FAS-PASS Maths

CSEC MATHEMATICS MAY 2019 PAPER 2

SECTION I

Answer ALL questions.
All working must be clearly shown.

1. (a) Using a calculator, or otherwise, evaluate EACH of the following:

(i) $\dfrac{2\frac{1}{4}-1\frac{3}{5}}{3}$

SOLUTION:

Required to evaluate: $\dfrac{2\frac{1}{4}-1\frac{3}{5}}{3}$

Solution:

Working the numerator first: $2\frac{1}{4}-1\frac{3}{5}=\frac{9}{4}-\frac{8}{5}$

$$=\frac{5(9)-4(8)}{20}$$

$$=\frac{45-32}{20}$$

$$=\frac{13}{20}$$

So, $\dfrac{2\frac{1}{4}-1\frac{3}{5}}{3}=\dfrac{\frac{13}{20}}{3}$

$$=\frac{13}{20\times 3}$$

$$=\frac{13}{60}\text{ (in exact form)}$$

(ii) $2.14\sin 75°$, giving your answer to 2 decimal places.

SOLUTION:
Required to evaluate: $2.14\sin 75°$ correct to 2 decimal places
Solution:

$$2.14\sin 75° = 2.14\times 0.965\,9$$

$$=2.06\underline{\underline{7}}$$

$$=2.07\text{ (correct to 2 decimal places)}$$

(b) Irma's take-home pay is $4 320 per fortnight (**every** two weeks). Each fortnight Irma's pay is allocated according to the following table.

Item	Amount Allocated
Rent	$\$x$
Food	$629
Other living expenses	$\$2x$
Savings	$1 750
Total	**$4 320**

(i) What is Irma's **annual** take-home pay? (Assume she works 52 weeks in any given year.)

SOLUTION:
Data: Table showing the allocation of Irma's $4 320 per fortnight pay on various items.
Required to find: Irma's annual take-home pay
Solution:
Irma's pay is $4 320 per fortnight.

There are 52 weeks in a year and which is $\frac{52}{2} = 26$ fortnights.

So, Irma's annual take-home pay $= \$4\ 320 \times 26$
$= \$112\ 320$

(ii) Determine the amount of money that Irma allocated for rent each month.

SOLUTION:
Required to determine: The amount of money Irma spends on rent each month
Solution:

$$x + 629 + 2x + 1750 = 4320 \quad \text{(data)}$$
$$x + 2x + 2379 = 4320$$
$$3x = 4320 - 2379$$
$$3x = 1941$$
$$x = \frac{1941}{3}$$
$$x = 647$$

$\therefore$ Allocation for rent per fortnight $= \$x = \$647$
There are 2 fortnights per month.
So Irma's rent per month =$647 × 2 = $ 1 2 94

= $ 1 2 94

(iii) All of Irma's savings is used to pay her son's university tuition cost, which is $150 000.

If Irma's pay remains the same and she saves the same amount each **month**, what is the minimum number of years that she must work in order to save enough money to cover her son's tuition cost?

SOLUTION:
Data: Irma's son's tuition costs $150 000 and her pay and the amount of money she saves each month remains the same.
Required to find:
Solution:
Irma saves $1 750 per fortnight.
So, each year, Irma saves $\$1\ 750 \times 26 = \$45\ 500$

To save $150 00 the number of years will be $\dfrac{150\ 000}{45\ 500} = 3.296$

If the number of years is to be taken as a positive integer then the number of years will be the next integer after 3.296 which is 4.
$\therefore$ Irma must work for 4 years in order to save enough money to cover her son's tuition.
(After 3 years, Irma would not have saved up the amount)

2. (a) Simplify completely:

(i) $3p^2 \times 4p^5$

SOLUTION:
Required to simplify: $3p^2 \times 4p^5$
Solution:
$$\begin{aligned} 3p^2 \times 4p^5 &= 3 \times 4 \times p^{2+5} \\ &= 12p^7 \end{aligned}$$

(ii) $\dfrac{3x}{4y^3} \div \dfrac{21x^2}{20y^2}$

SOLUTION:
Required to simplify: $\dfrac{3x}{4y^3} \div \dfrac{21x^2}{20y^2}$
Solution:
$$\frac{3x}{4y^3} \div \frac{21x^2}{20y^2} = \frac{3x}{4y^3} \times \frac{20y^2}{21x^2}$$

$$= \frac{\cancel{3} \times \cancel{20}^{5}}{\cancel{4} \times \cancel{21}_{7}} y^{2-3} x^{1-2}$$

$$= \frac{5x^{-1}y^{-1}}{7} \text{ or } \frac{5}{7xy}$$

(b) Solve the equation $\frac{3}{7x-1} + \frac{1}{x} = 0$.

SOLUTION:

Required to solve: $\frac{3}{7x-1} + \frac{1}{x} = 0$

Solution:

$$\frac{3}{7x-1} + \frac{1}{x} = 0$$

$$\frac{3(x)+1(7x-1)}{x(7x-1)} = 0$$

$$\frac{3x+7x-1}{x(7x-1)} = 0$$

So $$\frac{10x-1}{x(7x-1)} = 0$$

$$10x-1 = 0(x)(7x-1)$$

$$10x-1 = 0$$

$$x = \frac{1}{10}$$

(c) When a number, x, is multiplied by 2, the result is squared to give a new number. y.

(i) Express y in terms of x.

SOLUTION:

Data: A number, x, when multiplied by 2, the result is squared to give a new number. y.

Required to express: y in terms of x

Solution:

$$(2x)^2 = y$$

$$y = 4x^2$$

(ii) Determine the two values of x that satisfy the equation $y = x$ AND the equation derived in (c) (i).

SOLUTION:
Required to determine: the two values of x that satisfy the equations $y = x$ and $y = 4x^2$.
Solution:
$y = x$ (data)
Substituting $y = x$ in the equation of (i) we get:

$$x = 4x^2$$

So $4x^2 - x = 0$
$x(4x - 1) = 0$
And $x = 0$ or $4x - 1 = 0$
and $x = \frac{1}{4}$

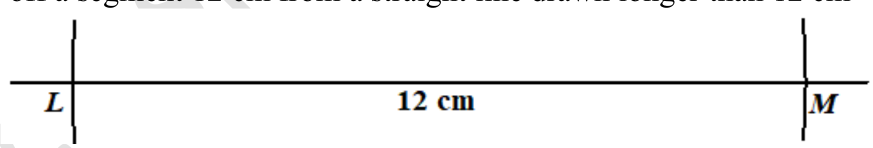
Hence, $x = 0$ or $\frac{1}{4}$.

3. (a) Using a ruler, a pencil and a pair of compasses only, construct the triangle NLM, in which $LM = 12$ cm, $\angle MLN = 30°$ and $\angle LMN = 90°$.

(Credit will be given for clearly drawn construction lines.)

SOLUTION:
Required to construct: Triangle NLM with $LM = 12$ cm, $\angle MLN = 30°$ and $\angle LMN = 90°$.
Construction:

We cut off a segment 12 cm from a straight line drawn longer than 12 cm

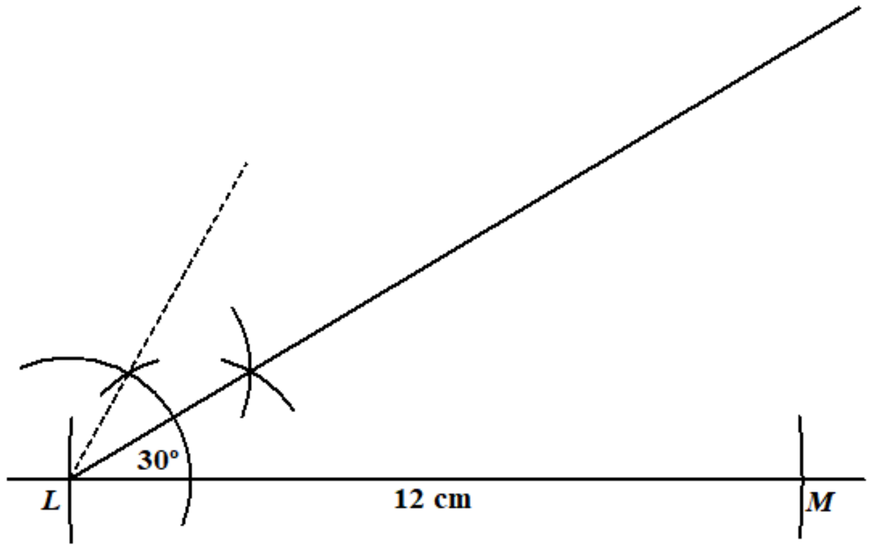


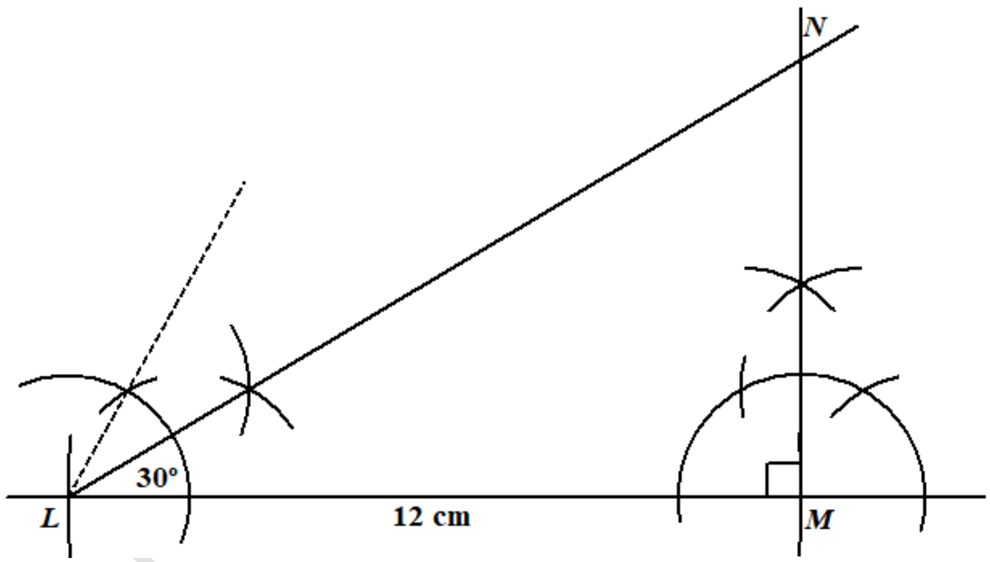
At the point L, we construct an angle of 60^0 and bisect this angle to obtain $\angle MLN = 30°$

N has not yet been obtained but will lie on the line of bisection

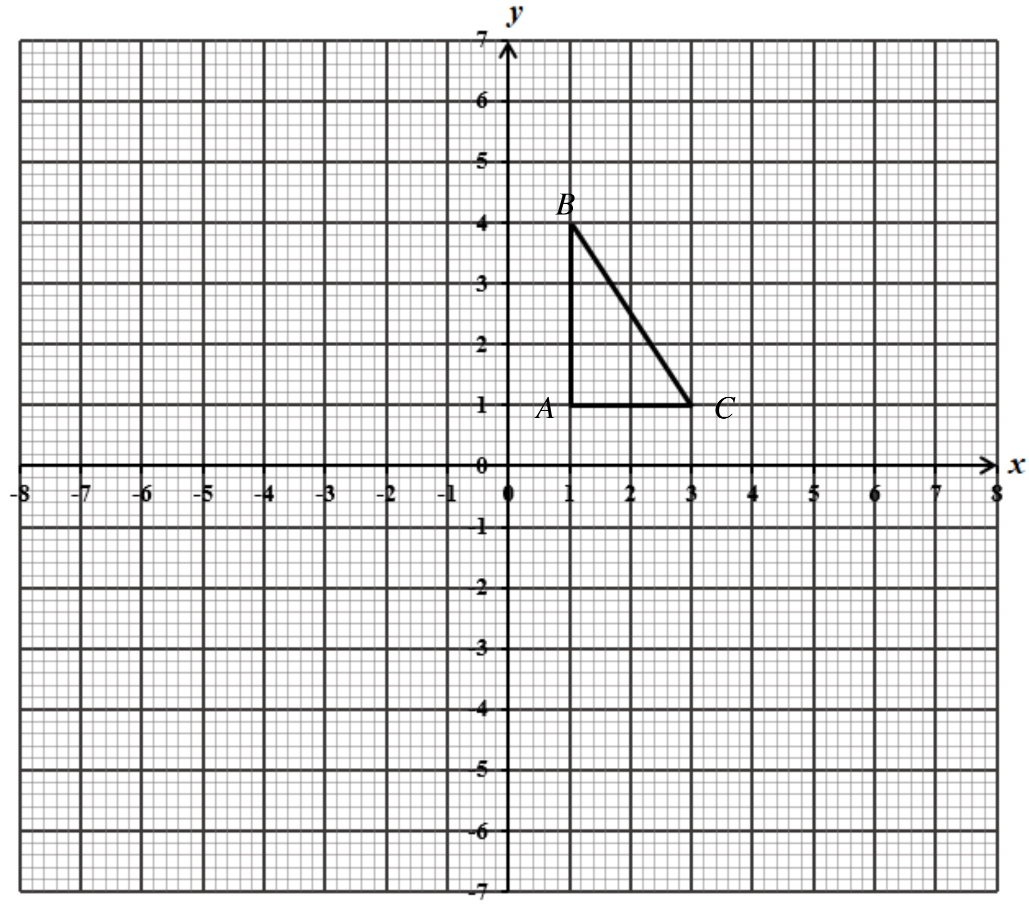
At the point M, we construct an angle of 90^0

The line from M and the line drawn from L will meet at N.





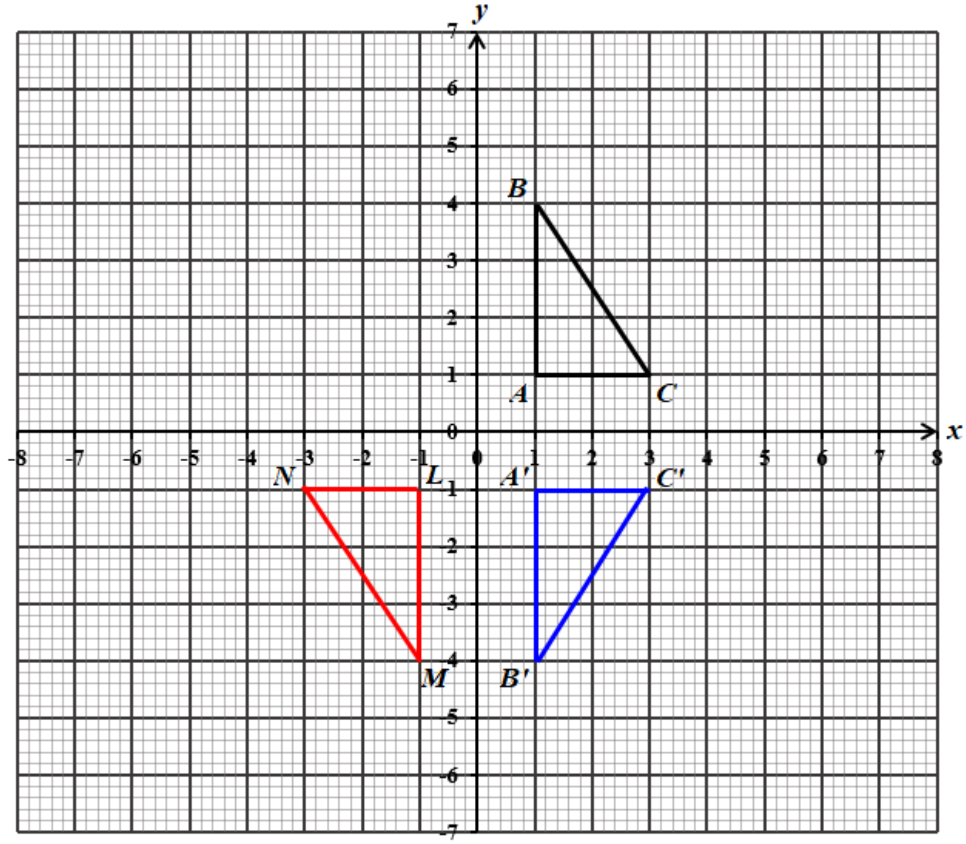
(b) Triangle ABC with vertices $A\left(1,\,1\right)$, $B\left(1,\,4\right)$ and $C\left(3,\,1\right)$ is shown on the diagram below.



ΔABC is mapped onto ΔLMN by a reflection in the x – axis followed by a reflection in the y – axis.

