# Cross Section Practice

Name_____

Directions: Read carefully. For multiple choice questions, choose the best answer.

1. Given a right circular cone.

a) What is the shape of a cross section parallel to the base of the cone?

b) What is the shape of a cross section perpendicular to the base of the cone?

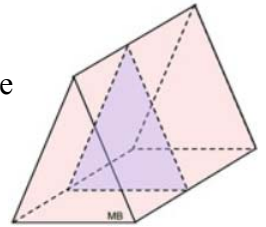**2.** A right square pyramid sits atop a right circular cylinder as shown at the right. A sword will make a vertical slice through both figures starting at the vertex of the pyramid. Which of the following shapes represents the cross section?
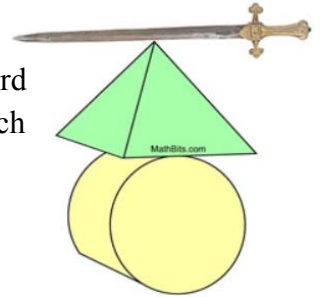
1)   2)   3)   4)

3. Of the choices "square", "rectangle" and "triangle", which are possible cross sections obtained when slicing a cube?

4. Given a right triangular prism as shown at the right. A cross section is made parallel to the bases. Which of the following statements is true regarding the area of the cross section and the area of the base?
1) The area of the cross section is less than the area of the base.
2) The area of the cross section is greater than the area of the base.
3) The area of the cross section is equal to the area of the base.
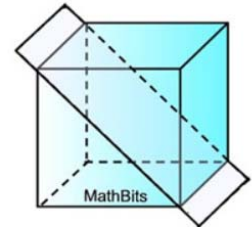4) There is insufficient data to determine these area relationships.

5. Given a right circular cylinder with a radius of 8 inches and a height of 20 inches. Which of the following choices is NOT a possible cross section of this cylinder?
1) circle   2) oval   3) square   4) rectangle

6. Which of the following statements is FALSE regarding possible cross sections when slicing a right square pyramid?
1) A slice perpendicular to the base yields a triangle.
2) A slice perpendicular to the base yields a square.
3) A slice parallel to the base yields a square.
4) A non-perpendicular slice intersecting the base yields a triangle.

7. Given a cube. A cross section is sliced diagonally through the center of the cube, as shown. Which of the following statements is true regarding the area of the cross section and the area of the cube's face?
1) The area of the cross section equals the area of the cube's face.
2) The area of the cross section is larger than the area of the cube's face.
3) The area of the cross section is smaller than the area of the cube's face.
4) There is insufficient data to determine the relationship between the area of the cross section and the area of the cube's face.

8. A plane intersects a sphere. (The plane is not tangent to the sphere.) Which of the following statements is true regarding the cross section formed by the plane?
1) The cross section is always a circle.
2) The cross section is sometimes a circle.
3) The cross section is always an oval.
4) The cross section is sometimes a square.

9. A solid has a triangle as a cross section. Which of the following solids could NOT have a triangle as a cross section?
1) cube   2) square pyramid   3) triangular prism   4) rectangular prism   5) All have triangular cross sections.

10. A cube and a right square pyramid are sitting on a flat surface. Which of the following statements are true? (*Choose ALL that apply!*)
It is possible to slice . . .
1) a triangular cross section from the cube.
2) a trapezoidal cross section from the pyramid.
3) a rectangular cross section (not a square) from the pyramid.
4) a hexagonal cross section from the cube.
5) a square cross section from the pyramid.
6) a hexagonal cross section from both the pyramid and the cube.
7) a square cross section from both the pyramid and the cube.
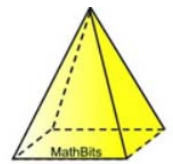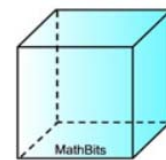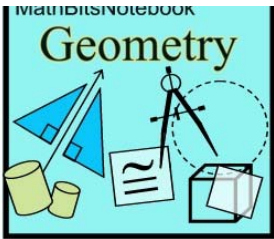
Rotations – Solids of Revolution Practice

Name _____

*Directions: Read carefully. Choose best answer when needed.*

1. What 3-D solid is formed when this rectangle is rotated about the vertical line, as shown at the right? *Choose:* 1) right, rectangular prism 2) right, circular cylinder   3) right, oval cylinder   4) right, square prism
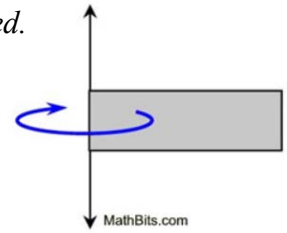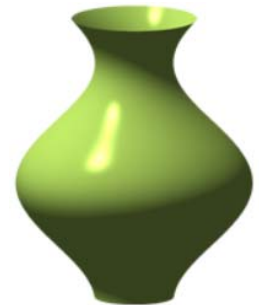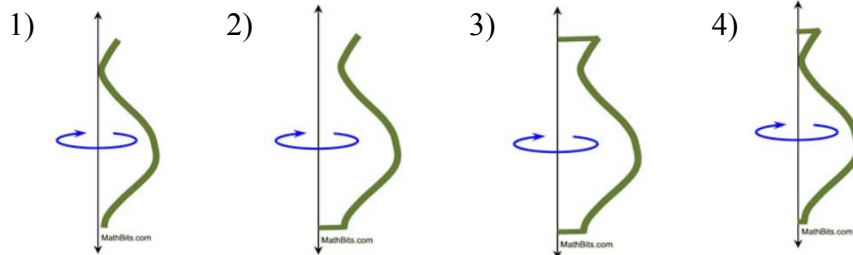
**2.** Which 2-D shape could be revolved about the vertical line to create the 3-D solid at the right?

1)   2)   3)   4)
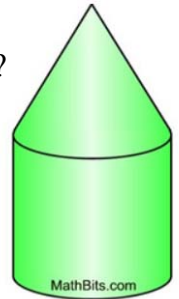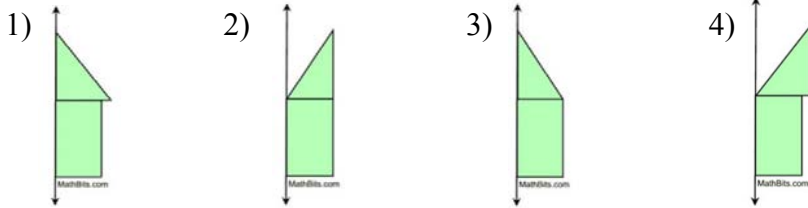
3. Which 3-D solid is created when the rectangle shown at the right is rotated about the horizontal line?

