CURENT Course

Power System Coherency and Model Reduction

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Slow Coherency

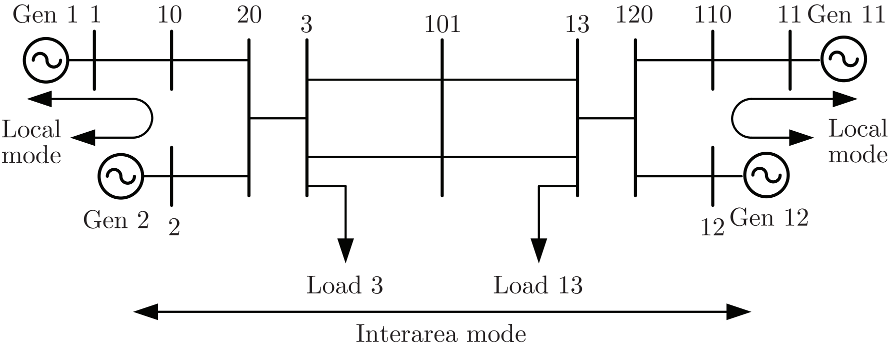
- A large power system usually consists of tightly connected control regions with few interarea ties for power exchange and reserve sharing
- The oscillations between these groups of strongly connected machines are the interarea modes
- These interarea modes are lower in frequency than local modes and intra-plant modes
- Singular perturbations method can be used to show this time-scale separation

References:

J H. Chow, G. Peponides, B. Avramovic, J. R. Winkelman, and P. V. Kokotovic, *Time-Scale Modeling of Dynamic Networks with Applications to Power Systems*, Springer-Verlag, 1982.

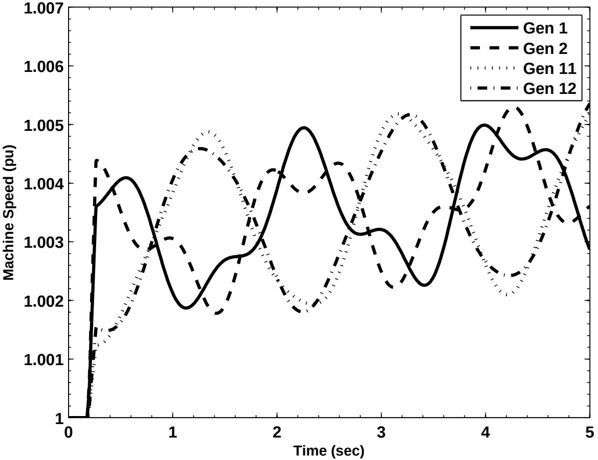
J. H. Chow, ed., *Power System Coherency and Model Reduction*, Springer, 2013.

Klein-Rogers-Kundur 2-Area System



Coherency in 2-Area System

Disturbance: 3-phase fault at Bus 3, cleared by removal of 1 line between Buses 3 and 101



Power System Model

An n-machine, N-bus power system with classical electromechanical model and constant impedance loads:

$$m_i \ddot{\delta}_i = P_{mi} - P_{ei} = P_{mi} - \frac{E_i V_j \sin(\delta_i - \theta_j)}{x'_{di}} = f_i(\delta, V) \tag{1}$$

where

- machine i modeled as a constant voltage E_i behind a transient reactance x'_{di}
- $m_i = 2H_i/\Omega$, H_i = inertia of machine i
- $\Omega = 2\pi f_o$ = nominal system frequency in rad/s
- damping $D = 0$
- δ = n-vector of machine angles
- P_{mi} = input mechanical power, P_{ei} = output electrical power

Power System Model

- the bus voltage

$$V_j = \sqrt{V_{j\mathrm{re}}^2 + V_{j\mathrm{im}}^2}, \quad \theta_j = \tan^{-1}\left(\frac{V_{j\mathrm{im}}}{V_{j\mathrm{re}}}\right) \tag{2}$$

- $V_{j\mathrm{re}}$ and $V_{j\mathrm{im}}$ are the real and imaginary parts of the bus voltage phasor at Bus j, the terminal bus of Machine i
- $V = 2N$-vector of real and imaginary parts of load bus voltages