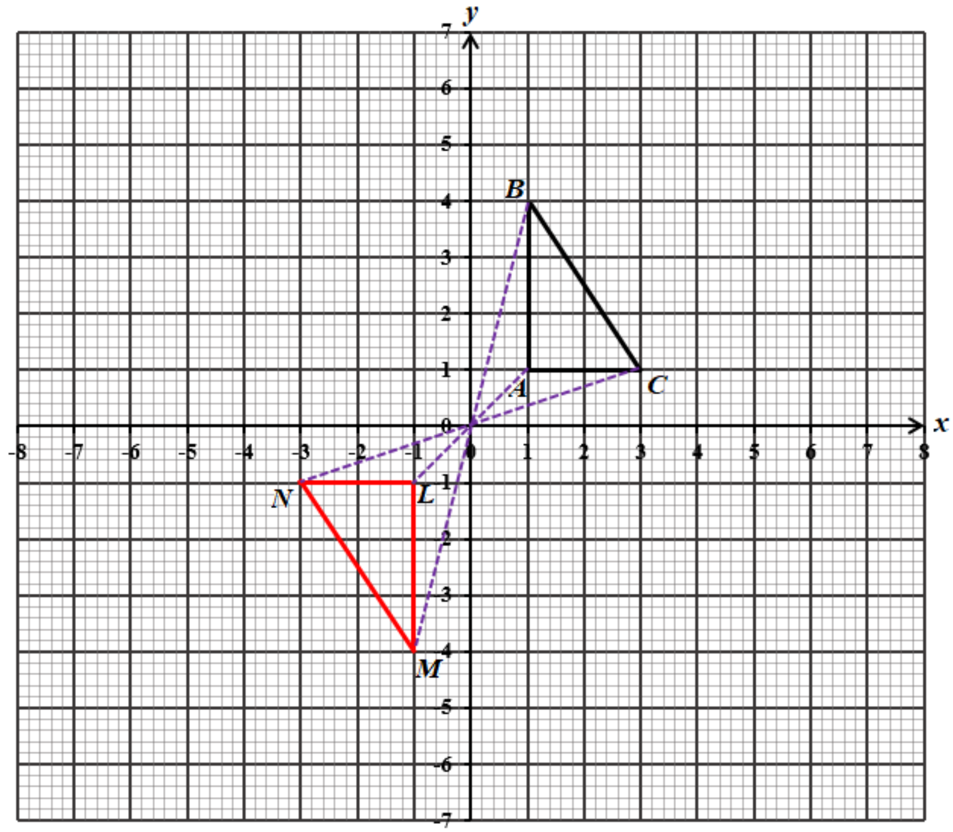
(i) On the diagram, draw and label ΔLMN.

SOLUTION:
Data: Diagram showing triangle ABC with vertices $A\left(1, 1\right)$, $B\left(1, 4\right)$ and $C\left(3, 1\right)$. ΔABC is mapped onto ΔLMN by a reflection in the x – axis followed by a reflection in the y – axis.
Required to draw: ΔLMN
Diagram:



(ii) Describe fully a single transformation that maps ΔABC onto ΔLMN.

SOLUTION:
Required to describe: The single transformation that maps ΔABC onto ΔLMN.
Solution:



ΔLMN is congruent to ΔABC and re-oriented with respect to ΔABC. By joining the object points to their corresponding image points we note that these lines all pass though O and which is the center of rotation. The angles CON or BOM or AOL are all 180^0

The transformation is a 180° clockwise or anti-clockwise rotation about O.

(iii) State the 2×2 matrix for the transformation that maps ΔABC onto ΔLMN.

SOLUTION:

Required to state: The 2×2 matrix for the transformation that maps ΔABC onto ΔLMN.

Solution:

The matrix maps $A(1,1)$ onto $L(-1,-1)$; $B(1,4)$ onto $M(-1,-4)$ and $C(3,1)$ onto $N(-3,-1)$. Consider:

This transformation preserves order but changes direction. By inspection, we notice that:

$B(1,4) \rightarrow M(-1,-4)$ and $C(3,1) \rightarrow N(-3,-1)$

$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$

The 2×2 matrix which represents this transformation is $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$.

4. (a) The quantity P varies inversely as the square of V.

(i) Using the letters P, V and k, form an **equation** connecting the quantities P and V.

SOLUTION:
Data: P varies inversely as the square of V.
Required to write: An equation connecting P and V
Solution:

So $P \propto \frac{1}{V^2}$

Hence $P = k \times \frac{1}{V^2}$, where k is the constant of proportionality

$$P = \frac{k}{V^2}$$

(ii) Given that $V = 3$ when $P = 4$, determine the positive value of V when $P = 1$.

SOLUTION:
Data: When $P = 4,\ V = 3$
Required to determine: The value of V when $P = 1$.
Solution:

$V = 3$ when $P = 4$

$$4 = \frac{k}{(3)^2}$$

So $k = 4 \times (3)^2$

$= 36$

$$P = \frac{36}{V^2}$$

When $P = 1$:

$$1 = \frac{36}{V^2}$$

$$V^2 \times 1 = 36$$

$$V^2 = 36$$

$$V = \sqrt{36}$$

$$= \pm 6$$

$V > 0$ (data)

So, $V = 6$ only

(b) **(i)** Given that x is a real number, solve the inequality $-7 < 3x + 5 \leq 7$.

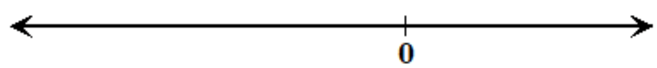
SOLUTION:
Data: $-7 < 3x + 5 \leq 7$ and x is a real number.
Required to solve: For x
Solution:

$$-7 < 3x + 5$$
$$-7 - 5 < 3x$$
$$-12 < 3x$$
$$(\div 3)$$
$$-4 < x$$

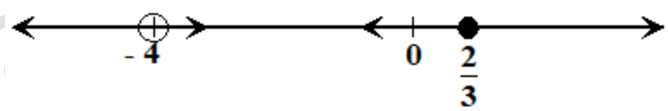
$$3x + 5 \leq 7$$
$$3x \leq 7 - 5$$
$$3x \leq 2$$
$$(\div 3)$$
$$x \leq \frac{2}{3}$$

Hence, $-4 < x \leq \frac{2}{3}$.

(ii) Represent your answer in (b) (i) on the number line shown below.



SOLUTION:
Required to represent: The solution to $-7 < 3x + 5 \leq 7$ on a number line
Solution:



(c) The equation of a straight line is given as $\frac{x}{3} + \frac{y}{7} = 1$. This line crosses the y – axis at Q.

(i) Determine the coordinates of Q.

SOLUTION:
Data: The line with equation $\frac{x}{3} + \frac{y}{7} = 1$ crosses the y – axis at Q.
Required to determine: The coordinates of Q
Solution:
A line crosses the y – axis at $x = 0$
Let $x = 0$

$$\frac{x}{3}+\frac{y}{7}=1$$
$$\frac{0}{3}+\frac{y}{7}=1$$
$$\frac{y}{7}=1$$
$$y=1\times 7$$
$$y=7$$

$$Q=(0,\ 7)$$

(ii) What is the gradient of this line?

SOLUTION:

Required to find: The gradient of the line $\frac{x}{3}+\frac{y}{7}=1$.

Solution:

$$\frac{x}{3}+\frac{y}{7}=1$$
$$\frac{y}{7}=-\frac{x}{3}+1$$
$$(\times 7)$$

$y=-\frac{7}{3}x+7$ is of the form $y=mx+c$, where $m=-\frac{7}{3}$ is the gradient.

5. The cumulative frequency distribution of the volume of petrol needed to fill the tanks of 150 different vehicles is shown below.

Volume (litres)	Cumulative Frequency
11 – 20	24
21 – 30	59
31 – 40	101
41 – 50	129
51 – 60	150

(a) For the class 21 – 30, determine the

(i) the lower class boundary

SOLUTION:
Data: Cumulative frequency table showing the distribution of the volume of petrol needed to fill the tanks of 150 different vehicles.
Required to find: The lower class boundary for the class 21 – 30

Solution:
For the class 21 – 30:
21 – Lower class limit
30 – Upper class limit
If the volume is V, then $20.5 \leq V < 30.5$, where 20.5 is the lower class boundary and 30.5 is the upper class boundary.

$\therefore$ The lower class boundary of the class interval 21 – 30 is 20.5.

