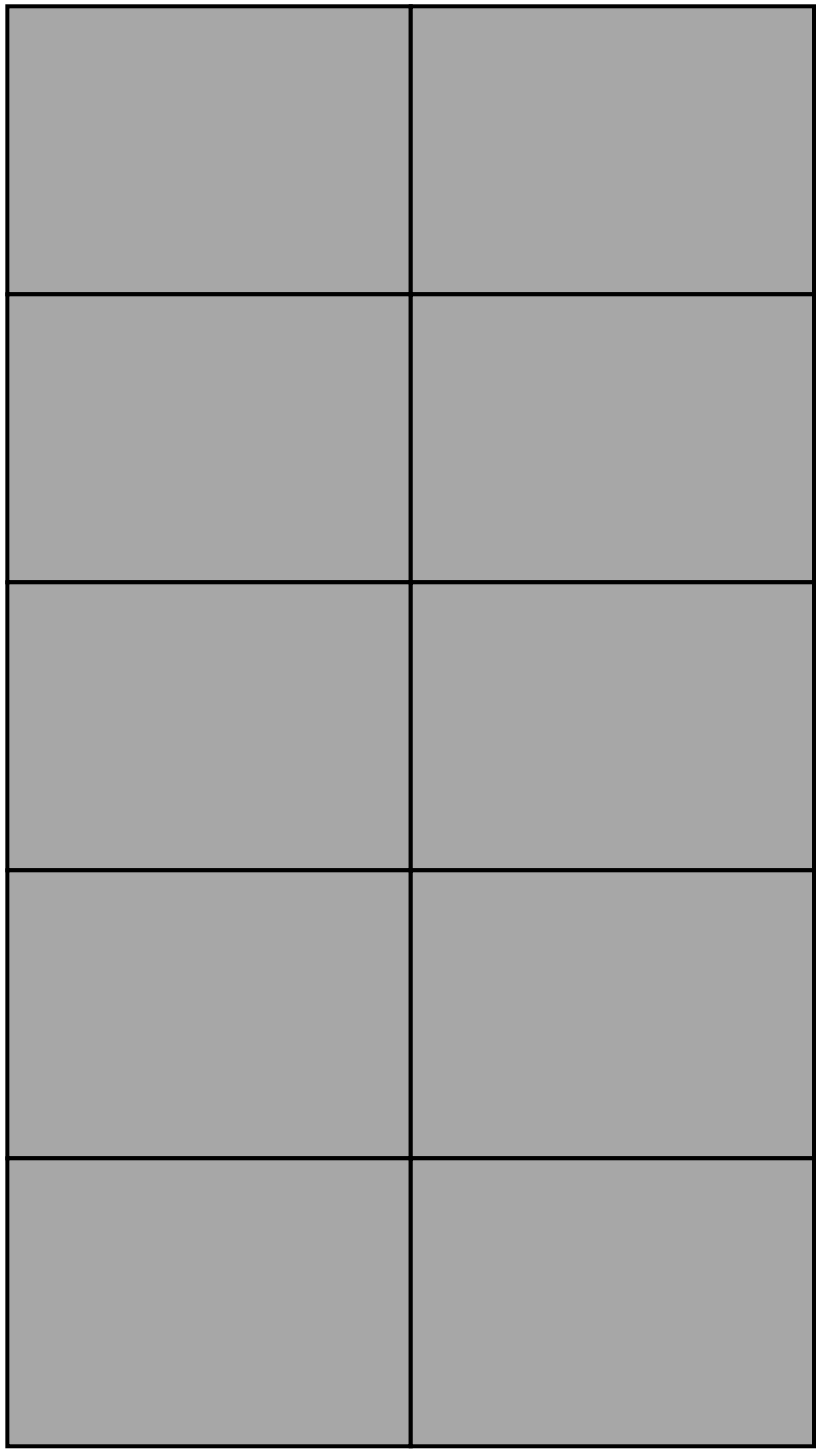
a. How many unit tiles are in the perfect square?

