Reinforced Concrete Buildings Series

Design Booklet RCB-3.1(1)

Cross-section Strength of Columns

Part 1: AS 3600 Design

OneSteel Reinforcing

Guide to Reinforced Concrete Design

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PREFACE

This design booklet is a part of OneSteel Reinforcing' Guide to Reinforced Concrete Design that has been produced to promote the superiority of OneSteel Reinforcing' reinforcing steels, products and technical support services. The Guide covers important issues concerning the design and detailing of Reinforced Concrete Buildings, Residential Slabs and Footings, and Concrete Pavements. The use of 500PLUS reinforcing steels is featured in the booklets. Special attention is given to showing how to get the most benefit from these new high-strength steels.

The design booklets of the Reinforced Concrete Buildings series have each been written to form two separate parts: Part 1- AS 3600 Design which provides insight into major new developments in AS 3600; and Part 2 – Advanced Design using OneSteel 500PLUS Rebar which leads to significant economic advantages for specifiers of OneSteel steel. These booklets are supported by 500PLUS computer software that will prove to be indispensable to design engineers who use AS 3600.

Columns are an important structural element in reinforced concrete structures. They are usually constructed integrally with framing concrete beams and slabs although precast columns can been used in appropriate situations. They have to provide resistance to both axial forces and bending moments generally resulting from loads applied to the floor beams and slabs. A key aspect in the design of columns is the strength of the column cross-section subjected to combined axial force and bending moment. Considering all the possible load and moment combinations for a given cross-section, the manual determination of this strength can be a tedious process [1] , even more so when all the possible different reinforced cross-sections that might be used are taken into account. Therefore, it has been the practice for designers to use load-moment interaction diagrams that have been published [2] for a range of standard cross-section shapes and reinforcement patterns and steel ratios.

A new Australian/New Zealand Standard for reinforcing steels will shortly be introduced with the specification of 500 MPa reinforcing steels covered for the first time [3]. A revised version of AS3600-1994 to take account of the use of 500 MPa reinforcing steels has already gone to public review as document DR99193CP but no changes were then proposed to the rules for the design of columns. Subsequent research associated with the production of this booklet now recommends that some changes be made to the design provisions for columns in AS3600-1994 and these are included in this booklet. Revised load-moment interaction diagrams have been developed for typical column cross-sections used in practice and have been based on considerations of equilibrium, compatibility and appropriate constitutive relationships for the steel and concrete. The method is applicable to column cross-sections with bonded reinforcement assuming sufficient column ties to prevent buckling of the reinforcement.

This design booklet contains both worked examples and important explanatory information about the strength design method in AS3600-1994 and the recommended changes. It is intended that this information will assist structural design engineers to understand the engineering principles on which the design method is based and to better realise the benefits of the increase in yield strength to be gained leading to a significant reduction in steel area. Further research is proceeding that will allow these design provisions to be improved upon when using OneSteel 500PLUS Rebar, and more advanced rules will be found in Part 2 of this design booklet.

1. SCOPE AND GENERAL

1.1 Scope

This design booklet is concerned with the strength of reinforced concrete cross-sections of concrete columns subjected to combined axial force and bending moment and designed in accordance with AS 3600-1994 and some proposed changes contained herein. It provides rules essential for designers to efficiently detail the main (longitudinal) reinforcement. The need for these rules is largely the result of the introduction of 500 MPa as a standard strength grade. This is a significant increase on a current grade of 400 MPa for reinforcing bars.

The types of reinforced concrete columns that have been considered are shown in Figure 1.1

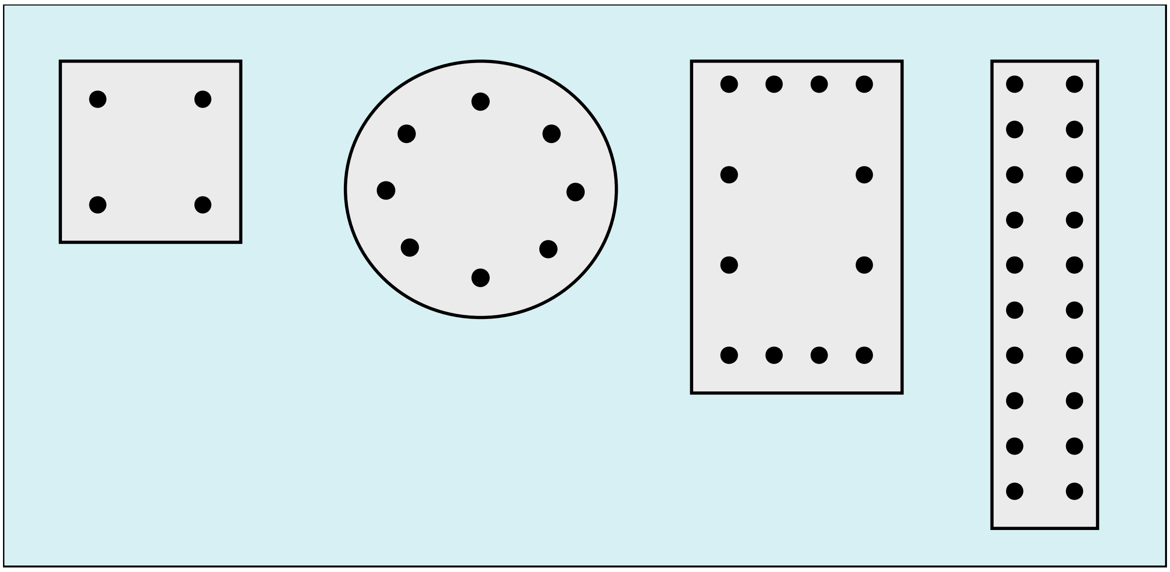

Figure 1.1 Typical Column Cross-sections

The reinforcement must consist of deformed bars with a rib geometry that provides adequate bond. This booklet does not directly cover the design of slender columns, the effects of slenderness and reinforcement requirements. Nor does the booklet cover the calculation of the design action effects resulting from the application of live and dead loads, construction loads, foundation movements, temperature changes and gradients, and creep and shrinkage. Suffice it too say that axial forces and/or bending moments need to be calculated at critical sections for comparison with the axial force and bending strengths that are determined using the methods in this booklet. Some of these aspects will be covered in later booklets. However, the design data in this booklet can be used in conjunction with AS3600-1994 in the consideration of these aspects.

1.2 General

The design rules presented herein are based on considerations of equilibrium and strain compatibility (plane sections normal to the axis remain plane after bending) to determine the load and moment strength for reinforced concrete cross-sections subjected to combined bending and axial force. The column cross-sections are doubly symmetric and are comprised of concrete and fully-bonded longitudinal reinforcement as indicated in Figure 1.1 above.

Two design approaches have been used concerning the distribution of stress in the concrete and steel:

(a) the stress-strain curves for both the steel and concrete are assumed to be of a form defined by recognised simplified equations, or determined from suitable test data; and

(b) a simple rectangular stress block of $0.85f'_c$ is used for the concrete at the strength limit state subject to the limitations of Clause 10.6.2 of AS3600-1994, and the steel is assumed to be linear elastic-plastic in nature with a constant yield stress.

1.2.1 Concrete

The characteristic compressive strengths f'_c of the standard concrete grades covered in this booklet are 20 MPa, 25 MPa, 32 MPa, 40 MPa and 50 MPa. The design properties of the concrete are covered in Section 6.1 of AS3600-1994. It should be noted that if a curvilinear stress-strain relationship is used for the concrete, the curve is modified for design purposes so that the maximum stress is $0.85f'_c$. The concrete is assumed to have no tensile strength.

1.2.2 Reinforcement

The yield strengths f_{sy} of reinforcement covered in this booklet are: 400 MPa for deformed bar to AS1302-1991 with a designated grade 400Y; and 500 MPa for deformed bar with a proposed designated grade of 500N [3]. The other design properties of the reinforcement are covered in Section 6.2 of AS3600-1994. The steel has been assumed to be linear elastic-plastic with an elastic modulus of 200,000 MPa and with a constant yield plateau at the yield strength f_{sy} .

2. TERMINOLOGY

Some important terminology used in this booklet is summarised in this section.

Action

Any agent, such as imposed load, foundation movement or temperature gradient, which may act on a structure.

Action effects

The axial forces and bending moments which are produced in a structure or in its component elements (column) by an action.

Braced column

A column in a structure in which the lateral loads are resisted not by the column but rather by masonry infill panels, shear walls and bracing

Balanced condition for combined bending and compression

The so-called "balanced" condition is where the ultimate strength in combined bending and compression is reached when the stress in the outermost layer of the tensile reinforcement has just reached the yield stress.

Limit condition for combined bending and compression

The "limit" condition, as used in this booklet, is where the ultimate strength in combined bending and compression is reached when the stress in the stress in the outermost layer of the tensile reinforcement is zero, and therefore the rest of the reinforcement is in compression.

Rectangular stress block

Where the neutral axis lies within the cross-section at the strength limit-state, a uniform concrete compressive stress of $0.85f'_c$ can be assumed to act on an area as defined in Section 10.6.2 in AS3600-1994.

Short column

A column where the additional bending moments due to slenderness are small and can be taken as zero.

Slender column

A column in which slenderness effects cause additional bending moments within the length of the column (and also at the column ends in sway columns)

Sway column

A column in which the lateral loads on the structure are resisted by bending of the column.

3. DESIGN CONCEPTS & MODELS

3.1 General

Typically columns are vertical members with large length to depth (*L/D*) ratios subject to predominantly compressive loads although some columns may be subjected to significant bending. The strength of a column cross-section can be determined from the geometry of the cross-section, the constitutive relationships of the concrete and steel and a consideration of equilibrium and strain compatibility. This strength is usually expressed in the form a load-moment strength interaction diagram which plots the locus of ϕM_u and ϕN_u values where M_u is the ultimate strength in bending at a cross-section of an eccentrically loaded compression member, N_u is the corresponding ultimate strength in compression at the same cross-section of the eccentrically loaded compression member, ϕ is the strength reduction factor to account for variability in geometric and material properties, and e is the eccentricity of loading at the cross-section where $e = M^*/N^*$. A typical load-moment interaction diagram is shown in Figure 3.1.

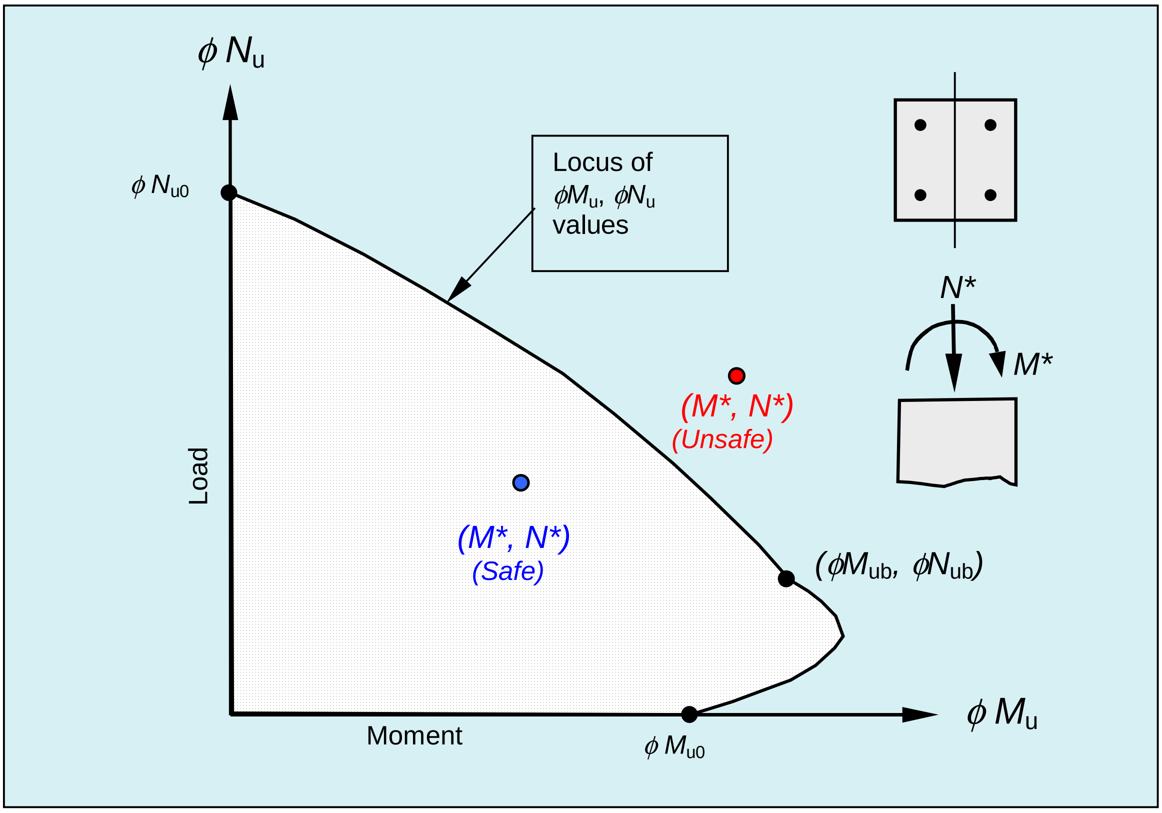

Figure 3.1 Typical Load-Moment Strength Interaction for a Column Cross-section

In determining the strength of the column, the effects of column slenderness must also be considered. Consequently AS 3600 - 1994 classifies columns as either ***short columns*** or ***slender columns*** based on the slenderness ratio L_e/r (Clause 10.3 of AS3600). When the design loads (typically 1.25*G* +1.5*Q*) are applied to a structure, bending moments *M** and axial forces *N** are generated at every cross-section of each column. For slender columns, *M** will include additional bending moments due to slenderness of the columns (second-order effects) which can be calculated directly from a second-order analysis of the loads (Clause 7.7 of AS3600-1994) and/or indirectly using moment magnifiers (Clause 10.4 of AS3600-1994). For short columns, the additional bending moments are deemed negligible and the design values for combinations of *M** and *N** are those applied at the cross-sections at the end of the columns (assuming no lateral loading along the length of the columns). For the design of a column to be considered adequate (safe), the combination of action effects (M^*, N^*) must be less than the combination of design strengths (ϕM_u , ϕN_u) at every cross-section along the length of the column. This process is indicated in Figure 3.1. Usually there is one critical cross-section for any individual column and often this can be determined by simple observation of the action effects.

The design of columns is generally governed by ultimate strength requirements, resulting in the use of ultimate strength design methods. However, serviceability must also be considered, in particular the long term effects such as creep and shrinkage

3.2 Cross-section strength

3.2.1 Introduction

The strength of a cross-section depends on the dimensions of the cross-section, the relative configuration of the steel and concrete components and the material properties of the steel and concrete, in particular the stress-strain relationships for the steel and concrete.

A well authenticated stress-strain relationship for concrete is that given by the Comite Europeen de Beton (CEB [4]). Typical stress-strain curves, for the current grades of concrete up to 50 MPa, are shown in Figure 3.2 where it should be noted that: the maximum strength of the concrete is taken as $0.85f'_c$ to account for the effects of long term loading and other site conditions; and the strain ε_0 corresponding to the maximum strength of the concrete has a constant value of 0.0022 for all the concrete strengths.

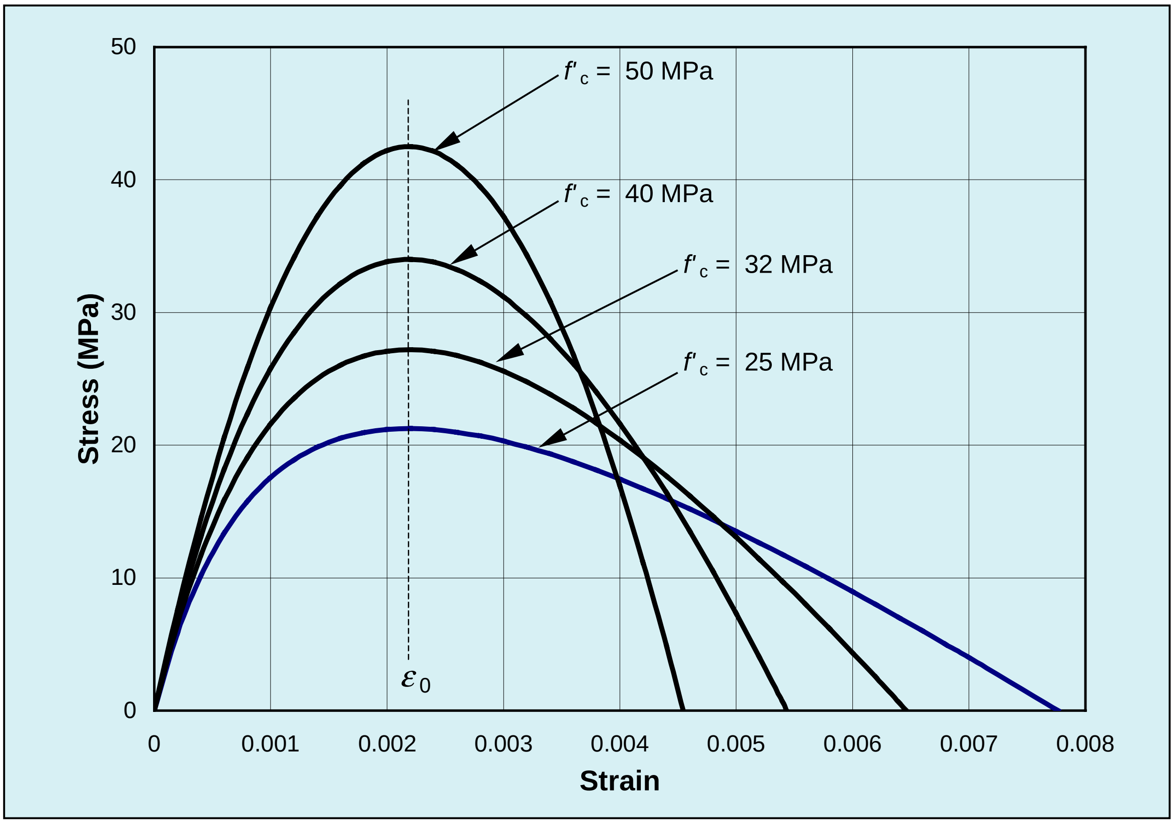

Figure 3.2 Stress-Strain Relationship for Concrete (CEB [4])

For reinforcing steels such as the current 400Y grade and the new 500PLUS® Rebar (500N grade), the assumption of a linear-elastic plastic stress-strain response for design is appropriate as indicated in Figure 3.3. In Figure 3.3, f_{sy} is the yield strength, ε_{sy} is the yield strain where $\varepsilon_{sy} = f_{sy}/E_s$ and E_s is the elastic modulus which can be taken as 200,000 MPa for design purposes. It is normal in strength design to ignore the strain hardening although Turner [3] has suggested an amended stress-strain curve (bi-linear) to take account of strain-hardening.

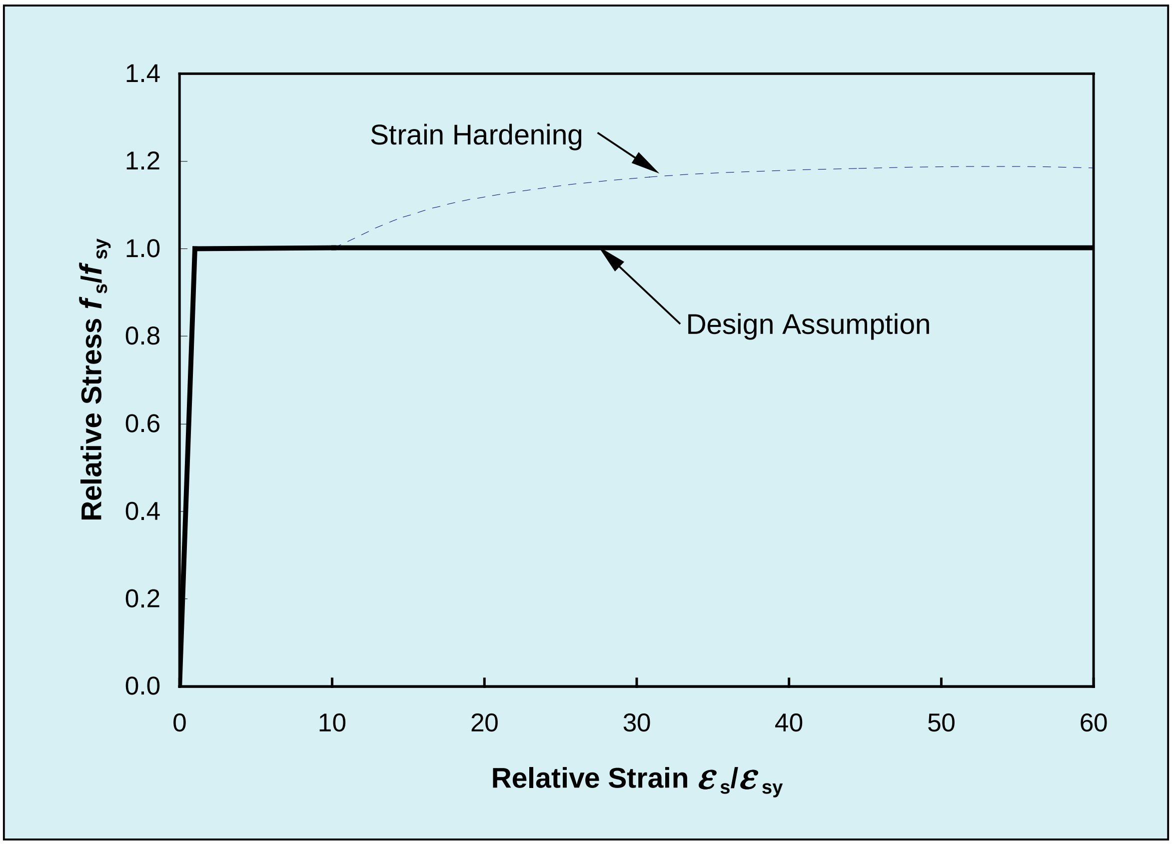

Figure 3.3 Stress-Strain Relationship for Reinforcing Bar

For a cross-section subjected to bending and/or axial force, there are a number of assumptions that are usually made in calculating the strength of the cross-section. These are:

(a) the resulting strain is assumed to have a linear distribution over the cross-section i.e. plane sections remain plane;

(b) the rebar is fully bonded to the concrete such that there is no slip between the concrete and the rebar;

(c) the tensile strength of the concrete is ignored;

(d) the stress in the materials within the cross-section are related to the applied strain by stress-strain relationships for the materials determined under uniaxial stress conditions as shown in Figure 3.2 and Figure 3.3; and

(e) equilibrium and strain compatibility considerations are satisfied.

3.2.2 Axial Compression

Shown in Figure 3.4 is a reinforced concrete cross-section with an arbitrary uniform axial strain ε_a applied to the cross-section. Also shown is the resulting distribution of the concrete stresses and steel stresses. These can be determined using the stress-strain relationships for the concrete (Figure 3.2 non-linear) and the steel (Figure 3.3 linear-elastic plastic) respectively using the value of the axial strain ε_a.

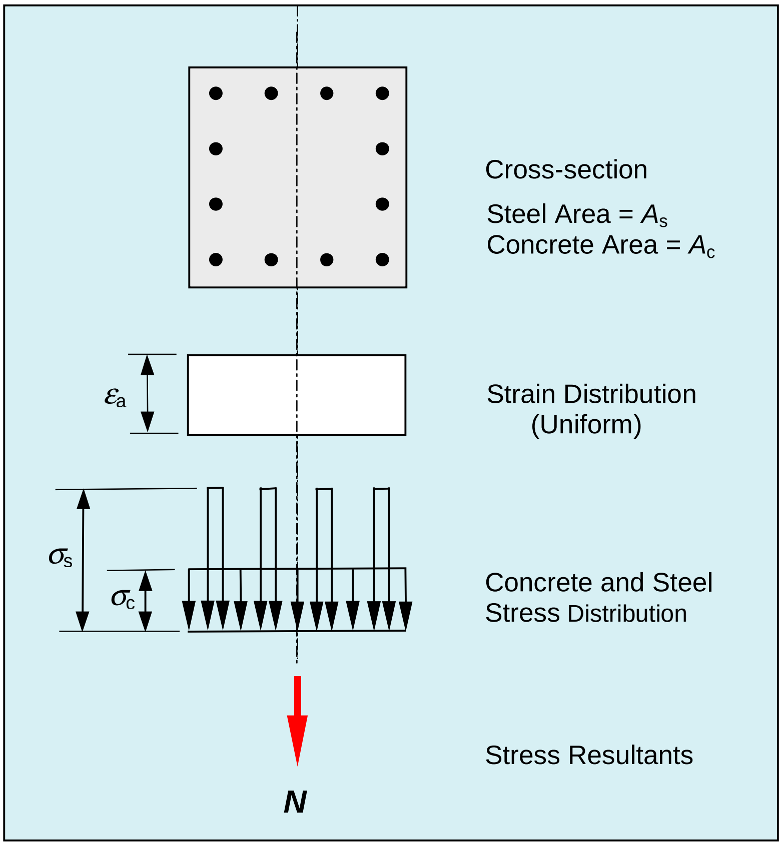

Figure 3.4 Axial Force at a Cross-section

The axial force N resulting from the application of an axial strain ε_a to both the concrete and the steel is given by

$$N = \sigma_c A_c + \sigma_s A_s \qquad 3.2.2(1)$$

The concrete stress σ_c is a non-linear function of the strain ε_a where

$$\sigma_c = f(\varepsilon_a) \qquad 3.2.2(2)$$

as indicated in Figure 3.2 where the strain ε_o at the maximum strength of $0.85f'_c$ has a value of 0.0022. The steel stress σ_s is a simple linear function of the strain ε_a as indicated in Figure 3.3 where the steel is either elastic with an elastic modulus E_s of 200,000 MPa, or plastic with a yield stress of f_{sy} .

To determine the ultimate strength N_{uo} in axial compression, the strain ε_a can be increased until the axial force N given by Eq. 3.2.2(1) reaches a maximum. This process is indicated in Figure 3.5 where the axial force behaviour for the composite reinforced concrete section is compared with the corresponding stresses in the steel and concrete that make up the composite section. The strain corresponding to the ultimate strength N_{uo} in axial compression is defined as ε_{uo} .

When the yield strain ε_{sy} of the steel is less than the strain ε_o (= 0.0022) at the maximum strength of $0.85f'_c$ for the concrete (i.e. the steel yields before the concrete has reached its maximum strength), the ultimate strength N_{uo} in axial compression is simply given by

$$N_{uo} = 0.85f'_c A_c + f_{sy} A_s \qquad 3.2.2(3)$$

This will be the case for 400Y grade rebar having a yield stress f_{sy} = 400 MPa with a yield strain ε_{sy} = 0.002 which is less than the strain ε_o (= 0.0022) at the maximum strength of $0.85f'_c$ for the concrete. The ultimate strength N_{uo} in axial compression is reached at a strain ε_{uo} = 0.0022. This is reflected in Clause 10.6.3 of AS3600-1994 where N_{uo} can be calculated assuming a uniform concrete compressive stress of $0.85f'_c$ and a maximum strain in the steel and concrete of 0.002 (which is approximately equal to 0.0022).

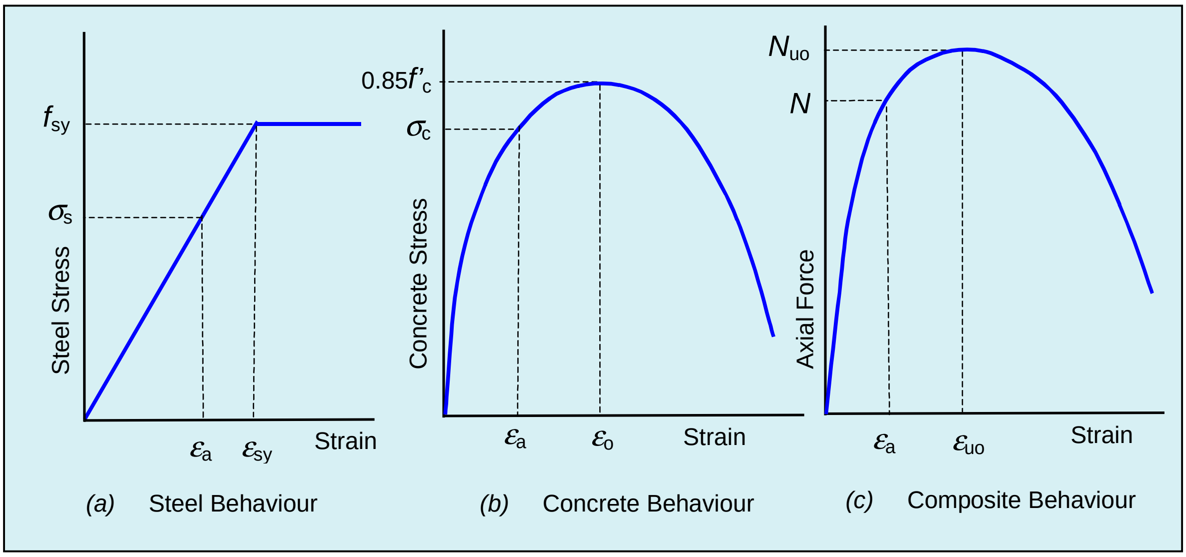

Figure 3.5 Behaviour In Axial Compression

However, 500N grade reinforcing steel such as 500PLUS Rebar has a yield stress f_{sy} = 500 MPa and a yield strain ε_{sy} = 0.0025 which is greater than the strain ε_o (= 0.0022) at the maximum strength of $0.85f'_c$ for the concrete. Hence, the ultimate strength N_{uo} in axial compression is reached at a strain ε_{uo} greater than 0.0022 but not greater than 0.0025 for the following reason. With increasing strains greater than 0.0022, the stress in the concrete (and hence the axial force in the concrete) will commence to decrease at an ever increasing rate. However, the stress in the steel (and hence the axial force in the steel) will continue to increase at a linear rate up to the yield strain of 0.0025. Beyond this strain of 0.0025, the stress (and hence the force) in the steel will remain constant while the stress in the concrete will continue to decrease.

Depending on the geometric properties and the shape of the concrete stress-strain relationship, two solutions are possible for 500N rebar.

(i) The ultimate axial compressive strength N_{uo} is reached when the steel yields where

$$\varepsilon_{uo} = \varepsilon_{sy} = 0.0025 \qquad 3.2.2(4)$$

Using Eq. 3.2.2(1)

$$N_{uo} = \sigma_c A_c + f_{sy} A_s \qquad 3.2.2(5)$$

and using Eq. 3.2.2(2) for σ_c gives

$$N_{uo} = f(\varepsilon_{sy}) A_c + f_{sy} A_s \qquad 3.2.2(6)$$

(ii) The ultimate axial compressive strength N_{uo} is reached before the steel yields. In this elastic region, the stress in the steel σ_s given by

$$\sigma_s = E_s\varepsilon_a \qquad 3.2.2(7)$$

where E_s is the elastic modulus of the steel. From Eqs 3.2.2(1), (2) and (7) then the axial axial force N corresponding to a strain ε_a is given by

$$N = f(\varepsilon_a)A_c + E_s\varepsilon_a A_s \qquad 3.2.2(8)$$

Differentiating N in Eq. 3.2.2(8) with respect to strain ε_a and equating to zero gives the condition for maximum axial compressive strength N_{uo} where

$$\frac{dN}{d\varepsilon_a} = E_s A_s + \frac{d(f(\varepsilon_a))}{d\varepsilon_a}A_c = 0 \qquad 3.2.2(9)$$

That is when

$$\frac{d(f(\varepsilon_a))}{d\varepsilon_a} = -E_s\frac{A_s}{A_c} \qquad 3.2.2(10)$$

indicating that the maximum strength is reached on the unloading (negative slope) portion of the concrete stress-strain curve. The solution of Eq. 3.2.2(10) yields the value of $\varepsilon_a = \varepsilon_{uo}$ and the corresponding maximum axial compressive strength N_{uo} is given by

$$N_{uo} = f(\varepsilon_{uo})A_c + E_s\varepsilon_{uo}A_s \qquad 3.2.2(11)$$

Equation 3.2.2(10) can be solved analytically if the stress-strain relationship $\sigma_c = f(\varepsilon_a)$ is in a closed form solution and amenable to differentiation. Alternatively, Eq. 3.2.2(11) can be solved numerically by varying ε_{uo} until a maximum value is obtained for N_{uo} .

There is no prior way of knowing which of the two solutions (i) and (ii) above is applicable. However, the value of N_{uo} from Eq. 3.2.2(6) will be less than or equal the value from Eq. 3.2.2(11) and could be used conservatively for design purposes.

3.2.3 Combined Compression and Bending

3.2.3.1 Introduction

The capacity of a column cross-section depends on the eccentricity of the applied load, with the load decreasing as the eccentricity increases. General practice is to represent an eccentric load as an axial load and a moment equivalent to the product of the applied axial load and the eccentricity.

Shown in Figure 3.6 is a reinforced concrete cross-section with an arbitrary linear distribution of strain defined by the centroidal axis strain ε_a and the curvature $\rho = 1/R$ where R is the radius of curvature. Also shown are the resulting distributions of concrete stress and steel stress which are related to the strain by the stress-strain relationships for the steel and the concrete as shown in Figure 3.2 and Figure 3.3 respectively. The axial force N and bending moment M about the centroidal axis resulting from the stresses are shown at the foot of Figure 3.6.

Hence, values of axial force N and moment M corresponding to a given value of axial strain ε_a and curvature $1/R$ can be calculated as follows. The stresses are integrated over the concrete and steel areas to determine the axial forces in the steel and concrete components of the area and then summed to give the axial force N. Moments of the individual forces are taken about the centroidal axis and then summed to determine the moment M about that axis.

This process can be done manually providing the stress-strain function for the concrete is amenable to integration and the geometry of the cross-section is not too complex. Simplifying assumptions can be made such as: treating the steel reinforcement as concentrated at one point and ignoring the concrete area displaced by the steel and fitting a simple function to the concrete stress-strain curve. However, this process can still be tedious and the most convenient approach is to use a computer based analytical method such as that by Wheeler and Bridge[5].

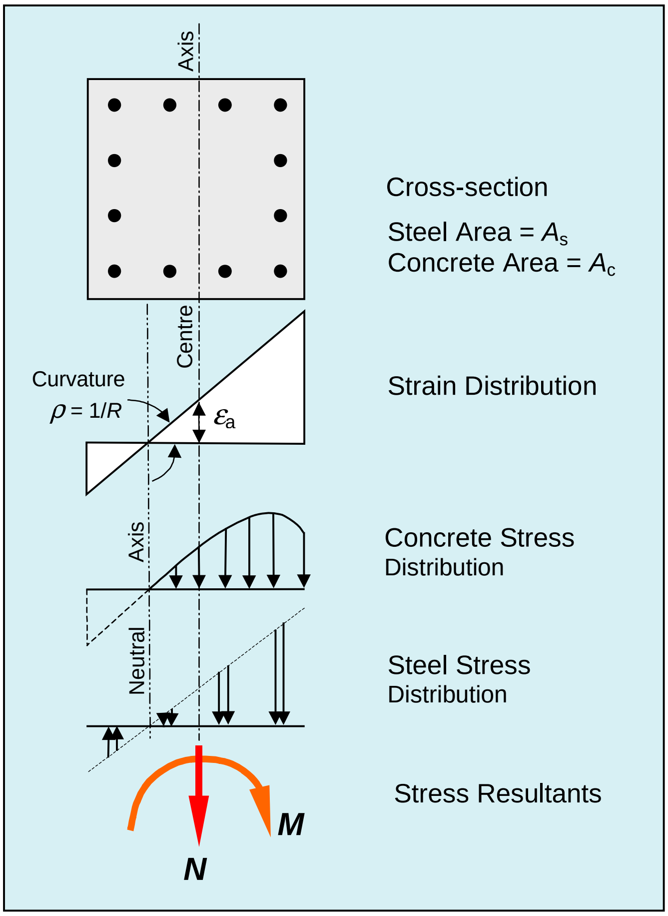

Figure 3.6 Axial Force and Bending Moment at a Cross-section

3.2.3.2 Load-Moment-Curvature Relationship

By keeping the curvature ρ *(=1/R)* constant and varying the centroidal axial strain ε_a, the value of axial force *N* and moment *M* corresponding to each value of centroidal axial strain ε_a can be calculated and then plotted as a contour of constant curvature as shown in Figure 3.7. The values of constant curvature that have been plotted in Figure 3.7 have first been made non-dimensional by multiplying by the depth *D* of the cross-section and then further multiplied by 1000 to give convenient values for notation on the Figure 3.7.

It can be seen from Figure 3.7 that by varying both strain ε_a and curvature ρ (= *1/R*) over a full range of values, the complete load-moment-curvature relationship can be developed and can be displayed as a set of contours of constant curvature *D/R* (x1000) on a plot of axial force *N* against moment *M*.

The outer boundary (or failure locus) of points of maximum *M, N* combination shown in Figure 3.7 is the well-known interaction curve for cross-section strength, similar to those shown in the design charts published jointly by the Cement and Concrete Association and Standards Australia [2].

The calculations to derive the load-moment-curvature relationship in Figure 3.7 were carried out using a computer programme solution originally developed for composite columns by Bridge and Roderick [6]. This was later modified to account for all types of column cross-sections by Wheeler and Bridge [5]. The analysis included equilibrium and compatibility considerations and assumed that: plane sections remained plane; concrete had no tensile strength; the concrete material stress-strain curve was of the type shown in Figure 3.2; and the stress-strain curve for the reinforcement was linear-elastic fully-plastic of the type shown in Figure 3.3.

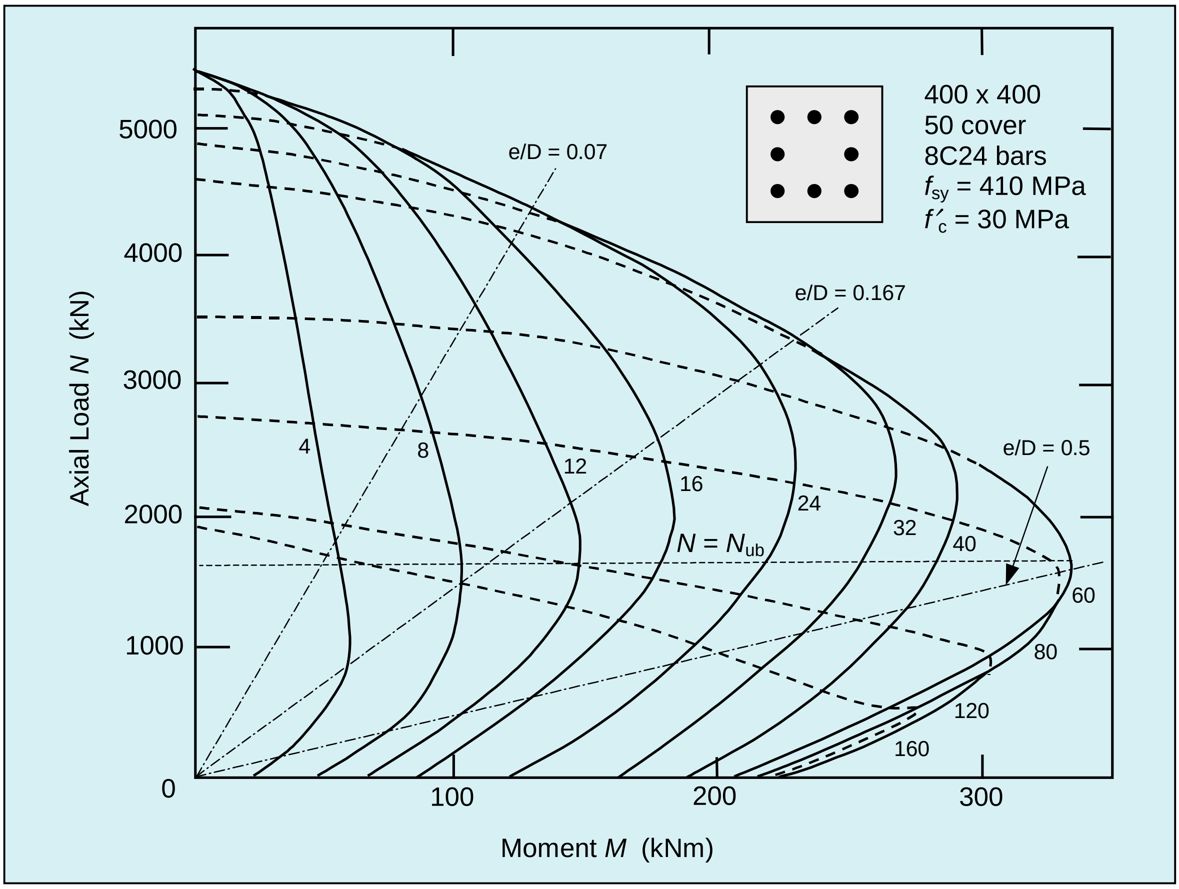

Figure 3.7 Load-Moment-Curvature Diagram with Contours of Constant Curvature D/R

Figure 3.8 Model of Load-Moment-Curvature Diagram

It should be noted that Figure 3.7 actually represents a three dimensional surface of all equilibrium points of load-moment-curvature. This surface has been modelled in timber and is shown in Figure 3.8 where the contours of constant curvature can be clearly seen.

While designers are mainly interested in the outermost boundary to the load and moment values (known as the load-moment strength interaction diagram), it should be realised that equilibrium solutions exist for curvatures both less than and greater than those corresponding to the maximum value of the load-moment combination which defines the strength.

For the same column cross-section as that used in Figure 3.7 and loaded at a constant eccentricity of e by a load N, the resulting moment-curvature relationships are shown in Figure 3.9 for four different values of eccentricity to depth ratio e/D. The maximum moment M_u for each e/D ratio is shown marked by a solid point and the corresponding curvature is indicated by a vertical dashed line. The maximum moment M_u is a point on the boundary of the maximum M, N values corresponding to the intersection of the constant e/D eccentricity line with the boundary as shown in Figure 3.1. (Note: $e/D = \infty$ corresponds to the case of pure moment with zero axial force).

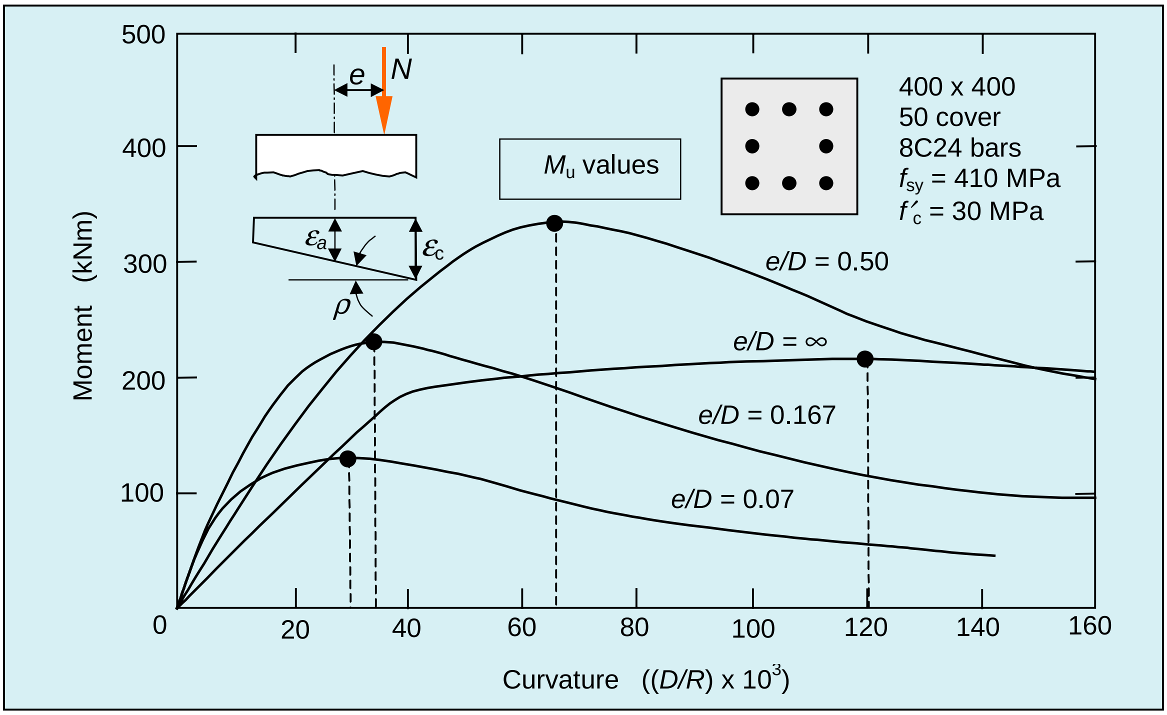

Figure 3.9 Moment-Curvature Relationships for Cross-Section Loaded at Constant Eccentricity

It can be seen that the curvature corresponding to the maximum moment varies with the eccentricity of loading. However, it is interesting to note that for strain distributions where the neutral axis lies within the cross-section ($e/D < 0.167$), the maximum moment (a point on the locus of maximum M, N values) is obtained when the compressive strain ε_c at the extreme fibre (see insert in Figure 3.9) has an essentially constant value of approximately 0.003 [7] irrespective of the eccentricity. This is the basis for the requirement of the rectangular compressive stress block in Section 10.6.2 of AS3600-1994 where, provided the maximum strain in the extreme compression fibre of the concrete is taken as 0.003, the strength of the cross-section can be calculated using a uniform rectangular compressive stress block of $0.85f'_c$ acting on an area bounded by the edges of the cross-section and a line parallel to the neutral axis under the loading concerned and located at a distance $\gamma k_u d$ from the extreme compressive fibre where $k_u d$ is the depth of the neutral axis for the compressive region of the cross-section. It should be noted that using a constant value 0.003 for the extreme fibre strain ε_{cu} is only strictly valid for cross-sections with low percentages of steel [7] such that the tensile steel is yielded at the ultimate strength M_{uo} in bending without axial force (i.e. pure bending). This type of cross-section is sometimes referred to in texts [1] as "under-reinforced". However, AS3600-1994 uses the value 0.003 for all types of cross-sections and steel configurations. It should be noted that when computer analyses [5] [6], are used to generate the load-moment-curvature relationship and hence the load-moment interaction diagram such as that in Figure 3.7, there is no need to use a value of ε_{cu} = 0.003 as the analyses can determine the ultimate (maximum) bending strength M_u for any given ultimate axial strength N_u. The use of a value of ε_{cu} = 0.003 is only an approximation.

3.2.3.3 Load-Moment Strength

There are four key points on the load-moment strength interaction diagram that are of particular interest and use to designers as indicated in Figure 3.10. The four points are clearly marked and are also identified by their corresponding strain distribution in Figure 3.10.

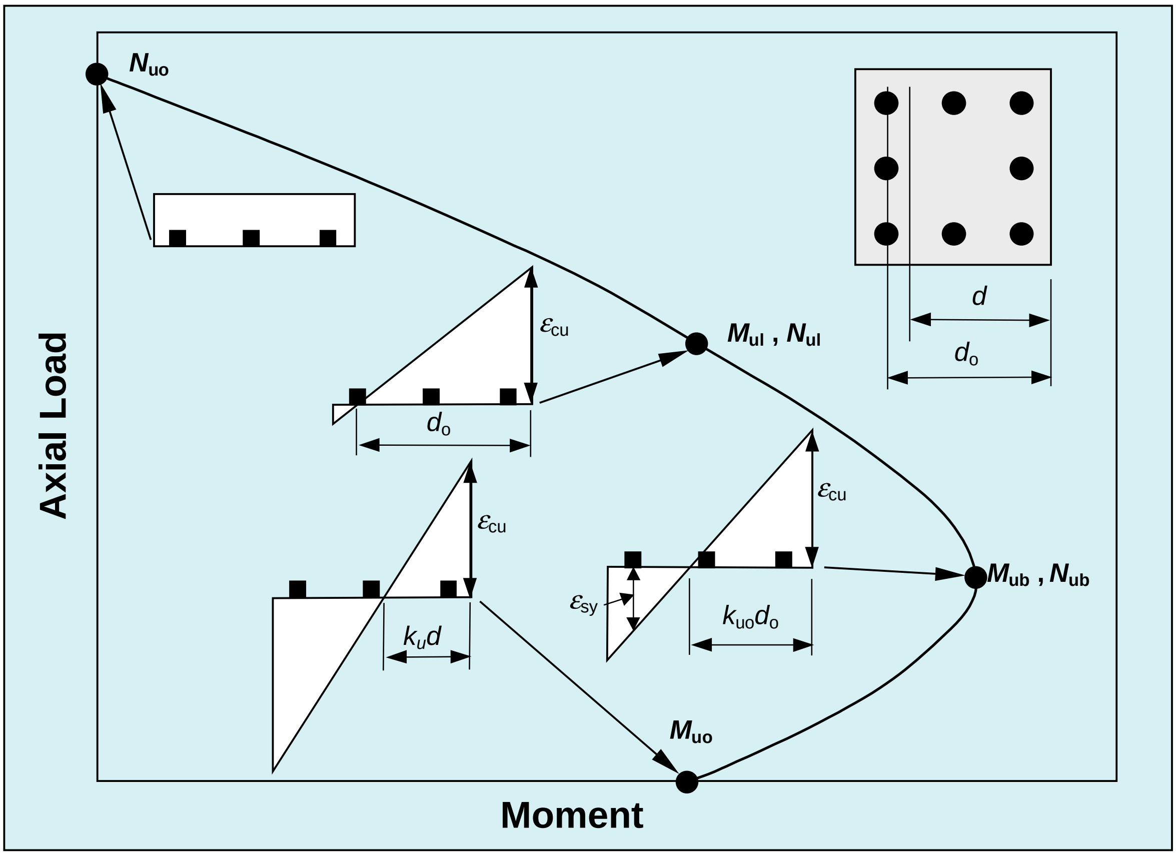

Figure 3.10 Key Points on the Load-Moment Interaction Diagram

(a) The ultimate strength M_{uo} in bending without axial force (pure bending).

This is the point where the axial compressive strength N_u = 0. This condition of pure bending is used to define the effective depth d of the cross-section. The effective depth d is calculated as the distance from the extreme compressive fibre of the concrete to the resultant tensile force in the reinforcing steel that is in the tension zone. Where there are two or more layers of steel in tension under pure bending, the effective depth d will be less than the depth d_o from the extreme compressive fibre to the centroid of the outermost layer of tensile reinforcement. The depth to the neutral axis is k_ud where k_u will be in the range $0 < k_u < 1$ depending on the percentage of steel p (= A_s/bD) used in the cross-section, the higher the value of p , the higher the value of k_u . High values of p and hence k_u lead to poor ductility [1] [8] and should be avoided if possible. This is not always possible in heavily reinforced columns.

In design, the design strength is taken as ϕM_{uo} where ϕ is a strength reduction factor to account for variability in geometric and material properties of the cross-section. For cross-sections with low k_u values where k_u is less than 0.4, a value of ϕ = 0.8 is required by AS3600-1994 (Table 2.3). For cross-sections where k_u exceeds 0.4, AS3600-1994 (Table 2.3) requires the use of a reduced strength reduction factor ϕ to reflect the reduced ductility and hence the reduced ability for any moment redistribution which may be required in the event of overload or change in load pattern, where

$$\phi = 0.8\frac{M_{ud}}{M_{uo}} \geq 0.6 \qquad 3.2.3.3(1)$$

and M_{ud} is the reduced ultimate strength in pure bending. The value of M_{ud} can calculated in accordance with Clause 8.1.3 of AS3600-1994 by assuming that: the neutral axis depth is reduced to 0.4d (i.e. k_u = 0.4); and the resultant of the tensile forces in the reinforcement is reduced to equal the reduced compressive force corresponding to a neutral axis depth 0.4d such the axial force is zero. When using computer based analytical methods [5] [6], an alternative approach is to reduce the percentage of steel until k_u = 0.4 for the condition of pure bending in which case $M_{ud} = M_{uo}$.

(b) "Balanced" ultimate strength M_{ub} in bending and ultimate strength N_{ub} in axial compression

This so-called "balanced" condition is where the particular ultimate strength M_{ub} in bending and the corresponding particular ultimate strength N_{ub} in axial compression are determined for a particular value of neutral axis depth $k_{uo}d_o$ where d_o is the depth from the extreme compressive fibre to the centroid of the outermost layer of tensile reinforcement and k_{uo} is such that this outermost layer of steel has just reached yield at a strain of ε_{sy} . This point is usually at or close to the "nose" of the load-moment interaction diagram. From the strain distribution corresponding to this point as shown in Figure 3.10, the value of k_{uo} is given by

$$k_{uo} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{sy}} \qquad 3.2.3.3(2)$$

In AS3600-1994, the normal type of bar reinforcement used in columns is 400Y with a design yield stress f_{sy} = 400 MPa and a yield strain ε_{sy} = 0.002. The maximum compressive strain ε_{cu} in the concrete at ultimate strength is taken as 0.003. Using these values in Eq. 3.2.3.3(2) gives a value of k_{uo} = 0.6 which is the value that used in AS3600-1994 (see definitions of M_{ub} and N_{ub} in Clause 1.7). For 500N grade steel such as the new 500PLUS® Rebar with a design yield stress f_{sy} = 500 MPa and a yield strain ε_{sy} = 0.0025, then Eq. 3.2.3.3(2) gives a value of k_{uo} = 0.545.

