

MODULE 6.3

**CURRICULUM AND
INSTRUCTION:
THE TEACHING OF
MATHEMATICS**

Teacher Induction Program

Teacher Education Council, Department of Education

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TEACHER INDUCTION PROGRAM

MODULE 6.3

Curriculum and Instruction:

MATHEMATICS

INTRODUCTION

Hello, dear teacher! Welcome to this module on curriculum and instruction in mathematics. This module tries to walk you through the foundations of mathematics and the various teaching strategies and assessment techniques that you can employ in teaching the discipline. The module is self-instructional and allows you to learn in your own space and pace. So, relax and enjoy!

To get the most out of this module, here are a few reminders:

1. Take your time in reading the lessons.
2. Write down points for clarification. You may discuss these points with other teachers or with your mentor.
3. Perform all activities and answer all worksheets. The activities and worksheets are designed to enhance your understanding of the ideas and concepts being discussed.
4. Answer all tests in this module, including the Self-Check Questions, and check your answers against the answer key. The tests will give you an idea how well you understand the lessons. Review the lessons if necessary, until you have achieved a sufficient level of proficiency.
5. As a courtesy to future users, **PLEASE DO NOT WRITE ANYTHING ON ANY PART OF THIS MODULE.** Write the answers to SCQs and worksheets in a notebook/journal. This shall be part of your formative evaluation. TEC shall provide the post-assessment booklet for summative evaluation.

Overview

This module is designed to provide beginning mathematics teachers like you with background knowledge and understanding of some basic contemporary issues in mathematics education. It focuses on the framework of teaching mathematics, and on instructional strategies and assessment approaches.

According to Van de Walle (2001), for teachers of mathematics to be truly effective, they should bring together these four basic components: (1) an

appreciation of the discipline of mathematics itself – what it means to “do mathematics”, (2) an understanding of how students learn and construct ideas, (3) an ability to design and select tasks so that students learn mathematics in a problem-solving environment, and (4) the ability to integrate assessment with the teaching process in order to enhance learning and improve daily instruction.

Basically, this module tries to cover all four grounds.

Lesson 1 sets out to help shape and define your framework for teaching mathematics. It discusses how mathematics has been redefined to address the current demands of society and to accommodate breakthroughs in research and cognitive psychology. More importantly, it explores the implications these changes have on the teaching and learning of mathematics in your own classrooms.

Lesson 2 focuses on instructional strategies. It presents various research-based, learner-centered instructional strategies that you can adopt in your teaching. Activities are provided to help you select and design tasks using these strategies.

Finally, Lesson 3 presents both traditional and authentic forms of assessment, and argues for the adoption of a balanced assessment of student learning.

It is hoped that the module has achieved its aim of producing a concise self-learning kit which nevertheless considers all the significant issues in mathematics education comprehensively and coherently enough to be useful to you.

OBJECTIVES

After going through this module, you should have:

1. obtained greater insights and understanding into the nature of mathematics and its importance;
2. resolved to move away from teacher-centered behaviorist practices towards more learner-centered and constructivist classroom practices;
3. become acquainted with some of the significant and controversial issues in mathematics education for you to consider and reflect on;
4. acquired a repertoire of effective instructional strategies to improve your practice;
5. gained skills in assessing your students' learning more meaningfully using both pen-and-paper tests and authentic assessment practices such as portfolios and performance-based assessment; and,
6. developed an open and willing attitude towards change and innovation.

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PREASSESSMENT

You must be excited to flip over the pages and begin with the lessons just as I am. But first, you need to answer the pre-test to measure how much you know about the topics. The test has two parts. *Good Luck!*

A. DIRECTIONS: Read each item carefully. Choose the letter of the best answer.

Write your answers on a separate sheet.

1. Which ability should you develop in your students foremost?
 - A. using formulas appropriately
 - B. thinking and analyzing problems
 - C. applying rules and laws correctly
 - D. computing quickly and accurately
2. Which of these statements about mathematics is **true**?
 - A. Learning mathematics means mastering a fixed set of basic skills.
 - B. Males are better in mathematics than females.
 - C. Mathematics is about getting the right answers.
 - D. Mathematics is a mental discipline.
3. Which of these statements is characteristic of the behaviorist perspective?
 - A. A learner learns best through repetition, drill and practice.
 - B. Students learn best when left alone to discover concepts and relationships from some given tasks.
 - C. Group work encourages students to learn from each other and make connections.
 - D. Students learn best when the teacher uses situations and contexts that they experience in real life.

4. What is the role of the learner in a constructivist classroom?
- A. actively makes meaning of their experiences and the environment
 - B. passively receives stimuli from their teacher and environment
 - C. constantly helps in the performance of classroom activities
 - D. obediently follows assigned tasks
5. Which of the following strategies is essentially an inquiry approach?
- A. Lecture
 - B. Discovery
 - C. Socratic
 - D. Deductive
6. Concept attainment sharpens the students' skills in the following EXCEPT one. Which is it?
- A. Recalling definitions and rules
 - B. Defining and explaining concepts
 - C. Separating important from unimportant information
 - D. Searching for patterns and making generalizations
7. In developing the concept of surface area of solid figures, Ms. Abalajon makes use of actual boxes, cans, balls and pyramids for students to explore and investigate. What principle of mathematics teaching is she applying?
- A. Communication is an integral part of mathematics learning.
 - B. Mathematics learning is a developmental process.
 - C. Multi-embodiment aids learning mathematics.
 - D. Metacognition affects mathematics learning.
8. Ms. Buzon is introducing the addition and multiplication rules of equality. Which of the following techniques would help her students acquire a deeper conceptual understanding of the rules of equality?
- A. Present the rules clearly and in an orderly manner
 - B. Use a balance scale to illustrate the rules of equality.

- C. Ask the students to state the rules in their own words
- D. Use the rules of equality to solve different equations.
9. Mr. Anselmo wants to present his lesson on radicals in a meaningful way. Which of the following techniques would be best for his purpose?
- A. Define a radical clearly and provide specific examples.
- B. Begin with a problem that shows the real-life application of radicals.
- C. Start the lesson with a game on finding powers of numbers and relate it to the concept of radicals.
- D. Review powers of numbers using the drill method or with flashcards and relate it to the concept of radicals.
10. Ms. Sioson noticed that her students tend to forget the lessons after the test, especially when the course is over. What could be the most likely reason for this?
- A. Students have short memory recall.
- B. Students are required to remember the lesson only for the test.
- C. The lessons lack relevance and application in the students' lives.
- D. The lessons are too difficult and complicated to be remembered later.
11. Which of these statements about teaching and assessing is true?
- A. Teaching is used to improve assessment.
- B. Assessment is done only after teaching occurs.
- C. Teaching and assessing are interdependent processes.
- D. Teaching and assessing are independent processes.
12. What is the primary reason why we need to assess?
- A. To give a grade at the end of the grading period
- B. To identify our students' strengths and weaknesses
- C. To identify which students are good, average or poor

D. To identify the weaknesses and misconceptions of our students

13. What are the features of performance assessment?

- A. Tasks, rubrics, collaboration
- B. Tests, quizzes, assignments
- C. Reflection, goal-setting, self-evaluation
- D. Criterion-reference, group activities, competition

14. How should one assess?

- A. Use standardized tests as these are valid and reliable.
- B. Assess what students do not know about mathematics.
- C. Use various techniques to get a more accurate picture of the learner.
- D. Use multiple choice items as these are most versatile, economical and efficient.

15. Which one of the following is characteristic of authentic assessment?

- A. Is highly objective and reliable
- B. Involves counting of scores and correct answers
- C. Uses machines for accurate and speedy checking
- D. Uses rubrics and the judgment of human evaluators

B. DIRECTIONS: Read and reflect on each item. Answer each question briefly.

1. What is the most important thing a student can learn in your mathematics class?
2. How do students learn mathematics?
- 3-4. Discuss two effective strategies in teaching mathematics.
5. How should one assess students' learning in mathematics?

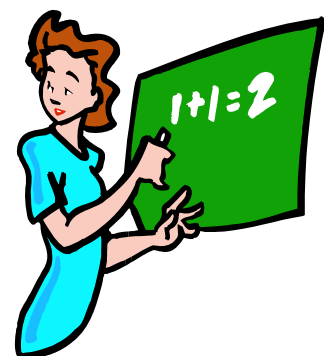
Turn to page 109 to check your answers.

LESSON 1

FOUNDATIONS OF MATHEMATICS TEACHING AND LEARNING

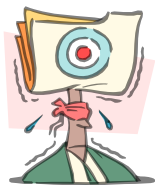
INTRODUCTION

Teaching is both an art and a science. Have you ever thought about what makes an effective mathematics teacher? Try to recall the teachers who influenced your life. What made them stand out?



An effective mathematics teacher reflectively integrates theory with practice. Why? Because it is the theories about teaching and learning that provide a framework for analyzing learning situations and improving classroom instruction.

To develop one's philosophical framework for teaching mathematics, we begin by understanding the nature of mathematics and the goals of mathematics education. Research has shown that the teachers' beliefs and conceptions influence the way they teach mathematics and the way their students perceive the discipline. Hence, this lesson aims to clarify these beliefs and conceptions, and to enrich or modify them, if necessary.



OBJECTIVES

After going through this lesson, you should be able to:

- discuss the nature of mathematics and its importance;
- state the goals of mathematics education;
- compare and contrast behaviorism and cognitivism;
- reflect on the theory of constructivism and its implications to mathematics teaching and learning; and,
- reflect, clarify and enrich your philosophy of mathematics teaching.



ACTIVITY 1.1

Some Views about the Nature of Mathematics and Mathematics Teaching and Learning

DIRECTIONS: Read and reflect on each item carefully. State whether you agree or disagree to each of the statements. The questionnaire asks for your opinion, hence, there is no right or wrong answer.

Statements
1. Learning mathematics means mastering a fixed set of basic skills.
2. Mathematics is a series of arbitrary rules, handed down by the teacher, who in turn got them from some very smart source.
3. Mathematics is about getting the right answers.

Statements

4. There is only one way to solve any problem.
5. Every problem must have a predetermined solution.
6. Mathematics is boring and nothing you can do will make it interesting.
7. Mathematics never changes.
8. Only very intelligent people can understand mathematics. Others cannot do it at all.
9. Males are better in mathematics than females.
10. The harder mathematics is, the better it is – if it is too easy, it cannot be really mathematics.
11. Mathematics requires the memorization of a lot of rules and formulas.
12. There is no room for opinions in mathematics. Everything is right or wrong, true or false.
13. Mathematics is made up of a number of unrelated topics.
14. If you are good in language, you are not good in mathematics.
15. You have to be really good in math to appreciate it.

Did you agree to all or most of the questions? Set aside your responses to this questionnaire for later use. Your responses may just reveal your views about the nature of mathematics!

The Nature of Mathematics and Its Implications

This activity will help clarify your idea of the nature of mathematics

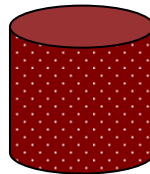
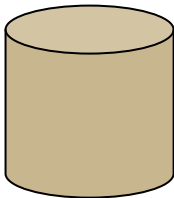


ACTIVITY 1.2

What's in a Circle?

Do This

Materials: cans of different sizes, string, ruler



Steps:

1. Wrap the string around the can to measure the circumference of its base.
Note that the base of the can is a circle.
2. Measure the diameter of the can's base using a ruler.
3. Divide the circumference by the diameter.
4. Repeat steps 1-3 using the other cans. Fill the table below.

Can	Circumference (C)	Diameter (D)	C/D
A			
B			
C			
D			

What do you observe? Do you see any pattern? If you do, can you state the relationship between the circumference and diameter of a circle?



SCQ 1.1

1. Did the activity ask you to recall a formula and ask you to substitute appropriate values in the formula?
2. Did the activity allow you to observe patterns and find out for yourself the relationship between the circumference and diameter of a circle?
3. From this activity, what does it mean to do mathematics?

Your answers should lead you to the definition of the nature of mathematics that is put forth by the National Council of Teachers of Mathematics (NCTM), the largest organization of mathematics teachers in the world.



READ

NCTM (1989) defines the nature of mathematics as follows:

1. *Mathematics is a study of patterns and relationships.*

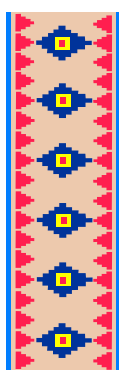
Mathematical ideas are interwoven with each other. Students must explore the recurring ideas or the patterns and discover the relationships between and among them, like what you did in the activity.

2. Mathematics is a way of thinking.

The problem solving activities and various lessons in mathematics train us to think logically, analytically, critically and systematically. In a way, mathematics provides us with the thinking skills needed to confront everyday problems.



3. Mathematics is an art.



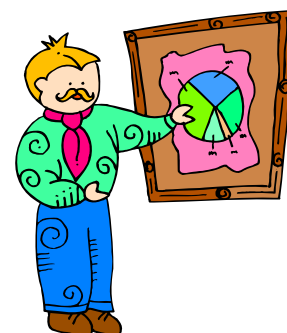
Mathematics is characterized by order and internal consistency. Numerous patterns can be found in numbers and geometric figures. Tessellations, weaving and tiling are a few explicit examples of mathematics in art. By exploring the orderliness and consistency of mathematics, we learn to appreciate its beauty.

4. Mathematics is a language.

It is used to communicate complex processes and thoughts efficiently using symbols and specific and precise terms. Mathematics has its own register, or special vocabulary, which students have to learn to be able to communicate well about mathematics and to speak and think like mathematicians. For instance, mathematicians would not use 'equal', 'congruent' and 'similar' interchangeably as these terms mean different things.

5. Mathematics is a tool.

Many occupations require the knowledge of mathematics. Scientists, engineers, businessmen, and many other professions use a great deal of mathematics to do their work.





SCQ 1.2

1. Based on the discussions on the nature of mathematics, what should you emphasize in teaching mathematics? Why?
2. Consider your responses to Activity 1. All statements in the questionnaire are **myths**, meaning, they are not held as true by most mathematics educators in the world. Did you agree to any of the statements? Justify your answer.



READ

The Goals of Mathematics Education



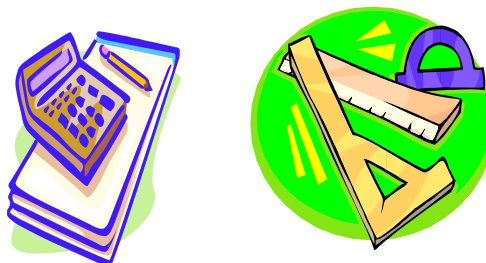
If mathematics is not just about performing operations, using the right formulas, and getting the right answers, what then should be our goals as mathematics teachers?

The National Council of Teachers of Mathematics' *Standards for School Mathematics* (NCTM, 1989) identified five broad goals required to meet the students' mathematical needs for the 21st century.

NCTM recommends that mathematics teachers enable students to:

1. *Value mathematics*

Students will value mathematics if they see how it plays a role in their real lives and in society. Thus, your task is to make mathematics learning meaningful to the students by connecting the lesson to their real life experiences and allowing students to experience mathematics through actual measurements and explorations.



2. *Reason mathematically*

Mathematics trains the mind to think analytically and logically. As the teacher, your task is to provide activities that will provide students opportunities to reason logically, make conjectures, gather evidence, build arguments, and arrive at informed and sound decisions. The process of obtaining the correct answers should be emphasized.

3. *Communicate mathematics*

To be able to communicate well in mathematics, students must be familiar with the mathematics register, or the special vocabulary of mathematics. You must be a good model in the use of correct and precise mathematical terms and phrases. You must also encourage students to verbalize and defend their answers.



