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Customer Based Two Stage Credit Policies In A Supply Chain

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Abstract

In general, a supplier/retailer frequently offers trade credit to its credit risk downstream member in order to stimulate their respective sales. This trade credit may either be full or partial, which depends on the past profile of the customers. The full credit period is being offered to existing customers whereas new customers are given partial credit. In the light of this very fact, an attempt has been made to derive an optimal ordering policy for the retailer, who receives full credit from his supplier and offers distinct credit to his customers. An easy- to- use closed form expressions have been obtained. Results have been validated with relevant example. In a way, the proposed model provides the generalized framework of previous related works.

Keywords: EOQ, Inventory, Supply chain, Partial trade credit, Two- level trade credit.

1. Introduction

In the conventional inventory models it is assumed that the buyer's capitals are unrestricted and must be paid for the items as soon as the items were received. But in practice, the supplier allows a certain fixed credit period which is known as trade credit period to settle the account for stimulating retailer's demand. During this period, the retailer can accumulate revenues on the sales and earn interest on that revenue, but beyond this period the supplier charges interest as has been agreed upon. Hence, paying later indirectly reduces the cost of holding stock because it reduces the amount of capital invested in stock for the duration of permissible delay period. These type of inventory models fall under the category of one level trade credit policy. An existing amount of literature exists in this area. Goyal [6] studied the effect of the trade credit period on the optimal inventory policy with the exclusion of the penalty cost due to late payment. Further, Aggarwal and Jaggi [2] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Jamal et.al [13] further generalized the model with shortages. Teng [17] amended Goyal's model by considering the difference between unit price and unit cost. Chung and Huang [4] developed an EPQ inventory model for a retailer when the supplier offers a permissible delay in payments. Many related articles can be found in Hwang & Shinn [12], Jamal et.al [14], Abad and Jaggi [1], Chung et.al [5] & their references.

Basically, the one stage credit policy works on the assumption that the supplier offers a credit period to the retailer but the retailer would not offer any credit period to its customers,. Whereas, in the present scenario, this assumption is quite unrealistic, as nowadays the retailer also passes on this credit period to his customers. The trade credit period serves as a powerful promotional tool for the retailer to attract new customer and stimulate the demand as has been the case of the supplier. This phenomenon is called as two level trade credit policies. Huang [7] presented a model assuming that the retailer also offers a credit period to his customer which is shorter than the credit period offered by the supplier. Huang [8] extended Huang's[7] model to investigate the retailer's inventory policy under two levels of trade credit and limited storage capacity. Huang [9] incorporated Huang [8] to investigate the two level trade credit policies in the EPQ framework. Jaggi et.al [15] incorporated the concept of credit-linked demand for the retailer and developed an inventory model under two levels of trade credit policy to determine the optimal credit as well as replenishment policy jointly. Teng and Chang [18] established an EPQ model under two levels of trade credit policy. Another realistic phenomenon getting momentum in supply chain model is partial trade credit financing, which refers to paying partial amount for the purchased items as soon as the items are received and remaining amount should be settled at the end of the trade credit period. Huang [11] developed an optimal retailer's policy when the supplier offers partial trade credit to the retailer and retailer offers full trade credit to his/her customer. Further, Huang and Hsu [10] expanded the model where the retailer gets full trade credit but offers partial trade credit to his/her customer. Thangam [21] developed the partial trade credit financing in an EPQ model under the same environment.

Recently, Teng [16] explored optimal ordering policies for a retailer who offers distinct trade credits (i.e. full and partial trade credit) to its good and bad credit customers where a good credit customer is the existing customer and a bad credit customer is the new customer. In his paper he has claimed that he first establishes an EOQ model for a retailer who receives full trade credit by its supplier, and offers either a partial trade credit to the bad (new) customer or a full trade credit to its good (existing) customer. However, on the contrary he concentrated only on the new customer throughout his paper, and the role of the existing customers who enjoys full trade credit has been oversighted.

The present study derives an optimal ordering policy for the retailer, who receives full credit from his supplier and offers partial trade credit to the new customers and full credit to the existing customers. It can easily be estimated from the past profiles of the total customers that some fraction is going to be the existing customers and rest is new customers. An easy-to-use closed form optimal solution for the retailer has been obtained. A sensitivity analysis on the fraction of existing customers and interest earning rate has also been performed.

