Section 1 Computation

1. a) $\frac{1}{8} = 0.125$ [2]

b) $\frac{2}{3} = 0.667$ [2]

c) $\frac{1}{5} = 0.2$ [2]

2. a) $0.375 = \frac{375}{1000} = \frac{3}{8}$ [2]

b) $0.75 = \frac{75}{100} = \frac{3}{4}$ [2]

c) $0.02 = \frac{2}{100} = \frac{1}{50}$ [2]

3. a) $\frac{5}{8} = \frac{5}{8} \times \frac{100}{1} = 62.5\%$ [2]

b) $0.135 = 0.135 \times 100 = 13.5\ \%$ [2]

c) $\frac{5}{7} = \frac{5}{7} \times \frac{100}{1} = 71.4\%$ [2]

4. a) 3770 [1]

b) 1.07 [1]

c) 1.2 [1]

d) 25 000 [1]

5. a) $2\,500\,000 = 2.5 \times 10^6$ [1]

b) $0.003\,251 = 3.251 \times 10^{-3}$ [1]

c) $362\,000 = 3.62 \times 10^5$ [1]

d) $0.000\,009 = 9.0 \times 10^{-6}$ [1]

6. a) Angle A $= \frac{2}{6} \times 180 = 60°$ [1]

Angle B $= \frac{3}{6} \times 180 = 90°$ [1]

Angle C $= \frac{1}{6} \times 180 = 30°$ [1]

b) Length of longest piece $= \frac{5}{10} \times 40 = 20$ m [2]

c) Melissa receives $\frac{4}{5}$

Kerry receives $\frac{1}{5}$

Let x represent the total sum shared

Kerry received $\frac{1}{5}$ of x which is \$45

Then, x = \$45 (5) = \$225. [2]

7. a) Duration = 9 hr 30 min − 7 hr 25 min = 2 hr 5 min [2]

b) Time taken (hours) = 2 hr + $\left(\frac{5}{60}\right)$ hr = 2.08 hours

Average speed $= \frac{\text{distance}}{\text{time}} = \frac{92\text{ km}}{2.08} = 44.2$ km/hr [2]

8. a) $\frac{2\frac{1}{2} \times \frac{3}{5}}{\frac{1}{2} - \frac{1}{5}} = \frac{\frac{5}{2} \times \frac{3}{5}}{\frac{3}{10}} = \frac{\frac{15}{10}}{\frac{3}{10}} = \frac{15}{10} \div \frac{3}{10} = \frac{15}{10} \times \frac{10}{3} = 5$ [4]

b) $14.25 - (1.24)^2$
$= 14.25 - 1.5376$
$= 12.7124$
$= 12.7$ (to 3 significant figures) [3]

9. a) $2.14(3 - 1.26)$
$= 6.42 - 2.6964$
$= 3.7236$
$= 3.72$ (to 3 significant figures) [2]

b) $\frac{2.15}{0.8^2 - 0.22} = \frac{2.15}{0.64 - 0.22} = \frac{2.15}{0.42} = 5.119 = 5.12$
(to 3 significant figures) [3]

10. 1 cm = 10 000 000 cm (from scale given)
1 cm = 100 km
4.2 cm = 420 km [2]

11. a) 20 000 cm [1]

b) 0.2 km [1]

c) 0.9 km [1]

12. a) $0.125 \times 80 = 10$ [1]

b) 20% = 25

$1\% = \frac{25}{20}$

Therefore, $100\% = \frac{25}{20} \times 100 = 125$ [1]

c) $\frac{12}{60} \times 100 = 20\%$ [1]

d) $\frac{10}{50} = \frac{1}{5}$ [1]

13. a) $2.5 \times 1000 = 2500$ m [1]

b) 3000 cm = 30 m

30 m $= \frac{30}{1000} = 0.03$ km [1]

c) 1 litre = 1000 ml
2 litres = 2000 ml [1]

Section 2 Number theory

1. a) Whole numbers – B [1]

b) Integers – C [1]

c) Natural numbers – A [1]

2. a) Factors of 12 – 1, 2, 3, 4, 6, 12 [2]

b) Factors of 10 – 1, 2, 5, 10 [2]

c) Factors of 21 – 1, 3, 7, 21 [2]

3. a) Factors of 12 – 1, 2, 3, 4, 6, 12 [2]
Factors of 18 – 1, 2, 3, 6, 9, 18 [2]
HCF – 6 [1]

b) Factors of 30 – 1, 2, 3, 5, 6, 10, 15, 30 [2]
Factors of 15 – 1, 3, 5, 15 [2]
HCF – 15 [1]

c) Factors of 15 – 1, 3, 5, 15 [2]
Factors of 25 – 1, 5, 25 [2]
Factors of 40 – 1, 2, 4, 5, 8, 10, 20, 40 [2]
HCF – 5 [1]

4. a) The first four multiples of 5 – 5, 10, 15, 20 [2]

b) The first four multiples of 6 – 6, 12, 18, 24 [2]

c) The first four multiples of 12 – 12, 24, 36, 48 [2]

5. a) Multiples of 9 – 9, 18, 27, 36 [2]
Multiples of 12 – 12, 24, 36 [2]
LCM – 36 [1]

b) Multiples of 5 – 5, 10, 15, 20, 25, 30, 35, 40 [2]
Multiples of 8 – 8, 16, 24, 32, 40 [2]
LCM – 40 [1]

c) Multiples of 6 – 6, 12, 18, 24, 30, 36, 42 [2]
Multiples of 7 – 7, 14, 21, 28, 35, 42 [2]
LCM – 42 [1]

6. a) 8, 13, 21 [2]

b) 22, 26, 30 [2]

c) 25, 36, 49 [2]

7. a) Distributive law **[1]**

b) Associative law **[1]**

c) Commutative law **[1]**

8. a) $11011_2 = (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$
$= 16 + 8 + 0 + 2 + 1$
$= 27$ **[2]**

b) $232_4 = (2 \times 4^2) + (3 \times 4^1) + (2 \times 4^0)$
$= 32 + 12 + 2$
$= 46$ **[2]**

c) $1242_5 = (1 \times 5^3) + (2 \times 5^2) + (4 \times 5^1) + (2 \times 5^0)$
$= 125 + 50 + 20 + 2$
$= 197$ **[2]**

9. a)

2	15		
2	7	R	1
2	3	R	1
2	1	R	1
	0	R	1

1111_2 **[2]**

b)

5	27		
5	5	R	2
5	1	R	0
	0	R	1

102_5 **[2]**

c)

8	90		
8	11	R	2
8	1	R	3
	0	R	1

132_8 **[2]**

10. a) $110110_2 + 11001_2$

1001111

Answer – 1001111_2 **[3]**

b) $1111_2 - 1001_2$

0110

Answer – 110_2 **[3]**

Section 3 Consumer arithmetic

1. a) Total hire purchase price = 800 + (300 × 10)
= 800 + 3000 = $3800 **[2]**

b) Money that would be saved = 3800 − 3000 = $800 **[1]**

2. a) Amount received by the grocer = 120 × 32 = $3840 **[1]**

b) Profit = 3840 − 3360 = $480 **[1]**

c) Percentage profit = $\frac{480}{3360} \times 100 = 14.3\%$ **[2]**

3. a) Hourly rate = $\frac{600}{40}$ = $15 per hour **[1]**

b) Overtime rate = 1.5 × $15 = $22.50
Overtime wage = 8 × $22.50 = $180 **[2]**

c) Wage for 40 hours = 40 × $15 = $600
Wage for 10 hours overtime = 10 × $22.50 = $225
Total wage = $600 + $225 = $825 **[3]**

4. a) TT $6.30 = US $1.00
TT $5000 = US $ $\left(\frac{5000 \times 1.00}{6.30}\right)$ = US $793.65 **[2]**

b) Amount left = US $793.65 − US $650 = US $143.65
US $1.00 = TT $6.30
US $143.65 = TT $(143.65 × 6.30) = TT $905 **[3]**

5. a) Percentage profit = $\frac{\text{selling price} - \text{cost price}}{\text{cost price}} \times 100\%$
$= \frac{420\,000 - 350\,000}{350\,000} \times 100$
$= 20\%$ **[2]**

b) Loss = $75 000 − $40 000 = $35 000
Percentage loss = $\frac{\text{loss}}{\text{cost price}} \times 100\% = \frac{35\,000}{75\,000} \times 100 = 46.7\%$ **[2]**

c) i) Depreciation after 1 year = 0.10 × $180 000
= $18 000
Value of car after 1 year = $180 000 − $18 000
= $162 000 **[2]**

ii) Depreciation after 2 years = 0.10 × $162 000
= $16 200
Value of car after 2 years = $162 000 − $16 200
= $145 800 **[2]**

6. a) i) Total interest repaid = $\frac{P \times R \times T}{100} = \frac{120\,000 \times 10 \times 5}{100}$
= $60 000 **[2]**

ii) Total amount of money repaid = $60 000 + $120 000
= $180 000 **[2]**

iii) Monthly instalment = $\frac{180\,000}{5 \times 12}$ = $3000 per month **[2]**

b) Amount = $P\left(1 + \frac{R}{100}\right)^n$
$= 10\,000\left(1 + \frac{2.5}{100}\right)^5$
= $11 314.08
Compound interest = $11 314.08 − $10 000 = $1314.08 **[2]**

