CS 2336 Discrete Mathematics

Lecture 3 Logic: Rules of Inference

Outline

- Mathematical Argument
- Rules of Inference

Argument

- In mathematics, an argument is a sequence of propositions (called premises) followed by a proposition (called conclusion)
- A valid argument is one that, if all its premises are true, then the conclusion is true
- Ex: "If it rains, I drive to school." "It rains."
 - .: "I drive to school."

Valid Argument Form

• In the previous example, the argument belongs to the following form:

 $p \rightarrow q$ p $\therefore q$

- Indeed, the above form is valid no matter what propositions are substituted to the variables
- This is called a valid argument form

Valid Argument Form

- By definition, if a valid argument form consists
 - -premises: p_1, p_2, \dots, p_k
 - -conclusion: q

then ($p_1 \wedge p_2 \wedge ... \wedge p_k$) $\rightarrow q~$ is a tautology

- Ex: (($p \rightarrow q$) $\land p$) $\rightarrow q$ is a tautology
- Some simple valid argument forms, called rules of inference, are derived and can be used to construct complicated argument form

- 1. Modus Ponens (method of affirming) premises: p, $p \rightarrow q$ conclusion: q
- 2. Modus Tollens (method of denying)
 premises: ¬q, p→q
 conclusion: ¬p

- 3. Hypothetical Syllogism premises: $p \rightarrow q, q \rightarrow r$ conclusion: $p \rightarrow r$
- 4. Disjunctive Syllogism
 premises: ¬p, p∨q
 conclusion: q

5. Addition

premises: p

conclusion: $p \lor q$

6. Simplification
premises: p ∧ q
conclusion: p

7. Conjunction

premises: p, q

conclusion: $p \land q$

8. Resolution premises: $p \lor q$, $\neg p \lor r$ conclusion: $q \lor r$

Rules of Inference with Quantifiers

- 9. Universal Instantiation
 premises: ∀x P(x)
 conclusion: P(c), for any c
- 10. Universal Generalization premises: P(c) for any arbitrary c conclusion: $\forall X P(X)$

Rules of Inference with Quantifiers

- 11. Existential Instantiation premises: $\exists x P(x)$ conclusion: P(c), for some element c
- 12. Existential Generalization premises: P(c) for some element c conclusion: $\exists x P(x)$

Applying Rules of Inferences

- Example 1: It is known that
 - 1. It is not sunny this afternoon, and it is colder than yesterday.
 - 2. We will go swimming only if it is sunny.
 - 3. If we do not go swimming, we will play basketball.
 - 4. If we play basketball, we will go home early.
- Can you conclude "we will go home early"?

- To simplify the discussion, let
 - p := It is sunny this afternoon
 - q := It is colder than yesterday
 - r := We will go swimming
 - s := We will play basketball
 - t := We will go home early
- We will give a valid argument with
 premises: ¬p∧r, r→p, ¬r→s, s→t
 conclusion: t

Step

1. $\neg p \land r$

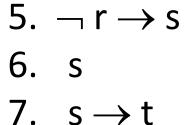
2. ¬ p

4. ¬ r

3. $r \rightarrow p$

Reason

- Premise
- Simplification using (1)
- Premise
- Modus Tollens using (2) and (3)
- Premise
 - Modus Ponens using (4) and (5)
- Premise
 - Modus Ponens using (6) and (7)



8. t

Applying Rules of Inferences

- Example 2: It is known that
 - 1. If you send me an email, then I will finish my program.
 - 2. If you do not send me an email, then I will go to sleep early.
 - 3. If I go to sleep early, I will wake up refreshed.
- Can you conclude "If I do not finish my program, then I will wake up refreshed"?

- To simplify the discussion, let
 - p := You send me an email
 - q := I finish my program
 - r := I go to sleep early
 - s := I wake up refreshed
- We will give a valid argument with premises: p→q, ¬p→r, r→s conclusion: ¬q→s

Step	Reason
1. $p \rightarrow q$	Premise
2. $\neg q \rightarrow s$	Contrapositive of (1)
3. $\neg p \rightarrow r$	Premise
4. $\neg q \rightarrow r$	Hypothetical Syllogism by (2) and (3)
5. $r \rightarrow s$	Premise
6. $\neg q \rightarrow s$	Modus Ponens by (4) and (5)

Applying Rules of Inferences

- Example 3: It is known that
 - 1. A student in this class has not read the book.
 - 2. Everyone in this class passed the first exam.

 Can you conclude that "Someone who passed the first exam has not read the book"?

• To simplify the discussion, let

C(x) := x is a student in the class B(x) := x has read the book P(x) := x passed the first exam

• We will give a valid argument with premises: $\exists X (C(X) \land \neg B(X)), \forall X (C(X) \rightarrow P(X))$ conclusion: $\exists X (P(X) \land \neg B(X))$

Step

1. $\exists X (C(X) \land \neg B(X))$ 2. C(a) $\wedge \neg B(a)$ 3. C(a) 4. $\forall \chi (C(\chi) \rightarrow P(\chi))$ 5. C(a) \rightarrow P(a) 6. P(a) 7. ¬B(a) 8. $P(a) \wedge \neg B(a)$

Reason Premise **Existential Instantiation** Simplification by (2) Premise Universal Instantiation Modus Ponens by (3) and (5) Simplification by (2) Conjunction by (6) and (7) 9. $\exists \chi (P(\chi) \land \neg B(\chi))$ Existential Generalization

From Sherlock Holmes

• The following is from Silver Blaze, one of Sherlock Holmes stories (written by Sir Arthur Conan Doyle):

Gregory: Is there any other point to which you would wish to draw my attention?

- Holmes: To the curious incident of the dog in the night-time.
- Gregory: The dog did nothing in the night-time.
- Holmes: That was the curious incident.