4. *Solve problems*

Problem solving is the heart of mathematics. Students must be exposed to a variety of problems – problems that vary in context, in level of difficulty and in mathematical methods required for their solutions. Students must learn to analyze the conditions in a problem, to restate them, to plan strategies for solving it, to develop several solutions, and to work collaboratively with others in search of the solution. Most of all, students must develop the discipline and perseverance to solve a problem no matter how complex it is.

5. *Develop confidence*

Taking pride in one's competence in mathematics is all-important. Sadly, a number of people find it fashionable to boast of their incompetence in mathematics. To go further in mathematics, students must develop confidence in their ability to learn and do mathematics. Such confidence is built on success in mathematical tasks in the classroom.



SCQ 1.3

1. Do your students value mathematics? If yes, what is it about mathematics that they find important? If no, why not?
2. Do you ask students to explain their solution or their reasoning on a particular task? Do you encourage them to present and defend their answers in class? Why or why not?
3. Is problem solving central to the way you teach mathematics? If yes, how do you do it? If not, what are the emphases of your lessons?
4. Are your students happy and confident about their ability to do mathematics? What strategies do you believe help boost their confidence?



ACTIVITY 1.3

What is Your Teaching Style?

DIRECTIONS: Do you want to know your teaching style? This questionnaire lists some views, assumptions and practices of teachers about teaching and learning. Read each statement carefully. Then state whether you agree or disagree with the statement.

STATEMENTS
1. The most practical way of teaching students is through lectures and discussions.
2. A student retains the most knowledge by memorizing definitions and facts.
3. A learner learns best through repetition, drill and practice.
4. A student gains knowledge by listening to the teachers' explanation.
5. The teacher must answer right away all questions of students that are related to the lesson and explain them.
6. When a pupil gives a wrong answer, the teacher must correct it right away.
7. The teacher should ask students to memorize rules, laws, theorems and formulas.
8. A student's mind is like a dry sponge that absorbs what the teacher explains or discusses.
9. The teachers teach best when they define terms, state the laws/rules, explain the lesson in detail, and give specific examples or illustrations.

STATEMENTS

10. Students learn best when left alone to discover concepts and relationships from some given tasks.

11. Learners construct understanding by linking new information with prior knowledge.

12. Learners are creatures of will and purpose who actively make meaning as they interact with objects and events.

13. Students learn best when left alone to discover concepts and relationships from some given tasks.

14. The teacher should provide as little guidance, explanations, and lectures as possible to encourage learner autonomy and initiative.

15. Students should explain terms, concepts or rules/laws in their own words.

16. Group work encourages students to learn from each other and make connections.

17. The teacher should encourage learner inquiry, debate and discussion in the classroom.

18. Students learn best when the teacher uses situations and contexts that they experience in real life.

19. The teacher should emphasize novel, investigative and open-ended problems rather than drill, practice and rote exercises.

20. The teacher should ask probing questions that require students to justify their claims, provide evidence and uncover new ideas.

Would you like to know your teaching style?



If you agreed to the first 10 items, it is likely you are a behaviorist teacher and you subscribe to the philosophy of **Behaviorism**. If you agreed to the second half of the questionnaire, then it is likely you are a constructivist teacher and you subscribe to the **Constructivist Theory of Learning**, or more broadly, to **Cognitivism**. If you agreed to several points from both schools of thought, it is perhaps because (1) you have an eclectic teaching style; or, (2) you have yet no definite or clear philosophy about teaching and learning.

There are two predominant schools of thought when it comes to teaching and learning, and these are behaviorism and cognitivism. Do Activity 4 to check how well you can differentiate one from the other.



ACTIVITY 1.4

Comparison of Behaviorist and Cognitive Perspectives

Directions: Complete the table that follows which compares the behaviorist and cognitive perspectives. The pool of ideas provides the answers. For each pair of views, decide which one is behaviorist and which is cognitive. Write each idea in the correct box, or you may simply write the number corresponding to the idea.

	BEHAVIORISM	COGNITIVISM
View of Learning		
View of Learner		
Role of Teacher		
Role of Learner		

POOL OF IDEAS

View of Learning

1. Accumulation of responses through selective reinforcement
2. Development of strategies to encode and retrieve information

View of Learner

3. Creators of understanding
4. Empty receptacle

Role of Teacher

5. Partner in the process of meaning making; helps students organize and make sense of information
6. Controller of the environment through reinforcement and cues for appropriate student behavior

Role of Learner

7. Passive recipient of stimuli from teacher and environment
8. Active meaning maker through strategy use

Turn to page 113 for the answer key

Were you able to differentiate the two correctly? Great! Now, let us move on to a theory of learning that has taken the world by storm in the past two decades—CONSTRUCTIVISM. Are you ready?



READ THIS

Constructivism

Constructivism is a theory about knowledge and learning that is influenced by the work of Piaget and Vygotsky. It is anchored on the following principles from research on cognitive psychology:

- **Learners construct understanding.** They do not simply mirror what they are told or what they read.
- **To understand something is to know relationships.** Bits of information isolated from these structures are forgotten or become inaccessible to memory.
- **All learning depends on prior knowledge.** Learners try to link new information to what they already know in order to interpret the new material in terms of established schemata.
- **Learning is enhanced by social interaction.** Thoughts and ideas are enriched and clarified when these are verbalized in the course of the learners' interaction with each other.



Do you know how a classroom guided by constructivist ideas differs from traditional classrooms?

Some of the ideas in the table below are adapted from Brooks and Brooks (1993).

TRADITIONAL CLASSROOM	CONSTRUCTIVIST CLASSROOM
Curriculum emphasizes basic skills and proceeds from the parts to the whole.	Curriculum emphasizes BIG concepts; proceeds from the whole and expanding to include the parts
Adheres strictly to fixed curriculum	Values student questions and interests
Primary materials are the textbook and workbook.	Materials are varied and include manipulatives.
Learning is based on repetition.	Learning is interactive, building on what the student already knows.
Teachers disseminate information; students receive knowledge.	Teachers facilitate the students' construction of knowledge.
Teacher's role is directive, rooted in authority.	Teacher's role is interactive, rooted in negotiation.
Assessment is through pen-and-paper testing.	Assessment focuses on both process and product; emphasizes authenticity of tasks.
Knowledge is seen as inert.	Knowledge is seen as dynamic, ever changing with our experiences.

Students work alone.	Students work in groups.
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In the constructivist classroom, teaching must give learners the opportunity for concrete, contextually meaningful experience through which they can search for patterns, raise their own questions, and construct their own models, concepts and strategies.

In general, contemporary learning theories in mathematics place emphasis on the following:

1. the extended use of concrete experiences and a more gradual move to abstraction;
2. the support of meaningful practical activities;
3. the use of manipulative materials;
4. the importance of integrating knowledge in a meaningful way;
5. the value of discussion; and,
6. the need to cater for individual differences.



SCQ 1.4

1. Is your classroom primarily traditional or constructivist?
2. Do you see some advantages of shifting from a traditional classroom to a constructivist one? If yes, what are these? If no, why not?
3. Do you see some difficulties in adopting the constructivist perspective in teaching? If yes, what are these?
4. Which teaching style would a teacher likely adopt if they see mathematics as a study of patterns and relationships? Why do you say so?



ACTIVITY 1.5

What the BEC Says

As a beginning teacher, you must be familiar with the Basic Education Curriculum (BEC). Do you know what the BEC says about the teaching and learning of mathematics? Here is an excerpt from the philosophy of the 2002 Basic Education Curriculum (BEC). Read it carefully and answer the reflection questions that follow.



READ

Philosophy of the 2002 Basic Education Curriculum

- The emphasis is on the empowerment of learners who are competent in learning how to learn and have life skills so that they become a self-developed person who is *makabayan*, *makatao*, *makakalikasan*, and *makadiyos*.
- Curriculum is an interactive and collaborative one. Hence, it is based on the principle that there are two main sources of reliable and meaningful knowledge for contemporary basic education: expert systems of knowledge (teachers, textbooks and other resources) and the learner's experience in their context.

- The teacher is not the authoritarian instructor but a facilitator or manager of the learning process. Thus the teacher helps students to learn not primarily answers but how to reflect on, characterize and discuss problems and how, on their initiative, they can form or find valid answers.



SCQ 1.5

1. In your view, is the philosophy of the 2002 BEC inclined towards the behaviorist or the constructivist perspective? Why do you say so?
2. What implications does the philosophy of the 2002 BEC have on your teaching?
3. Who are you as a mathematics teacher? Write down your views about mathematics teaching and learning.

SCQ 1.6

Answer the following questions briefly to check your understanding of the lesson.

1. Describe how a teacher who subscribes to a behaviorist perspective views the teaching and learning of mathematics.
2. If you subscribe to constructivism, how would you teach math? Why?
3. Discuss the nature and goals of mathematics and their implications to the teaching and learning of mathematics.



POINTS TO REMEMBER

- Mathematics is a study of patterns and relationships; a way of thinking; an art, characterized by order and internal consistency; a language, using carefully defined terms and symbols; and, a tool (NCTM, 1989).
- The five broad goals of mathematics education to meet students' mathematical needs for the 21st century are: to value mathematics; to reason mathematically; to communicate mathematics; to solve problems; and to develop confidence.
- **Behaviorism** is characterized by the use of stimulus-response situations through which connections are practiced.
- **Cognitivism** is characterized by the emphasis on the learners and in providing them with an environment in which they investigate and perhaps discover and in which understandings might be constructed through their own efforts
- **Constructivism** is a theory about knowledge and learning that is anchored on the following principles: (1) *Learners construct understanding*; (2) *To understand something is to know relationships*; (3) *All learning depends on prior knowledge*; and, (4) *Learning is enhanced by social interaction*.
- In general, contemporary learning theories in mathematics place emphasis on the following: the use of concrete experiences and a more gradual move to abstraction; the support of meaningful practical activities; the use of manipulatives; and the value of discussion and integrating knowledge in a meaningful way.

LESSON 2

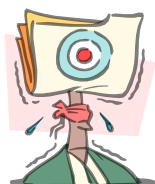
STRATEGIES IN TEACHING MATHEMATICS

INTRODUCTION

Now you know what mathematics is, why it is important, and what the theories say about how students learn. But how do you teach it? As they say, good teachers understand theory, and they translate theory into practice.

There is no single best strategy or method in teaching mathematics since the choice of teaching strategy depends on a number of factors. As teachers we are expected to have a repertoire of teaching strategies and methods that cater to the specific purposes and needs of our class. Research has shown that teaching strategies that are interactive, integrated, experiential, varied, and that tries to connect the lesson to the students' daily lives are effective.

This lesson will walk you through some of these instructional strategies such as Inquiry, Problem-based Learning (PBL), Cooperative Learning, Use of Rathmell Triangle Model, Concept Attainment, and Peer Practice. Emphasis will be placed on instructional learner-centered strategies that promote a constructivist classroom environment.



OBJECTIVES

After going through this lesson, you should be able to:

1. compare the direct instructional approach to the interactive, learner centered-approaches;
2. discuss various strategies in teaching mathematics with focus on the roles of the teacher and the learners, and the procedure;
3. apply the Rathmell-Triangle Model in developing a concept; and,
4. construct sample lessons using concept attainment.



READ

Research-Based Strategies



What instructional strategies best enhance student achievement? After many years of teaching, seasoned teachers can identify some of these strategies based on their experiences.

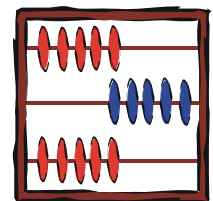
To help answer this question, Marzano, Gaddy and Dean (2000, in Feden & Vogel, 2003) conducted a meta-analysis of what works in the classroom. A 'meta-analysis' requires summarizing a large number of research studies and combining their results. The results of their study yielded nine (9) categories of good

instructional practices. Most of them are self-explanatory and not new to you! In fact you may be practicing most of them. We will just go over each one briefly.

- 1. Identifying similarities and differences.** Recall that mathematics is defined as a study of patterns and relationships. Identifying similarities and differences is a crucial step towards identifying patterns and relationships. The Concept Attainment Strategy on page (55) is one good example of this instructional strategy.
- 2. Summarizing and note taking.** To summarize concepts learned, you may use tables, graphic organizers and concept mapping.
- 3. Reinforcing effort and providing recognition.** These are extrinsic ways of motivating learners, which, if done well, will develop intrinsic motivation among the learners.
- 4. Homework and practice.** These develop procedural knowledge and are best if the practice is followed by specific feedback.
- 5. Nonlinguistic representations.** The use of visual representations and manipulative models in mathematics help learners understand concepts that are otherwise abstract.
- 6. Cooperative learning.** These are effective promoters of student achievement and will be discussed in greater detail in this lesson.
- 7. Setting goals and providing feedback.** Setting goals provides direction to student learning and providing feedback tells them how far they have gone in reaching their goals.



Very good!



- 8. Generating and testing hypothesis.** The discovery or inquiry approaches in teaching mathematics, as well as the Concept Attainment Strategy and Concept Formation Strategy, encourage learners to generate and test hypothesis.
- 9. Activating prior knowledge.** This can be connected to our discussion of constructivism in Lesson 1. Learners learn best when they build on previous knowledge and when they are able to make meaningful connections between what they already know and what the new learning.



Do you think you can integrate all of these good instructional practices in your teaching? You can start by identifying those that you can readily integrate in your teaching and working on the rest one at a time. Good luck!



READ

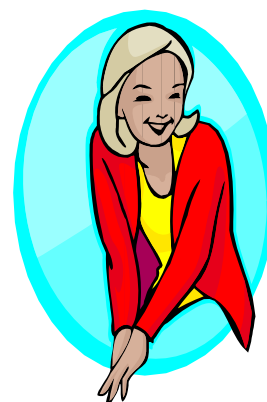
The Two Main Methods: Deductive and Inductive

You often hear of emerging and innovative strategies like interactive, integrated, multi-disciplinary, reflective or brain-based. The variety of strategies in the educational literature seems endless, but all these can be classified either as deductive or inductive strategies. Can you differentiate between the two?



Deductive method begins with what is abstract, general, and unknown to the learners and proceeds to what is concrete, specific and what is known to the learner. Teaching begins with the rule or principle, then examples are given to explain the rule or principle.

Inductive method starts with what is specific, concrete and what is known to the learners and ends with what is abstract, general and unknown. Teaching begins from the concrete experiences of the learners and from there moves to the rule or principle.



SAMPLE LESSONS USING DEDUCTIVE AND INDUCTIVE METHODS

A. USING THE DEDUCTIVE METHOD

Lesson Topic: Converting Mixed Numbers to Improper Fractions and Vice-Versa

Objectives: To convert mixed numbers to improper fractions
To convert improper fractions to mixed numbers

Materials: flash cards

Procedure:

Starter/Motivation

Use flash cards on fractions and mixed numbers. Let the pupils identify whether the number on the card is a proper fraction, an improper fraction, or a mixed number.

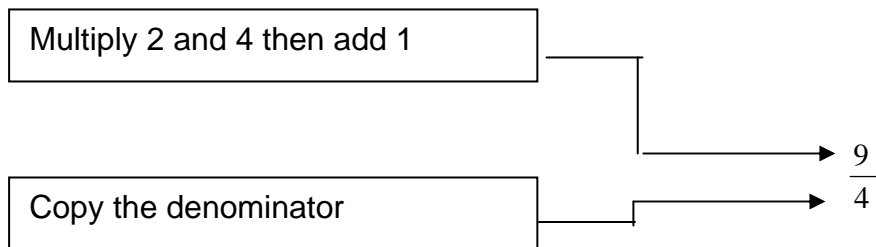
Lesson Proper

1. Discuss how a mixed number can be written as an improper fraction by stating the rule.

(Multiply the denominator of the mixed number to its whole number, then add the numerator to the product. This will be the numerator of the improper fraction. Use the same denominator.)