In design, the design strength is taken as ϕM_{ub} and ϕN_{ub} where ϕ is a strength reduction factor to account for variability in geometric and material properties of the cross-section. A value of ϕ = 0.6 is required by AS3600-1994 (Table 2.3). For values of $0 \leq N_u < N_{ub}$, linear interpolation is used to evaluate ϕ where

$$\phi = 0.6 + \left[(\phi_0 - 0.6)(1 - \frac{N_u}{N_{ub}})\right] \qquad 3.2.3.3(3)$$

and ϕ_o is the value of ϕ that is obtained for pure bending (see case (a) above).

(c) "Limit" ultimate strength M_{ul} in bending and ultimate strength N_{ul} in axial compression

This condition is defined herein as the "limit" condition where the particular ultimate strength M_{ul} in bending and the corresponding particular ultimate strength N_{ul} in axial compression are determined for a particular value of neutral axis depth $k_{uo}d_o$. The value k_{uo} is taken as 1.0 and d_o is the depth from the extreme compressive fibre to the centroid of the outermost layer of tensile reinforcement Therefore, the outermost layer of steel has zero stress and hence the rest of the steel is in compression. This is taken as the limit to the validity of the use of a constant value of compressive strain ε_{cu} = 0.003 at ultimate strength in combined bending and compression as assumed in Clause

10.6.2 of AS3600-1994. Clause 10.6.2 actually allows the neutral axis for this limit to be at the extreme tension edge of the cross-section such that k_{uo} will be greater than 1.0, the value depending on the concrete cover. Using this value of k_{uo} simply defines a slightly different point on the load-moment interaction diagram. It should be noted that beyond this limit point for $k_{uo} > 1.0$ where all the steel is in compression, the maximum strength will be the ultimate strength N_{uo} in pure axial compression where $k_{uo} = \infty$ and the ultimate strain will have values ranging from 0.0022 to 0.0025 depending on the grade of the steel used (see Section 3.2.2 above). This is less than the value of $\varepsilon_{cu} = 0.003$ for bending where the neutral axis is within the cross-section.

In design, the design strength is taken as ϕM_{ul} and ϕN_{ul} where ϕ is a strength reduction factor to account for variability in geometric and material properties of the cross-section. A value of $\phi = 0.6$ is required by AS3600-1994 (Table 2.3).

(d) The ultimate strength N_{uo} *in compression without bending (pure axial force)*

This is the point where the moment strength $M_u = 0$. This case has already been covered separately in Section 3.2.2 above.

In design, the design strength is taken as ϕN_{uo} where ϕ is a strength reduction factor to account for variability in geometric and material properties of the cross-section. A value of $\phi = 0.6$ is required by AS3600-1994 (Table 2.3).

The selection of the appropriate values of strength reduction factor ϕ in AS3600-1994 was based on reliability studies carried out by Bridge and Pham [9] using target safety indices appropriate to reinforced concrete construction and consistent with other material based standards.

3.2.4 Effects of Slenderness

This booklet is concerned only with the strength of reinforced concrete cross-sections and the effects of slenderness will be covered in detail in another booklet. However, a brief discussion on the effects of slenderness for braced columns is included here to allow designers to appreciate the overall design process in which the cross-section strength is an integral part.

Shown in Figure 3.11 are columns of length L subjected to an axial load N^* and end moments M^*_1 and M^*_2 where the magnitude of M^*_2 is greater than the magnitude of M^*_1. It can be seen that the columns can be bent in constant (symmetrical) curvature, single curvature or double curvature. The deformation Δ of the columns results in additional bending moments $N^*\Delta$ being applied to the columns. For the case of constant symmetric curvature, the maximum moment will be at the centre of the column with a value of $M^*_2 + N^*\Delta$ and so for any finite length of column, the maximum moment will be greater than the end moment M^*_2. For the case of double curvature and even for the case of single curvature, it can be seen that the additional moments $N^*\Delta$ may not result in the maximum moment being greater than the larger end moment M^*_2. This will depend on the amount of deformation Δ which is directly related to the length of the column (columns with small L will have small values of deformation Δ). An extensive study by Bridge and Seevaratnam [10] found that there was a wide range of column slenderness and end moment ratio (single and double curvature) for which the maximum moment remained at the end of the column with a value M^*_2.

The results of this study have been incorporated in Clause 10.3.1 of AS3600-1994 which defines the conditions for a column to be deemed "short". A "short" column can be simply defined as *"a column which can be designed for its cross-section strength with out the need to consider stability (slenderness) effects"* [10]. Therefore there is a maximum length of a column below which the column strength is not reduced by slenderness and the strength of the column is governed by the strength of the cross-section. For columns bent in single and double curvature as shown in Figure 3.11 and which are deemed to be short, the critical cross-section for design will be the end where the larger end moment M^*_2 is applied.

For an adequate design, the following conditions must be met concurrently:

$$M^*_2 \le \phi M_u \qquad 3.2.4(1)$$

and

$$N^* \le \phi N_u \qquad 3.2.4(2)$$

where values of ϕM_u and ϕN_u are obtained from the load-moment interaction diagram (see Section 3.2.3.3 and also Figure 3.1.

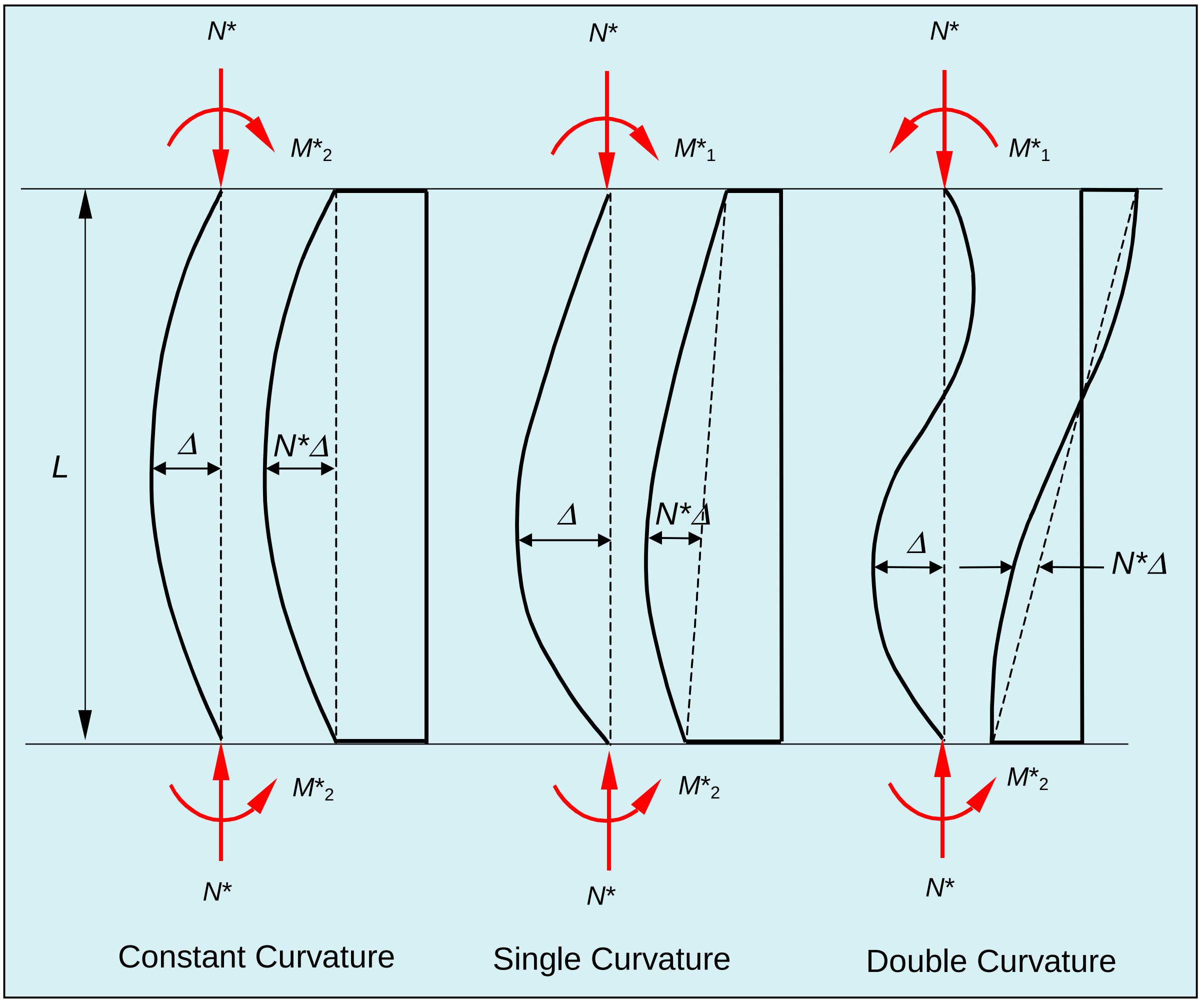

Figure 3.11 Braced Columns Subjected End Moment and Axial Load

In Clause 10.3.1 of AS3600-1994, a braced column is deemed to be short provided the column slenderness L_e/r meets the following conditions:

$$\frac{L_e}{r} \le 25 \qquad 3.2.4(3)$$

or

$$\frac{L_e}{r} \le 60\left(1 + \frac{M_1^*}{M_2^*}\right)\left(1 - \frac{N^*}{0.6N_{uo}}\right) \qquad 3.2.4(4)$$

whichever is the greater. The ratio of M^*_1/M^*_2 is taken as positive when the column is bent in double curvature and negative when bent in single curvature.

The radius of gyration for a reinforced column can be defined as

$$r = \sqrt{\frac{E_s I_s + E_c I_c}{E_s A_s + E_c A_c}} \qquad 3.2.4(5)$$

but this requires the determination of the elastic moduli E_c and E_s and the second moment of areas I_c and I_s of the concrete and steel respectively. The value of radius of gyration r in Equation 3.2.4(5) can be reasonably approximated [10], for rectangular reinforced concrete cross-sections with practical steel percentages, by

$$r = 0.3D \qquad 3.2.4(6)$$

where D is the depth of the overall section in the direction in which stability is being considered (see Clause 10.5.2 of AS3600-1994), and for circular reinforced concrete cross-sections with practical steel percentages by

$$r = 0.25D \qquad 3.2.4(7)$$

where D is the diameter. There are a number of different approaches for determining the effective length L_e of a column which is part of a frame or structure and these have been discussed in detail by Bridge [11]. Clause 10.5.3 of AS3600-1994 uses an approach based on the well-known method for a column with simple rotational end restraint coefficients γ_1 and γ_2 for the two cases of translational end restraint of zero (free to sway) or infinity (fully braced). It is identical to that used for a column in a steel structure designed to AS4100-1998. No account is taken of axial forces in the beam members, intermediate values of translational restraint, or interaction frame buckling with other columns in the frame or structure.

There are two important points that can be drawn from Eqs. 3.2.4(3) and 3.2.4(4). Firstly, a column of finite length and bent in constant curvature can still be deemed to be short even though the moment at the centre of the column will be greater than M^*_2. This is tantamount to accepting a design strength up to approximately 5% greater than the actual strength for values of slenderness ratio L_e/r up to 25 [10]. While such an approach may be considered as slightly unconservative, it is one which reduces the design effort and hence the probability of error for a number of practical columns. Secondly, for a column bent in bent in double curvature with equal end moments, a column with a slenderness ratio L_e/r as high as 120 could be considered as short provided the axial load N^* required to be supported is small.

It should be noted that Clause 10.1.2 of AS 3600-1994 requires that, at any cross-section of the column, the design bending moment M^* should not be less than $0.05DN^*$. This implies a minimum load eccentricity e of $0.05D$ where D is the depth of the column in the plane of the applied bending moment. This allows for geometric and material imperfections in the column. It also allows the axial strength of slender columns (in the absence of any other applied end moments) to be determined. This is one of the major differences between the design of concrete and steel columns [12], the latter using a column curve (variation of strength with slenderness, Clause 6.3 of AS4100-1998).

Where the columns are not short in accordance with Eqs. 3.2.4(3) and 3.2.4(4), they are deemed to be slender and slenderness effects have to be taken into account by multiplying the largest first-order (initial) design bending moment by a moment magnifier δ to determine the design bending moment M^*. The design axial force N^* is not altered and this procedure is similar to that used for the design of steel columns to AS4100-1998. For the column with constant curvature shown in Figure 3.11 where the additional bending moment is given by $N^*\Delta$, the design bending moment M^* (at the centre) is given by

$$M^* = M^*_2 + N^*\Delta \equiv \delta M^*_2 \qquad 3.2.4(8)$$

Hence, the value of the magnifier δ has to be calibrated to give the correct additional bending moment $N^*\Delta$. The moment magnifier for a braced column δ_b is given in Clause 10.4.2 of AS3600-1994 as

$$\delta_b = \frac{k_m}{1 - N^*/N_c} \geq 1.0 \qquad 3.2.4(9)$$

where

$$k_m = \left(0.6 - 0.4M_1^*/M_2^*\right) \geq 0.4 \qquad 3.2.4(10)$$

The coefficient k_m is used to convert a column with unequal end moments into an equivalent column bent in constant symmetrical curvature with equal end moments of $k_m M^*_2$. The ratio of M^*_1/M^*_2 is taken as positive when the column is bent in double curvature and negative when bent in single curvature. The effects of varying end moments on the maximum moment are shown Figure 3.11. For the constant curvature case, M^*_1/M^*_2 = -1 resulting in a coefficient k_m = 1.0. For single curvature the coefficient k_m decreases to 0.6 as M^*_1 approaches zero. Finally for double curvature, the coefficient k_m decreases to 0.2 (taken as 0.4) as the value of M^*_1 approaches the value of M^*_2. While the coefficient k_m may reduce to a low value less than 1.0, the amplification factor δ_b must be at least equal to 1.0 such that the minimum design moment has a value at least equal to the larger end moment of M^*_2 .

Figure 3.12 Slender Column in Practice

In determining the moment amplification factor δ_b from Eq.3.2.4(9), it is necessary to determine the column buckling load N_c where

$$N_c = \frac{\pi^2 EI}{L_e^2} \qquad 3.2.4(11)$$

The effective length L_e can be determined in accordance with Clause 10.5.3 of AS 3600-1994. However the stiffness *EI* of the column cross-section varies according to the level of axial load and moment applied to the column. To simplify the design process, the secant stiffness for the column, based on the stiffness of the column cross-section at the balance point (M_{ub}, N_{ub}) is utilised to define the column stiffness [13],[14]. The secant stiffness has been shown to be relative constant for a wide range of points (M_u, N_u) other than the balanced point [15]. Shown in Figure 3.13 is a typical moment-curvature relationship at a constant axial force equal to the balanced value N_{ub}.

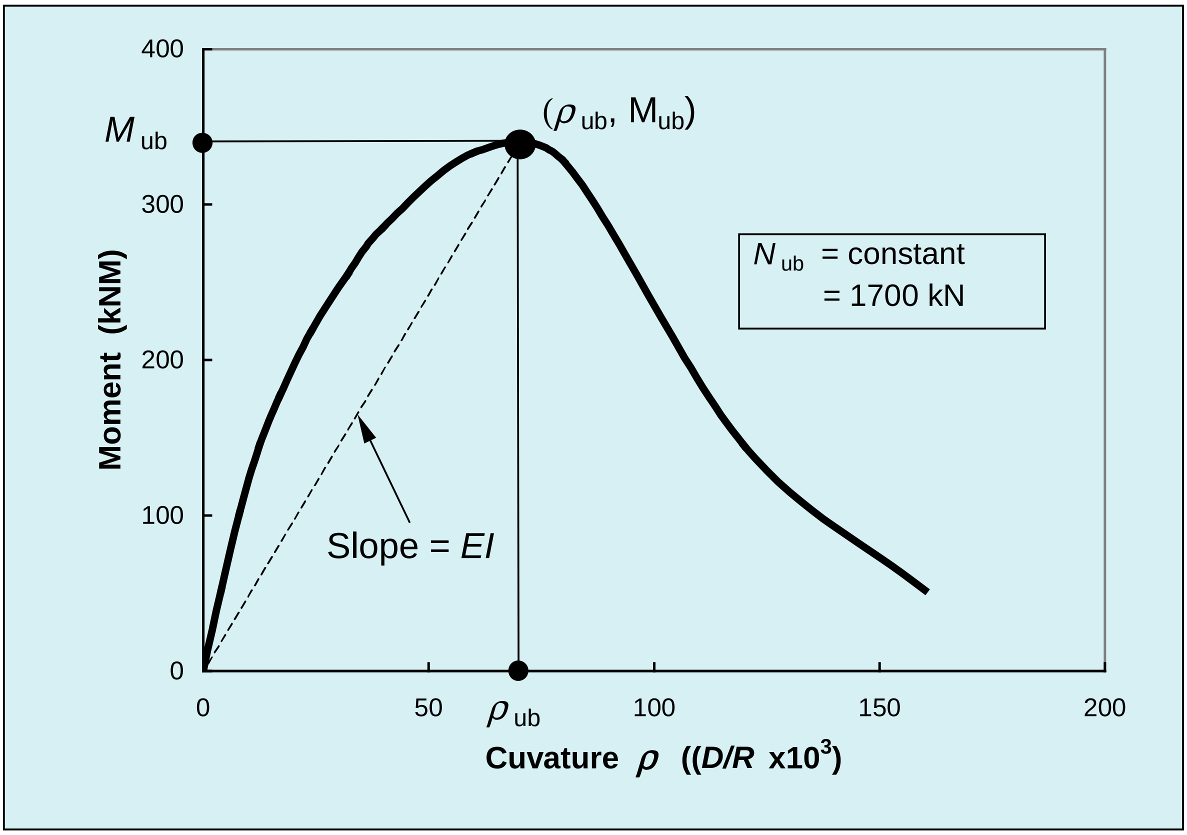

Figure 3.13 Moment-Curvature Relationship for Constant Balanced Axial Force N_{ub}

Figure 3.13 was derived from the load-moment-curvature relationship for the cross-section shown in Figure 3.7 for $N = N_{ub}$. The secant stiffness *EI* at the balance point is given by

$$EI = \frac{M_{ub}}{\rho_{ub}} \qquad 3.2.4(12)$$

From the strain diagram shown in Figure 3.10 for the balance point, the curvature ρ_{ub} (slope of the strain distribution) is given by

$$\rho_{ub} = \frac{\varepsilon_{cu}}{k_{uo} d_o} \qquad 3.2.4(13)$$

Substituting the value of k_{uo} from Eq. 3.2.3.3(2) into Eq 3.2.4(13) then

$$\rho_{ub} = \frac{\varepsilon_{cu} + \varepsilon_{sy}}{d_o} \qquad 3.2.4(14)$$

and then substituting this value of ρ_{ub} into Eq. 3.2.4(12) gives the secant stiffness *EI* where

$$EI = \frac{M_{ub}\, d_o}{\varepsilon_{cu} + \varepsilon_{sy}} \qquad 3.2.4(15)$$

In AS3600-1994, the normal type of bar reinforcement used in columns is 400Y with a design yield stress f_{sy} = 400 MPa and a yield strain ε_{sy} = 0.002, and the maximum compressive strain ε_{cu} in the concrete at ultimate strength is taken as 0.003. Using these values in Eq. 3.2.4(15) then

$$EI = 200 d_o M_{ub} \qquad 3.2.4(16)$$

To account for creep due to sustained loading, a reduced concrete elastic modulus of $E/(1+\beta_d)$ is used instead of the short term elastic modulus E . The creep factor $\beta_d = G/(G+Q)$ where G is the dead load and Q is the live load. In design, the design strength ϕM_{ub} is used instead of the calculated strength M_{ub}. Using these values in Eq. 3.2.4(16) gives

$$EI = 200 d_o(\phi M_{ub})/(1+\beta_d) \qquad 3.2.4(17)$$

which is the design value of EI given in Clause 10.4.4 of AS3600-1994

For 500N grade steel such as the new 500PLUS® Rebar with a design yield stress f_{sy} = 500 MPa and a yield strain ε_{sy} = 0.0025, and taking the strain ε_{cu} in the concrete at ultimate strength as 0.003, then substitution in Eq. 3.2.4(15) gives the design value for EI where

$$EI = 182 d_o(\phi M_{ub})/(1+\beta_d) \qquad 3.2.4(18)$$

To demonstrate the effects of slenderness, a simple example is shown in Figure 3.14 for a braced column, loaded with equal end eccentricity and bent in symmetrical single curvature.

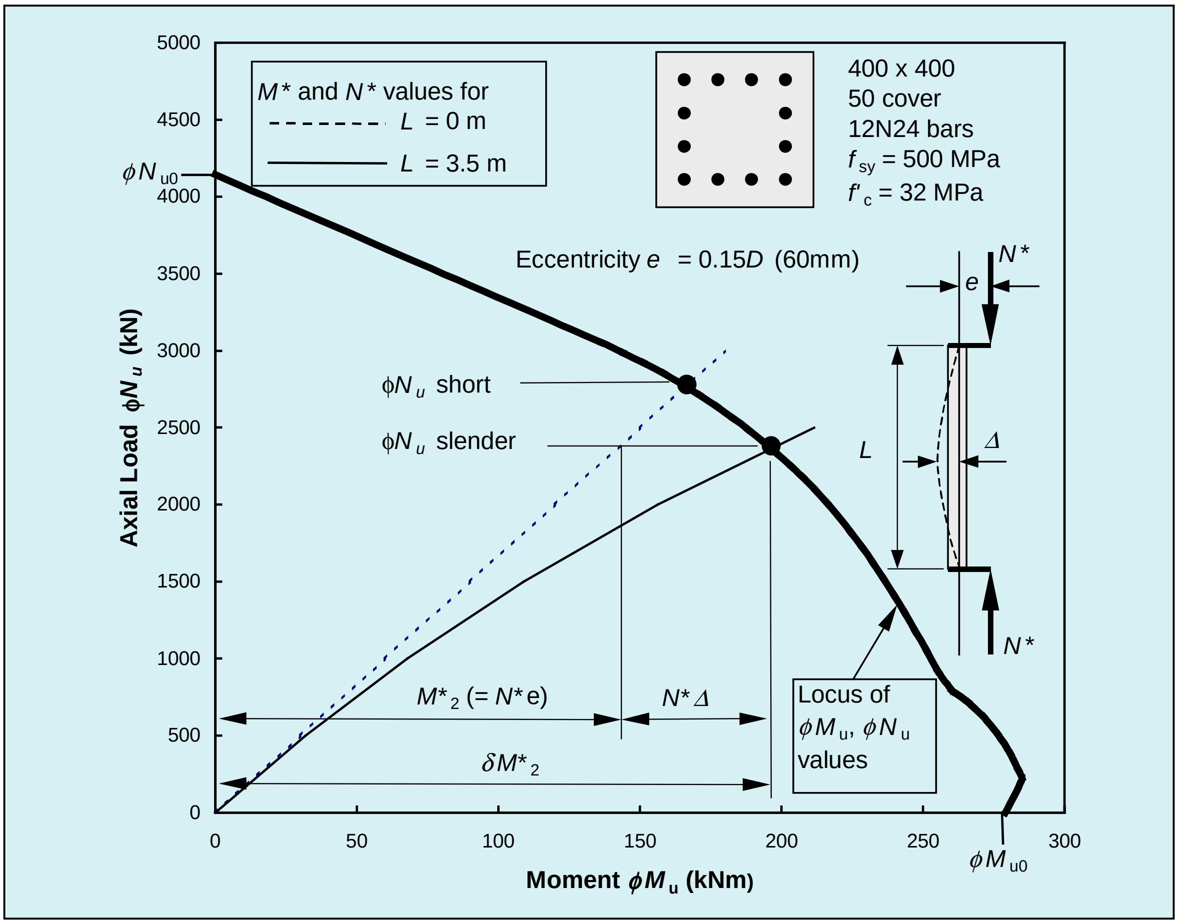

Figure 3.14 The Effect of Slenderness for an Eccentrically Loaded Column

In Figure 3.14, the design values of M^* and N^* for the cross-section at the centre of the column have been plotted for both a short column (L = 0 and shown as a linear dashed line) and a slender column (L = 3.5m, L_e/r = 29 and shown as a curved solid line). The effect of moment magnification for the slender column can be clearly seen by comparison with the short column. The axial strength ϕN_u for the column is reached when the line of M^*, N^* values intersects the load-moment strength interaction curve defined in Figure 3.14 as the locus of ϕM_u , ϕM_u values which were determined using rectangular stress block theory in accordance with Clause 10.6.2 of AS3600-1994 but with the modifications to k_{uo} and EI derived above to account for the use of 500N grade steel for the reinforcement such as the new 500PLUS® Rebar.

The column in Figure 3.15 is the same column as that used in Figure 3.14 but with an additional eccentricity e = 0.05D which is the minimum eccentricity required by Clause 10.1.2 of AS3600-1994.

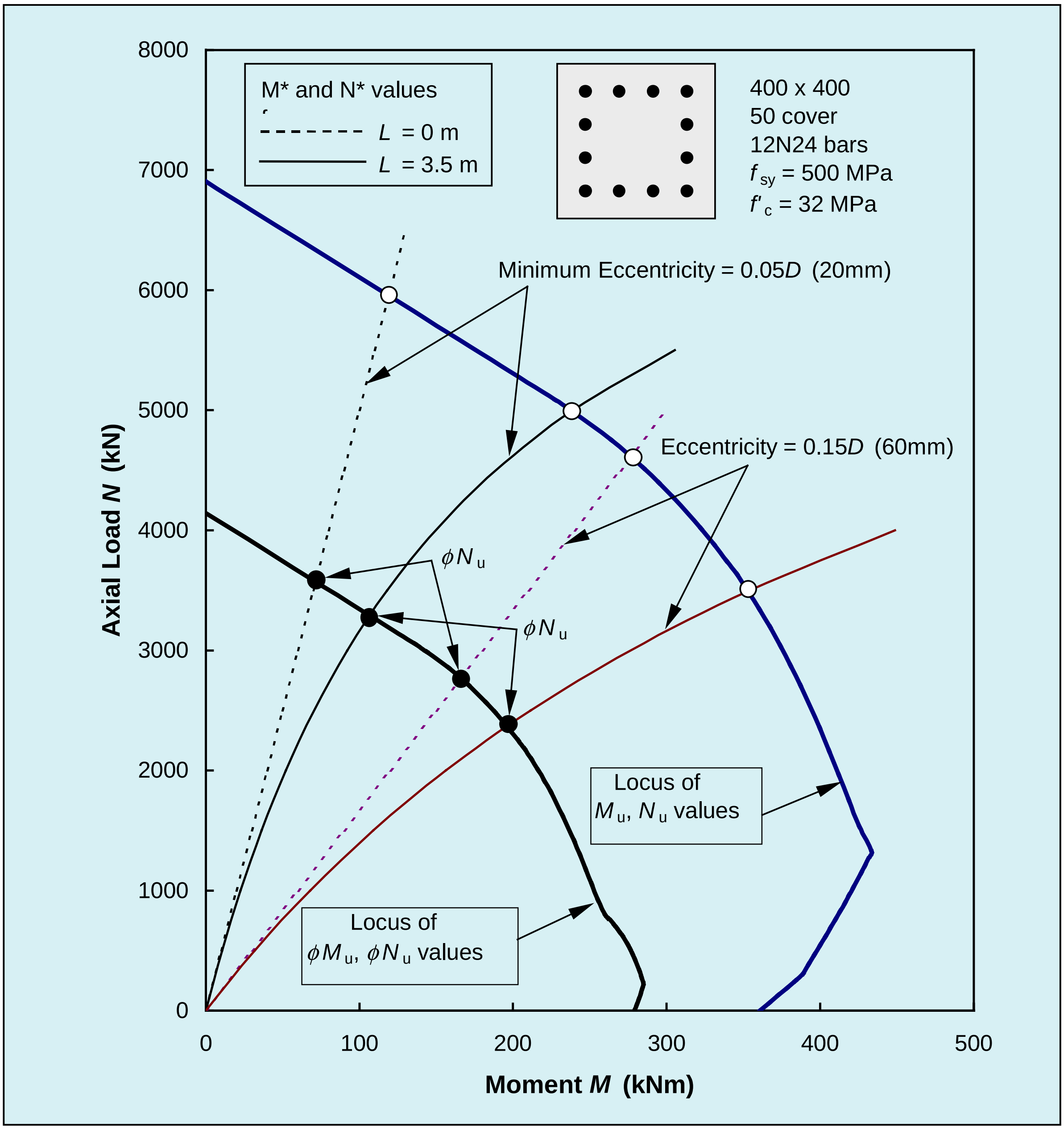

Figure 3.15 Strength of an Eccentrically Loaded Column

The effect of the capacity reduction factor ϕ on the strength can be clearly seen by comparing the locus of the ϕM_u , ϕN_u values with the locus of the M_u , N_u values.

When a slender column is nominally axially loaded without any externally applied end moments, the use of the minimum end eccentricity e = 0.05D (bent in symmetrical single curvature where k_m = 1) for this case enables the column strength ϕN_u for a slender axially loaded column to be determined. This is marked as a solid point in Figure 3.15 as ϕN_u for a column with a length of 3.5 m and a minimum eccentricity of 0.05D. As the length L of the column increases, the moment magnification increases and the value of ϕN_u for a slender axially loaded column will decrease.

The accuracy of the moment magnifier method has been checked by comparison with the results of an accurate computer model for slender columns [6]. This study by Smith and Bridge [15] covered a wide range of column types and slenderness. The method generally gives conservative results, and tends to be more conservative for columns with small end eccentricities and large slenderness rations L_e/r.

3.2.5 Biaxial Bending

Biaxial bending may be considered for rectangular cross-sections subjected to simultaneous axial force N* and bending moments M^*_x and M^*_y about both principal axes (x-axis and y-axis respectively) by using the following limits.

$$\left[\frac{M_x^*}{\phi M_{ux}}\right]^{\alpha_n} + \left[\frac{M_y^*}{\phi M_{uy}}\right]^{\alpha_n} \le 1.0 \qquad 3.2.4(19)$$

where

$$\alpha_n = 0.7 + \frac{1.7N^*}{0.6N_{uo}} \quad \text{and} \quad 1 \le \alpha_n \le 2 \qquad 3.2.4(19)$$

The design strengths ϕM_{ux} , ϕM_{uy} corresponding to the applied axial force N^* are determined from separate load-moment interaction diagrams for x-axis bending and y-axis bending respectively where the column is rectangular. For symmetric square columns, then $\phi M_{ux} = \phi M_{uy}$. The design bending moments M^*_x and M^*_y should include the additional bending moments produced by the slenderness effects i.e. magnified moments.

4. DESIGN APPROACH

4.1 General

The design approach for the load-moment strength of reinforced concrete cross-sections is mainly contained within Clause 10.6 of AS3600-1994 with reference to Table 2.3 for the strength reduction factors ϕ and Clause 1.7 for the definition of the terms used in the design approach. The effect of the change from 400 MPa to 500 MPa reinforcing bars such as the new 500PLUS® Rebar with a design yield stress f_{sy} = 500 MPa has been discussed in Section 3 Design Concepts and Models above. Proposed changes to the design rules in AS3600-1994 to accommodate the 500 MPa reinforcing bars are given in the following Section 5 Design Rules. However, the design approach is essentially the same as that in AS3600-1994. The objective of presenting the design approach in this Section is to ensure that full advantage is taken of the savings associated with the introduction of 500 MPa reinforcing bars.

Two methods for determining the strength of reinforced concrete cross-sections are given in AS3600-1994. The first method is based on the use of a simplified rectangular stress block to represent the concrete stress distribution in the concrete at ultimate strength when the neutral axis lies within the cross-section and the maximum strain ε_{cu} at the extreme compression fibre is taken as 0.003. These assumptions make the method amenable to hand calculation [1] but the calculations become tedious when the geometry of the cross-section becomes more complex with multiple layers of steel. In this case, the use of a spreadsheet becomes essential. The second method is based on considerations of equilibrium and compatibility and the distributions of stress in the concrete and steel are determined from realistic stress-strain relationships such as those shown in Figure 3.2 and Figure 3.3 respectively. Such stress-strain relationships are based on test data. This method generally requires the use of computer-based analyses [5] [6]. In this method, the ultimate strength in bending and axial force can be calculated as maximum values without requiring the assumption that the maximum strain ε_{cu} at the extreme compression fibre be taken as 0.003 for the determination of ultimate strength when the neutral axis lies within the cross-section.

4.2 Rectangular Stress Block Method

4.2.1 Introduction

The rectangular stress block provides a simple approach to the manual calculation of the load-moment strength of reinforced concrete cross-sections. The method is based on the general provisions in Clause 10.6.1 of AS3600-1994 and the particular provisions in Clause 10.6.2 of AS3600-1994. In Clause 10.6.2, the maximum strain ε_{cu} in the extreme compression fibre of the concrete is taken as 0.003 when the neutral axis lies within the cross-section. For this case, the distribution of the stress in the concrete is assumed to be a uniform compressive stress of $0.85f'_c$ acting on an area bounded by: the edges of the cross-section; and a line parallel to the neutral axis and located a distance $\gamma k_u d$ from the extreme compressive fibre of the concrete. The value of γ is given by

$$\gamma = [0.85 - 0.007(f'_c - 28)] \quad \text{where } 0.85 \geq \gamma \geq 0.65 \qquad 4.2.1(1)$$

The procedure is illustrated in Figure 4.1 below for a cross-section with four layers of bars. The reinforcement is assumed to be linear-elastic fully-plastic with an elastic modulus E_s of 200,000 MPa a yield stress of f_{sy} and a yield strain of $\varepsilon_{sy} = f_{sy}/E_s$ with the values of f_{sy} and ε_{sy} depending on the strength grade of the reinforcement. For deformed bar to AS1302-1991 with a designated grade 400Y, the design yield stress f_{sy} is 400 MPa. For 500N grade steel such as the new 500PLUS® Rebar, the design yield stress f_{sy} will be 500 MPa.

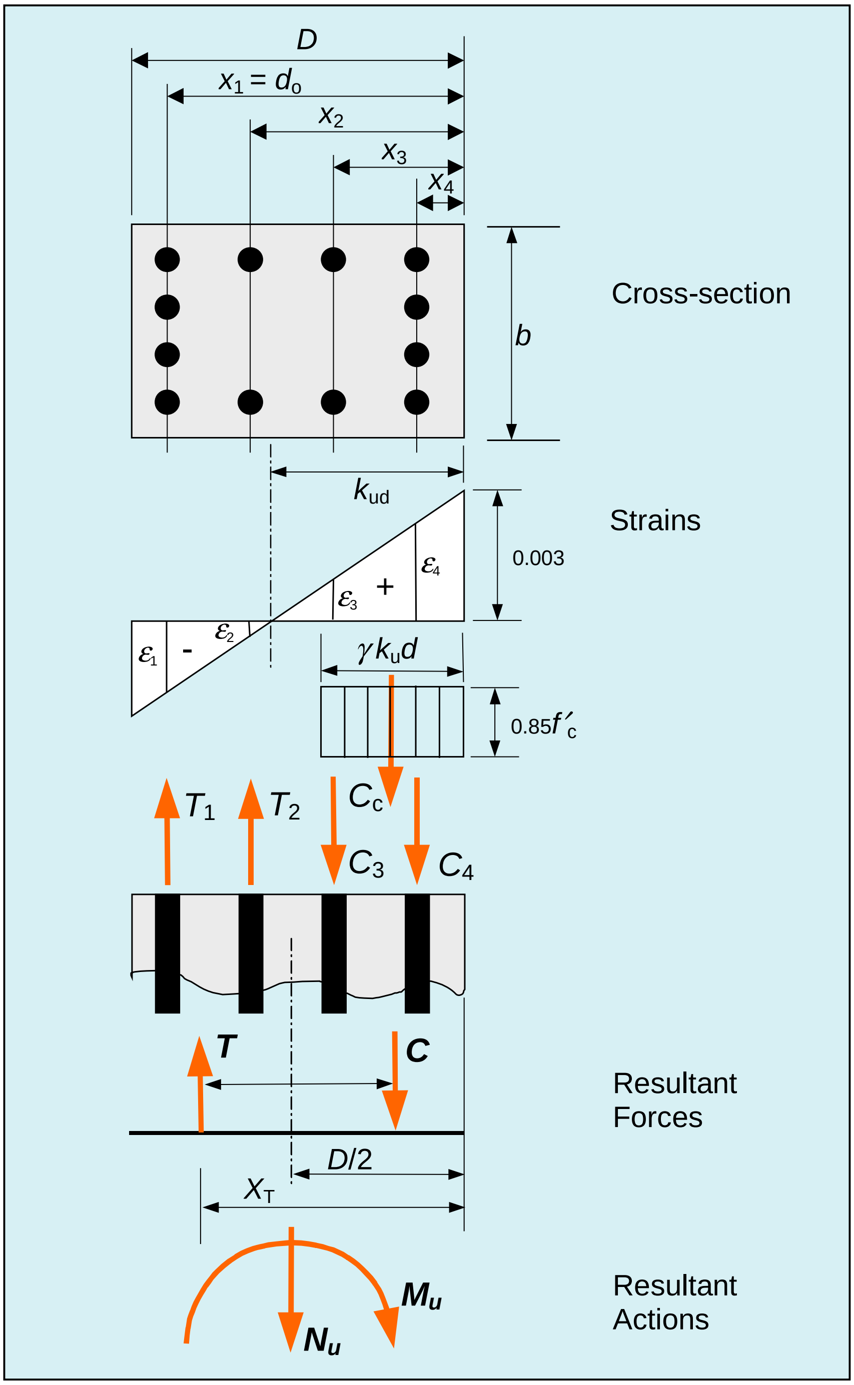

Figure 4.1 Example of Load-Moment Strength Calculation for a Cross-section

Figure 4.1 shows a general rectangular cross-section of width b and depth D with an arbitrary strain distribution at ultimate strength defined by a strain ε_{cu} = 0.003 at the extreme compression fibre of the concrete and a neutral axis depth $k_u d$ where d is the effective depth of the cross-section determined for the condition of pure bending. Compressive strains are taken as positive and tensile strains as negative. The compressive force C_c in the concrete (ignoring the holes due to the compressive reinforcement) is given by

$$C_c = 0.85 f'_c b(\gamma k_u d) \qquad 4.2.1(2)$$

The force F_i in layer i of the reinforcement is given by

$$F_i = \sigma_i A_i \tag{4.2.1(3)}$$

where A_i is the area of the reinforcement in layer i and σ_i is the stress in the reinforcement in layer i. The magnitude of the stress σ_i is determined from

$$|\sigma_i| = |f_{sy}| \quad \text{when} \quad |\varepsilon_i| \geq |\varepsilon_{sy}| \tag{4.2.1(4)}$$

and

$$|\sigma_i| = |E_s \varepsilon_i| \quad \text{when} \quad |\varepsilon_i| < |\varepsilon_{sy}| \tag{4.2.1(5)}$$

but where the sign of the stress corresponds to the sign of the strain, compression being positive and tension being negative. If the total number of layers of reinforcement is n (n = 4 in Figure 4.1) and the number of layers in tension is m (m = 2 in Figure 4.1) where $m \leq n$, then the ultimate strength N_u in axial compression is given by

$$N_u = C_c + \sum_{i=1}^{n} F_i - \sum_{i=m+1}^{n} 0.85 f'_c A_i \tag{4.2.1(6)}$$

where a correction has been made to allow for the concrete displaced by the steel in compression. The ultimate strength M_u in bending is given by

$$M_u = C_c\left(\frac{D}{2} - \frac{\gamma k_u d}{2}\right) + \sum_{i=1}^{n} F_i\left(x_i - \frac{D}{2}\right) - \sum_{i=m+1}^{n} 0.85 f'_c A_i\left(x_i - \frac{D}{2}\right) \tag{4.2.1(7)}$$

The distance x_T of the resultant tensile force T in the tensile reinforcement from the extreme compressive fibre is given by

$$x_T = \frac{\sum_{i=1}^{m} F_i x_i}{\sum_{i=1}^{m} F_i} \tag{4.2.1(8)}$$

4.2.2 The Ultimate Strength M_{uo} for Pure Bending

This condition of pure bending (N_{uo} = 0) is used to define the effective depth d of the cross-section. The determination of the ultimate strength M_{uo} under pure bending, despite the simplifying assumptions of the rectangular stress block, remains an iterative process. It requires the designer to assume a finite value of the neutral axis depth $k_u d$ (which with ε_{cu} = 0.003 defines a strain distribution over the cross-section) and then to vary this value until the value of N_u given in Eq. 4.2.1(6) is zero. This is equivalent to having $C = T$ as shown at the foot of Figure 4.1.

Once this has been achieved, then M_{uo} is calculated from Eq. 4.2.1(7) and the effective depth d is obtained from Eq. 4.2.1(8) where $d = x_T$ for the case of pure bending, and hence the value of k_u can be obtained.

In design, the design strength is taken as ϕM_{uo} where ϕ is a strength reduction factor to account for variability in geometric and material properties of the cross-section. For cross-sections where $k_u \leq 0.4$, a value of ϕ = 0.8 is required by AS3600-1994 (Table 2.3). For cross-sections where $k_u > 0.4$, AS3600-1994 (Table 2.3) requires the use of a reduced strength reduction factor ϕ where

$$\phi = 0.8 \frac{M_{ud}}{M_{uo}} \geq 0.6 \tag{4.2.2(1)}$$

and M_{ud} is the reduced ultimate strength in pure bending. The value of M_{ud} can calculated in accordance with Clause 8.1.3 of AS3600-1994. In this approach, the neutral axis depth $k_u d$ is reduced to a value 0.4d (i.e. k_u = 0.4). A reduced compressive force C_R resulting from the concrete and steel in compression corresponding to a neutral axis depth of 0.4d can be calculated from

$$C_R = 0.85 f'_c\, b\,(0.4d) + \sum_{i=m+1}^{n} F_i - \sum_{i=m+1}^{n} 0.85 f'_c A_i \qquad 4.2.2(2)$$

The resultant of the tensile forces in the reinforcement acting at the effective depth d is then reduced to a value T_R equal such C_R = T_R and the axial force N_u is zero. The value of M_{ud} is the couple obtained using the lever arm between C_R and T_R. An alternative approach is to reduce the percentage of steel in the cross-section until k_u = 0.4 for the condition of pure bending in which case M_{ud} = M_{uo} where M_{uo} is calculated from Eq. 4.2.1(7) for k_u = 0.4. This approach is iterative but can be useful when using a computer based analytical method.

The effect of the variation of the percentage p of steel on the ultimate strength M_{uo} in pure bending, the capacity reduction factor ϕ and the design ultimate strength ϕM_{uo} in pure bending is shown in Figure 4.2 for a typical column cross-section with reinforcement in all four faces.

There is change in the capacity reduction factor ϕ and hence the corresponding design ultimate strength ϕM_{uo} at the point where the neutral axis parameter k_u = 0.4. This point can be considered the limit for adequate ductility [1] .

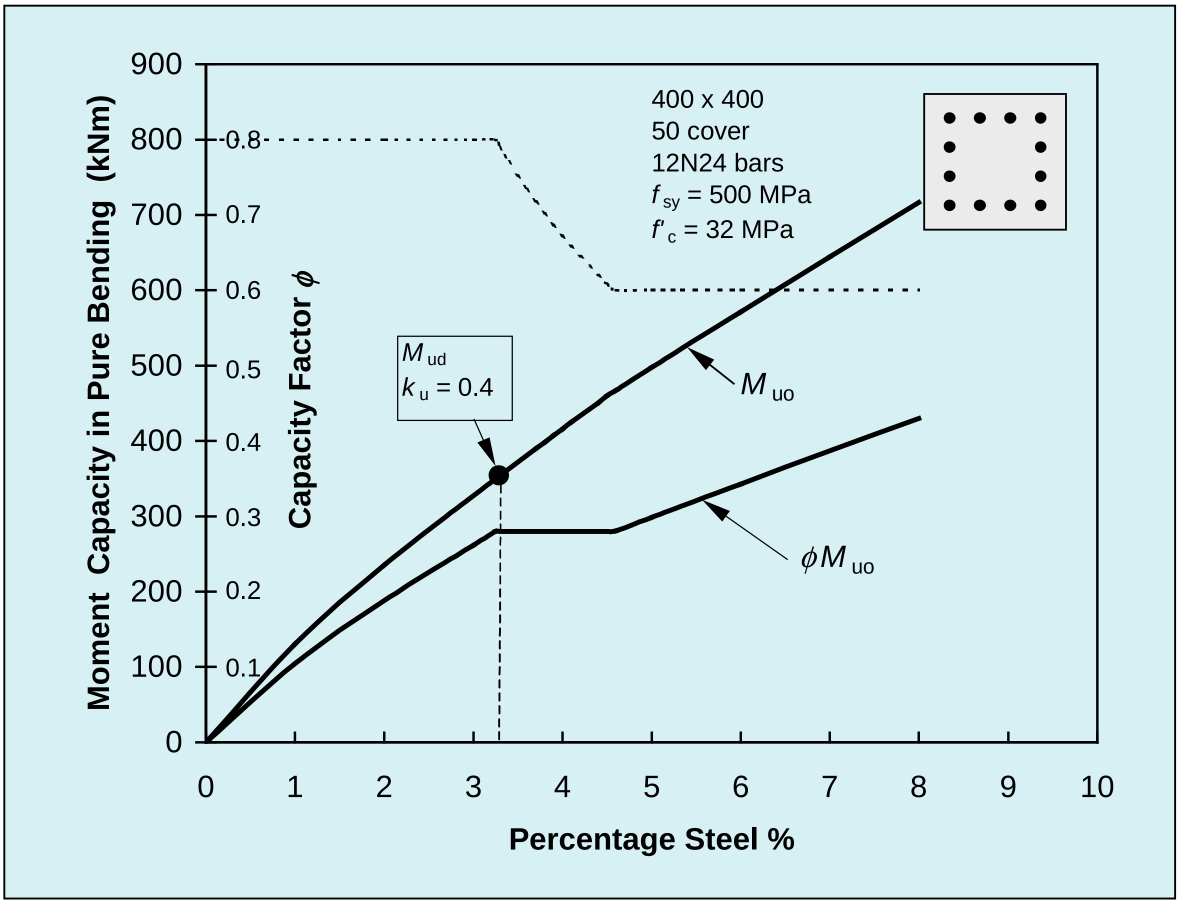

Figure 4.2 Variation of the Ultimate Strength M_{uo} in Pure Bending and the Capacity Reduction Factor ϕ with Steel Percentage

4.2.3 The Balanced Condition M_{ub}, N_{ub} for Ultimate Strength

For this condition, the neutral axis depth $k_u d$ is set to a value of $k_{uo}d_o$ where d_o (= x_1 in Figure 4.1) is the depth from the extreme compressive fibre to the centroid of the outermost layer of tensile reinforcement and k_{uo} is such that this outermost layer of steel has just reached yield at a strain of ε_{sy}. Values of M_{ub} and N_{ub} are then calculated from Eqs. 4.2.1(6) and 4.2.1(7) respectively.

As discussed in Section 3.2.3.3 above, the value of k_{uo} = 0.6 is for 400Y grade steel with a design yield stress f_{sy} = 400 MPa and a yield strain ε_{sy} = 0.002. However, for 500N grade steel such as the new 500PLUS® Rebar with a design yield stress f_{sy} = 500 MPa and a yield strain ε_{sy} = 0.0025, then the value of k_{uo} = 0.545.

In design, the design strength is taken as ϕM_{ub} and ϕN_{ub} where ϕ is a strength reduction factor to account for variability in geometric and material properties of the cross-section. At the balanced point where $N_u = N_{ub}$, a value of $\phi = 0.6$ is required by AS3600-1994 (Table 2.3). For other values of ϕ where $0 \le N_u < N_{ub}$, linear interpolation is used to evaluate ϕ where

$$\phi = 0.6 + \left[(\phi_0 - 0.6)(1 - \frac{N_u}{N_{ub}}) \right] \qquad 4.2.3(1)$$

and ϕ_0 is the value of ϕ for pure bending as determined from Section 4.2.2 above.

4.2.4 The Limit Condition M_{ul}, N_{ul} for Ultimate Strength

For this condition, the neutral axis depth k_{ud} is set to $1.0d_o$ where d_o is the depth from the extreme compressive fibre to the centroid of the outermost layer of tensile reinforcement. Therefore the outermost layer of steel has zero stress and hence the rest of the steel is in compression. Values of M_{ul} and N_{ul} are then calculated from Eqs. 4.2.1(6) and 4.2.1(7) respectively.

In design, the design strength is taken as ϕM_{ul} and ϕN_{ul} where ϕ is a strength reduction factor to account for variability in geometric and material properties of the cross-section. A value of $\phi = 0.6$ is required by AS3600-1994 (Table 2.3).

4.2.5 The Ultimate Strength N_{uo} for Pure Axial Force

This condition (M_{uo} = 0) has been fully discussed in Section 3.2.2 above. For 400Y grade rebar having a yield stress f_{sy} = 400 MPa with a yield strain ε_{sy} = 0.002 which is less than the strain ε_o (= 0.0022) at the maximum strength of $0.85f'_c$ for the concrete (see Figure 3.2), then the steel yields before the concrete has reached its maximum strength and the ultimate strength N_{uo} in axial compression is simply given by

$$N_{uo} = 0.85f'_c A_c + f_{sy}A_s \qquad 4.2.5(1)$$

The ultimate strength N_{uo} in axial compression is therefore reached at an axial strain ε_{uo} = 0.0022. This is reflected in Clause 10.6.3 of AS3600-1994 where N_{uo} can be calculated assuming a uniform concrete compressive stress of $0.85f'_c$ and a maximum strain in the steel and concrete of 0.002 (which is approximately equal to 0.0022).

For 500N grade reinforcing steel such as 500PLUS Rebar which has a yield stress f_{sy} = 500 MPa, the yield strain ε_{sy} (= 0.0025) is greater than the strain ε_o (= 0.0022) at the maximum strength of $0.85f'_c$ for the concrete. Hence, the ultimate strength N_{uo} in axial compression is reached at a strain ε_{uo} greater than or equal to 0.0022 but not greater than 0.0025.

The ultimate strength N_{uo} in axial compression is given by

$$N_{uo} = f(\varepsilon_{uo})A_c + E_s \varepsilon_{uo} A_s \qquad 4.2.5(2)$$

and the stress σ_c in the concrete is a function of the strain ε_a in the concrete (see Eq. 3.2.1(2)) where

$$\sigma_c = f(\varepsilon_a) \qquad 4.2.5(3)$$

A typical stress-strain relationship for concrete is shown in Figure 3.2. Equation 4.2.5(2) can be solved by an iterative process in which the strain ε_{uo} is varied within the limits 0.0022 to 0.0025 until a maximum is obtained for N_{uo} which then defines the ultimate strength N_{uo} in axial compression.

A simple and direct yet slightly unconservative estimate for the axial strength N_{uo} for cross-sections containing 500N grade reinforcing steel such as 500PLUS Rebar is given by

$$N_{uo} = 0.85 f'_c A_c + f_{sy} A_s \qquad 4.2.5(4)$$

which assumes a maximum strain ε_{uo} = 0.0025 in the steel and concrete and a uniform compressive stress in the concrete of $0.85f'_c$ which is identical to Eq. 4.2.5(1). The degree of unconservatism is slight. A study has been made of the axial strength of a rectangular column cross-section with steel percentages ranging from 0% to 8% and for two different concrete strengths, f'_c = 25 MPa and f'_c = 50 MPa . It can be seen from Figure 3.2 that the 50 MPa concrete has steeper unloading region for strains larger than the strain ε_o corresponding to the maximum stress and hence more likely to be of concern. The study revealed that the value of N_{uo} from the simple Eq. 4.2.5(4) exceeded the value of N_{uo} from the more accurate Eq. 4.2.5(2) by not more than 1.5% (for approx. 2% steel with f'_c = 50 MPa).

In design, the design strength in axial compression is taken as ϕN_{uo} where ϕ is a strength reduction factor to account for variability in geometric and material properties of the cross-section. A value of ϕ = 0.6 is required by AS3600-1994 (Table 2.3).