(ii) class width

SOLUTION:
Required to find: The class width for the class 21 – 30.
Solution:

$$\begin{aligned}\text{Class width} &= \text{Upper class boundary} - \text{Lower class boundary} \\ &= 30.5 - 20.5 \\ &= 10\end{aligned}$$

(b) How many vehicles were recorded in the class 31 – 40?

SOLUTION:
Required to find: The number of vehicles in the class 31 – 40.
Solution:

Volume (litres)	Frequency	Cumulative Frequency
$\vdots$	$\vdots$	$\vdots$
21 – 30		59
31 – 40	x	$59 + x = 101$
$\vdots$	$\vdots$	$\vdots$

So, the number of vehicles recorded in the class 31 – 40 will be $101 - 59 = 42$.

(c) A vehicle is chosen at random from the 150 vehicles. What is the probability that the volume of petrol needed to fill the tank is **more** than 50.5 litres? **Leave your answer as a fraction**.

SOLUTION:
Required to find: The probability the volume of petrol needed to fill the tank is more than 50.5 litres
Solution:

Volume (litres)	Class Boundaries	Cumulative Frequency
$\vdots$	$\vdots$	$\vdots$
41 – 50	$40.5 \leq V < 50.5$	129
51 – 60	$50.5 \leq V < 60.5$	150

So, the number of vehicles that required more than 50.5 litres is $150-129=21$.

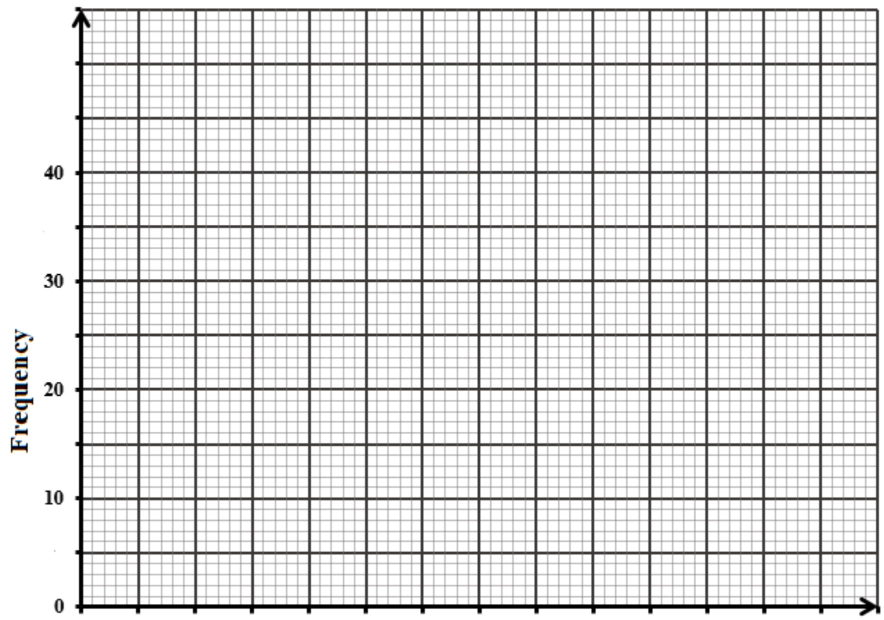
P(vehicle chosen at random requires more than 50.5 litres of petrol to be filled)
$$= \frac{\text{No. of vehicles requiring more than 50.5 litres}}{\text{Total no. of vehicles}}$$
$$= \frac{21}{150}$$
$$= \frac{7}{50}$$

(d) Byron estimates the median amount of petrol to be 43.5 litres. Explain why Byron's estimate is INCORRECT.

SOLUTION:
Data: Byron estimates the median amount of petrol to be 43.5 litres.
Required to explain: Why Byron's estimate is INCORRECT.
Solution:
$$\frac{1}{2}\left(\text{Cumulative frequency}\right) = \frac{1}{2}(150)$$
$$= 75$$

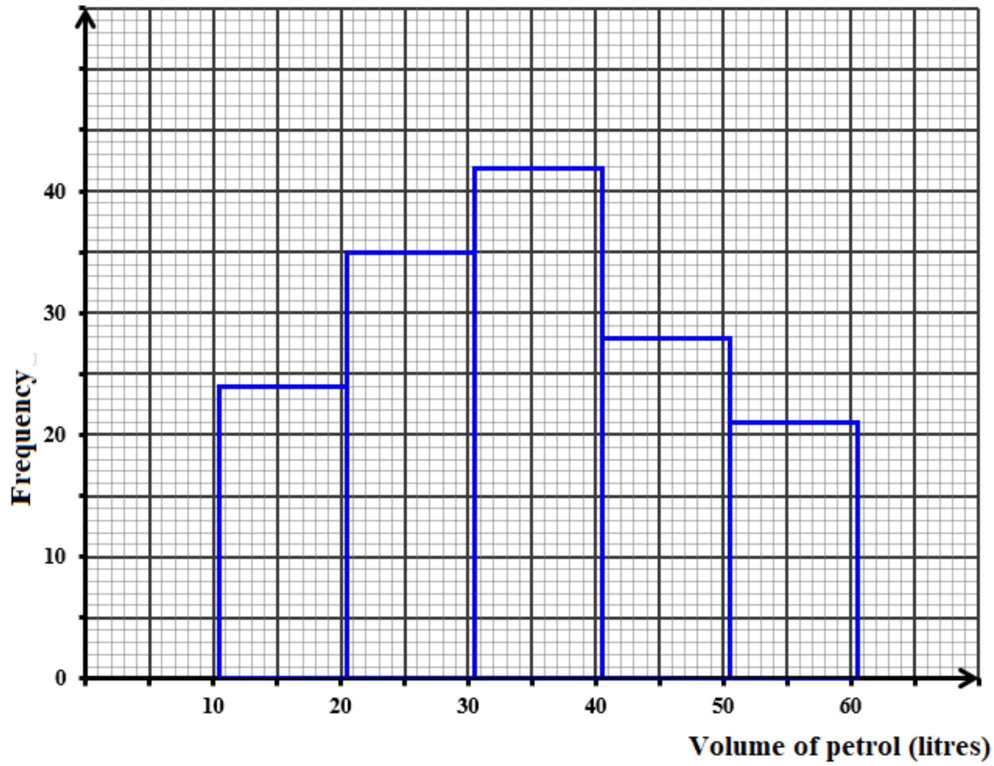
So the 75th value corresponds to the median. The 75th value lies in the class 31 – 40 or more precisely $30.5 \le V < 40.5$. The median would be $\frac{31+40}{2}$ or $\frac{30.5+40.5}{2}$ which is the mid-class interval of the class $=35.5\,\text{l}$. So Byron's estimate of 43.5 is incorrect.

(e) On the partially labelled grid below, construct a histogram to represent the distribution of the volume of petrol needed to fill the tanks of the 150 vehicles.



SOLUTION:
Required to construct: A histogram to represent the distribution of the volume of petrol needed to fill the tanks of the 150 vehicles
Solution:

Volume in litres **L.C.L. U.C.L.**	**Class Boundaries** **L.C.B. U.C.B.**	**Frequency**	**Cumulative Frequency**
11 – 20	$10.5 \le V < 20.5$	24	24
21 – 30	$20.5 \le V < 30.5$	$59 - 24 = 35$	59
31 – 40	$30.5 \le V < 40.5$	$101 - 59 = 42$	101
41 – 50	$40.5 \le V < 50.5$	$129 - 101 = 28$	129
51 – 60	$50.5 \le V < 60.5$	$150 - 129 = 21$	150



6. (a) The scale on a map is $1:25\,000$.

(i) Determine the actual distance, in km, represented by 0.5 cm on the map.

SOLUTION:
Data: The scale of a map is $1:25\,000$.
Required to determine: The actual distance, in km, represented by 0.5 cm on the map
Solution:
Scale is $1:25\,000$

$$\therefore \quad 1 \text{ cm} \equiv 25\,000 \text{ cm}$$

$$0.5 \text{ cm} \equiv 0.5 \times 25\,000 \text{ cm}$$

$$= 12\,500 \text{ cm}$$

$$1 \text{ km} \equiv 100\,000 \text{ cm}$$

$$100\,000 \text{ cm} \equiv 1 \text{ km}$$

$$1 \text{ cm} \equiv \frac{1}{100\,000} \text{ km}$$

$$\text{And } 12\,500 \text{ cm} \equiv \frac{1}{100\,000} \times 12\,500 \text{ km}$$

$$= 0.125 \text{ km or } \frac{1}{8} \text{ km}$$

(ii) Calculate the actual area, in km^2, represented by $2.25\ \text{cm}^2$ on the map.