2. Illustrate the rule by showing examples.

To convert $2\frac{1}{4}$ to improper fraction:

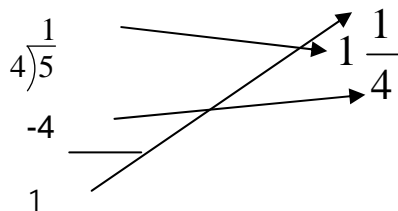


3. State how an improper fraction can be written as a mixed number.

(Divide the numerator by the denominator. The remainder becomes the numerator of the mixed number and the partial quotient becomes the whole number of the mixed number. Use the same denominator.)

4. Illustrate the rule by showing an example.

To convert $5/4$ to mixed number:

*Practice*

1. Give additional exercises for the pupils to work on.
2. Call on volunteers to show the answers. Discuss the answers.

Evaluation

A. Convert each of the following mixed numbers to improper fractions.

- a. $1 \frac{1}{2}$ b. $4 \frac{3}{4}$ c. $7 \frac{1}{4}$ d. $8 \frac{5}{6}$

B. Convert each of the following improper fractions to mixed numbers.

- a. $\frac{17}{2}$ b. $\frac{15}{4}$ c. $\frac{11}{3}$ d. $\frac{9}{5}$

C. Explain briefly.

- a. How do you convert a mixed number to an improper fraction?
b. How do you convert an improper fraction to a mixed number?

Homework

Assign the page number in the workbook or give a worksheet to be answered.

B. USING THE INDUCTIVE METHOD

Lesson Topic: Converting Mixed Numbers to Improper Fractions and Vice-Versa

Objectives: To convert mixed numbers to improper fractions

To convert improper fractions to mixed numbers

Materials: fraction discs, whole-part chart

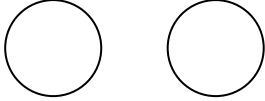

Procedure:*Starter/Motivation*

Ask pupils to work in pairs. Each pupil should have his or her fraction discs and whole-part chart.

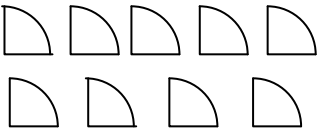
Mario has $2\frac{1}{4}$ hotcakes left. He slices the hotcakes into fourths so he can give them away to his friends. How many fourths does he make?

Lesson Proper

1. Let the pupils show $2\frac{1}{4}$ on their whole-part chart. Let them identify the whole and the part.

Whole	Part
	

2. Ask the pupils to convert the 2 wholes into fourths. How many fourths are there now? (There are 9 one-fourths or 9 (1/4), thus $2\frac{1}{4} = \frac{9}{4}$)

Whole	Part
	

3. Let them convert each of these mixed numbers to improper fractions using the same procedure:

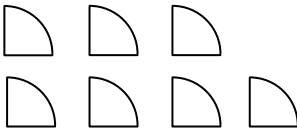
a) $1\frac{1}{4}$

b) $2\frac{1}{3}$

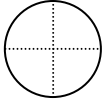

c) $3\frac{1}{2}$

4. Ask them to find a pattern of converting mixed numbers to improper fractions (without using the fraction discs).

5. Let the pupils show seven fourths (that is, $\frac{7}{4}$).

Whole	Part
	

6. Place the 7 fourths on the whole-part chart. Use a whole circle disc for the fourths that form a whole.

Whole	Part
	

7. Read the number on the chart now (Ans: $1\frac{3}{4}$)
8. Let the pupils convert the following to mixed numbers using the same procedure:
- a) $\frac{5}{4}$ b) $\frac{7}{3}$ c) $\frac{10}{4}$
9. Ask them to find a pattern of converting mixed numbers to improper fractions (without using the fraction discs).
10. State the generalizations.

Practice

- Give additional exercises for the pupils to work on.
- Call on volunteers to show the answers. Discuss the answers.

Evaluation

- A. Convert each of the following mixed numbers to improper fractions. You may use your fraction discs and whole-part chart.
- a. $1\frac{1}{2}$ b. $4\frac{3}{4}$ c. $7\frac{1}{4}$ d. $8\frac{5}{6}$
- B. Convert each of the following improper fractions to mixed numbers. You may use your fraction discs and whole-part chart.
- a. $\frac{17}{2}$ b. $\frac{15}{4}$ c. $\frac{11}{3}$ d. $\frac{9}{5}$
- C. Explain briefly.
- How do you convert a mixed number to an improper fraction?
 - How do you convert an improper fraction to a mixed number?

Homework

Assign the page number in the workbook or give a worksheet to be answered.



SCQ 2.1

1. Identify at least two advantages and two disadvantages of the deductive method.
2. Identify at least two advantages and two disadvantages of the inductive method.



READ



How do you choose the right method or strategy for teaching a lesson? Is there a “best” method?

Factors in Choosing a Method or Strategy

1. **Instructional Objective.** If the objective is to master multiplication facts, the strategy may be more on drill and practice. However, if the objective is to develop the concept of fraction, the lesson may require a more activity-oriented strategy involving lots of manipulative materials and reflective thinking.

2. **The nature of the subject matter.** Generally, the inductive approach is favored over the deductive approach, but certain topics are more efficiently and effectively taught using the deductive method. When the topic is too difficult, the use of the inductive method may require too much time and effort and may not be successful. On the other hand, when the topic is too easy and routine, it is best taught deductively to save time.
3. **The learners.** The learner's readiness is a factor we cannot ignore. Complex, investigative strategies may be more suitable for more competent students while drill and practice for mastery may be more appropriate for the less competent students.
4. **The teacher.** Beginning teachers, or those who are still groping with the subject matter, are usually more confident using the deductive method. The inductive method requires a solid and deep understanding of the subject matter as it involves more processing of ideas and student responses.
5. **School Policies.** Some schools adopt a particular approach to teaching. For instance, certain private schools claim that their schools employ learner-centered strategies while others boast of brain-based strategies or thematic, integrated approaches to teaching. On the other hand, there are schools that put emphasis on mastery of skills.



SCQ 2.2

1. Cite a lesson objective that is best taught using the deductive method.
Justify your choice.
2. Cite a lesson objective that is best taught using the inductive method.
Justify your choice.

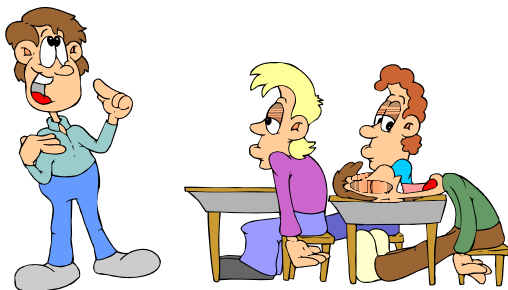


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STRATEGIES IN TEACHING MATHEMATICS

Here are some common strategies in teaching mathematics. The first one uses a deductive approach while the rest are variations of the inductive approach.

Interactive Direct Instruction



Do you recall how your teachers taught you? Chances are it was through direct instruction.

Direct instruction is the most traditional approach in teaching. It is teacher-centered and is an efficient means of covering content. However, while it is the most widely used by teachers, it is criticized as the **least effective** in promoting real learning among the learners.

Gunter, Estes, and Schwab (1995) describe the steps in the direct instructional approach as follows:

1. Review previously learned material.
2. State objectives for the lesson.
3. Present the new material.
4. Provide guided practice with corrective feedback.
5. Assign independent practice with corrective feedback.
6. Review periodically and provide corrective practice.

Do these steps sound too familiar? Learners will certainly be better off if you can provide variety to this all-time favorite! Can you do that?



SCQ 2.3

1. Cite at least two advantages of the interactive direct instruction strategy.
2. Cite at least two disadvantages of the interactive direct instruction strategy.
3. Describe the lecture method that your teachers used when you were a student. Is it similar to the interactive direct instruction strategy? In what ways?



READ

Powerful Questions

Questioning is one of the most powerful modes of teaching. It has the potential to greatly facilitate the learning process when used appropriately. Using questions and answers to challenge assumptions, expose contradictions and lead to a new understanding is an undeniably powerful teaching approach. In fact, it is widely believed that in order to teach well, one must be able to question well.

Knowing what to ask is one fundamental tool of effective teachers. What kinds of questions do teachers usually ask?

Convergent Vs Divergent Questions



Convergent questions require single responses and there is usually one correct or best response.

Examples: a) What is the value of pi?

b) Find the product of 6 and 7.

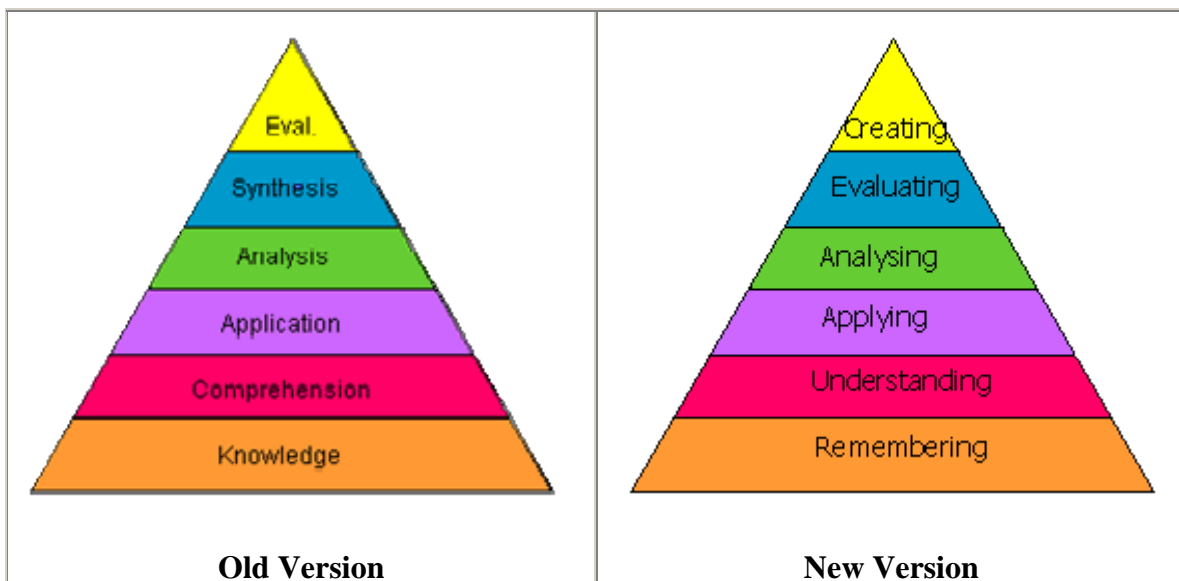
Divergent questions require students to give more general or open responses to questions that may have more than one appropriate answer.

Examples: a) What are the possible dimensions of a rectangle whose area is 100 cm^2 ?

b) Show different ways of adding 28 and 67.



Questions may be categorized according to the level of thinking they are likely to stimulate using Bloom's taxonomy, which incidentally has been recently revised. In the new version, Bloom's six major categories were changed from noun to verb forms. Additionally, the lowest level of the original, knowledge was renamed and became remembering. Finally, comprehension and synthesis were retitled to understanding and creating. In an effort to minimize the confusion, comparison images appear below.



Source: http://web.odu.edu/educ/lischult/blooms_taxonomy.htm

The new terms are defined as:

- **Remembering:** Retrieving, recognizing, and recalling relevant knowledge from long-term memory.

Example: *What is the answer in subtraction?*

- **Understanding:** Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining.

Example: *State in your own words the steps in subtracting numbers with regrouping.*

- **Applying:** Carrying out or using a procedure through executing, or implementing.

Example: *Subtract 128 from 400.*

- **Analyzing:** Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing.

Example: *Joy received P50 from her mother. She bought a book for P75 and still had P28 in her wallet. How much money did she have in her wallet at the start?*

- **Evaluating:** Making judgments based on criteria and standards through checking and critiquing.

Example: *Here is Jo's solution to a subtraction problem: $96 - 58 = 48$. Is her solution correct? Why or why not?*

- **Creating:** Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing.

Example: *How many different ways can you subtract 58 from 96? Discuss each way.*

(Anderson & Krathwohl, 2001, pp. 67-68).

Bloom's Taxonomy

The Knowledge Dimension	The Cognitive Process Dimension					
	Remember	Understand	Apply	Analyze	Evaluate	Create
<u>Factual Knowledge</u>	<u>List</u>	<u>Summarize</u>	<u>Classify</u>	<u>Order</u>	<u>Rank</u>	<u>Combine</u>
<u>Conceptual Knowledge</u>	<u>Describe</u>	<u>Interpret</u>	<u>Experiment</u>	<u>Explain</u>	<u>Assess</u>	<u>Plan</u>
<u>Procedural Knowledge</u>	<u>Tabulate</u>	<u>Predict</u>	<u>Calculate</u>	<u>Differentiate</u>	<u>Conclude</u>	<u>Compose</u>
<u>Meta-Cognitive Knowledge</u>	<u>Appropriate Use</u>	<u>Execute</u>	<u>Construct</u>	<u>Achieve</u>	<u>Action</u>	<u>Actualize</u>

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Questions in the remembering and understanding categories are considered to be **lower-order questions** as these require lower-order thinking skills only. Those in the other categories are considered to be **higher-order questions** as these require higher-order thinking skills (HOTS).

For questioning to be a powerful strategy, a teacher should take note of the following (Feden and Vogel, 2003):

1. Try to ask as many higher-order questions as lower-order questions, if possible. Sadly, teachers tend to ask mostly lower-order questions.
2. Know how to time and pace your questions.
3. Use prompting questions (hints or clues) to help students respond to your questions.
4. Ask probing questions to seek clarification and to lead students to a more complete and thoughtful answer.
5. Allow enough time for students to answer a question.
6. Provide verbal and non-verbal reinforcements to students' responses such as "OK", "Good work!".

ACTIVITY 2.1

A. Directions: Write L if the question is a lower-order question and H if it is a higher-order question.

1. What is the numerator of $\frac{1}{2}$?
2. What must be added to -5 to get 4?
3. Is your answer to the problem logical?
4. Which is a better buy: a 50 g butter for P12 or a 100 g butter for P23?
5. What is a triangle with 3 equal sides?
6. What is the rule in multiplying integers?
7. State the Pythagorean theorem.
8. Jo is thrice older than Ray. If Jo's age is c , what is Ray's age in terms of c ?
9. How is addition related to subtraction?
10. Allan bought a shirt at 25% discount. If he paid P240 for the shirt, what was its original price?

B. Directions: Do what is asked.

1. Write (1) convergent question
2. Write (1) divergent question.

Turn to page 113 for the answer key.