Lastly, the major characteristics of the previous related work have been summarized in Table1. It is apparent from Table1 that the present model is a generalized model.

Author(s) & published Year	Two Stage	Assuming $p \ge c$	$I_p \ge I_e$	Allowing for Partial Payment	Bifurcation of Customers
Goyal [6]	No	No	Yes	No	No
Chung [3]	Yes	No	Yes	No	No
Teng [17]	No	Yes	No	No	No
Teng & Goyal [20]	Yes	Yes	No	No	No
Teng [16]	Yes	Yes	No	Yes	No
Present Paper	Yes	Yes	Yes	Yes	Yes

Table1. Major attribute of inventory models on selected researches

2. Notations and Assumptions

Following notations are used throughout the paper.

- *D* the annual demand rate .
- *A* the ordering cost per order.
- *c* the unit purchasing cost.
- p the unit selling price, $p \ge c$.
- *h* the unit stock holding cost per year excluding interest charge.
- I_p the interest charged per dollar in stock per year.
- I_e the interest earned per dollar per year.
- *M* the retailer's partial trade credit period offered by the supplier in years.
- *N* the customer's trade credit period offered by the retailer in years.
- *T* the cycle time in years.
- *k* the fraction of the existing customers who are offered full trade credit by the retailer.
- 1-k the fraction of the new customers who are offered partial trade credit by the retailer.
- α the fraction of the purchase cost in which the customer must pay the retailer at the time of placing an order, $0 \le \alpha \le 1$.
- $1-\alpha$ the fraction of the purchase cost in which the retailer offers its customer a permissible delay of N periods. TRC(T) the annual total relevant cost, which is a function of T.

$$T^{\dagger}$$
 the optimal replenishment cycle time of $TRC(T)$.

 Q^* the optimal order quantity = $D T^*$.

The mathematical model proposed in this paper is based on the following assumptions:

- 1. The present model is restricted to a single supplier, single retailer and multiple customers.
- 2. The inventory system deals with only one type of item.

- 3. Shortages are not allowed.
- 4. The lead time is zero.
- 5. Time period is infinite.
- 6. The supplier offers a full trade credit of M periods to the retailer. When $T \ge M$, the account is settled to the supplier at M and the retailer starts paying for the interest charges on the items in stock with rate I_p .
- 7. The retailer observes two types of customers namely the existing customers and the new customers. An existing customer is one who pays all his dues on time, so he is offered full trade credit by the retailer. A new customer is a debtor and therefore, the retailer offers partial trade credit to him. It can be estimated that some fraction k, of total customers is the existing customers and rest (1-k) is the new customers.
- 8. The retailer offers a full trade credit of N periods to his existing customers. The new customers have to make an initial payment on α units to the retailer at the time of purchasing and rest of the payment would be made at N. The retailer receives his revenue from N to T + N. He earns interest on the α units of every purchases up to T. Also when $N \le M$, the retailer can accumulate revenue and earn interest on rest of $(1-\alpha)$ units at the rate I_e .
- 9. At the end of permissible delay period M, the retailer pays all of the purchasing cost to the supplier and incurs a capital opportunity cost at a rate of I_p for the items still in stock and for the items sold but have not been paid for yet.
- 10. The interest rate charged may not necessarily higher than the retailer's investment rate.

3. Mathematical Model

The retailer's annual total relevant cost is given by

TRC(T) = (a)Annual ordering cost + (b) Annual stock holding cost + (c) Annual interest payable

- (d) Annual interest earned where
- (a) Annual ordering $\cot = \frac{A}{T}$

(b) Annual stock holding cost (excluding interest charges) = $\frac{DTh}{2}$

(c) To calculate annual interest charged and annual interest earned (according to assumption (6) and (9)) broadly two cases may arise (i) $M \ge N$ and (ii) M < N.