4.2.6 Load - Moment Interaction Diagrams

A load-moment strength interaction diagram has been generated for a typical reinforced column cross-section in Figure 4.3 using the rectangular stress block theory. The four key points are shown.

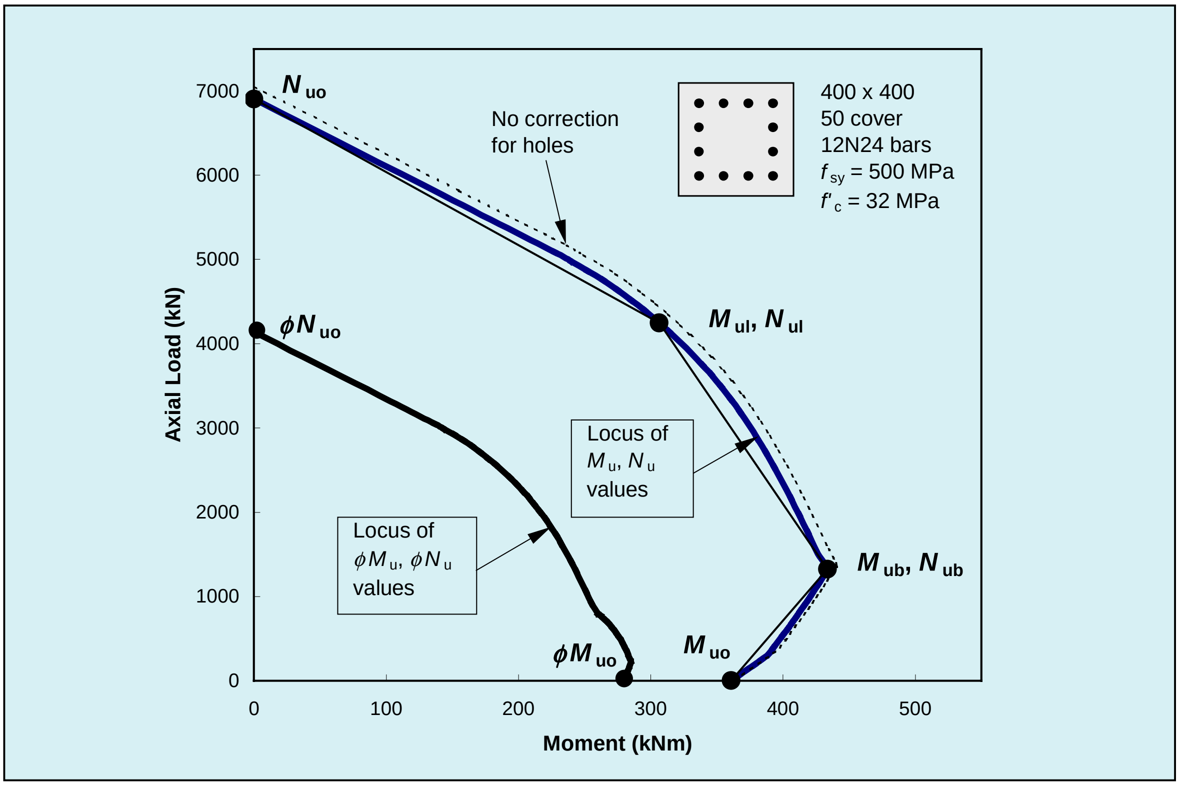

Figure 4.3 Load-Moment Strength Interaction Diagram for a Typical Column Cross-section

It can be seen in Figure 4.3 that three straight lines joining the key points on the interaction is a reasonable approximation to the load-moment strength interaction diagram. This approach is used in texts [1] and is suitable for hand calculation. The effect of ignoring the holes in the concrete formed by the compression reinforcement (i.e. the gross concrete cross-section in compression is used in the calculations) can be seen as the dashed line in Figure 4.3.

4.3 Computer Based Analytical Method

Computer based analytical methods should meet the provisions of Clause 10.6.1 in AS3600-1994 and should incorporate equilibrium and compatibility considerations and be consistent with the following assumptions. Plane sections remain plane. The concrete has no tensile strength. The concrete material stress-strain curve is of a recognised type such as that shown in Figure 3.2. The stress-strain curve for the reinforcement is also of a recognised type such as linear-elastic fully plastic of the type shown in Figure 3.3. In this method, the load-moment strength interaction diagram is taken as the locus of the maximum values of axial load and moment from the load-moment-curvature relationship such as that shown in Figure 3.7. This does not require the use of the assumption used in the rectangular stress block theory in Section 4.2 that the strain ε_{cu} at ultimate strength in the extreme fibre in compression has a constant value of 0.003 when the neutral axis is within the cross-section.

The analytical method developed by Bridge and Roderick [6] and later modified for all types of column cross-sections by Wheeler and Bridge [5] has been used to generate the load-moment interaction diagrams in Figure 4.4 for a typical column cross-section reinforced on all four sides with 500N grade steel such as the new 500PLUS® Rebar. It can be seen that there are differences between the curve generated using the simplified rectangular stress block theory and the curve generated using an accurate analytical method. For some ranges of axial load, the rectangular stress block theory is conservative yet unconservative for other ranges of load.

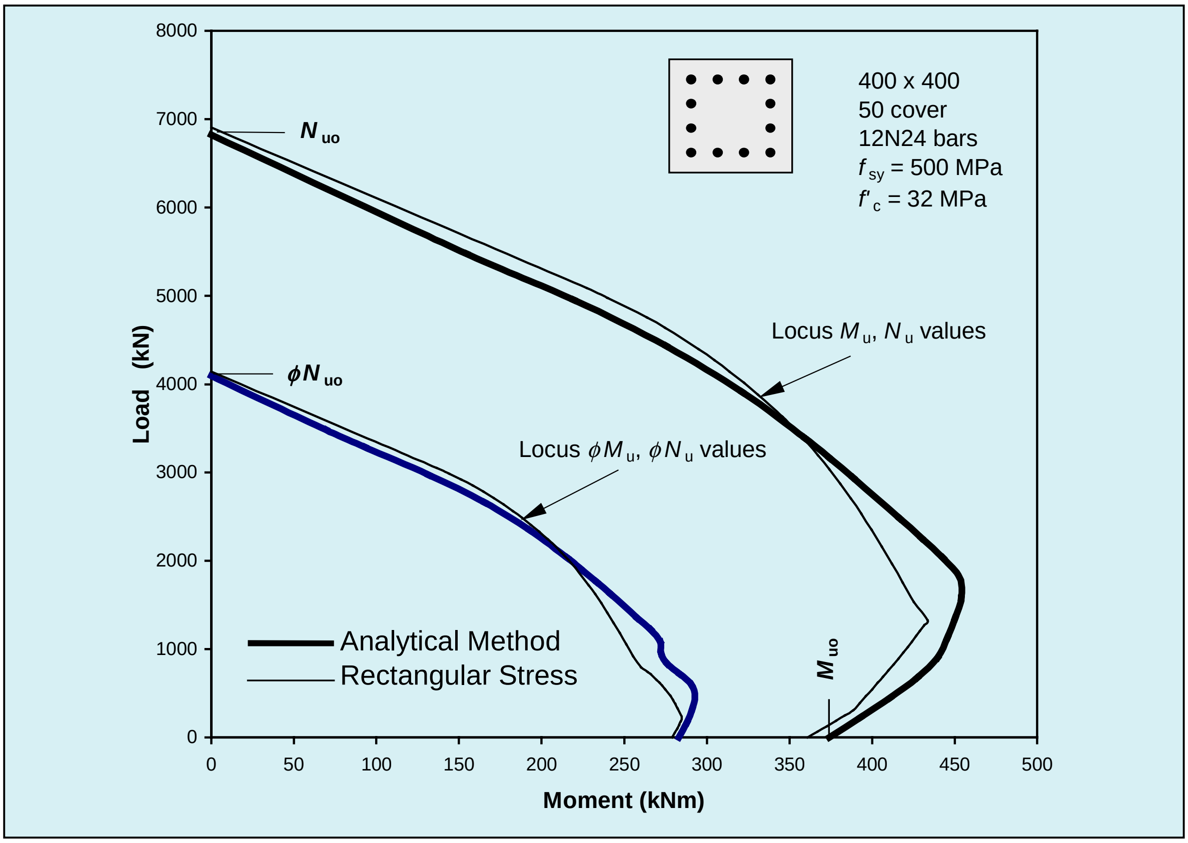

Figure 4.4 Comparison of Analytical and Rectangular Stress Block Approaches

Further comparisons can be found in Smith and Bridge [13] and Bridge and Smith [6] for a range of different column types and configurations of reinforcement including prestressed tendons. Again the rectangular stress block theory is shown to be generally conservative but can still be unconservative relative to the analytical method depending on the column type, reinforcement configuration and range of axial load.

The analytical method has been used to generate the design charts in Appendix C for three types of columns incorporating 500 MPa reinforcing steels such as the new 500PLUS® Rebar (500N grade). These charts can be used directly in design and include the proposed changes to the design rules for columns in AS3600-1994 as detailed in the following Section 5. The charts cover the normal types of rectangular and circular columns for a range of practical steel percentages and practical values of the ratio *g* of the distance between the outermost bars to the depth *D* of the cross-section.

5. DESIGN RULES

5.1 General

The design rules for the load-moment strength of reinforced concrete cross-sections are mainly contained within Clause 10.6 of AS3600-1994 with reference to Table 2.3 for the strength reduction factors ϕ and Clause 1.7 for the definition of the terms used in the design approach. The effect of the change from 400 MPa to 500 MPa reinforcing bars such as the new 500PLUS® Rebar with a design yield stress f_{sy} = 500 MPa has been discussed in Section 3 Design Concepts and Models above. Proposed changes to the design rules in AS3600-1994 to accommodate the 500 MPa reinforcing bars are given in the following Section 5.2 AS3600-1994 Changes to Design Rules.

5.2 AS3600-1994 Proposed Changes to Design Rules

The following changes to the design rules in AS3600-1994 are proposed and are based on the information contained in Sections 3 and 4 above.

Clause 1.7 Notation

M_{ub} = the particular ultimate strength in bending of a cross-section when k_{uo} = 0.545 .

N_{ub} = the particular ultimate strength in compression of a cross-section when k_{uo} = 0.545 .

Clause 10.4.4 Buckling Load

The buckling load N_c shall be taken as

$$N_c = (\pi^2 / L_e^2)\left[182 d_0 (\phi M_{ub}) / (1 + \beta_d)\right] \qquad 5.2(1)$$

where

ϕM_{ub} = the design strength in bending of the cross-section when k_{uo} = 0.545 and ϕ = 0.6.

Clause 10.6.3 Calculation of N_{uo}

The ultimate strength in compression N_{uo} shall be calculated by assuming –

(a) uniform concrete compressive stress in the concrete of $0.85 f'_c$; and

(b) a maximum strain in the steel and concrete of 0.0025.

6. WORKED EXAMPLES

6.1 General

Several worked examples are used to illustrate how design engineers can use the design rules in AS3600-1994, in conjunction with the proposed changes to the design rules in Section 5, to take account of the use of the higher strength 500 MPa reinforcing steels in the design of columns. The opportunity is also taken to show the benefits obtained by using the higher strength 500 MPa reinforcing steels such as the new 500PLUS® Rebar (500N grade) in columns. This can lead to a significant reduction in steel areas and less congestion, the latter resulting in benefits not related directly to strength.

6.2 Rectangular Column Reinforced on Four sides – Analytical Method

Shown in Figure 6.1 is the load-moment strength interaction diagram for a column taken from an actual design incorporating 500PLUS® Rebar (500N grade) and having a rectangular cross-section 700 mm by 450 mm reinforced on four sides with approximately the same spacing of bars in each face of the cross-section and bent about the stronger axis. The ratio *g* of the distance between the outermost bars to the depth of the cross-section was 0.87. The results in Figure 6.1 were generated using a computer based analytical method (see Section 4.3). The areas of the bars were varied to give selected percentages of steel. The use of 400Y bars is compared with the use of 500N bars with 20% more strength but 20% less area. Based on cross-section strength alone, the strength M_u, N_u using 500N bars with 20% more strength but 20% less area is directly equivalent to the use of 400Y bars over the full range of values for moment M_u and axial force N_u .

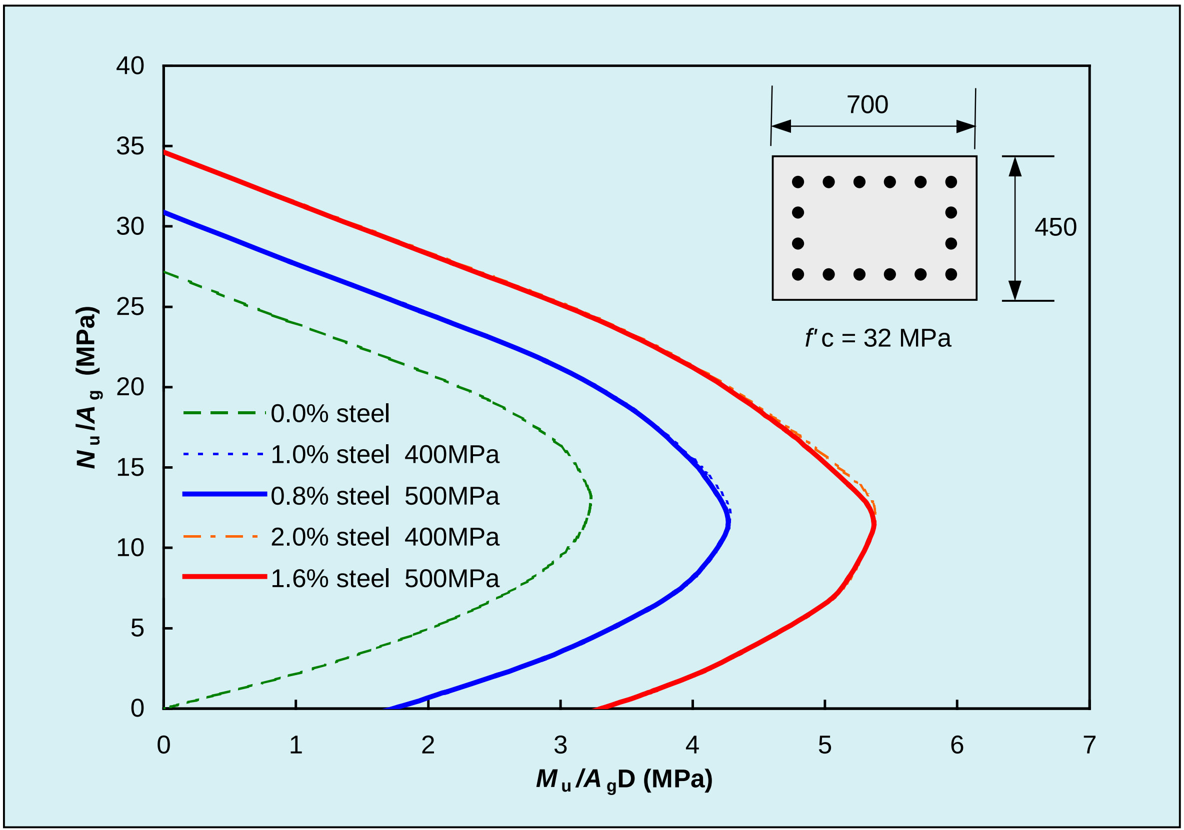

Figure 6.1 Load (N_u)-Moment(M_u) Strength for Column Reinforced on Four Sides

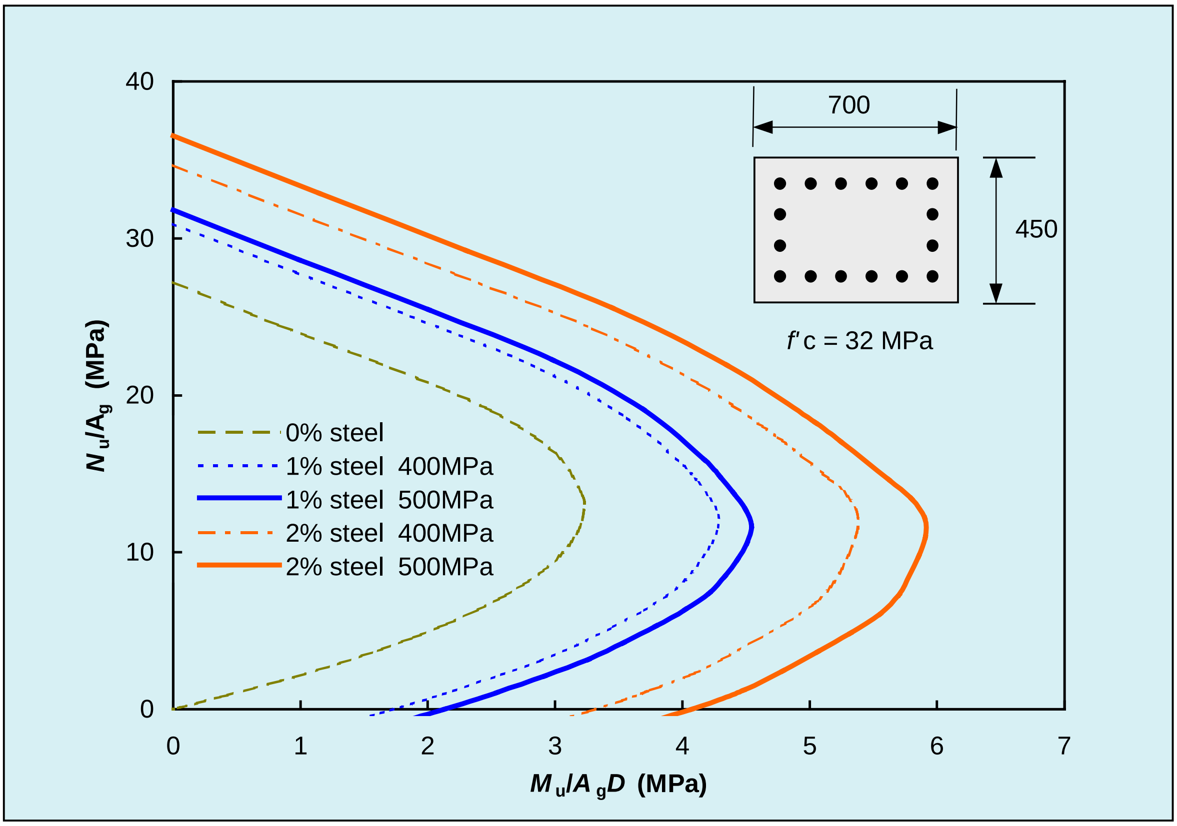

Figure 6.2 Direct Comparison of Column Strength using 400Y and 500N steel reinforcement

The results in Figure 6.2 were generated for the same column as in Figure 6.1 except that the percentage of steel used for both 400Y reinforcing bars and 500N reinforcing bars was kept the same. It can be clearly seen from Figure 6.2 that, for a given fixed percentage of steel used in a typical rectangular column reinforced with 400Y reinforcing bars, the strength of the cross-section can be increased by the use 500N reinforcing bars.

6.3 Column Reinforced on Four sides – Rectangular Stress Block Method

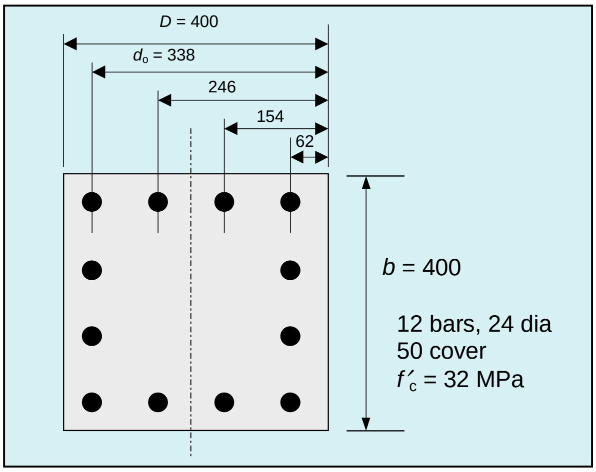

Figure 6.3 Square Column Reinforced on Four Sides

For the cross-section shown in Figure 6.3, manual calculations are used to determine the four key points on the load-moment strength interaction diagram, as indicated on Figure 3.10, using the rectangular stress block theory as described in Section 4.2. The calculations are first done using 500N grade reinforcing steel such as 500PLUS Rebar having a design yield stress f_{sy} = 500 MPa and a yield strain ε_{sy} = 0.0025. They are then repeated (in shortened form) for 400Y grade reinforcing steel having a yield stress f_{sy} = 400 MPa and a yield strain ε_{sy} = 0.002 to demonstrate the benefits that can be achieved in using the higher grade rebar.

The following properties for the cross-section are used in the calculations:

b = 400.0 mm

D = 400.0 mm

f'_c = 32 MPa

γ = 0.822

d_b = 24 mm

A_b = 450 mm^2

p = 3.375%

6.3.1 Ultimate Strength M_{uo} for Pure Bending

500 MPa Rebar

The strain distribution and resultant forces in the steel and concrete for this condition are shown in Figure 6.4. The procedure to determine the value of M_{uo} is iterative and requires the value of $k_u d$ to be varied until the axial force N_u is zero (i.e. $C_c + C_4 = T_1 + T_2 + T_3$ in Figure 6.4).

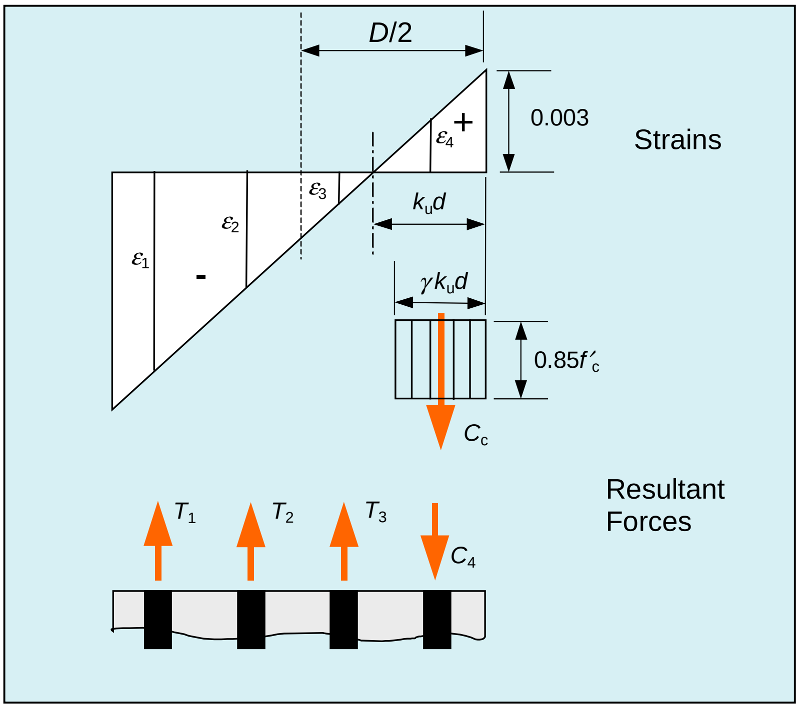

Figure 6.4 Strain Distribution and Resultant Forces for Condition of Pure Bending - M_{uo}

The calculations are displayed in tabular form where:

x_i = distance from line of action of the force to the extreme compression fibre;
i = identifier for steel or concrete layer;
ε_i = strain in the steel at layer i ;
σ_i = stress in the steel at the layer i (= $E_s\varepsilon_i$ or f_{sy} whichever is the smaller);
A_i = area of layer i of the steel in tension or compression, and area of concrete in compression;
F_i = the force in layer i of the steel in tension or compression, and the concrete in compression;
M_i = the moment of the force F_i about the centroidal axis; and
$D/2$-x_i = distance from the force F_i to the centroidal axis.

Compressive strains, stresses and forces are taken as positive and tensile strains, stresses and forces are taken as negative.

Try a value of $k_u d$ = 130.00 mm

Layer	i	x_i (mm)	ε_i	σ_i (MPa)	A_i (mm^{2})	F_i (kN)	$D/2$ - x_i (mm)	M_i (kNm)
Steel	1	338.00	-0.0048000	-500.00	1800	-900.0	-138.00	124.2
Steel	2	246.00	-0.0026769	-500.00	900	-450.0	-46.00	20.7
Steel	3	154.00	-0.0005538	-110.77	900	-99.7	46.00	-4.6
Steel	4	62.00	0.0015692	313.85	1800	[b]516.0	138.00	71.2
Concrete	5	[a]53.43		27.20	[a]42744	1162.6	146.57	170.4
						Σ = 228.9		Σ = 381.9

[a] Note: For the concrete area, $A_i = b\gamma k_u d$ and $x_i = \gamma k_u d/2$.
[b] Note: For bars in compression where the value of γk_ud (= 106.86 mm in this case) is greater than the value of x_i (= 62 mm in this case), a correction of -0.85$f'_c A_i$ has been made to the force at this layer to account for the holes in the concrete from the rebars as the gross section was used to determine the area of the concrete A_5.

It can be seen from the table above that $N_u = \Sigma F_i$ = 228.8 kN in compression which is not zero. To reduce the compression, the value of the neutral axis has to be decreased until N_u = 0. This was achieved for the following trial value of $k_u d$.

Try a value of $k_u d$ = 117.79 mm

Layer	i	x_i (mm)	ε_i	σ_i (MPa)	A_i (mm^{2})	F_i (kN)	$D/2$ - x_i (mm)	M_i (kNm)
Steel	1	338.00	-0.0056086	-500.00	1800	-900.0	-138.00	124.2
Steel	2	246.00	-0.0032654	-500.00	900	-450.0	-46.00	20.7
Steel	3	154.00	-0.0009222	-184.45	900	-166.0	46.00	-7.6
Steel	4	62.00	0.0014209	284.18	1800	462.6	138.00	63.8
Concrete	5	48.41		27.20	38729	1053.4	151.59	159.7
						Σ = 0.0		Σ = 360.8

As the value of $N_u = \Sigma F_i$ = 0 kN, then the value of M_{uo} is determined as

$$M_{uo} = \Sigma M_i = 360.8 \text{ kN}.$$

The next step is to determine the effective depth d. Considering only the tensile forces, then

$$\Sigma T_i = 900.0 + 450.0 + 166.0 = 1516.00 \text{ kN}$$

Taking moments of these forces about the extreme compressive fibre, then

$$\Sigma T_i x_i = 900.0 \times 338.0 + 450.0 \times 246.0 + 166.0 \times 154.0 = 440464 \text{ kNmm}$$

The effective depth d is given by

$$d = (\Sigma T_i x_i)/(\Sigma T_i) = 440464/1516 = 290.54 \text{ mm}$$

The value of the neutral axis parameter k_u is given by

$$k_u = k_u d/d = 117.79/290.54 = 0.4054$$

As the value of $k_u > 0.4$, the value of M_{ud} has to be calculated (Clause 8.1.3 of AS3600-1994) so that a reduced value of capacity reduction factor ϕ can be determined (Table 2.3 of AS3600-1994).

Taking a value of $k_u d = 0.4d = 0.4 \times 290.54 = 116.22$

Layer	i	x_i (mm)	ε_i	σ_i (MPa)	A_i (mm^2)	F_i (kN)	$D/2 - x_i$ (mm)	M_i (kNm)
Steel	1	338.00	-0.0057248	-500.00	1800	-900.0	-138.00	124.2
Steel	2	246.00	-0.0033500	-500.00	900	-450.0	-46.00	20.7
Steel	3	154.00	-0.0009752	-195.04	900	-175.5	46.00	-8.1
Steel	4	62.00	0.0013996	279.92	1800	454.9	138.00	62.8
Concrete	5	47.77		27.20	38213	1039.4	152.23	158.2
						$\Sigma = -31.2$		$\Sigma = 357.8$

The sum T of the tensile forces is calculated as

$$T = 900.0 + 450.0 + 175.5 = 1525.5 \text{ kN}$$

The sum C_R of the compressive forces reduced as a result of reducing the neutral axis depth to $0.4d$ is given by

$$C_R = 454.9 + 1039.4 = 1494.3 \text{ kN}$$

which gives a nett axial force on the cross-section of 31.2 kN in tension (also see the summation of F_i in the table above) such that the cross-section is not in pure bending. Therefore, the resultant tension force T is reduced to a value T_R such that

$$T_R = C_R = 1494.3 \text{ kN}$$

and M_{ud} is given by the moments about the centroidal axis of the compression forces and the reduced tension force T_R where T_R is assumed to act at the effective depth d determined for the case of pure bending. Therefore,

$$M_{ud} = 454.9 \times 138.00 + 1039.4 \times 152.23 + 1494.3 \times (290.54 - 400/2) \text{ kNmm}$$

$$= 356.3 \text{ kNm}$$

The reduced capacity reduction factor ϕ is given by

$$\phi = 0.8(M_{ud})/(M_{uo}) = 0.8 \times 356.3/360.8 = 0.790$$

While this reduction may not seem great, if larger percentages of steel had been used, the value could drop to as low as 0.6 (see Figure 4.2).

Therefore the design strength ϕM_{uo} for pure bending is determined as

$$\phi M_{uo} = 0.79 \times 360.8 = 285.0 \text{ kNm}$$

400 MPa Rebar

Only the final result is shown for the iterative process to determine the value of $k_u d$ such that the value of the axial force $N_u = 0$.

Try a value of $k_u d$ = 104.98 mm

Layer	i	x_i (mm)	ε_i	σ_i (MPa)	A_i (mm^2)	F_i (kN)	$D/2 - x_i$ (mm)	M_i (kNm)
Steel	1	338.00	-0.0066590	-400.00	1800	-720.0	-138.00	99.4
Steel	2	246.00	-0.0040299	-400.00	900	-360.0	-46.00	16.6
Steel	3	154.00	-0.0014008	-280.17	900	-252.1	46.00	-11.6
Steel	4	62.00	0.0012282	245.65	1800	393.2	138.00	54.3
Concrete	5	43.15		27.20	34518	938.9	156.85	147.3
						$\Sigma = 0$		$\Sigma = 306.0$

As the value of $N_u = \Sigma F_i = 0$ kN, then the value of M_{uo} is determined as

$$M_{uo} = \Sigma M_i = 306.0 \text{ kN.}$$

The next step is to determine the effective depth d. Considering only the tensile forces, then

$$\Sigma T_i = 720.0 + 360.0 + 252.1 = 1332.1 \text{ kN}$$

Taking moments of these forces about the extreme compressive fibre, then

$$\Sigma T_i x_i = 720.0 \times 338.0 + 360.0 \times 246.0 + 252.1 \times 154.0 = 370744 \text{ kNmm}$$

The effective depth d is given by

$$d = (\Sigma T_i x_i)/(\Sigma T_i) = 370744/1332.1 = 278.32 \text{ mm.}$$

The value of the neutral axis parameter k_u is given by

$$k_u = k_u d/d = 104.98/278.32 = 0.3772$$

As the value of $k_u < 0.4$, the value of $\phi = 0.8$ (Table 2.3 of AS3600-1994).

Therefore the design strength ϕM_{uo} for pure bending is determined as

$$\phi M_{uo} = 0.8 \times 306.0 = 244.8 \text{ kNm (cf 285.0 kNm for 500N rebar)}$$

It can be seen that the use of 500N grade reinforcing steel such as 500PLUS Rebar for the same area of steel results in a strength that is 16.4% higher than that achieved with the use of 400Y rebar.

6.3.2 The Balanced Condition M_{ub}, N_{ub} for Ultimate Strength

500 MPa Rebar

The strain distribution and resultant forces in the steel and concrete for this condition are shown in Figure 6.5. The procedure to determine the values of M_{ub} and N_{ub} for the balanced condition is direct in which the neutral axis depth $k_u d$ is set to the value of $k_{uo} d_o$ (see Section 4.2.3) where k_{uo} = 0.545 for 500N grade rebar and d_o = 338 mm for this cross-section. The distance d_o is the distance from the extreme compressive fibre to the centroid of the outermost layer of tensile reinforcement.

Again, compressive strains, stresses and forces are taken as positive and tensile strains, stresses and forces are taken as negative.

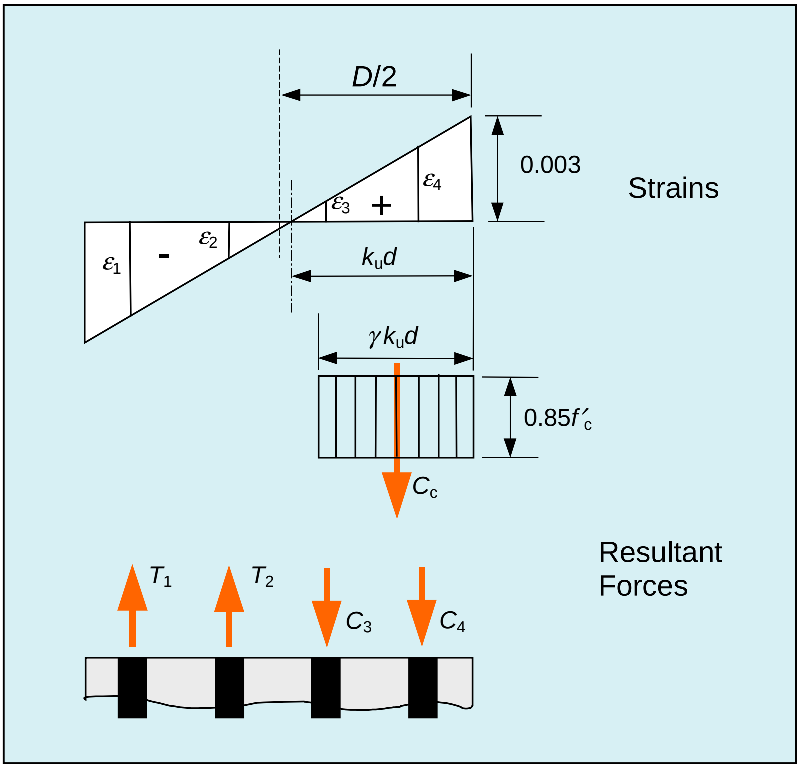

Figure 6.5 Strain Distribution and Resultant Forces for the Balanced Condition - M_{ub}, N_{ub}

Set the value of $k_u d = 0.545 d_o = 0.545 \times 338.0 = 184.21$

Layer	i	x_i (mm)	ε_i	σ_i (MPa)	A_i (mm²)	F_i (kN)	$D/2 - x_i$ (mm)	M_i (kNm)
Steel	1	338.00	-0.0025046	-500.00	1800	-900.0	-138.00	124.2
Steel	2	246.00	-0.0010063	-201.26	900	-181.1	-46.00	8.3
Steel	3	154.00	0.0004920	98.40	900	88.6	46.00	4.1
Steel	4	62.00	0.0019903	398.06	1800	667.5	138.00	92.1
Concrete	5	75.71		27.20	60568	1647.5	124.29	204.8
						Σ = 1322.5		Σ = 433.5

The values of M_{ub} and N_{ub} are determined as

$$M_{ub} = \Sigma M_i = 433.5 \text{ kNm}$$

$$N_{ub} = \Sigma F_i = 1322.5 \text{ kN}$$

The value of $\phi = 0.6$ (Table 2.3 of AS3600-1994) and the design strengths are

$$\phi M_{ub} = 0.6 \times 433.5 = 260.1 \text{ kNm}$$

$$\phi N_{ub} = 0.6 \times 1322.5 = 793.5 \text{ kN}$$

400 MPa Rebar

The neutral axis depth $k_u d$ is set to the value of $k_{uo} d_o$ (see Section 4.2.3) where k_{uo} = 0.6 for 400Y grade rebar. Again, compressive strains, stresses and forces are taken as positive and tensile strains, stresses and forces are taken as negative. The results of the calculations are shown in the following table.

Set the value of $k_u d = 0.6 d_o = 0.6 \times 338.0 = 202.8$ mm

Layer	i	x_i (mm)	ε_i	σ_i (MPa)	A_i (mm^2)	F_i (kN)	$D/2 - x_i$ (mm)	M_i (kNm)
Steel	1	338.00	-0.0020000	-400.00	1800	-720.0	-138.00	99.4
Steel	2	246.00	-0.0006391	-127.81	900	-115.0	-46.00	5.3
Steel	3	154.00	0.0007219	144.38	900	105.5	46.00	4.9
Steel	4	62.00	0.0020828	400.00	1800	671.0	138.00	92.6
Concrete	5	83.35		27.20	66681	1813.7	116.65	211.6
						Σ = 1755.2		Σ = 413.8

The values of M_{ub} and N_{ub} are determined as

$$M_{ub} = \Sigma M_i = 413.8 \text{ kNm}$$

$$N_{ub} = \Sigma F_i = 1755.2 \text{ kN}$$

The value of $\phi = 0.6$ (Table 2.3 of AS3600-1994) and the design strengths are

$$\phi M_{ub} = 0.6 \times 413.8 = 248.3 \text{ kNm}$$

$$\phi N_{ub} = 0.6 \times 1755.2 = 1053.1 \text{ kN}$$

6.3.3 The Limit Condition M_{ul}, N_{ul} for Ultimate Strength

500 MPa Rebar

The strain distribution and resultant forces in the steel and concrete for this condition are shown in Figure 6.6. To determine the values of M_{ul} and N_{ul} for the limit condition, the neutral axis depth $k_u d$ is set to the value of d_o (see Section 4.2.4). Again, compressive strains, stresses and forces are taken as positive and tensile strains, stresses and forces are taken as negative.

Set the value of $k_u d = d_o = 338.0$ mm

Layer	i	x_i (mm)	ε_i	σ_i (MPa)	A_i (mm^2)	F_i (kN)	$D/2 - x_i$ (mm)	M_i (kNm)
Steel	1	338.00	0.0000000	0.00	1800	0.0	-138.00	0.0
Steel	2	246.00	0.0008166	163.31	900	122.5	-46.00	-5.6
Steel	3	154.00	0.0016331	326.63	900	269.5	46.00	12.4
Steel	4	62.00	0.0024497	489.94	1800	832.9	138.00	114.9
Concrete	5	138.92		27.20	111134	3022.9	61.08	184.6
						Σ = 4247.8		Σ = 306.3

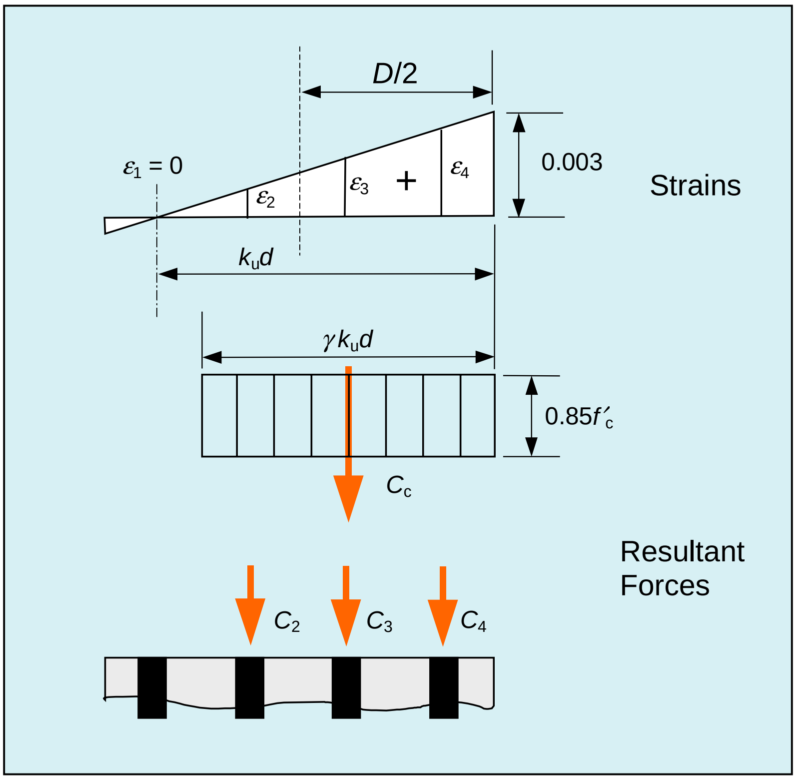

Figure 6.6 Strain Distribution and Resultant Forces for the Limit Condition - M_{ul}, N_{ul}

The values of M_{ul} and N_{ul} are determined as

$$M_{ul} = \Sigma M_i = 306.3 \text{ kNm}$$

$$N_{ul} = \Sigma F_i = 4247.8 \text{ kN}$$

The value of ϕ = 0.6 (Table 2.3 of AS3600-1994) and the design strengths are

$$\phi M_{ul} = 0.6 \times 306.3 = 183.8 \text{ kNm}$$

$$\phi N_{ul} = 0.6 \times 4247.8 = 2548.7 \text{ kN}$$

400 MPa Rebar

Set the value of $k_u d = d_o$ = 338.0 mm

Layer	i	x_i (mm)	ε_i	σ_i (MPa)	A_i (mm^2)	F_i (kN)	$D/2 - x_i$ (mm)	M_i (kNm)
Steel	1	338.00	0.0000000	0.00	1800	0.0	-138.00	0.0
Steel	2	246.00	0.0008166	163.31	900	122.5	-46.00	-5.6
Steel	3	154.00	0.0016331	326.63	900	269.5	46.00	12.4
Steel	4	62.00	0.0024497	400.00	1800	671.0	138.00	92.6
Concrete	5	138.92		27.20	111134	3022.9	61.08	184.6
						Σ = 4085.9		Σ = 284.0

The values of M_{ul} and N_{ul} are determined as

$$M_{ub} = \Sigma M_i = 284.0 \text{ kNm}$$

$$N_{ub} = \Sigma F_i = 4085.9 \text{ kN}$$

The value of $\phi = 0.6$ (Table 2.3 of AS3600-1994) and the design strengths are

$$\phi M_{ub} = 0.6 \times 284.0 = 170.4 \text{ kNm}$$

$$\phi N_{ub} = 0.6 \times 4085.9 = 2451.5 \text{ kN}$$

6.3.4 The Ultimate Strength N_{uo} for Pure Axial Force

500 MPa Rebar

The strain distribution and resultant forces in the steel and concrete for this condition are shown in Figure 6.7. The neutral axis depth $k_u d$ is at infinity. There are only uniform compressive strains and stresses in the concrete and steel.

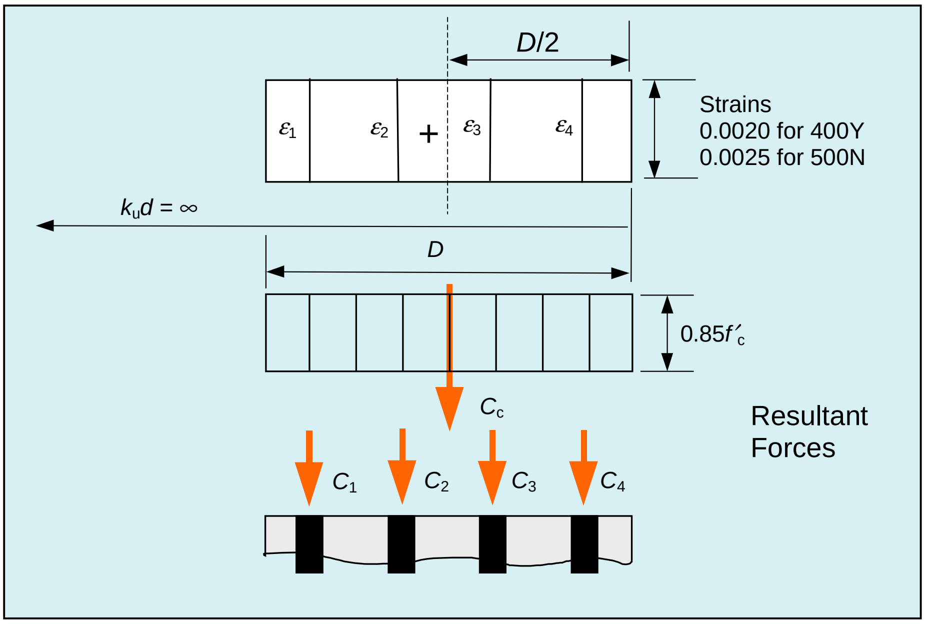

Figure 6.7 Strain Distribution and Resultant Forces for Pure Axial Force N_{uo}

The gross area A_g of the cross-section has a value

$$A_g = 400 \times 400 = 160000 \text{ mm}^2$$

The area A_s of the steel rebar has a value

$$A_s = 12 \times 450 = 5400 \text{ mm}^2$$

and hence the area A_c of the concrete has a value

$$A_c = A_g - A_s = 160000 - 5400 = 154600 \text{ mm}^2$$

The value of the ultimate strength N_{uo} in pure compression is given by

$$N_{uo} = 0.85 f'_c A_c + f_{sy} A_s = 0.85 \times 32 \times 154600 + 500 \times 5400 \text{ N} = 6905.1 \text{ kN}$$

The value of $\phi = 0.6$ (Table 2.3 of AS3600-1994) and hence the design strength ϕN_{uo} is

$$\phi N_{uo} = 0.6 \times 6905.1 = 4143.1 \text{ kN}$$

400 MPa Rebar

The value of the ultimate strength N_{uo} in pure compression is given by

$$N_{uo} = 0.85f'_c A_c + f_{sy} A_s = 0.85 \text{ x } 32 \text{ x } 154600 + 400 \text{ x } 5400 \text{ N} = 6365.1 \text{ kN}$$

The value of ϕ = 0.6 (Table2.3 of AS3600-1994) and hence the design strength ϕN_{uo} is

$$\phi N_{uo} = 0.6 \text{ x } 6365.12 = 3819.1 \text{ kN}$$

It can be seen that the use of 500N grade reinforcing steel such as 500PLUS Rebar for the same area of steel results in an axial strength that is 8.5% higher than that achieved with the use of 400Y rebar.

6.3.5 Load - Moment Strength Interactions Diagrams

The load-moment strength interaction diagrams generated using the rectangular stress block approach are shown plotted in Figure 6.8. It can be seen that the use of 500N grade reinforcement generally leads to asignificant increase in strength over the full range of load and moment. However, around the balance point, there appears to be little advantage in using the higher strength grade. It should be noted that there is a change in the capacity reduction factor ϕ for loads below the balance point (see Eq. 3.2.3.3(3) and Table 2.3 of AS3600-1994). The cross-section is also slightly "over"-reinforced (k_u > 0.4 for pure bending) when using the 500N rebar resulting in a value of ϕ slightly less than 0.8 for pure bending. However, the main reason for this anomaly results from the assumptions used in the simplified rectangular stress block theory (see the comparisons between an accurate analytical method and the rectangular stress block method in Figure 4.4 of Section 4.3).

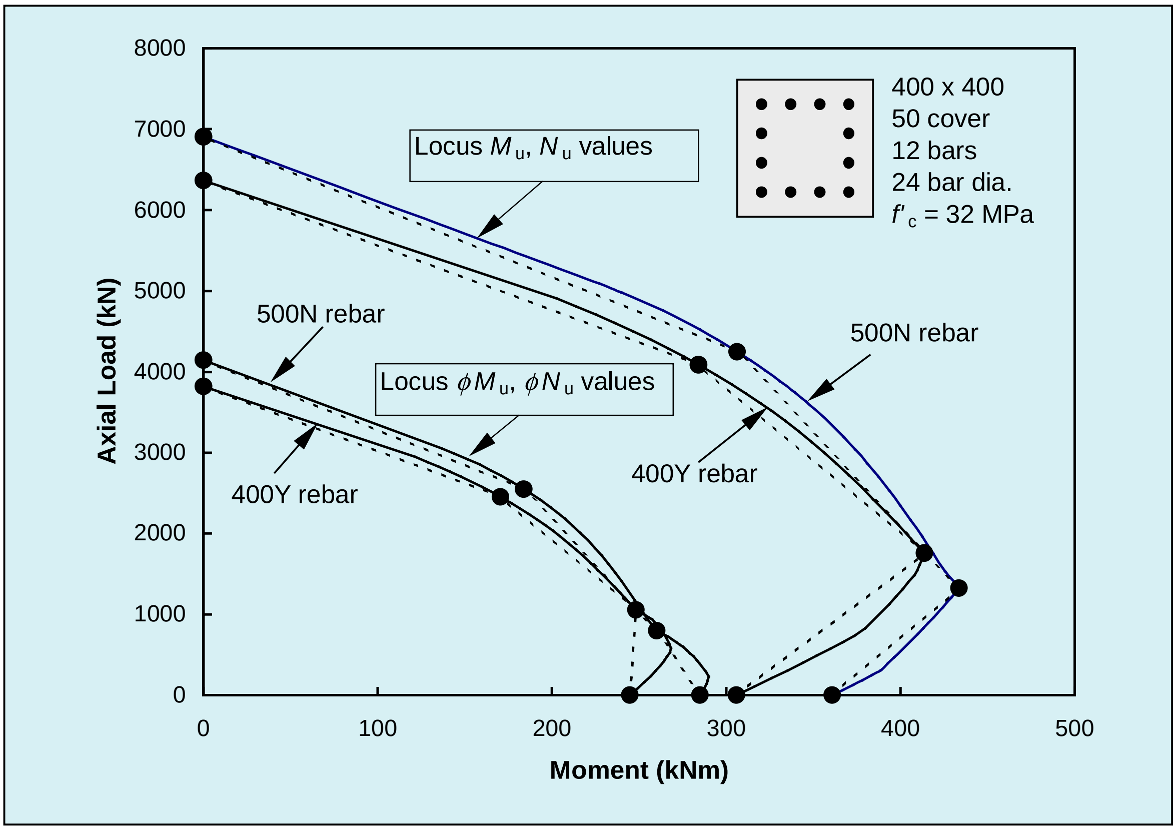

Figure 6.8 Comparison of Steel Strengths Using Rectangular Stress Block

When the accurate analytical method is used to generate the load-moment strength interaction diagrams, the full advantage of the higher grade reinforcement is obtained over the full range of load and moment as demonstrated in the examples in Section 6.2 above and the following Section 6.4. It should be noted that the analytical method was used to generate the load-moment strength design charts in Appendix C.

6.4 Column Reinforced on Four sides – Analytical Method

The analytical method of Section 4.3 has also been used to generate the load-moment strength interaction diagram for the cross-section shown in Figure 6.3. The calculations were first done using 500N grade reinforcing steel such as 500PLUS Rebar having a design yield stress f_{sy} = 500 MPa and a yield strain ε_{sy} = 0.0025. They were then repeated for 400Y grade reinforcing steel having a yield stress f_{sy} = 400 MPa and a yield strain ε_{sy} = 0.002 to demonstrate the benefits that can be achieved in using the higher grade rebar. The results are shown in Figure 6.9. The values can also be determined using the design chart RCB 3.1 – C22 for g = 0.7 (close to actual g = 0.69) and interpolating between the values for 3% and 4% steel (actual percentage p = 3.375%). This process of interpolating in the design chart is normal practice for designers.