7. a) Discount = 10% × $6500 = $ 650
Amount paid = $6500 − $650 = $5850 **[2]**

b) Tax = 15% × $3000 = $450
Amount paid = $3000 + $450 = $3450 **[2]**

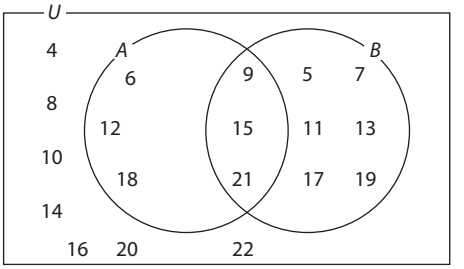
Section 4 Sets

1. a) $n(U) = 19$ **[1]**

b) A = {6, 9, 12, 15, 18, 21} **[2]**

c) B = {5, 7, 9, 11, 13, 15, 17, 19, 21} **[2]**

d)



[3]

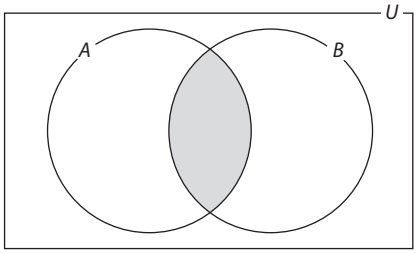
2. a) $n(A) = 8$ **[1]**

b) $n(B) = 5$ **[1]**

c) $A \cap B = \{9, 11, 15\}$ **[1]**

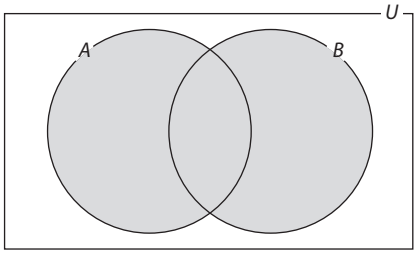
d) $A \cup B = \{2, 3, 4, 5, 7, 9, 11, 13, 15, 17\}$ **[1]**

3. a)



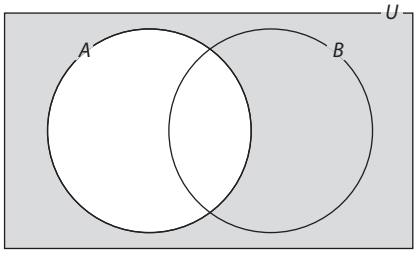
[1]

b)



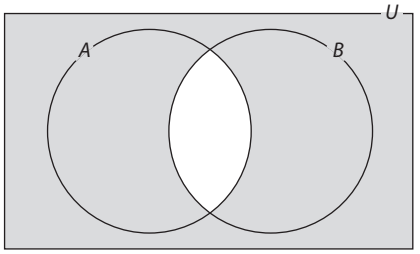
[1]

c)



[1]

d)



[1]

4. a) Number of subsets = $2^3 = 8$ [1]

b) { }, {2}, {4}, {6}, {2, 4}, {2, 6}, {4, 6}, {2, 4, 6} [2]

5. a) Infinite [1]

b) Finite [1]

c) Finite [1]

d) Infinite [1]

6. a) B and E [1]

b) D is a subset of B [1]

c) B and C OR C and D [1]

d) A [1]

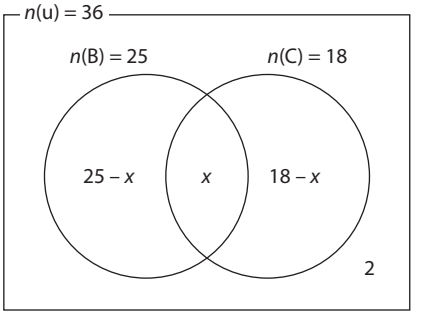
e) B, D OR E [1]

f) C [1]

g) 4 [1]

h) Number of subsets = $2^4 = 16$ [1]

7. a)



[3]

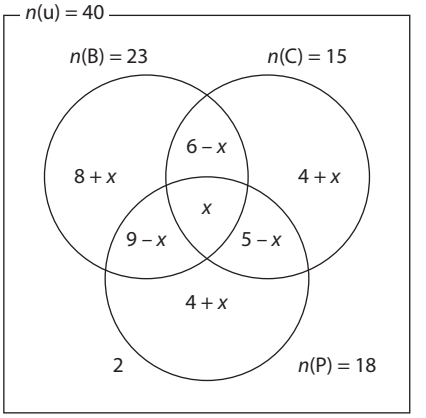
b) $25 - x + x + 18 - x + 2 = 36$

$45 - x = 36$

$x = 45 - 36$

$x = 9$ [2]

8. a)



[6]

b) $8 + x + 6 - x + x + 9 - x + 4 + x + 5 - x + 4 + x + 2 = 40$

$x + 38 = 40$ [2]

c) $x + 38 = 40$

$x = 40 - 38$

$x = 2$ [2]

d) n(Biology only) $= 8 + x = 8 + 2 = 10$ [1]

e) n(Chemistry and Biology only) $= 6 - x = 6 - 2 = 4$ [1]

Section 5 Measurement

1. a) $C = 2\pi r = 2 \times 3.14 \times 6 = 37.68$ cm [2]

b) $A = \pi r^2 = 3.14 \times 6^2 = 113$ cm^2 [2]

c) Area of minor sector $= \frac{\theta}{360} \times A = \frac{120}{360} \times 113 = 37.7$ cm^2 [2]

d) Area of triangle AOB $= \frac{1}{2}ab \sin C = \frac{1}{2} \times 6 \times 6 \times \sin 120°$

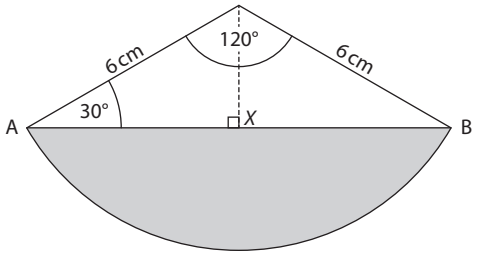
$= 15.6$ cm^2 [2]

e) Area of shaded region $= 37.7 - 15.6 = 22.1$ cm^2 [2]

f) Length of minor arc $= \frac{\theta}{360} \times C = \frac{120}{360} \times 37.68 = 12.56$ cm [2]

g) Length of major arc $= \frac{\theta}{360} \times C = \frac{240}{360} \times 37.68 = 25.12$ cm [2]

h)



$AX = 6 \times \cos 30° = 5.2$ cm
$AB = 2 \times 5.2 = 10.4$ cm
Perimeter of shaded region = length of minor arc + AB
$= 12.56 + 10.4 = 22.96$ cm **[2]**

2. a) $A = l \times b = 6 \times 8.2 = 49.2$ cm^2 **[2]**

b) $V = A \times h = 49.2 \times 12 = 590$ cm^3 **[2]**

c) Total surface area $= 2 \times (12 \times 6) + 2 \times (12 \times 8.2) + 2 \times (6 \times 8.1)$
$= 144 + 196.8 + 98.4$
$= 439.2$ cm^2 **[4]**

3. a) Total area = Area of triangle + Area of square + Area of semi-circle
$= \left(\frac{1}{2} \times 2 \times 4\right) + (4 \times 4) + \left(\frac{\pi(2)^2}{2}\right)$
$= 4 + 16 + 6.28$
$= 26.28$ cm^2 **[3]**

b) Total area = Area of rectangle ABCD − Area of semi-circle
$= (8 \times 6) - \left(\frac{\pi(4)^2}{2}\right)$
$= 48 - 25.12$
$= 22.88$ m^2 **[4]**

4. a) Area of shaded cross-section $= \pi r^2 = 3.14 \times 2^2$
$= 12.56$ cm^2 **[2]**

b) Volume of cylinder $= \pi r^2 h = 12.56 \times 8 = 100.48$ cm^3 **[2]**

c) Area of curved part of cylinder $= h \times 2\pi r$
$= 8 \times 2 \times 3.14 \times 2$
$= 100.48$ cm^2 **[2]**

5. a) i) Length of one side of square $= \sqrt{196} = 14$ cm **[2]**

ii) Perimeter of square $= 4 \times 14 = 56$ cm **[1]**

b) i) Circumference = 56 cm **[1]**

ii) $2\pi r = 56$
$r = \frac{56}{2\left(\frac{22}{7}\right)} = 8.91$ cm **[2]**

iii) Area of circle $= \pi r^2 = \frac{22}{7} \times (8.91)^2 = 249.5$ cm^2 **[2]**