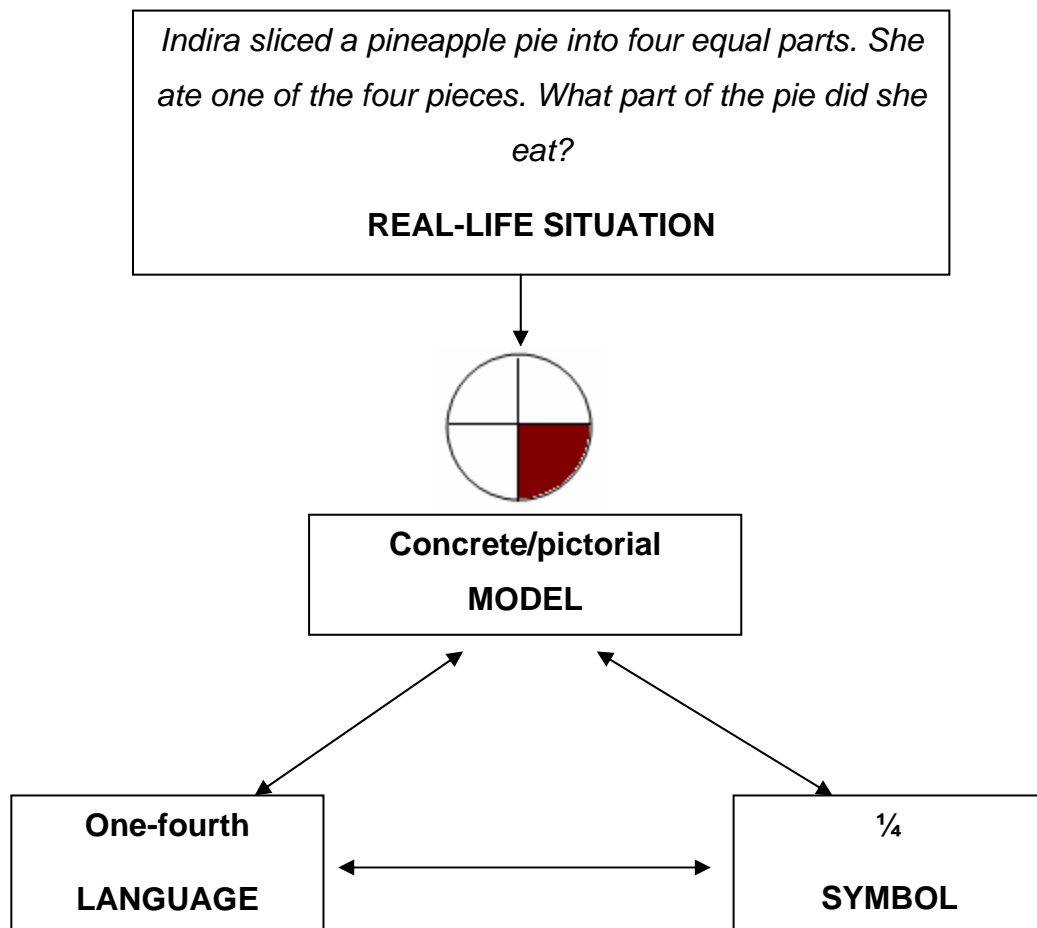


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Rathmell Triangle Model

The Rathmell Triangle Model is a versatile framework for teaching mathematics. It states that relationships must be discussed between and among real-life situations, materials, language and symbols to develop strong mathematical ideas. Study the model below.

CONCEPT: *Fraction as Part of a Whole*



This triangle model suggests six interactions that need to be facilitated:

Model to Language

Language to Model

Model to Symbol

Symbol to Model

Language to Symbol

Symbol to Language

A strategy that is anchored on this model begins with a real-life application of the concept to make the lesson more meaningful to the learners. It uses concrete or visual materials to illustrate the concept before proceeding to the operation of symbols. Also, emphasis is placed on the mathematical language used.

Here are sample activities that focus on each interaction.

Interaction	Activity
Model to Language	The teacher shades 3 of 10 equal parts of a circle and asks, "What fraction is shaded?"
Language to Model	The teacher asks: "Shade $\frac{1}{6}$ of a circle."
Model to Symbol	The teacher shows 5 parts shaded out of 6 equal parts and asks: "Write down the fraction shaded in symbols."
Symbol to Model	The teacher directs the students to shade a rectangle to show this fraction. She writes $\frac{3}{8}$ on the board.
Language to Symbol	The teacher says, "Write down four-sevenths in symbols."
Symbol to Language	The teacher shows a card with $\frac{2}{5}$ written on it and says: "Say this out loud."

Can you integrate all these activities in teaching a concept?



ACTIVITY 2.2

Applying the Rathmell Triangle Model

Procedure:

1. Choose a concept or lesson that you want to develop.
2. Pose a real-life situation that requires the application of the concept.
3. Identify the concrete or pictorial model to be used to help develop the concept.
4. Complete the grid below by describing the activity that develops each of the six relationships in the Rathmell Triangle.

Concept: _____

Real-Life Situation

Concrete/pictorial Model of the Concept:

Development of the Six Relationships

Interaction	Activity
Model to Language	
Language to Model	
Model to Symbol	
Symbol to Model	
Language to Symbol	
Symbol to Language	



SCQ 2.4

1. What are the benefits of starting a lesson with a real-life situation?
2. Why should you always try to start with a model/pictorial or concrete representation of the concept before going to abstract representations?
3. Why is there a need to emphasize the language appropriate to the concept being developed?



READ

Cooperative Learning

When performing a mathematical task, do you prefer to work alone or in groups? A lot of us feel more confident when working in groups.



Cooperative learning is an educational format that is useful in many strategies in teaching mathematics. In this format, students work together in small mixed ability groups to achieve a particular goal or to complete an academic task. It fosters social skills, positive peer relationships and a high level of self-esteem, thus reducing competition and increasing cooperation among students.

This strategy serves the following purposes:

1. to increase achievement through group collaboration that enables students to learn from each other;
2. to provide an alternative to the competitive structure of most classrooms today that discourages poorer students; and,
3. to improve human relations in the classroom by promoting interdependent activities that teach collaborative skills (Wilén, et al , 2000)

Cooperative Learning is appropriate to use when:

1. the task demands collaborative effort;
2. the open-ended problem-solving activity calls for clarification and a range of strategies for the solution; and,
3. the resources/sources are limited.

Roles of the Teacher and Learners



TEACHER

The teacher takes on these roles:

1. Acts as facilitator by forming groups whose members work together on shared goals.
2. Plans the tasks, and explains them and the goals of each task to the students
3. Monitors the groups, provides assistance, intervenes whenever necessary, and evaluates the groups and the students

Learners take on the following roles:

1. Give and receive assistance, feedback, reinforcement and support each other
2. Take responsibility for each group member's learning as well as for one's academic success



LEARNERS



SCQ 2.5

1. Why is it good for students to work cooperatively?
2. Do you think cooperative learning will work with your students? Why or why not?
3. Identify 2 advantages and 2 disadvantages of cooperative learning.
4. Cite a lesson or a learning situation that may require cooperative learning.



READ

Five common formats used for cooperative learning are the following (Wilén, et al, 2000):

1. *Student Teams-Achievement Division (STAD)* – Teacher presents content or skill. Students complete common tasks in groups and then are tested individually. Individual student's improvements in performance are calculated to get team score.
2. *Think-Pair-Share* – Students first try to work-out the task by themselves. Then they form pairs and interacts and discuss their thoughts with each other. Finally they share their understandings with the class.
3. *Jigsaw* – Students are assigned to teams. The academic material is divided into several sections. The members of different teams who have studied the same sections meet in “expert groups” to discuss their sections. Then the students return to their teams and take turns teaching their teammates about their section.
4. *Team Assisted Instruction* – Students are assigned materials at their own achievement level and are assisted by their group members in learning the material. Group points are obtained through improvement on individual tests.
5. *Group Investigation* – Students take responsibility for their own learning as each group decides what to investigate, what contribution each will make and how each will communicate what they have learned (Lewis and Doorlag, 1991).

Structure

The following are the steps in implementing cooperative learning (Johnson, Johnson and Smith, 1991, in Wilen, et al 2000):

1. Planning
2. Preparing Students
3. Monitoring and Intervening
4. Evaluating and Processing

Research has shown the positive effects of grouping students for collaborative work. Not only do they feel more relaxed, they also learn more!



SCQ 2.6

1. Recall an instance when you were a student that you were asked to work in groups. What task was assigned to your group?
2. Did you assign roles for the members in the group? Why do you think this is important?
3. Does working in groups really help the learners learn? Justify your answer.
4. How do you think learners should be assessed when working in groups?



READ

Inquiry Method



Are you familiar with these approaches: discovery, inductive thinking, reflective teaching, and problem solving? Inquiry is viewed as synonymous with these approaches and is like a generic term for all of them.

Inquiry is a flexible instructional method used to involve students in a process to analyze a problem or issue in a logical and systematic way (Wilén, et al, 2000). It is appropriate to use when inductive thinking and learning the inquiry process itself are among the objectives. This method teaches both content and an investigative process that has application to life. Notice how similar the phases of the inquiry method are to the **scientific method!**

Practices–Phases of the Inquiry Method (Wilén, et al, 2000).

Phase I- Entry: Presentation and Clarification of a Problem, an Issue or a Question

- a. State objectives; provide rationale.
- b. Identify a problem, an issue or a query
- c. Relate to students' experiences and lives.
- d. Clarify the problem.



Phase II – Formation of Hypotheses

- a. Encourage the formation of tentative explanations of solutions.
- b. Clarify hypotheses.



Phase III – Collection of Data

1. Facilitate the identification of sources of evidence.
2. Assist in the evaluation of the evidence.

Phase IV – Test Hypotheses

1. Assist in organizing the data.
2. Assist in the analysis and evaluation of the data.



Phase V – Closure: Draw Conclusions

- a. Facilitate the reaching of a generalization, an explanation or a solution.
- b. Integration and transition

Admittedly, using the inquiry approach takes up so much time. But, though you get to cover less material, more is retained in the minds of the learner.



SCQ 2.7

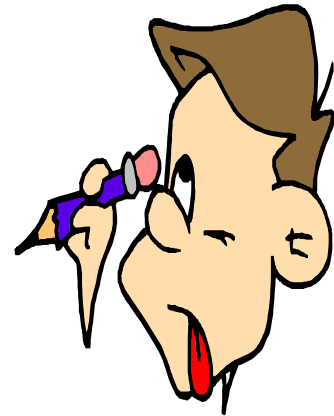
1. Have you tried the inquiry method in class? If yes, how did you find it?
2. Why do you think the inquiry method is applicable to mathematics and not just to sciences?



READ

Problem-Based Learning (PBL)

Problem-based learning is closely aligned with the inquiry and discovery methods of teaching and performance assessment (for more information about performance assessment, see Lesson 3 of this module). It is centered on posing a problem and leading students through an investigative procedure to solve it. The problem should be complex enough to encourage critical thought and group effort. PBL provides students with the opportunity to understand and resolve ill-structured problems under the guidance, rather than the direction, of the teacher (Feden & Vogel, 2003). Through PBL, students learn content and skills in the context of the types of problems encountered in the “real world” (O Neil, 1992).



Steps:

1. Present the problem statement.

Introduce an “ill-structured” problem. Students should not have enough prior knowledge to solve the problem.

Example:

You are interested in buying a new vehicle.

What should your annual salary be to afford the car that you want?



2. List what is known.

- Students list what they know to solve the problem.
- This may include data from the situation as well as information based on prior knowledge.
- Write this information under the heading “What do we know?”

3. Develop a problem statement.

- A problem statement should come from the students’ analysis of what they know.
- The problem statement will probably have to be refined as new information is discovered and brought to bear on the situation.

4. List what is needed.

- Students will list the information they need to fill in missing gaps under the heading “What do we need to know”.
- These questions will guide searches that may take place on-line, in the library, and in other out-of-class searches.

5. List possible actions, recommendations, solutions, or hypotheses.

- Students list actions to be taken (e.g. questioning an expert) under the heading “What should we do?”
- Formulate and test tentative hypotheses.

6. Present and support the solution.

- As part of closure, teachers may require students to communicate orally and/or in writing, their findings and recommendations.

The product should include:

- Problem statement
- Questions
- Data gathered
- Analysis of data
- Support for solutions or recommendations based on analysis

Students should write their plans in the organizer below.

What do we know?	What do we need to know?	What should we do?

Evaluating the Strategy:

The use of PBL is successful when:

1. Students are able to solve the problem on their own.
2. Students work together and are actively engaged in solving the problem.
3. Students use various resources and try out different means to solve the problem.
4. Students use mapping and other visual organizers to organize information and visualize relationships among variables.

Useful Tips:

1. Select problems that are interesting and relevant to the students.
2. Use questioning effectively to guide the students.



SCQ 2.8

1. Compare Problem-Based Learning to the Inquiry Method. Do you think PBL is part of the Inquiry Method? Justify your answer.
2. Why is there a need to emphasize problem solving in your teaching of mathematics?
3. When is the PBL strategy most suitable to use?



READ

Concept Attainment Strategy (CAS)

Another strategy that is applicable in teaching a variety of mathematical concepts is concept attainment. This strategy is used when you want students to discover the essential attributes of a concept. It can sharpen students' skills in

- separating important from unimportant information;
- searching for patterns and making generalizations; and,
- defining and explaining concepts.

Steps:

1. Select a concept and identify its essential attributes.
2. Present examples and non-examples of the concept.
3. Let students identify or define the concept based on its essential attributes.
4. Ask students to generate additional examples.

Evaluating the Strategy:

The use of concept attainment strategy is successful when:

1. Students are able to identify the essential attributes of the concept.
2. Students are able to generate their own examples.
3. Students are able to describe the process they used to find the essential attributes of the concept.

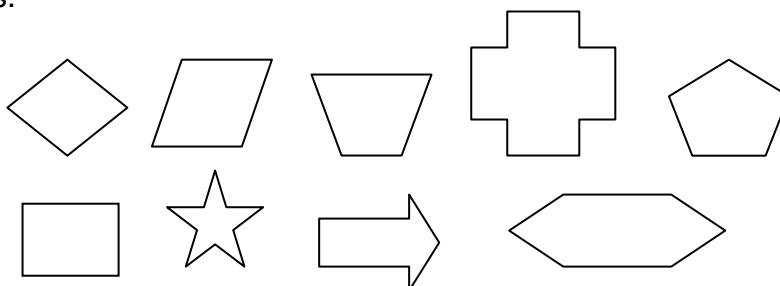
Useful Tips:

1. Each example contains all the essential attributes of the concept.
2. The set of non-examples helps students distinguish between essential and non-essential attributes of the concept.

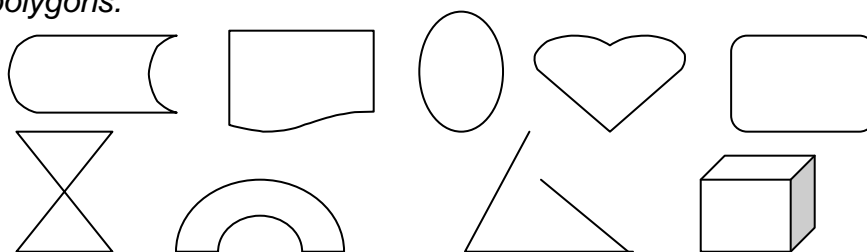
Sample of Concept Attainment Strategy
for Upper Primary and Secondary Classes

Concept: Polygon

These are polygons.

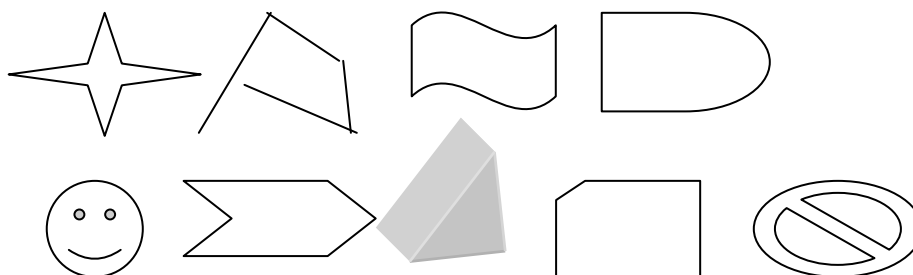


These are **not** polygons.



