Chapter 7

IMPULSE AND MOMENTUM

PREVIEW

The *momentum* of an object is the product of its mass and velocity. If you want to change the momentum of an object, you must apply an *impulse*, which is the product of force and the time during which the force acts. If there are no external forces acting on a system of objects, the momentum is said to be conserved, that is, the total momentum of the system before some event (like a collision) is equal to the total momentum after that event. In this chapter, we will discuss examples of both one- and two-dimensional collisions.

QUICK REFERENCE

Important Terms

impulse
: The product of the average force acting on an object and the time during which it acts. Impulse is a vector quantity, and can also be calculated by finding the area under a force versus time curve.

linear momentum
: The product of the mass of an object and its velocity. Momentum is a vector quantity, and thus the total linear momentum of a system of objects is the vector sum of the individual momenta of the objects in the system.

internal forces
: The forces which act between the objects of a system

external forces
: The forces which act on the objects of a system from outside the system, that is by an agent which is not a part of the system of objects which are being studied.

inelastic collision
: A collision between two or more objects in which momentum is conserved but kinetic energy is not conserved, such as two railroad cars which collide and lock together.

elastic collision
: A collision between two or more objects in which both momentum and kinetic energy are conserved, such as in the collision between two steel balls.

center of mass
: The point at which the total mass of a system of masses can be considered to be concentrated.

Equations and Symbols

$\mathbf{p} = m\mathbf{v}$

$\mathbf{J} = \boldsymbol{F}\Delta t = \Delta\mathbf{p} = m\mathbf{v}_f - m\mathbf{v}_0$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

where

p = momentum
m = mass
v = velocity
J = impulse
F = force
Δt = time interval during which a force acts
x_{cm} = position of the center of mass of a system of particles
x_1 = position of a mass relative to a chosen origin

DISCUSSION OF SELECTED SECTIONS

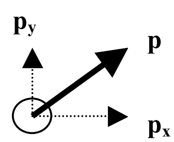
7.1 The Impulse – Momentum Theorem

The momentum **p** of an object is the product of the mass *m* of the object and its velocity **v**:

$\mathbf{p} = m\mathbf{v}$

The momentum of a moving mass is a vector which has a direction that is the same as the velocity of the mass. Thus, the momentum of an object can be broken down into its components:

$\mathbf{p} = \mathbf{p_x} + \mathbf{p_y}$



where $p_x = p\cos\theta$ and $p_y = p\sin\theta$.

The magnitude of the momentum vector can be found by the Pythagorean theorem:

$$|p| = \sqrt{p_x^{\,2} + p_y^{\,2}}$$

Newton's second law states that an unbalanced (net) force acting on a mass will accelerate the mass in the direction of the force. Another way of saying this is that a net force acting on a mass will cause the mass to change its momentum. We can rearrange the equation for Newton's second law to emphasize the change in momentum:

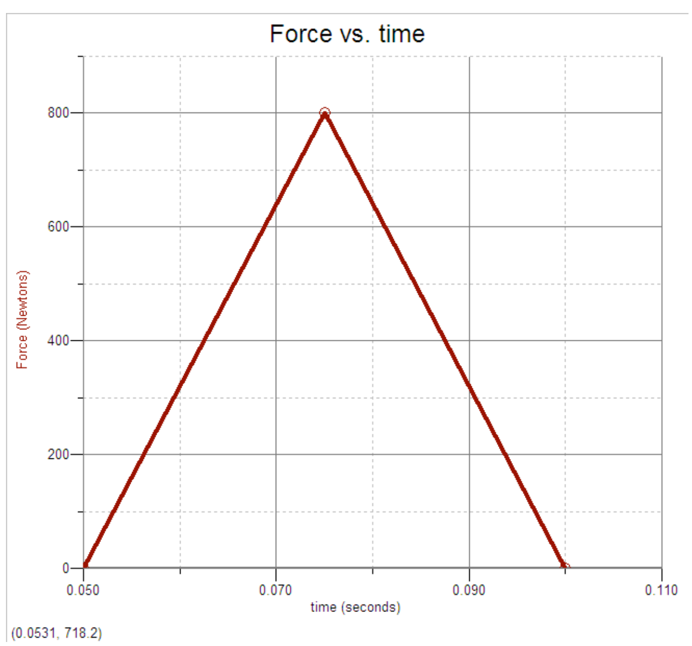
$$\mathbf{F}_{net} = m\mathbf{a} = m\left(\frac{\Delta \mathbf{v}}{\Delta t}\right)$$

Rearranging this equation by dividing both sides by Δt gives

$$\mathbf{F}\Delta t = m\Delta \mathbf{v} = m\mathbf{v}_f - m\mathbf{v}_0$$

The left side of the equation ($\mathbf{F}\Delta t$) is called the *impulse*, and the right side is the *change in momentum*. This equation reflects the *impulse-momentum theorem*, and in words can be stated *"a force acting on a mass during a time causes the mass to change its momentum"*. The force **F** in this equation is the *average force* acting over the time interval.

Example 1
A 2-kg block slides along a floor of negligible friction with a speed of 20 m/s when it collides with a 3-kg block, which is initially at rest. The graph below represents the force exerted on the 3-kg block by the 2-kg block as a function of time.



Find
(a) the initial momentum of the 2-kg block,
(b) the impulse exerted on the 3-kg block,and
(c) the momentum of the 2-kg block immediately after the collision.