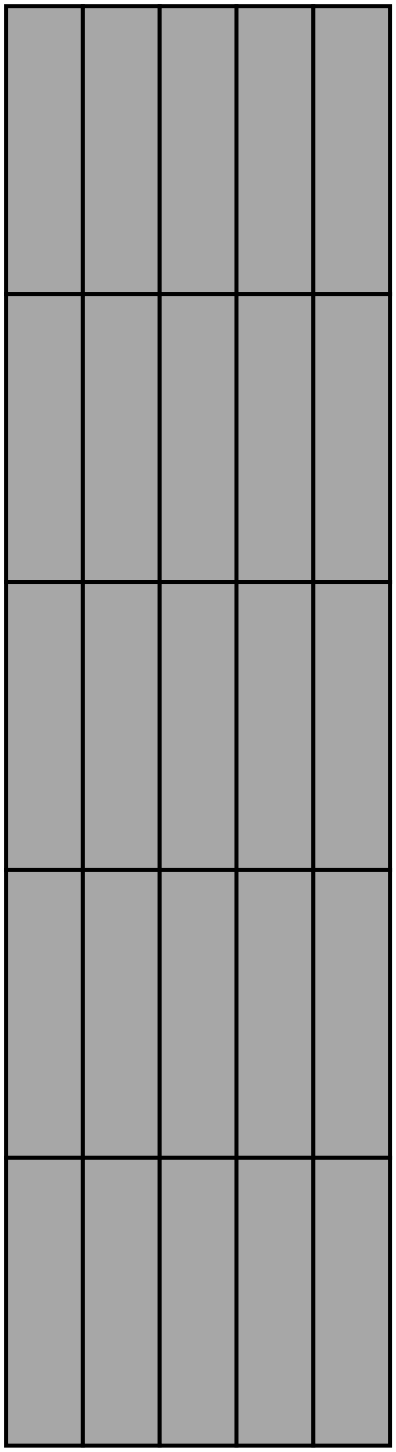
b. Does Jessica have too many or too few tiles in her original expression? How many?

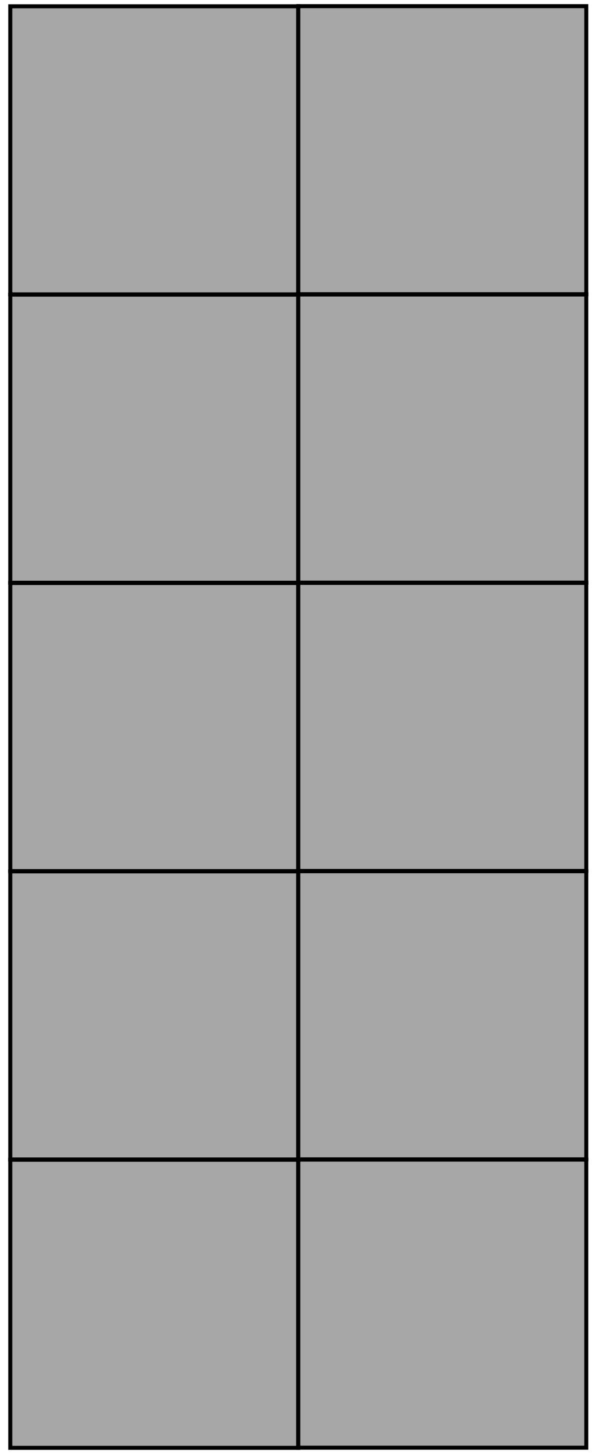
c. Write the graphing form of the function.

  MORE MATH FOR MORE PEOPLE









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Left
Which is greater?
Right

EQUATION MAT