Case 1: $N \le M$

In the beginning of the cycle, the retailer starts selling the item to all his customers i.e. existing as well as the new customers. The existing customers make all the payment at the end of the credit period offered to them. The new customers have to make an initial payment of αpDT at the time of purchasing and rest of the payment will be made at the end of the credit period. From the values of M (i.e. the time at which the retailer must pay the supplier to avoid interest charges), T (i.e. the replenishment cycle time) and T + N (i.e. the time at which the retailer receives the payment from the last customer (either existing or new), we have three possible sub-cases: (i) $M \le T$, (ii) $T \le M \le T + N$, (iii) $T + N \le M$. We discuss the detailed formulation in each sub-case.

Sub-Case 1.1: $M \le T$

In this case, the retailer starts getting an initial amount of αpDT of every unit sold from the new customers and the rest of the payment will be received from N up to M. Also the actual sales revenue from the existing customer will be received during the period [N,M] (Figure 1). Thus he earns interest on average sales revenue for the time period M - N. Also, he earns interest for the fraction of new customer for the instant payment for the period [0,M] (Figure 2) and for the credit payment in the period for the period [N,M] (Figure 3). Therefore, the total interest earned by the retailer is

$$pI_e D\left[\frac{k(M-N)^2}{2} + (1-k)\left[\frac{\alpha M^2}{2} + \frac{(1-\alpha)(M-N)^2}{2}\right]\right]$$
(1)



Figure 3: Credit Payment $N \le M$ and $M \le T$

Further, the account is to be settled at M. If all the credit sales are not realized then the finances are to be arranged to make the payment to the supplier. Therefore, the retailer pays interest for the portion of existing customer for the period [M, T + N] (Figure1). Also, for the fraction of new customer, he pays interest for the instant payment for the period for the period [M, T + N] (Figure2) and for the credit payment in the period for the period [M, T + N] (Figure3). Thus, the total interest paid by the retailer is

$$cI_p D \left[\frac{k(T+N-M)^2}{2} + (1-k) \left[\frac{\alpha(T-M)^2}{2} + \frac{(1-\alpha)(T+N-M)^2}{2} \right] \right]$$
(2)

Hence, the total annual relevant cost is given by

$$TRC_{11}(T) = \frac{A}{T} + \frac{hDT}{2} + cI_p D \left[\frac{k(T+N-M)^2}{2T} + (1-k) \left[\frac{\alpha(T-M)^2}{2T} + \frac{(1-\alpha)(T+N-M)^2}{2T} \right] \right]$$
(3)
$$- pI_e D \left[\frac{k(M-N)^2}{2T} + (1-k) \left[\frac{\alpha M^2}{2T} + \frac{(1-\alpha)(M-N)^2}{2T} \right] \right]$$

Sub-Case 1.2: $T \le M \le T + N$

Here also, the retailer can accumulate revenue from N through M from the existing credit customer (Figure 4). From the new customer the revenue can be added from the initial payment in the period [0, M] (Figure 5) and for the rest of the payment in the period [N, M] (Figure 6). Hence, the total interest earned by the retailer is

$$pI_e D \left[\frac{k(M-N)^2}{2} + (1-k) \left[\frac{\alpha T^2}{2} + \alpha T(M-T) + \frac{(1-\alpha)(M-N)^2}{2} \right] \right]$$
(4)

Also, the retailer has to finance all items sold after M - N. Therefore he pays interest for the fraction of new customer for the credit payment during the period for the interval [M, T + N]. Thus, the total interest paid by the retailer is

$$cI_p D \left[\frac{k(T+N-M)^2}{2} + \frac{(1-k)(1-\alpha)(T+N-M)^2}{2} \right]$$
(5)



Figure 6: Credit payment $N \le M$ and $T \le M \le T + N$

Consequently, the total annual relevant cost is given by

$$TRC_{12}(T) = \frac{A}{T} + \frac{hDT}{2} + cI_p D \left[\frac{k(T+N-M)^2}{2T} + \frac{(1-k)(1-\alpha)(T+N-M)^2}{2T} \right] - pI_e D \left[\frac{k(M-N)^2}{2T} + (1-k) \left[\frac{\alpha T^2}{2T} + \alpha (M-T) + \frac{(1-\alpha)(M-N)^2}{2T} \right] \right]$$
(6)