Power Flow Equations

For each load bus j, the active power flow balance

$$P_{ej} - \text{Real}\left\{\sum_{k=1,k\neq j}^{N} (V_{j\text{re}} + jV_{j\text{im}} - V_{k\text{re}} - jV_{k\text{im}})\left(\frac{V_{j\text{re}} + jV_{j\text{im}}}{R_{Ljk} + jX_{Ljk}}\right)^*\right\} - V_j^2 G_j = g_{2j-1} = 0 \quad (3)$$

and the reactive power flow balance

$$Q_{ej} - \text{Imag}\left\{\sum_{k=1,k\neq j}^{N} (V_{j\text{re}} + jV_{j\text{im}} - V_{k\text{re}} - jV_{k\text{im}})\left(\frac{V_{j\text{re}} + jV_{j\text{im}}}{R_{Ljk} + jX_{Ljk}}\right)^*\right\} - V_j^2 B_j + V_j^2 \frac{B_{Ljk}}{2} = g_{2j} = 0 \quad (4)$$

- R_{Ljk}, X_{Ljk}, and B_{Ljk} are the resistance, reactance, and line charging, respectively, of the line j-k
- P_{ej} and Q_{ej} are generator active and reactive electrical output power, respectively, if bus j is a generator bus
- G_j and B_j are the load conductance and susceptance at bus j

Note that j denotes the imaginary number if it is not used as an index.

Electromechanical Model

$$M\ddot{\delta} = f(\delta, V), \quad 0 = g(\delta, V) \tag{5}$$

- M = diagonal machine inertia matrix
- f = vector of acceleration torques
- g = power flow equation

Linearized Model

Linearize (5) about a nominal power flow equilibrium (δ_o, V_o) to obtain

$$
\begin{aligned}
M\Delta\ddot{\delta} &= \left.\frac{\partial f(\delta, V)}{\partial \delta}\right|_{\delta_o, V_o} + \left.\frac{\partial f(\delta, V)}{\partial V}\right|_{\delta_o, V_o} = K_1\Delta\delta + K_2\Delta V \quad (6)\\
0 &= \left.\frac{\partial g(\delta, V)}{\partial \delta}\right|_{\delta_o, V_o} + \left.\frac{\partial g(\delta, V)}{\partial V}\right|_{\delta_o, V_o} = K_3\Delta\delta + K_4\Delta V \quad (7)
\end{aligned}
$$

- $\Delta\delta = n$-vector of machine angle deviations from δ_o
- $\Delta V = 2N$-vector of the real and imaginary parts of the load bus voltage deviations from V_o
- K_1, K_2, and K_3 are partial derivatives of the power transfer between machines and terminal buses, K_1 is diagonal
- $K_4 =$ network admittance matrix and nonsingular.
- the sensitivity matrices K_i can be derived analytically or from numerical perturbations using the Power System Toolbox

Linearized Model

Solve (6) for

$$\Delta V = -K_4^{-1} K_3 \Delta\delta \tag{8}$$

to obtain

$$M\Delta\ddot{\delta} = K_1 - K_2 K_4^{-1} K_3 \Delta\delta = K\Delta\delta \tag{9}$$

where

$$K_{ij} = E_i E_j \left(B_{ij}\cos(\delta_i - \delta_j) - G_{ij}\sin(\delta_i - \delta_j)\right)\Big|_{\delta_o, V_o}, \quad i \neq j \tag{10}$$

and $G_{ij} + jB_{ij}$ is the equivalent admittance between machines i and j. Furthermore,

$$K_{ii} = -\sum_{j=1, j\neq i}^{n} K_{ij} \tag{11}$$

Thus the row sum of K equals to zero. The entries K_{ij} are known as the synchronizing torque coefficients.

Slow Coherent Areas

Assume that a power system has r slow coherent areas of machines and the load buses that interconnect these machines

Define

- $\Delta\delta_i^\alpha$ = deviation of rotor angle of machine i in area α from its equilibrium value
- m_i^α = inertia of machine i in area α

Order the machines such that $\Delta\delta_i^\alpha$ from the same coherent areas appears consecutively in $\Delta\delta$.