6. a) Curved surface area of the cylinder $= 2\pi rh$
$= 2 \times 3.14 \times 2 \times 6 = 75.36$ cm^2 **[2]**

b) TOTAL surface areas of the two hemispheres (or one sphere) $= 4\pi r^2$
$= 4 \times 3.14 \times 2^2$
$= 50.24$ cm^2 **[2]**

c) TOTAL surface area of perfume bottle = 75.36 + 50.24
$= 125.6$ cm^2 **[1]**

d) Volume of cylinder $= \pi r^2 h = 3.14 \times 2^2 \times 6 = 75.36$ cm^3 **[2]**

e) TOTAL volume of the two hemispheres (or one sphere)
$= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2)^3$
$= 33.49$ cm^3 **[2]**

f) TOTAL volume of perfume bottle = 75.36 + 33.49
$= 108.85$ cm^3 **[1]**

7. a) BDC = 30° (BDC is an isosceles triangle) **[2]**

b) DBC = 180 − (30 + 30) = 180 − 60 = 120°
ABD = 150 − 120 = 30° **[2]**

c) ADB $= \frac{180 - 30}{2} = 75°$ **[2]**

8. a) Area of trapezium $= \frac{1}{2}(8 + 10) \times 6 = 54$ cm^2 **[2]**

b) $110 + x + 120 + 60 = 360$
$x + 290 = 360$
$x = 360 - 290$
$x = 70°$ **[1]**

Section 6 Statistics

1. a)

Score (x)	Tally	Frequency (f)	$x \times f$
1	|||	3	3
2	||||	4	8
3	||||	4	12
4	||||	4	16
5	||	2	10
6	𝍸 |||	8	48
7	|	1	7
8	|||	3	24
9	|	1	9

[5]

b) Mode = 6 **[1]**

c) Median $= \frac{4 + 5}{2} = 4.5$ **[1]**

d) Mean $= \frac{137}{30} = 4.6$ **[3]**

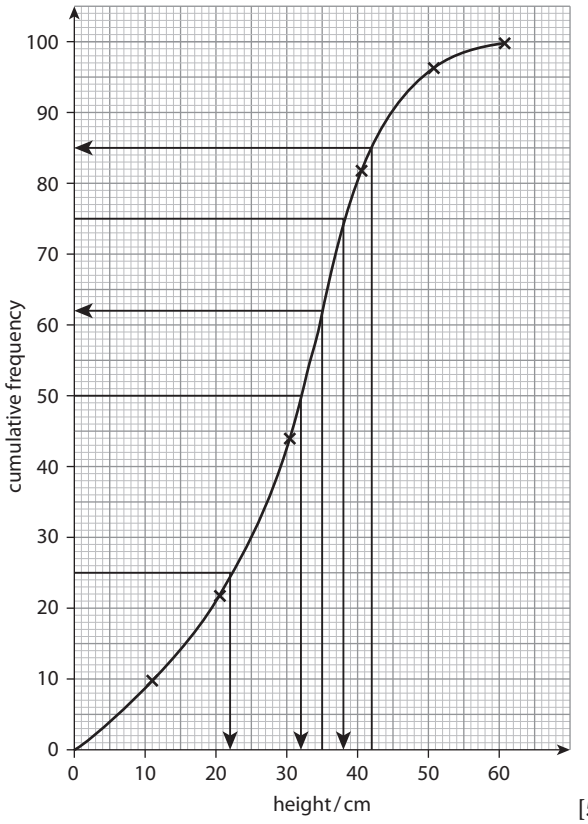
e) Probability (score > 5) $= \frac{8 + 1 + 3 + 1}{30} = \frac{13}{30}$ **[2]**

2. a)

Height (cm)	Number of seedlings	Cumulative frequency
1–10	10	10
11–20	12	22
21–30	22	44
31–40	38	82
41–50	15	97
51–60	3	100

[3]

b) (See graph)



[5]

c) i) Lower quartile = 22 cm **[1]**

ii) Median = 32 cm **[1]**

iii) Upper quartile = 38 cm **[1]**

iv) Interquartile range = 38 − 22 = 16 cm **[1]**

v) Semi-interquartile range = $\frac{16}{2} = 8$ cm **[1]**

d) i) P (less than 35 cm) = $\frac{62}{100} = 0.62$ **[1]**

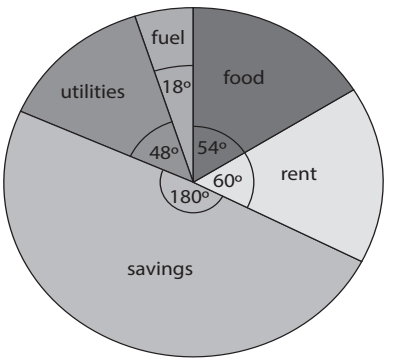
ii) P (greater than 42 cm) = $\frac{100 - 85}{100} = \frac{15}{100} = 0.15$ **[1]**

3. a)

Item	Budgeted amount	Angle of sector in pie chart
rent	\$1000	$\frac{1000}{6000} \times 360 = 60°$
food	\$900	$\frac{900}{6000} \times 360 = 54°$
fuel for her car	\$300	$\frac{300}{6000} \times 360 = 18°$
utilities	\$800	$\frac{800}{6000} \times 360 = 48°$
savings	\$3000	$\frac{3000}{6000} \times 360 = 180°$
Total	\$6000	360°

[4]

b)



[3]

4. a) Mean score = $\frac{6+5+7+7+6+5+6+6+6+7}{10} = \frac{61}{10} = 6.1$ **[2]**

b) Arrange the scores in ascending order to find the median – 5, 5, 6, 6, **6**, **6**, 6, 7, 7, 7

Median score = 6 **[2]**

c) Modal score = 6 (most frequent score) **[1]**

5. a) 14–16 **[1]**

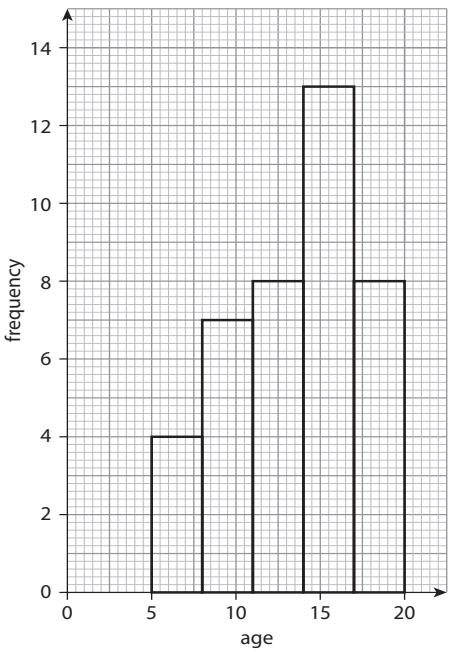
b) 14–16 **[1]**

c) 10.5 **[1]**

d) 16.5 **[1]**

e) 7.5 − 4.5 = 3 **[1]**

f) See graph



[4]

6. a) Number of letter Ms = 2

Number of letters = 11

P (selecting an M) = $\frac{2}{11}$ **[2]**

b) Number of yellow balls = $\frac{5}{8} \times 40 = 25$ **[2]**

c) P (student scores more than 6) = $\frac{1+2+1+1}{30} = \frac{5}{30} = \frac{1}{6}$ **[2]**

Section 7 Algebra

1. a) i) $\frac{x-3}{4}+\frac{2x+1}{3}$

$=\frac{3(x-3)+4(2x+1)}{12}$

$=\frac{3x-9+8x+4}{12}$

$=\frac{11x-5}{12}$ **[3]**

ii) $\frac{2x+1}{2}-\frac{x-3}{3}$

$=\frac{3(2x+1)-2(x-1)}{6}$

$=\frac{6x+3-2x+2}{6}$

$=\frac{4x+5}{6}$ **[3]**

b) $\frac{7x+1}{5}+\frac{2x-1}{3}=4$

$3(7x+1)+5(2x-1)=15\times 4$

$21x+3+10x-5=60$

$31x-2=60$

$31x=60+2$

$31x=62$

$x=\frac{62}{31}$

$x=2$ **[3]**

2. a) $4x^2y(1-3xy^2)$ **[2]**

b) $(1+3x)(1-3x)$ **[2]**

c) $(3x-1)(x+2)$ **[2]**

d) $6p^2+3pq-2p-q=3p(2p+q)-(2p+q)$

$=(3p-1)(2p+q)$ **[2]**

3. a) $mx+c=y$

$mx=y-c$

$x=\frac{y-c}{m}$ **[2]**

b) $pq=\frac{a}{x}$

$pqx=a$

$x=\frac{a}{pq}$ **[2]**

c) $T=k\sqrt{\frac{x}{a}}$

$\frac{T}{k}=\sqrt{\frac{x}{a}}$

$\frac{T^2}{k^2}=\frac{x}{a}$

$xk^2=aT^2$

$x=\frac{aT^2}{k^2}$ **[2]**

4. a) $3x^2y-6x^2y^2+3xy^2$ **[2]**

b) $2x-2y-6x^2+3x=5x-2y-6x^2$ **[2]**

c) $4x^2-\frac{5}{y}$ **[2]**

5. a) $2*3=(2+3)^2-3(2)(3)=25-18=7$ **[2]**

b) $\frac{(1)(-2)-3}{(-2)^2+1}=\frac{-5}{5}=-1$ **[2]**

6. a) $y\propto x$

$y=kx$

$12=k(4)$

$k=\frac{12}{4}$

$k=3$

Therefore, $y=3x$

When $y=9$, $x=a$

$9=3a$

$a=\frac{9}{3}$

$a=3$

When $x=7$, $y=b$

$b=3(7)=21$ **[3]**

b) $y\propto x^2$

$y=kx^2$

$16=k\,(2)^2$

$k=\frac{16}{4}$

$k=4$

Therefore, $y=4x^2$

When $x=a$, $y=64$

$64=4a^2$

$a^2=\frac{64}{4}$

$a^2=16$

$a=\sqrt{16}$

$a=4$

When $x=5$, $y=$ b

$b=4(5)^2$

$b=100$ **[3]**

c) $y\propto\frac{1}{x}$

$y=\frac{k}{x}$

$6=\frac{k}{2}$

$k=6\times 2$

$k=12$

Therefore, $y=\frac{12}{x}$

When $x=a$, $y=3$

$3=\frac{12}{a}$

$3a=12$

$a=\frac{12}{3}$

$a=4$

When $x=4$, $y=b$

$b=\frac{12}{4}=3$ **[3]**

7. a) $2x+3y=8$……..(1)