Sub-Case 1.3: $T + N \le M$

In this case, the retailer earns interest on average sales revenue received during the period [N, T + N] and on full sales revenue for a period of M - T - N from the existing customers (Figure 7). Also, the interest earned from the new customers will be from the instant payment for the period [0, M] (Figure 8) and from the credit payment for the period [N, M] (Figure 9). So, the total interest earned by the retailer is

$$pI_e D\left[\frac{kT^2}{2} + kT(M - T - N) + (1 - k)\left[\frac{\alpha T^2}{2} + \alpha T(M - T) + \frac{(1 - \alpha)T^2}{2} + (1 - \alpha)T(M - T - N)\right]\right]$$
(7)

For this case the interest paid by the retailer for both types of customer is 0. As a result, the total annual relevant cost is

$$TRC_{13}(T) = \frac{A}{T} + \frac{hDT}{2} - \frac{pI_eD}{T} \left[\frac{kT^2}{2} + kT(M - T - N) + \frac{(1 - \alpha)T^2}{2} + (1 - \alpha)T(M - T - N) \right]$$
(8)





Figure 8: Instant Payment $N \le M$ and $T + N \le M$



Figure 9: Credit Payment $N \le M$ and $T + N \le M$

From the above arguments, the annual total relevant cost for the retailer can be expressed as

$$TR(T) = \begin{cases} TRC_1(T) & if M \le T & (9a) \\ TRC_2(T) & if T \le M \le T + N & (9b) \\ TRC_3(T) & if T + N \le M & (9c) \end{cases}$$

Since, $TRC_{11}(M) = TRC_{12}(M)$ and $TRC_{12}(M - N) = TRC_{13}(M - N)$, TRC(T) is continuous and well defined on T > 0.

Case 2: N > M

From the values of M and T, we have the following two possible sub-cases: (i) $M \le T$ and (ii) $M \ge T$. We discuss the detailed formulation in each sub-case.

Sub-case 2.1: *M* < *T*

Now, for this case there will be no interest earned by the retailer for the fraction of existing customer. The retailer earns interest for the portion of new customer during the period [0, M] (Figure 11) from the initial payment. Here, the retailer has to pay interest on full order quantity for the period [M, T + N] for the existing customers (Figure 10). Also in case of the new customers, he has to finance all items sold after M for the initial payment (Figure 11) and the entire amount of delayed payment for the period [M, T + N] (Figure 12). Thus, the total interest paid by the retailer is

$$cI_{p}D\left[\frac{k(T-M)^{2}}{2} + \frac{kT^{2}}{2} + (1-k)\left[\frac{\alpha(T-M)^{2}}{2} + (1-\alpha)\left(\frac{T^{2}}{2} + T(N-M)\right)\right]\right]$$
(10)

Accordingly, the total annual relevant cost is given by

$$TRC_{21}(T) = \frac{A}{T} + \frac{hDT}{2} + \frac{cI_p D}{T} \left[\frac{k(T-M)^2}{2} + \frac{kT^2}{2} + (1-k) \left[\frac{\alpha(T-M)^2}{2} + (1-\alpha) \left(\frac{T^2}{2} + T(N-M) \right) \right] \right]$$

$$- \frac{pI_e D}{T} \left((1-k) \frac{\alpha M^2}{2} \right)$$
(11)





Figure 12. Credit Payment $N \ge M$ and $M \le T$

Sub-case 2.2. *M* > *T*

Again, there will be no interest earned for the fraction of existing customers and the retailer earns interest for the fraction of new customers for the instant payment during the period for the interval [M,T] (Figure 14). Therefore total interest earned by the retailer will be

$$pI_e D(1-k)\alpha \left[\frac{T^2}{2} + T(M-T)\right]$$
(12)

Also, the total interest paid by the retailer for the fraction of existing customers (Figure 13) and for the fraction of new customers (Figure 15) is

$$cI_{p}D\left[kT(N-M) + \frac{kT^{2}}{2} + (1-k)(1-\alpha)\left(\frac{T^{2}}{2} + T(N-M)\right)\right]$$
(13)