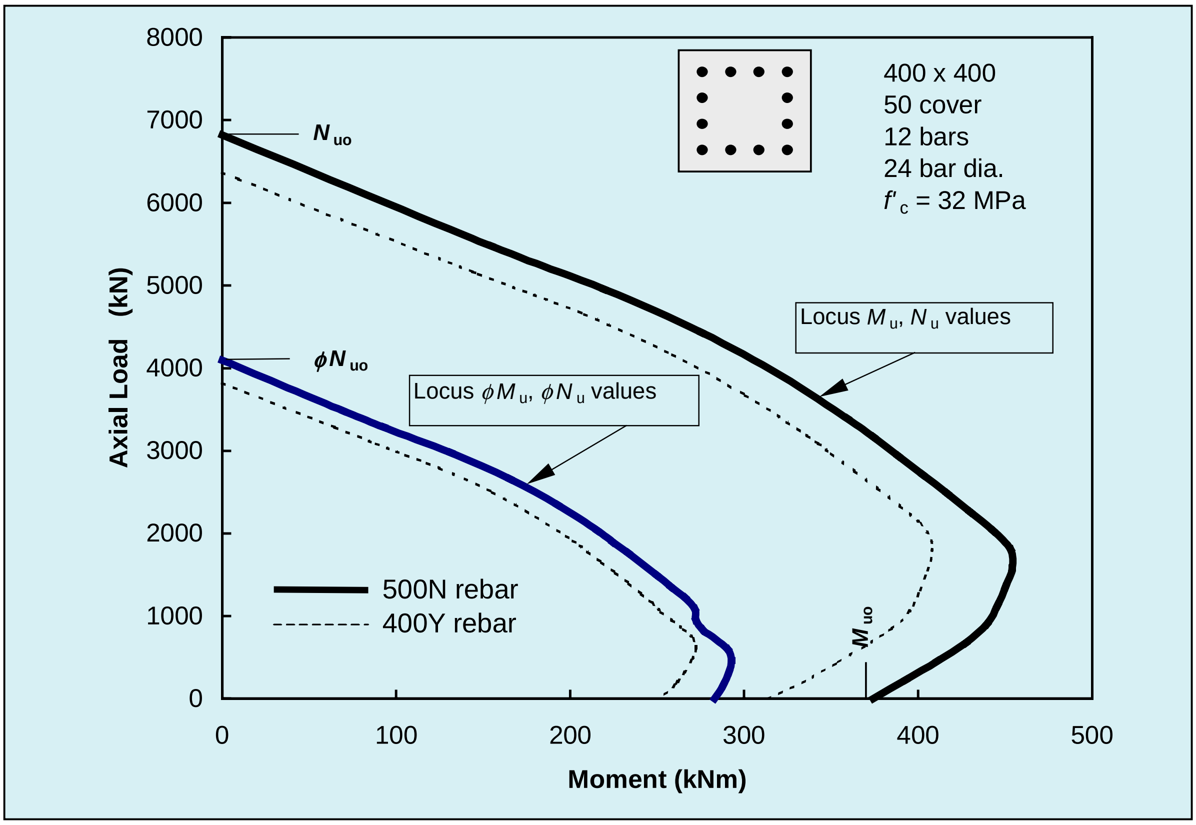

Figure 6.9 Comparison of Steel Strengths Using Analytical Method

6.5 Eccentrically Loaded Column

As part of the design process, design (factored) loads are placed on the reinforced concrete structure and an analysis is carried, assuming certain stiffnesses for the members, in order to determine values of design axial force N^* and bending moment M^* at cross-sections within the members. If an elastic analysis is carried out (which is usually the case), and the members are deemed to be short (see Section 3.2.4), then the moment M^* is related directly to the axial force N^*

such that the axial force N^* at a cross-section can be considered acting at an eccentricity e where where $e = M^*/N^*$. Hence, values of eccentricity will be known and a design situation could be to determine value of the axial strength ϕN_u for a given eccentricity e (= 120mm) and a trial cross-section (such as that in Figure 6.3).

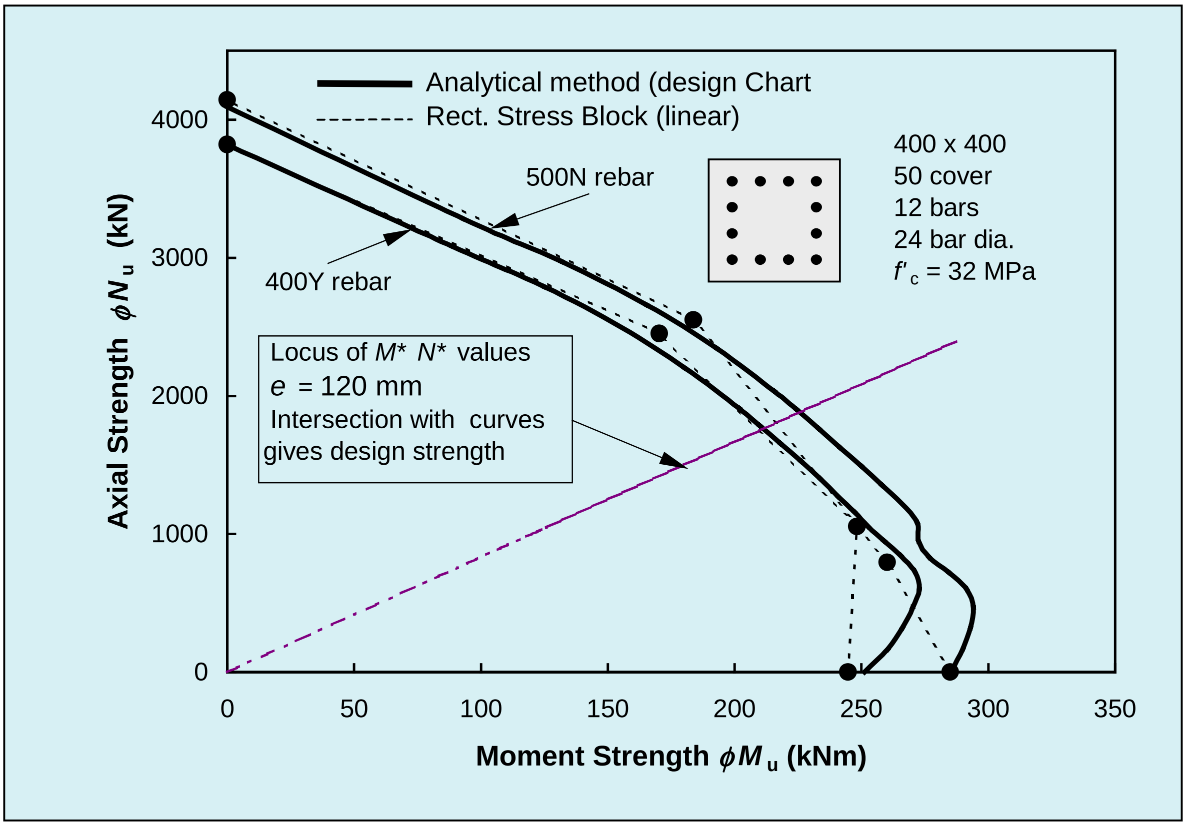

Figure 6.10 Strength of an Eccentrically Loaded Cross-section

The intersection of the eccentricity line with the load-moment strength interaction diagrams in Figure 6.10 gives the design axial strength ϕN_u and the corresponding design bending strength ϕM_u (= $\phi N_u e$) . For the rectangular stress block method, only the linear approximation passing through the four key points (see Section 6.3) is shown in Figure 6.10 as this is the approach used for manual calculations. If these linear approximations are graphed in a spreadsheet, the cursor on the graph can be used to determine the value at the intersection rather than by numerical means.

500 MPa Rebar

For the analytical method:

ϕN_u = 1880 kN

and ϕM_u = $\phi N_u e$ = 1880 x 120 kNmm = 225.6 kNm

For the rectangular stress block method:

ϕN_u = 1800 kN

and ϕM_u = $\phi N_u e$ = 1800 x 120 kNmm = 216.0 kNm

400 MPa Rebar

For the analytical method:

ϕN_u = 1770 kN (cf. 1880 kN for 500N rebar which is 6.2% higher)

and $\phi M_u = \phi N_u e$ = 1770 x 120 kNmm = 212.4 kNm

For the rectangular stress block method:

ϕN_u = 1750 kN

and $\phi M_u = \phi N_u e$ = 1750 x 120 kNmm = 210.0 kNm

7. CASE STUDIES

7.1 Introduction

Four different types of columns that were originally designed using Grade 400Y reinforcement have been redesigned using OneSteel 500PLUS Rebar (Grade 500N reinforcement) to give a similar strength in both major axis and minor axis bending. This can be achieved using a number of different strategies including: keeping the number of bars the same and reducing the bar size; keeping the bar size the same and reducing the number of bars; and keeping the bar size and number of bars the same and reducing the overall dimensions of the concrete cross-section. These strategies are examined in detail by comparing the load-moment design strength interaction diagram for the original Grade 400Y bar design (shown as a broken dashed line in all the comparisons) with the load-moment interaction design strength diagram for an alternative Grade 500N bar design (shown as a bold solid line in all the comparisons). The interaction diagrams have been generated using the accurate analytical method of Section 4.3 and the CEB [4] stress-strain curve for the concrete as shown in Figure 3.2. This approach was used for the both the new design charts in Appendix C for 500PLUS Rebar and the current C&CAA [2] design charts.

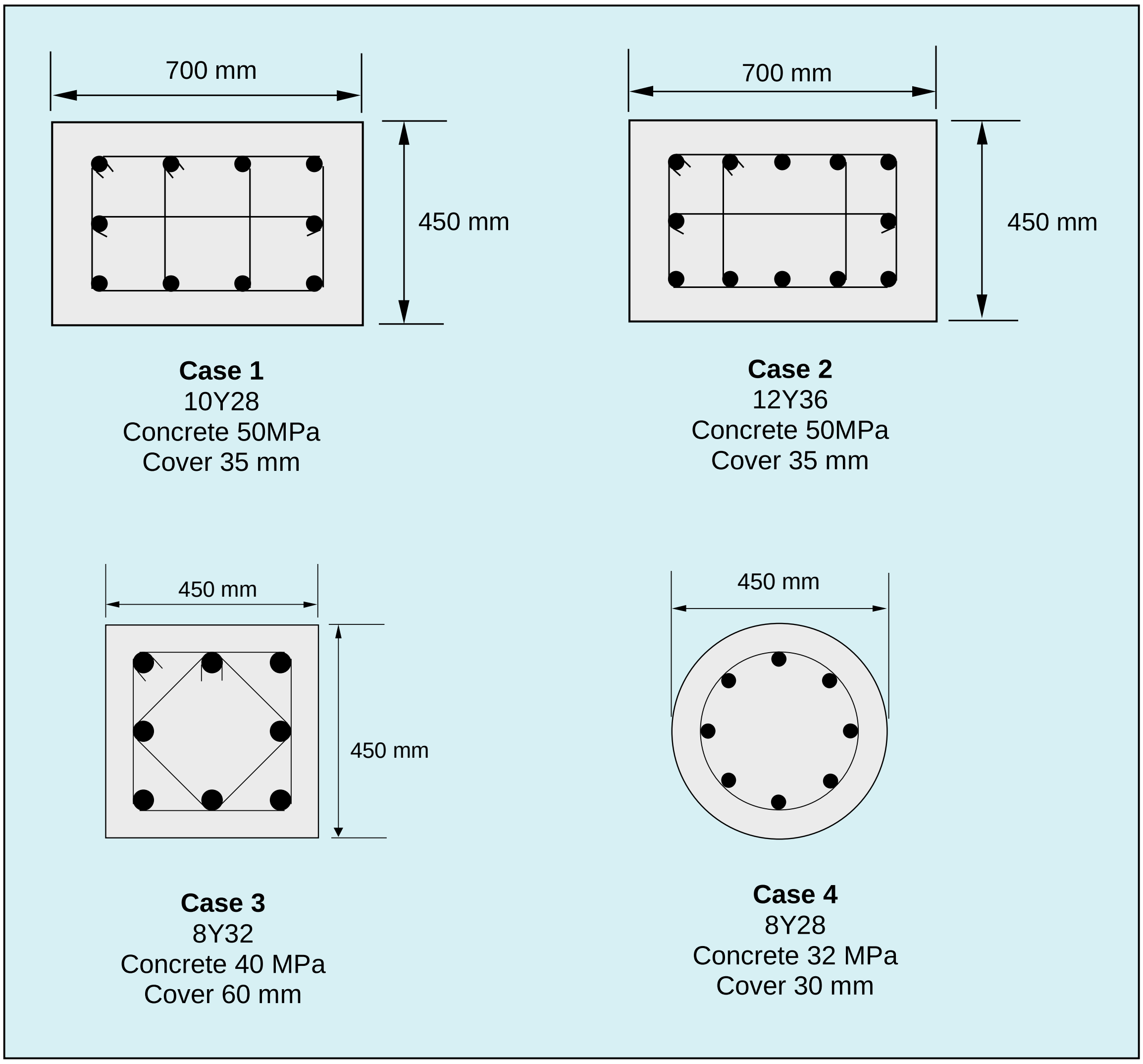

Fig. 7.1 Cross-sections of Case Study Columns – Original Design

7.2 Case 1 – 700 mm x 450 mm Rectangular Column with 10Y28 bars

7.2.1 Major Axis Bending - Change in Steel Grade

The bar size and number of bars are kept the same and the steel is changed from 400 grade to 500 grade.

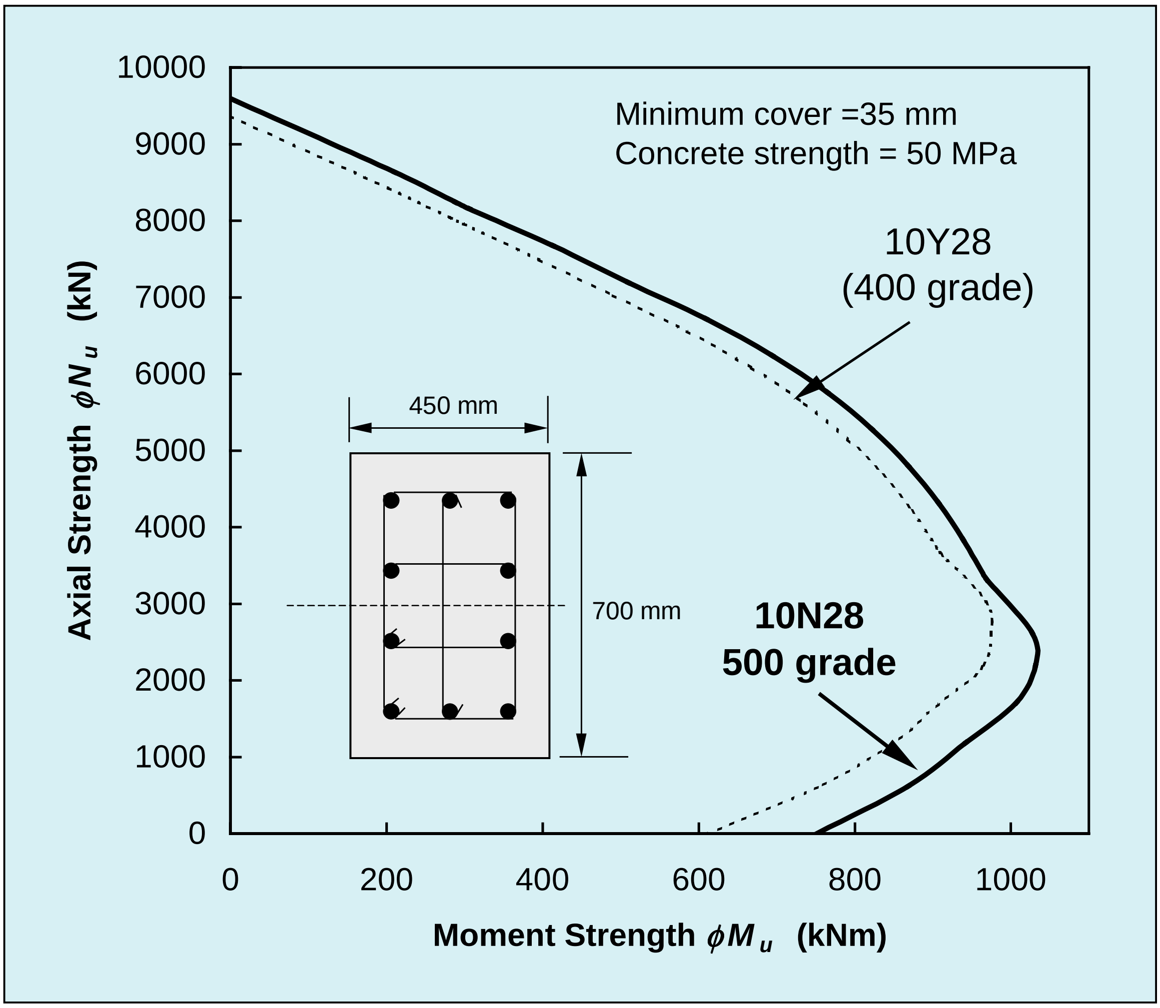

Fig. 7.2 Major Axis Bending - Effect of Changing Steel Grade

It can be seen from Figure 7.2 that changing the steel grade from 400 MPa to 500 MPa increases the strength over the whole range of load-moment combinations from pure bending (22.9% increase) to pure axial force (2.5% increase). With the high concrete strength of 50MPa, changing the steel grade does not have a large effect on the axial load carrying capacity. However, it does have a significant effect on the bending strength for low values of axial force.

7.2.2 Major Axis Bending - Change in Bar Size and Steel Grade

The Y28 bars have been changed to N24 bars but the number of bars and the concrete cover have been kept the same.

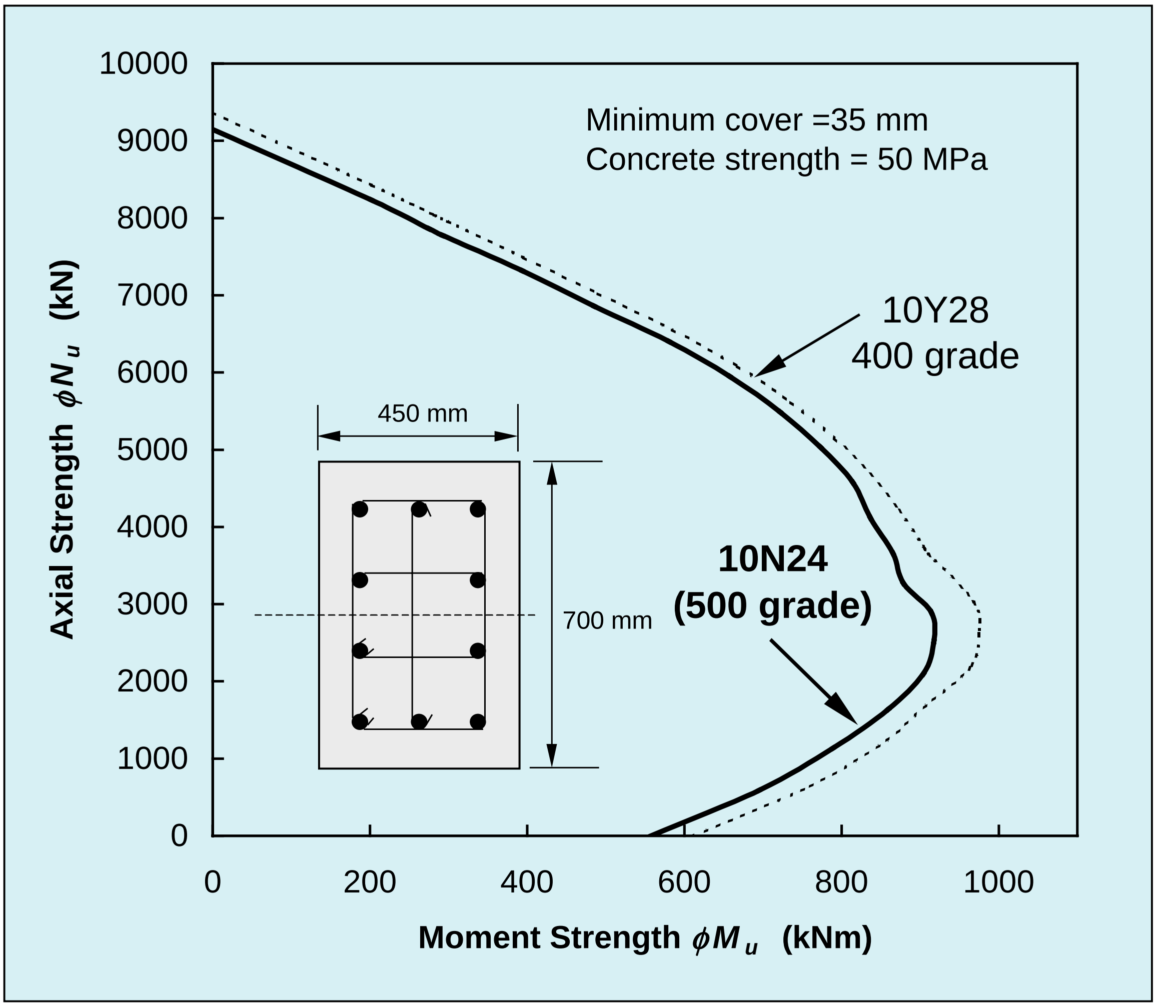

Fig. 7.3 Major Axis Bending - Effect of Changing the Bar Size and Steel Grade

It can be seen from Figure 7.3 that the reduction in one bar size from 28 mm diameter to 24 mm diameter was not completely compensated by the increase in steel grade from 400 MPa to 500 MPa. This is to be expected as the area of steel has been decreased by 27.4% whereas the steel strength has been increased by only 20%.

This redesign using 10N24 bars may still be adequate if the design action effects M^* and N^* on the cross-section (which should include the second-order effects) resulting from the application of the design loads still lay within the envelope of the 10N24 interaction diagram *and* the original design using 10Y28 bars was slightly conservative.

7.2.3 Major Axis Bending - Change in Number of Bars and Steel Grade

The Y28 bars have been changed to N28 bars (i.e. the bar size has been kept the same), the concrete cover has been kept the same but the number of bars has been reduced from ten to eight.

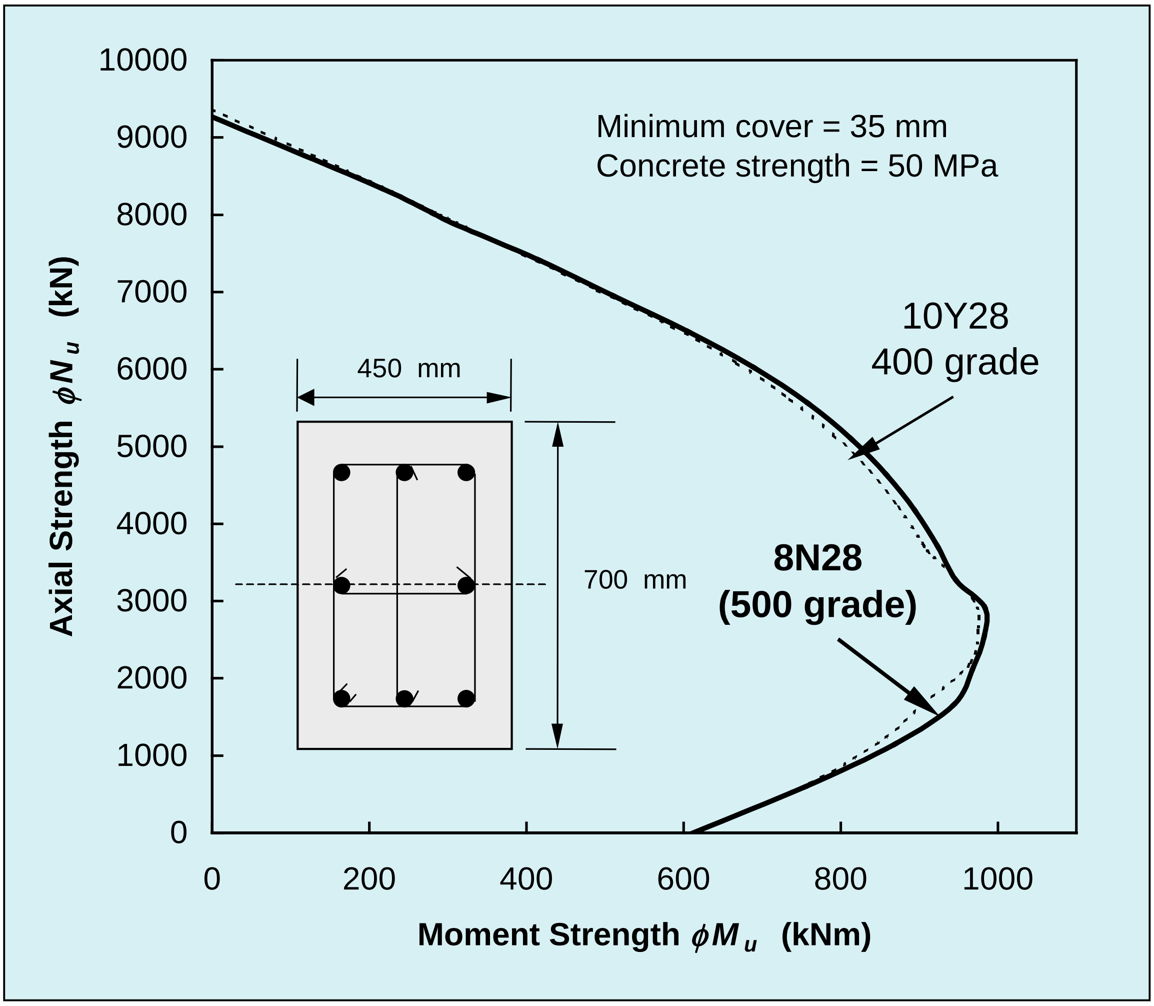

Fig. 7.4 Major Axis Bending - Effect of Changing the Number of Bars and Steel Grade

It can be seen from Figure 7.4 that the reduction in the number of bars has been compensated by the increase in steel grade from 400 MPa to 500 MPa. In this case, the area of steel has been decreased by 20% and the steel strength has been increased by 20%. The distribution of steel in the side faces has been altered but this did not effect the major axis strength.

This redesign using 8N28 bars could be considered to be adequate for major axis bending if the design action effects *M** and *N** on the cross-section (which should include the second-order effects) resulting from the application of the design loads were within the envelope of the interaction diagram for the original design using 10Y28 bars.

7.2.4 Minor Axis Bending - Change in Steel Grade

The bar size and number of bars are kept the same and the steel is changed from 400 grade to 500 grade.

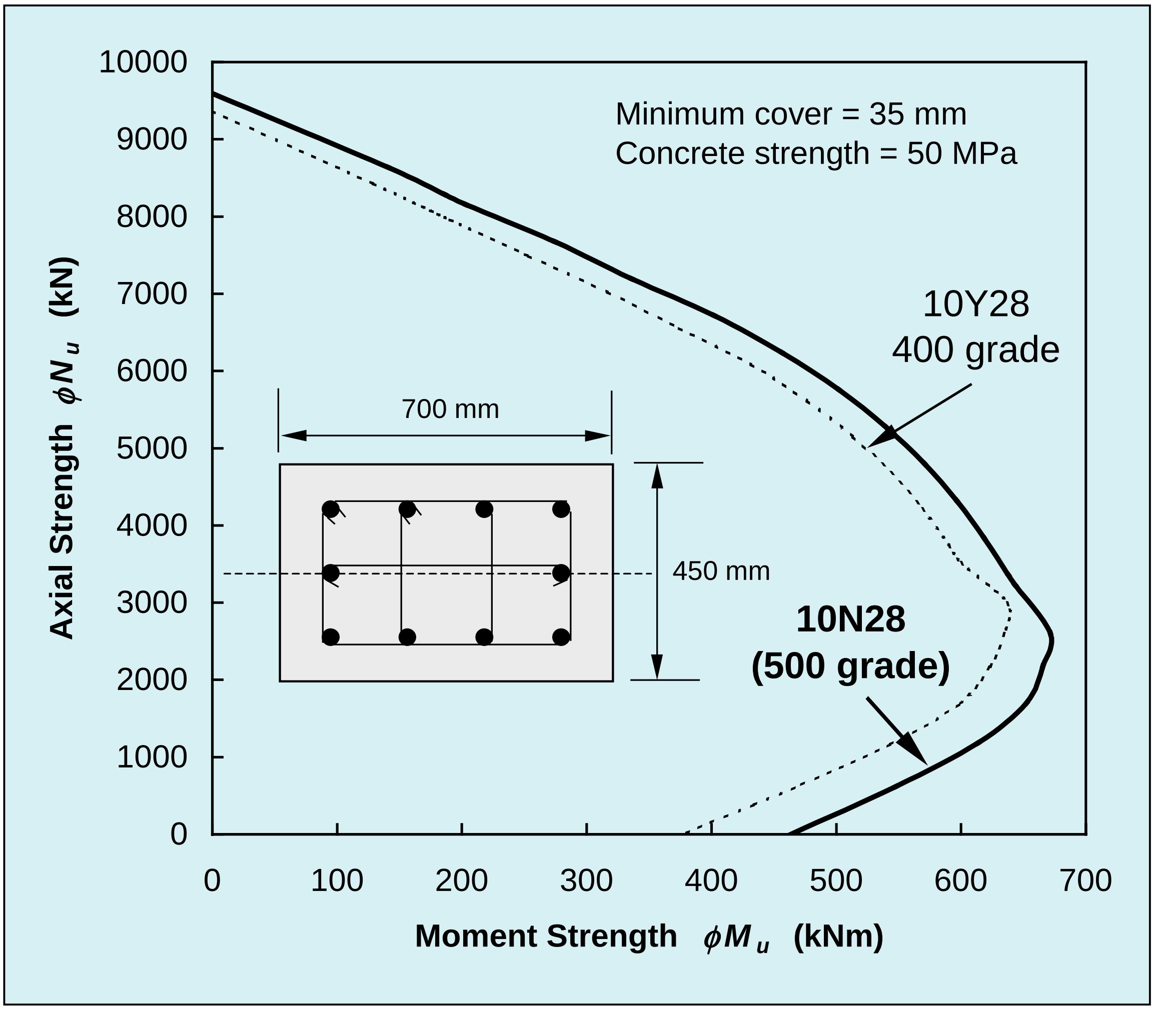

Fig. 7.5 Minor Axis Bending - Effect of Changing Steel Grade

It can be seen from Figure 7.5 that changing the steel grade from 400 MPa to 500 MPa increases the strength over the whole range of load-moment combinations from pure bending (23.1% increase) to pure axial force (2.5% increase). With the high concrete strength of 50MPa, changing the steel grade does not have a large effect on the axial load carrying capacity. However, it does have a significant effect on the bending strength for low values of axial force.

7.2.5 Minor Axis Bending - Change in Bar Size and Steel Grade

The Y28 bars have been changed to N24 bars but the number of bars and the concrete cover have been kept the same.

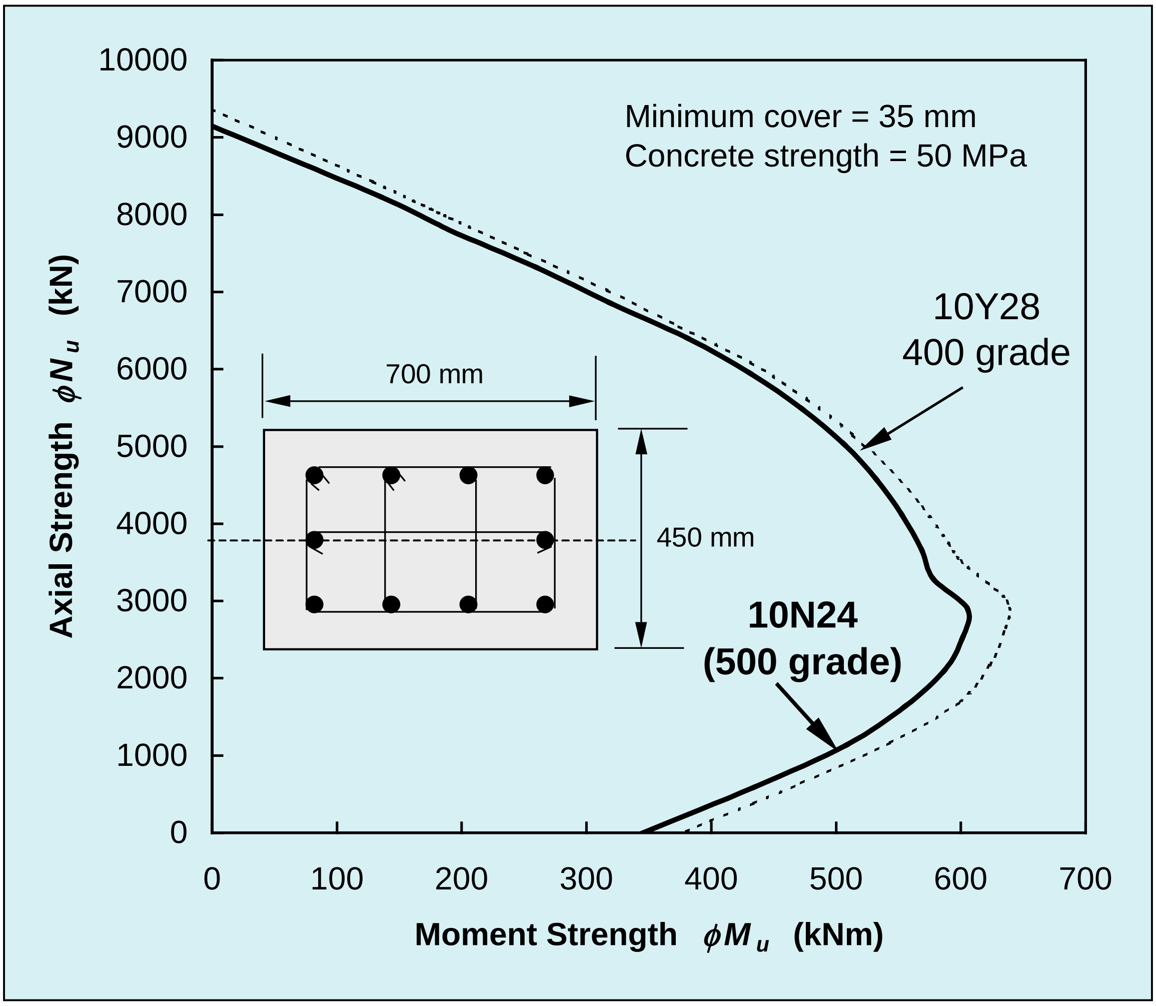

Fig. 7.6 Minor Axis Bending - Effect of Changing the Bar Size and Steel Grade

It can be seen from Figure 7.6 that the reduction in one bar size from 28 mm diameter to 24 mm diameter was not completely compensated by the increase in steel grade from 400 MPa to 500 MPa. This is to be expected as the area of steel has been decreased by 27.4% whereas the steel strength has been increased by only 20%.

This redesign using 10N24 bars may still be adequate if the design action effects *M** and *N** on the cross-section (which should include the second-order effects) resulting from the application of the design loads still lay within the envelope of the 10N24 interaction diagram *and* the original design using 10Y28 bars was slightly conservative.

7.2.6 Minor Axis Bending - Change in Number of Bars and Steel Grade

The Y28 bars have been changed to N28 bars (i.e. the bar size has been kept the same), the concrete cover has been kept the same but the number of bars has been reduced from ten to eight.

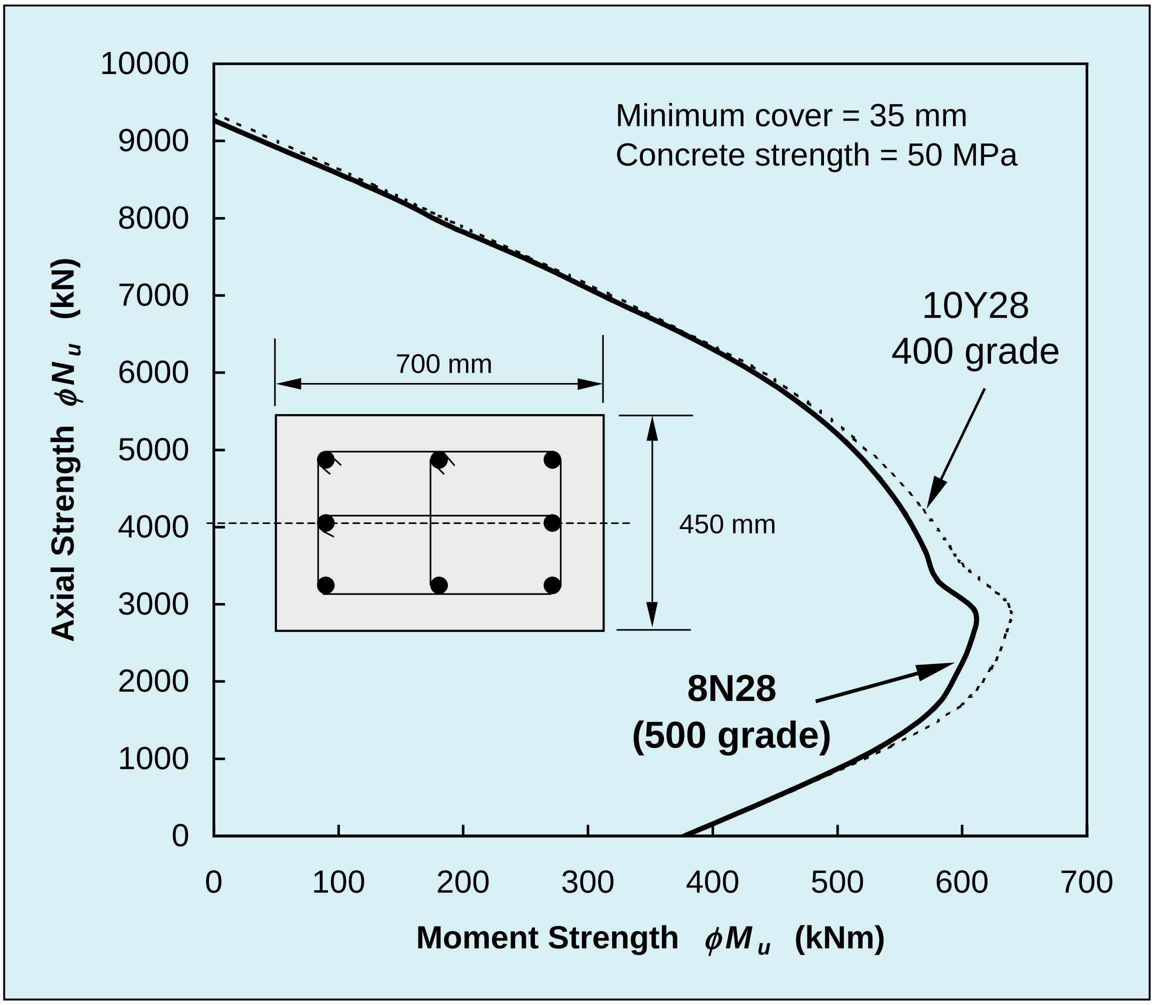

Fig. 7.7 Minor Axis Bending - Effect of Changing the Number of Bars and Steel Grade

It can be seen from Figure 7.7 that the reduction in the number of bars has mostly been compensated by the increase in steel grade from 400 MPa to 500 MPa. In this case, the area of steel has been decreased by 20% and the steel strength has also been increased by 20%. The distribution of steel in the top and bottom faces has been altered and this did have a slight effect on the minor axis strength around the “nose” of the interaction diagram where the strength for 8N28 bars was less than the strength for 10Y28 bars.

This redesign using 8N28 bars may still be adequate if the design action effects *M** and *N** on the cross-section (which should include the second-order effects) resulting from the application of the design loads still lay within the envelope of 8N28 interaction diagram *and* the original design using 10Y28 bars was slightly conservative, particularly if the values were around the “nose” of the interaction diagram.

7.3 Case 2 – 700 mm x 450 mm Rectangular Column with 12Y36 bars

7.3.1 Major Axis Bending - Change in Steel Grade

The bar size and number of bars are kept the same and the steel is changed from 400 grade to 500 grade.

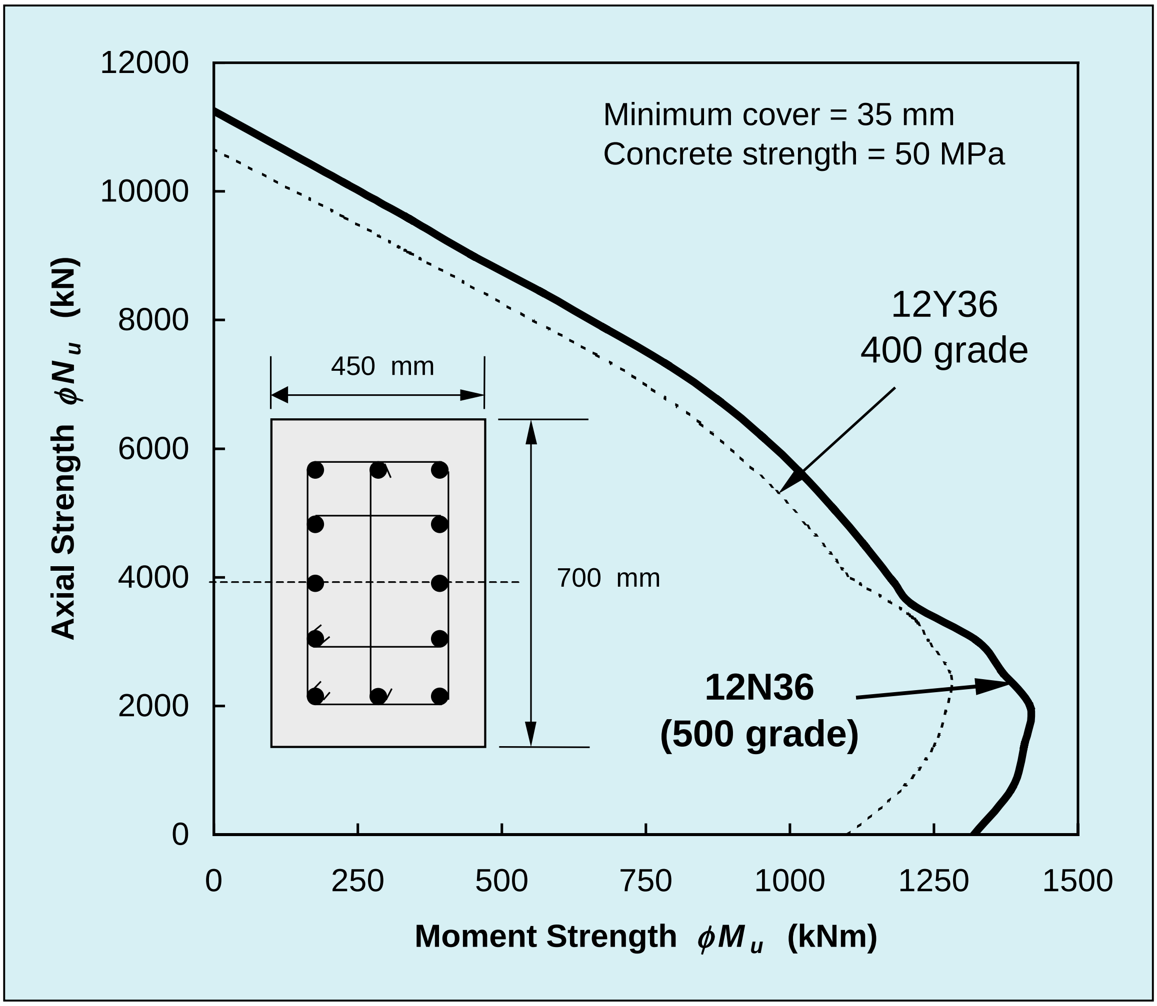

Fig. 7.8 Major Axis Bending - Effect of Changing Steel Grade

It can be seen from Figure 7.8 that changing the steel grade from 400 MPa to 500 MPa increases the strength over the whole range of load-moment combinations from pure bending (20.0% increase) to pure axial force (5.5% increase). With the high concrete strength of 50MPa, changing the steel grade does not have a large effect on the axial load carrying capacity. However, it does have a significant effect on the bending strength for low values of axial force.

7.3.2 Major Axis Bending - Change in Bar Size and Steel Grade

The Y36 bars have been changed to N32 bars but the number of bars and the concrete cover have been kept the same.

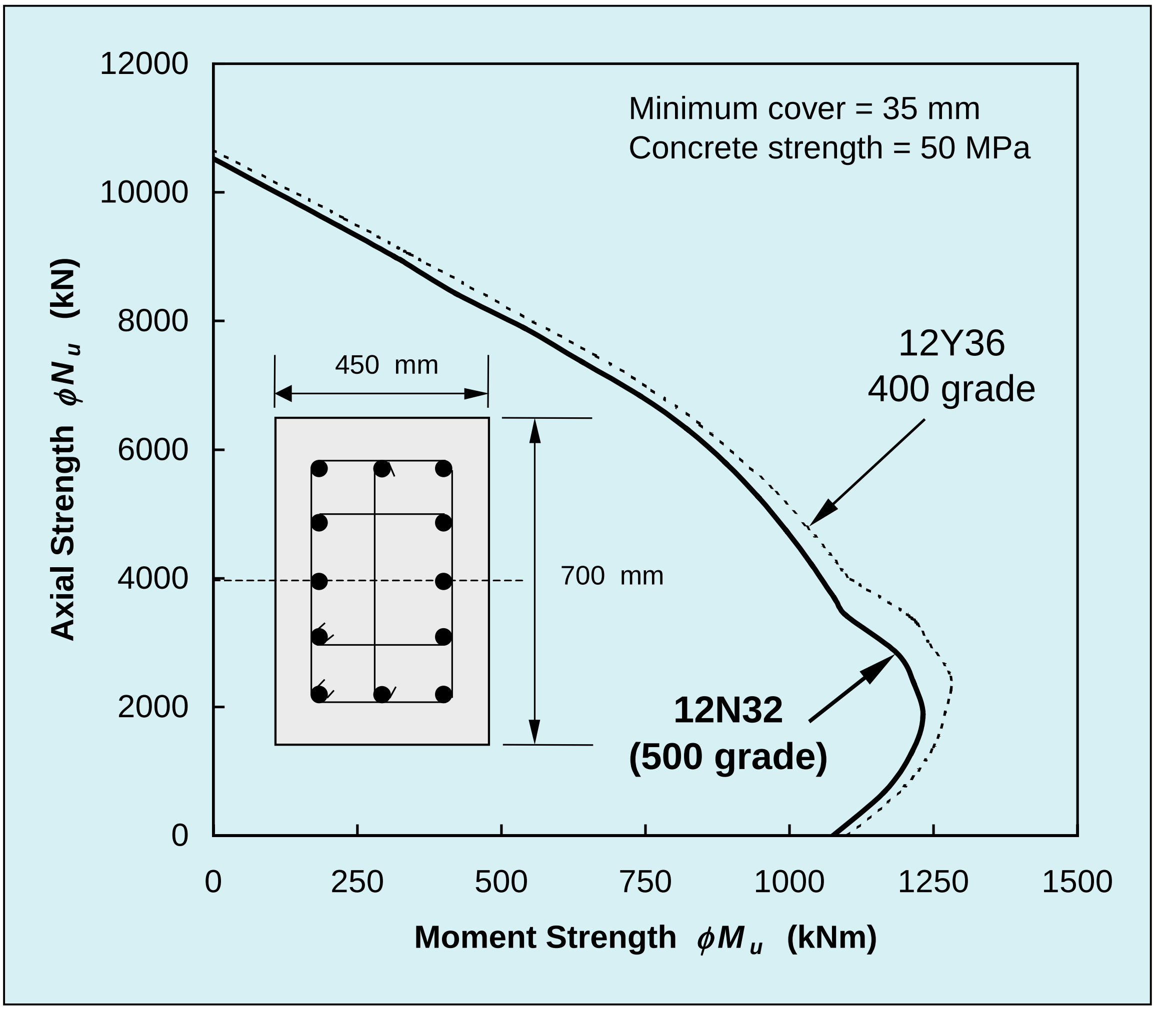

Fig. 7.9 Major Axis Bending - Effect of Changing the Bar Size and Steel Grade

It can be seen from Figure 7.9 that the reduction in one bar size from 36 mm diameter to 32 mm diameter was almost but not completely compensated by the increase in steel grade from 400 MPa to 500 MPa. This is to be expected as the area of steel has been decreased by 21.6% whereas the steel strength has been increased by 20%.

This redesign using 12N32 bars may still be adequate if the design action effects *M** and *N** on the cross-section (which should include the second-order effects) resulting from the application of the design loads still lay within the envelope of the 12N32 interaction diagram *and* the original design using 12Y36 bars was slightly conservative.

7.3.3 Major Axis Bending - Change in Number of Bars and Steel Grade

The Y36 bars have been changed to N36 bars (i.e. the bar size has been kept the same), the concrete cover has been kept the same but the number of bars has been reduced from twelve to ten.

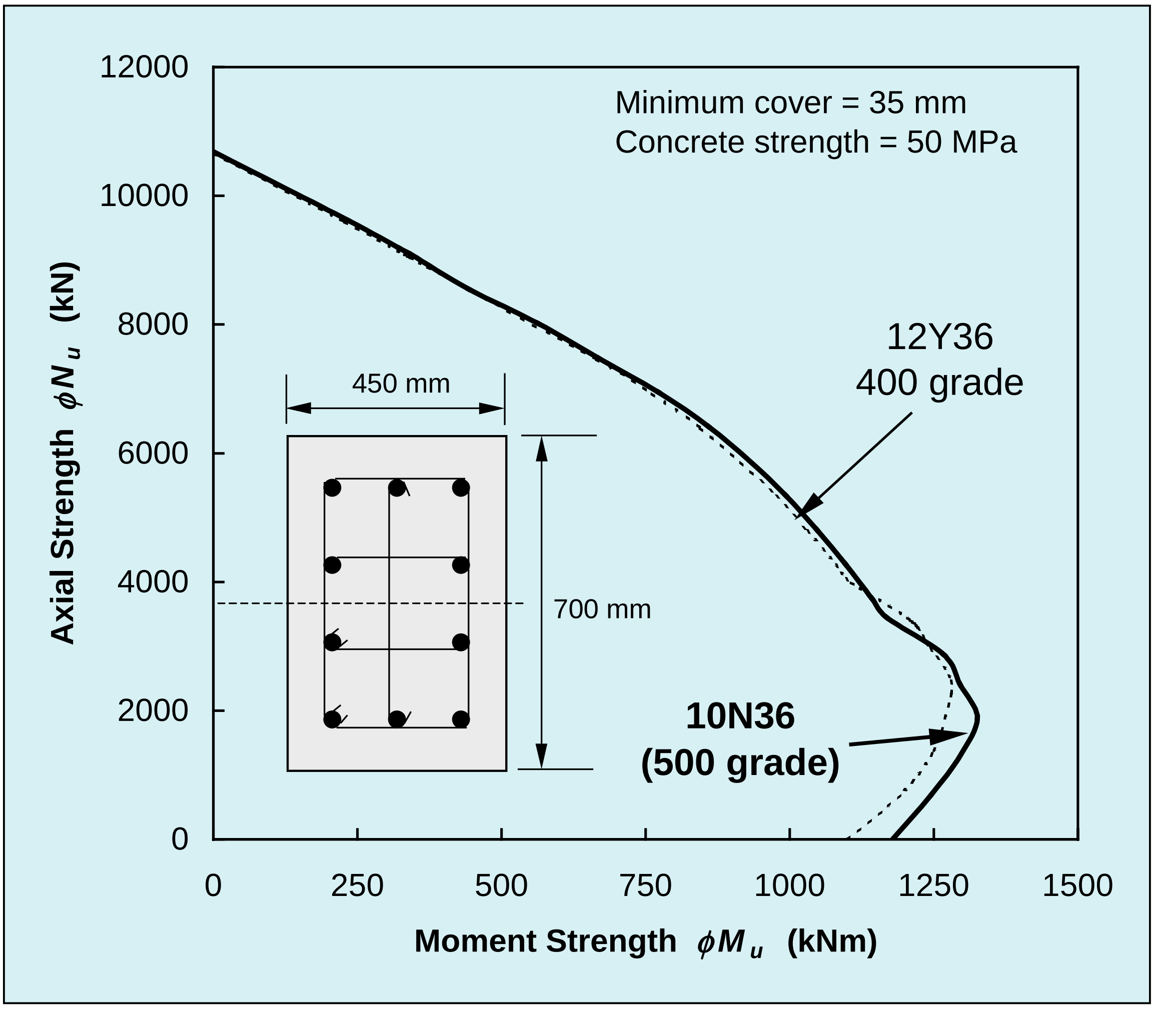

Fig. 7.10 Major Axis Bending - Effect of Changing the Number of Bars and Steel Grade

It can be seen from Figure 7.10 that the reduction in the number of bars has been more than compensated by the increase in steel grade from 400 MPa to 500 MPa. In this case, the area of steel has been decreased by 16.7% and the steel strength has been increased by 20%. The distribution of steel in the side faces has been altered but this did not effect the major axis strength.

This redesign using 10N36 bars could be considered to be adequate for major axis bending if the design action effects *M** and *N** on the cross-section (which should include the second-order effects) resulting from the application of the design loads were within the envelope of the interaction diagram for the original design using 12Y36 bars.

7.3.4 Minor Axis Bending - Change in Steel Grade

The bar size and number of bars are kept the same and the steel is changed from 400 grade to 500 grade.