$3x-y=1$……(2)

From equation (2), $y=3x-1$

Substiting equation (2) into (1)

$2x+3(3x-1)=8$

$2x+9x-3=8$

$11x-3=8$

$11x=8+3$

$11x=11$

$x=\frac{11}{11}$

$x=1$

Substituting $x=1$ into equation (2)

$y=3(1)-1=3-1=2$ **[3]**

b) $y+2x=7$ ……..(1)

$x^2-xy=6$ ……(2)

From equation (1), $y=7-2x$

Substituting equation (1) into equation (2)

$x^2-x\,(7-2x)=6$

$x^2-7x+2x^2=6$

$3x^2-7x-6=0$

$(3x+2)(x-3)=0$

Either $3x+2=0$

$3x=-2$

$x=-\frac{2}{3}$

Or $x-3=0$

$x=3$

When $x=-\frac{2}{3}$, $y=7-2\left(-\frac{2}{3}\right)=8\frac{1}{3}$

When $x=3$, $y=7-2(3)=1$ **[6]**

8. a) $2x - 8 = 12$

$2x = 12 + 8$

$2x = 20$

$x = \frac{20}{2}$

$x = 10$ **[2]**

b) $\frac{2x}{3} + \frac{x}{2} = 6$

$4x + 3x = 36$

$7x = 36$

$x = \frac{36}{7}$ **[3]**

c) $8 - 2x < 2$

$-2x < 2 - 8$

$-2x < -6$

$2x > 6$

$x > \frac{6}{2}$

$x > 3$ **[2]**

9. a) i) $50 - x$ **[1]**

ii) $50x + 35(50 - x)$ **[1]**

b) i) $50x + 1750 - 35x = 2200$

$15x = 2200 - 1750$

$15x = 450$

$x = \frac{450}{15}$

$x = 30$ **[3]**

ii) $50 - 30 = 20$ **[1]**

10. a) $\frac{1}{2}x + 400$ **[2]**

b) i) First piece = x

Second piece = $x + 2$

Third piece = $3x$ **[2]**

ii) $x + (x + 2) + 3x = 5x + 2$ **[1]**

iii) $5x + 2 = 42$

$5x = 42 - 2$

$5x = 40$

$x = \frac{40}{5}$

$x = 8$ **[3]**

11. a) x^8y^4 **[1]**

b) x^3y **[1]**

c) $\frac{a^3b^4}{ab} = a^2b^3$ **[2]**

d) y^2 **[1]**

12. a) $x^2 + 7x + 10 = 0$

$(x + 5)(x + 2) = 0$

Either $x + 5 = 0$

$x = -5$

OR $x + 2 = 0$

$x = -2$ **[2]**

b) $3x^2 + 10x + 8 = 0$

$(3x + 4)(x + 2) = 0$

Either $3x + 4 = 0$

$3x = -4$

$x = -\frac{4}{3}$

OR $x + 2 = 0$

$x = -2$ **[2]**

c) $6x^2 - 13x + 5 = 0$

$(2x - 1)(3x - 5) = 0$

Either $2x - 1 = 0$

$2x = 1$

$x = \frac{1}{2}$

OR $3x - 5 = 0$

$3x = 5$

$x = \frac{5}{3}$ **[2]**

13. a) $2x^2 + 5x + 1 = 0$

$a = 2, b = 5, c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(1)}}{2(2)}$

$x = \frac{-5 \pm \sqrt{17}}{4}$

Either $x = \frac{-5 + \sqrt{17}}{4} = -0.22$

OR $x = \frac{-5 - \sqrt{17}}{4} = -2.28$ **[4]**

b) $x^2 + 7x - 2 = 0$

$a = 1, b = 7, c = -2$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{-7 \pm \sqrt{57}}{2}$

Either $x = \frac{-7 + \sqrt{57}}{2} = 0.27$

OR $x = \frac{-5 - \sqrt{57}}{2} = -7.27$ **[4]**

14. a) $2 - 18x^2 = 2(1 - 9x^2) = 2(1 + 3x)(1 - 3x)$ **[2]**

b) $(2x + 3)(3x - 4)$ **[2]**

c) $(2p + 3q)(4r - s)$ **[2]**

Section 8 Relations, functions and graphs

1. a) Gradient = 2 **[1]**

b) Gradient = $-\frac{1}{2}$ **[1]**

c) $y = mx + c$

$y = 2x + c$

When $x = 2, y = -3$

$-3 = 2(2) + c$

$c + 4 = -3$

$c = -3 - 4$

$c = -7$

Therefore, the equation of the line is $y = 2x - 7$. **[3]**

2. a) Gradient = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 1}{3 - (-2)} = \frac{10}{5} = 2$ **[1]**

b) Midpoint = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 3}{2}, \frac{1 + 11}{2}\right) = \left(\frac{1}{2}, 6\right)$ **[2]**

c) Length = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(3 - (-2))^2 + (11 - 1)^2}$

$= \sqrt{(5)^2 + (10)^2}$

$= 11.2$ units **[2]**

d) Gradient of perpendicular bisector $= -\frac{1}{2}$

$y = mx + c$

When $x = \frac{1}{2}$, $y = 6$

$$6 = -\frac{1}{2}\left(\frac{1}{2}\right) + c$$

$$6 = -\frac{1}{4} + c$$

$$c = 6 + \frac{1}{4}$$

$$c = \frac{25}{4}$$

Therefore, $y = -\frac{1}{2}x + \frac{25}{4}$ **[3]**

3. a) When line meets x-axis, $y = 0$

$3x - 6 = 0$

$3x = 6$

$x = \frac{6}{3}$

$x = 2$

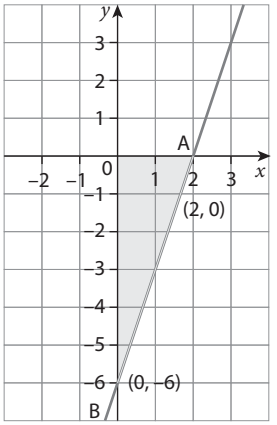
Therefore, A (2, 0) **[1]**

b) When the line meets the y-axis, $x = 0$

$y = 3(0) - 6 = -6$

Therefore, B (0, −6) **[1]**

c) Area of triangle OAB $= \frac{1}{2} \times 2 \times 6 = 6$ units squared **[1]**



4. a) i) One-to-one **[1]**

ii) Many-to-one **[1]**

iii) One-to-many **[1]**

iv) Many-to-many **[1]**

b) Functions – one-to-one, many-to-one **[2]**

5. a) $f(2) = 5(2) - 2 = 10 - 2 = 8$ **[1]**

b) $f(-1) = 5(-1) - 2 = -5 - 2 = -7$ **[2]**

c) $g(4) = \frac{1}{3(4)} = \frac{1}{12}$ **[1]**

d) $fg(x) = 5\left(\frac{1}{3x}\right) - 2 = \frac{5}{3x} - 2$ **[2]**

e) $gf(x) = \frac{1}{3(5x-2)} = \frac{1}{15x-6}$ **[2]**

f) $y = 5x - 2$

Step 1 – Interchange x and y

$x = 5y - 2$

Step 2 – Make y the subject of the formula

$5y = x + 2$

$y = \frac{x+2}{5}$

Therefore, $f^{-1}(x) = \frac{x+2}{5}$ **[2]**

6. a) $x - 3 = 0$

$x = 3$

Therefore, when $x = 3$, $f(x)$ is undefined. **[1]**

b) $g(2) = 2(2) + 3 = 7$

$f(7) = \frac{2(7)+1}{7-3} = \frac{15}{4}$

Therefore, $fg(2) = \frac{15}{4}$ **[3]**

c) $f(x) = \frac{2x+1}{x-3}$

Let $y = \frac{2x+1}{x-3}$

Step 1 – Interchange x and y

$x = \frac{2y+1}{y-3}$

Step 2 – Make y the subject of the formula

$x(y-3) = 2y + 1$

$xy - 3x = 2y + 1$

$xy - 2y = 1 + 3x$

$y(x-2) = 1 + 3x$

$y = \frac{1+3x}{x-2}$

Therefore, $f^{-1}(x) = \frac{1+3x}{x-2}$ **[3]**

7. a) $2x^2 + 5x - 3 = a(x+b)^2 + c$

$= a(x^2 + 2bx + b^2) + c$

$= ax^2 + 2abx + ab^2 + c$

Equating coefficients:

$a = 2$

$2ab = 5$

$2(2)b = 5$

$b = \frac{5}{4}$

$ab^2 + c = -3$

$2\left(\frac{5}{4}\right)^2 + c = 3$

$c = -3 - \frac{25}{8}$

$c = -\frac{49}{8}$

So $f(x) = 2\left(x + \frac{5}{4}\right)^2 - \frac{49}{8}$ **[3]**

b) Axis of symmetry $x = -\frac{5}{4}$ **[1]**

c) Coordinates of minimum point $\left(-\frac{5}{4}, -\frac{49}{8}\right)$ **[1]**

d) $2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

Either $2x - 1 = 0$

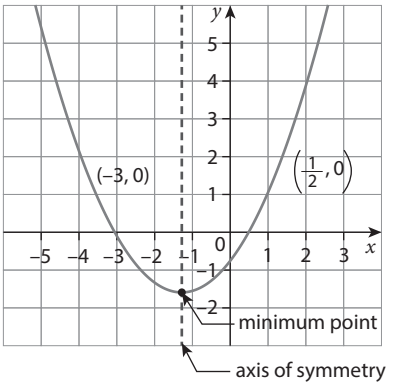
$2x = 1$

$x = \frac{1}{2}$

Or $x + 3 = 0$

$x = -3$ **[2]**

e) f)



[4]

8. a) $x = -3, x = 1$ **[2]**

b) $f(x) = (x + 3)(x - 1) = x^2 + 3x - x - 3 = x^2 + 2x - 3$ **[2]**

c) $f(x) = 5$ **[2]**

d) $(-1, -4)$ **[2]**

e) $x = -1$ **[2]**

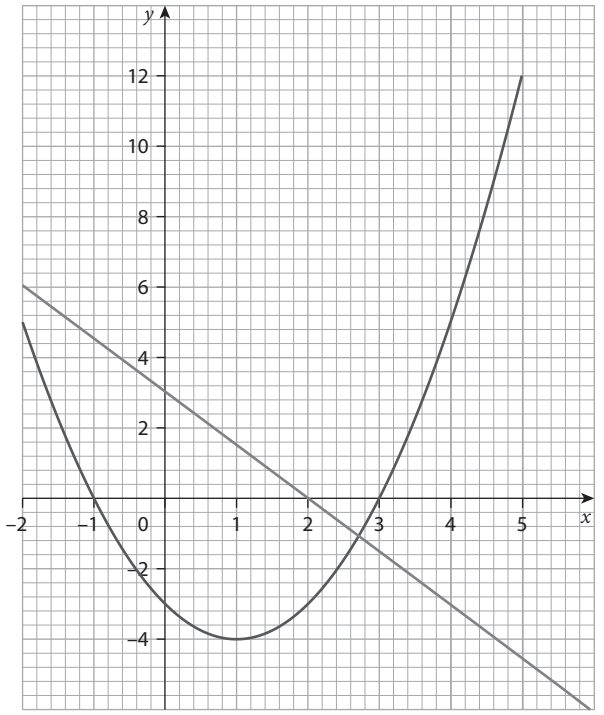
f) $x = 0, x = -2$ **[2]**

g) $-2 < x < 0$ **[2]**

9. a)

x	−2	−1	0	1	2	3	4	5
$f(x)$	5	0	−3	−4	−3	0	5	12

[10]



b) $a = -2$
$b = 5$ **[2]**

c) $x = -1$
$x = 3$ **[2]**

d) $(1, -4)$ **[1]**

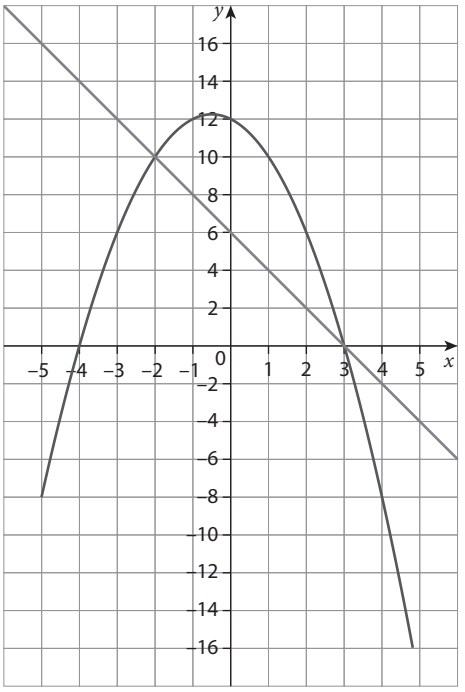
e) Choose any two points on the tangent drawn at the point $x = 2$.
$(5, 3)$ and $(1, -5)$
Gradient $= \frac{-5-3}{1-5} = \frac{-8}{-4} = 2$ **[3]**

10. a)

x	−5	−4	−3	−2	−1	0	1	2	3	4
$f(x)$	−8	0	6	10	12	12	10	6	0	−8
$g(x)$	16	14	12	10	8	6	4	2	0	−2

[6]

b)



[4]

c) From the graph, solutions are:
$(-2, 10)$ and $(3, 0)$ **[2]**

d) $x = -4, x = 3$ **[2]**

e) $-4 < x < 3$ **[2]**

f) $x > 3, x < -2$ **[2]**

11. a) $a = \frac{20-0}{30-0} = \frac{2}{3}$ ms^{-2} **[2]**

b) $a = \frac{40-20}{50-30} = \frac{20}{20} = 1$ ms^{-2} **[2]**

c) $d = \frac{40-0}{100-80} = \frac{40}{20} = 2$ ms^{-2} **[2]**

d) $a = 0$ ms^{-2} **[1]**

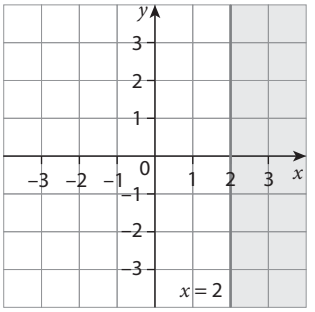
e) Distance = Area of trapezium
$= \frac{1}{2}(30 + 50) \times 40$
$= 1600$ m **[2]**

12. a) Average speed $= \frac{d}{t} = \frac{10}{60} = 0.167$ ms^{-1} **[2]**

b) $3.5 - 1 = 2.5$ minutes **[1]**

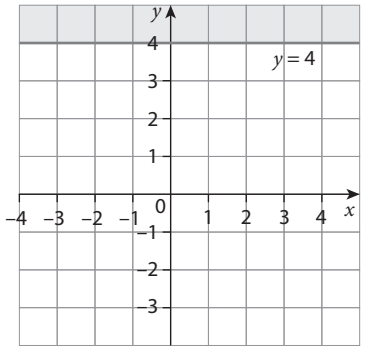
c) Average speed $= \frac{8}{30} = 0.267$ ms^{-1} **[2]**

13. a) $x \geq 2$



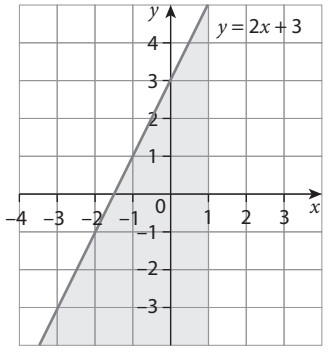
[1]

b) $y \geq 4$



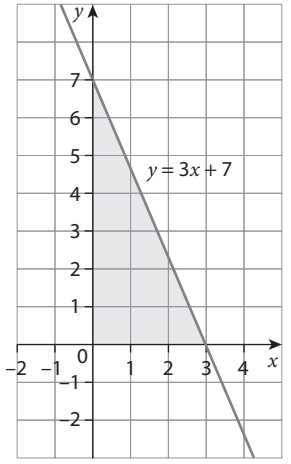
[1]

c) $y \leq 2x + 3$



[1]

d) $y + 3x \leq 7$

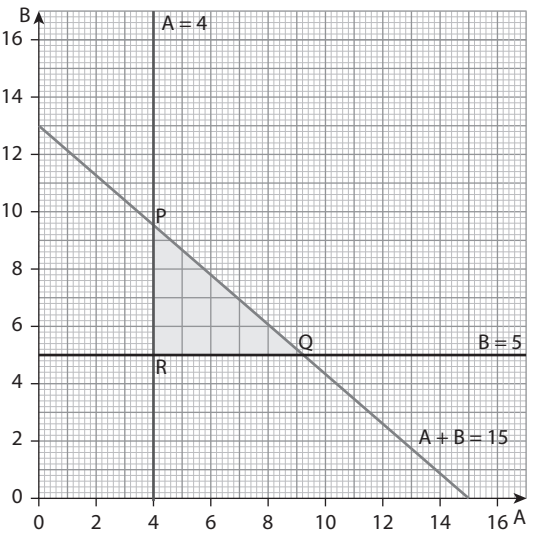


[1]