The total annual relevant cost is

$$TRC_{22}(T) = \frac{A}{T} + \frac{hDT}{2} + \frac{cI_p D}{T} \left[kT(N-M) + \frac{kT^2}{2} + (1-k)(1-\alpha) \left(\frac{T^2}{2} + T(N-M)\right) \right] - \frac{pI_e D(1-k)\alpha}{T} \left[\frac{T^2}{2} + T(M-T)\right]$$
(14)

Thus, the annual total relevant cost for the retailer can be expressed as

$$TRC(T) = \begin{cases} TRC_{21}(T) & \text{if } M \le T \\ TRC_{22}(T) & \text{if } T \le M \end{cases}$$
(15a)
(15b)

Again, $TRC_{21}(M) = TRC_{22}(M)$, TRC(T) is continuous and well defined on T > 0.

4. Optimal order Quantity

Case 1. N < M

Our problem is to determine the optimum value of T which minimizes TRC(T). The necessary and sufficient condition for optimality is TRC'(T) = 0 and TRC''(T) < 0. Taking the first-order and the second -order derivatives of $TRC_{11}(T), TRC_{12}(T)$ and $TRC_{13}(T)$ with respect to T, we obtain the following expressions:

$$TRC'_{11}(T) = -\frac{1}{T^2} \left[A + \frac{(cI_p - pI_e)D((M - N)^2(1 - \alpha(1 - k)) + (1 - k)\alpha M^2)}{2} \right] + \frac{D(h + cI_p)}{2}$$
(16)

$$TRC_{11}^{"}(T) = \frac{2A + (cI_p - pI_e)D((M - N)^2(1 - \alpha(1 - k)) + (1 - k)\alpha M^2)}{T^3}$$
(17)

$$TRC'_{12}(T) = -\frac{1}{T^2} \left[A + \frac{(cI_p - pI_e)D(M - N)^2(1 - \alpha(1 - k))}{2} \right] + \frac{D(h + cI_p - \alpha(1 - k)(cI_p - pI_e))}{2}$$
(18)

$$TRC_{12}''(T) = \frac{2A + (cI_p - pI_e)D(M - N)^2(1 - \alpha(1 - k))}{T^3}$$
(19)

$$TRC'_{13}(T) = -\frac{A}{T^2} + \frac{D(h+pI_e)}{2}$$
(20)

$$TRC_{13}''(T) = \frac{2A}{T^3} > 0$$
(21)

Equation (21) clearly indicates that $TRC_{13}(T)$ is strictly convex on T > 0. However, from (19) and (17) it can be clearly seen that $TRC_{12}(T)$ and $TRC_{11}(T)$ will be a strictly convex function on T > 0 if

$$2A + (cI_p - pI_e)D(M - N)^2(1 - \alpha(1 - k)) > 0 \text{ and}$$

$$2A + (cI_p - pI_e)D((M - N)^2(1 - \alpha(1 - k)) + (1 - k)\alpha M^2) > 0$$
(22)

By equating the first order derivative to zero we obtain the optimal value of T. Thus, there exists a unique value of T say T_{13}^* which minimizes $TRC_{13}(T)$ as

$$I_{13}^* = \sqrt{\frac{2A}{D(h+pI_e)}}$$
(23)

 T_{13}^* will satisfy the condition $T + N \le M$ provided

 Δ_1

$$\equiv 2A - (M - N)^2 D(h + pI_e) \le 0$$
⁽²⁴⁾

Substituting Eq. (23) into (8), we can get the optimal value of $TRC_{13}(T)$ (say $TRC_{13}^*(T)$) and the optimal order quantity Q_{13}^* is given by

$$Q_{13}^* = \sqrt{\frac{2AD}{(h+pI_e)}}$$
(25)

Similarly, there exists a unique value of T say T_{12}^* which minimizes $TRC_{12}(T)$ as

$$T_{12}^{*} = \sqrt{\frac{2A + (cI_{p} - pI_{e})D(M - N)^{2}(1 - \alpha(1 - k))}{D(h + cI_{p} - \alpha(1 - k)(cI_{p} - pI_{e}))}}$$
(26)

 T_{12}^* will satisfy the condition $T \le M \le T + N$ if and only if

$$\Delta_1 \equiv 2A - (M - N)^2 D(h + pI_e) \ge 0 \text{ and}$$

$$\Delta_2 \equiv 2A - M^2 D(h + pI_e) + (N^2 - 2MN)(1 - \alpha(1 - k))(cI_p - pI_e) \le 0$$
(27)