SOLUTION:
Required to calculate: the actual area, in km^2, represented by $2.25\ \text{cm}^2$ on the map
Calculation:

$$1\ \text{cm} \equiv \frac{25\,000}{100\,000}\ \text{km}$$
$$= \frac{1}{4}\ \text{km}$$

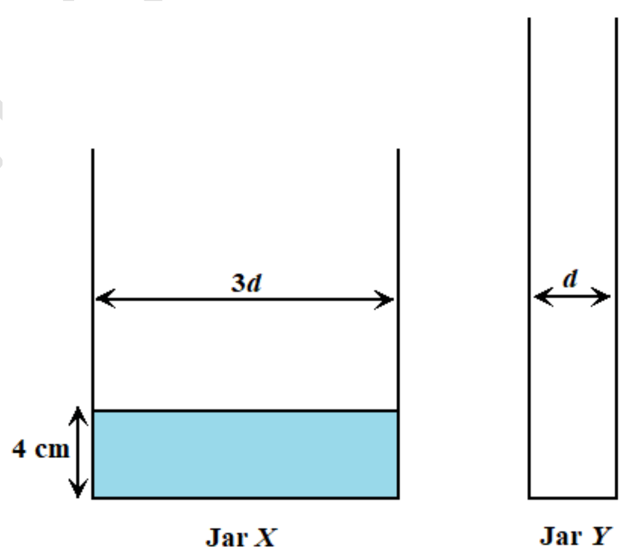
So $$1\ \text{cm}^2 \equiv \left(\frac{1}{4} \times \frac{1}{4}\right)\ \text{km}^2$$

And $$2.25\ \text{cm}^2 \equiv \left(\frac{1}{4} \times \frac{1}{4}\right) \times 2.25\ \text{km}^2$$
$$= \frac{1}{16} \times \frac{9}{4}\ \text{km}^2$$
$$= \frac{9}{64}\ \text{km}^2$$

$$= 0.140625\ \text{km}^2$$

(b) The diagram below (**not drawn to scale**) shows the cross-section of two cylindrical jars, Jar X and Jar Y. The diameters of Jar X and Jar Y are $3d$ cm and d cm respectively.

Initially, Jar Y is empty and Jar X contains water to a height (depth) of 4 cm.



(i) Determine, in terms of π and d, the volume of water in Jar X.

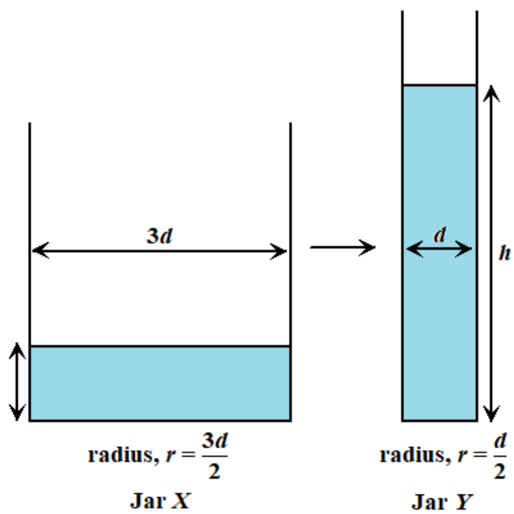
SOLUTION:
Data: Diagram showing two jars X and Y with diameters $3d$ cm and d cm respectively. Jar Y is empty and Jar X contains water to a height (depth) of 4 cm.
Required to determine: The volume of Jar X, in terms of π and d
Solution:
Volume of water in $X = \pi r^2 h$, where $r =$ radius and $h =$ height

$$= \pi\left(\frac{3d}{2}\right)^2 \times 4$$

$$= 9\pi d^2 \text{ cm}^3$$

(ii) If all the water from Jar X is now poured into Jar Y, calculate the height it will reach.

SOLUTION:
Required to find: The height of water in Jar Y if the contents of Jar X is poured into it
Solution:



Water from X is poured into Y.
Let the height reached be h cm.

Volume of water in $Y = \pi\left(\frac{d}{2}\right)^2 h$

$$= \frac{\pi d^2 h}{4}$$

Hence, $9\pi d^2 = \dfrac{\pi d^2 h}{4}$

$$9 = \frac{h}{4} \qquad [\div \pi d^2]$$

$$h = 36 \text{ cm}$$

So the height of the water in Jar Y is 36 cm.

7. (a) The nth term, T_n, of a sequence is given by $T_n = 3n^2 - 2$.

(i) Show that the first term of the sequence is 1.

SOLUTION:
Data: $T_n = 3n^2 - 2$, where T_n is the nth term in a sequence.
Required to show: The first term of the sequence is 1
Solution:
$T_n = 3n^2 - 2$
When $n = 1$
$T_1 = 1^{st}$ term
$= 3(1)^2 - 2$
$= 3(1) - 2$
$= 3 - 2$
$= 1$
Q.E.D.

(ii) What is the third term of the sequence?

SOLUTION:
Required to find: The third term in the sequence
Solution:
When $n = 3$
$T_3 = 3(3)^2 - 2$
$= 3(9) - 2$
$= 27 - 2$
$= 25$

(iii) Given that $T_n = 145$, what is the value of n?

SOLUTION:
Data: $T_n = 145$
Required to find: n
Solution:
$T_n = 145$
So $3n^2 - 2 = 145$

$$3n^2 = 145 + 2$$
$$3n^2 = 147$$
$$n^2 = \frac{147}{3}$$
$$n^2 = 49$$
$$n = \sqrt{49}$$
$$= \pm 7$$

n is a positive integer so $n = 7$

(b) The first 8 terms of another sequence with n^{th} term, $U(n)$ are 1, 1, 2, 3, 5, 8, 13, 21, where $U(1) = 1$, $U(2) = 1$ and $U(n) = U(n-1) + U(n-2)$ for $n \geq 3$.

For example, the fifth and seventh terms are
$U(5) = U(4) + U(3) = 3 + 2 = 5$
$U(7) = U(6) + U(5) = 8 + 5 = 13$

(i) Write down the next two terms in the sequence, that is, $U(9)$ and $U(10)$.

SOLUTION:
Data: The first 8 terms of another sequence with n^{th} term, $U(n)$ are 1, 1, 2, 3, 5, 8, 13, 21, where $U(1) = 1$, $U(2) = 1$ and $U(n) = U(n-1) + U(n-2)$ for $n \geq 3$.
Required to find: $U(9)$ and $U(10)$
Solution:
The first 8 terms of the sequence 1,1, 2, 3, 5, 8, 13, 21
$U(1) = 1$
$U(2) = 1$
$U(3) = 2$
$U(n) = U(n-1) + U(n-2)$

$U(9) = U(9-1) + U(9-2)$ $= U(8) + U(7)$ $= 21 + 13$ $= 34$	$U(10) = U(10-1) + U(10-2)$ $= U(9) + U(8)$ $= 34 + 21$ $= 55$

(ii) Which term in the sequence is the sum of $U(18)$ and $U(19)$.

SOLUTION:
Required to find: The term in the sequence that is the sum of $U(18)$ and $U(19)$.
Solution:
$U(n) = U(n-1) + U(n-2)$
$U(n) = U(19) + u(18)$
$n - 1 = 19$ and $n - 2 = 18$
$n = 19 + 1 = 20$ OR $n = 18 + 2 = 20$

Hence, $U(18)+U(19)=U(20)$
Therefore, the 20th term of the sequence is the sum of $U(18)$ and $U(19)$.

(iii) Show that $U(20)-U(19)=U(19)-U(17)$.