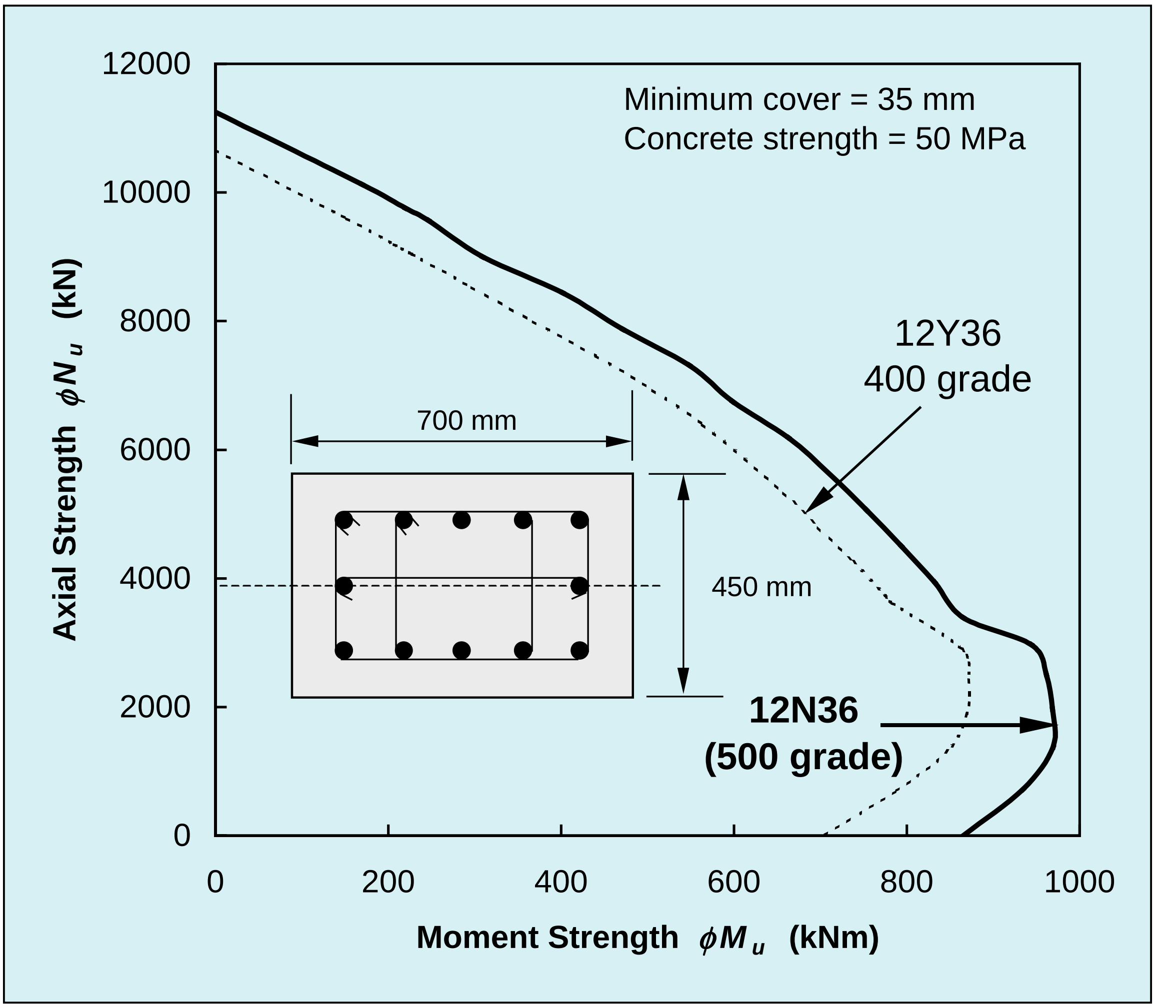

Fig. 7.11 Minor Axis Bending - Effect of Changing Steel Grade

It can be seen from Figure 7.11 that changing the steel grade from 400 MPa to 500 MPa increases the strength over the whole range of load-moment combinations from pure bending (22.9% increase) to pure axial force (5.5% increase). With the high concrete strength of 50MPa, changing the steel grade does not have a large effect on the axial load carrying capacity. However, it does have a significant effect on the bending strength particularly for low values of axial force.

7.3.5 Minor Axis Bending - Change in Bar Size and Steel Grade

The Y36 bars have been changed to N32 bars but the number of bars and the concrete cover have been kept the same.

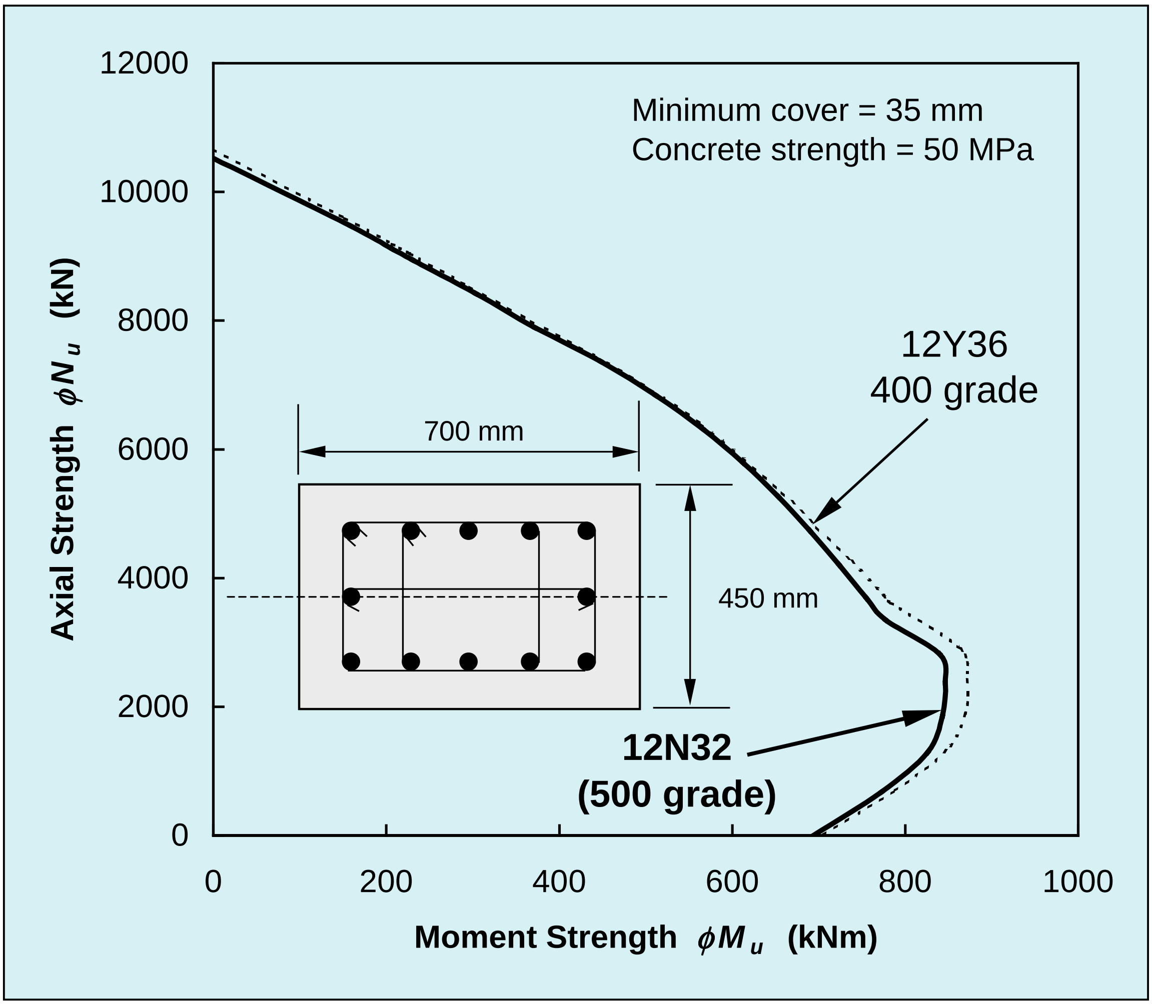

Fig. 7.12 Minor Axis Bending - Effect of Changing the Bar Size and Steel Grade

It can be seen from Figure 7.12 that the reduction in one bar size from 36 mm diameter to 32 mm diameter was almost but not completely compensated by the increase in steel grade from 400 MPa to 500 MPa. This is to be expected as the area of steel has been decreased by 21.6% whereas the steel strength has been increased by 20%.

This design using 12N32 bars may still be adequate if the design action effects *M** and *N** on the cross-section (which should include the second-order effects) resulting from the application of the design loads still lay within the envelope of the 12N32 interaction diagram *and* the original design using 12Y36 bars was just slightly conservative, particularly for values near the "nose" of the interaction diagram.

7.3.6 Major Axis Bending - Change in Number of Bars and Steel Grade

The Y36 bars have been changed to N36 bars (i.e. the bar size has been kept the same), the concrete cover has been kept the same but the number of bars has been reduced from twelve to ten.

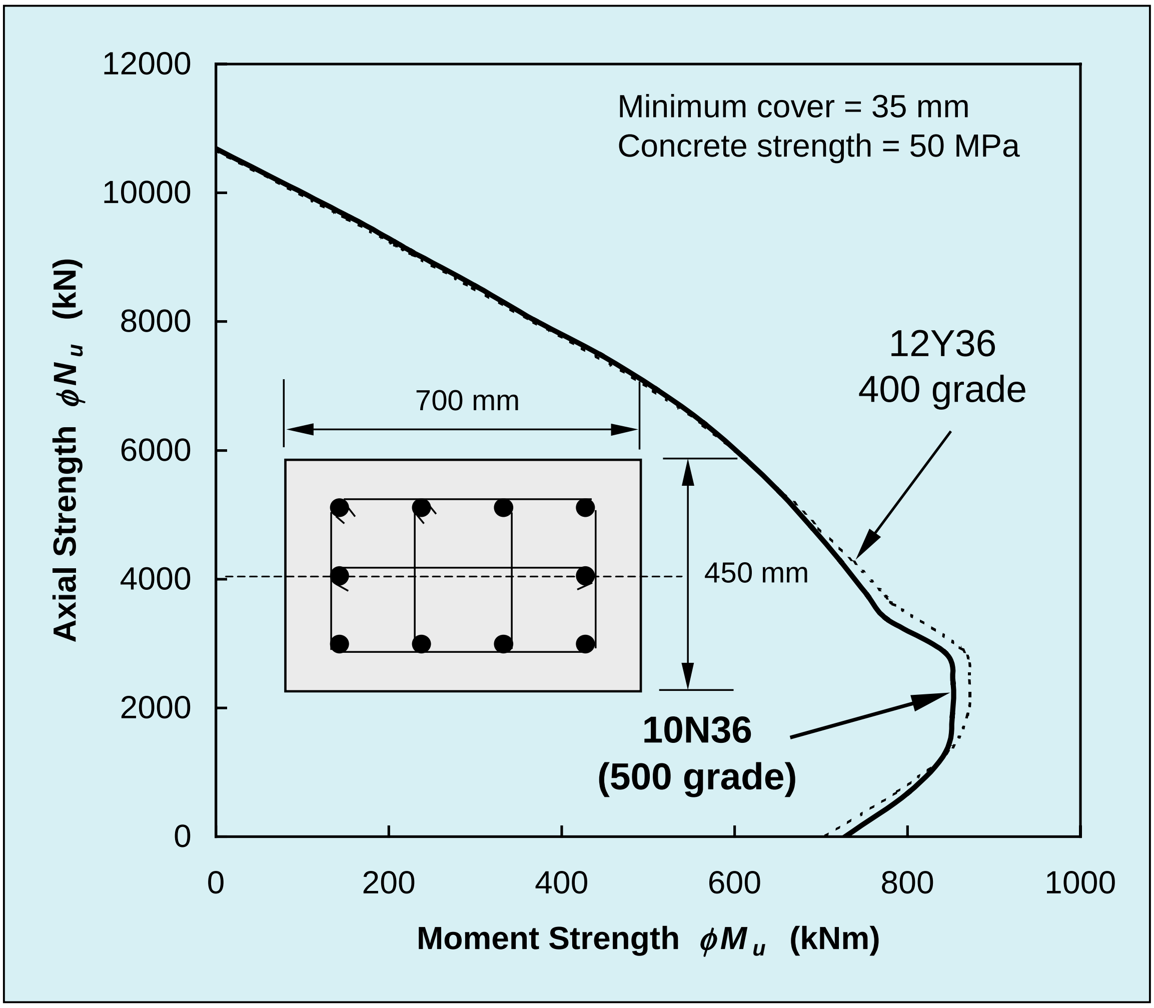

Fig. 7.13 Minor Axis Bending - Effect of Changing the Number of Bars and Steel Grade

It can be seen from Figure 7.13 that the reduction in the number of bars has mostly been compensated by the increase in steel grade from 400 MPa to 500 MPa. In this case, the area of steel has been decreased by 16.7% but the steel strength has also been increased by 20%. The distribution of steel in the top and bottom faces has been altered and this did have a slight effect on the minor axis strength around the "nose" of the interaction diagram where the strength for 10N36 bars was just slightly less than the strength for the 12Y36 bars.

This redesign using 10N36 bars could be considered to be adequate for minor axis bending if the design action effects *M** and *N** on the cross-section (which should include the second-order effects) resulting from the application of the design loads were within the envelope of the interaction diagram for the original design using 12Y36 bars.

7.4 Case 3 – 450 mm Square Column with 8Y32 bars

7.4.1 Change in Steel Grade

The bar size and number of bars are kept the same and the steel is changed from 400 grade to 500 grade.

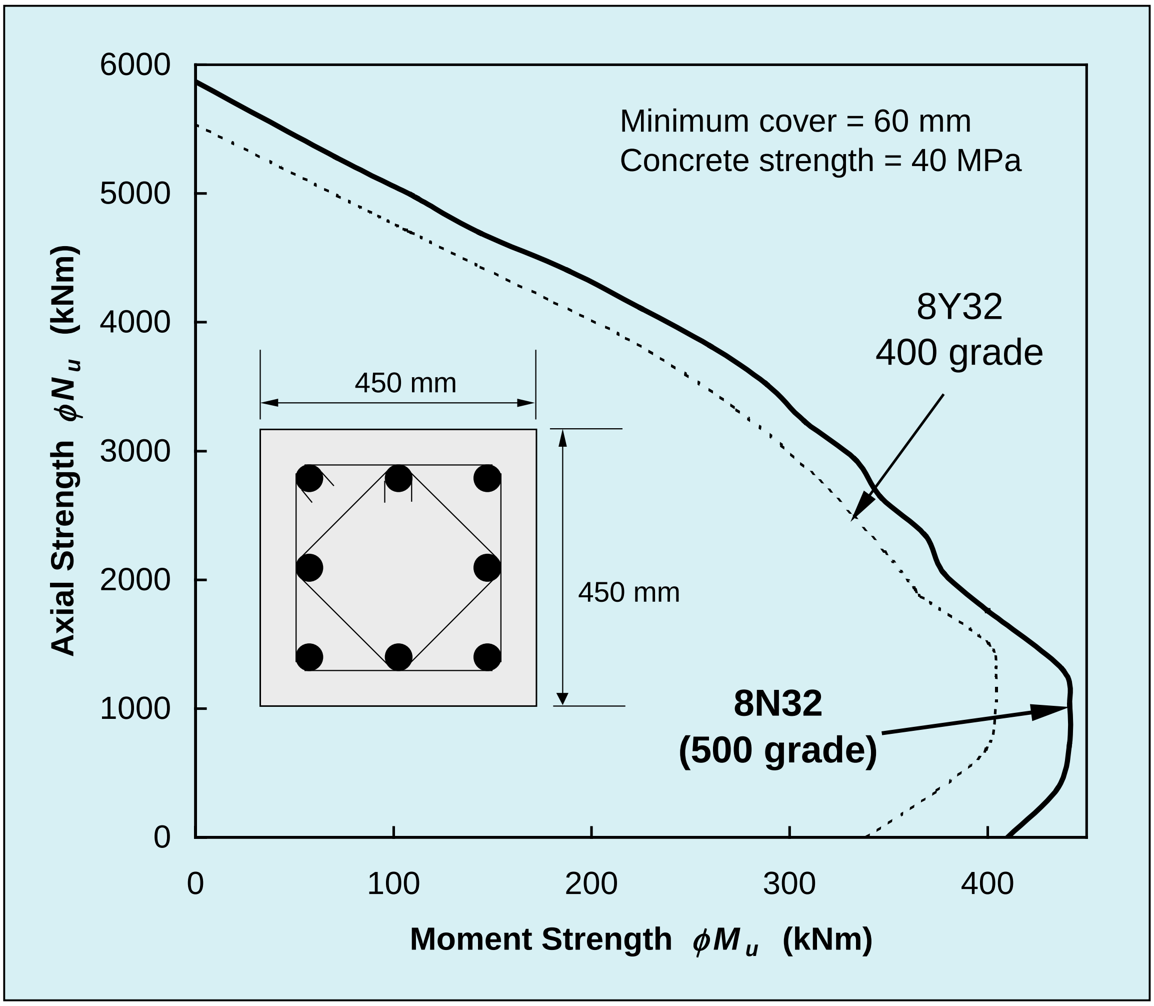

Fig. 7.14 Effect of Changing Steel Grade

It can be seen from Figure 7.14 that changing the steel grade from 400 MPa to 500 MPa increases the strength over the whole range of load-moment combinations from pure bending (20.9% increase) to pure axial force (6.0% increase). With a concrete strength of 40MPa, changing the steel grade did have a noticeable effect on the axial load carrying capacity. However, there was a more significant effect on the bending strength, particularly for low values of axial force.

7.4.2 Change in Bar Size and Steel Grade

The Y32 bars have been changed to N28 bars but the number of bars and the concrete cover have been kept the same.

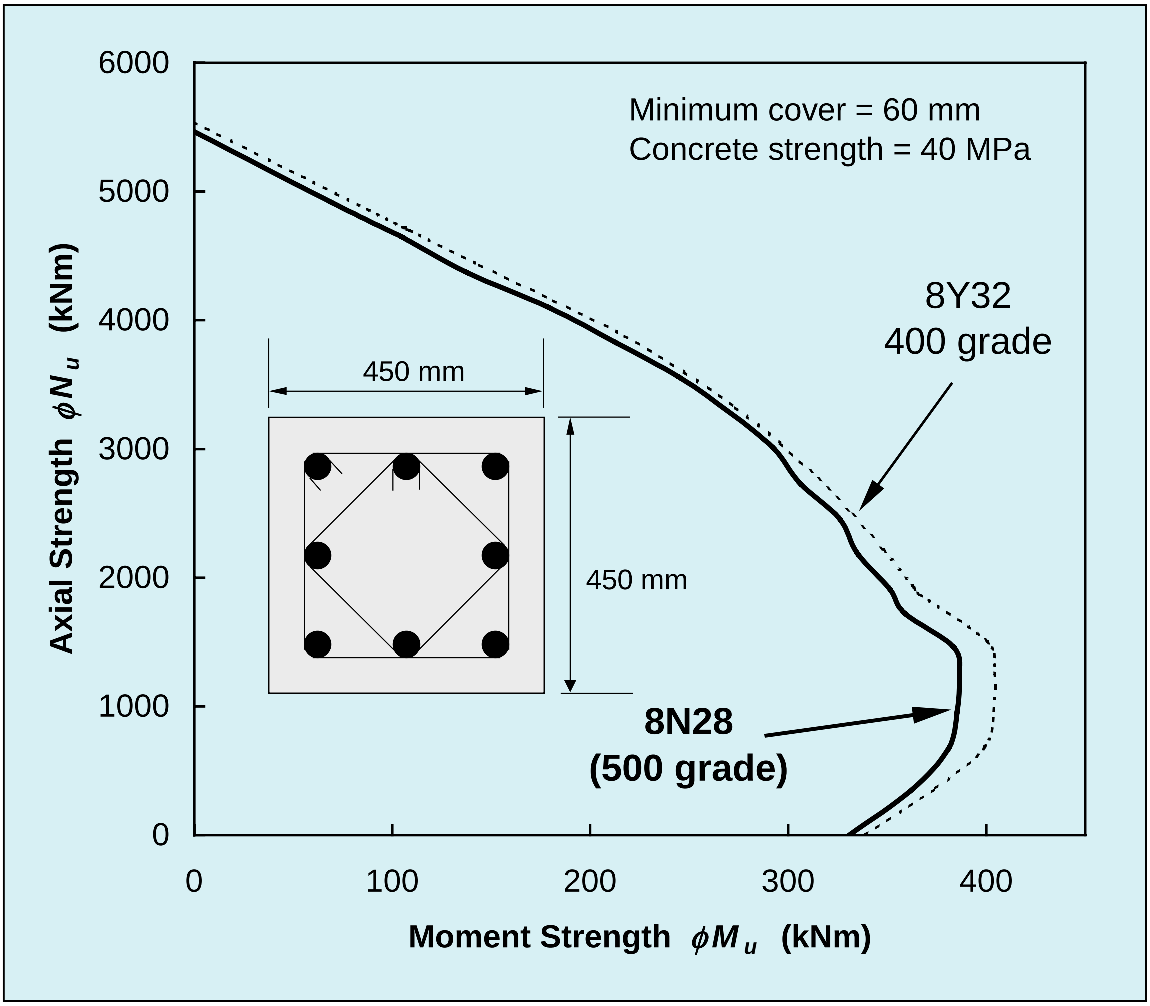

Fig. 7.15 Major Axis Bending - Effect of Changing the Bar Size and Steel Grade

It can be seen from Figure 7.15 that the reduction in one bar size from 32 mm diameter to 28 mm diameter has not been completely compensated by the increase in steel grade from 400 MPa to 500 MPa. This is to be expected as the area of steel has been decreased by 22.5% whereas the steel strength has been increased by 20%.

This redesign using 8N28 bars may still be adequate if the design action effects M^* and N^* on the cross-section (which should include the second-order effects) resulting from the application of the design loads still lay within the envelope of the 8N28 interaction diagram *and* the original design using 8Y32 bars was slightly conservative.

7.4.3 Mixed Bar Sizes and Change in Steel Grade

The eight Y32 bars have been changed to four N32 bars and four N28 bars but the concrete cover has been kept the same.

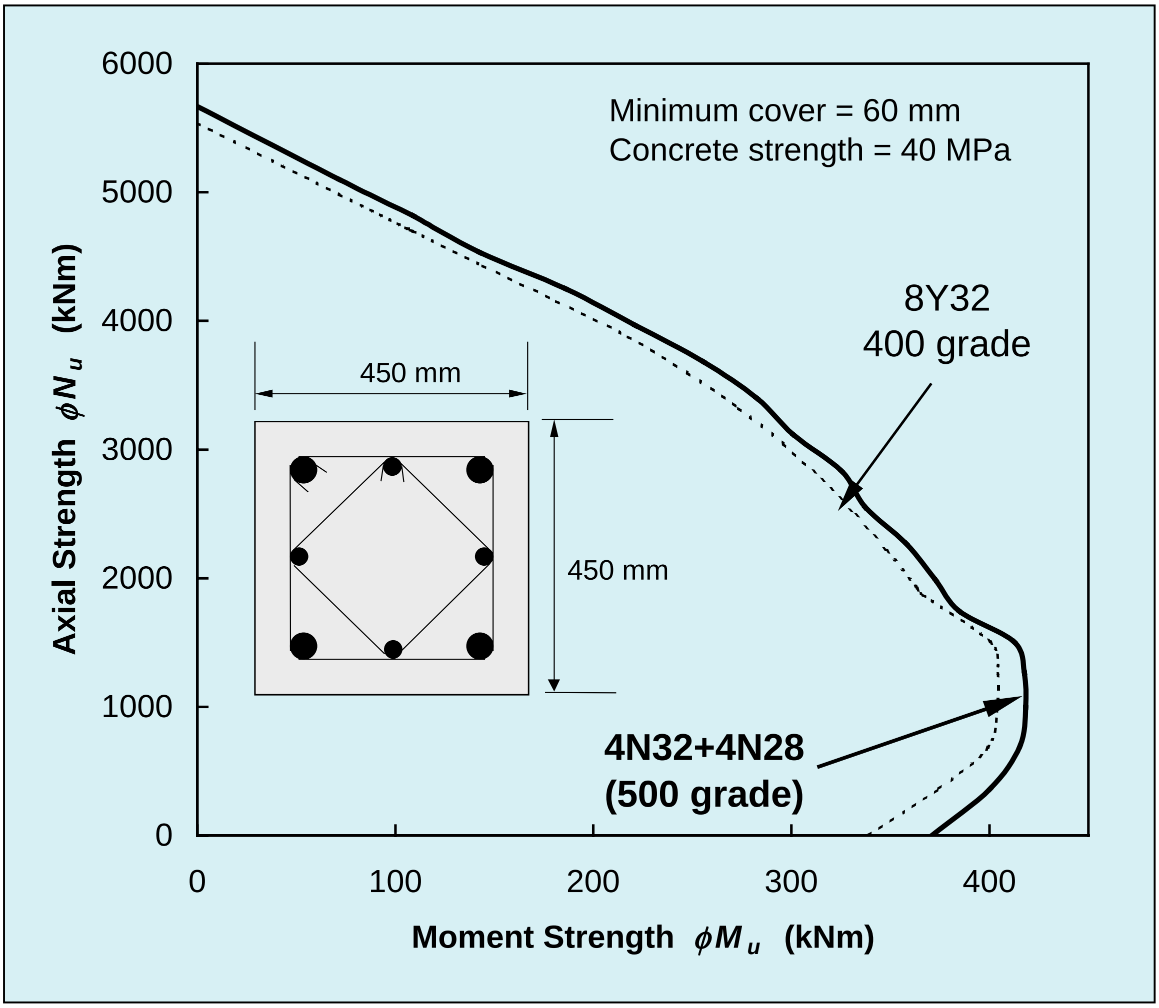

Fig. 7.16 Effect of Mixed Bar Sizes and Change in Steel Grade

It can be seen from Figure 7.16 that the use of mixed bar sizes has been more than compensated by the increase in steel grade from 400 MPa to 500 MPa. In this case, the area of steel has been decreased by only 11.2% yet the steel strength has been increased by 20%.

This redesign using 4N32 and 4N28 bars could be considered to be more than adequate if the design action effects M^* and N^* on the cross-section (which should include the second-order effects) resulting from the application of the design loads were within the envelope of the interaction diagram for the original design using 8Y32 bars.

7.4.4 Change in Section Size and Steel Grade

The eight Y28 bars have been changed to eight N28 bars (i.e. same size and number but different steel grade) but the overall dimensions of the concrete section have been reduced to compensate for the increase in steel strength. The concrete cover has been kept the same.

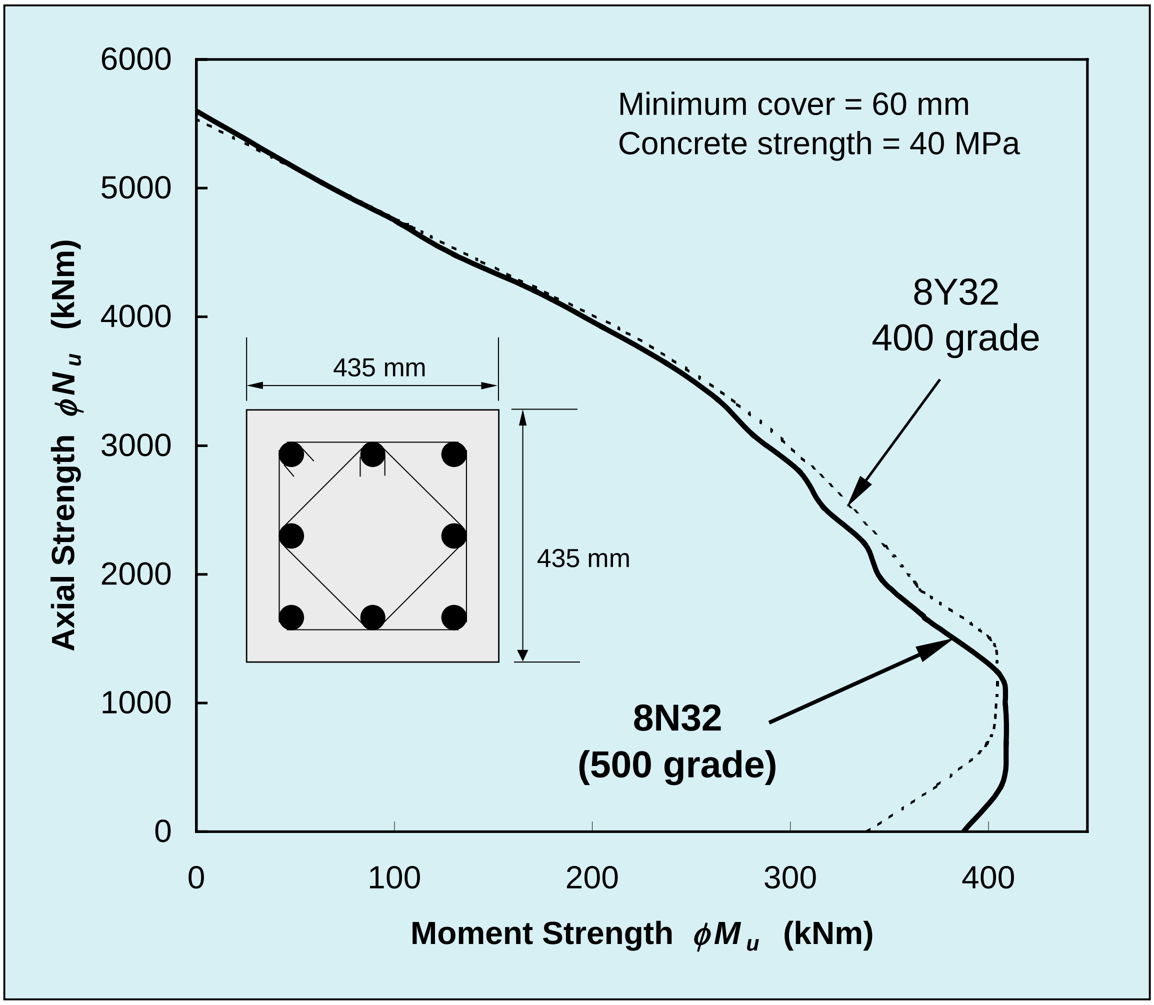

Fig. 7.17 Effect of Changing the Section Size and Steel Grade

In Figure 7.17, the concrete section with 8N32 bars has been reduced to a 435 mm square section such that the axial strength is the same as the axial strength of the original section 450 mm square section using 8Y32 bars. It can be seen that this results in an increase in moment strength in the region of low values of axial force but with some slight reduction in strength for intermediate values of axial force. It is interesting to note how the shape of the interaction diagram can be altered by changing the dimensions and the steel grade.

This redesign using 8N32 bars and a reduced section could be considered, in general, to be adequate if the design action effects M^* and N^* on the cross-section (which should include the second-order effects) resulting from the application of the design loads were within the envelope of the interaction diagram for the original design using 8Y32 bars.

7.5 Case 4 – 450 mm Circular Column with 8Y28 bars

7.5.1 Change in Steel Grade

The bar size and number of bars are kept the same and the steel is changed from 400 grade to 500 grade.

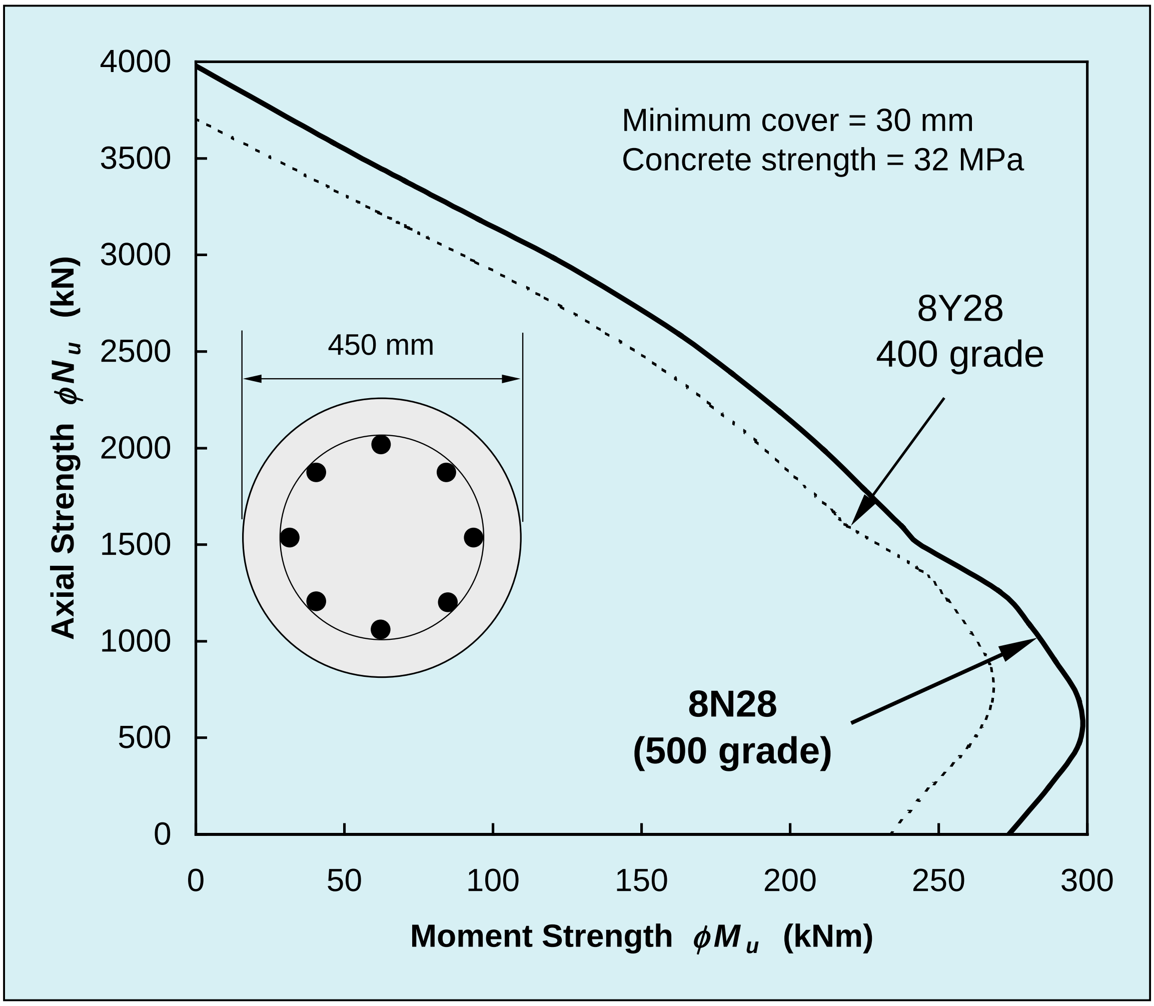

Fig. 7.18 Effect of Changing Steel Grade

It can be seen from Figure 7.18 that changing the steel grade from 400 MPa to 500 MPa increases the strength over the whole range of load-moment combinations from pure bending (17.6% increase) to pure axial force (7.4% increase). With a lower concrete strength of 32MPa, changing the steel grade can have a noticeable effect on the axial load carrying capacity. However, there was still a more significant effect on the bending strength, particularly for low values of axial force.

7.5.2 Change in Bar Size and Steel Grade

The Y28 bars have been changed to N24 bars but the number of bars and the concrete cover have been kept the same.

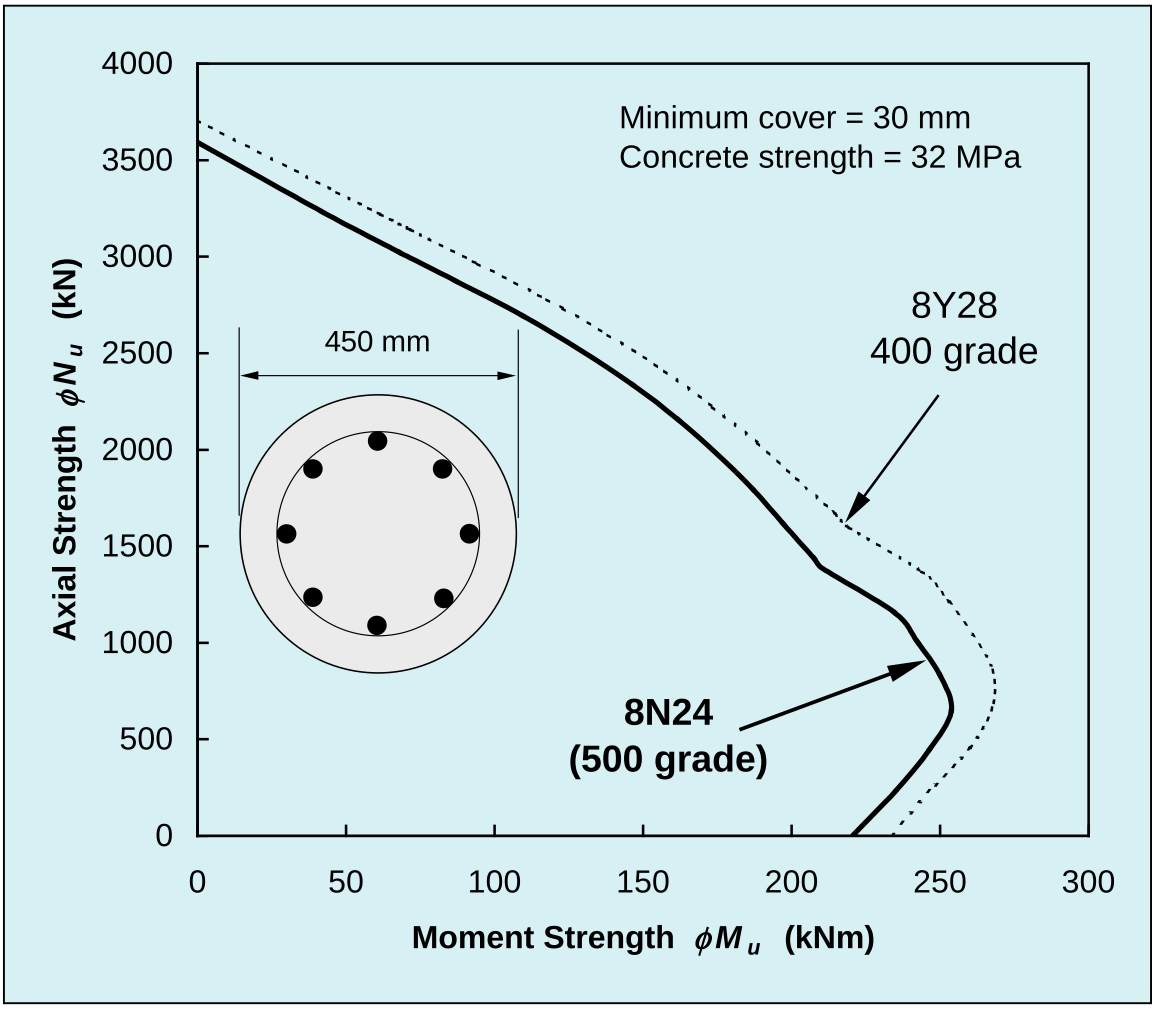

Fig. 7.19 Major Axis Bending - Effect of Changing the Bar Size and Steel Grade

It can be seen from Figure 7.19 that the reduction in one bar size from 28 mm diameter to 24 mm diameter has not been compensated by the increase in steel grade from 400 MPa to 500 MPa. This is to be expected as the area of steel has been decreased by 27.4% whereas the steel strength has only been increased by 20%.

This redesign using 8N24 bars may still be adequate if the design action effects M^* and N^* on the cross-section (which should include the second-order effects) resulting from the application of the design loads still lay within the envelope of the 8N24 interaction diagram *and* the original design using 8Y28 bars was quite conservative.

7.5.3 Mixed Bar Sizes and Change in Steel Grade

The eight Y28 bars have been changed to four N28 bars and four N24 bars but the concrete cover has been kept the same.

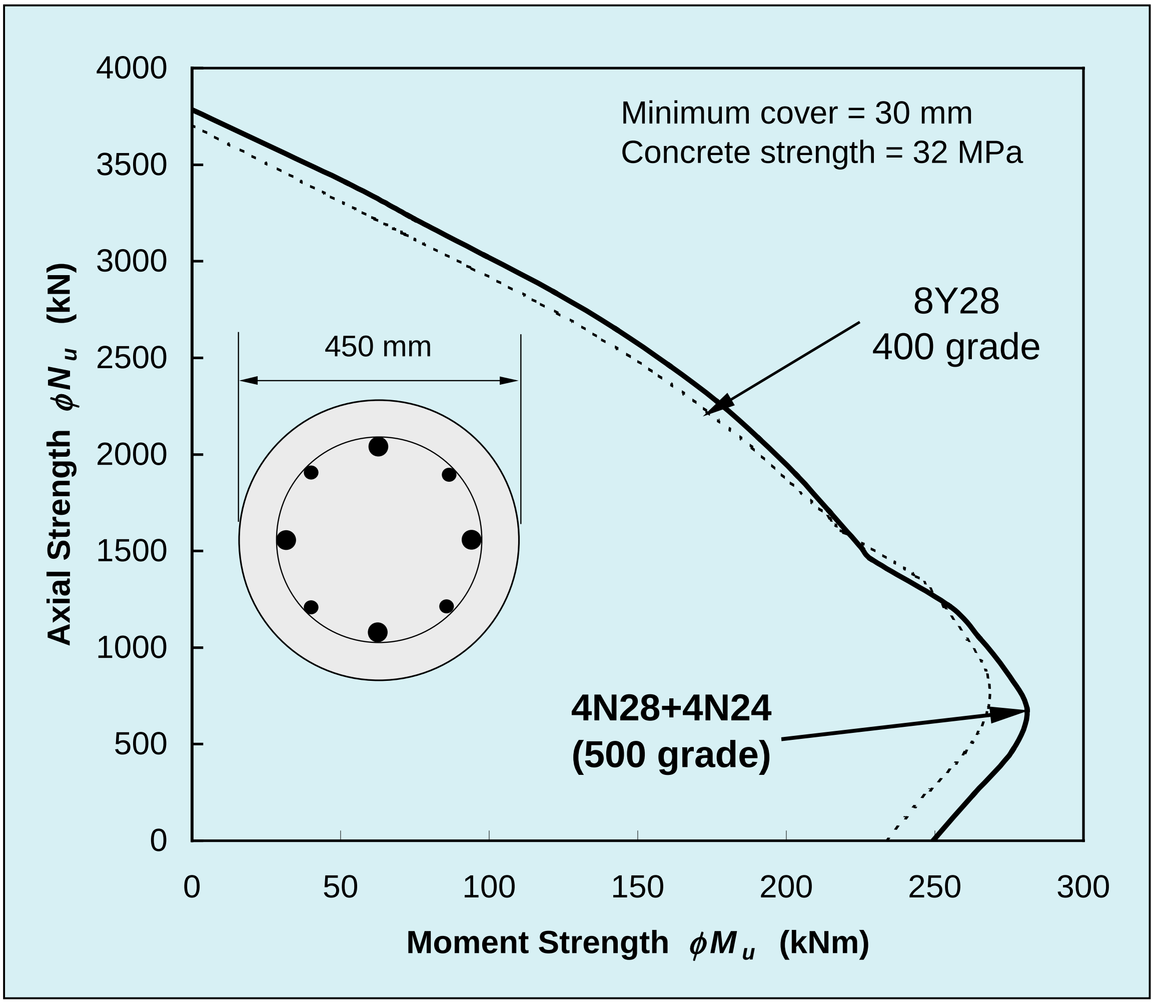

Fig. 7.20 Effect of Mixed Bar Sizes and Change in Steel Grade

It can be seen from Figure 7.20 that the use of mixed bar sizes has, in general, been more than compensated by the increase in steel grade from 400 MPa to 500 MPa. In this case, the area of steel has been decreased by only 13.7% yet the steel strength has been increased by 20%.

This redesign using 4N28 and 4N24 bars could be considered to be more than adequate if the design action effects *M** and *N** on the cross-section (which should include the second-order effects) resulting from the application of the design loads were within the envelope of the interaction diagram for the original design using 8Y28 bars.

7.5.4 Change in Section Size and Steel Grade

The eight Y28 bars have been changed to eight N28 bars (i.e. same size and number but different steel grade) but the overall dimensions of the concrete section have been reduced to compensate for the increase in steel strength. The concrete cover has been kept the same.

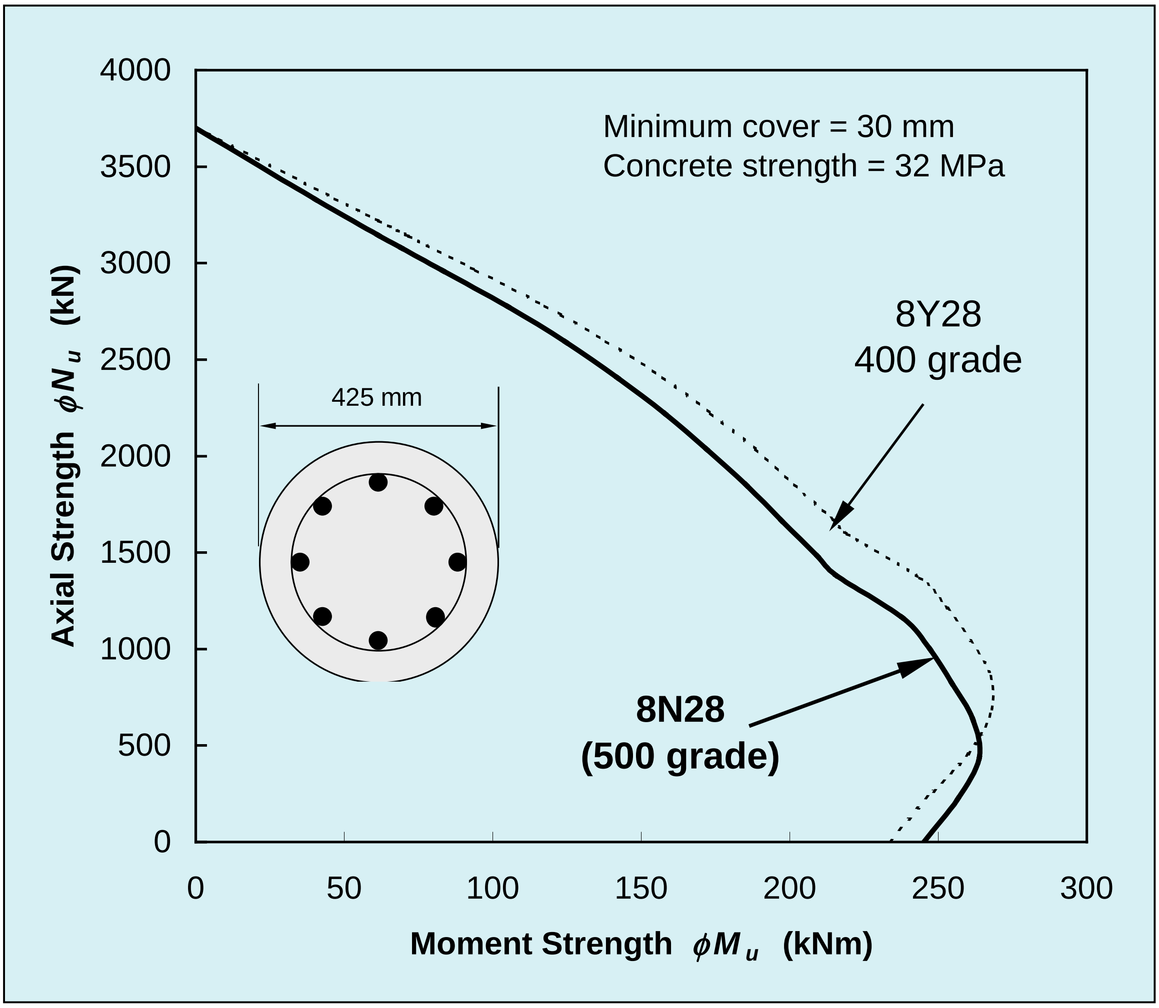

Fig. 7.21 Effect of Changing the Section Size and Change in Steel Grade

In Figure 7.21, the concrete section with 8N28 bars has been reduced to a 425 mm circular section such that the axial strength is the same as the axial strength of the original section 450 mm square section using 8Y28 bars. It can be seen that while this results in an increase in moment strength in the region of very low values of axial force, there is a significant reduction in strength for intermediate values of axial force.

This redesign using 8N24 bars may still be adequate if the design action effects M^* and N^* on the cross-section (which should include the second-order effects) resulting from the application of the design loads still lay within the envelope of the 8N24 interaction diagram *and* the original design using 8Y28 bars was quite conservative, particularly in the region of intermediate values of axial force.

7.5.5 Change in Section Size, Bar Size and Steel Grade

The eight Y28 bars have been changed to eight N24 bars (i.e. same size and number but different steel grade) and the concrete cover has been kept the same. This option was considered in Section 7.5.2 above where it can be seen from Figure 7.19 that the section with 8N24 bars had less strength than the section with 8Y28 bars. To compensate for this reduction in strength, the concrete section was increased in size.

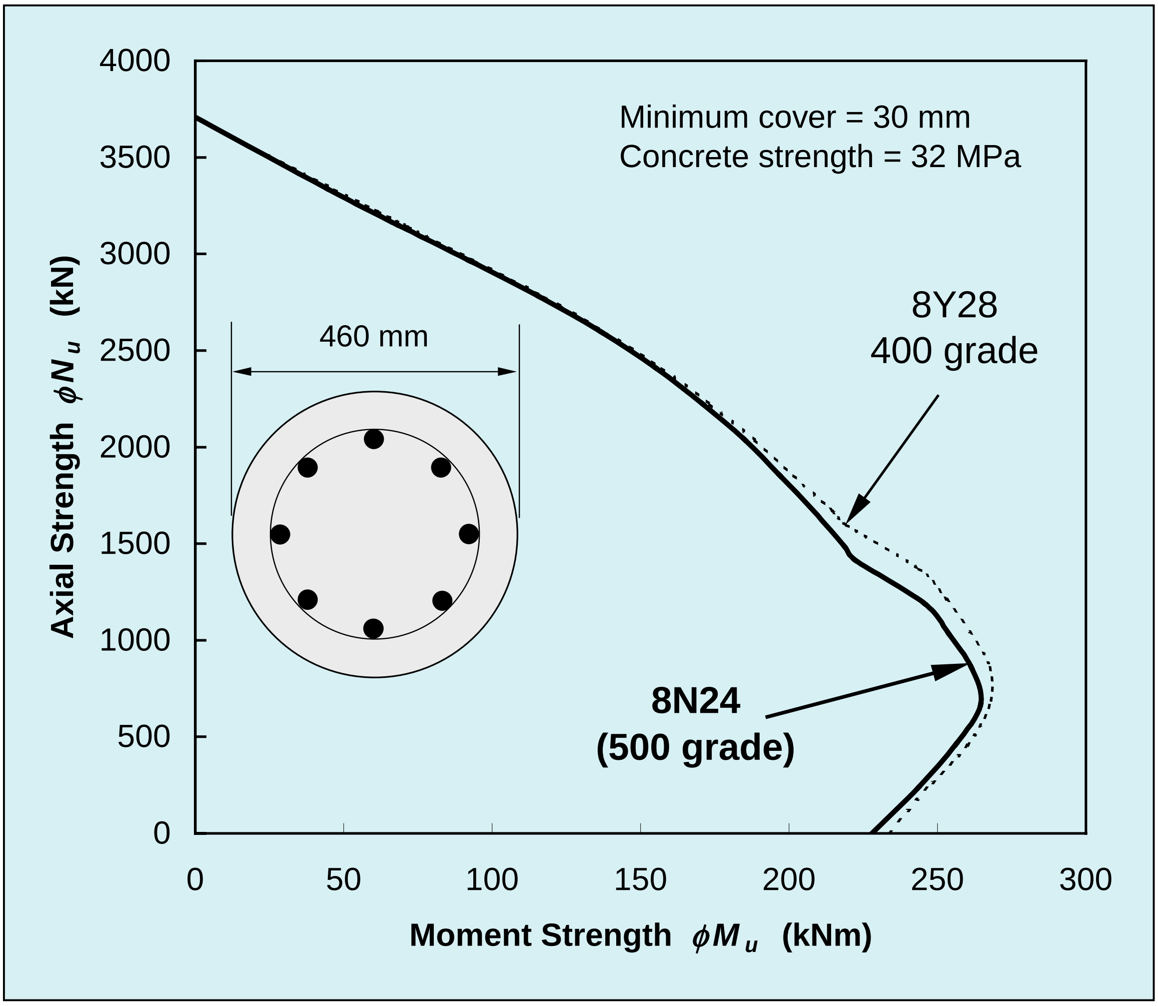

Fig. 7.22 Effect of Changing the Section Size, Bar Size and Steel Grade

In Figure 7.22, the concrete section with 8N24 bars has been increased to a 460 mm circular section such that the axial strength is the same as the axial strength of the original section 450 mm circular section using 8Y28 bars. It can be seen that this results in similar interaction diagrams with only a slight reduction in strength for low to intermediate values of axial force.

This redesign using 8N24 bars and a slightly increased section size (only 10 mm increase in diameter) could therefore be considered, in general, to be adequate if the design action effects *M** and *N** on the cross-section (which should include the second-order effects) resulting from the application of the design loads were within the envelope of the interaction diagram for the original design using 8Y28 bars.

7.6 Summary

In changing from 400 grade to 500 grade reinforcement (20% increase in steel strength), the design approach could follow a number of steps, depending on the outcome of the previous step.