1)   2)   3)   4)
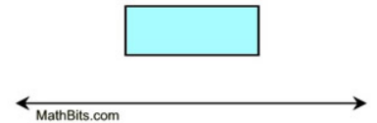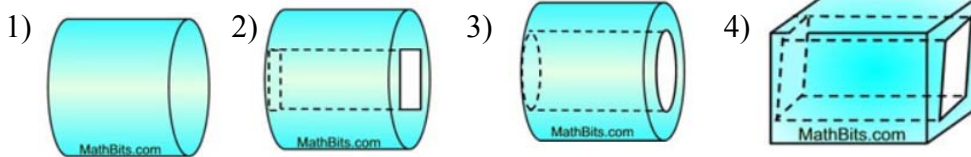
4. Which 2-D shape could be revolved about the vertical line to create the 3-D vase shown at the right?   Assume the vase is open at the top and closed at the bottom.

1)   2)   3)   4)

5. A 3-D solid is formed by rotating the rectangle shown at the right about the vertical line. The dimensions of the rectangle are stated in inches. What is the volume of the solid formed by this rotation, in cubic inches?

3

5
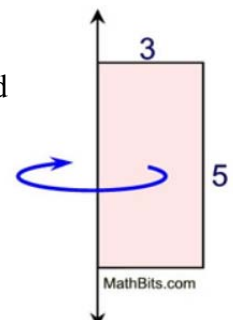
6. A rectangle and a right triangle, as shown at the right, are rotated about the vertical line. What is the volume of the solid formed by this rotation, in cubic units?
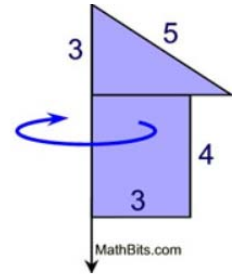
7. The figure shown at the right was the result of a 2-D figure revolved about a line. Which of the following figures, when rotated about the line, could NOT produce this 3-D shape?

1)   2)   3)   4)
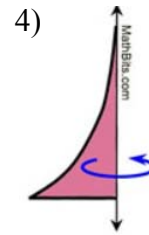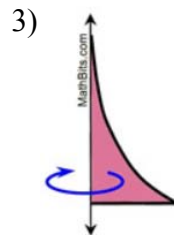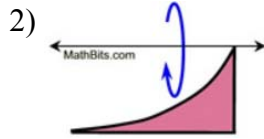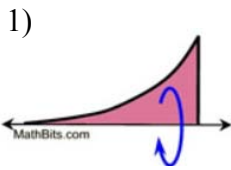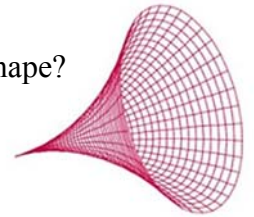
8. The 2-D figure at the right is to be revolved about the vertical line. Which of the following descriptions best describes the 3-D solid formed by the rotation?

1) Nail   2) Pencil   3) Water Glass   4) Gun
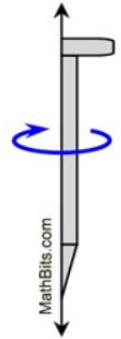
9. The right triangle shown at the right is revolved about its hypotenuse. Find the volume of the solid formed by this rotation, in cubic units.   *Hint:* Find the altitude of the triangle drawn to the hypotenuse using the area of the triangle.
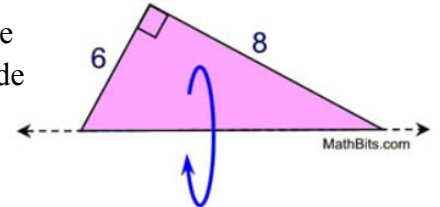
10. Given: the 2-D figure at the right is a rectangle, and line a is perpendicular to line b. Which of the following statements are true regarding this diagram?
(*Choose ALL that apply!*)

1) When the rectangle is revolved about line b, a solid cylinder is formed.

2) When the rectangle is revolved about line a, a rectangular prism is formed.

3) The volume of the solid formed when the rectangle is rotated about line a is 72π cubic units.

4) The volume of the solid formed when the rectangle is rotated about line a will equal the volume of the solid formed when the rectangle is rotated about line b.
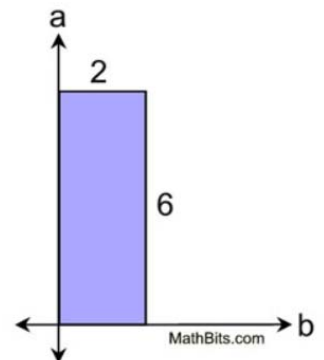
Examining Rotations

Name_____

Directions: Determine the shape of the 3-D figure generated by rotating the 2-D shape about the stated axis of rotation. Sketch the 3-D figure on the graph and describe or name the shape. Questions #1-8 will also ask for the volume of the 3-D figure to the *nearest tenth* of a cubic unit.