SOLUTION:
Required to show: $U(20)-U(19)=U(19)-U(17)$
Proof:
$U(n) = U(n-1) + U(n-2)$

LHS	RHS
$U(20) - U(19)$	$U(19) - U(17)$
$= [U(19) + U(18)] - U(19)$	$= [U(18) + U(17)] - U(17)$
$= U(18)$	$= U(18)$

$$LHS = RHS = U(18)$$
$$U(20)-U(19)=U(19)-U(17)$$

Q.E.D

OR

$U(20) - U(19) = \{U(19) + U(18)\} - \{U(18) + U(17)\} = U(19) - U(17)$

Q.E.D

OR

$$U(1)=1$$
$$U(2)=1$$
$$U(3)=2$$
$$U(4)=3$$
$$U(5)=5$$
$$U(6)=8$$
$$U(7)=13$$
$$U(8)=21$$
$$U(9)=34$$
$$U(10)=55$$
$$U(11)=55+34=89$$
$$U(12)=89+55=144$$
$$U(13)=144+89=233$$
$$U(14)=233+144=377$$
$$U(15)=377+233=610$$
$$U(16)=610+377=987$$
$$U(17)=987+610=1597$$
$$U(18)=1597+987=2584$$
$$U(19)=2584+1597=4181$$
$$U(20)=4181+2584=6765$$

$$\begin{aligned} U(20)-U(19) &= 6765-4181 \\ &= 2584 \end{aligned}$$

$$\begin{aligned} U(19)-U(17) &= 4181-1597 \\ &= 2584 \end{aligned}$$

So $U(19)-U(17)=U(20)-U(19)$

Q.E.D.

SECTION II
Answer ALL questions.
All working must be clearly shown.

ALGEBRA, RELATIONS, FUNCTIONS AND GRAPHS

8. (a) The functions *f* and *g* are defined by $f(x)=\frac{9}{2x+1}$ and $g(x)=x-3$.

(i) State a value of *x* that CANNOT be in the domain of *f*.

SOLUTION:
Data: $f(x)=\frac{9}{2x+1}$ and $g(x)=x-3$
Required to state: A value of *x* that cannot be in the domain of *f*
Solution:
As $2x+1\to 0$

$2x\to -1$

$x\to -\frac{1}{2}$

$f(x)\to \frac{9}{0}=\infty(\text{undefined})$

So $x=-\frac{1}{2}$ cannot be in the domain of $f(x)$. We say that $f(x)$ is undefined or not defined or discontinuous or not continuous for $x=-\frac{1}{2}$.

(ii) Find, in its simplest form, expressions for:

a) $fg(x)$

SOLUTION:
Required to find: $fg(x)$
Solution:

$f(x)=\frac{9}{2x+1}$ and $g(x)=x-3$

$fg(x)=f[g(x)]=f(x-3)$

$$=\frac{9}{2[g(x)]+1}$$

$$= \frac{9}{2(x-3)+1}$$

$$= \frac{9}{2x-6+1}$$

$$= \frac{9}{2x-5},\ x \neq 2\frac{1}{2}$$

b) $f^{-1}(x)$

SOLUTION:
Required to find: $f^{-1}(x)$
Solution:
Let $y = f(x)$

$$y = \frac{9}{2x+1}$$

$$y(2x+1) = 9$$

$$2xy + y = 9$$

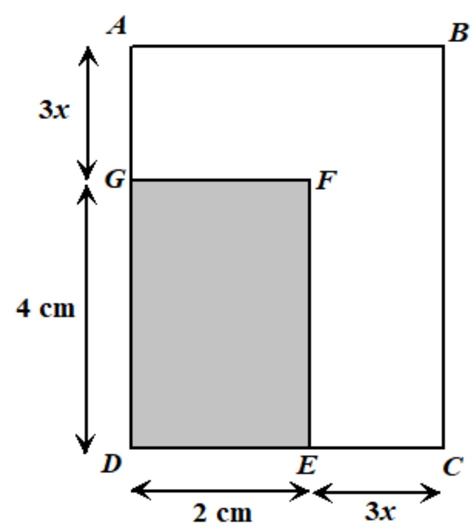
$$2xy = 9 - y$$

$$x = \frac{9-y}{2y}$$

Replace y by x to get:

$$f^{-1}(x) = \frac{9-x}{2x},\ x \neq 0$$

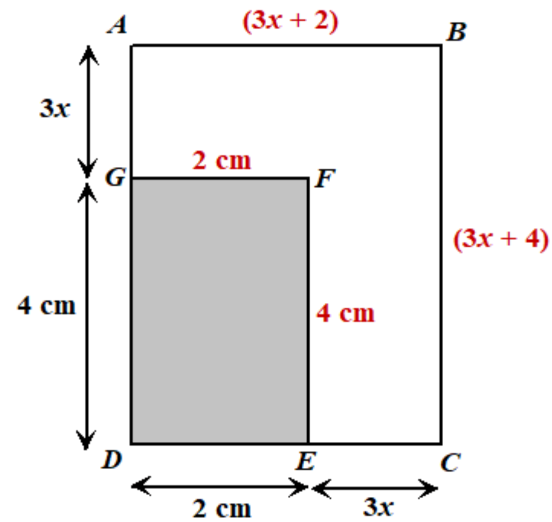
(b) The diagram below shows two rectangles, *ABCD* and *GFED*. *ABCD* has an area of 44 cm^2. *GFED* has sides 4 cm and 2 cm. $AG = EC = 3x$ cm.



(i) By writing an expression for the area of rectangle $ABCD$, show that $x^2 + 2x - 4 = 0$.

SOLUTION:
Data: Diagram showing two rectangles *ABCD* and *GFED*, such that *ABCD* has an area of 44 cm^2, *GFED* has sides 4 cm and 2 cm and $AG = EC = 3x$ cm.

Required To Show: $x^2 + 2x - 4 = 0$
Proof:



Length of $DC = (2 + 3x)$
Length of $AD = (4 + 3x)$

$$\text{Area of } ABCD = (2+3x)(4+3x)$$
$$= 8 + 12x + 6x + 9x^2$$
$$= 8 + 18x + 9x^2$$

Hence, $9x^2 + 18x + 8 = 44$
$$9x^2 + 18x - 36 = 0$$
$$(\div 9)$$
$$x^2 + 2x - 4 = 0$$

Q.E.D

(ii) Calculate, to 3 decimal places, the value of x.

SOLUTION:
Required to calculate: x, correct to 3 decimal places.
Calculation:

Recall if $ax^2+bx+c=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

When $x^2+2x-4=0$

$$x=\frac{-(2)\pm\sqrt{(2)^2-4(1)(-4)}}{2(1)}$$

$$=\frac{-2\pm\sqrt{4+16}}{2}$$

$$=\frac{-2\pm\sqrt{20}}{2}$$

$$=\frac{-2\pm4.4721}{2}$$

$x=\dfrac{2.4721}{2}$ or $x=\dfrac{-6.4721}{2}$

$=1.23605$ or $=-3.23605$

$=1.236$ or $=-3.236$ (correct to 3 decimal places)

(iii) Calculate the perimeter of the UNSHADED region.

SOLUTION:
Required to calculate: The perimeter of the unshaded region
Calculation:
Perimeter of the unshaded region

$$=(3x)+(3x+2)+(3x+4)+(3x)+(4)+(2)$$

$$=12x+12$$

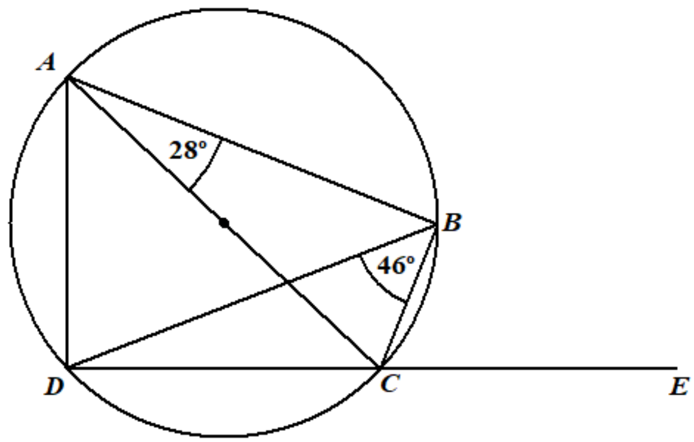
$$=12(1.23605)+12 \quad \text{(since } x \text{ is positive)}$$

$$=26.8326\text{ cm}$$

$$=26.833\text{ cm (correct to 3 decimal places)}$$

GEOMETRY AND TRIGONOMETRY

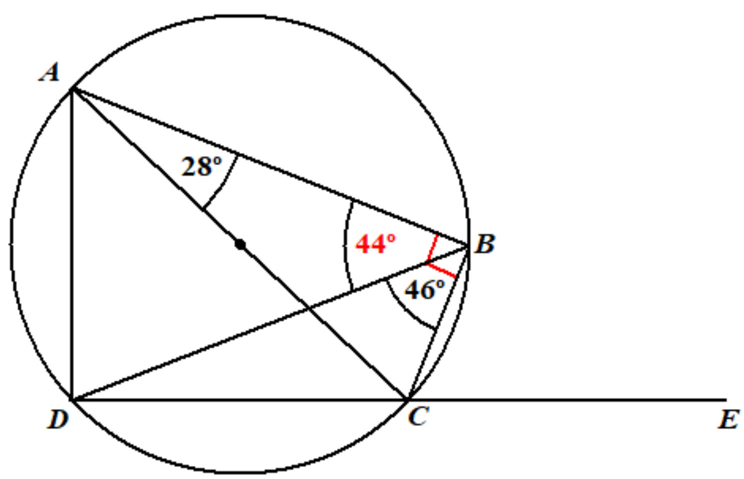
9. (a) The diagram below shows a circle where AC is a diameter. B and D are two other points on the circle and DCE is a straight line. Angle $CAB = 28°$ and $\angle DBC = 46°$.