14. a) Inequality 1 : $A \geq 4$ [1]
Inequality 2: $B \geq 5$ [1]
Inequality 3: $A + B \leq 15$ [1]

b) See graph [4]

c) See shaded region [2]



d) **i)** P – (4, 11) Profit = (4 × 6000) + (11 × 7000)
= $101 000 [1]
Q – (10, 5) Profit = (10 × 6000) + (5 × 7000)
= $95 000 [1]
R – (4, 5) Profit = (4 × 6000) + (5 × 7000)
= $59 000 [1]
Quantity of refrigerator A = 4
Quantity of refrigerator B = 11

ii) Maximum profit = $101 000 [2]

15. a) **i)** $y = 0$ [1]

ii) $x = 0$ [1]

iii) $y = 4$ [1]

b) $x \geq 0$ [1]
$y \geq 0$ [1]
$y \leq 4$ [1]
$y \leq 6 - x$ [1]

16. a) P: $x = 10$, Q: $y = 2$ [2]

b) $x \leq 10$ [1]
$y \geq 2$ [1]
$x + y \leq 15$ [1]
$y \leq x$ [1]

17. a) $y = -2\left(x + \frac{5}{4}\right)^2 + \frac{49}{8}$ [2]

b) $x = -\frac{5}{4}$ [1]

c) Two [1]

d) $x = \frac{1}{2}$ and $x = -3$ [2]

e) Maximum point. Coordinates of maximum point $\left(-\frac{5}{4}, \frac{49}{8}\right)$ [2]

18. a) Let $y = 3x + 2$
Interchanging x and y
$x = 3y + 2$
$3y = x - 2$
$y = \frac{x-2}{3}$
Therefore, $f^{-1}(x) = \frac{x-2}{3}$ [2]

b) Let $y = 6 - x$
Interchanging x and y
$x = 6 - y$
$y = 6 - x$
Therefore, $g^{-1}(x) = 6 - x$ [2]

c) Let $y = \frac{x+3}{2x-1}$

Interchanging x and y

$$x = \frac{y+3}{2y-1}$$

$x(2y-1) = y+3$

$2xy - x = y + 3$

$2xy - y = 3 + x$

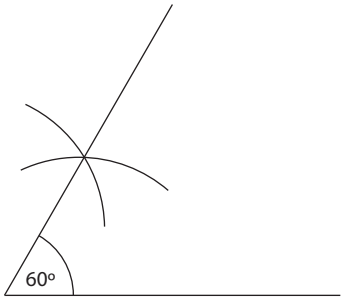
$y(2x-1) = 3 + x$

$$y = \frac{3+x}{2x-1}$$

Therefore, $f^{-1}(x) = \frac{3+x}{2x-1}$ **[3]**

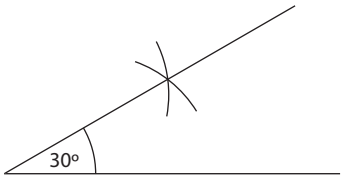
Section 9 Geometry and trigonometry

1. a)



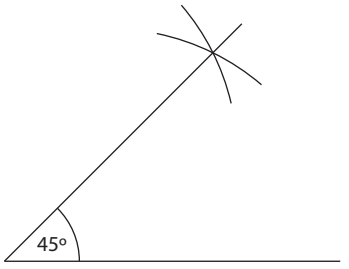
[3]

b)



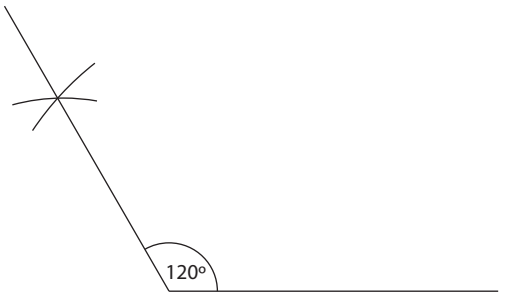
[3]

c)



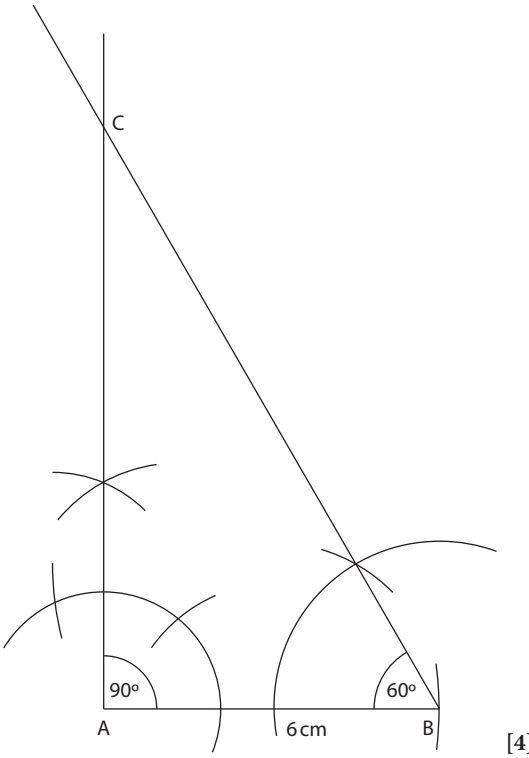
[3]

d)



[3]

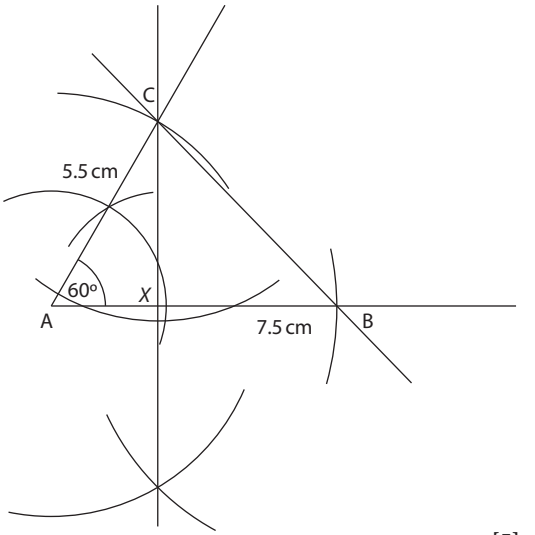
2. a)



[4]

b) AC = 10.4 cm **[1]**

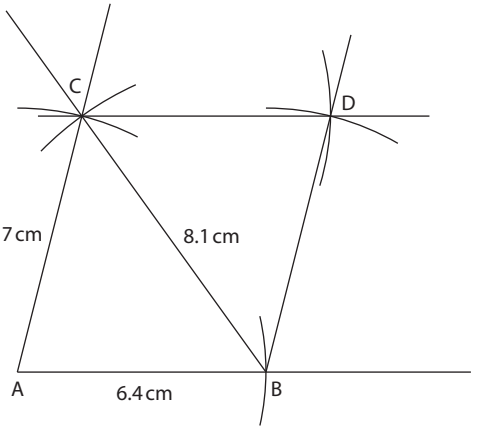
3. a) b)



[5]

Angle BCX = 45° **[1]**

4.



[5]

5. a) $2x + x = 180$

$3x = 180$

$x = \frac{180}{3}$

$x = 60°$ **[2]**

b) $x + 60 = 90$

$x = 90 - 60$

$x = 30°$ **[1]**

c) $x + 90 + 120 + 100 = 360$

$x + 310 = 360$

$x = 360 - 310$

$x = 50°$ **[2]**

d) $d = 60°$ (vertically opposite) **[1]**

$a = 180 - 60 = 120°$ **[1]**

$c = 120°$ (vertically opposite) **[1]**

$e = 120°$ (alternate) **[1]**

$b = 60°$ **[1]**

6. a) 90°

The angle in a semi-circle is a right angle. **[1]**

b) 60°

The angles subtended by a chord at the circumference of a circle and standing on the same arc are equal. **[1]**

c) $180 - 80 = 100°$

The opposite angles of a cyclic quadrilateral are supplementary. **[1]**

d) $\frac{60}{2} = 30°$

The angle subtended by a chord at the centre of a circle is twice the angle that the chord subtends at the circumference, standing on the same arc. **[1]**

e) 70°

The angle formed by the tangent to a circle and a chord, at the point of contact, is equal to the angle in the alternate segment. **[1]**

7. a) QPR = 80° **[2]**

The angle formed by the tangent to a circle and a chord, at the point of contact, is equal to the angle in the alternate segment.

b) QOR = 160° **[2]**

The angle subtended by a chord at the centre of a circle is twice the angle that the chord subtends at the circumference, standing on the same arc.

c) QSR = 180 − (80 + 80)

= 180 − 160

= 20° **[2]**

Tangents QS and RS are equal lengths. Triangle RQS is isosceles, making RQS = 80°.