1. Rotate about *y*-axis:

V =

2. Rotate about *x*-axis:

V =
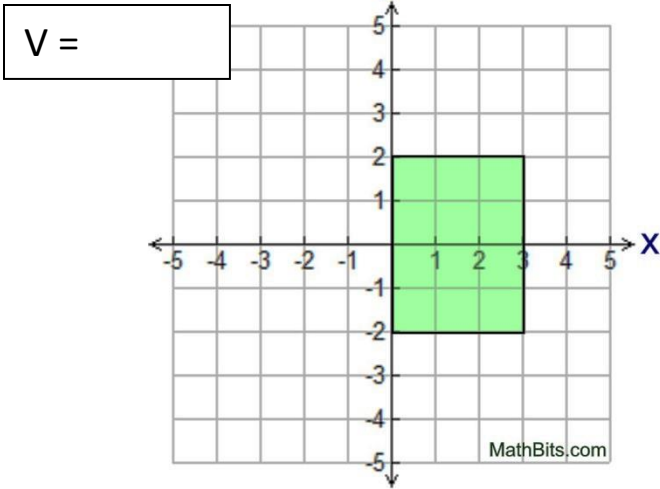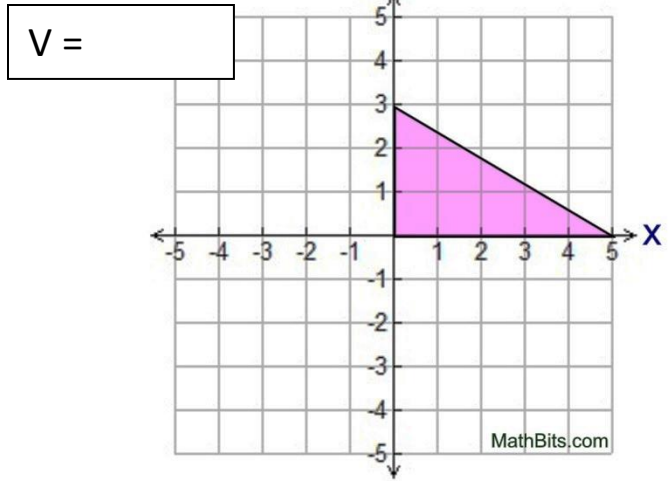
3. Rotate about *y*-axis:

V =
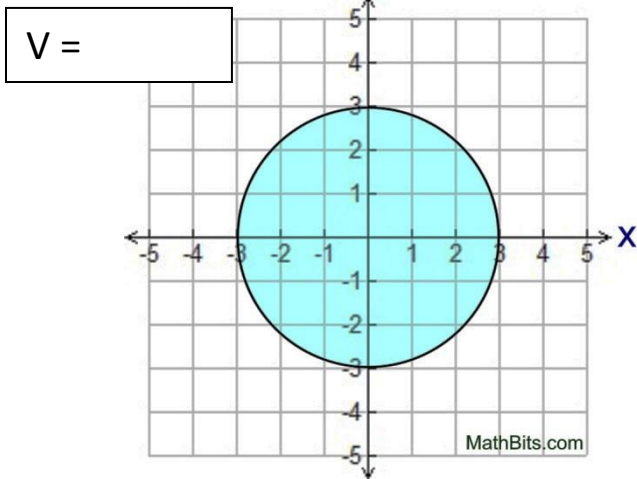
4. Rotate about *y*-axis

V =
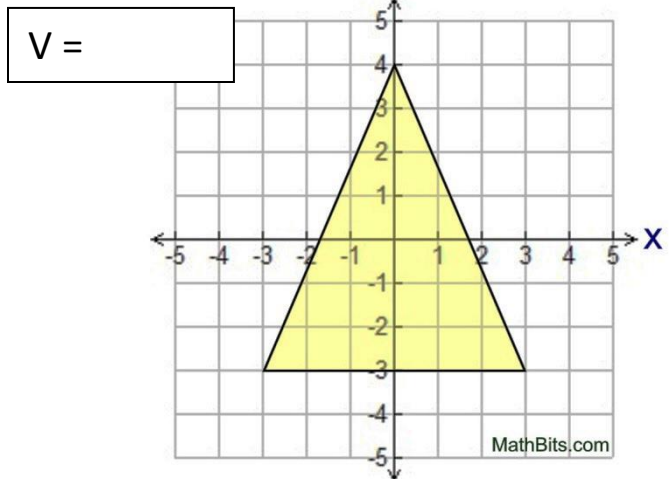
5. Rotate about *x*-axis:

V =
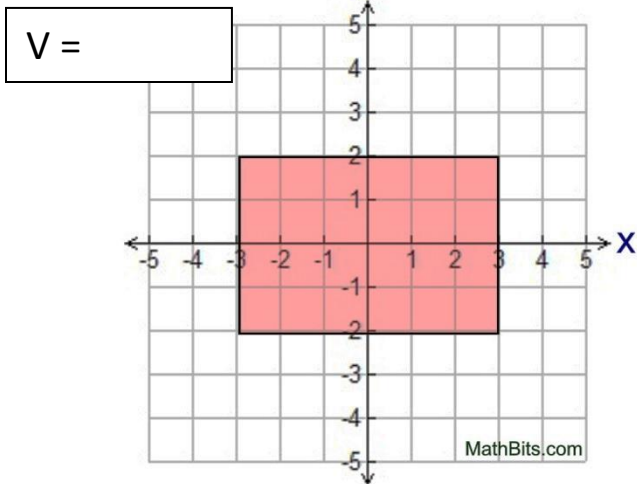
6. Rotate about *y*-axis:

V =

7. Rotate about $x = -1$:

V =
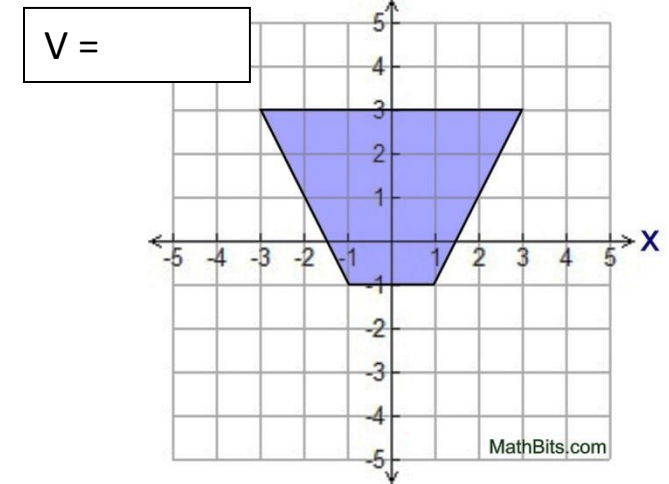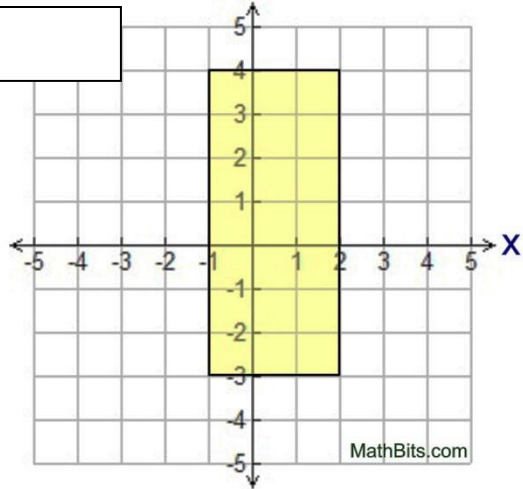
8. Rotate about $y = 2$:

V =
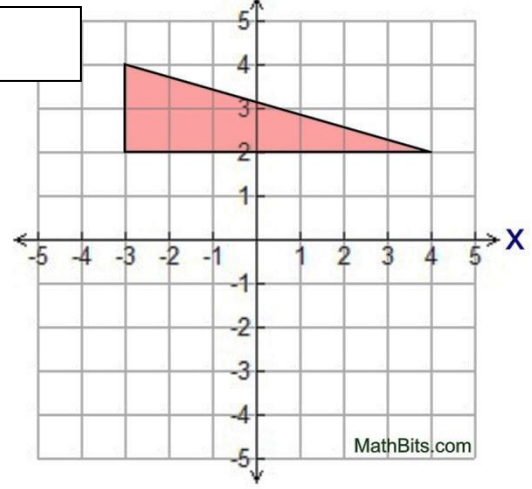
Directions: For questions 9-12, sketch the 3-D figure on the graph and describe the shape. List one real life item that possesses this shape.

9. Rotate about *y*-axis:
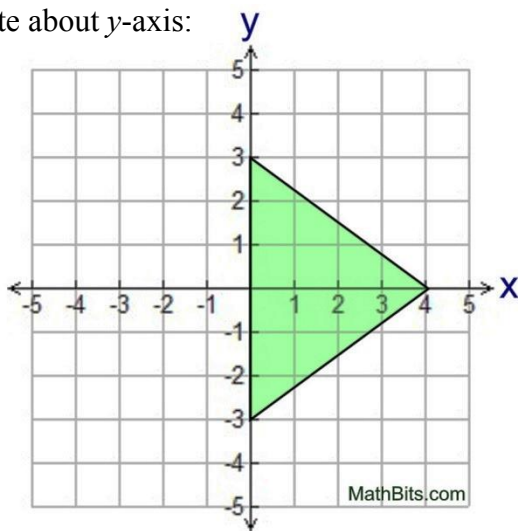
10. Rotate about *y*-axis:
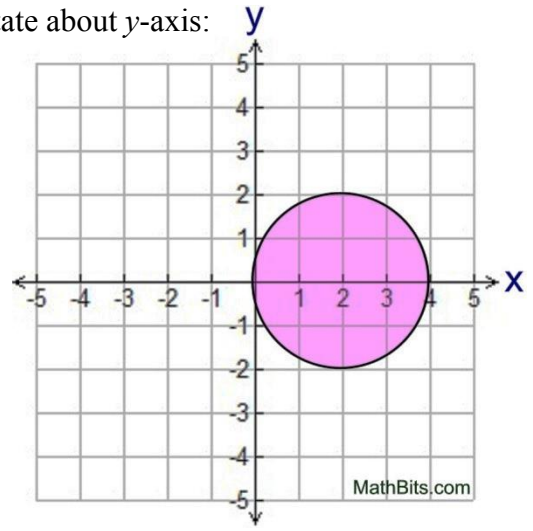
11. Rotate about *x*-axis:
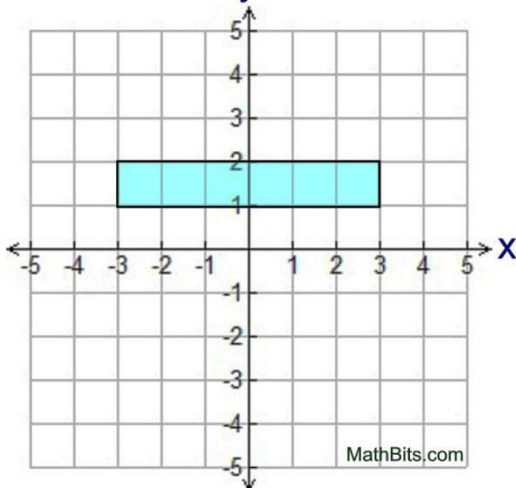
12. Rotate about *x*-axis:
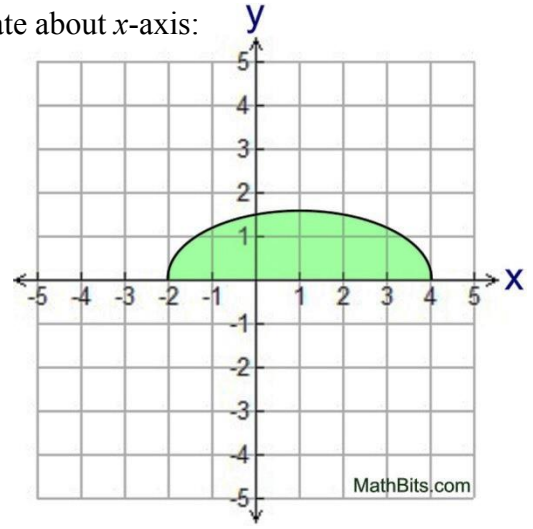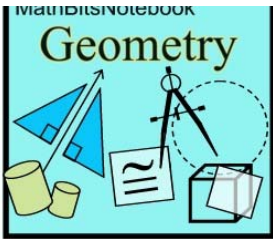
Practice with Cavalieri's Principle

Name _____

Directions: Read carefully. Show your work.