Weakly Coupled Areas

We attribute the slow coherency phenomenon to be primarily due to the connections between the machines in the same coherent areas being stiffer than those between different areas, which can be due to two reasons:

- The admittances of the external connections B_{ij}^E much smaller than the admittances of the internal connections B_{pq}^I

$$\varepsilon_1 = \frac{B_{ij}^E}{B_{pq}^I} \tag{12}$$

  where E denotes external, I denotes internal, and i, j, p, q are bus indices. This situation also includes heavily loaded high-voltage, long transmission lines between two coherent areas.

- The number of external connections is much less than the number of internal connections

$$\varepsilon_2 = \frac{\bar{\gamma}^E}{\underline{\gamma}^I} \tag{13}$$

where

$$\bar{\gamma}^E = \max_{\alpha}\{\gamma_\alpha^E\}, \quad \underline{\gamma}^I = \min_{\alpha}\{\gamma_\alpha^I\}, \quad \alpha = 1, ..., r \tag{14}$$

$\gamma_\alpha^E =$ (the number of external connections of area α)$/N^\alpha$
$\gamma_\alpha^I =$ (the number of internal connections of area α)$/N^\alpha$
where N^α is the number of buses in area α.

Internal and External Connections

For a large power system, the weak connections between coherent areas can be represented by the small parameter

$$\varepsilon = \varepsilon_1 \varepsilon_2 \tag{15}$$

Separate the network admittance matrix into

$$K_4 = K_4^I + \epsilon K_4^E \tag{16}$$

where K_4^I = internal connections and K_4^E = external connections. The synchronizing torque or connection matrix K is

$$\begin{aligned} K &= K_1 - K_2(K_4^I + \varepsilon(K_4^I))^{-1}K_3 \\ &= K_1 - K_2(K_4^I(I + \varepsilon(K_4^I)^{-1}K_4^E))^{-1}K_3 = K^I + \epsilon K^E \end{aligned} \tag{17}$$

where

$$K^I = K_1 - K_2(K_4^I)^{-1}K_3, \quad K^E = -K_2 K_{4\varepsilon}^E K_3 \tag{18}$$

In the separation (17), the property that each row of K^I sums to zero is preserved.

Slow Variables

Define for each area an inertia weighted *aggregate variable*

$$y^\alpha = \sum_{i=1}^{n_\alpha} m_i^\alpha \Delta\delta_i^\alpha / m^\alpha, \quad \alpha = 1, 2, \ldots, r \tag{19}$$

where m_i^α = inertia of machine i in area α, n_α = number of machines in area α, and

$$m^\alpha = \sum_{i=1}^{n_\alpha} m_i^\alpha, \quad \alpha = 1, 2, \ldots, r \tag{20}$$

is the aggregate inertia of area α.

Denote by y = the r-vector whose αth entry is y^α.
The matrix form of (19) is

$$y = C\Delta\delta = M_a^{-1} U^T M \Delta\delta \tag{21}$$

where

$$U = \text{blockdiag}(u_1, u_2, \ldots, u_r) \tag{22}$$

is the grouping matrix with $n_\alpha \times 1$ column vectors

$$u_\alpha = \begin{bmatrix} 1 & 1 & \ldots & 1 \end{bmatrix}^T, \quad \alpha = 1, 2, \ldots, r \tag{23}$$

$$M_a = \text{diag}(m^1, m^2, \ldots, m^r) = U^T M U \tag{24}$$

is the $r \times r$ diagonal aggregate inertia matrix.

Fast Variables

Select in each area a reference machine, say the first machine, and define the motions of the other machines in the same area relative to this reference machine by the *local variables*

$$z_{i-1}^{\alpha} = \Delta\delta_i^{\alpha} - \Delta\delta_1^{\alpha}, \quad i = 2, 3, \ldots, n_{\alpha}, \quad \alpha = 1, 2, \ldots, r \tag{25}$$