8. a) ROC = 60° **[2]**

The angle subtended by a chord at the centre of a circle is twice the angle that the chord subtends at the circumference, standing on the same arc.

b) ABC = 30°

CAB = 90 − 30 = 60° **[2]**

c) OCQ = 30°

QCP = 90 + 30 = 120°

QPS = 180 − (120 + 30) = 30° **[2]**

d) QOC = 120°

COR = 60°

$OCR = \frac{180 - 60}{2} = 60°$

RCA = 90 − 60 = 30° **[2]**

9. $AB^2 = AC^2 + BC^2$

$7^2 = AC^2 + 4.5^2$

$AC^2 = 7^2 - 4.5^2$

$AC = \sqrt{28.75}$

AC = 5.4 cm **[2]**

10. $AB^2 = AC^2 + BC^2$

$AB^2 = 4^2 + 5.2^2$

$AB^2 = 43.04$

$AB = \sqrt{43.04}$

AB = 6.6 cm **[2]**

11. $\tan Q = \frac{PR}{RQ}$

$\tan 60° = \frac{12}{RQ}$

$RQ = \frac{12}{\tan 60°}$

RQ = 6.9 cm **[2]**

12. $\tan Q = \frac{PR}{RQ}$

$\tan Q = \frac{6}{8}$

$Q = \tan^{-1}\left(\frac{6}{8}\right)$

$Q = 36.9°$ **[2]**

13. $\sin RPQ = \frac{RQ}{PQ}$

$\sin 30° = \frac{RQ}{12}$

$RQ = 12 \times \sin 30°$

RQ = 6 cm **[2]**

14. $\sin PQR = \frac{PR}{PQ}$

$\sin PQR = \frac{6}{9}$

$PQR = \sin^{-1}\left(\frac{6}{9}\right)$

$PQR = 41.8°$ **[2]**

15. $\cos PQR = \frac{RQ}{PQ}$

$\cos 42° = \frac{RQ}{8.2}$

$RQ = 8.2 \times \cos 42°$

RQ = 6.1 cm **[2]**

16. $\cos RPQ = \frac{PR}{PQ}$

$\cos RPQ = \frac{3.8}{9.4}$

$RPQ = \cos^{-1}\left(\frac{3.8}{9.4}\right)$

$RPQ = 66.2°$ **[2]**

17. Using the sine rule

$\frac{AB}{\sin C} = \frac{BC}{\sin A}$

$\frac{4}{\sin C} = \frac{6}{\sin 42°}$

$6 \times \sin C = 4 \times \sin 42°$

$\sin C = \frac{4 \times \sin 42°}{6}$

$C = \sin^{-1}(0.446)$

$C = 26.5°$ **[2]**

18. Using the sine rule

$\frac{BC}{\sin A} = \frac{AB}{\sin C}$

$\frac{BC}{\sin 20°} = \frac{9}{\sin 120°}$

$BC = \frac{9 \times \sin 20°}{\sin 120°}$

BC = 3.6 cm **[2]**

19. Using the cosine rule

$c^2 = a^2 + b^2 - 2ab \cos C$

$BC^2 = 8.5^2 + 6.2^2 - 2(8.5)(6.2) \cos 40°$

$BC^2 = 29.95$

$BC = \sqrt{29.95}$

$BC = 5.5$ cm **[2]**

20. Using the cosine rule

$c^2 = a^2 + b^2 - 2ab \cos C$

$5.6^2 = 7^2 + 6.2^2 - 2(7)(6.2) \cos ABC$

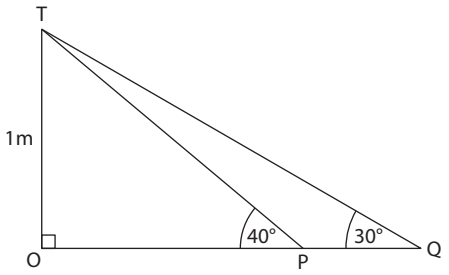
$\cos ABC = \frac{7^2 + 6.2^2 - 5.6^2}{2(7)(6.2)}$

$\cos ABC = 0.646$

$ABC = \cos^{-1}(0.646)$

$ABC = 49.8°$ **[2]**

21. a)



[2]

b) $\tan 40° = \frac{OT}{OP}$

$\tan 40° = \frac{11}{OP}$

$OP = \frac{11}{\tan 40°} = 13.1$ m **[2]**

c) $\tan 30° = \frac{11}{OQ}$

$OQ = \frac{11}{\tan 30°}$

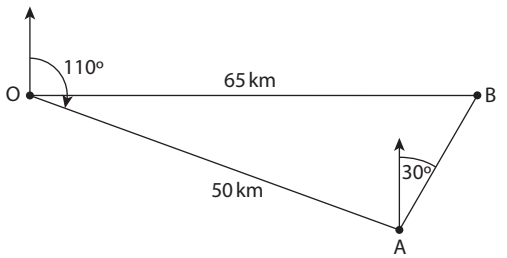
$OQ = 19.1$ m

$PQ = OQ - OP$

$= 19.1 - 13.1$

$= 6$ m **[3]**

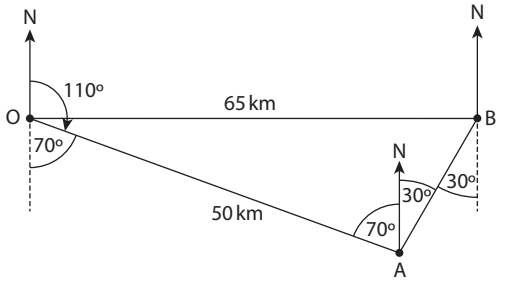
22. a)



[5]

b) i) OAB = 70 + 30

= 100° **[1]**



ii) Using the sine rule

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{65}{\sin 100°} = \frac{50}{\sin OBA}$

$\sin OBA = \frac{50 \times \sin 100°}{65}$

$\sin OBA = 0.758$

$OBA = \sin^{-1}(0.758)$

$OBA = 49.2°$ **[3]**

iii) Bearing of O from B = 180 + 30 + 49.2 = 259.2° **[1]**

23. T – Translation $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ **[3]**

24. Enlargement of scale factor 3

Centre of enlargement (−4, −8) **[3]**

25. Reflection in the line $x = 4$ **[2]**

26. a) i) (0, 0) **[1]**

ii) 90° **[1]**

iii) Anticlockwise **[1]**

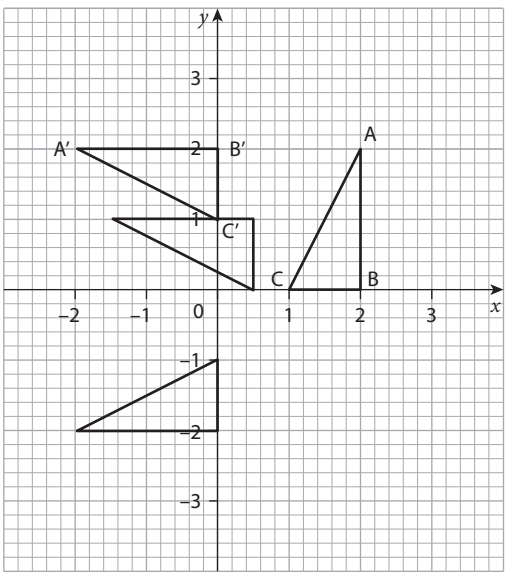
b) Congruent triangles **[1]**

A' – (−2, 2) , B' – (0, 2), C' – (0, 1)

c) Image after the transformation

A" – (−1, 0), B" – (1, 0) , C" – (1, −1) **[3]**

d)



[3]

27. a) Sum of the interior angles of a triangle = 180°.

Triangle BCD is an isoseles triangle.

$CDB = \frac{180 - 64}{2} = 58°$ **[2]**

b) BAD = 64°. The angles subtended by a chord at the circumference of a circle and standing on the same arc are equal. **[2]**

c) ADB = 90°. The angle in a semi-circle is a right angle.

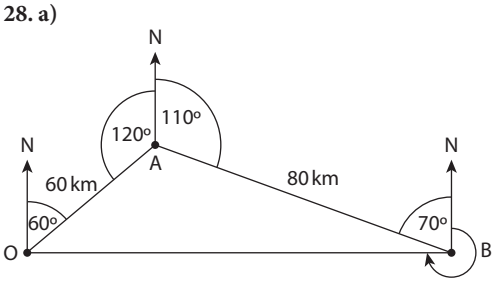
ABD = 180 − (64 + 90) = 26° **[2]**

d) BDT = 64°. The angle formed by the tangent to a circle and a chord, at the point of contact, is equal to the angle in the alternate segment.

DT = BT, therefore, triangle DBT is isosceles.