Define a polygon.

Which of the following are polygons? Tick (/) your answers.



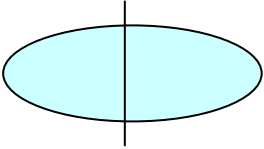
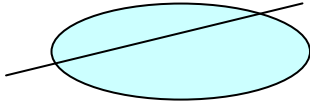

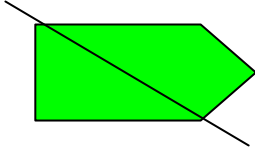
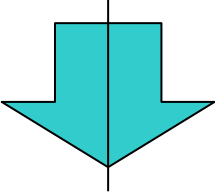
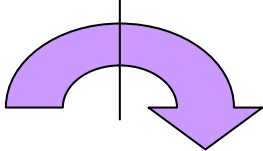
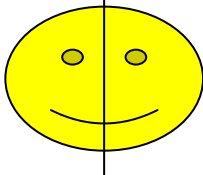
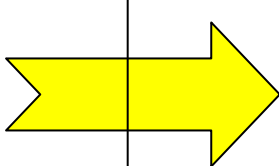
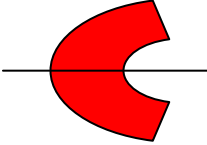
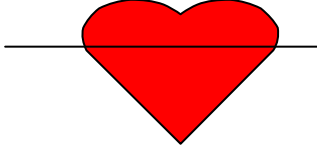
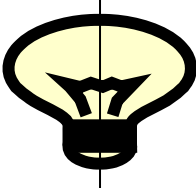
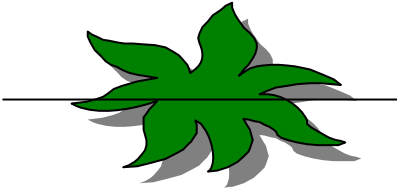
Give additional examples of a polygon.

Sample of Concept Attainment Strategy

for Lower Primary Classes

Concept: *One-Half or Symmetry*

Can You Guess My Idea?

YES Examples	NO Examples
	
	
	
	
	
	



ACTIVITY 2.3

Directions: Prepare a sample plan using the concept attainment strategy similar to the examples given by answering these questions. Follow the format that suits the grade level of your students.

1. *What is the concept you want to develop?*
2. *Give examples of the concept:*
3. *Give non-examples of the concept.*
4. *List the essential characteristics of the concept. Are these characteristics clearly shown in the examples given?*
5. *Ask the students to define the concept.*
6. *Ask the students to provide more examples of the concept.*



SCQ 2.9

1. Were you able to define the concept of polygon in the first sample plan? If yes, how did you do it? If no, why not?
2. Were you able to write your own concept attainment plan? How did you find that activity?
3. How is CAS for lower primary different from the CAS for upper levels?



READ

Concept Formation Strategy

The concept formation strategy is used when you want students to make connections between and among essential elements of the concept (ASCD, 1987).

Steps:

1. Present a particular question or problem.
2. Ask students to generate data relevant to the question or problem.
3. Let the students group data with similar attributes.
4. Ask students to label each group of data with a word or phrase.
5. Have students explore the relationships between and among the groups.
They may group the data in various ways and some groups may be subsumed in other groups based on their attributes.

Evaluating the Strategy:

The use of concept formation strategy is successful when:

1. Students are able to group the data in one way or in different ways.
2. Students are able to label the different groups.
3. Students are able to identify relationships and hierarchies between and among groups.

Useful Tips:

1. Provide guide questions to help students group the data.
2. Place each data on cards or paper cut-outs to allow students to group and re-group them easily.

Sample Plan Using the Concept Formation Strategy

Concept: Real numbers

Generating data: Ask the students to give as many different kinds of numbers as possible.

Grouping and labeling: Ask the students to group data with similar attributes and label them.

Possible groups and labels:

counting numbers	zero	whole numbers
integers	fractions	positive integers
negative integers	decimals	irrational numbers
real numbers		

Regrouping and Subsuming:

0 + counting numbers = whole numbers

whole numbers + negative integers = integers

fractions + repeating, terminating decimals = rational numbers

rational numbers + irrational numbers = real numbers

WORKSHEET FOR CONCEPT FORMATION STRATEGY

DIRECTIONS: Place the numbers that seem to go together in the same circle. Then name the circles. You can re-group and make additional circles.

The worksheet contains the following numbers in boxes:

- -1
- $\frac{1}{2}$
- -1.2
- 2
- -7
- -5
- 3
- 2.15
- $\frac{1}{5}$
- $\sqrt{2}$
- $-\frac{2}{3}$
- π
- $3.\overline{33}$
- 9
- -3.2
- 0



READ

Peer Practice Strategy

The peer practice strategy is used when you want students to develop a deeper understanding of the concepts by practicing previously taught information (ASCD, 1987). They do so by teaching their peers and honing their helping skills in the process. This strategy allows students to learn how to help each other and how to communicate effectively.

Steps:

1. Prepare parallel worksheets for students.
2. Form peer practice partners.
3. Discuss the roles of the Doer and the Helper.

DOER

1. Answers the worksheet.
2. Communicates their answers and questions with the Helper.



HELPER

1. Observes and listens to the Doer.
2. Checks the Doer's answers against the criteria
3. Responds to the Doer's questions and commends correct answers.

Evaluating the Strategy:

The use of peer practice strategy is successful when:

1. Students are able to work together as partners.
2. Students are able to help their partner learn the material using the answer key.
3. Students are able to perform their roles correctly.

Useful Tips:

1. The answer key must include the solutions to the problems.
2. The answer key must be clear and easy to understand.
3. Peer partners swap roles regularly.
4. Students are trained to guide their partners and provide positive feedback.

**Sample Plan Using the Peer Practice Strategy for
Upper Primary Grades**

Concept: Ratio and Proportion

Skill: Solving ratio and proportion problems

Worksheets 1 and 2 will be provided to each pair, one for each student.

WORKSHEET 1

TASK A	Answers to Task B
<p>1. There are 25 mangoes and 15 chicos in a basket. What is the ratio of mangoes to chicos?</p>	<p>56 chairs: 8 tables</p> $56 : 8$ <p>Dividing both by 8, the GCF,</p> $\frac{56}{8} : \frac{8}{8}$ $= 7 : 1$
<p>2. What is the ratio of 72 hours to 2 days?</p>	<p>3 weeks : 28 days (since 1 week = 7 days)</p> <p>21: 28</p> <p>Dividing both by 7, the GCF,</p> $\frac{21}{7} : \frac{28}{7}$ $= 3 : 4$
<p>3. Find x :</p> <p>6: x = 54 : 45</p>	<p>10: 18 = 25 : x</p> <p>10x = 18 (25)</p> $x = \frac{18(25)}{10}$ <p>x = 45</p> <p>The product of the extremes is equal to the product of the means.</p>
<p>4. In a school meeting, the ratio of teachers to parents is 2: 15. If there are 14 teachers, how many parents are there?</p>	<p>Cows: Goats = Cows: Goats</p> <p>4: 5 = 16 : x</p> $5(16) = 4x$ $\frac{5(16)}{4} = x$ $20 = x$
<p>5. In a group of 65 students, the ratio of dancers to singers is 3:2. How many were dancers?</p>	<p>Boy scouts: Girl scouts</p> <p>5 : 3</p> $5 + 3 = 8$ <p>Dividing 72 by 8, we get 9.</p> <p>5 (9) = 45 boy scouts</p> <p>3 (9) = 27 girl scouts</p>

WORKSHEET 2

TASK B	Answers to Task A
1. There are 56 chairs and 8 tables in a room. What is the ratio of tables to chairs?	<p>25 mangoes: 15 chicos $25 : 15$ Dividing both by 5, the GCF</p> $\frac{25}{5} : \frac{15}{5}$ $= 5 : 3$
2. What is the ratio of 3 weeks to 28 days?	<p>72 hours : 2 days (since 1 day = 24 hours)</p> <p>72 hours : 48 hours $72 : 48$ Dividing both by 24, the GCF</p> $\frac{72}{24} : \frac{48}{24}$ $= 3 : 2$
3. Find x : 10: 18 = 25 : x.	<p>6: x = 54 : 45 $54x = 45(6)$</p> $x = \frac{45(6)}{54}$ <p style="text-align: center;">$x = 5$</p> <p>Note: The product of the extremes is equal to the product of the means.</p>
4. The ratio of cows to goats in a class is 4:5. If there are 16 cows, how many goats are there?	<p>$T : P = T : P$ $2 : 15 = 14 : x$</p> $\frac{15(14)}{2} = x$ $105 = x$
5. There are 72 scouts in a camp. If the ratio of boy scouts to girl scouts is 5:3, how many boy scouts are there?	<p>Dancers : Singers $3 : 2$ $3 + 2 = 5$ Dividing 65 by 5, we get 13. $3(13) = 39$ dancers $2(13) = 26$ singers</p>



ACTIVITY 2.4

Do the Peer Practice Strategy

Procedure:

1. Follow the steps for doing the peer practice strategy.
2. Complete the peer practice worksheets 1 and 2. Remember the answers to a task are written on the other worksheet.

Sample Plan Using the Peer Practice Strategy

Concept/skill: Identifying and naming fractions

Parts of the Lesson:

A. Presenting the Roles

The teacher discusses the roles of the doer and the helper. Peer practice pairs are formed.

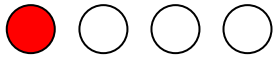
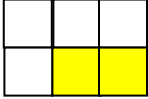
B. Implementing the Roles

The teacher provides the worksheets. The students perform their tasks.

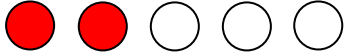
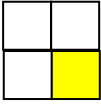
C. Discussing the Roles

The teacher guides the students to reflect on and discuss their roles.

WORKSHEET 1

TASK A Directions: What fraction of the figure or set is shaded?	Answers to Task B		
	No. of parts or objects in the set	Parts or objects shaded	Fraction shaded
1. 	5	2	$\frac{2}{5}$
2. 	4	1	$\frac{1}{4}$
3.			
4.			
5.			

WORKSHEET 2

TASK A Directions: What fraction of the figure or set is shaded?	Answers to Task B		
	No. of parts or objects in the set	Parts or objects shaded	Fraction shaded
1. 	4	1	$\frac{1}{4}$
2. 	6	2	$\frac{2}{6}$ or $\frac{1}{3}$
3.			
4.			
5.			



SCQ 2.10

1. Were you able to complete the peer practice worksheet? How did you find that activity?
2. Is the peer practice strategy useful in introducing a concept? Why or why not?
3. In your view, what are the advantages and limitations of the peer practice strategy?

Let's Summarize

The two main teaching methods are:

- **Deductive method.** Teaching begins with the rule or principle, then examples are given to explain the rule or principle.
- **Inductive method.** Teaching begins from the concrete experiences of the learners and from there moves to the rule or principle.

Some commonly used teaching strategies, models and formats are:

- **Interactive Direct Instruction.** A traditional approach in teaching, it is teacher-centered and is an efficient means of covering content.
- **Cooperative Learning.** It is an educational format in which students work together in small mixed ability groups to achieve a particular goal or to complete an academic task.
- **Rathmell Triangle Model.** A teaching strategy anchored in this model makes learning meaningful as it shows the relationships between real-world situations, models, language and symbols.
- **Inquiry.** Inquiry is a versatile instructional method used to involve students in a process to analyze a problem or issue in a logical and systematic way.
- **Problem-based Learning (PBL).** It is centered on posing a problem and leading students through an investigative procedure to solve it.
- **Concept Attainment.** It sharpens students' skills in identifying essential information, in searching for patterns and making generalizations; and, in defining and explaining concepts.
- **Concept Formation.** Students are asked to generate data relevant to the question or problem. Then they label each group of data with a word or phrase. It is used when you want students to make connections between and among essential elements of the concept.
- **Peer Practice.** It is used when you want students to develop a deeper understanding of the concepts by practicing previously taught information. Students teach their peers and hone their helping skills in the process.

**SCQ 2.11**

A. Directions: Read each item carefully. Choose the letter of the best answer.

1. Mr. Perez presented the laws of exponents and provided examples for each law. What strategy is he using?
A. inquiry B. discovery C. PBL D. direct instruction
2. In a group activity, Ms. Infante observes that only 3 of the 5 students in the group are actually working on the learning task. What is the best thing for her to do?
A. Scold the students who are not participating.
B. Assign specific roles to each member in the group.
C. Move the students who are not participating to another group.
D. Remind everyone that only those who are working will be graded.
3. Which strategy is anchored on the idea that students learn from each other when working together on an engaging task?
A. inquiry C. direct instruction
B. discovery D. cooperative learning
4. Why is the use of real-life problems seen as good lesson starters?
A. They motivate the learners to listen.
B. They make the learners reflect on the lesson.
C. They make the lesson meaningful to the learners.
D. They are a good substitute for good motivational games.

5. Mr. Miguel wants to develop inquisitiveness, perseverance and a scientific attitude among his students. Which strategy will NOT serve his purpose?

A. lecture

C. discovery

B. inquiry

D. Problem-based learning

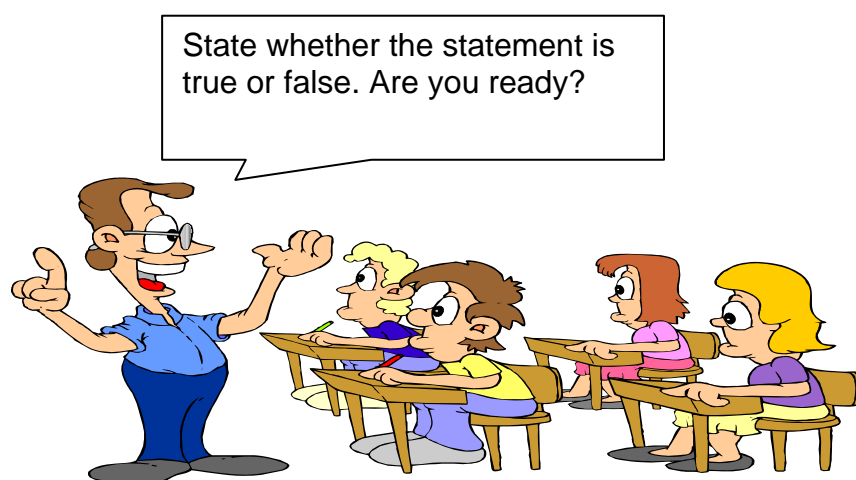
B. Describe in one or two paragraphs how you would teach the area of a rectangle using each of these two methods:

1. Deductive Method

2. Inductive Method

LESSON 3

ASSESSMENT



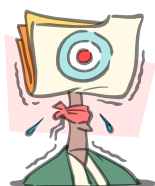
Writing tests, checking test papers, and computing grades are some of the essential tasks of a teacher.

While assessment is often viewed as being at the tail-end of the teaching-learning process, it is the area that has been at the center of debates and discussion for the past couple of decades. Issues concerning the validity and reliability of pen-and-paper tests versus those of authentic assessment procedures evoke strong sentiments from various sectors of the school community. Indeed, assessment is riddled with so many misconceptions and traditional views and practices.

This lesson aims to identify these misconceptions, provide an overview of the assessment process, review some commonly used testing formats, and offer some alternatives to pen-and-paper testing through portfolio and performance assessment.