Step 1. Keep the number of bars the same and reduce the diameter of the bars by one bar size. If this satisfies the design strength requirements, then a satisfactory design is obtained (see Section 7.3.5 Figure 7.12). Note: For bar diameters of 36 mm or less, the cross-sectional area reduce by more than 20% for each decrease in bar size (21.6% for changing from 36 mm to 32mm bar, and 45.0% for changing from 16 mm to 12 mm bar) and a reduction in bar size may be unconservative (see Section 7.5.2, Figure 7.19).

Step 2. Keep the bar size the same and reduce the number of bars by approximately 20%. If this satisfies the design strength requirements, then a satisfactory design is obtained (see Sections 7.2.3 and 7.3.3, Figures 7.4 and 7.10). Note: for columns with small number of bars and where symmetry is to be achieved, this may not be possible.

Step 3. Consider using two sizes of bar in the one cross-section such that the reduction in area is approximately 20%. If this satisfies the design strength requirements, then a satisfactory design is obtained (see Sections 7.4.3 and 7.5.3, Figures 7.16 and 7.20). This may not be considered practical for some forms of construction.

Step 4. Keep the number of bars and the size of the bars the same and reduce the concrete cross-sectional area to compensate for the increase in steel strength. If this satisfies the design strength requirements, then a satisfactory design is obtained (see Sections 7.4.4 and 7.5.4, Figures 7.17 and 7.21). This can change the shape of the load-moment interaction diagram. Again, this may not be considered practical where standard column sizes are required for particular forms of construction.

An important point to consider is the degree of conservatism used in the original design i.e. if the design action effects M^* and N^* on the cross-section (which should include the second-order effects) resulting from the application of the design loads lay well within the envelope of the load-moment interaction diagram for the original design, then the scope for making savings using OneSteel 500PLUS Rebar by reducing bar sizes and/or number of bars is enhanced.

8. REFERENCES

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APPENDIX A

REFERENCED AUSTRALIAN STANDARDS

REFERENCE NO.	TITLE
AS 3600-1994	Concrete Structures (including Amendment No. 1)
AS 4100-1999	Steel Structures
AS 1302-1991	Steel Reinforcing Bars for Concrete

APPENDIX B

NOTATION

Much of the notation used in this booklet has been taken AS 3600-1994

Latin letters

A_b	design area for a steel bar used as reinforcement
A_c	total area of concrete in a cross-section
A_g	gross cross-sectional area (= bD for rectangular sections, or $\pi D^2/4$ for circular sections)
A_i	area of steel in layer i of the reinforcement
A_s	total area of steel reinforcement in a cross-section
b	width of a rectangular cross-section
C	total force in the steel and concrete in the zone which is in compression
C_c	compressive force in the concrete
C_i	compressive force in layer i of the steel reinforcement
C_R	reduced total force in the steel and concrete in the zone which is in compression, taking k_u = 0.4 for cross-sections where k_u > 0.4 in pure bending
d	effective depth of a cross-section, taken as the distance from the extreme compressive fibre of the concrete to the resultant tensile force in the reinforcing steel in the zone that is in tension at the ultimate strength condition (M_{uo}) in pure bending
d_b	diameter of steel bar reinforcement
d_o	distance from the extreme compressive fibre of the concrete to the centroid of the outermost layer of tensile reinforcement
D	depth of a rectangular cross-section, or diameter of circular section
e	eccentricity of loading at a cross-section = M^*/N^*
EI	secant stiffness of a cross-section at the "balanced" condition for ultimate strength
E_c	elastic modulus of the concrete
E_s	elastic modulus of the steel reinforcement , taken as 200000 MPa
f'_c	characteristic compressive strength of concrete
f_s	steel stress
f_{sy}	yield strength of reinforcing steel
F_i	force in layer i of the steel reinforcement
G	dead load
I	second moment of area
I_c	second moment of area of the concrete area A_c
I_s	second moment of area of the steel area A_s
k_m	coefficient to account for unequal values of the end moments M^*_1 and M^*_2
k_u	neutral axis parameter being the ratio, at ultimate strength of the cross-section, of the depth to the neutral axis from the extreme compressive fibre to the effective depth d
k_{uo}	ratio, at ultimate strength of the cross-section, of the depth to the neutral axis from the extreme compressive fibre to the distance d_o
	lever arm between C and T

m	number of layers of reinforcement in tension
M	bending moment
M^*	the design bending moment calculated using the design load for strength
M^*_x	design bending moment about the principal x-axis
M^*_y	design bending moment about the principal y-axis
M^*_1	smaller of the design bending moments at the ends of a column
M^*_2	larger of the design bending moments at the ends of a column
M_i	moment about the centroidal axis of the force F_i in layer i of the steel reinforcement
M_u	ultimate strength in bending at a cross-section of an eccentrically loaded member
M_{ub}	particular ultimate strength in bending for the "balanced" condition
M_{ud}	reduced ultimate strength in bending, without axial force, for cross-sections where $k_u > 0.4$ and can be calculated by taking $k_u = 0.4$ and reducing the tensile force to balance the reduced compressive force
M_{ul}	particular ultimate strength in bending for the "limit" condition
M_{ux}	ultimate strength in bending about the principal x-axis for a given value of axial force N^*
M_{uy}	ultimate strength in bending about the principal y-axis for a given value of axial force N^*
M_{uo}	ultimate strength in bending, without axial force, of a cross-section
n	total number of layers of steel reinforcement in a cross-section
N	axial force
N^*	the design axial force calculated using the design load for strength
N_c	design buckling load of a column
N_u	ultimate strength in compression at a cross-section of an eccentrically loaded member
N_{ub}	particular ultimate strength in compression for the "balanced" condition
N_{ul}	particular ultimate strength in compression for the "limit" condition
N_{uo}	ultimate strength in compression, without bending, of a cross-section
L	length of a column
L_e	effective length of a column
p	percentage of steel in gross cross-section
Q	live load
r	radius of gyration of a cross-section
R	radius of curvature
T	total force in the steel in the zone which is in tension
T_i	tension force in layer i of the steel reinforcement
T_R	reduced total force in the steel in the zone which is in tension where $T_R = C_R$
x_i	distance from the extreme compressive fibre in the concrete to layer i of the steel reinforcement

Greek letters

α_n	coefficient used in the determination of the ultimate strength in biaxial bending
β_d	creep factor used to determine a reduced elastic modulus for the concrete
δ	moment magnifier for slenderness effects
δ_b	moment magnifier for slenderness effects in a braced column
Δ	lateral deflection of a column
ε	strain
ε_a	strain at the centroidal axis of the cross-section (axial strain)
ε_{cu}	strain in the extreme compressive fibre of the concrete at the ultimate strength of the cross-section
ε_i	strain in layer *i* of the steel reinforcement
ε_o	strain at maximum strength of the concrete
ε_{sy}	strain at first yield of the steel = f_{sy}/E_s
ε_{uo}	strain at the ultimate strength in compression, without bending
ϕ	strength reduction factor
ϕ_o	reduced strength reduction factor for the case of pure bending where $k_u > 0.4$
ϕM_u	design ultimate strength in bending of a cross-section of an eccentrically loaded member
ϕM_{uo}	design ultimate strength in bending, without axial force, of a cross-section
ϕN_u	design ultimate strength in axial force of a cross-section of an eccentrically loaded member
ϕN_{uo}	design ultimate strength in compression, without bending, of a cross-section
γ	ratio of the depth of the assumed rectangular compressive stress block to the effective depth *d*
ρ	curvature = $1/R$ (slope of the linear strain distribution)
ρ_{ub}	curvature at the "balanced" condition for ultimate strength
σ_c	stress in the concrete
σ_i	stress in layer *i* of the steel reinforcement
σ_s	stress in the steel reinforcement

APPENDIX C

COLUMN DESIGN CHARTS

Combined Compression and Bending

The column design charts provided in this appendix give the design strength in compression and bending for three standard cross-sections as shown in Figure C1. A computer based analytical method (see Section 4.3) was used to generate the charts.

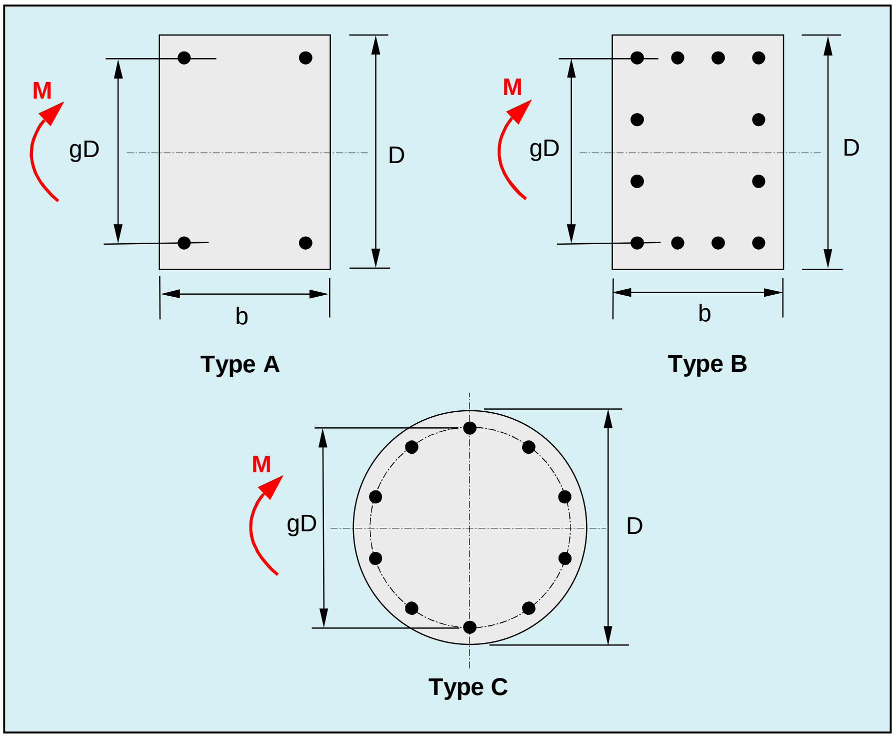


Figure C.1 Cross-section Layouts

Type A is a rectangular cross-section reinforced on two faces only and is covered in Charts RCB3.1-C1 to RCB3.1C16 inclusive for values of g = 0.6, 0.7, 0.8 and 0.9 and concrete strengths f'_c = 25, 32, 40 and 50 MPa.

Type B is a rectangular cross-section with equal reinforcement on all four faces with twelve or more bars and is is covered in Charts RCB3.1-C17 to RCB3.1-C32 inclusive for values of g = 0.6, 0.7, 0.8 and 0.9 and concrete strengths f'_c = 25, 32, 40 and 50 MPa.

Type C is a circular column reinforced symmetrically with a minimum of eight bars and is covered in Charts RCB3.1-C17 to RCB3.1-C32 inclusive for values of g = 0.6, 0.7, 0.8 and 0.9 and concrete strengths f'_c = 25, 32, 40 and 50 MPa.

To determine the locus of strength for the design charts, the stress distributions in the concrete were determined from the CEB [4] stress strain relationship, with a maximum stress of $0.85f'_c$. The stresses in the reinforcing steel were determined using a bi-linear stress-strain relationship utilising a modulus of elasticity E_s of 200,000 MPa and a yield stress f_{sy} of 500MPa.

The balance moment ϕM_{ub} and corresponding axial load ϕN_{ub} were determined when k_{uo} =0.545. For cross-sections where the value of k_u > 0.4 in pure bending, the value of M_{ud} is derived from the point at which the percentage of steel results in a k_u in pure bending being equal to 0.4. This value M_{ud} is needed to determine values of the capacity reduction factor ϕ.

Combined Tension and Bending

For a given design axial load ϕN_{ut} in tension, the corresponding design moment ϕM_u may be determined using the following linear equation

$$\frac{\phi\, M_u}{A_g D} = \left(\frac{\phi\, M_{uo}}{A_g D}\right)\left[1 - \frac{(\phi\, N_{ut}/A_g)}{(0.8)(500)\mathrm{p}}\right]$$

where for the selected value of percentage steel p in the design charts:

$\dfrac{\phi M_{uo}}{\mathrm{A}_g D}$ is the non-dimensional bending moment at zero axial load

A_g is the gross sectional area (= bD for rectangular or $\pi D^2/4$ for circular sections)

D is the depth of a rectangular section, or the diameter of a circular section.