Denote by z^{α} the $(n_{\alpha} - 1)$-vector of z_i^{α} and conside z^{α} as the αth subvector of the $(n - r)$-vector z. Eqn. (25) in matrix form is

$$z = G\Delta\delta = \text{blockdiag}(G_1, G_2, \ldots, G_r)\Delta\delta \tag{26}$$

where G_{α} is the $(n_{\alpha} - 1) \times n_{\alpha}$ matrix

$$G_{\alpha} = \begin{bmatrix} -1 & 1 & 0 & . & 0 \\ -1 & 0 & 1 & . & 0 \\ . & . & . & . & . \\ -1 & 0 & 0 & . & 1 \end{bmatrix} \tag{27}$$

Slow and Fast Variable Transformation

A transformation of the original state $\Delta\delta$ into the aggregate variable y and the local variable z

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} C \\ G \end{bmatrix} \Delta\delta \tag{28}$$

The inverse of this transformation is

$$\Delta\delta = \begin{pmatrix} U & G^{+} \end{pmatrix} \begin{bmatrix} y \\ z \end{bmatrix} \tag{29}$$

where

$$G^{+} = G^{T}(GG^{T})^{-1} \tag{30}$$

is block-diagonal.

Slow Subsystem

Apply the transformation (28) to the model (9), (17)

$$\begin{aligned} M_a\ddot{y} &= \epsilon K_a y + \epsilon K_{ad} z \\ M_d\ddot{z} &= \epsilon K_{da} y + (K_d + \epsilon K_{dd}) z \end{aligned} \qquad (31)$$

where

$$\begin{aligned} M_d &= (GM^{-1}G^T)^{-1}, \quad K_a = U^T K^E U \\ K_{da} &= U^T K^E M^{-1} G^T M_d, \quad K_{da} = M_d G M^{-1} K^E U \\ K_d &= M_d G M^{-1} K^I M^{-1} G^T M_d, K_{dd} = M_d G M^{-1} K^E M^{-1} G^T M_d \end{aligned} \qquad (32)$$

- K_a, K_{ad}, and K_{da} are independent of K^I
- System (31) is in the *standard singularly perturbed form*
- ε is both the weak connection parameter and the singular perturbation parameter

Neglecting the fast dynamics, the slow subsystem is

$$M_a\ddot{y} = \varepsilon K_a y \qquad (33)$$

Formulation including Power Network

Apply (28) to the model (6)-(7)

$$\begin{aligned} M_a \ddot{y} &= K_{11} y + K_{12} z + K_{13} \Delta V \\ M_d \ddot{z} &= K_{21} y + K_{22} z + K_{23} \Delta V \\ 0 &= K_{31} y + K_{32} z + (K_4^I + \varepsilon K_4^E) \Delta V \end{aligned} \tag{34}$$

where

$$\begin{aligned} K_{11} &= U^T K_1 U, \quad K_{12} = U^T K_1 G^+, \quad K_{13} = U^T K_2, \quad K_{21} = (G^+)^T K_1 U \\ K_{22} &= (G^+)^T K_1 G^+, \quad K_{23} = (G^+)^T K_2, \quad K_{31} = K_3 U, \quad K_{32} = K_3 G^+ \end{aligned} \tag{35}$$

Eliminating the fast variables, the slow subsystem is

$$\begin{aligned} M_a \ddot{y} &= K_{11} y + K_{13} \Delta V \\ 0 &= K_{31} y + K_4 \Delta V \end{aligned} \tag{36}$$

This is the *inertial aggregate model* which is equivalent to linking the internal nodes of the coherent machines by infinite admittances.

2-Area System Example

Connection matrix

$$K = \begin{bmatrix} -9.4574 & 8.0159 & 0.5063 & 0.9351 \\ 8.7238 & -11.3978 & 0.9268 & 1.7472 \\ 0.6739 & 0.9520 & -9.6175 & 7.9917 \\ 1.3644 & 1.9325 & 8.1747 & -11.4716 \end{bmatrix} \tag{37}$$

Decompose K into internal and external connections

$$K^I = \begin{bmatrix} -8.0159 & 8.0159 & 0 & 0 \\ 8.7238 & -8.7238 & 0 & 0 \\ 0 & 0 & -7.9917 & 7.9917 \\ 0 & 0 & 8.1747 & -8.1747 \end{bmatrix} \tag{38}$$

$$\varepsilon K^E = \begin{bmatrix} -1.4414 & 0 & 0.5063 & 0.9351 \\ 0 & -2.6739 & 0.9268 & 1.7472 \\ 0.6739 & 0.9520 & -1.6258 & 0 \\ 1.3644 & 1.9325 & 0 & -3.2969 \end{bmatrix} \tag{39}$$