The topics will follow this sequence:

- Assessment, Evaluation and Testing
- Call for Changes in Student Assessment
- Authentic Assessment
- Balanced Assessment
- Portfolio
- Performance-Based Assessment

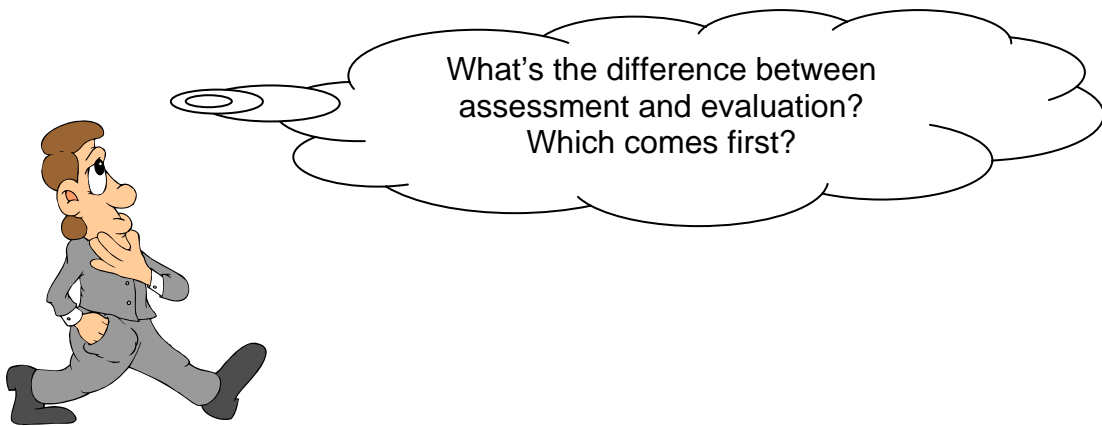


OBJECTIVES

After going through this module, you are expected to:

1. define the terms assessment, evaluation and testing;
2. review the different types of teacher-made tests and the tips on writing each type of test;
3. discuss the criticisms against pen-and-paper tests, in general, and multiple-choice testing in particular;
4. discuss authentic assessment and its advantages;
5. define balanced assessment;
6. define a mathematics portfolio, its purposes and process;
7. illustrate the kinds of performance tasks; and,
8. differentiate the two types of rubrics: holistic and analytic.

Assessment, Evaluation and Testing



The terms *testing*, *assessment* and *evaluation* are often used interchangeably, but strictly speaking, the three terms mean differently and cannot be used interchangeably.



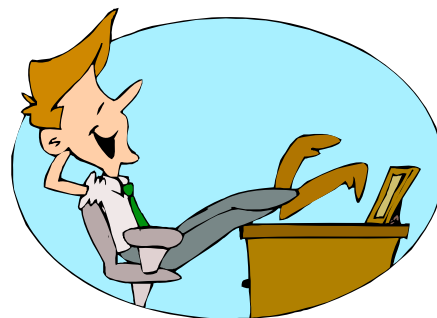
READ

Assessment is the process of gathering information about students – what they know and can do.

Evaluation is the process of interpreting and making judgments about assessment information.

Testing is a means of assessment.

You must be very familiar with the different test formats. But let me give you some helpful tips on testing.





READ

Tests

Paper-and-pencil tests are often made up of essay and objective items.

An **essay** test measures higher-order thinking, including analysis, synthesis and evaluation. It may require short answers such as a sentence or two, or extended responses which may involve paragraphs.

Here are examples of essay items in mathematics:

- ▶ *How are milligrams and milliliters similar? How are they different?*
- ▶ *Is $(-3)^2$ equal to $-(3)^2$? Explain your answer.*

Tips in Grading Essays

1. Be clear about your expectations by preparing a rubric for each essay.
2. Discuss with students the components that will be assessed and the percentages: for example, comprehensiveness- 40%, accuracy, 40%, clarity of expression – 20%.



SCQ 3.1

1. Is there a place for essay questions in mathematics tests? Explain.
2. Write two (2) essay questions on any topic in mathematics.



READ

Objective Tests

The most common types of **objective items** are multiple-choice, true-false, matching, and completion.

In writing any objective test item, the teacher must constantly watch out to avoid ambiguity.

The **multiple-choice** is the most useful objective test item but is also the most difficult to write.

Tips on Writing Multiple-Choice Test Items

1. Keep the language at the students' level using good grammatical form.
2. Express as much of the substance of the item as possible in the stem.
3. Make certain that one option is clearly the best answer.
4. Make sure that each option is plausible.
5. Keep the options in an item consistent in type and length.
6. Avoid specific determiner words and absolutes such as *every*, *none*, *always*, and *never*.
7. Avoid using a double negative or a word used both in the stem and in the response.
8. Do not use *all of the above* as a distractor.

(Wilén, Ishler, Hutchison & Kindsvatter, 2000)

**SCQ 3.2**

Identify the weakness of each item, then rewrite the item to improve it.

1. A fraction whose numerator is less than its denominator is called a
 - A. proper fraction
 - B. improper fraction
 - C. unit fraction
 - D. similar fractions
2. What is the place value of 2 in 0.123?
 - A. tenths
 - B. hundredths
 - C. thousandths
 - D. all of the above
3. None of the following is larger than 0.65 EXCEPT
 - A. 0.5
 - B. 0.641
 - C. 0.7
 - D. 0.6000
4. Which of the following is a polygon?
 - A. circle
 - B. square
 - C. cube
 - D. horse
5. A square is
 - A. a polygon with 4 right angles and 4 equal sides.
 - B. a 4-sided polygon.
 - C. a quadrilateral with 4 equal sides.
 - D. a quadrilateral with 4 right angles.



READ

Tips on Writing True-False Items

1. Every item should be wholly true or wholly false.
2. Each item must be short and significant.
3. Whenever possible, avoid such items as *generally* and *usually*.
4. Avoid using double negative.
5. Avoid trivia.



SCQ 3.3

Identify the weakness of each item, and then rewrite the item to improve it.

TRUE OR FALSE

1. A circle is a closed plane figure and a polygon.
2. No equiangular triangle is not equilateral.
3. A diameter of a circle is twice longer than the length of any radius of the circle.
4. An isosceles triangle is usually an acute triangle.
5. Euclid was born on 300 B.C.



READ

Tips on Writing Matching Items

1. Make sure all items concern one topic.
2. Include more possible answers than questions or state that some answers may be used more than once.
3. Arrange the options in some logical order such as chronological or alphabetical.
4. State clearly in the direction the properties that are to be matched (Examples: (1) Match the scientist to his or her famous invention, (2) Match the definition to the term being defined)



SCQ 3.4

Identify the weaknesses of this matching type test. Modify the test to improve it.

Directions: Match Column A with Column B.

A	B
1. triangle	a. pentagon
2. polygon with five sides	b. square
3. special triangle	c. 30° - 60° - 90°
4. congruent	d. 180°
5. rectangle with four equal sides	e. \cong



READ

Tips on Writing Completion Items

1. Write an item that can be completed with a single word or a short phrase.
2. Be sure that only one word or phrase can correctly complete the sentence.
3. Make all blanks the same length.
4. Do not give grammatical clues.
5. Do not put more than two blanks in any item.



SCQ 3.5

Identify the weakness of each item, and then rewrite the item to improve it.

COMPLETION TYPE

1. A _____ is a triangle with at least one acute angle.
2. A parallelogram is _____.
3. A square is a parallelogram with 4 _____ angles and 4 _____ sides.
4. A triangle with two congruent sides is an _____ triangle.
5. The _____ of a circle is a _____ that contains the _____ of the circle.

Testing is just one means of assessing students. Other sources of information about the students' strengths and weakness include students' participation in class, homework, group work, project, portfolio and performance. Portfolio and performance-based assessments are discussed in detail in this module.

The Department of Education provides the criteria for evaluating students.



READ

Criticisms against Multiple-Choice Testing

Three main points have been raised against standardized, multiple choice tests (Hart, 1994):

- 1) The results of standardized testing are often inconsistent, inaccurate and biased.
- 2) The tests are a poor measure of anything except a student's test-taking abilities.
- 3) They corrupt the very process they are supposed to improve.
- 4) Teachers' obsession with multiple-choice test has damaged teaching and learning by:
 - putting too much value on recall and rote learning at the expense of understanding and reflection;
 - promoting the misleading impression that there is a single answer for most every problem or question;
 - turning students into passive learners who need only to recognize, not construct, answers and solutions;
 - forcing teachers to focus more on what can be tested easily than on what is important for students to learn, thus, trivializing content and skill development.



Do you agree with these criticisms against multiple-choice testing? Why or why not?

To address these concerns, the NCTM (1989) recommends the following changes in student assessment.

Recommendations for Student Assessment	
<i>Increased Attention Should Be Given To:</i>	<i>Decreased Attention Should Be Given To:</i>
1. Assessing what students know and how they think	1. Assessing what students do not know about mathematics
2. Making assessment be an integral part of teaching	2. Simply counting correct answers on tests for the purpose of assigning grades
3. Focusing on a broad range of mathematical tasks and taking a holistic view of mathematics	3. Focusing on a large number of specific and isolated skills
4. Developing problem situations that require the application of a number of mathematical ideas.	4. Using exercises or word problems requiring only one or two skills
5. Using multiple assessment techniques, including written, oral and demonstration formats	5. Using only written tests
6. Using calculators, computers and manipulative models in assessment	6. Excluding calculators, computers and manipulative models from the assessment process.

Source: From Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989, p.191)



READ

Authentic Assessment

Thoughtful educators agree that if the goal is to move education away from trivialized or rote learning, then teachers will have to change the way they assess students. But while many teachers are aware that traditional tests alone fail to give a complete picture of students' mathematical reasoning, they seem to be stuck to their old habits for lack of competence in and understanding of authentic assessment.

This lesson provides a bird's eye view of where the assessment revolution is heading, that is, towards authentic assessment. It tries to define what makes an assessment approach authentic in terms of its design, structure and grading. It argues for authentic assessment by citing its benefits to the students and the teachers. Lastly, it presents the processes of authentic assessment.

This lesson presents the variety of performance tasks, some examples of each that apply to the mathematics classroom, and the rubrics that may be used for marking such tasks.



ACTIVITY 3.1

What Makes Assessment Authentic?

DIRECTIONS: Read each item carefully. Tick the statements that, in your opinion, characterize authentic assessment.

Tick (/)	Statements
	1. It involves students in tasks that are worthwhile, significant and meaningful.
	2. It highlights what students do not know.
	3. It measures competence rather than performance.
	4. It is a standardized assessment tool.
	5. It involves varied activities such as oral interviews, group problem-solving tasks or the creation of portfolios.
	6. It involves higher order thinking skills and the integration of a broad range of knowledge.
	7. Its primary aim is to provide each student a grade at the end of the semester.

Tick ()	Statements
	8. It is scored using clearly stated and agreed upon performance standards, not norms or counting of errors.
	9. It empowers students by providing for a significant degree of student choice.
	10. It needs to be done by students in the classroom at the same time and under time pressure.
	11. It reflects real-life interdisciplinary tasks/problems.
	12. Its design, structure and grading rigidly follow prescribed standards.
	13. It is educational, engaging and is worth practicing for and repeating.
	14. It is teacher-centered with very little student choice and participation.
	15. It encourages self-assessment.

Check your answers against mine on page 118.



READ

Changing Roles of Students and Teachers in Authentic Assessment

Authentic assessment changes the roles of students and teachers in the assessment process.

A. Changing Roles of Students

1. Students are active participants in the assessment process, rather than passive test takers.
2. Students do tasks that are interesting, worthwhile and relevant to their lives rather than specific pen-and-paper tasks that often have no meaning in their day-to-day activities.
3. Students pose questions, not just answer them.
4. Students evaluate themselves and their peers, instead of just being at the receiving end.
5. Students have choices to accommodate individual differences.

B. Changing Roles of Teachers

In authentic assessment, the teachers' role changes as follows:

1. from “transmitter of knowledge” to “facilitator of classroom activities designed to promote learning,”
2. from taking on the responsibility for student learning to assisting students take responsibility for their own learning;
3. from being “enemies” to becoming “allies” of the students in the assessment process; and,
4. from being the sole evaluator of student's performance to being just one of the many possible evaluators of student's performance.



SCQ 3.6

1. Based on your experience, are the students' roles in authentic assessment different from their current roles in the classroom? In what ways?
2. Do you favor these changes? Why or why not?
3. Based on your experience, are the teachers' roles in authentic assessment different from their current roles in the classroom? In what ways?
4. Do you favor these changes? Why or why not?



READ

Balanced Assessment

While criticisms have been heaped against standardized and teacher-made tests, these assessment tools cannot be done away with. After all, testing is still the most economical and efficient way to assess. On the other hand, while praises have been heaped on portfolio and performance assessment, they are not perfect nor insufficient. Many educators agree that no single assessment tool provides all the needed information regarding a student's knowledge, skills, strengths and weakness. To paint a complete picture of a student, they need all the tools at their disposal.

For a balanced assessment, Burke (1999) suggests that educators use standardized and teacher-made tests to measure knowledge and content, portfolios to measure process and growth, and performances to measure application. Such combination provides a "union of insufficiencies" which will provide a more accurate portrait of the individual learner.

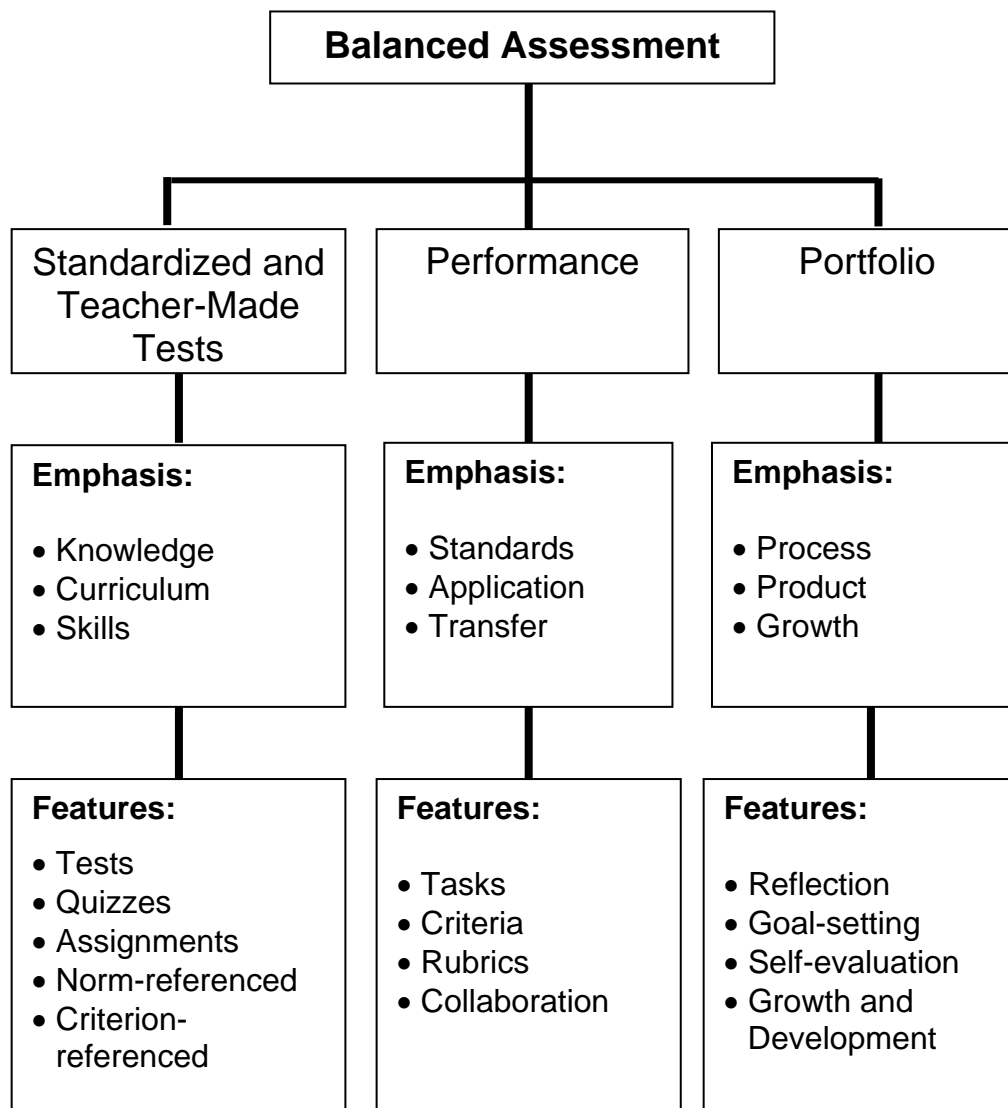


Diagram of a Balanced Assessment

The integration of the three types of assessment represents a balanced assessment based on the model by Fogarty and Stoehr (1996).