1. A right circular cylinder and an oblique circular cylinder are given.
True or False? If the radii of both cylinders are equal, the volumes of the cylinder will be equal because they have the same height.
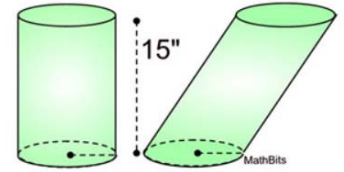
2. A right circular cylinder and a right rectangular prism are given.
True or False? Cavalieri's Principle does not apply to these solids because their bases are not the same shape.
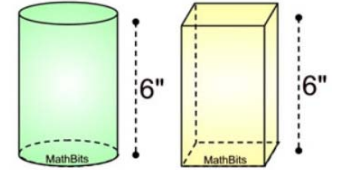
3. The three solids shown at the right have the same height and matching cross sectional areas parallel to the bases. Which of the solids have the same volume?
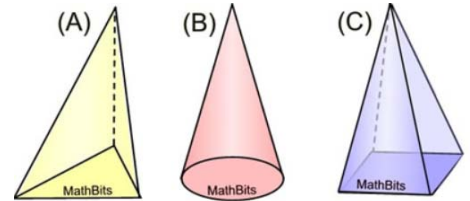
4. A right circular cylinder, *A*, and an oblique circular cylinder, *B*, are shown at the right. Find the volume of cylinder *B* in cubic inches.
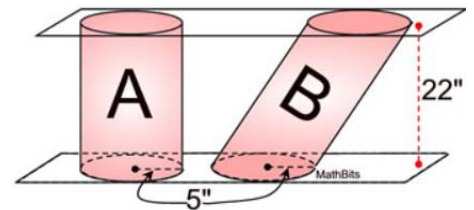
5. By Cavalieri's Principle, this right triangular prism and right rectangular prism have the same volume. If the center plane intersects the solids parallel to their bases, which of the following choices could be the base and height of the triangular cross section?
1) $h = 4; b = 12$
2) $h = 8; b = 12$
3) $h = 4; b = 8$
4) $h = 8; b = 14$
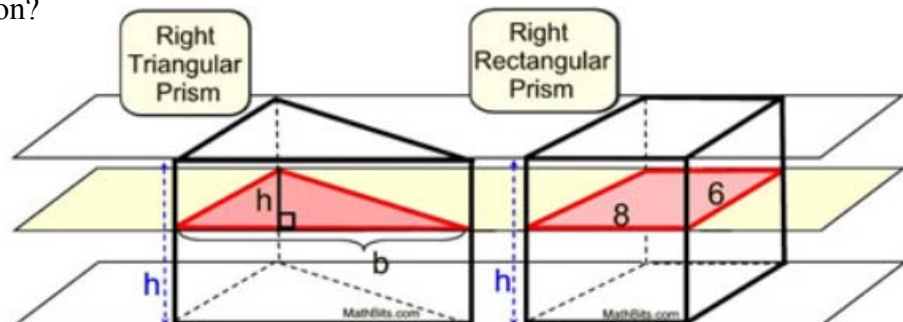
6. What is the volume of an oblique cylinder with a radius of 3 and a height of 9?

9
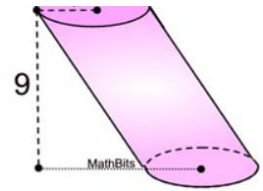
7. The right circular cylinder and the right rectangular prism have the same heights and the same base areas. A plane, parallel to the bases slices the two solids. If the rectangular cross section has a base length of 16 inches and a width of 4π inches, what is the radius of the cylinder's cross section?
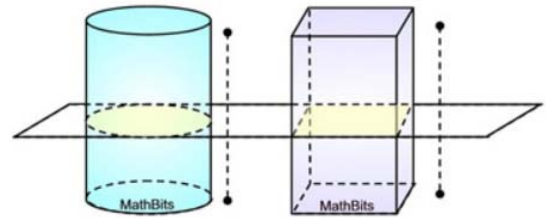
8. An oblique hexagonal prism has a base area of 68 square meters and a lateral (slant) side length of $10\sqrt{2}$ meters. The lateral side makes an angle of 45° with the horizontal plane containing the base, as shown. What is the volume of the prism?

$10\sqrt{2}$

45°
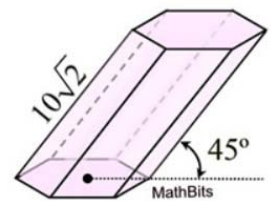
9. A right triangular prism and a right square prism are given. The base of the triangular prism is an isosceles right triangle with a hypotenuse of $6\sqrt{2}$. Both solids have a height of 10, and their bases have equal areas. A plane slices both solids parallel to their bases. Find the length of the side of the square prism's cross section.
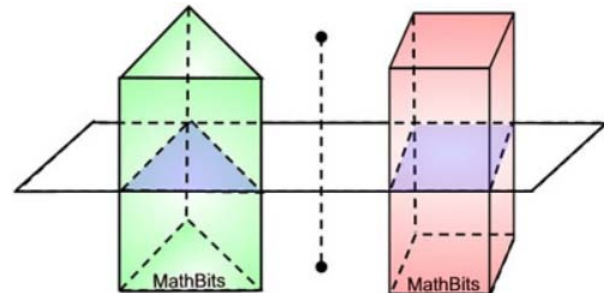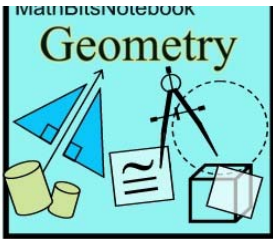
10. A series of coins are stacked to represent a right circular cylinder (on the left). The coins are then "slid" to represent a distorted cylinder (on the right). The same number of congruent coins was used in each stack.
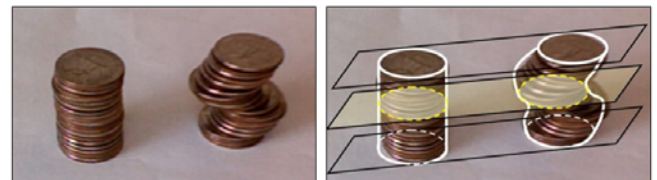
Which of the following statements will be **TRUE** regarding these stacks of coins?
(*Choose ALL that apply!*)
1) The volume of both stacks will be the same.
2) The area of a cross section parallel to the bases will not be equal due to the distorted nature of the second stack.
3) The height of the distorted stack will be slightly larger than that of the straight stack.
4) Cavalieri's Principle can be used in this situation to verify that the volumes of the stacks are equal.

Applied Solids Practice

Name _____

Directions: Read carefully. Show your work.

1. When balloons are inflated, they form spheres because they are trying to hold as much air as possible with as small a surface as possible. How much air, to the *nearest cubic inch*, is being held by a spherical balloon with a diameter of 10 inches?