DTB = (180 − (2 × 64)) = 52° **[2]**

28. a)



[3]

b) OAB = 360 − (120 + 110) = 130°
Using the cosine rule
$OB^2 = OA^2 + AB^2 - (2 \times OA \times AB \times \cos OAB)$
$OB^2 = 60^2 + 80^2 - (2 \times 60 \times 80 \times \cos 130)$
$OB^2 = 16\,171$
OB = 127.2 km [4]

c) Using the sine rule
$\frac{127.2}{\sin 130°} = \frac{60}{\sin OBA}$
$\sin OBA = \frac{60 \times \sin 130°}{127.2}$
$\sin OBA = 0.361$
$OBA = \sin^{-1}(0.361)$
$OBA = 21.2°$
The bearing of O from B = 360 − (70 + 21.2) = 268.8° [4]

Section 10 Vectors and matrices

1. a) $A + B = \begin{pmatrix} 1 & 3 \\ -2 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$
$= \begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix}$ [2]

b) $A + 2B = \begin{pmatrix} 1 & 3 \\ -2 & 2 \end{pmatrix} + 2\begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$
$= \begin{pmatrix} 5 & 5 \\ 0 & -4 \end{pmatrix}$ [2]

c) $B - 2A = \begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix} - 2\begin{pmatrix} 1 & 3 \\ -2 & 2 \end{pmatrix}$
$= \begin{pmatrix} 0 & -5 \\ 5 & -7 \end{pmatrix}$ [2]

d) $AB = \begin{pmatrix} 1 & 3 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix} = \begin{pmatrix} 5 & -8 \\ -2 & -8 \end{pmatrix}$ [2]

e) $A^2 = \begin{pmatrix} 1 & 3 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ -2 & 2 \end{pmatrix}$
$= \begin{pmatrix} -5 & 9 \\ -6 & -2 \end{pmatrix}$ [2]

$A^2B = \begin{pmatrix} -5 & 9 \\ -6 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & -3 \end{pmatrix}$
$= \begin{pmatrix} -1 & -32 \\ -14 & 0 \end{pmatrix}$ [2]

2. a) Determinant = (4 × 2) − (3 × −2)
= 8 − (−6)
= 14 [1]

b) Determinant = (2 × 8) − (3 × 5)
= 1 [1]

c) Determinant = (5 × 2) − (3 × 1)
= 13 [1]

d) Determinant = (6 × −2) − (3 × −1)
= −9 [1]

3. a) $\frac{1}{14}\begin{pmatrix} 2 & -3 \\ 2 & 4 \end{pmatrix}$ [2]

b) $\frac{1}{1}\begin{pmatrix} 8 & -3 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 8 & -3 \\ -5 & 2 \end{pmatrix}$ [2]

c) $\frac{1}{13}\begin{pmatrix} 2 & 3 \\ -1 & 5 \end{pmatrix}$ [2]

d) $-\frac{1}{9}\begin{pmatrix} -2 & -3 \\ 1 & 6 \end{pmatrix}$ [2]

4. a) $3x - (-12) = 15$
$3x + 12 = 15$
$3x = 15 - 12$
$3x = 3$
$x = \frac{3}{3}$
$x = 1$ [2]

b) $A^{-1} = \frac{1}{15}\begin{pmatrix} 1 & -6 \\ 2 & 3 \end{pmatrix}$ [2]

c) $\frac{1}{15}\begin{pmatrix} 1 & -6 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 6 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ [2]

5. The determinant of a singular matrix is zero.
$(8 \times 2) - (x \times 4x) = 0$
$16 - 4x^2 = 0$
$4x^2 = 16$
$x^2 = \frac{16}{4}$
$x^2 = 4$
Therefore $x = 2$ or -2 [4]

6. a) $\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ -1 \end{pmatrix}$ [2]

b) Determinant of matrix A = (2 × −2) − (3 × 5) = −19 [2]

c) $A^{-1} = -\frac{1}{19}\begin{pmatrix} -2 & -3 \\ -5 & 2 \end{pmatrix}$ [2]

d) $-\frac{1}{19}\begin{pmatrix} -2 & -3 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\frac{1}{19}\begin{pmatrix} -2 & -3 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 11 \\ -1 \end{pmatrix}$
$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$
$x = 1, y = 3$ [5]

7. a) $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ [2]

b) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$ [2]

c) $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$
$|\overrightarrow{AB}| = \sqrt{1^2 + (-3)^2} = 3.16$ units [2]

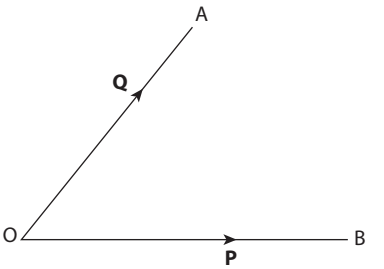
d) Unit vector $= \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{1}{3.16}\begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 3.16 \\ -0.949 \end{pmatrix}$ [1]

8. a) $\mathbf{a} + \mathbf{b} = (2\mathbf{i} + \mathbf{j}) + (\mathbf{i} - 3\mathbf{j}) = 3\mathbf{i} - 2\mathbf{j}$ [2]

b) $\mathbf{a} - 2\mathbf{b} = (2\mathbf{i} + \mathbf{j}) - 2(\mathbf{i} - 3\mathbf{j})$
$= 2\mathbf{i} + \mathbf{j} - 2\mathbf{i} + 6\mathbf{j}$
$= 7\mathbf{j}$ [2]

c) $\mathbf{a} - \mathbf{b} = (2\mathbf{i} + \mathbf{j}) - (\mathbf{i} - 3\mathbf{j})$
$= 2\mathbf{i} + \mathbf{j} - \mathbf{i} + 3\mathbf{j}$
$= \mathbf{i} + 4\mathbf{j}$
$|\mathbf{a} - \mathbf{b}| = \sqrt{1^2 + 4^2} = 4.12$ units [3]

9. a)



[2]

b) i) $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -\mathbf{b} + \mathbf{a}$ [2]

ii) $\overrightarrow{QB} = \overrightarrow{QO} + \overrightarrow{OB} = -\frac{2}{3}\overrightarrow{OA} + \overrightarrow{OB} = -\frac{2}{3}\mathbf{a} + \mathbf{b}$ [2]

iii) $\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP} = -\mathbf{a} + \frac{1}{2}\mathbf{b}$ [2]

iv) $\overrightarrow{QP} = \overrightarrow{QO} + \overrightarrow{OP} = -\frac{2}{3}\mathbf{a} + \frac{1}{2}\mathbf{b}$ [2]

10. a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \begin{pmatrix}2\\4\end{pmatrix} + \begin{pmatrix}1\\5\end{pmatrix} = \begin{pmatrix}3\\9\end{pmatrix}$ **[2]**

b) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = \begin{pmatrix}-1\\-5\end{pmatrix} + \begin{pmatrix}3\\11\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix}$ **[2]**

c) $\overrightarrow{AB} = \begin{pmatrix}3\\9\end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix}2\\6\end{pmatrix}$

$\overrightarrow{BC} = \frac{2}{3}\begin{pmatrix}3\\9\end{pmatrix} = \frac{2}{3}\overrightarrow{AB}$

Therefore, $\overrightarrow{BC}$ and $\overrightarrow{AB}$ are parallel. They share a common point B and hence A, B and C must be collinear. **[3]**

11. a) A (1, 3)
B (3, 1)
C (1, 1) **[2]**

b) i) Enlargement of scale factor 3 **[2]**

ii) $Q \times P = \begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}3 & 0\\0 & 3\end{pmatrix} = \begin{pmatrix}-3 & 0\\0 & 3\end{pmatrix}$ **[3]**

iii) $A\begin{pmatrix}-3 & 0\\0 & 3\end{pmatrix}\begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}-3\\9\end{pmatrix}$

A' (−3, 9) **[1]**

$B\begin{pmatrix}-3 & 0\\0 & 3\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}-9\\3\end{pmatrix}$

B' (−9, 3) **[1]**

$C\begin{pmatrix}-3 & 0\\0 & 3\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}-3\\3\end{pmatrix}$

C' (−3, 3) **[1]**

12. a) $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ **[2]**

b) $\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$ **[2]**

c) $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$ **[2]**

d) $\begin{pmatrix}-2\\4\end{pmatrix}$ **[1]**

e) $BD = \begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}-2\\4\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix}$

$P' = \begin{pmatrix}4\\10\end{pmatrix} + \begin{pmatrix}2\\4\end{pmatrix} = \begin{pmatrix}6\\14\end{pmatrix}$

P' (6, 14)

$AC = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$

$P'' = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}4\\10\end{pmatrix} = \begin{pmatrix}10\\4\end{pmatrix}$

P'' (10, 4) **[4]**

13. a) i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \mathbf{-a + b}$ **[2]**

ii) $\overrightarrow{BP} = \frac{1}{2}\overrightarrow{BA} = \mathbf{\frac{1}{2}(a - b) = \frac{1}{2}a - \frac{1}{2}b}$ **[2]**

iii) $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} = \mathbf{a + \frac{1}{2}(-a + b)}$
$\mathbf{= \frac{1}{2}a + \frac{1}{2}b}$ **[2]**

iv) $\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ} = \overrightarrow{AO} + \frac{1}{2}\overrightarrow{OB} = \mathbf{-a + \frac{1}{2}b}$ **[2]**

b) $\overrightarrow{AX} = \overrightarrow{AO} + \overrightarrow{OX} = \overrightarrow{AO} + \frac{3}{5}\overrightarrow{OP} = \mathbf{-a + \frac{3}{5}\left(\frac{1}{2}a + \frac{1}{2}b\right)}$
$\mathbf{= -\frac{7}{10}a + \frac{3}{10}b}$ **[3]**