Substituting Eq. (26) into (6), we can get the optimal values of $TRC_{12}(T)$ (say $TRC_{12}^*(T)$) and the optimal order quantity Q_{12}^* is given by

$$Q_{12}^{*} = \sqrt{\frac{2AD + (cI_{p} - pI_{e})D^{2}(M - N)^{2}(1 - \alpha(1 - k))}{(h + cI_{p} - \alpha(1 - k)(cI_{p} - pI_{e}))}}$$
(28)

Likewise, we can obtain the optimal value of T say T_{11}^* which minimizes $TRC_{11}(T)$ as

$$T_{11}^{*} = \sqrt{\frac{2A + (cI_{p} - pI_{e})D((M - N)^{2}(1 - \alpha(1 - k)) + \alpha(1 - k)M^{2})}{D(h + cI_{p})}}$$
(29)

 T_{11}^* will satisfy the condition $M \leq T$ provided

$$\Delta_2 = 2A - M^2 D(h + pI_e) + (N^2 - 2MN)(1 - \alpha(1 - k))(cI_p - pI_e) \ge 0$$
(30)

Substituting Eq. (29) into (3), we can get the optimal values of $TRC_{11}(T)$ (say $TRC_{11}^*(T)$) and the optimal order quantity Q_{11}^* can be obtained as

$$Q_{11}^{*} = \sqrt{\frac{2AD + (cI_p - pI_e)D^2 \left((M - N)^2 (1 - \alpha(1 - k)) + \alpha(1 - k)M^2 \right)}{(h + cI_p)}}$$
(31)

Combining the three possible cases, we obtain the following theorem.

Theorem1. For $N \leq M$,

- A. If $\Delta_2 \ge 0$, then $T^* = T_{11}^*$.
- B. If $\Delta_1 \ge 0$ and $\Delta_2 \le 0$, then $T^* = T_{12}^*$.
- C. If $\Delta_1 \le 0$, then $T^* = T_{13}^*$.

Proof. It immediately follows from (24), (27) and (30).

Further if we consider $k = 1, M = \alpha = N = 0$, the present result reduces to the classical EOQ model, i.e.

$$Q_E^* = \sqrt{\frac{2AD}{(h+cI_p)}}$$
(32)

As a result, we can obtain the following theoretical result.

Theorem 2. When $N \le M$, then A. If $pI_e < cI_p$, then all Q_{11}^*, Q_{12}^* and Q_{13}^* are greater than Q_E^* . B. If $pI_e > cI_p$, then all Q_{11}^*, Q_{12}^* and Q_{13}^* are less than Q_E^* . C. If $pI_e = cI_p$, then all $Q_{11}^* = Q_{12}^* = Q_{13}^* = Q_E^*$.

Proof. It follows from (25), (28), (31) and (32).

The above theorem clearly interpret that the retailer will take the benefit of partial trade credit offered to him, more frequently and will order less quantity if $pI_e > cI_p$ and vice versa.

Case 2. N > M

Again, in order to minimize the total annual relevant cost, we take the first-order and the second -order derivatives of $TRC_{21}(T)$ and $TRC_{22}(T)$ with respect to T and obtained the following expressions:

$$TRC'_{21}(T) = -\frac{1}{T^2} \left[A + \frac{cI_p DkM^2 + (cI_p - pI_e)D\alpha(1 - k)M^2}{2} \right] + \frac{D(h + cI_p(1 + k))}{2}$$
(33)

$$TRC''_{21}(T) = \frac{2A + cI_p DkM^2 + (cI_p - pI_e)D\alpha(1 - k)M^2}{T^3}$$
(34)

$$TRC'_{22}(T) = -\frac{A}{T^2} + \frac{D(h+cI_p - \alpha(1-k)(cI_p - pI_e))}{2}$$
(35)

$$TRC_{22}''(T) = \frac{2A}{T^3} > 0$$
(36)