2. A cone is filled to a height of 3 inches with a liquid. The liquid is then poured into an empty cylinder with the same height and same radius as the cone. What is the height reached by the liquid in the cylinder?
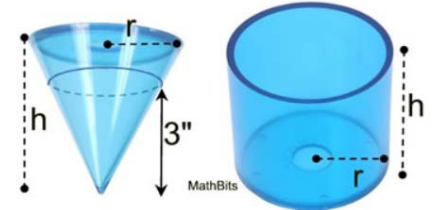
3. Otto is sitting on 10 blocks of gold. The blocks are in the shape of right trapezoidal prisms. The trapezoidal base of the prism has bases of 10 inches and 7 inches with a height of 8 inches. The lateral edge of the prism is 19 inches.
a) Upon how many total cubic inches of gold is he sitting?

b) Determine the worth of Otto's gold based upon the gold pricing of $13,067.76 per troy pound. Utilize the following information:
• there are 1728 cubic inches in 1 cubic foot.
• there are 1,206.83 pounds of gold in 1 cubic foot.
• there are 0.82 pounds in 1 troy pound.

4. A farmer is building a new silo for storing grain. He is comparing a silo with a cone top to a silo with a hemisphere top.
a) Which silo has the larger volume?
b) By how much does the volume of the larger silo exceed that of the smaller silo?

c) *Note:* 1 bushel = 1.24446 cubic feet
The cost of building a silo depends upon its holding capacity expressed in bushels. These silos fall into the price range of $2.21 per bushel capacity. What is the cost of the smaller silo?
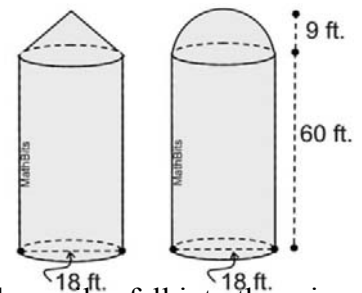
d) How much more will it cost to construct the larger silo?

5. At Theater A, popcorn is served in a box. At Theater B, popcorn is served in a cylindrical container. At home, Mom serves popcorn in a bowl (hemisphere in shape). Based upon the given dimensions, at which location are you getting the most popcorn?
(Disregard the thickness of the container, and popcorn piled above the container.)
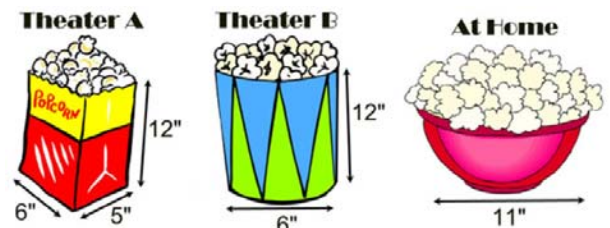
6. The Great Pyramid of Egypt, built for Pharaoh Khufu around 2560 BC, is the oldest and largest of the three pyramids in Giza. Its square base measures 230.4 meters on a side and is 146.5 meters tall.
The second tallest pyramid is the Pyramid of Khafre (son of Khufu) with a square base measuring 215.28 meters on a side and 136.4 meters tall. What is the difference in volume between these two pyramids, to the *nearest cubic meter*?

7. The county installed a metal pipe under the Jones' driveway to allow for drainage of run-off from the road. The pipe is a cylinder with a diameter of 2 feet and a length of 20 feet. To stop their dog, Skeeter, from running into the pipe, the Jones decide to build square steel grates for the ends of the pipe. The grates contain 5 vertical pipes, 5 horizontal pipes and 2 diagonal pipes, as shown. If the steel piping is sold by the foot, how many feet of steel piping will the Jones need to buy to build the two steel grates?

8. In Geometry class, you and your partner are given a cube of Play Dough (modeling clay) measuring 5 inches on a side. Your task is to create as many spherical balls as possible having a diameter of 2 inches each. What is the maximum number of spherical balls you can create from the cube of Play Dough?

9. "Happy" brand peanut butter is sold in cylindrical jars of varying sizes. As is usually the case, buying the larger jar has a reduced unit price. The larger jar has a unit price of 5.5 cents per cubic inch of peanut butter. The smaller jar has a unit price of 6.3 cents per cubic inch of peanut butter. Find the cost of each jar of peanut butter, to the *nearest penny*.

10. A manufacturing firm hired a company to design the pyramid-shaped container shown at the right. It was discovered that certain specifics regarding the design are incorrect. You have been hired to find the errors. The pyramid-shaped container is a right square pyramid with a base side length of 6 inches and a height of 9 inches. In the diagram, G is the midpoint of $\overline{AD}$. Which of the following statements are TRUE regarding the drawing of this container? (*Choose ALL that apply!*)
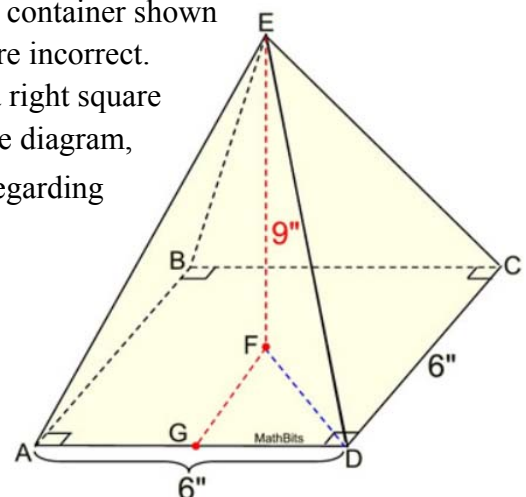
1) $GF = FD$

2) Volume $= 108$ in^3

3) $\triangle EFD$ is a right $\triangle$

4) $GF = 3$ inches

5) $ED = 3\sqrt{11}$ inches

6) Area $\triangle EDC = 27$ sq.in.

7) $\triangle EDC \cong \triangle EAD$

Applied 3D Problems

Name_____

Directions: Answer the following questions relating to three dimensional objects.
Please show all work.

1. Golf balls come in sleeves (rectangular solid packages) containing 3 balls per sleeve. A golf ball has a diameter of approximately 1.68 inches.

    a. What is the shortest possible length of a sleeve of 3 golf balls?

    b. What is the minimum surface area of materials needed to create the outside packaging of the sleeve?

    c. What is the minimum volume of the sleeve?
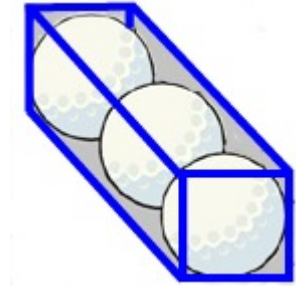
2. A beaver is harvesting bark from a fallen log. The 3 meter log has a consistent diameter of 0.4 meters. How many square meters of bark are on the log (disallowing for the thickness of the bark)? The ends of the log are sliced sections containing no bark.