Calculate the value of each of the following angles. Show detailed working where necessary and given a reason to support your answers.

(i) $\angle DBA$

SOLUTION:
Data: Diagram showing a circle where AC is a diameter. B and D are two other points on the circle and DCE is a straight line. Angle $CAB = 28°$ and $\angle DBC = 46°$.
Required to calculate: $\angle DBA$
Calculation:



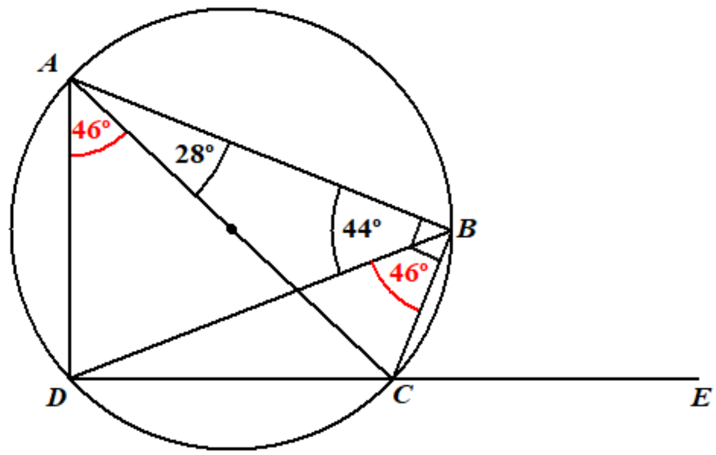
$A\hat{B}C = 90°$ (Angle in a semi-circle is a right angle.)

$A\hat{B}D = 90° - 46°$

$= 44°$

(ii) $\angle DAC$

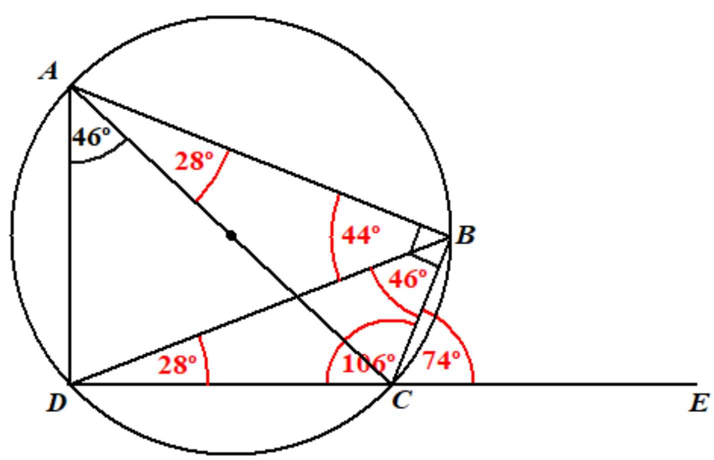
SOLUTION:
Required to calculate: $\angle DAC$
Calculation:



$D\hat{A}C = 46°$ (The angles subtended by a chord (DC) at the circumference of a circle ($D\hat{B}C$ and $D\hat{A}C$) and standing on the same arc are equal.)

(iii) $\angle BCE$

SOLUTION:
Required To Calculate: $\angle BCE$
Calculation:



$B\hat{D}C = 28^0$ (Angles subtended by a chord (BC) at the circumference of a circle ($B\hat{A}C$ and $B\hat{D}C$) and standing on the same arc are equal.)

Quadrilateral ABCD is cyclic and opposite angles of a cyclic quadrilateral are supplementary

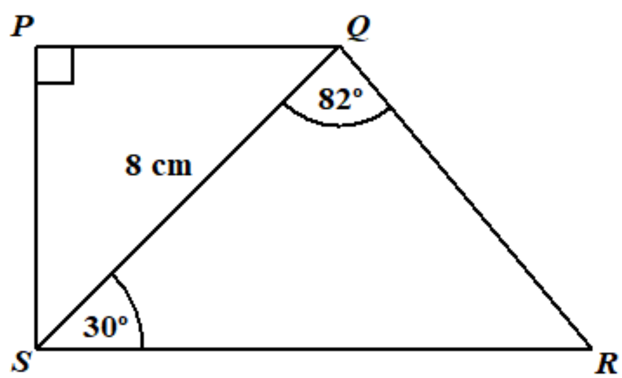
$B\hat{C}D = 180° - (46° + 28°)$
$= 180° - 74^0 = 106^0$

$B\hat{C}E = 180° - 106°$
$= 74°$ (Angles in a straight line add to 180^0.)

Alternative Method:

$B\hat{C}E = 28° + 46°$
$= 74°$ (Exterior angle of a triangle is equal to the sum of the interior opposite angles.)

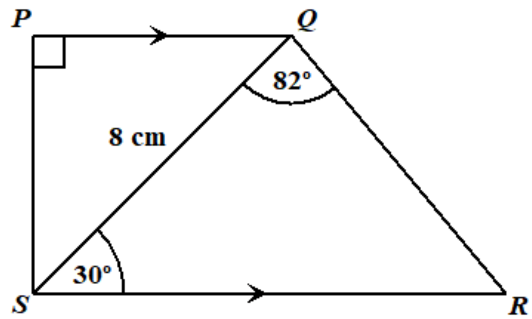
(b) The diagram below shows a quadrilateral $PQRS$ where PQ and SR are parallel. $SQ = 8\text{cm}$, $\angle SPQ = 90°$, $\angle SQR = 82°$ and $\angle QSR = 30°$.



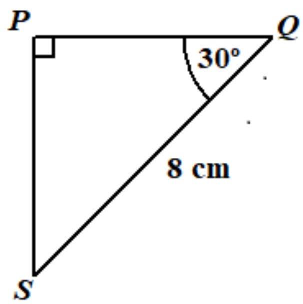
Determine

(i) the length PS

SOLUTION:
Data: Diagram showing a quadrilateral $PQRS$ where PQ and SR are parallel. $SQ = 8\text{cm}$, $\angle SPQ = 90°$, $\angle SQR = 82°$ and $\angle QSR = 30°$.
Required to determine: the length of PS
Solution:



Consider ΔPQS:



$$\sin 30° = \frac{PS}{8}$$
$$\therefore PS = 8 \times \sin 30°$$
$$= 4 \text{ cm}$$

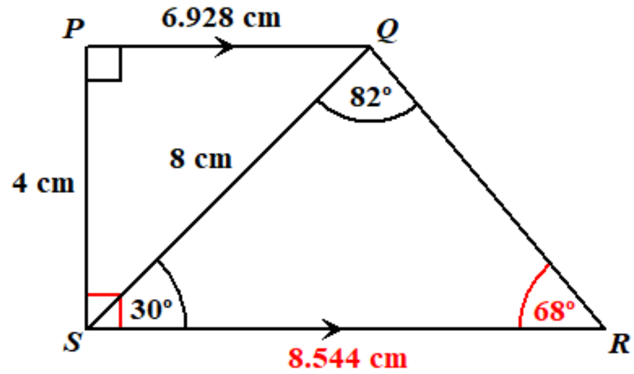
(ii) the length PQ

SOLUTION:
Required to determine: PQ
Solution:

$$\frac{PQ}{8} = \cos 30°$$
$$PQ = 8\cos 30°$$
$$= 8 \times \frac{\sqrt{3}}{2}$$
$$= 4\sqrt{3} \text{ cm (in exact form)}$$
$$\approx 6.93 \text{ cm (correct to 2 decimal places)}$$

(iii) the area of $PQRS$

SOLUTION:
Required to determine: The area of $PQRS$
Solution:



$$S\hat{R}Q = 180° - (30° + 82°)$$
$$= 68°$$

$P\hat{S}R = 90°$ (Co-interior angles)

$$\frac{SR}{\sin 82°} = \frac{8}{\sin 68°} \quad \text{(Sine rule)}$$
$$SR = \frac{8 \times \sin 82°}{\sin 68°}$$
$$= 8.544 \text{ cm}$$

$$\text{Area of } PQRS = \frac{1}{2}(6.928 + 8.544) \times 4$$
$$= 30.944 \text{ cm}^2$$
$$\approx 30.94 \text{ cm}^2 \text{ (correct to 2 decimal places)}$$

VECTORS AND MATRICES

10. (a) (i) a) Find the matrix product $\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix}$.