Activity 3.2

The Why, What and How of Assessment

Given your awareness of authentic assessment and its benefits and your experience with traditional forms of assessment, tick the statements that correspond to your answers to the following questions:

1. Why do we need to assess?

- To check our students' strengths and weakness and how well they are learning
- To come out with a grade at the end of the quarter or semester
- To identify which students are good, average or poor
- To know how well we are doing as teachers and as a class
- To identify which students should be promoted, retained, or placed in special or remedial programs
- Others _____

2. What should we assess?

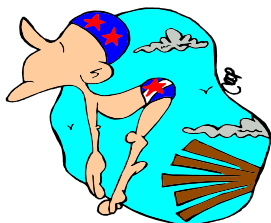
- Skills and content knowledge only
- Processes
- Skills and knowledge in a meaningful context
- Whatever is worth learning
- Whatever can be easily tested
- Others _____

3. How should we assess?

- By pen-and-paper tests
- By portfolios
- By observations _____
- By performance tasks and samples
- Others

**READ****Performance Assessment**

How do you know if a baker is good? By the quality of his or her baked goods.



How do you know if a diver is good? By his or her performance in competitions.



How do you know if a barber is good?
By the number of his loyal costumers.

Notice that we judge how well people do their jobs by their performance and not through a pen-and-paper test.

Can this idea be applied in the classroom?



READ

What is Performance Assessment?

Put simply, it is a manner of assessing one's knowledge and skills in an actual or realistic context. Hart (1994) notes that unlike traditional tests that focus on facts and discrete skills, performance assessment is designed to test what we care most about – the ability of students to use their knowledge and skills in a variety of realistic situations and contexts. It encompasses many skills and usually has a direct application to real tasks people are asked to do in everyday life (Burke, 1999). Students have no prescribed or memorized rules or specific correct solution method for doing such tasks (Van de Walle, 2001).

Characteristics of Performance Tasks

According to Gronlund (1998), tasks should:

1. be realistic to reflect those in the real world;
2. be complex and novel to encourage originality and multiple solutions;

3. require more time for assessment due to the difficulty in designing and evaluating the tasks; and
4. require greater use of judgment in scoring.



SCQ 3.7

Which of the following is a performance task? Write **Yes** if it is and **No** if it is not.

1. Add: $1345 + 2346 + 358 =$
2. Recite the multiplication table.
3. Add 52 and 39 in as many ways as possible. Show your solutions.
4. Why is division not commutative? Explain.
5. Prepare a business proposal for a company that will provide a capital of P1 million for the winning proposal.



READ

Types of Performance Tasks

Hart (1994) classifies the types of performance tasks according to format and uses into three broad categories:

1. Short Assessment Tasks

These are used to check on students' mastery of basic concepts, skills, procedures and thinking skills.

a. Open-ended Tasks

These tasks are often referred to as free-response questions. A problem or situation is presented and the student is asked to give a response by perhaps describing, solving, interpreting, graphing or predicting. It should allow students to solve it in different ways.

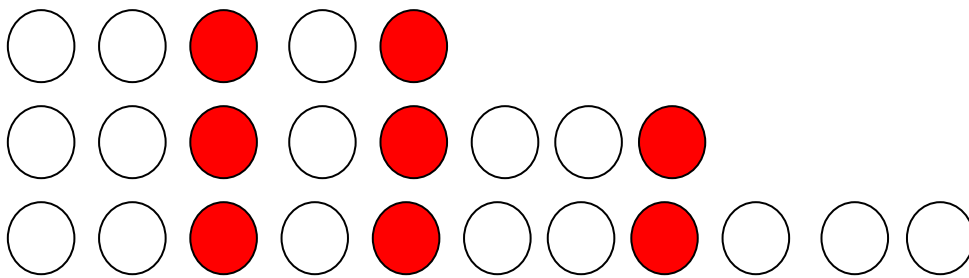
Example 1:

In how many ways can you multiply 16 and 24?

Illustrate these ways.

Example 2:

Study the figures. Complete the table then look for a pattern. Express the relationship you observed in terms of an equation.



Number of shaded circles	1	2	3	4	5	6	N
Number of unshaded circles	4	6	8				

b. Enhanced Multiple Choice Questions

When enhanced by making it more authentic and challenging or by requiring the students to explain or justify their answer, a question in a multiple-choice format can suit the characteristics of a performance task.

Example 1:

Clint plans to buy a number of white T-shirts for the school-year. A poster for a T-shirt sale reads:

SALE! SALE! Regular price: P60

Take 30% off on the second T-shirt purchased

Take 50% off on the third T-shirt purchased

He has P150 in his pocket. He needs to set aside P16 from the amount for his fare. Does Clint have enough money to buy three T-shirts from the store?

- A. Clint's money will be P2 short.
- B. Clint's money will be P20 short.
- C. Clint will have P2 extra.
- D. Clint will have P20 extra.

Concept mapping

A concept map is a cluster or web of information created by students to represent their understanding of concepts and relationships among ideas (Hart, 1994). It is used to reveal how students' understanding changes over time.

2. Event Tasks

These tasks assess broad competencies and skills and usually take a longer time to complete. Students perform better in event tasks when working together, rather than individually, which makes them well-suited for cooperative work.

a. Problem-solving and analytical tasks

Students are asked to plan and work out a solution to a realistic situation.

Example:

Your group is tasked to plan the Christmas party of the class. Prepare a detailed budget and menu for the Christmas party and suggest the amount to be collected from each member of the class for as long as the amount is not to exceed P100. Be ready to present your plan and justify each item in class in a week's time.

b. Long Term Projects

The project provides real-life context to the lessons learned.

Example:

Prepare a map of your school drawn to scale. Discuss the measurement procedures you employed and the processes you applied (e.g. estimation). The project is due in three week's time. Each group will be required to do an oral presentation aside from the written output of the project.

**SCQ 3.8**

1. Should a performance task always take a few days to do? Explain your answer.
2. Why is giving students enough time to do extensive performance tasks important?
3. Prepare one (1) performance task.



Rubrics

Assessments using alternative approaches require the use of rubrics and the judgment of human evaluators.

A rubric contains the criteria or indicators that are used in scoring students' performances, portfolios or responses to an open-ended task. It describes the levels of performance students may be expected to attain relative to a desired standard of achievement (Hart, 1994).

Kinds of Rubrics

- Holistic rubric – scoring is based on an overall impression of a sample of student work considered as a whole
- Analytic rubric – separate scores are given for different indicators or characteristics of a student's output or performance.

Function of Scoring Rubrics

1. Provides uniform, objective criteria for judging a performance assessment item.
2. Provides established expectations for teachers and students that help them identify the relationships among teaching, learning and assessment.

Steps in Creating a Rubric

1. Identify exactly what is to be scored.
2. Define the scale (point range) of the rubric.
3. Develop descriptors for each performance level that describes unique characteristics.

4. Assure that the rubric –
 - a. defines a continuum of quality
 - b. focuses on the same criteria
 - c. validly discriminates performance levels
 - d. can be reliably rated

Scale Construction

1. The scale should indicate the range of possible performance arranged in order from best to poorest performance.
2. The range of performance is then divided into various levels of performance e.g.

Levels	Score
Excellent	4
Very Good	3
Fair	2
Poor	1



SCQ 3.9

1. Differentiate an analytic rubric from a holistic rubric.
2. When is it appropriate to use the holistic rubric?
3. When is it appropriate to use the analytic rubric?
4. How can you establish inter-rater reliability when using rubrics?



Sample Task and Rubric

James knows that half the students from his school are accepted at the public university nearby. Also, half are accepted at the local private college. James thinks that these figures add up to 100 percent, so he will surely be accepted at one or the other institution. Is James' conclusion right or wrong? If possible, use a diagram in your explanation.



Sample Holistic Scoring Rubric for Problem Solving

4 Points: Exemplary Response

All of the following characteristics must be present.

- The answer is correct.
- The answer is clear and complete.
- The explanation includes a mathematically correct reason.
- Some sort of diagram is provided that relates directly and correctly to the information in the problem.

3 Points: Good Response

All of the following characteristics must be present:

- The answer is incorrect.

- The explanation lacks clarity.
- The explanation is incomplete.
- No diagram is provided that relates directly and correctly to the information in the problem.

2 Points: Good Response

Exactly one of the following characteristics is present:

- The answer is incorrect.
- The explanation lacks clarity or is incomplete but does indicate some correct and relevant reasoning.
- No diagram is provided that relates directly and correctly to the information in the problem.

1 Point: Poor Response

All of the following characteristics must be present:

- The answer is incorrect.
- The explanation, if any, uses irrelevant argument.
- No solution is attempted beyond just copying data given in the problem statement.
- No diagram is provided that relates directly and correctly to the information in the problem.

0 Point: No Response

- The student's paper is blank or it contains only work that appears to have no relevance to the problem

Source: California Assessment Program

Sample Analytic Scoring Rubric for Problem Solving

Understanding the Problem

- 0: Complete misunderstanding of the problem
- 1: Part of the problem misunderstood or misinterpreted
- 2: Complete understanding of the problem

Planning a Solution

- 0: No attempt, or totally inappropriate plan
- 1: Partially correct plan based on part of the problem being interpreted correctly
- 2: Plan could have led to a correct solution if implemented properly

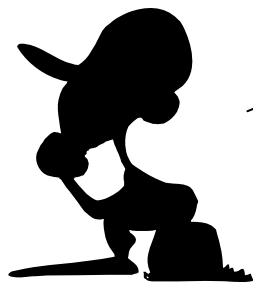
Getting an Answer

- 0: No answer, or wrong answer based on an incorrect plan
- 1: Copying error, computational error, partial answer for a problem with multiple answers
- 2: Correct answer and correct label



READ

PORTFOLIO ASSESSMENT



Have you heard about portfolio assessment?
This section answers the most frequently
asked questions about portfolio assessment.

What is a Portfolio?

- It is a purposeful collection of student's works that exhibits the student's efforts, progress and achievements in one or more areas (Carter and Spandel, 1992).
- It is a process that enables students to become active and thoughtful learners (Burke, Fogarty and Belgrad, 1995).

What are the Possible Contents of a Portfolio?

1. **A Creative Cover.** It gives the subject area, author, and title, and the general theme of the portfolio.
2. **A Letter to the Reader.** It welcomes the reader and explains the cover, the title and/or the theme.
3. **A Table of Contents.** This shows how the entire portfolio is organized.
4. The artifacts or evidences. These may vary depending on the type and purpose of the portfolio.
5. **Reflections.** This is meant to reveal student insight.
6. **Self-Evaluation.** This encourages students to assess their own strengths and weaknesses and monitor their own learning.

Why use Portfolios?

- To allow students to show their best
- To show change and growth over a period of time (Vavrus, 1990)
- To invite students and teachers to be allies in the assessment process.
- To provide an opportunity for richer, more authentic and more valid assessment of the students.
- To form an intersection of instruction and assessment (Paulson, Paulson & Meyer, 1991)

Types of Portfolios

The types of portfolios largely depend on the purpose for which they are made.

They include:

- **Best Work Portfolio** – showcases the students' best works as selected by the student and the teacher
- **Process Portfolio** – presents the progress of a certain work from the first draft to the final form to show growth
- **Working portfolio** – supplements traditional means of grading and is used for on-going informal assessment of students' progress and evidence of their learning.
- **Integrated Portfolio** – gathers different disciplines in one piece of work in line with a thematic approach to learning

Criteria for Portfolio Entries

The following are suggested by Steffy (1995).

- Reflect genuinely useful skills and knowledge
- Allow flexibility in student preparation and presentation
- Allow time for serious, comprehensive work
- Prompt students to stretch their minds and make connections
- Place knowledge in a context, which supports the value of what has been learned.

Possible Portfolio Evidences

- Papers that show student's correction of errors or misconceptions
- A solution to a complex open-ended question showing originality and unusual procedures
- A problem made up by the student
- A report of a group project with comments about the individual's contribution

- Journals that show students' reflections
- Artworks, photos, video or audiotapes that document their work in class or on a project
- A mathematical autobiography

Possible Portfolio Organizers

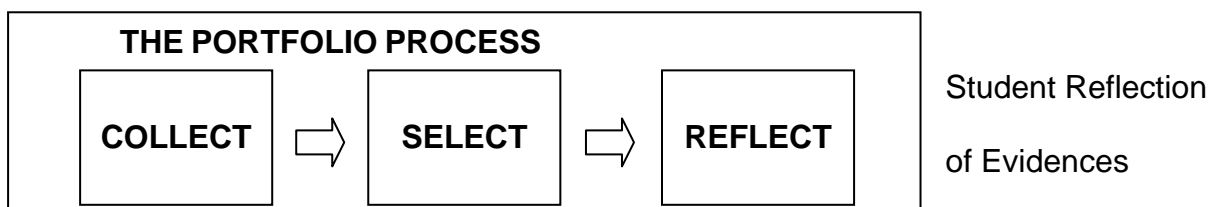
Organizers must be durable and economical. Some of the possible portfolio organizers are as follows: clear book, album, bag, envelope, box, bound sheets.

The Portfolio Process

Burke (1995) outlines the portfolio process as follows:

1. Orientation
2. Planning
3. Gathering of evidences
4. Selecting evidences based on criteria
5. Connecting and conferencing with others
6. Injecting and ejecting artifacts continually to update
7. Respecting work and exhibiting with pride

These steps can be simplified into three basic steps.



To help students' reflect on the items in their portfolio, any one of the following bridging questions may be used:

- Why have I chosen this piece?
- Why is this piece important?
- What does this piece show about what I know and what I learned?



SCQ 3.10

1. Identify three primary features of portfolio assessment.
2. What makes the portfolio product different from a scrapbook?
3. What is the role of the teacher in portfolio assessment?
4. How is the role of the student in portfolio assessment different from his or her role in traditional testing?



Do you know how portfolios are marked? Read this!

Marking the Portfolio

1. Evaluate portfolios in terms of growth demonstrated within an individual portfolio, rather than comparisons made among different students' work (Vavrus, 1990).
2. The teacher may grade none of the evidences, some of the evidences, all of the evidences or the whole portfolio itself.
3. Consult everyone involved in choosing the rubric.
4. An analytic or holistic rubric may be used in marking portfolios.

This is an example of an analytic scoring rubric for portfolios.