EM Modes in 2-Area System

$$\lambda(M^{-1}K) = 0, -14.2787, -60.7554, -62.2531 \tag{40}$$

with the corresponding eigenvector vectors

$$v_1 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0.4878 \\ 0.4031 \\ -0.5672 \\ -0.5271 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0.6333 \\ -0.7446 \\ 0.1924 \\ -0.0863 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0.1102 \\ -0.1494 \\ -0.8098 \\ 0.5566 \end{bmatrix} \tag{41}$$

- Interarea mode: $\sqrt{-14.279} = \pm j3.779$ rad/s
- Local modes: $\sqrt{-60.755} = \pm j7.795$ rad/s and $\sqrt{-62.253} = \pm j7.890$ rad/s

Slow and Fast Subsystems of 2-Area System

Slow dynamics:

$$M_a = \frac{1}{2\pi \times 60} \begin{bmatrix} 234 & 0 \\ 0 & 234 \end{bmatrix}, \quad \varepsilon K_a = \begin{bmatrix} -4.1154 & 4.1154 \\ 4.9227 & -4.9227 \end{bmatrix} \tag{42}$$

The eigenvalues of $M_a^{-1} K_a$ are 0 and $-14.561 \Rightarrow$ an interarea mode frequency of $\sqrt{-14.561} = \pm j3.816$ rad/s.
Fast local dynamics:

$$M_d = \frac{1}{2\pi \times 60} \begin{bmatrix} 58.500 & 0 \\ 0 & 55.611 \end{bmatrix}, \quad K_d = \begin{bmatrix} -8.3699 & 0 \\ 0 & -8.0628 \end{bmatrix} \tag{43}$$

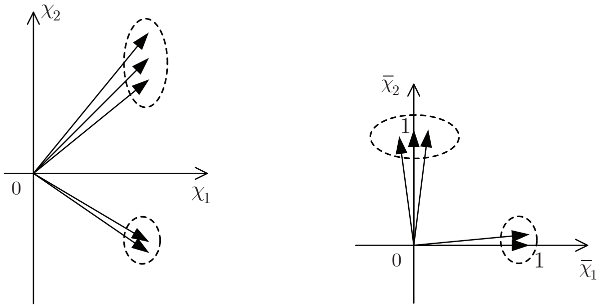
The eigenvalues of $M_d^{-1} K_d$ are -53.939 and $-54.660 \Rightarrow$ local modes of $\pm j7.3443$ and $\pm j7.3932$ rad/s.

Finding Coherent Groups of Machines

1. Compute the electromechanical modes of an N-machine power
2. Select the (interarea) modes with frequencies less than 1 Hz
3. Compute the eigenvectors (mode shapes) of these slower modes
4. Group the machines with similar mode shapes into slow coherent groups
5. These slow coherent groups have weak or sparse connections between them

Grouping Algorithm

Plot rows of the slow eigenvector V_s of the interarea modes



Use Gaussian elimination to select the r most separated rows as reference vectors and group them into V_{s1} and reorder V_s as

$$V_s = \begin{bmatrix} V_{s1} \\ V_{s2} \end{bmatrix} \Rightarrow \begin{bmatrix} V_{s1} \\ V_{s2} \end{bmatrix} V_{s1}^{-1} = \begin{bmatrix} I \\ L \end{bmatrix} = \begin{bmatrix} I \\ L_g \end{bmatrix} + \begin{bmatrix} 0 \\ O(\varepsilon) \end{bmatrix} \qquad (44)$$

Use V_{s1} to form a new coordinate system

Coherent Groups in 2-Area System

$$V_s = \begin{array}{c} \left[\begin{array}{cc} 0.5 & 0.4878 \\ 0.5 & 0.4031 \\ 0.5 & -0.5672 \\ 0.5 & -0.5271 \end{array} \right] \end{array} \begin{array}{l} \text{Gen 1} \\ \text{Gen 2} \\ \text{Gen 11} \\ \text{Gen 12} \end{array}, \quad V_{s1} = \left[\begin{array}{cc} 0.5 & 0.4878 \\ 0.5 & -0.5672 \end{array} \right] \begin{array}{l} \text{Gen 1} \\ \text{Gen 11} \end{array} \tag{45}$$