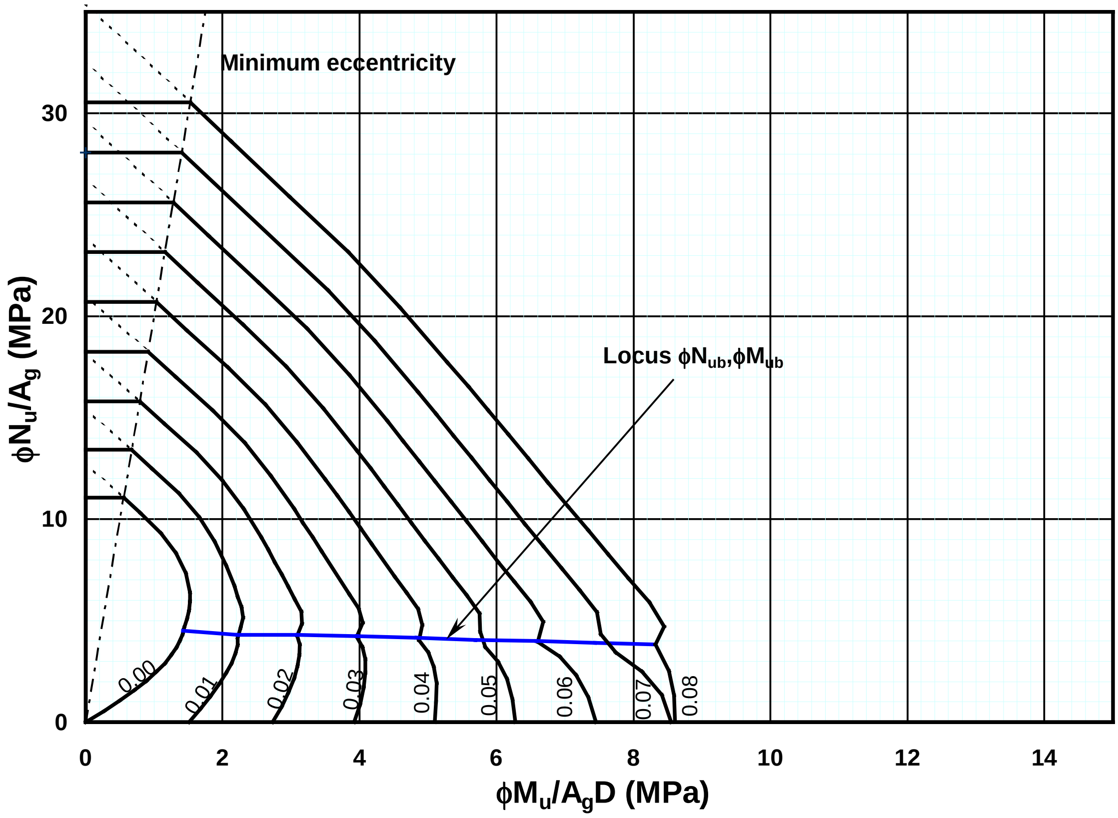


CHART RCB 3.1 - C1: Type A, f'_c = 25 MPa, g = 0.6

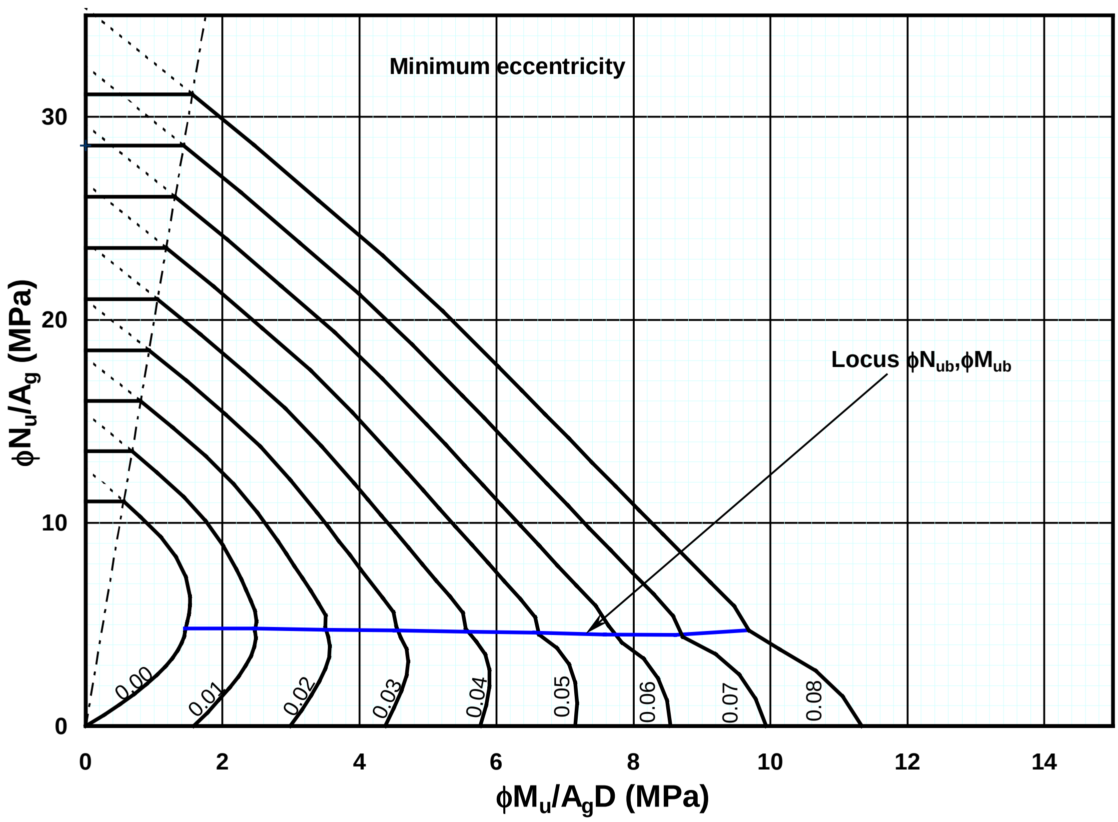


CHART RCB 3.1 - C2: Type A, f'_c = 25 MPa, g = 0.7

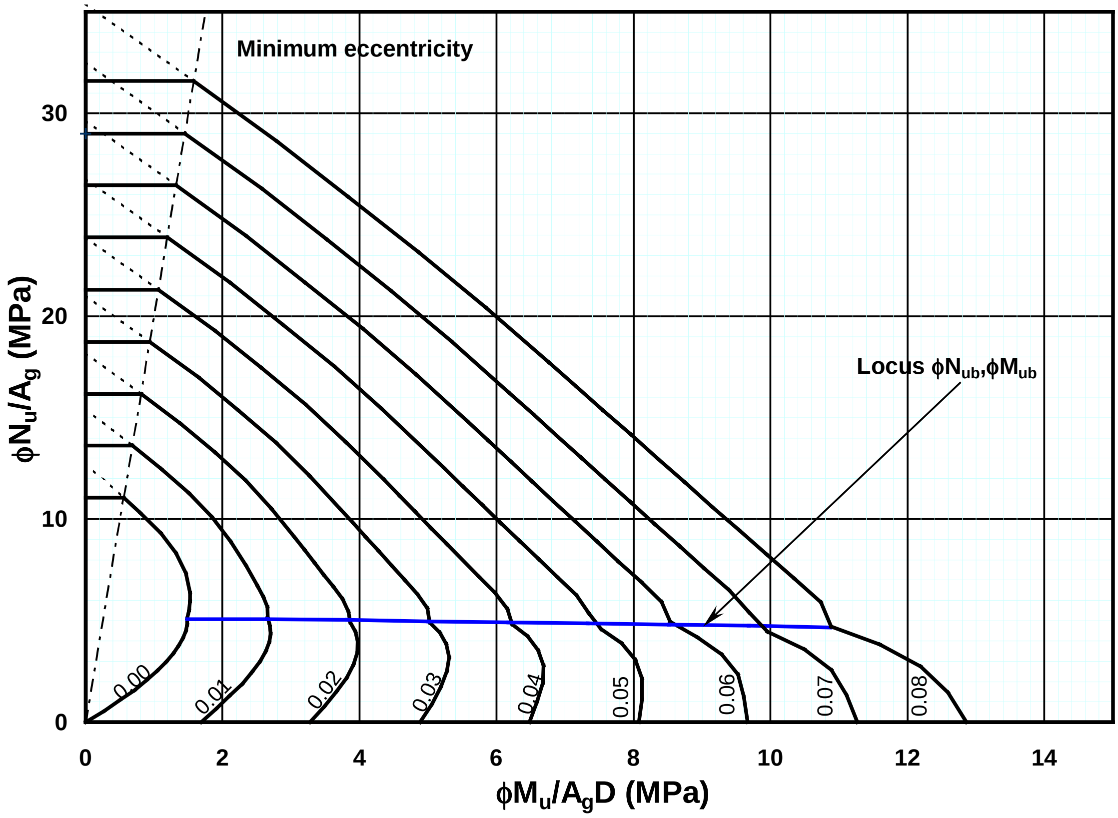


CHART RCB 3.1 - C3: Type A, f'_c = 25 MPa, g = 0.8

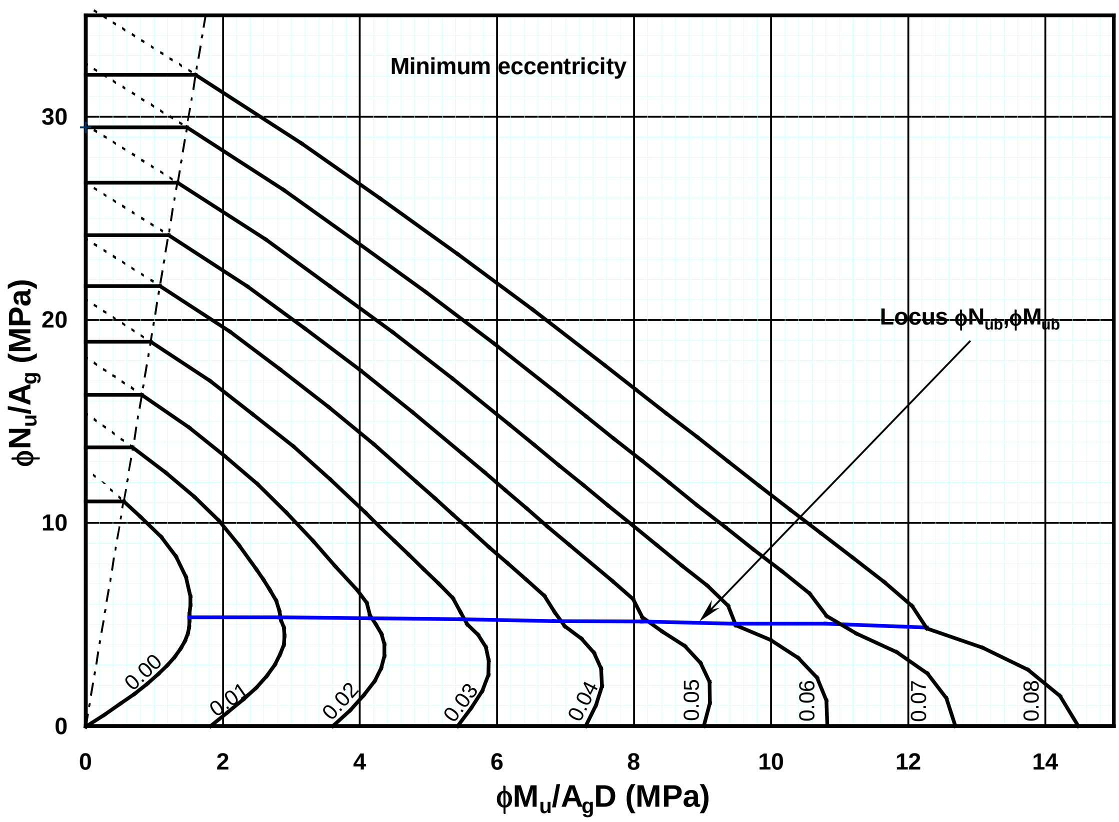


CHART RCB 3.1 - C4: Type A, f'_c = 25 MPa, g = 0.9

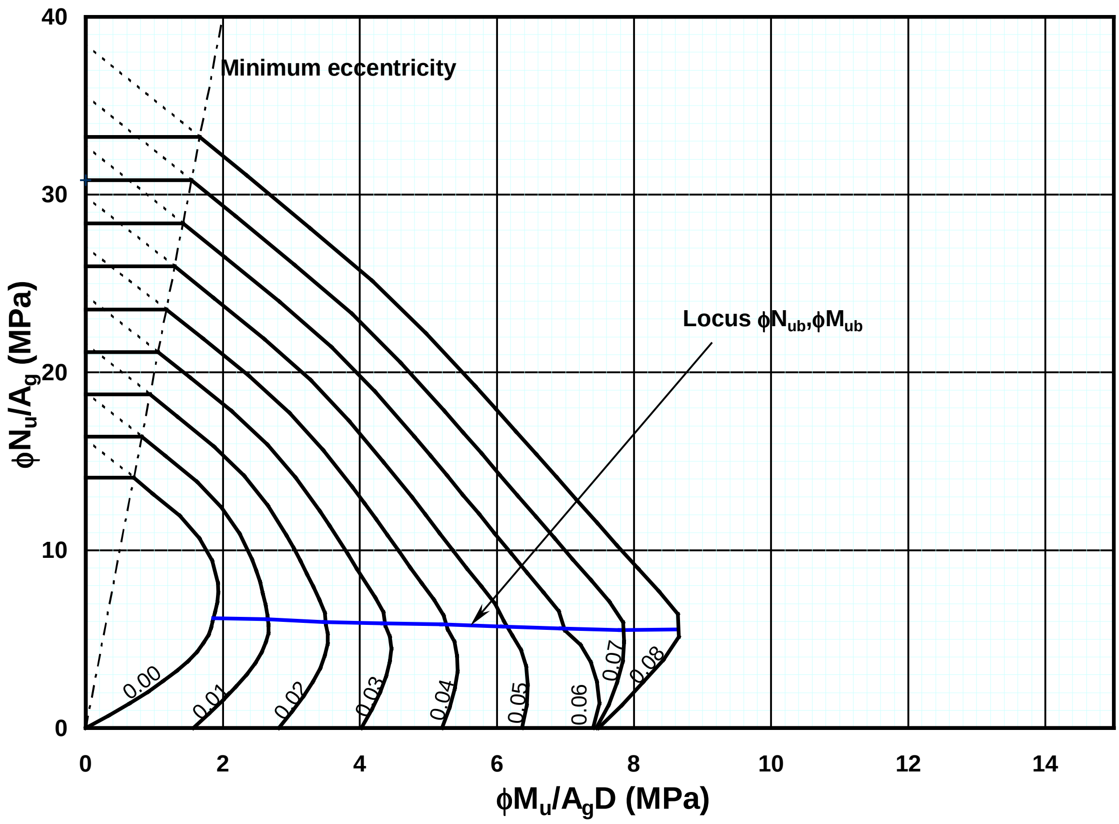


CHART RCB 3.1 - C5: Type A, f'_c = 32 MPa, g = 0.6

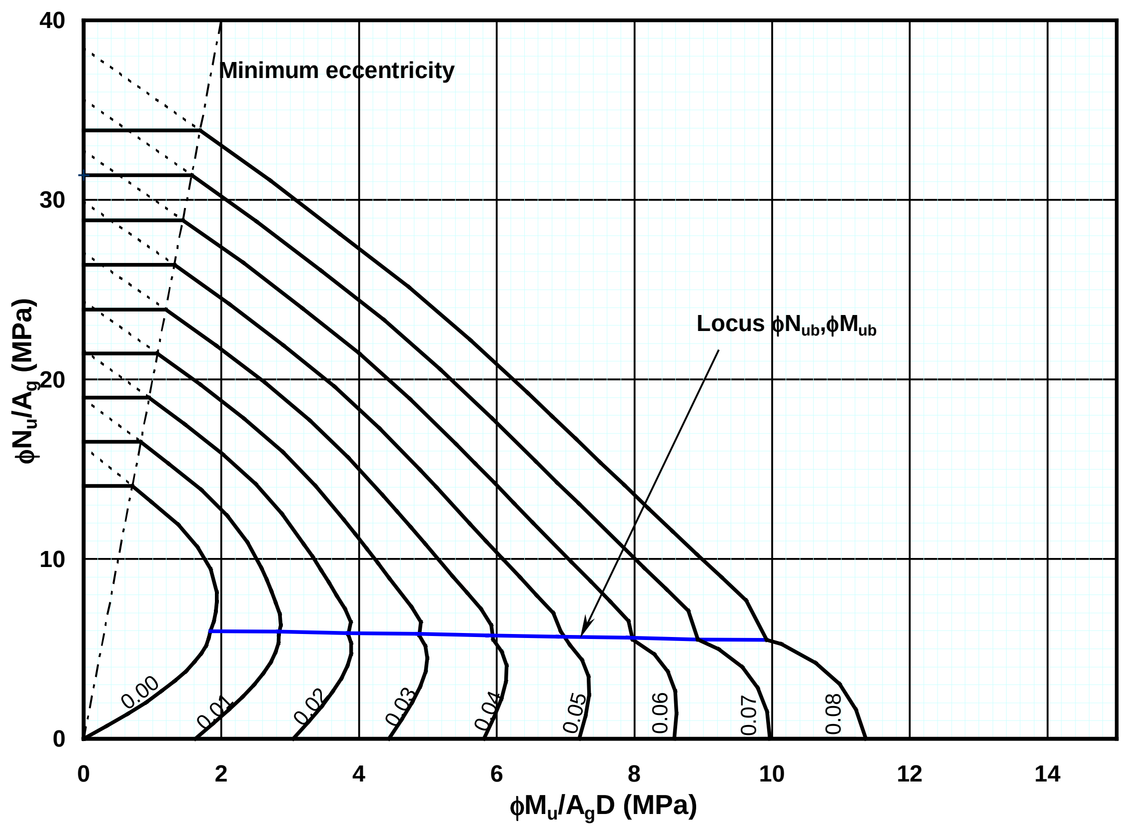


CHART RCB 3.1 - C6: Type A, f'_c = 32 MPa, g = 0.7

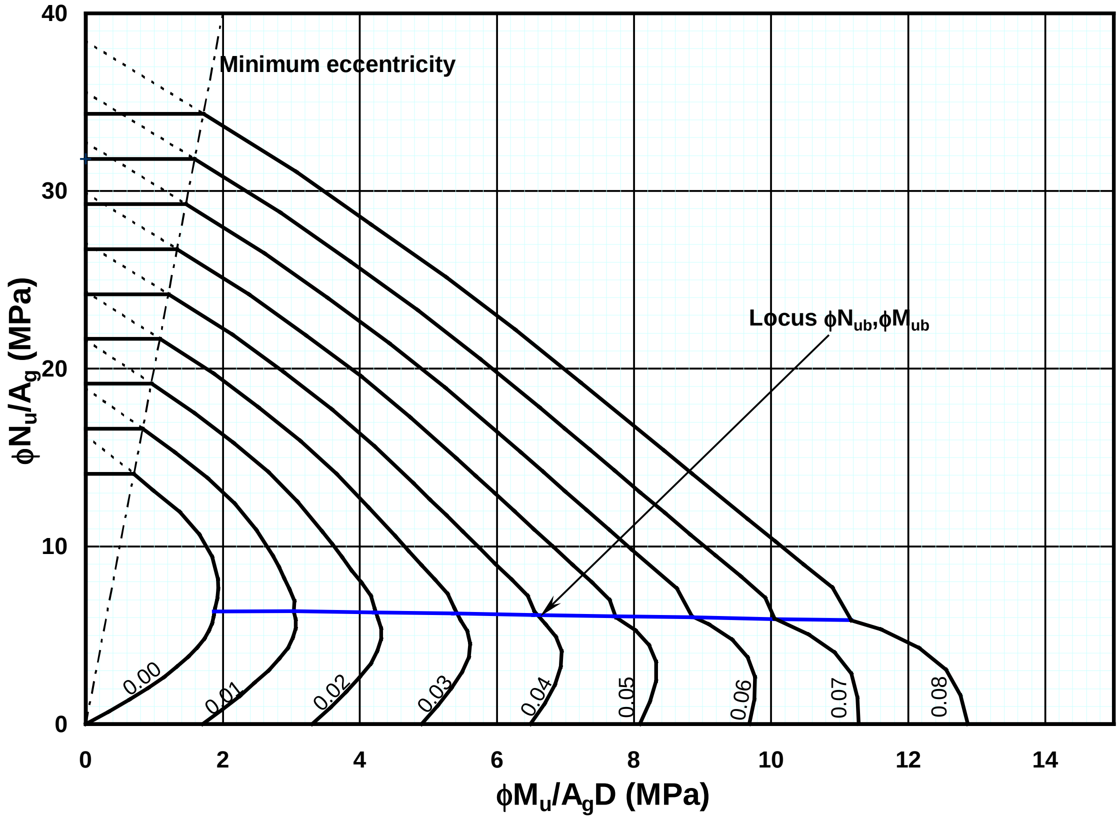


CHART RCB 3.1 - C7: Type A, f'_c = 32 MPa, *g* = 0.8

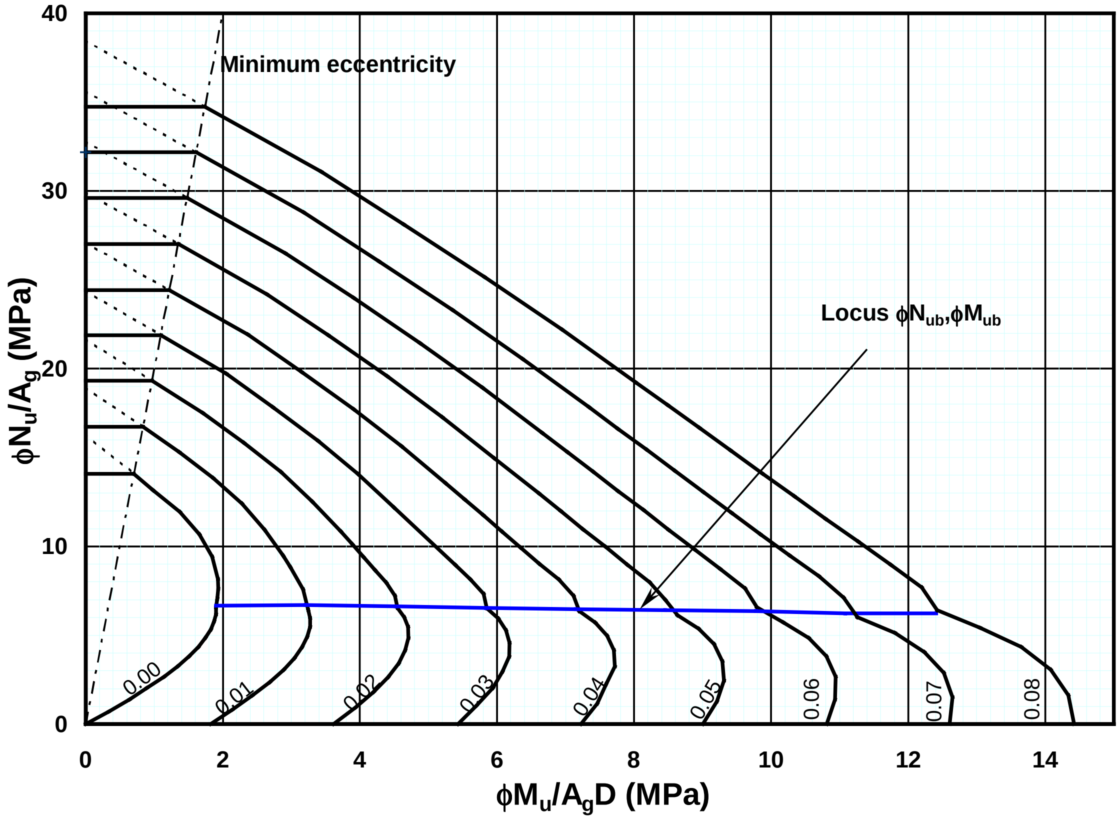


CHART RCB 3.1 - C8: Type A, f'_c = 32 MPa, *g* = 0.9

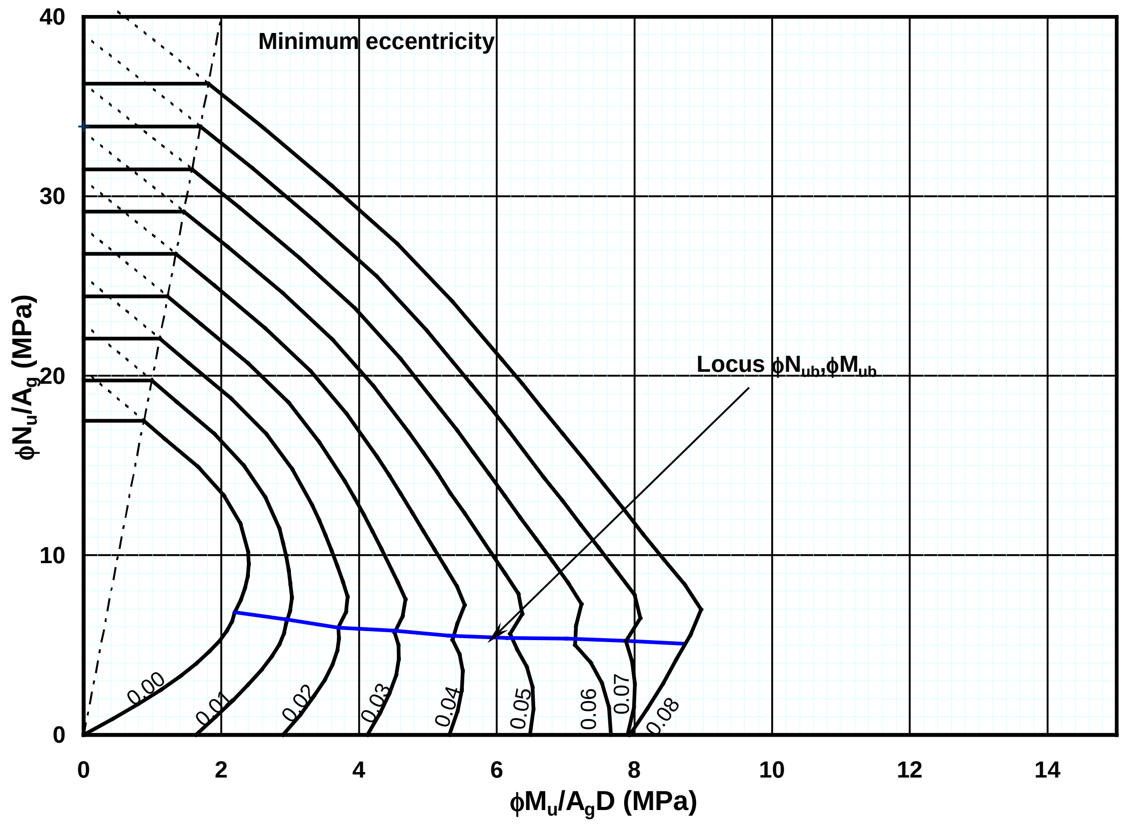


CHART RCB 3.1 - C9: Type A, f'_c = 40 MPa, g = 0.6

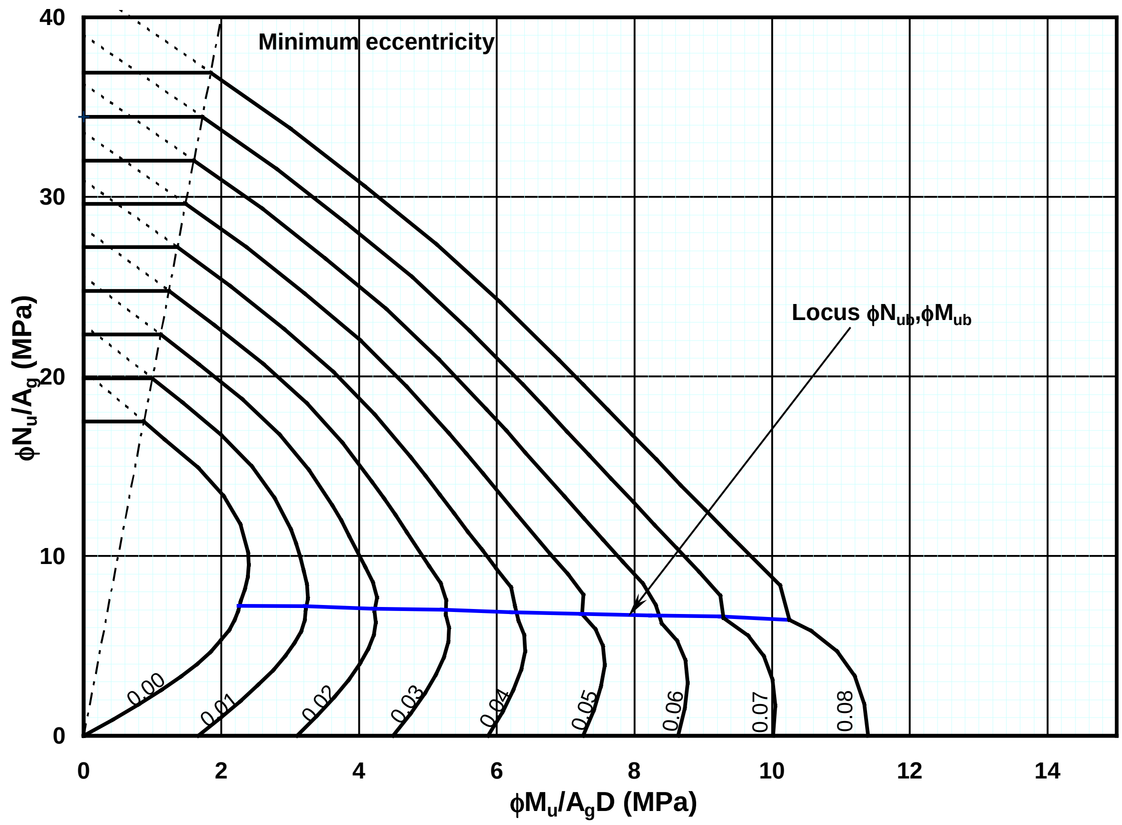


CHART RCB 3.1 - C10: Type A, f'_c = 40 MPa, g = 0.7

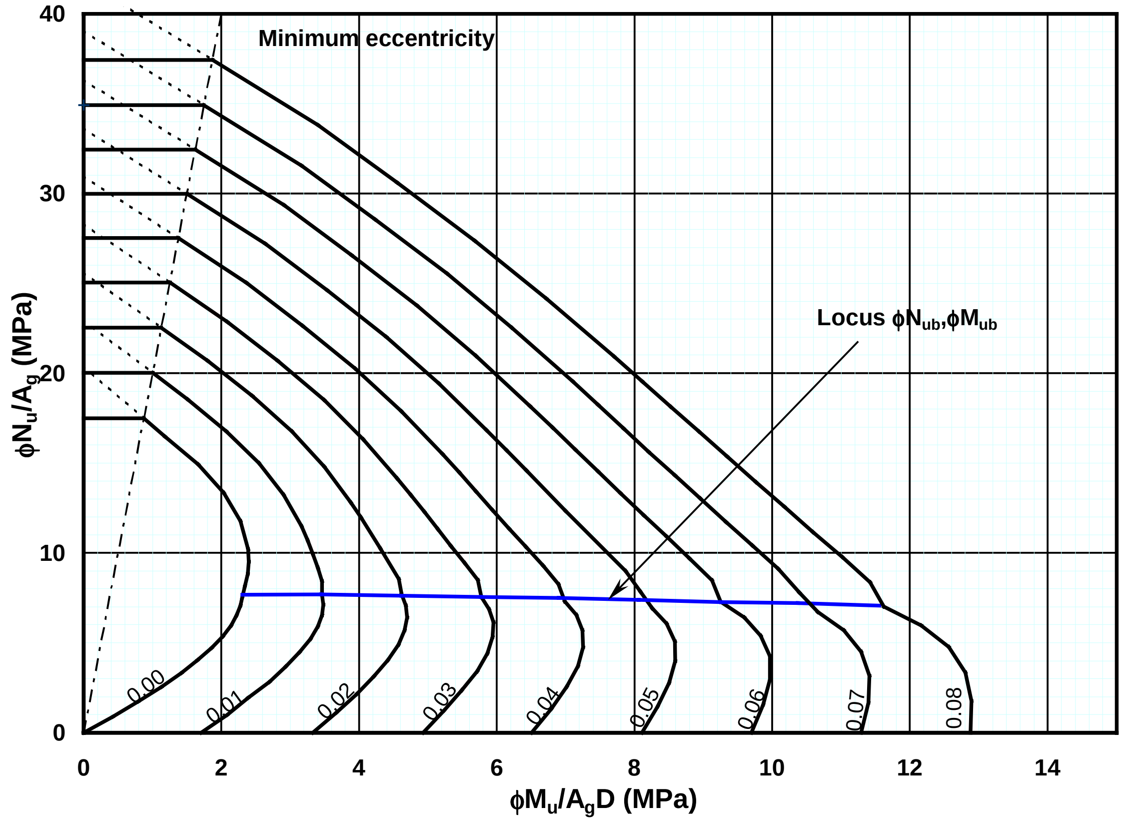


CHART RCB 3.1 - C11: Type A, f'_c = 40 MPa, g = 0.8

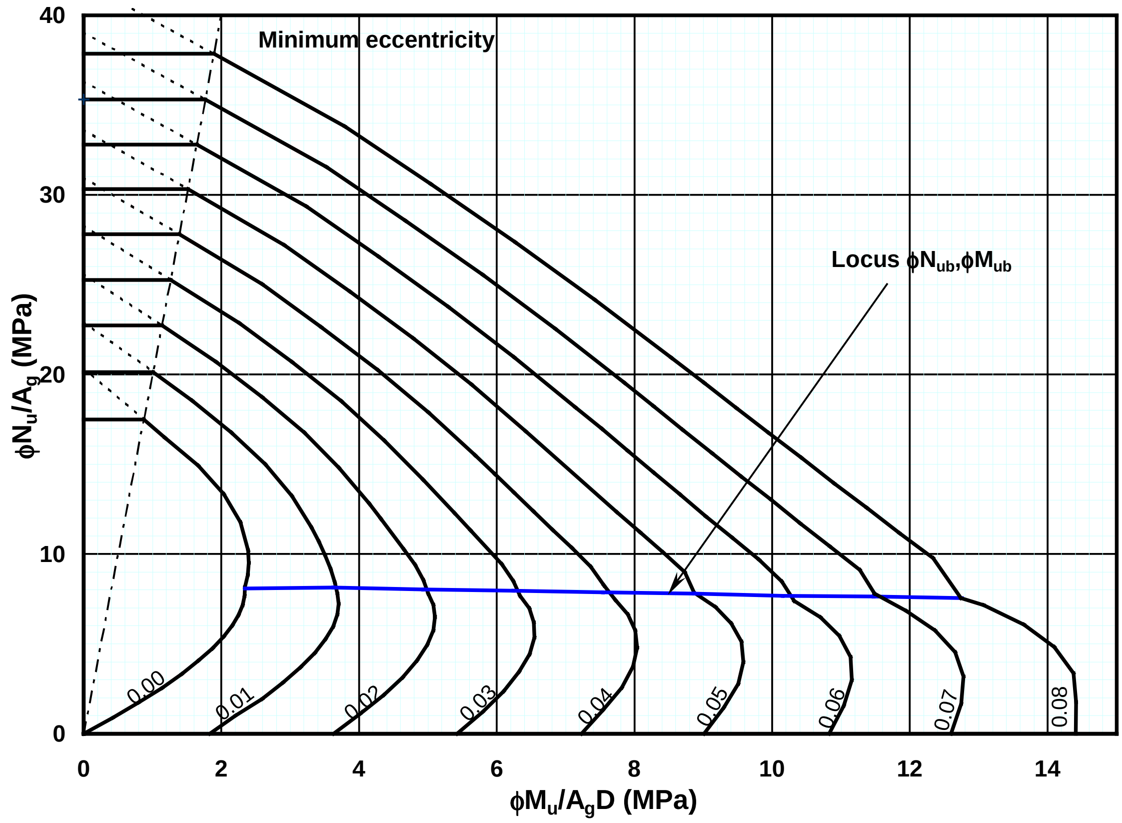


CHART RCB 3.1 - C12: Type A, f'_c = 40 MPa, g = 0.9

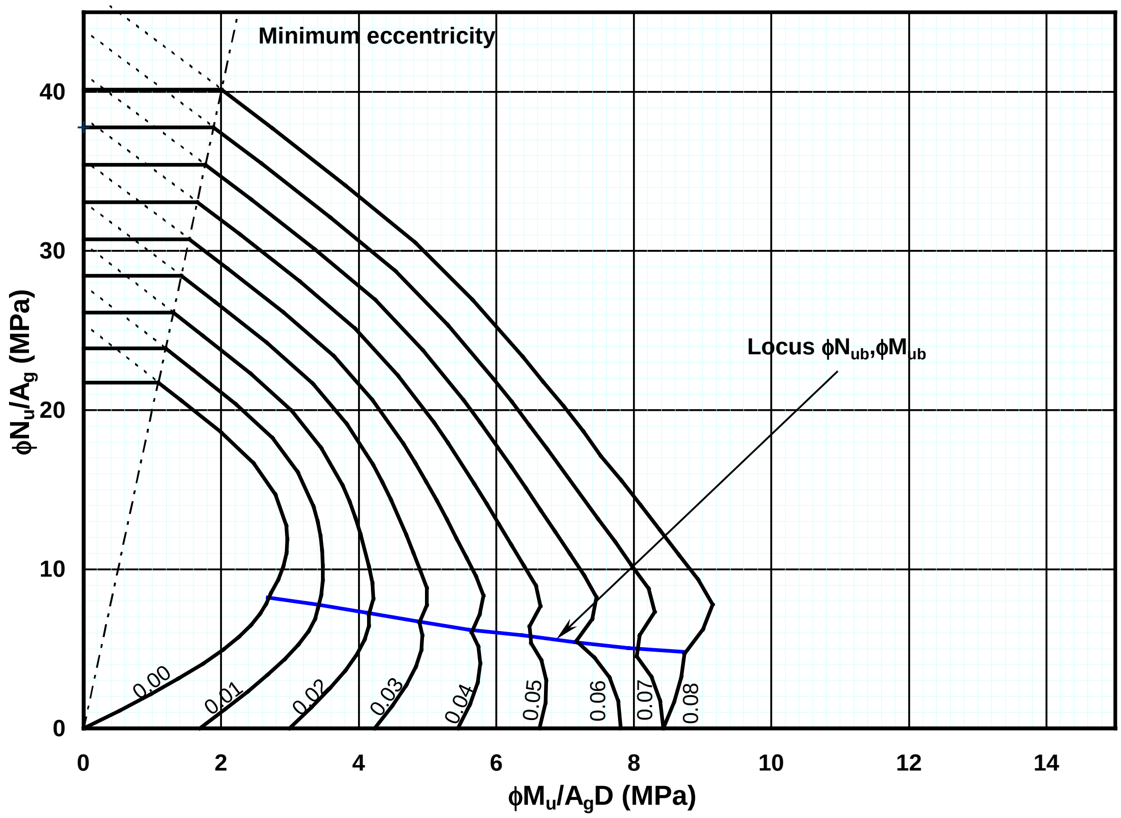


CHART RCB 3.1 - C13: Type A, f'_c = 50 MPa, g = 0.6

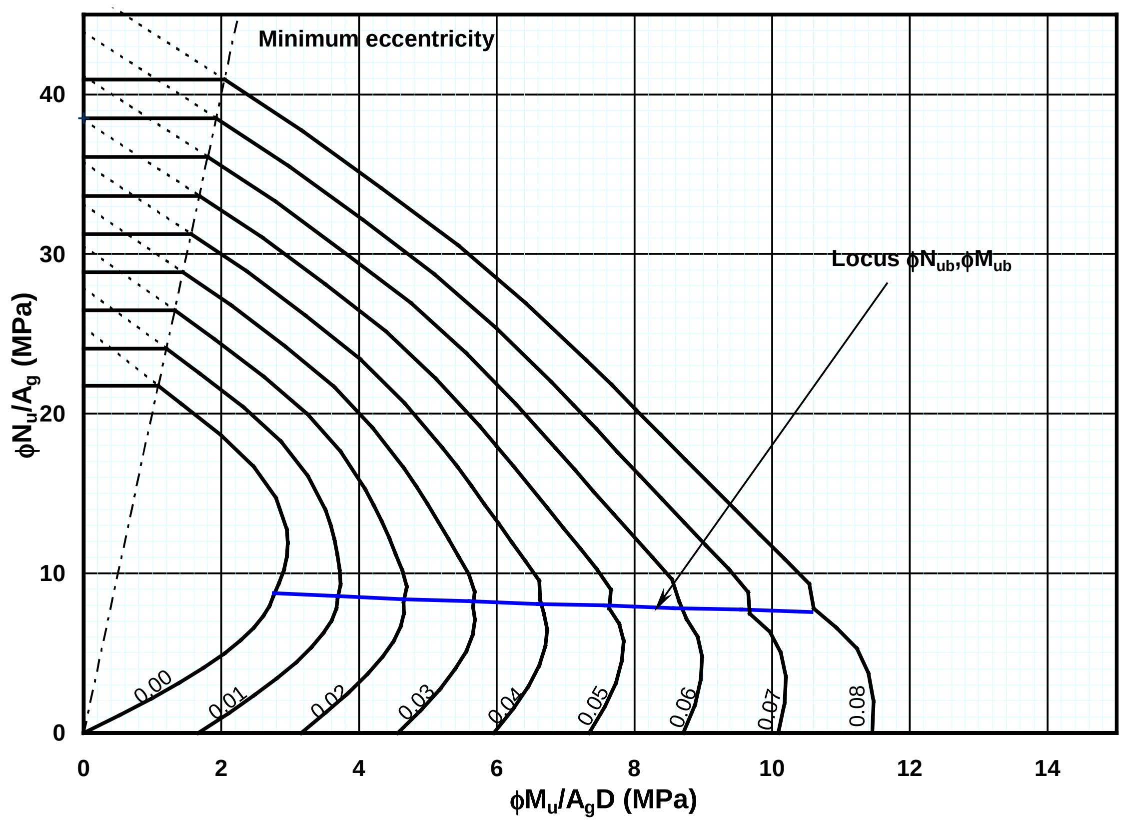


CHART RCB 3.1 - C14: Type A, f'_c = 50 MPa, g = 0.7

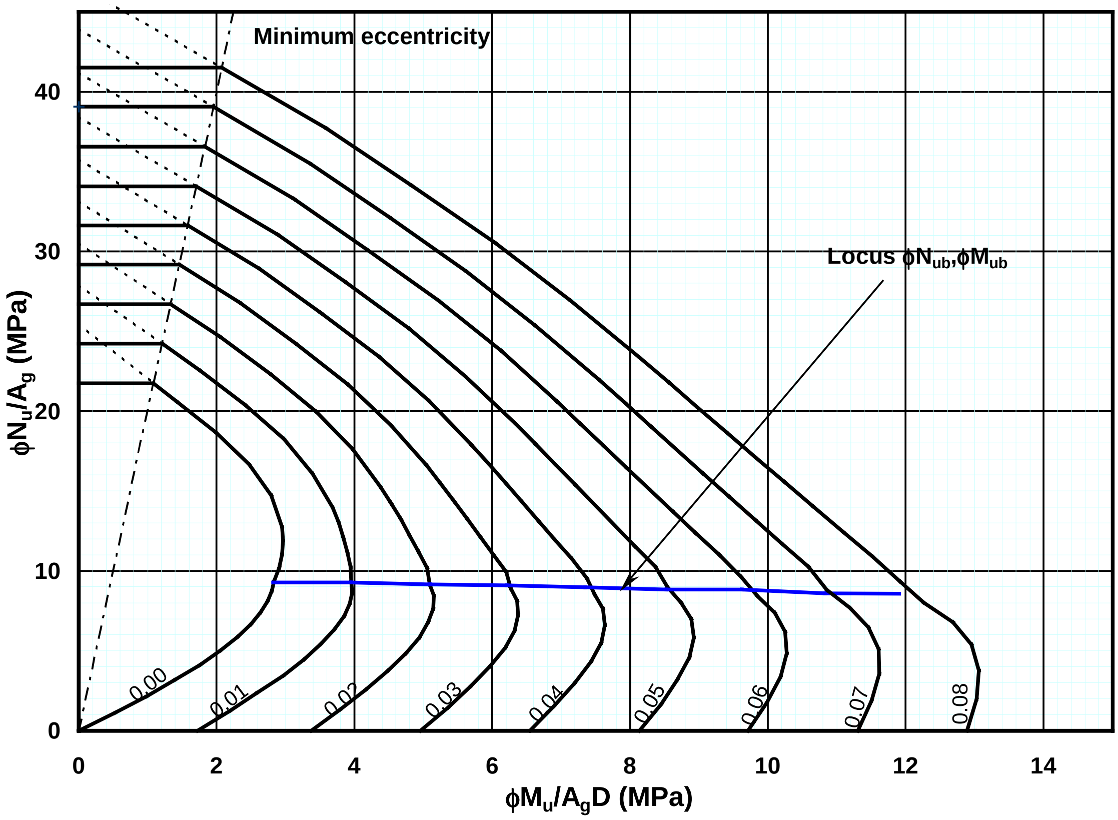


CHART RCB 3.1 - C15: Type A, f'_c = 50 MPa, g = 0.8

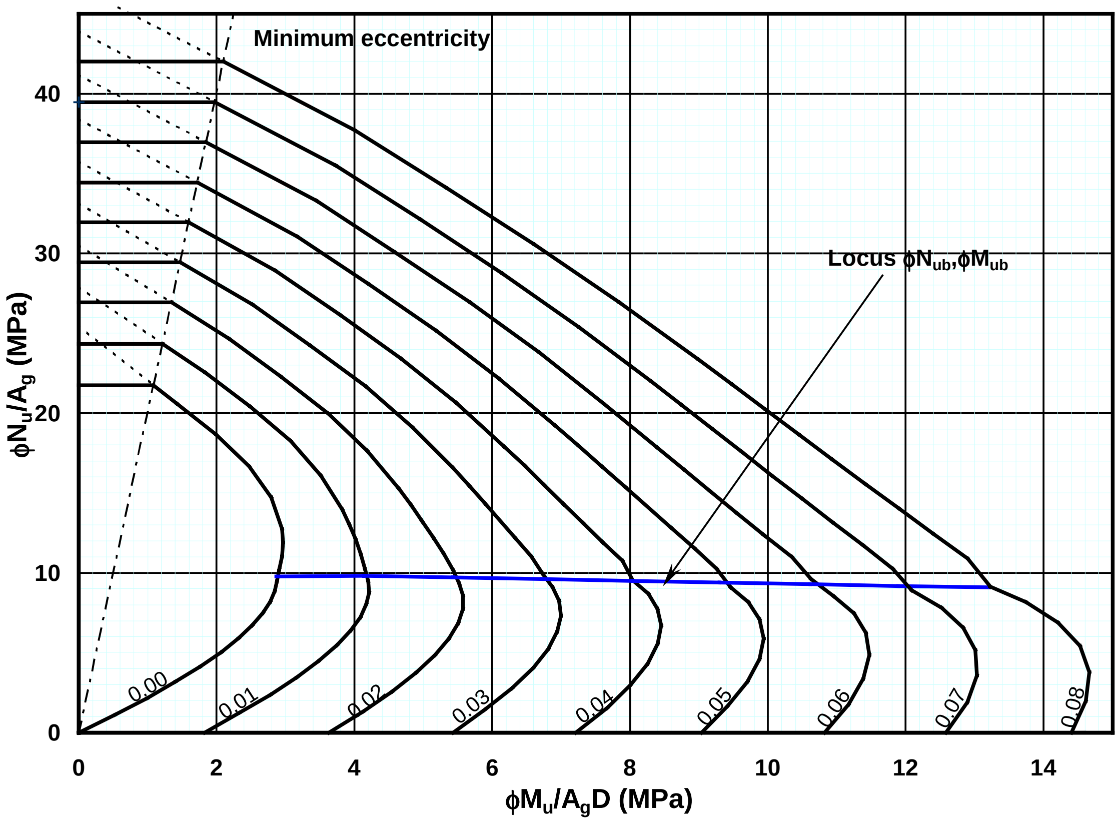


CHART RCB 3.1 - C16: Type A, f'_c = 50 MPa, g = 0.9

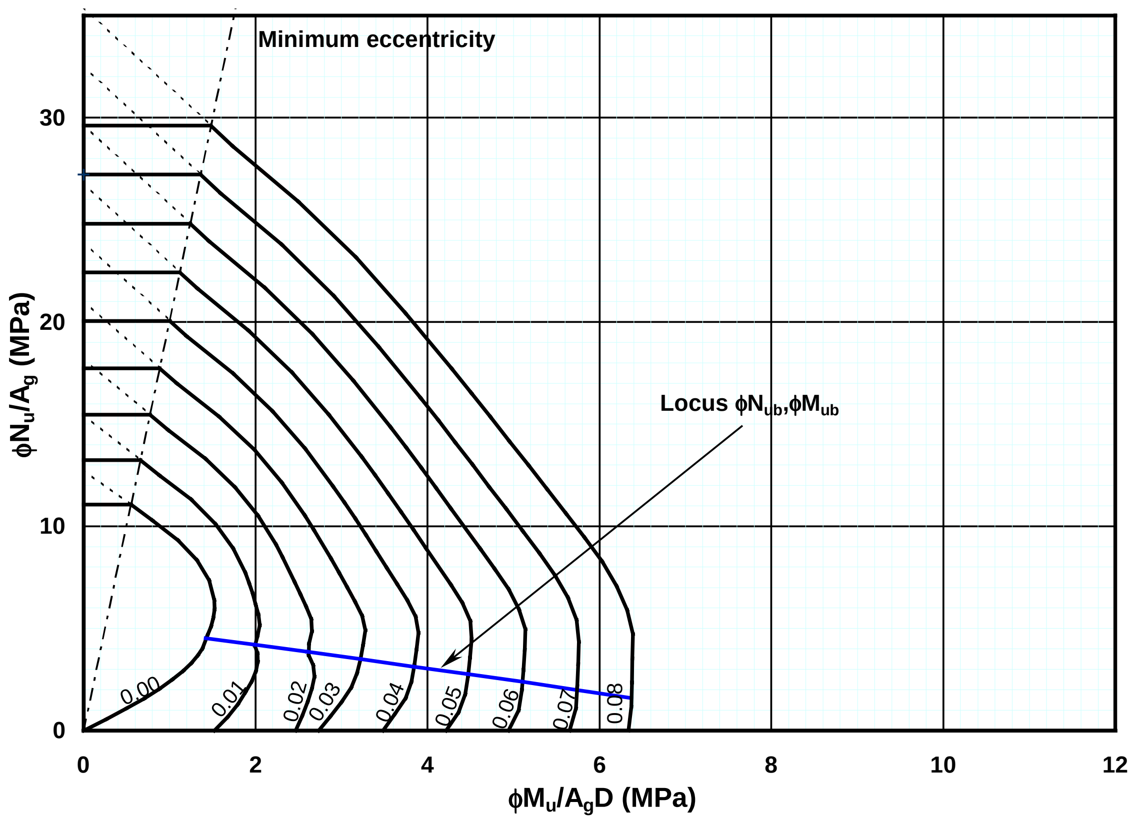


CHART RCB 3.1 - C17: Type B, f'_c = 25 MPa, g = 0.6

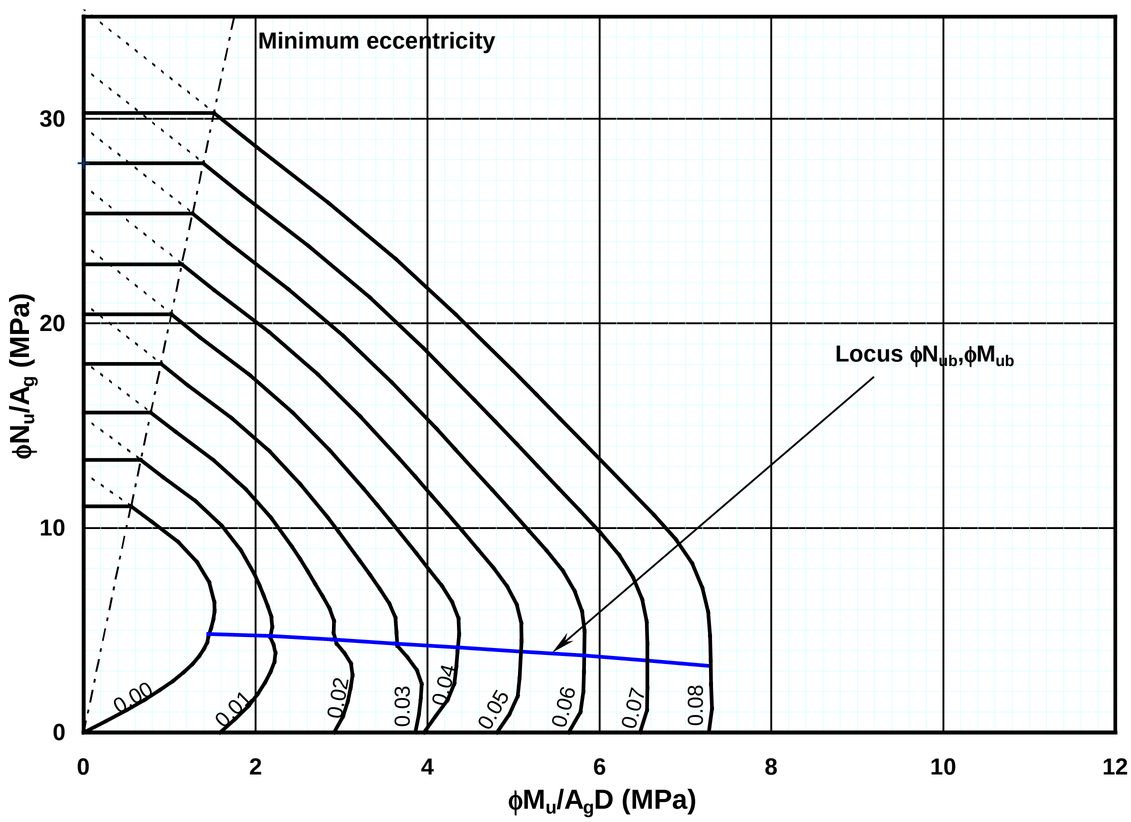


CHART RCB 3.1 - C18: Type B, f'_c = 25 MPa, g = 0.7

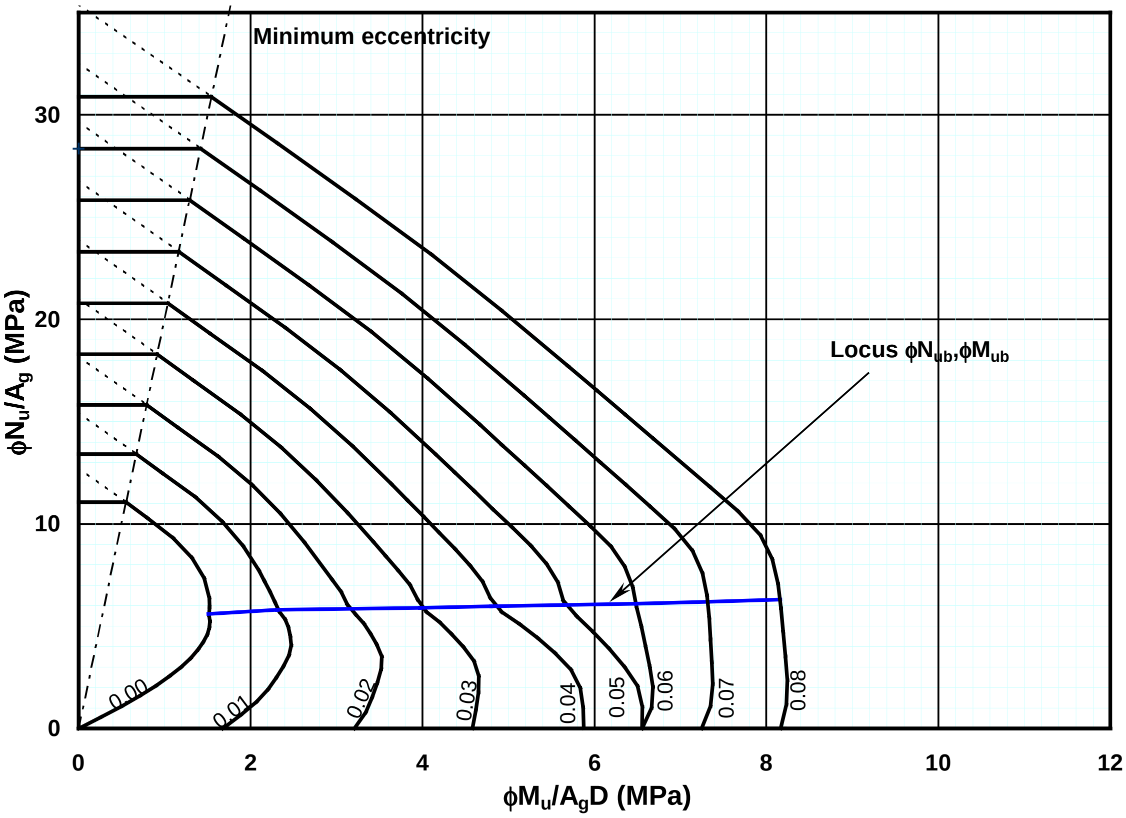


CHART RCB 3.1 - C19: Type B, f'_c = 25 MPa, g = 0.8

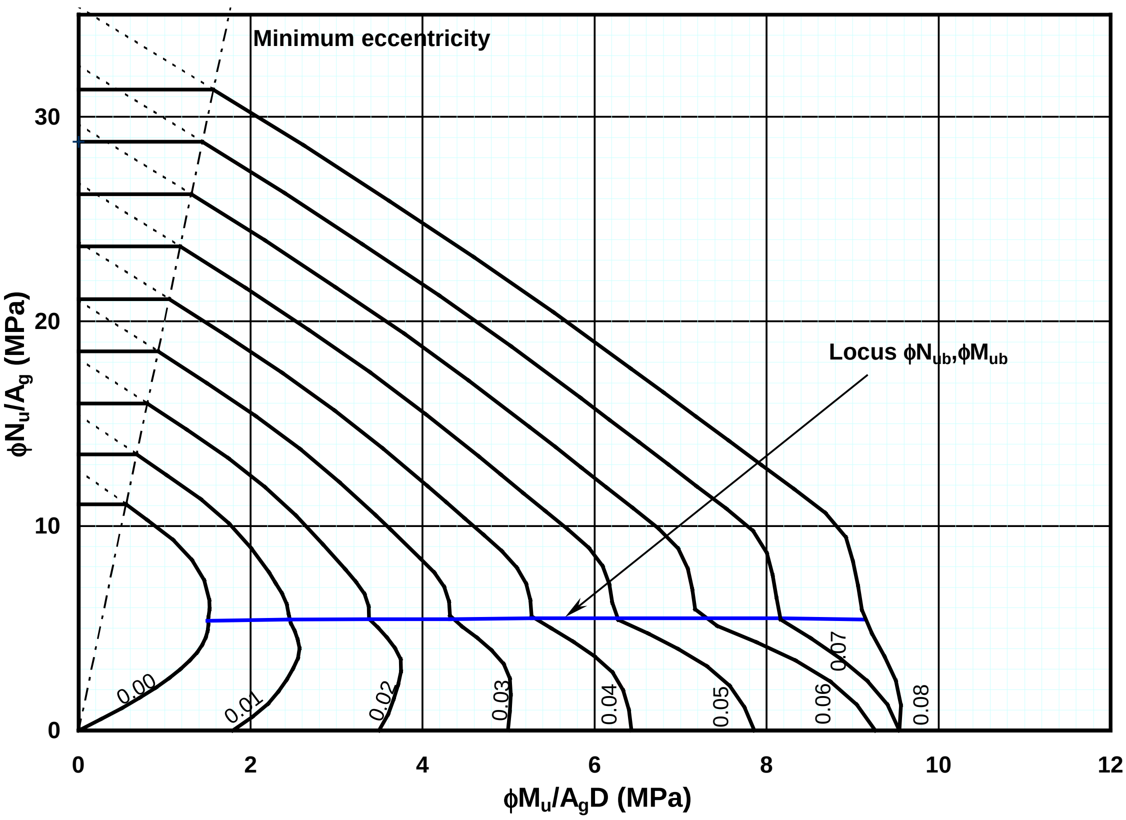


CHART RCB 3.1 - C20: Type B, f'_c = 25 MPa, g = 0.9

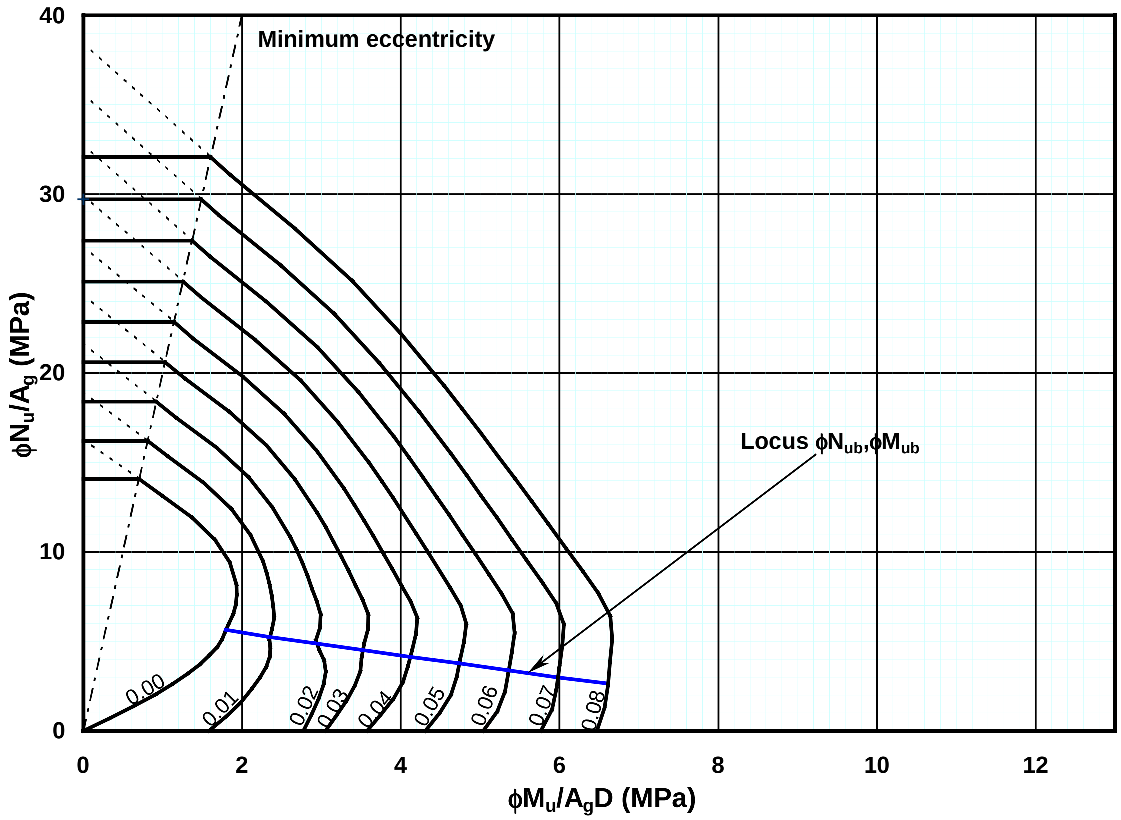


CHART RCB 3.1 - C21: Type B, f'_c = 32 MPa, g = 0.6

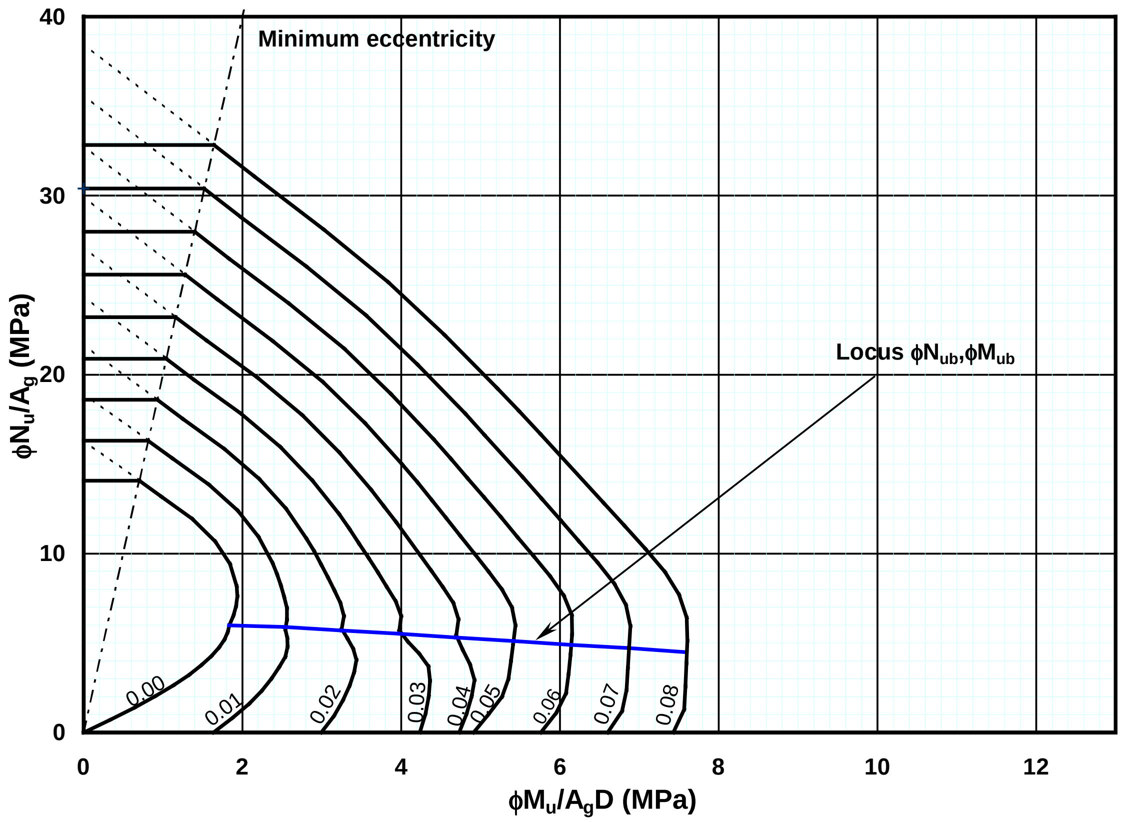


CHART RCB 3.1 - C22: Type B, f'_c = 32 MPa, g = 0.7

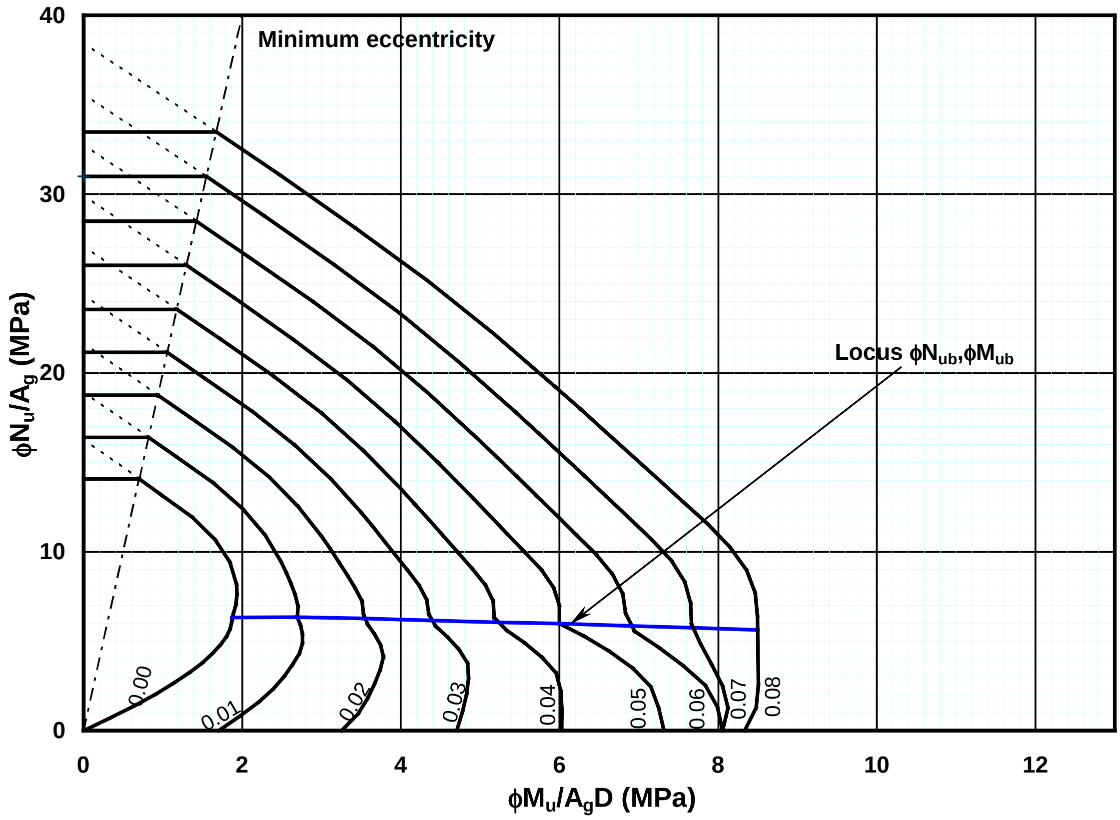


CHART RCB 3.1 - C23: Type B, f'_c = 32 MPa, g = 0.8

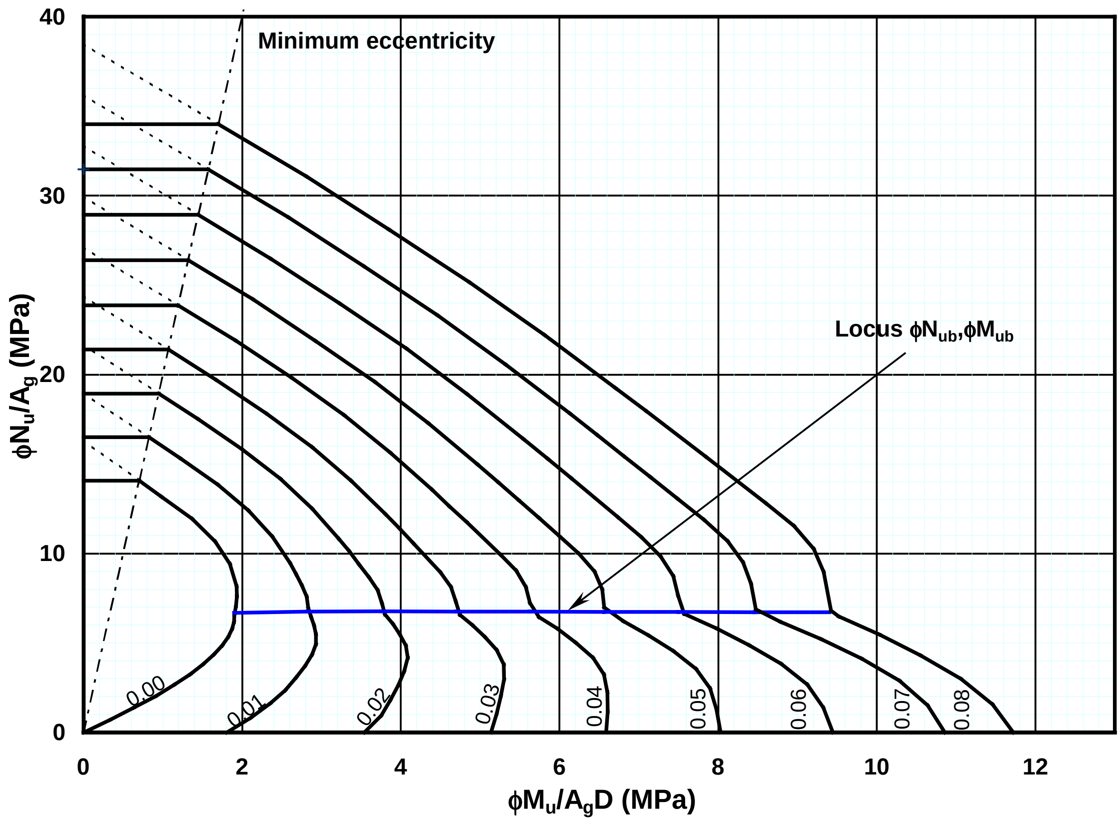


CHART RCB 3.1 - C24: Type B, f'_c = 32 MPa, g = 0.9