3. A cylindrical can of *Dog Munchies* is 14 cm high with a radius of 6 cm and sells for $3.40. Another can of the same height has a radius of 3 cm and sells for $1.20. Which can of *Dog Munchies* is the better buy?

4. A bathroom Dixie cup (shown at the right) is actually a "frustum" formed from a right circular cone. The formula for working with such a frustum is given below. How much water will the Dixie cup hold to the *nearest liquid ounce*? (*1 liquid ounce = 1.805 cubic inches*)

$$V_{frustum} = \frac{h\left(A_B + A_T + \sqrt{A_B A_T}\right)}{3}$$

$h = height$

$A_B = area\ of\ bottom\ base$

$A_T = area\ of\ top\ base$
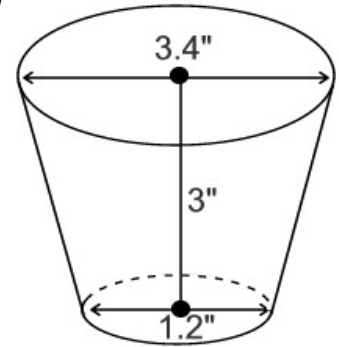
5. To complete a new sunroom addition to a home, concrete steps to the back door are being poured.
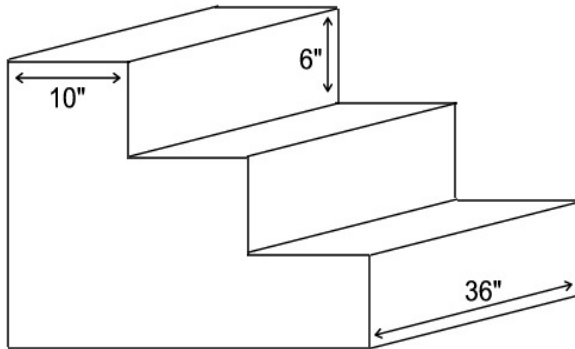
a. Find the volume of the stairs in cubic inches, assuming all of the steps to be of equal rise, depth and length.

b. Convert this answer to cubic yards, the measure normally used for working with concrete.

c. After the concrete has dried, the steps are to be painted with a special non-slip exterior paint. Find the surface area, in square feet, of the steps and the two sides. The back of the steps is placed against the house and will not be painted.

d. The paint for the steps is sold in pints, quarts and gallons. A pint covers 15 square feet. The pint sells for $14.95, the quart for $24.95 and the gallon for $75.95. Describe how to most economically purchase the paint for the steps.

6. A new home and driveway are constructed on a lot 100 feet by 200 feet. The home is 2400 square feet and the driveway is 380 square feet. The remaining areas of the lot are going to be covered with 4 inches of top soil for planting a new lawn. How many cubic feet of top soil are needed?
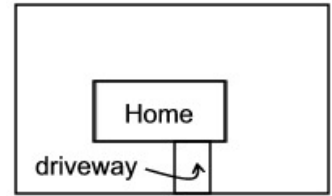
7. A concrete sidewalk is being constructed in front of your school. The width and thickness are as shown. If a concrete truck delivers 2.5 cubic yards of concrete, how many feet of sidewalk can be poured?

8.
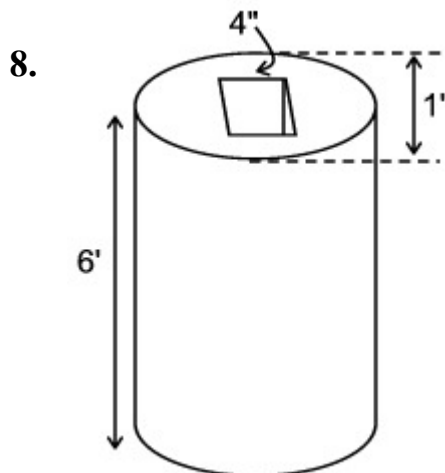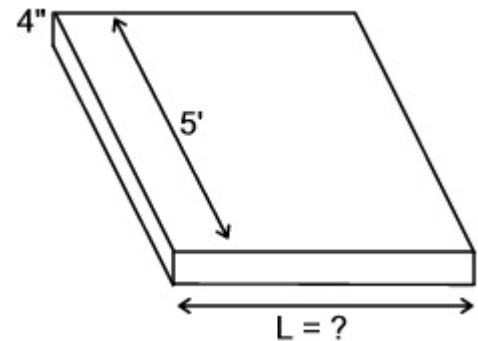
A protective cushion is used to cover the posts in a horseback riding arena. The cushions prevent the riders from injuring their legs on the posts should their horses pass too close to the posts. Each cushion is cylindrical in shape with an opening in the center to allow for the rectangular solid post. If the height of the cushion is 6 feet, the diameter of the cushion is 1 foot, and the post opening is 4 inches by 4 inches by 6 feet, find the volume of material needed to fill the mold for each cushion, to the *nearest hundredth* of a cubic foot.

Working in 3-D Sudoku

Name_____

Directions: Choose the best answer for each question. Place your numerical choice in the puzzle in the indicated row and column (A 4 means column A row 4). When finished, solve the remaining Sudoku puzzle. Every row of 9 numbers must include all digits 1 through 9 in any order, every column of 9 numbers must include all digits 1 through 9 in any order, and every 3 by 3 subsection of the 9 by 9 square must include all digits 1 through 9.