Equation (36) clearly indicates that $TRC_{22}(T)$ is strictly convex on T > 0. However, from (34) it can be seen that $TRC_{21}(T)$ will be a strictly convex function on T > 0 if

$$2A + cI_p DkM^2 + (cI_p - pI_e)D\alpha(1 - k)M^2 > 0$$
(37)

Thus, there exists a unique value of T say T_{22}^* which minimizes $TRC_{22}(T)$ as

$$T_{22}^{*} = \sqrt{\frac{2A}{D(h+cI_{p}-\alpha(1-k)(cI_{p}-pI_{e}))}}$$
(38)

 T_{22}^* will satisfy the condition $M \ge T$ if and only if

$$\Delta_3 = 2A - M^2 D(h + cI_p - \alpha(1 - k)(cI_p - pI_e)) \le 0$$
(39)

Substituting Eq. (38) into (14), we can get the optimal values of $TRC_{22}(T)$ (say $TRC_{22}^*(T)$) and the optimal order quantity Q_{22}^* as

$$Q_{22}^{*} = \sqrt{\frac{2AD}{(h+cI_{p}-\alpha(1-k)(cI_{p}-pI_{e}))}}$$
(40)

Similarly, we can obtain the optimal value of T say T_{21}^* which minimizes $TRC_{21}(T)$ as

$$T_{21}^{*} = \sqrt{\frac{2A + cI_{p}DkM^{2} + (cI_{p} - pI_{e})D\alpha(1 - k)M^{2}}{D(h + cI_{p}(1 + k))}}$$
(41)

 T_{21}^* will satisfy the condition $M \leq T$ provided

$$\Delta_3 = 2A - M^2 D(h + cI_p - \alpha(1 - k)(cI_p - pI_e)) \ge 0$$
(42)

Substituting Eq. (41) into (11), we can get the optimal values of $TRC_{21}(T)$ (say $TRC_{21}^*(T)$) and the optimal order quantity Q_{21}^* as

$$Q_{21}^{*} = \sqrt{\frac{2AD + cI_{p}D^{2}kM^{2} + (cI_{p} - pI_{e})D^{2}\alpha(1 - k)M^{2}}{(h + cI_{p}(1 + k))}}$$
(43)

Summarizing the above arguments, we obtain the following theorems.

Theorem 3. For $N \ge M$,

A. If $\Delta_3 \ge 0$, then $T^* = T_{21}^*$.

B. If $\Delta_3 \le 0$, then $T^* = T_{22}^*$.

Proof. It follows from (39) and (42).

Theorem 4. When $N \ge M$, and condition (37) holds, then

- A. If $pI_e < cI_p$, then both Q_{21}^* and Q_{22}^* are greater than Q_E^* .
- B. If $pI_e > cI_p$, then both Q_{21}^* and Q_{22}^* are less than Q_E^* .
- C. If $pI_e = cI_p$, then $Q_{21}^* = Q_{22}^* = Q_E^*$.

Proof. It follows from (32), (40) and (43).

5. Numerical Example

To gain more insight of the above theory, we consider the same example as Teng (2009) i.e.

A one-dollar store (i.e., p = \$1) buys nail cutters from a supplier at c = \$0.50 a piece. The supplier offers a permissible delay if the payment is made within 60 days (i.e., M = 2/12 = 1/6). This credit term in finance management is usually denoted as "net 60". However, if the payment is not paid in full by the end of 60 days, then 8% interest(i.e., $I_e = 0.08$) is charged on the outstanding amount. To avoid default risks, the store owner (or the retailer) offers a partial trade credit (e.g. $\alpha = 0.5$ and N = 1/12) to those credit- risk customers without credit cards. We assume that D = 3600 units, h = \$0.2/unit/year, A = \$12 per order, and $I_e = 2\%$ if the store deposits its revenue into a money-market account; or $I_e = 10\%$ if it invests its revenue into a mutual fund account.