PORTFOLIO SCORING RUBRIC					
Portfolio Owner: _____					
Evaluator: _____ Self _____ Peer _____ Teacher					
Directions: Tick (/) the box below the score that best describes the indicator. The legend below gives the description of each score.					
Legend: 5 – Outstanding		3 – Satisfactory			
4 – Very Satisfactory		2- Fair			
		1 – Needs Improvement			
Criteria	5	4	3	2	1
A. Visual Appeal (20%) 1. Cover 2. Lay-out 3. Tone/mood 4. Creativity 5. Resourcefulness 6. Neatness					
B. Organization (20%) 1. Order of entries 2. Coding technique 3. Readability of entries 4. Correctness of form (e.g. grammar)					
C. Content (30%) 1. Statement of purpose 2. Completeness of entries 3. Diversity of Selections					
D. Reflections (30%) 1. Depth of understanding 2. Application of ideas					

This is an example of a holistic scoring rubric for portfolios.

Portfolio Scoring Rubric

Directions: Read the entire portfolio carefully. Evaluate the entire work in terms of the scale indicators below.

SCALE INDICATORS

Scale 5 (Outstanding)

- Presents a variety of work done individually or in groups
- Uses many resources
- Shows good organization and a clear focus
- Displays evidences of self-assessment such as revisions, a letter on why one chose a certain entry, etc.
- Includes few, if any, errors in grammar, usage or mechanics
- Reflects enthusiasm, creativity, extensive investigation and analysis of information

Scale 4 (Very Satisfactory)

- Presents a variety of work done individually or in groups
- Uses many resources
- Contains minor organizational flaws
- Exhibits some errors in grammar, usage or mechanics
- Reflects enthusiasm, creativity, self-assessment, extensive investigations and analysis of information

Scale 3 (Satisfactory)

- Presents fewer works and some resources
- Includes confusing organization and a focus which is unclear
- Reflects some enthusiasm, creativity, self-assessment, extensive investigations and analysis of information

Scale 2 (Fair)

- Contains problems in mechanics that interfere with communication
- Reflects poor organization
- Lacks focus, enthusiasm, creativity and analysis of information

Scale 1 (Needs Improvement)

- Consists mainly of ditto sheets or pages copied from a textbook
- Contains no evidence of student thinking



SCQ 3.11

Complete these statements.

1. I think portfolio is
2. What I like most about it is
3. What I like least about it is...

LET'S SUMMARIZE

Assessment is the process of gathering information about students – what they know and can do.

Evaluation is the process of interpreting and making judgments using assessment information.

Testing is a means of assessment.

1. The standardized test as an assessment tool has many limitations, foremost of which is that it corrupts the very process it is supposed to improve as it puts too much value on recall and rote learning at the expense of understanding and reflection.
2. Assessment is **authentic** when it involves students in tasks that are worthwhile, significant and meaningful.
3. Teachers are encouraged to use multiple assessment techniques, to include calculators, manipulatives and computers in assessment, and to focus on a broad range of mathematical tasks for a holistic view of mathematics, among others.
4. **Balanced assessment** involves the use of standardized and teacher made tests to measure knowledge and content, portfolios to measure process and growth,

5. **Performance assessment** is a manner of assessing one's knowledge and skills in an actual or realistic context.
6. The **types of performance tasks** are:
 - a. Short Assessment Tasks (open-ended tasks, enhanced multiple choice questions, concept mapping)
 - b. Event Tasks (Problem-solving and analytical tasks, Long term projects)
7. A **rubric** contains the criteria or indicators that are used in scoring students' performances, portfolios or responses to an open-ended task.
 - **Holistic rubric**-scoring is based on an overall impression of a sample of student work considered as a whole
 - **Analytic rubric**- separate scores are given for different indicators or characteristics of a student's output or performance and performances to measure application.
8. **Portfolio assessment** is a purposeful collection of student's works that exhibit the student's efforts, progress and achievements in one or more areas.
9. The **types of portfolios** are: best work, process, working, and integrated.
The **portfolio process** is: collect, select and reflect.

ANSWER KEY

Pretest

- A.
- | | | |
|------|-------|-------|
| 1. B | 6. A | 11. C |
| 2. D | 7. C | 12. B |
| 3. A | 8. B | 13. A |
| 4. A | 9. B | 14. C |
| 5. B | 10. C | 15. D |

B.

1. Possible answers: problem solving or thinking skills
1. Possible answers: By exploring, investigating, making their own hypothesis and testing them; through exploration of numbers or models and searching for patterns and relationships; through problem solving and meaning making
- 3-4 Possible answers:

Inquiry- a versatile instructional method used to involve students in a process to analyze a problem or issue in a logical and systematic way.

Problem-based Learning (PBL)- centered on posing a problem and leading students through an investigative procedure to solve it.

Concept Attainment - lets students discover the essential attributes of a concept. It can sharpen students' skills in (a) separating important from unimportant information; (b) searching for patterns and making generalizations; and, (c) defining and explaining concepts.

Rathmell Triangle Teaching Strategy- makes use of the Rathmell triangle model where the lesson begins with a real life situation, proceeds to modeling and emphasizes the relationships between the model, language and symbol of the concepts involved.

Cooperative Learning- an educational format in which students work together in small mixed ability groups to achieve a particular goal or to complete an academic task.

5. By using a variety of techniques to obtain a more holistic picture of the learner. A balanced assessment includes pen-and-paper, performance and portfolio assessment.

Credit points shall be given to each item as follows:

Test A- 1 point each item - 15 points

Test B- 2 points each item - 10 points

Total: 25 points

24-25	Excellent	(E)
21-23	Very satisfactory	(VS)
19-20	Satisfactory plus	(SP)
17-18	Satisfactory	(S)
15-16	Moderately Satisfactory	(MS)
0-14	Needs Reinforcement	(NR)

If your score is...

24 – 25	You have the option to skip the module but you are still encourage to go through it.
15 –23	Go over the items that you find difficult and then you may proceed to the lessons in this module that you don't understand.
0 - 14	Don't worry about your score. Read this module. It has been prepared in order for you to understand the philosophy, strategies, and methods of assessment in mathematics.

Lesson 1

Activity 1.4

	Behaviorism	Cognitive
View of Learning	(1) Accumulation of responses through selective reinforcement	(2) Development of strategies to encode and retrieve information
View of Learner	(4) Empty receptacle	(3) Creators of understanding
Role of Teacher	(6) Controller of the environment through reinforcement and cues for appropriate student behavior	(5) Partner in the process of meaning making; helps students organize and make sense of information
Role of Learner	(7) passive recipient of stimuli from teacher and environment	(8) Active meaning-maker through strategy use

Self-Check Questions 1.6

1. He or she believes that math should be taught through a series of drills and practice, that if the lesson is laid down clearly and orderly, the students would learn; that he or she is the source of information and her or his task is to 'transfer' the information to the learners.
2. Use concrete experiences and move gradually to abstraction; support of meaningful practical activities; use manipulatives; integrate knowledge in a meaningful way; and value reflection and discussion.

3. Mathematics is a study of patterns and relationships; a way of thinking; an art, characterized by order and internal consistency; a language, using carefully defined terms and symbols; and, a tool (NCTM, 1989). The five broad goals of mathematics education to meet students' mathematical needs for the 21st century are: to value mathematics; to reason mathematically; to communicate mathematics; to solve problems; and to develop confidence. This implies that the teaching of mathematics must emphasize problem solving and the development of thinking skills and focus on process rather than product. This also implies a perspective of teaching and learning that is more in line with constructivism.

Lesson 2

Self-Check Questions 2.1

Advantages of the Deductive Method

1. Coverage of a wider scope of subject matter
2. Less preparation as there is less need for materials and activity sheets

Disadvantages of the Deductive Method

1. It is not supportive of the principle that learning is an active process
2. There is a tendency for the teacher to do chalk-and-talk

Advantages of the Inductive Method

1. The learners are more engaged in the learning process.
2. Learning becomes more interesting and meaningful because we begin with the experiences of the students.
3. It develops learners' higher order thinking skills.

Disadvantages of the Inductive Method

1. It requires more time, thus less content is covered.
2. It demands more preparation and skill on the part of the teacher

Self-Check Questions 2.2

Answers may vary. Possible answers:

1. Use synthetic division to apply polynomials. It would take a significant amount of time to attain this objective using the inductive method.
2. State the formula for finding the volume of a sphere. This concept is exciting to explore inductively as the use of manipulatives engages the students and leads them to the desired generalization.

Self-Check Questions 2.3

1. Advantages of the Interactive Direct Instruction strategy

- Easy and convenient to do
- Covers more content in a given time

Disadvantages of the Interactive Direct Instruction strategy

- Does not engage the learners in meaningful activities
- Encourages passive learning

Activity 2.1

- | | | | | |
|------|------|------|------|-------|
| 1. L | 2. H | 3. H | 4. H | 5. L |
| 6. L | 7. L | 8. H | 9. H | 10. H |

Self-Check Questions 2.4

1. It shows the students how the lesson applies to real life. It concretizes the lesson and makes it more meaningful for the students to learn.
2. So that students can have a visual and concrete representation of the concept, which makes the lesson more realistic and relevant. It also makes working with abstract representations of the concept easier. Because mathematics is a way of communicating ideas. It is a language

by itself with its own register. Students should learn the mathematics register for clarity in communicating ideas.

Self-Check Questions 2.5

1. To encourage cooperation and teamwork instead of competition and individualism and enhance students' social skills.
2. Answers may vary
3. *Advantages of Cooperative Learning*
 - Increases achievement through group collaboration that enables students to learn from each other;
 - provides an alternative to the competitive structure of most classrooms today that discourages poorer students; and,
 - improves human relations in the classroom by promoting interdependent activities that teach collaborative skills

Disadvantages of Cooperative Learning

- requires adequate preparation and facilitating skills
 - takes up a lot of class time
4. Cooperative learning will be suitable for the lesson on non-standard measures since this lesson usually requires students to work together in measuring objects in the classroom with their body parts and comparing the results of the measurements obtained.

Self-Check Questions 2.7

1. Problem-based learning is similar to the inquiry method in that both start with a real-life problem for the students to solve. Also, both methods are reflective of the scientific method in terms of the steps followed. Yes, problem-based learning is a type of inquiry method. The inquiry method is broader in the sense

that the problems can branch out and the students may investigate these new problems as well.

2. School mathematics is not detached from real-life mathematics. We study mathematics primarily to solve real-life problems.
3. It is most suitable when the problem that is involved in developing the concept is complex enough to encourage critical thought and group effort.

Self-Check Questions 2.10

1. Answers may vary
2. No. Since peer practice is used to develop deeper understanding of the concepts by practicing previously taught information, it is best used in the application or practice portion of the lesson.
3. Advantages:
 - Hones helping skills of students.
 - Encourages cooperation and teamwork rather than competition and individualism.
 - Trains students to communicate effectively.

Limitations:

- Not appropriate in developing or introducing a lesson
- Time consuming

Self-Check Questions 2.11

1.D

2.B

3.D

4.C

5.A

Lesson 3

Self-Check Questions 3.1

1. Yes. Essay questions can be used to enhance students mathematical reasoning. Students should be asked to justify their answers.

2. Answers may vary. Possible answers:
- a) Is division commutative? Justify your answer.
 - b) Is ab equal to ba ? Explain.

Self-Check Questions 3.2

1. There is a grammatical clue. Since the stem ends in a , it follows that the correct answer begins with a vowel.

A fraction whose numerator is less than its denominator is called **a/an**

- A. proper fraction
 - B. improper fraction
 - C. unit fraction
 - D. similar fractions
2. All of the above cannot possibly be an option and should be changed.

What is the place value of 2 in 0.123?

- A. tenths
 - B. Hundredths
 - C. thousandths
 - D. ten thousandths
3. The stem has double negative.

Which one of the following is larger than 0.65?

- A. 0.5
 - B. 0.6000
 - C. 0.7
 - D. 0.6000
4. Option D is not plausible since *horse* is not even a geometric figure.

Which of the following is a polygon?

- A. Circle
 - B. Square
 - C. Cube
 - D. Prism
5. Stem does not contain much of the substance of the item.

What do you call a polygon with 4 right angles and 4 equal sides?

- A. Quadrilateral
- B. Parallelogram
- C. Rectangle
- D. Rhombus

Self-Check Questions 3.3

True or False

1. The statement is not wholly true nor wholly false.
A circle is a polygon.
2. The statement uses double negatives.
An equiangular triangle is always equilateral.
3. The statement is wordy.
The radius of a circle is half its diameter.
4. The statement uses 'usually'.
An isosceles triangle is always an acute triangle.
5. It is a trivia.
Euclid organized the geometric system that is used today.

Self-Check Questions 3.4**Matching Type**

The directions do not specify the criteria to be matched. The material is not homogeneous. The number of responses is equal to the number of premises.

Directions: Match the kind of triangle in Column B that fits the description in Column

B. The response may be used once, more than once, or not at all.

- | A | B |
|------------------------|----------------|
| 1. Has a right angle | a. equiangular |
| 2. Has equal angles | b. equilateral |
| 3. Has equal sides | c. isosceles |
| 4. Has no equal sides | d. obtuse |
| 5. Has two equal sides | e. right |
| | f. scalene |

Self-Check Questions 3.4

Completion Type

1. The statement can be correctly completed in several ways.

A triangle with three acute angles is called (a/an) _____.

2. The item cannot be completed by a word or a short phrase.

A quadrilateral with 2 pairs of opposite sides parallel and congruent is (a/an) _____.

3. The blanks are not of uniform length.

A square is a parallelogram with four _____ sides and four _____ angles.

4. The item gives a grammatical clue.

A triangle with two congruent sides is (a/an) _____ triangle.

5. The item has more than two blanks.

A chord that contains the center of the circle is called _____.

Activity 3.1

The statements that characterize authentic assessment are 1, 2, 5, 6, 8, 9, 11, 13, and 15.

Activity 3.2

1. Why do we need to assess?

- To check our students' strengths and weakness and how well they are learning
- To know how well we are doing as teachers and as a class
- To identify which students should be promoted, retained, or placed in special or remedial programs

2. What should we assess?

- Processes

- Skills and knowledge in a meaningful context
- Whatever is worth learning

3. How should we assess?

- By pen-and-paper tests
- By portfolios
- By observations
- By performance tasks and samples

Self-Check Questions 3.7

1. No 2. No 3. Yes 4. Yes 5. Yes

Self-Check Questions 3.8

1. No the amount of time required for a task depends on the nature of the task.
Some tasks may take a few minutes to an hour to finish.
2. Enough time allows students to explore and plan the solution to the task extensively and provide the best answer. Thus, the students are able to show their best.

Self-Check Questions 3.9

1. In an **analytic rubric** separate scores are given for different indicators or characteristics of a student's output or performance whereas in a holistic rubric scoring is based on an overall impression of a sample of student work considered as a whole. An analytic rubric takes more time to do but it provides more information than the holistic rubric.
A holistic rubric is more appropriate to use when you need to give quick scores to the tasks or product and when the tasks are not complex.
2. Analytic rubric is more appropriate to use when the tasks are complex and/or extended and when you want to obtain more information.

3. By employing more raters of the output or task using the rubric, and checking the correlation of the ratings obtained.

Self-Check Questions 3.10

1. Reflection, self-assessment, growth and development
2. Each entry has a corresponding reflection that explains its relevance to the learner and to the course objectives.
3. The teacher becomes the student's ally in the assessment process. He or she guides the learners to show their best and becomes just one of the many possible evaluators.
4. From being passive test-takers, students will now be active participants in the assessment process, posing questions not just answering them, and evaluating themselves and their peers instead of just being at the receiving end.

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