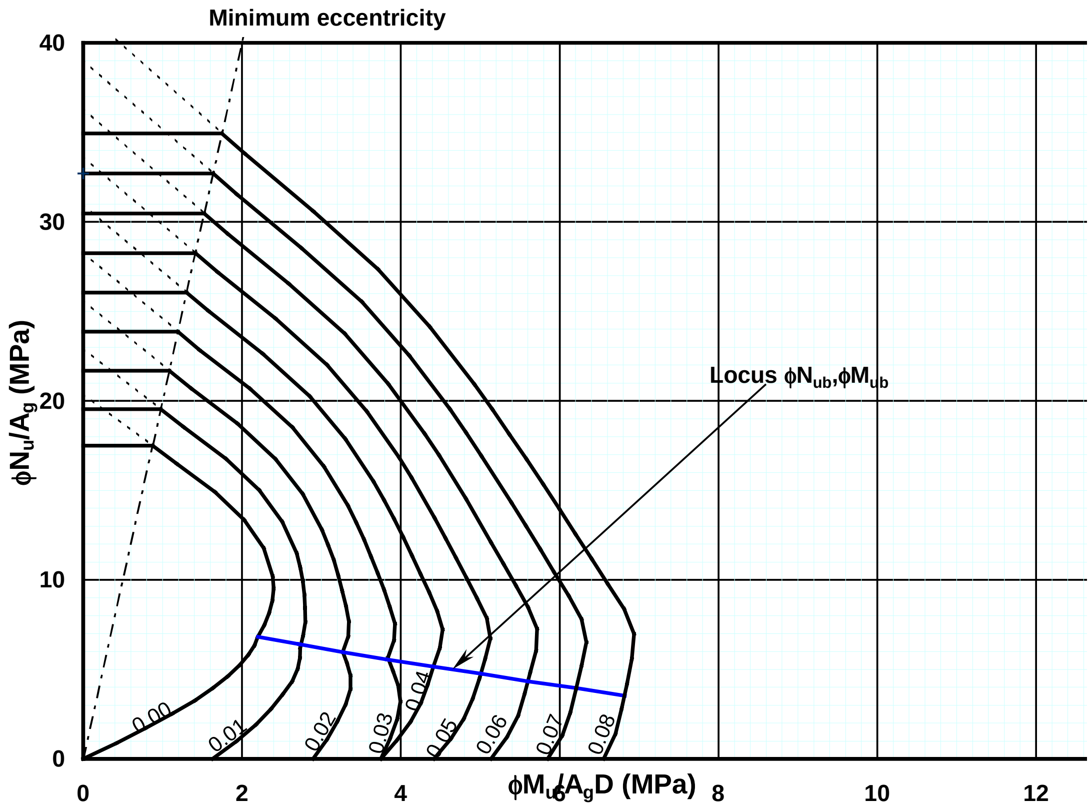


CHART RCB 3.1 - C25: Type B, f'_c = 40 MPa, g = 0.6

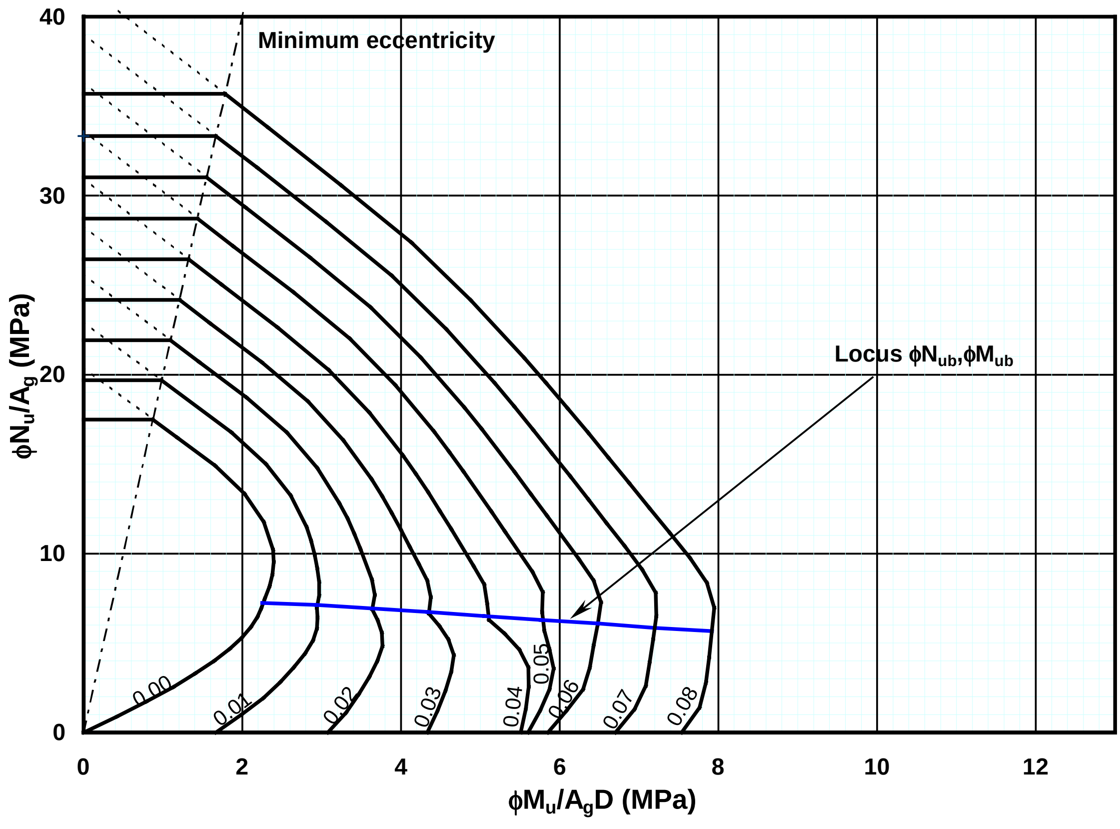


CHART RCB 3.1 - C26: Type B, f'_c = 40 MPa, g = 0.7

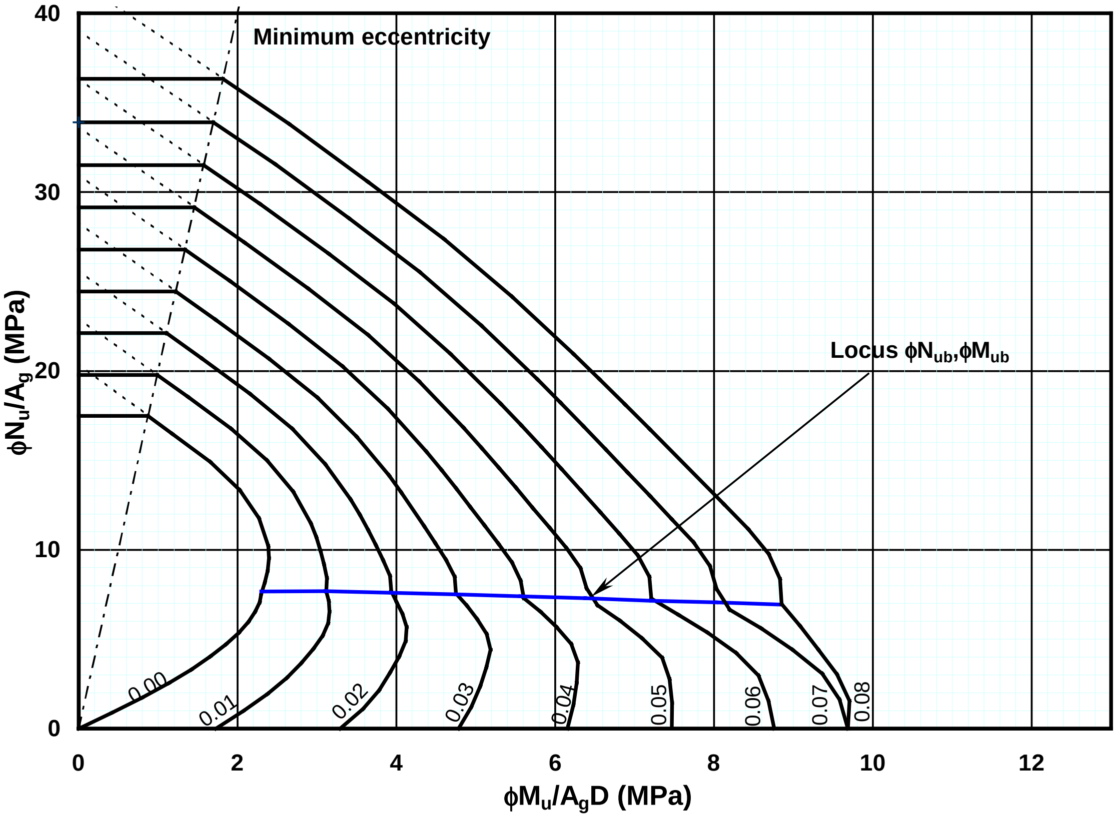


CHART RCB 3.1 - C27: Type B, f'_c = 40 MPa, g = 0.8

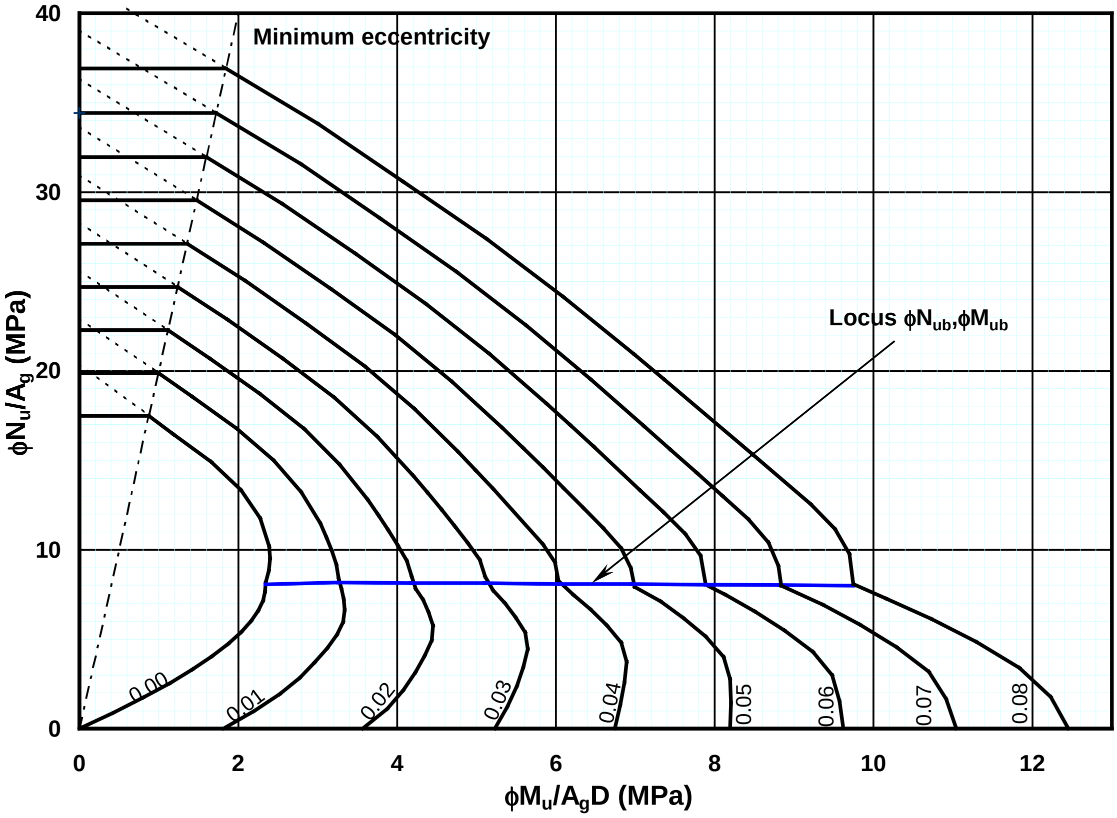


CHART RCB 3.1 - C28: Type B, f'_c = 40 MPa, g = 0.9

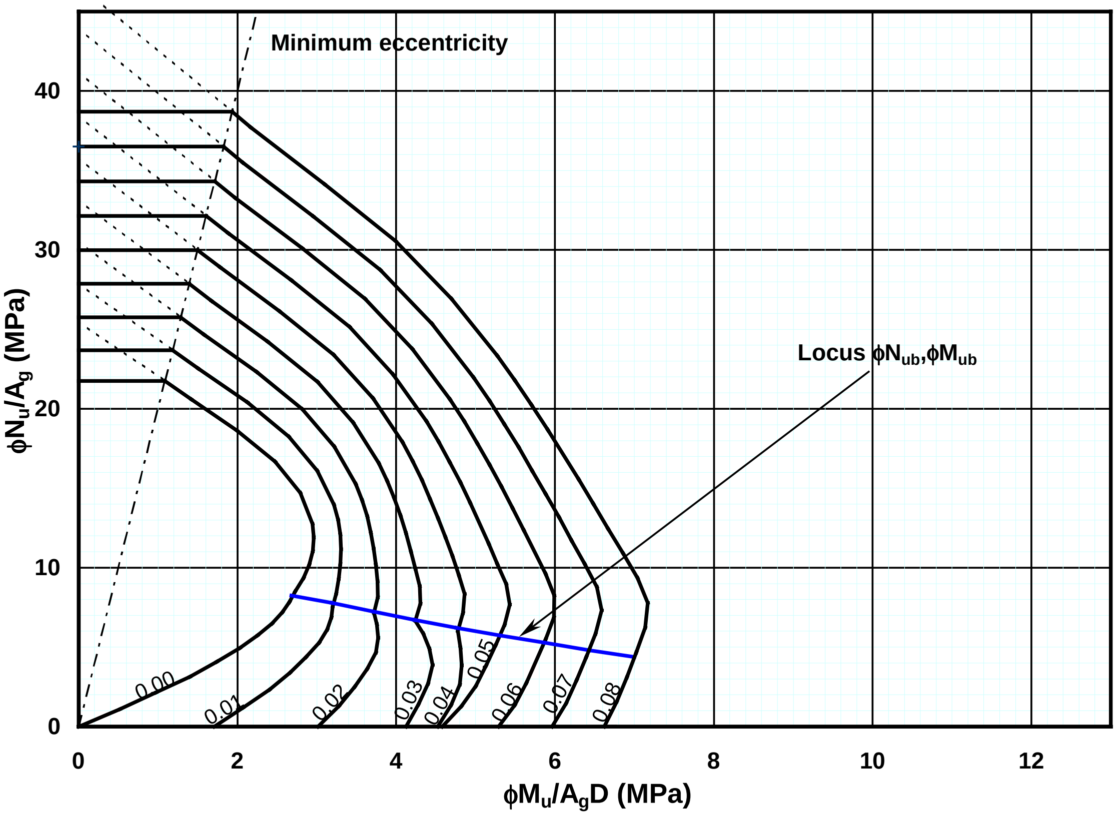


CHART RCB 3.1 - C29: Type B, f'_c = 50 MPa, g = 0.6

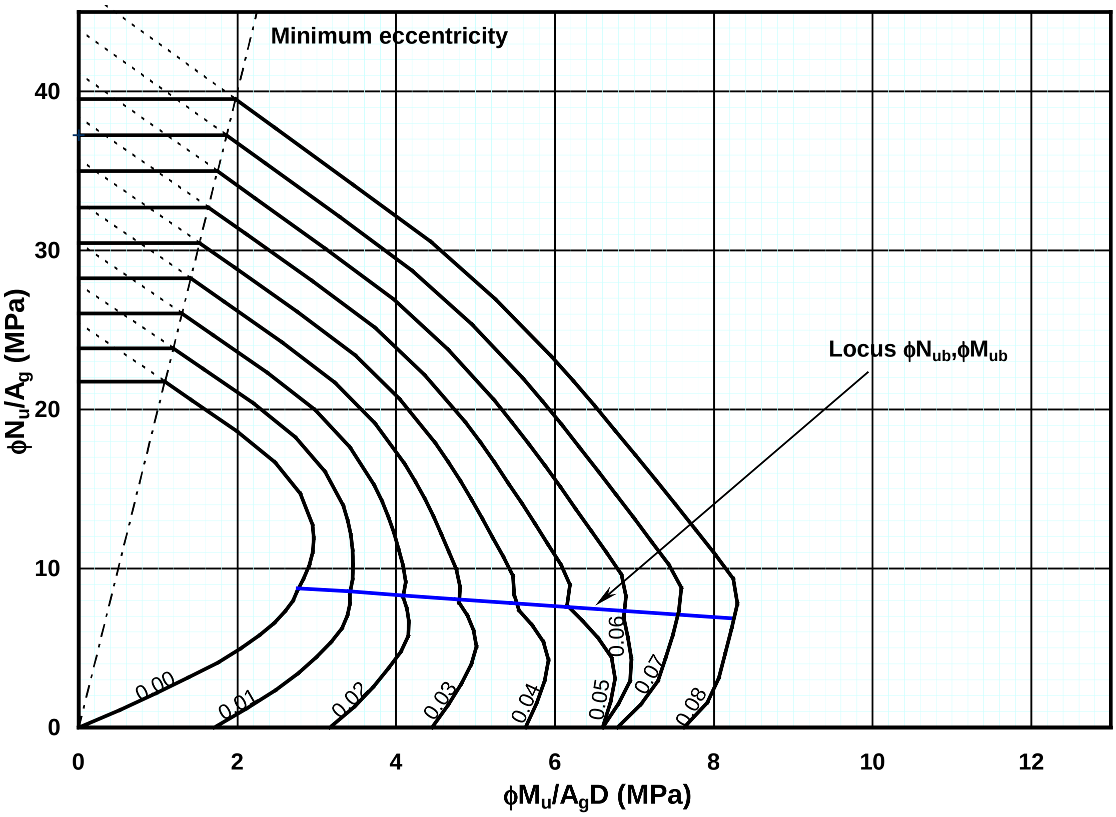


CHART RCB 3.1 - C30: Type B, f'_c = 50 MPa, g = 0.7

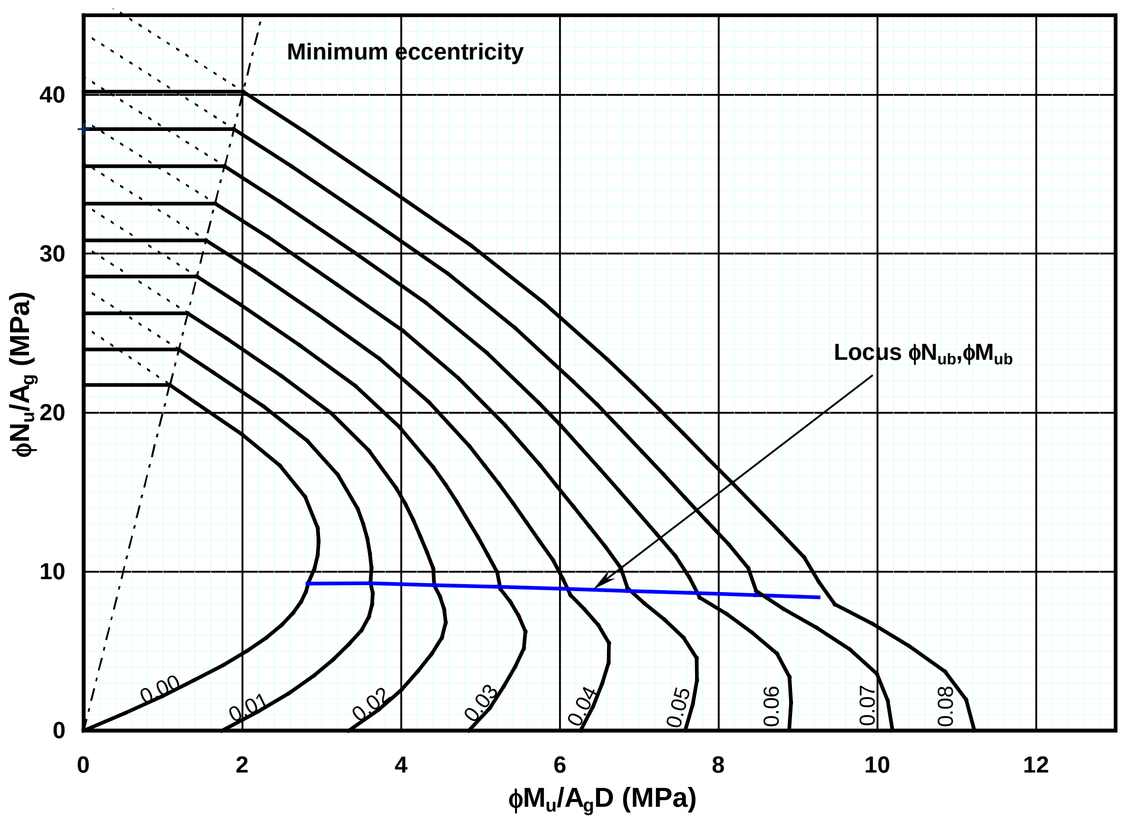


CHART RCB 3.1 - C31: Type B, f'_c = 50 MPa, g = 0.8

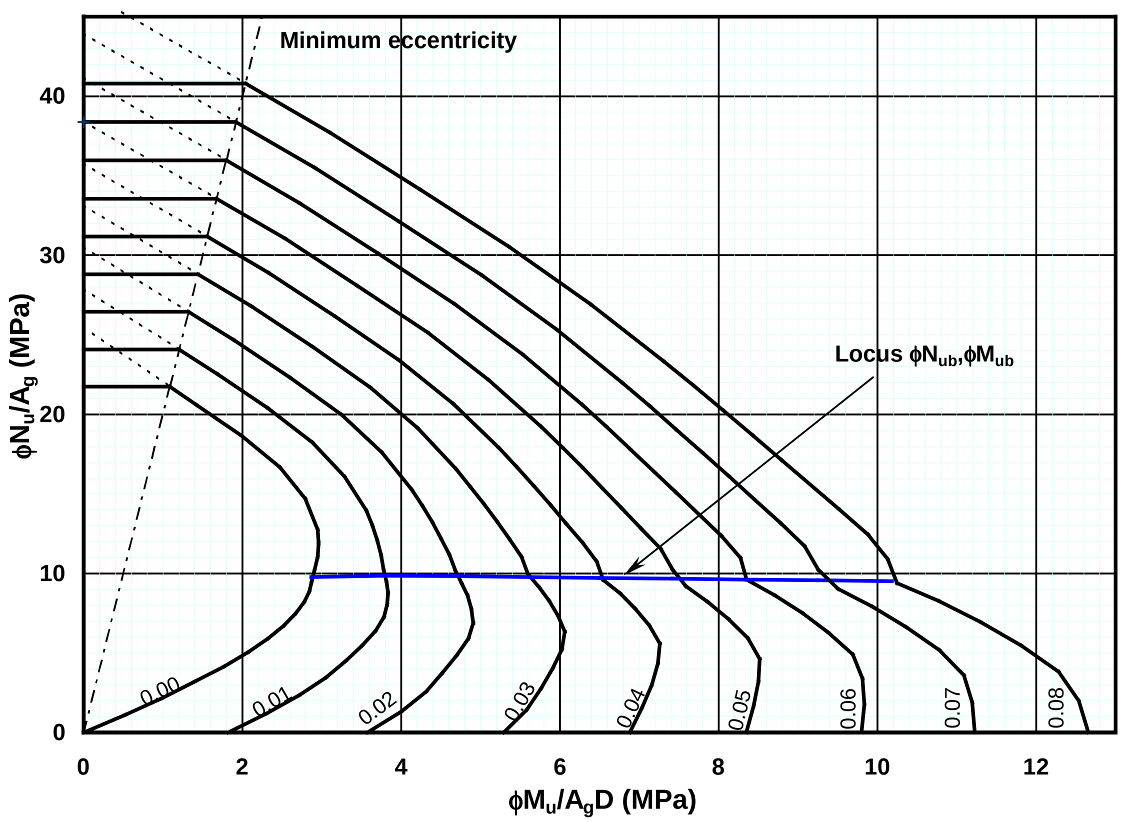


CHART RCB 3.1 - C32: Type B, f'_c = 50 MPa, g = 0.9

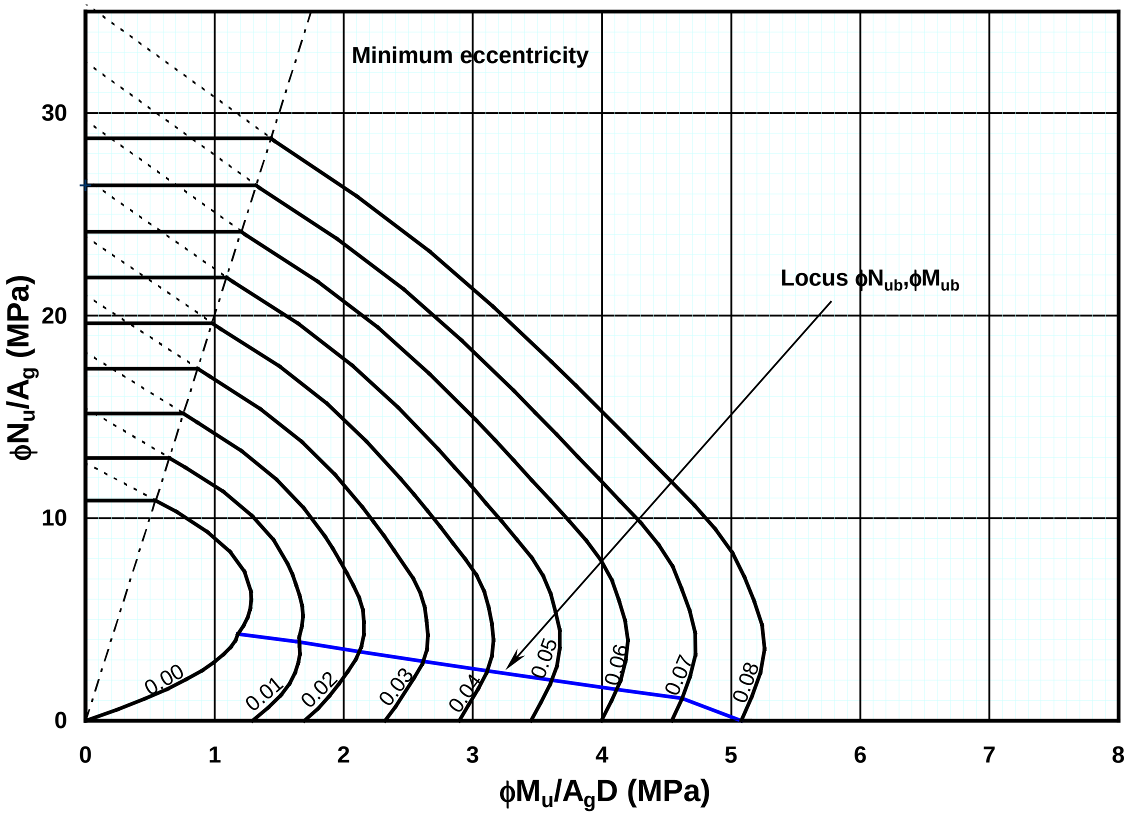


CHART RCB 3.1 - C33: Type C, f'_c = 25 MPa, g = 0.6

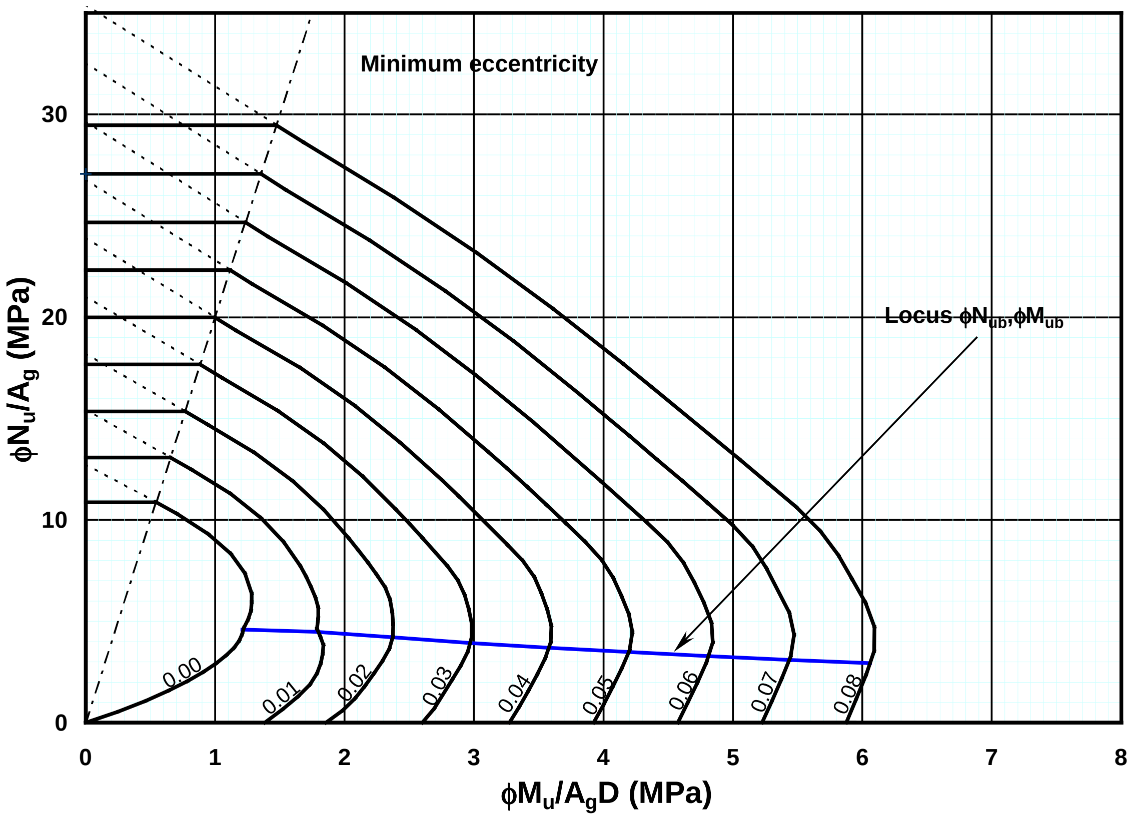


CHART RCB 3.1 - C34: Type C, f'_c = 25 MPa, g = 0.7

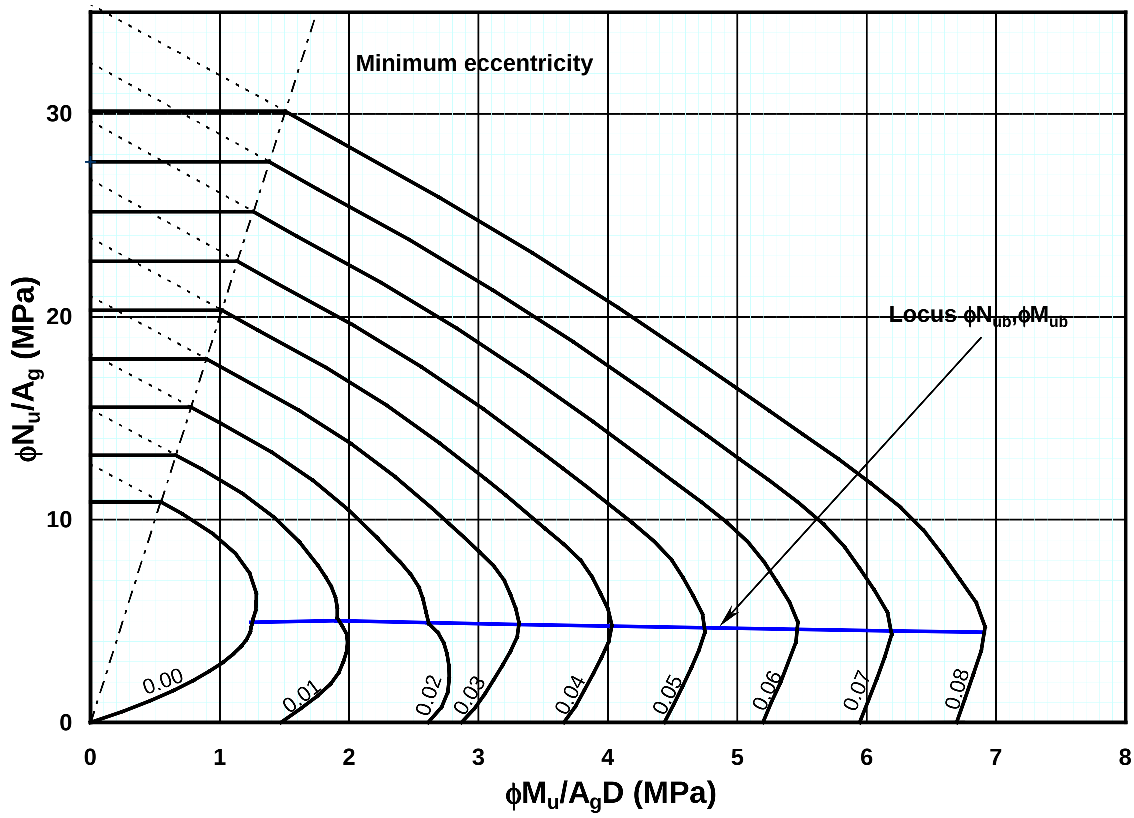


CHART RCB 3.1 - C35: Type C, f'_c = 25 MPa, g = 0.8

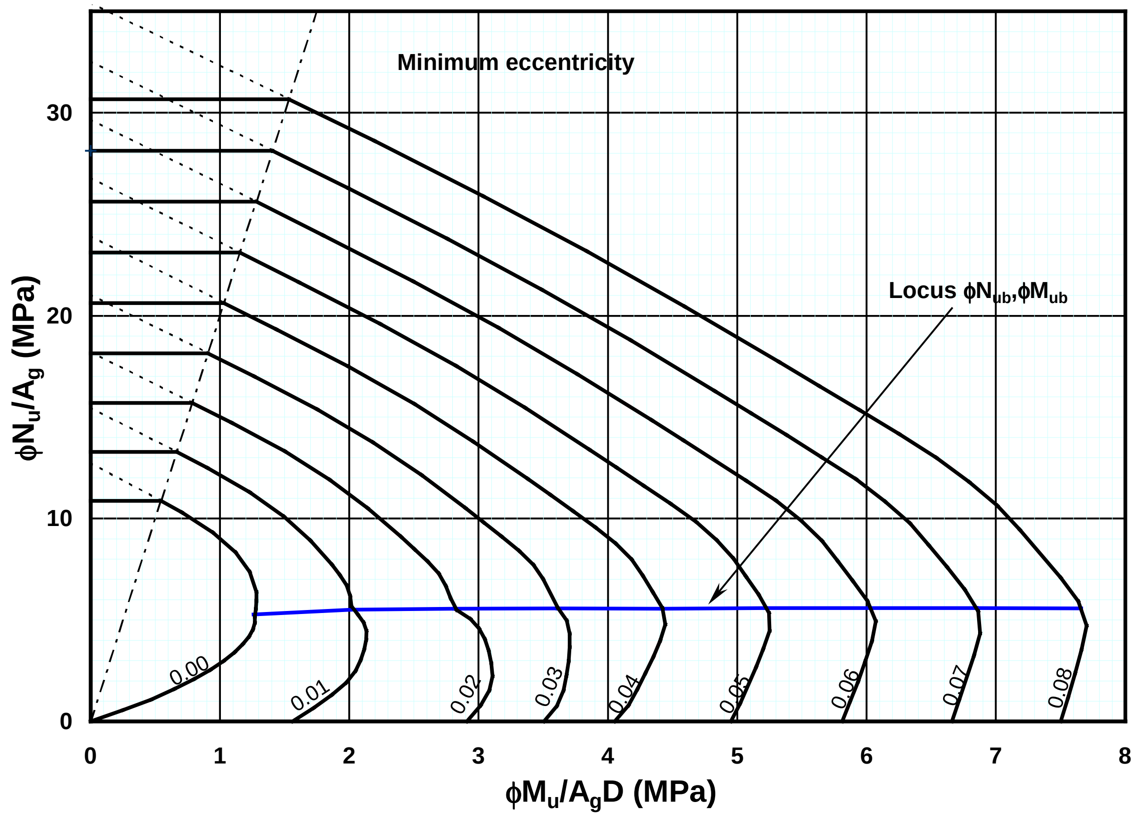


CHART RCB 3.1 - C36: Type C, f'_c = 25 MPa, g = 0.9

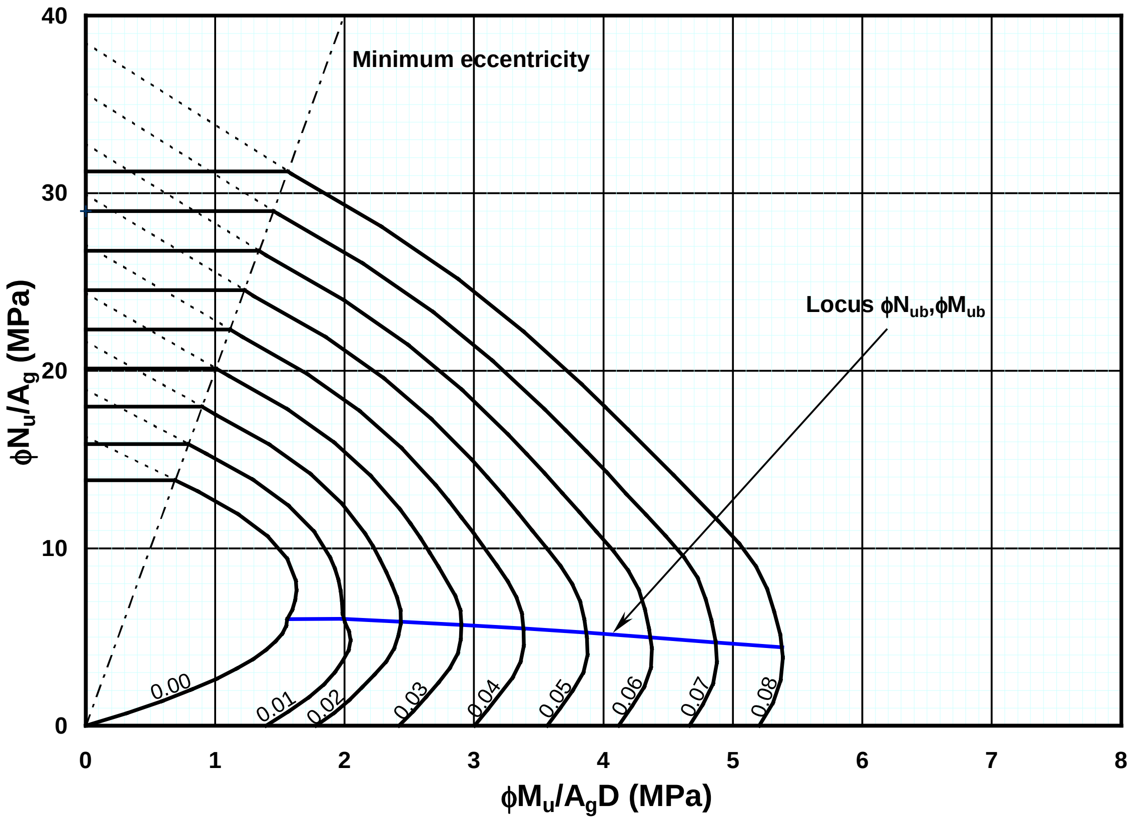


CHART RCB 3.1 - C37: Type C, *f*'$_c$ = 32 MPa, *g* = 0.6

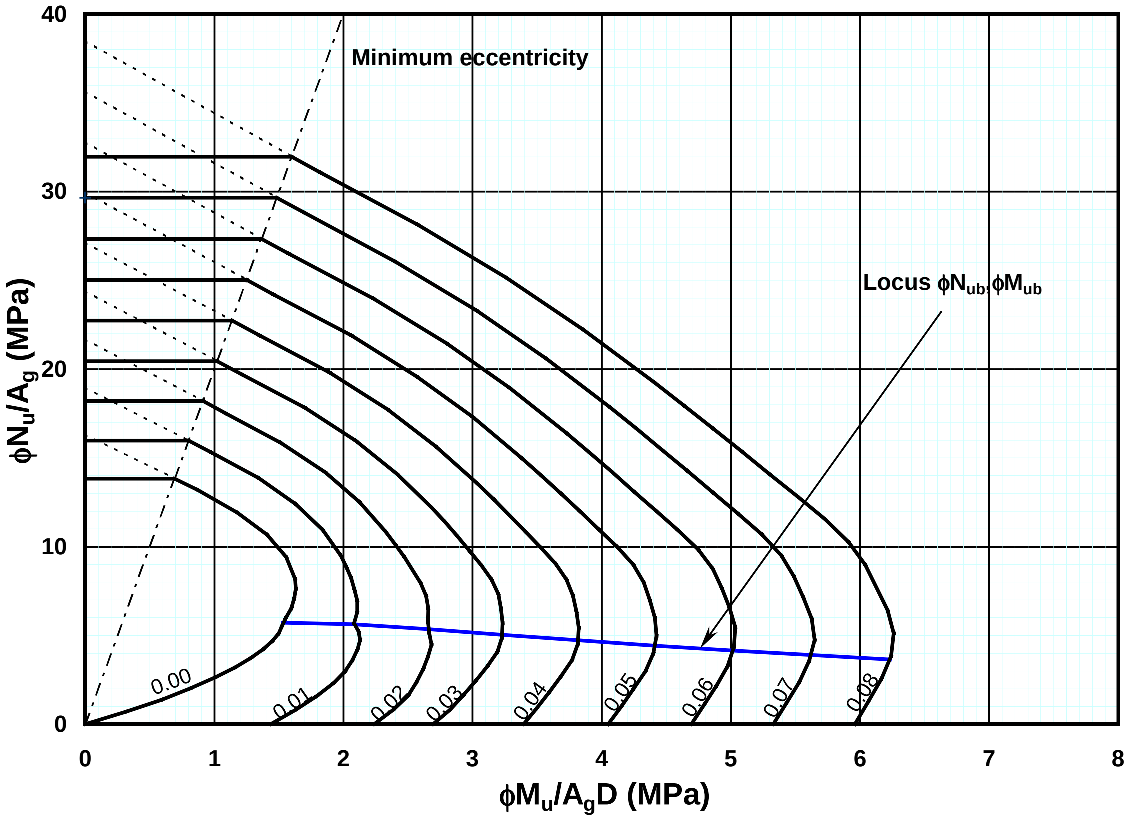


CHART RCB 3.1 - C38: Type C, *f*'$_c$ = 32 MPa, *g* = 0.7

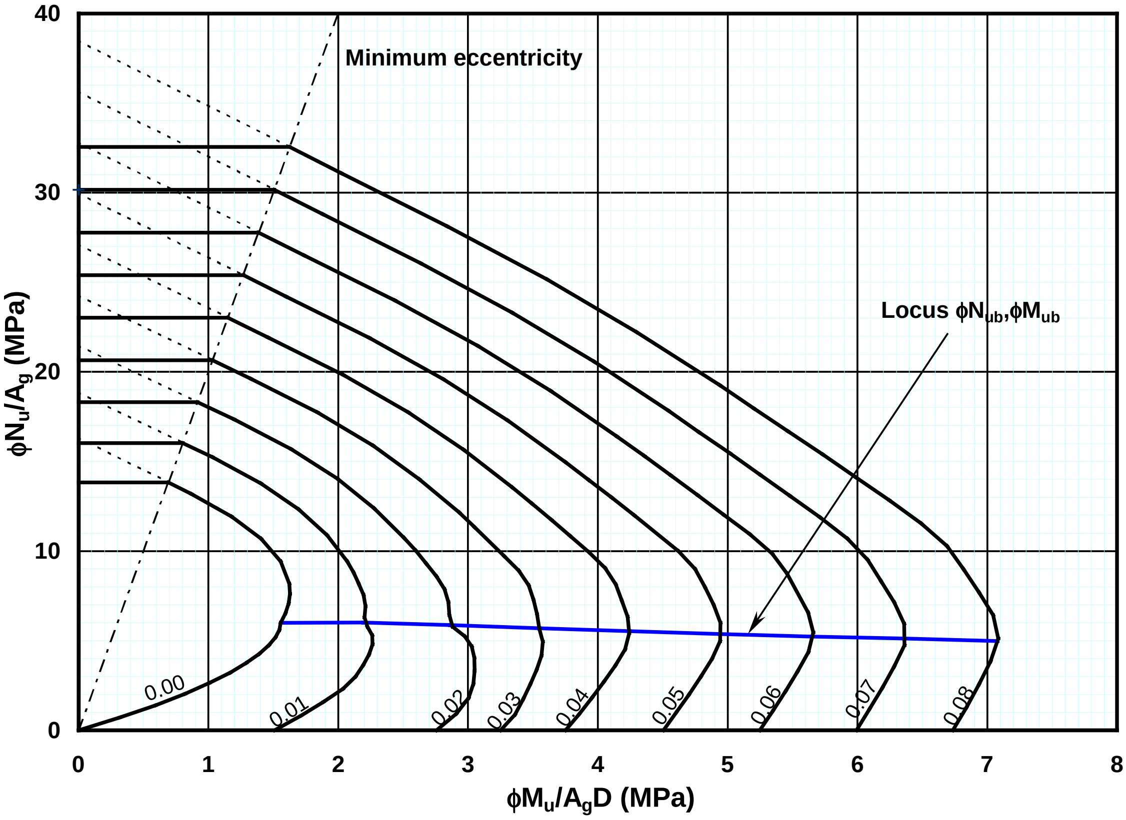


CHART RCB 3.1 - C39: Type C, f'_c = 25 MPa, g = 0.8

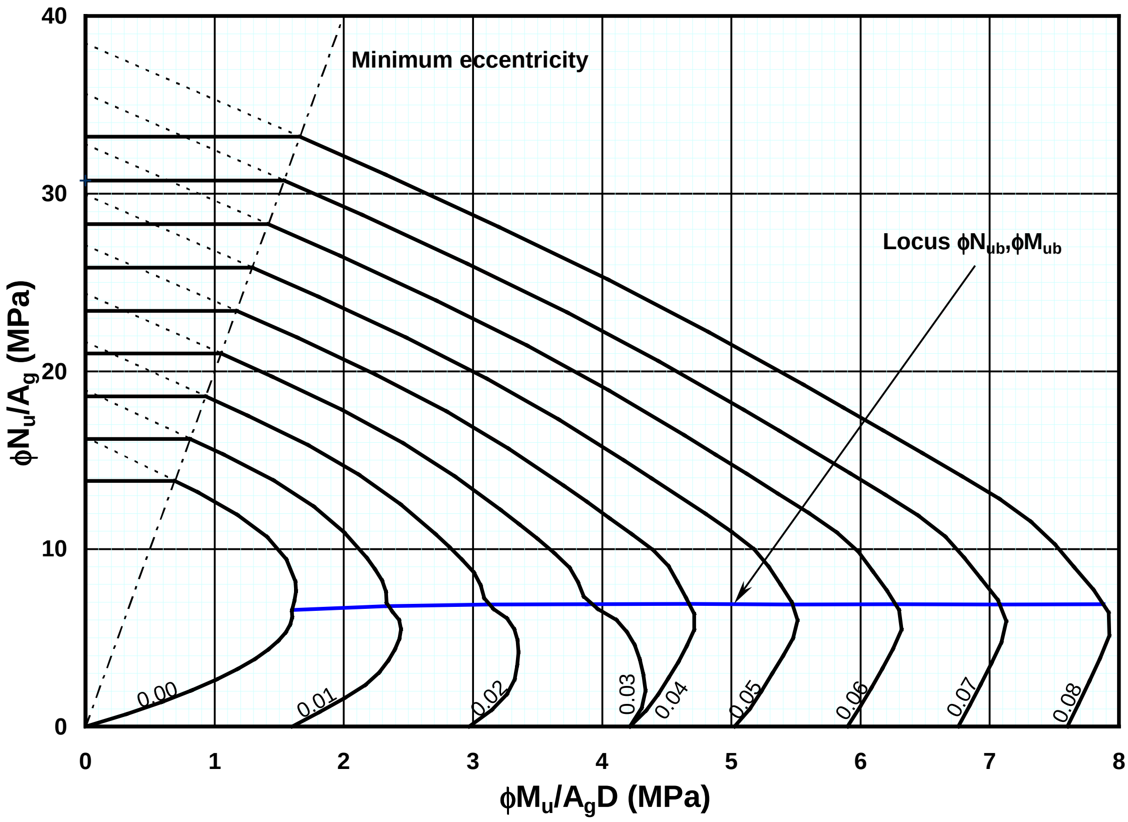


CHART RCB 3.1 - C40: Type C, f'_c = 32 MPa, g = 0.9

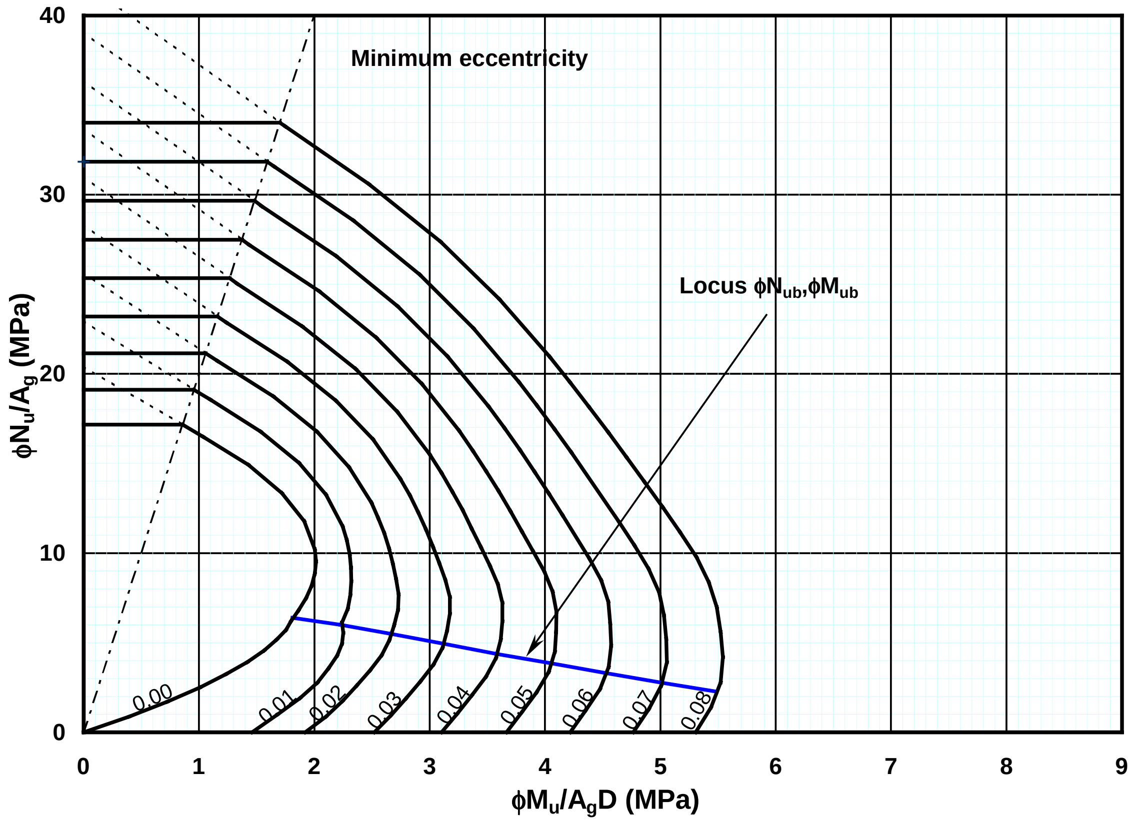


CHART RCB 3.1 - C41: Type C, f'_c = 40 MPa, g = 0.6

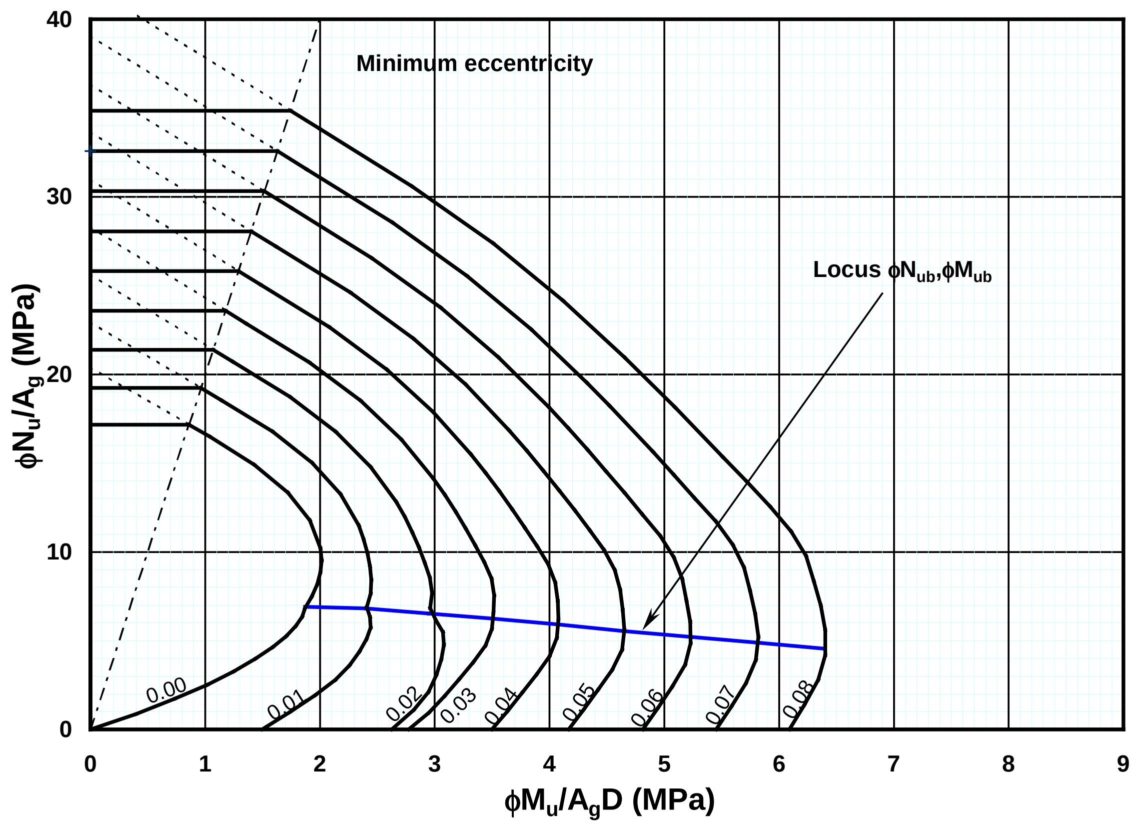


CHART RCB 3.1 - C42: Type C, f'_c = 40 MPa, g = 0.7

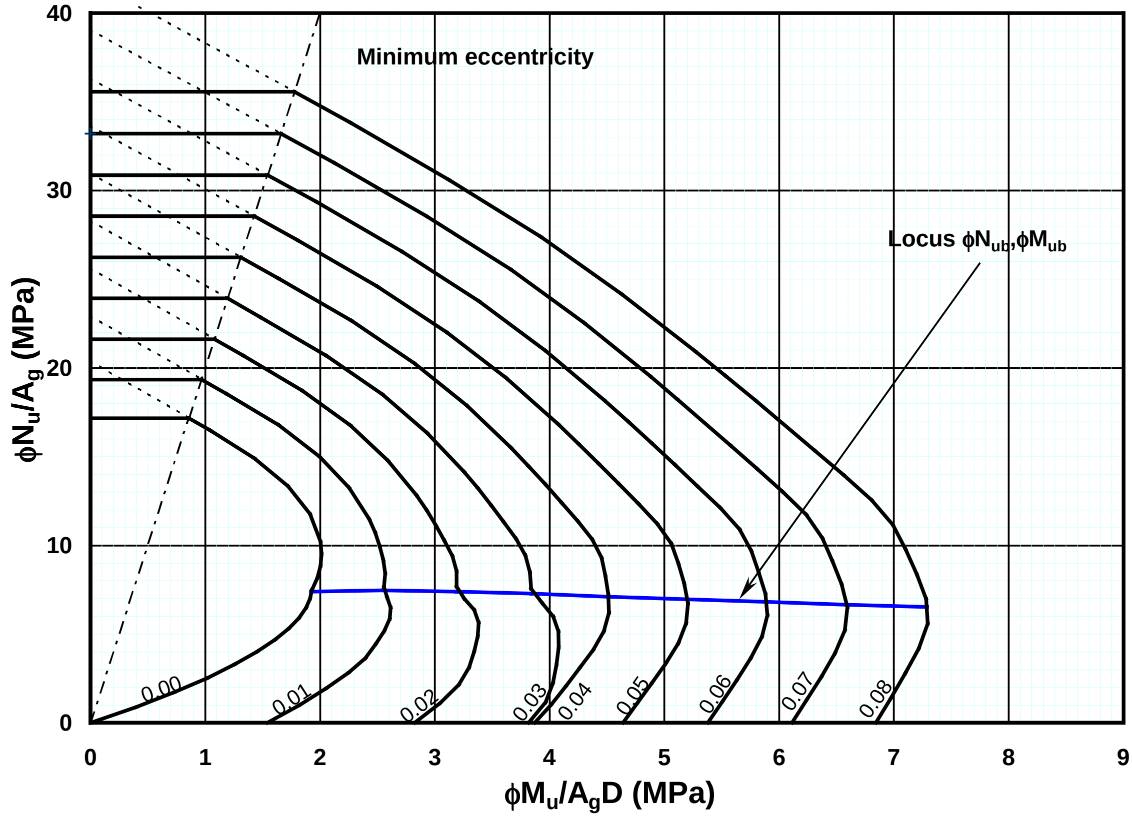


CHART RCB 3.1 - C43: Type C, f'_c = 40 MPa, g = 0.8

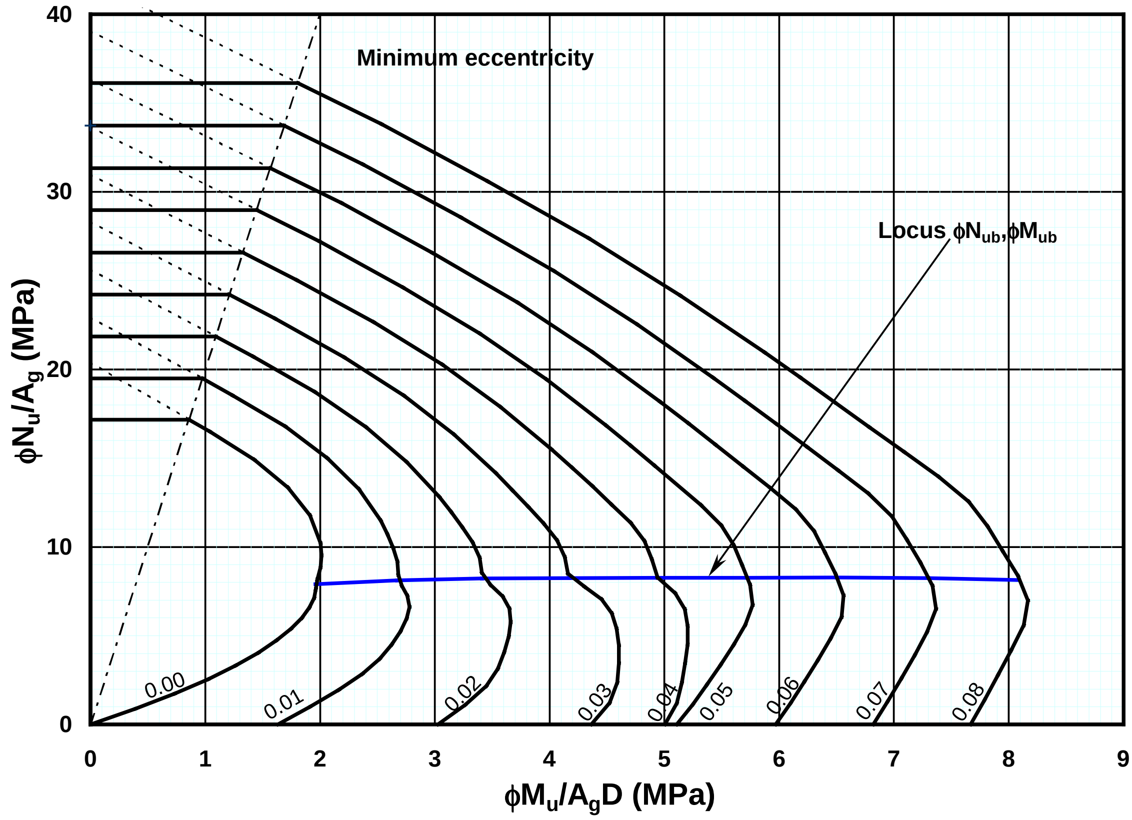


CHART RCB 3.1 - C44: Type C, f'_c = 40 MPa, g = 0.9

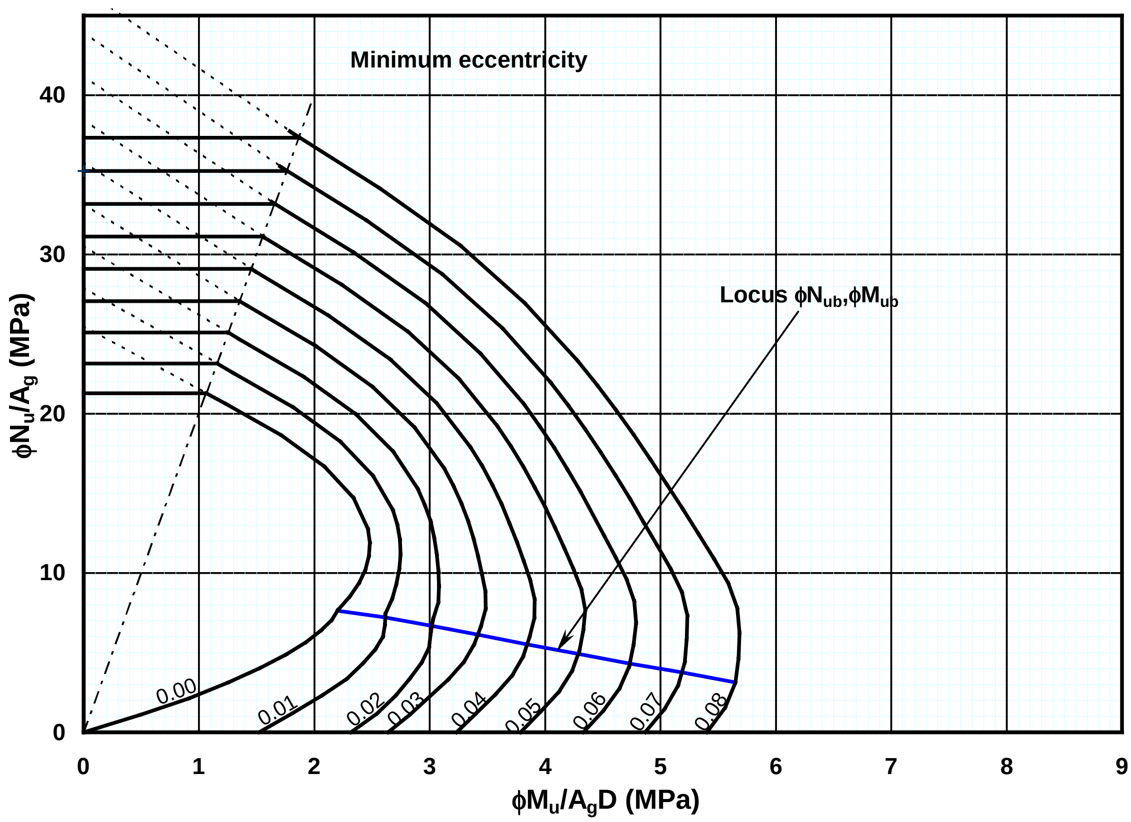


CHART RCB 3.1 - C45: Type C, *f'*$_c$ = 50 MPa, *g* = 0.6

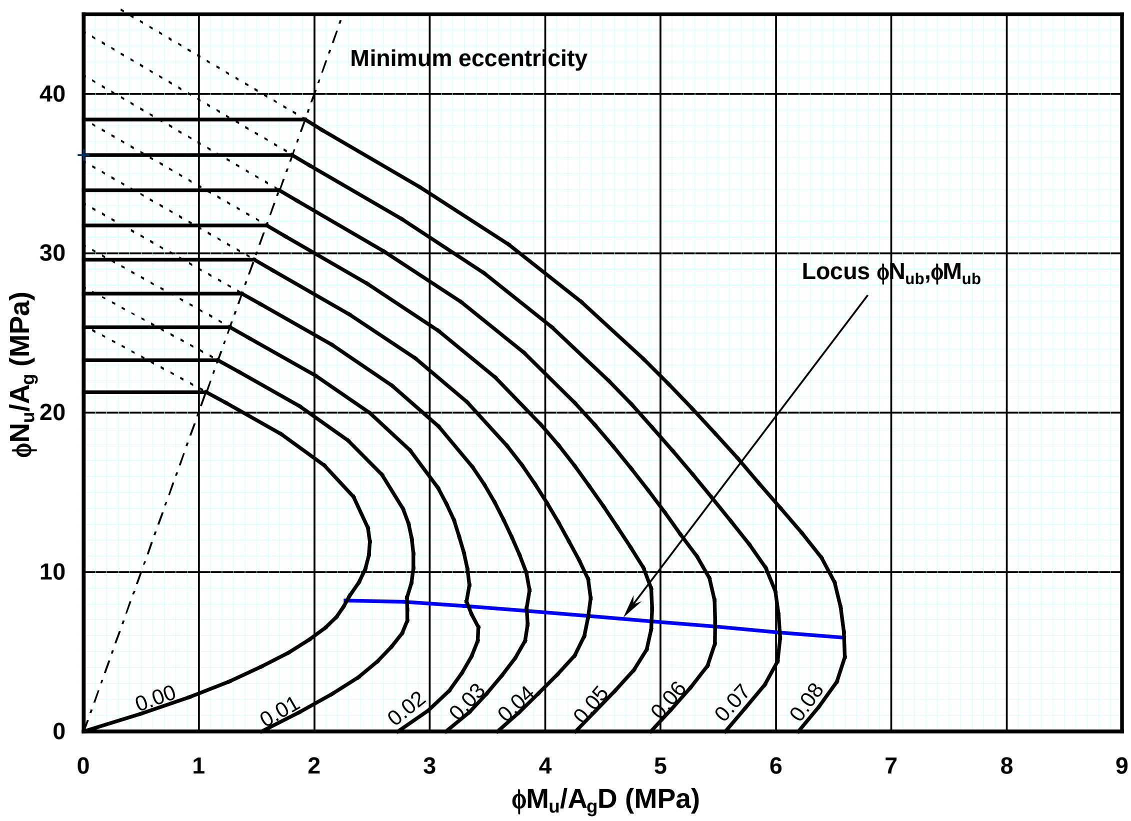
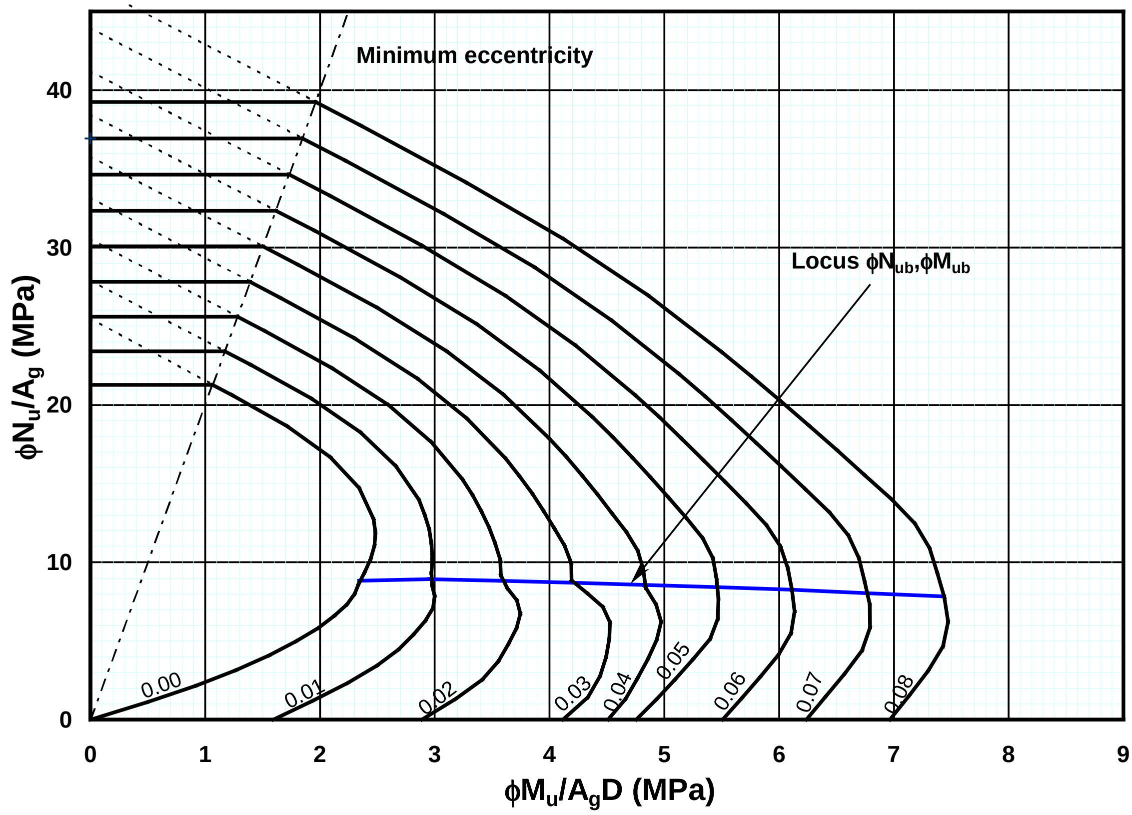


CHART RCB 3.1 - C46: Type C, *f'*$_c$ = 40 MPa, *g* = 0.7



C

CHART RCB 3.1 - C47: Type C, *f'*$_c$ = 50 MPa, *g* = 0.8

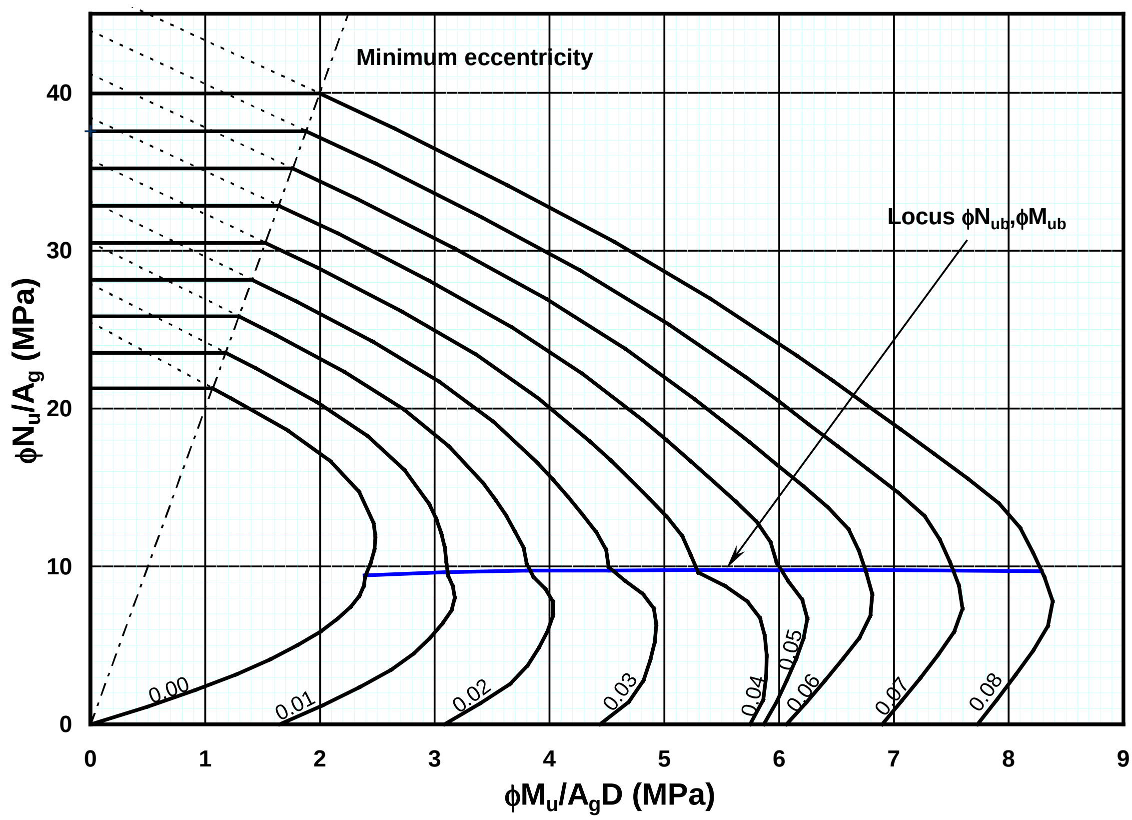


CHART RCB 3.1 - C48: Type C, *f'*$_c$ = 50 MPa, *g* = 0.9