Then

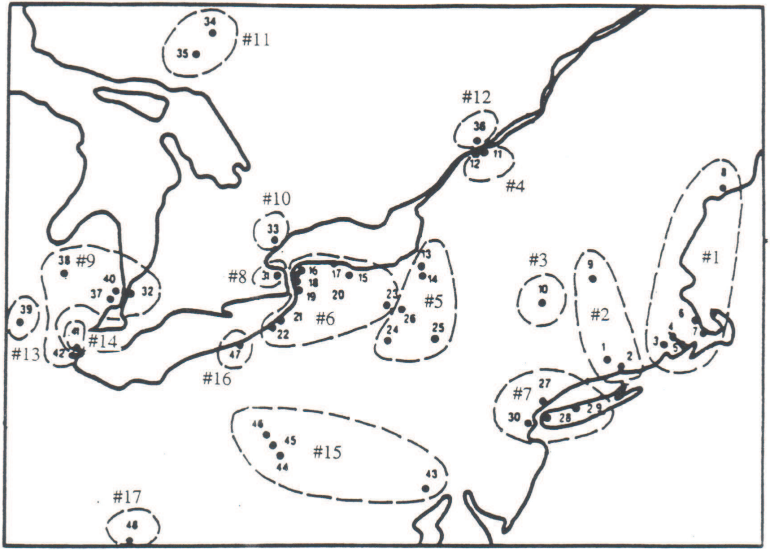
$$V_s' V_{s1}^{-1} = \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \\ \underline{0.9198} & 0.0802 \\ 0.0380 & \underline{0.9620} \end{array} \right] \begin{array}{l} \text{Gen 1} \\ \text{Gen 11} \\ \text{Gen 2} \\ \text{Gen 12} \end{array} \tag{46}$$

Coherency for Load Buses

$$V_\theta = \begin{bmatrix} 0.5 & 0.4283 \\ 0.5 & 0.3535 \\ 0.5 & 0.2556 \\ 0.5 & 0.3844 \\ 0.5 & -0.5018 \\ 0.5 & -0.4667 \\ 0.5 & -0.3556 \\ 0.5 & 0.3128 \\ 0.5 & -0.0523 \\ 0.5 & -0.4671 \\ 0.5 & -0.4125 \end{bmatrix} \begin{array}{l} \text{Bus 1} \\ \text{Bus 2} \\ \text{Bus 3} \\ \text{Bus 10} \\ \text{Bus 11} \\ \text{Bus 12} \\ \text{Bus 13} \\ \text{Bus 20} \\ \text{Bus 101} \\ \text{Bus 110} \\ \text{Bus 120} \end{array} \qquad V_\theta V_{s1}^{-1} = \begin{bmatrix} \underline{0.9436} & 0.0564 \\ \underline{0.8727} & 0.1273 \\ \underline{0.7800} & 0.2200 \\ \underline{0.9020} & 0.0980 \\ 0.0620 & \underline{0.9380} \\ 0.0953 & \underline{0.9047} \\ 0.2006 & \underline{0.7994} \\ \underline{0.8342} & 0.1658 \\ 0.4880 & 0.5120 \\ 0.0949 & \underline{0.9051} \\ 0.1466 & \underline{0.8534} \end{bmatrix} \tag{47}$$

- Bus 101

17-Area Partition of NPCC 48-Machine System



EQUIV and AGGREG Functions for Power System Toolbox

- `L_group`: grouping algorithm
- `coh-map`, `ex_group`: tolerance-based grouping algorithm
- `Podmore`: R. Pormore's algorithm of aggregating generators at terminal buses
- `i_agg`: inertial aggregation at generator internal buses
- `slow_coh`: slow-coherency aggregation, with additional impedance corrections

These functions use the same power system loadflow input data files as the Power System Toolbox.