However, since in the present study we have incorporated explicitly the existing and the new customers, therefore we assume that the store owner has 40% existing customers (i.e., k = 0.4) and rest are new customer. This example refers to Case1 where $N \le M$. Here $\Delta_1 \ge 0$ for $I_e = 2\%$ or 10%. Also,

 $|\geq 0 \quad \text{if } I_a = 2\%$

$$\Delta_2 \bigg| \le 0$$
 if $I_e = 10\%$

Incorporating the above theoretical results, the findings are

$$Q^* = \begin{cases} 612 & \text{if } I_e = 2\% \\ 566 & \text{if } I_e = 10\% \end{cases}$$

Further, from (32), the classical EOQ is given by $Q_E^* = 600$ units. This verifies Theorem (2) that if $pI_e < cI_p$, then the store owner should order more than classical EOQ and if $pI_e > cI_p$, and then he should order less to take the benefits of the trade credit more frequently.

Special Case.

For k = 0, the present paper reduces to Teng [16]. For the above example, from equation (41) and (44), the findings are

 $Q^* = \begin{cases} 615.42 & \text{if } I_e = 2\% \\ 556.77 & \text{if } I_e = 10\% \end{cases}$

whereas Teng [16] reports these values in his paper as

 $Q^* = \begin{cases} 600.00, \text{ if } I_e = 2\% \\ 565.68, \text{ if } I_e = 10\% \end{cases}$

which seems to be incorrect.

5.1 Sensitivity Analysis

Sensitivity analysis on the fraction of existing customer i.e. k and I_e has been performed and the findings has been summarized in Table 2 and Table3.

It is clearly evident from Table2 that for high I_e and $(pI_e > cI_p)$, as k increases, ordering quantity also increases. Under this scenario, the retailer should promote existing customers. On the other hand, Table 3 reflects that for low values of I_e and $(pI_e < cI_p)$, as k increases ordering quantity decreases, which suggests that the retailer should encourage new customers.

Table2. Sensitivity analysis of κ when $T_e = 0.1$					
k	Q^*	TRC(T)			
0	557	114.3			
0.2	561	116.9			
0.4	566	119.5			
0.6	571	122.2			
0.8	576	124.8			
1	581	127.4			

Table2. Sensitivity analysis on k when $I_a=0.1$

Table3. Sensitivity analysis on k when $I_e = 0.02$

k	Q^{*}	TRC(T)
0	615	129.7
0.2	614	130.4
0.4	612	131.2
0.6	610	131.9
0.8	608	132.7
1	606	133.4

6. Conclusion

The present study highlights the effect of the existing customers as well as the new customers on the ordering policies for the retailer. It is assumed that the retailer receives full credit from his supplier and offers full credit to its existing customers and partial credit to the new customers. An easy-to-use closed form expression for the optimal ordering quantity has been obtained. A sensitivity analysis on k i.e. the fraction of existing customers and interest earning rate has been presented. It reveals that the credit period offered to the customer has a positive impact on the unrealized demand. Also, for high I_e and $(pI_e > cI_p)$, the retailer should encourage existing customers. Whereas, for

less I_e and $(pI_e < cI_p)$, the retailer should promote new customers. The present paper provides the generalized framework of the previous related works.

For future research, the present model can be applied to the deteriorating items, limited storage space, time value of money, etc.

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Response Sheet

Review Comments

1. The abstract should highlight research findings and uniqueness of the approach. 2. The Introduction should include some latest journal article references (2007-2010) in order to highlight the scope and currency of the research subject. 3. Literature Review should include at least 5-7 latest journal article references (2007-2010) and discussions from popular industrial engineering, and operations management related journals. 4. Authors should have proper descriptions of all tables and figures of the results. It should have a proper validation of the approach/findings. 5. Conclusions should highlight the unique contributions of the paper, limitations of the research and some future research directions. 6. Update the reference list as per the comments. 7. Some references have missing details such as the volume, issue and page numbers. These must be fixed. The references were incorrectly cited both within the text and at the end. References should follow the style of IEOM Conference. 8. Please make sure that there are no grammatical and / or spelling errors. The language of the paper needs a careful editing for the international audience. If possible, get a complete proof read by a technical English writer.

Overall Comments:

The references have been appended with the issue numbers. Thanks

Final Comments:

All the equations are related to the context and can't be deleted further. Also the figures highlight the interest earned and interest payable for different sub-cases, which are necessary to understand the concept. Therefore, the figures can't be deleted. In spite of our best efforts, the length of the paper cannot be reduced further. Thanks.