SOLUTION:

Required to find: $\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix}$

Solution:

$$\underset{2\times 2}{\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}}\underset{2\times 1}{\begin{pmatrix} k \\ 5 \end{pmatrix}} = \underset{2\times 1}{\begin{pmatrix} e_{11} \\ e_{21} \end{pmatrix}}$$

$$e_{11} = (-1\times k) + (3\times 5) = -k + 15$$
$$e_{21} = (4\times k) + (h\times 5) = 4k + 5h$$

So, $\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} -k+15 \\ 4k+5h \end{pmatrix}$

b) Hence, find the values of *h* and *k* that satisfy the matrix equation $\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

SOLUTION:

Data: $\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Required to find: *h* and *k*

Solution:

$$\begin{pmatrix} -1 & 3 \\ 4 & h \end{pmatrix}\begin{pmatrix} k \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Hence, $\begin{pmatrix} -k+15 \\ 4k+5h \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Equating corresponding entries:

$$-k + 15 = 0$$
$$k = 15$$

$$4k + 5h = 0$$
$$4(15) + 5h = 0$$
$$5h = -60$$
$$h = -12$$

So $k = 15$ and $h = -12$.

(ii) Using a matrix method, solve the simultaneous equations

$$\begin{aligned} 2x+3y&=5 \\ -5x+y&=13 \end{aligned}$$

SOLUTION:
Required to solve: $2x+3y=5$ and $-5x+y=13$ using matrix method
Solution:

$$\begin{aligned} 2x+3y&=5 \\ -5x+y&=13 \end{aligned}$$

Expressing the given equations in a matrix form:

$$\begin{pmatrix} 2 & 3 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 5 \\ 13 \end{pmatrix} \qquad \ldots\text{matrix equation}$$

Let $A=\begin{pmatrix} 2 & 3 \\ -5 & 1 \end{pmatrix}$

Finding A^{-1}:
First fid the determinant, $|A|$

$$\begin{aligned} |A| &= (2\times 1)-(3\times -5) \\ &= 2+15 \\ &= 17 \end{aligned}$$

$$\begin{aligned} \therefore A^{-1} &= \frac{1}{17}\begin{pmatrix} 1 & -(3) \\ -(-5) & 2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{17} & -\frac{3}{17} \\ \frac{5}{17} & \frac{2}{17} \end{pmatrix} \end{aligned}$$

Multiply the matrix equation by A^{-1}:

$$A \times A^{-1} \times \begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \times \begin{pmatrix} 5 \\ 13 \end{pmatrix}$$

$$I \times \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{17} & -\frac{3}{17} \\ \frac{5}{17} & \frac{2}{17} \end{pmatrix} \begin{pmatrix} 5 \\ 13 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e_{11} \\ e_{21} \end{pmatrix}$$

$$e_{11} = \left(\frac{1}{17} \times 5 \right) + \left(\frac{3}{17} \times 13 \right) = -2$$

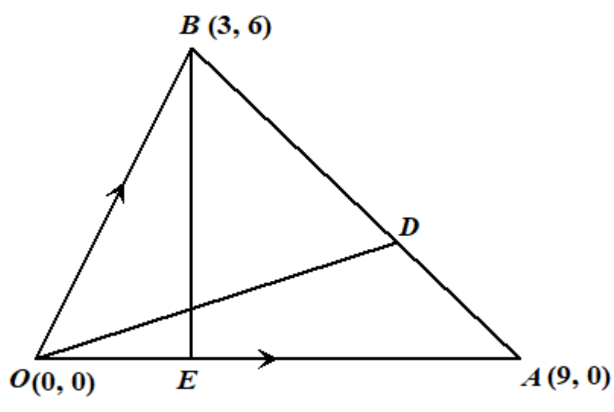
$$e_{21} = \left(\frac{5}{17} \times 5 \right) + \left(\frac{2}{17} \times 13 \right) = 3$$

So $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Equating corresponding entries:

$$x = -2, \; y = 3$$

(b) Relative to the origin $O\ (0,\ 0)$, the position vectors of the points A and B are $\overrightarrow{OA} = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ respectively. The points D and E are on AB and OA respectively and are such that $AD = \frac{1}{3}AB$ and $OE = \frac{1}{3}OA$. The following diagram illustrates this information.



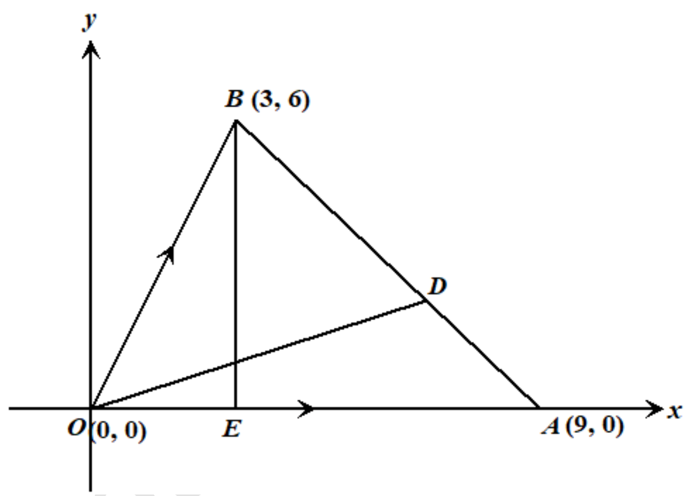
Express the following vectors in the form $\begin{pmatrix} a \\ b \end{pmatrix}$:

(i) $\overrightarrow{AB}$

SOLUTION:
Data: Diagram showing the points D and E are on AB and OA respectively and are such that $AD = \frac{1}{3}AB$ and $OE = \frac{1}{3}OA$. The position vectors of the points A and B are $\overrightarrow{OA} = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$.

Required to find: $\overrightarrow{AB}$ in the form $\begin{pmatrix} a \\ b \end{pmatrix}$

Solution:



$A = (9, 0)$

$\therefore \overrightarrow{OA} = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$

$B = (3, 6)$

$\therefore \overrightarrow{OB} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

$$= -\begin{pmatrix} 9 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$

$= \begin{pmatrix} -6 \\ 6 \end{pmatrix}$ is of the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a = -6$ and $b = 6$

(ii) $\overrightarrow{OD}$

SOLUTION:
Required to find: $\overrightarrow{OD}$
Solution:

$$\overrightarrow{AD} = \frac{1}{3}\overrightarrow{AB}$$
$$= \frac{1}{3}\begin{pmatrix} -6 \\ 6 \end{pmatrix}$$
$$= \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$$
$$= \begin{pmatrix} 9 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} 7 \\ 2 \end{pmatrix} \text{ is of the form } \begin{pmatrix} a \\ b \end{pmatrix}, \text{ where } a = 7 \text{ and } b = 2.$$

(iii) $\overrightarrow{BE}$

SOLUTION:
Required to find: $\overrightarrow{BE}$
Solution:

$$\overrightarrow{OE} = \frac{1}{3}\overrightarrow{OA}$$
$$= \frac{1}{3}\begin{pmatrix} 9 \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

$$\overrightarrow{BE} = \overrightarrow{BO} + \overrightarrow{OE}$$
$$= -\begin{pmatrix} 3 \\ 6 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$
$$= \begin{pmatrix} 0 \\ -6 \end{pmatrix} \text{ is of the form } \begin{pmatrix} a \\ b \end{pmatrix}, \text{ where } a = 0 \text{ and } b = -6.$$