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
1	9			4	6				3
2		6				5		2	
3				3					
4									
5			1			9		5	7
6				2	3		9	1	4
7	8	1			4				
8			5					3	
9			9		5	3			8

1. The radius of a sphere is 4 feet. What is the area of the great circle of this sphere to the *nearest tenth?*
1) 804.2 square feet
2) 50.3 square feet
3) 50.2 square feet
4) 50.1 square feet

[A 4]

2. A cylinder and a cone each have a radius of 4 cm. and a height of 10 cm. What is the ratio of the volume of the cone to the volume of the cylinder?
1) 1:2
2) 1:3
3) 3:1
4) 2:1

[F 7]

[G 4]
3. A rectangular gift box with whole number dimensions has a volume of 36 cubic inches. What are the dimensions of the box requiring the least amount of wrapping paper to cover the box?
1) 2 x 3 x 6 inches   2) 9 x 2 x 2 inches   3) 3 x 3 x 4 inches   4) 4 x 9 x 1 inches

[C 3]
4. When the dimensions of the radius and height of a cylinder are tripled, the volume is:
1) tripled   2) multiplied by 8   3) multiplied by 9   4) multiplied by 27

[E 4]
5. A swimming pool in the shape of a rectangular prism has dimensions 26 feet long, 16 feet wide, and 6 feet deep. How much water is needed to fill the pool to 6 inches from the top?
1) 2288 cu.ft.   2) 2496 cu.ft.   3) 2418 cu.ft.   4) 2216.5 cu.ft.

[B 5]
6. The Great Pyramid of Giza is a right pyramid with a square base. The length of a side of the base is approximately 230 meters and the slant height of the pyramid is approximately 464 meters. To the *nearest cubic meter*, what is the volume of the Great Pyramid?
1) 8181867 cu.m.   2) 8058661 cu.m.   3) 7926590 cu.m.   4) 23778550 cu.m.

[C 7]
7. A right circular cylinder with a height of 8 inches and a diameter of 3 inches was designed to hold water. If the container is filled to two-thirds full with water, what is the volume of the water in the container, to the *nearest cubic inch*?
1) 57 cu.in.   2) 19 cu.in.   3) 38 cu.in.   4) 151 cu.in.
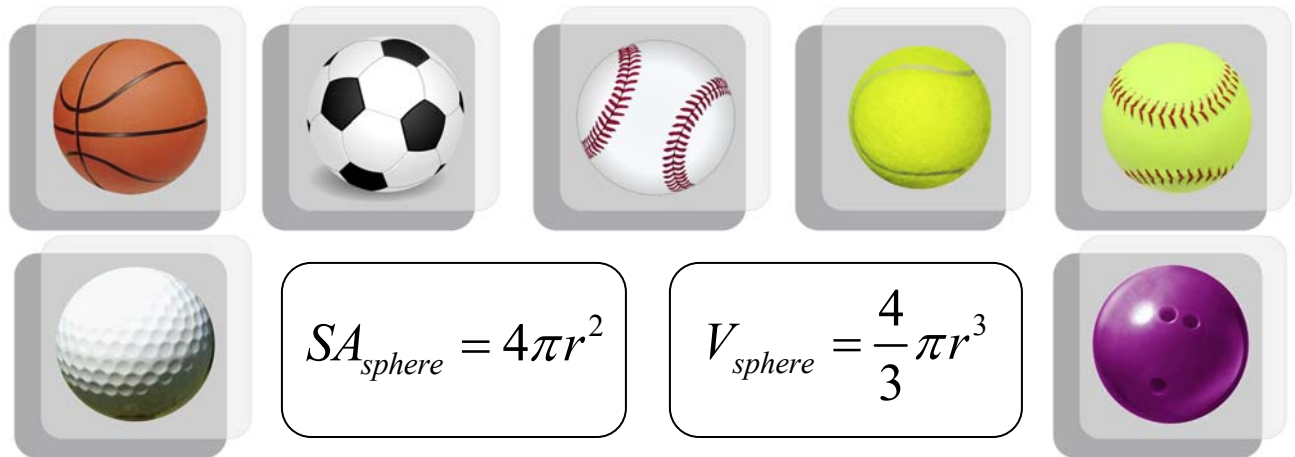
[H 2]
8. The volume, in cubic centimeters, of a sphere whose diameter is 6 centimeters is
1) 288π 2) 48π 3) 12π 4) 36π

Spheres in Play

Name_____

$$SA_{sphere} = 4\pi r^2$$

$$V_{sphere} = \frac{4}{3}\pi r^3$$

Part I: *Directions:* Using the formulas for Surface Area (*SA*) and Volume (*V*), complete the "Diameter", "Radius", "Surface Area" and "Volume" columns in the following chart. Measurements are made in centimeters and results are rounded to the *nearest tenth.*

	Ball	Diameter (cm)	Radius (cm)	Surface Area (cm²)	Volume (cm³)	Mass (grams)	Float ???
1	Basketball	23.9				620	
2	Soccer Ball		12.7			420	
3	Baseball			167.4		142	
4	Tennis Ball				150.5	56.7	
5	Softball	11.4				177	
6	Golf Ball		2.2			45	
7	Bowling Ball (10 pound)			1465.7		4500	
8	Bowling Ball (14 pound)		10.8			6300	

Part II: *Directions:* Using the mass from the chart and the "Density" formula below, determine which, if any, of the balls will float in water. Fill in "YES" or "NO" in the last column of the chart.

$$Density = \frac{Mass}{Volume}$$

If the density is >1, the object will sink.

If the density is <1, the object will float.

Density Worksheet

Please answer all questions as completely as possible showing all calculation and work needed. Also don't forget to include your units!

Density = Mass / Volume

1) Rearrange the density equation for the following:

   Mass =   Volume =

2) Calculate the density of a material that has a mass of 52.457 g and a volume of 13.5 cm³.

3) A student finds a rock on the way to school. In the laboratory he determines that the volume of the rock is 22.7 mL, and the mass in 39.943 g. What is the density of the rock?

4) The density of silver is 10.49 g/cm³. If a sample of pure silver has a volume of 12.993 cm³, what is the mass?

5) What is the mass of a 350 cm³ sample of pure silicon with a density of 2.336 g/cm³?

6) Pure gold has a density of 19.32 g/cm³. How large would a piece of gold be if it had a mass of 318.97 g?

7) The density of lead is 11.342 g/mL. What would be the volume of a 200.0 g sample of this metal?

8) The mass of a toy spoon is 7.5 grams, and its volume is 3.2 ml. What is the density of the toy spoon?

9) A mechanical pencil has the density of 3 grams per cubic centimeter. The volume of the pencil is 15.8 cubic centimeters. What is the mass of the pencil?

10) A screwdriver has the density of 5.5 grams per cubic centimeter. It also has the mass of 2.3 grams. What is the screwdriver's volume?