Solution: (a) The initial momentum of the 2-kg block is

$p = mv = (2kg)\left(20\frac{m}{s}\right) = 40\frac{kgm}{s}$ to the right.

(b) When the two blocks collide, they exert equal and opposite forces on each other for the same time interval, and thus exert equal and opposite impulses (**F***Δt*) on each other. The impulse exerted on the 3-kg block can be found by finding the area under the force vs. time graph:

Impulse = area under F vs. t graph = 20 N s.

(c) Since the 2-kg block is experiences – 20 N s of impulse, which acts in the opposite direction of the motion of the block, the impulse causes the block to lose momentum. This loss (change) in momentum is equal to the impulse exerted on the block. Thus, by the impulse-momentum theorem,

Impulse = change in momentum

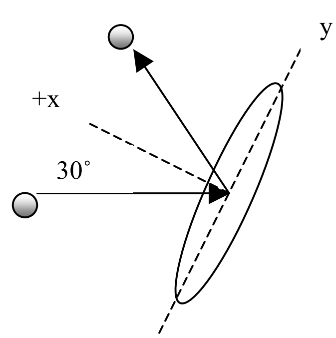
$F\Delta t = mv_f - mv_i$

$mv_f = mv_i + (F\Delta t) = 40\frac{kgm}{s} + (-20Ns) = 20\frac{kgm}{s}$

Note that a *N s* is equivalent to $\frac{kg\,m}{s}$.

Example 2

One of the photographs in chapter 7 of your textbook shows a tennis ball striking a tennis racquet, applying an impulse to it. Suppose the 0.1 kg ball strikes the racquet with a velocity of 60 m/s at an angle of 30° from a line which is perpendicular to the face of the racquet and rebounds with a speed of 60 m/s at 30° above the perpendicular line, as shown below. The ball is in contact with the strings of the racquet for 12 milliseconds.



Find

(a) the magnitude and direction of the average impulse exerted on the ball by the strings of the racquet, and

(b) the magnitude of the average acceleration of the ball while it is in contact with the strings.

Solution:
(a) The strings of the racquet exert a force and thus an impulse which is perpendicular to the face of the racquet, that is, along the +*x* – axis in the figure above. Therefore the change in momentum of the ball is also only along the +*x* – axis:
$\mathbf{F}\Delta t = \Delta\mathbf{p}_x = m\Delta\mathbf{v}_x = m(\mathbf{v}_{fx} - \mathbf{v}_{0x})$
where $v_{0x} = -v_0 \cos 30$ and $v_{0x} = +v_f \cos 30$
So the impulse is
$(0.1kg)[60\frac{m}{s}\cos 30 - (-60\frac{m}{s}\cos 30)] = 10.4\, N\, s$
(b) Since the force is applied in the +*x* – direction, the average acceleration is must also be directed along the +*x* – axis, that is, there is no acceleration along the *y*-axis.

$$a_x = \frac{\Delta v}{\Delta t} = \frac{[60\frac{m}{s}\cos 30 - (-60\frac{m}{s}\cos 30)]}{0.012s} = 8700\frac{m}{s^2}$$

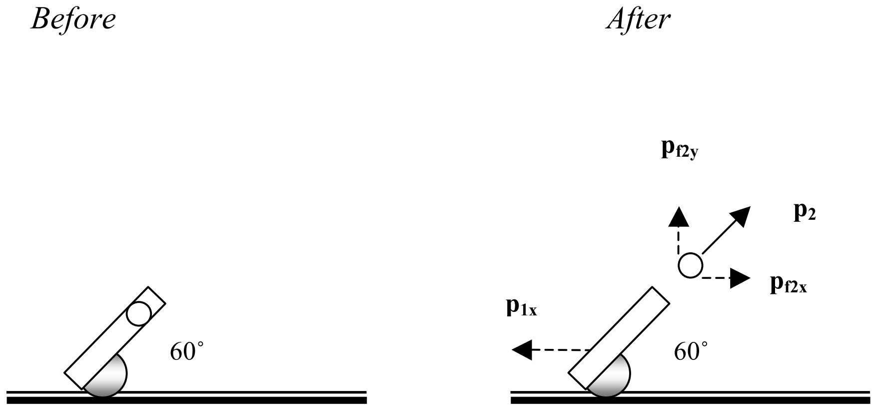
7.2 The Principle of Conservation of Linear Momentum

We've seen that if you want to change the momentum of an object or a system of objects, Newton's second law says that you have to apply an unbalanced force. This implies that if there are no unbalanced forces acting on a system, the total momentum of the system must remain constant. This is another way of stating Newton's first law, the law of inertia, discussed in chapter 4. If the total momentum of a system remains constant during a process, such as an explosion or collision, we say that the momentum is *conserved.* The **principle of conservation of linear momentum** states that *the total linear momentum of an isolated system remains constant (is conserved). An isolated system is one for which the vector sum of the external forces acting on the system is zero.*

Typically, the AP Physics B exam includes the following types of problems which use the principle of conservation of linear momentum: *recoil* in one and two dimensions, *inelastic collisions* in one and two dimensions, and *elastic collisions* in one and two dimensions. Remember, if a momentum vector is conserved, its components are also conserved.

Example 3
A toy cannon (m_1 = 2 kg) is mounted to two horizontal rails on which it can slide with negligible friction. The cannon fires a ball (m_2 = 0.025 kg) at a speed v_{f2} = 40 m/s at an angle of 60° above the horizontal.
(a) Is the total momentum of the system conserved during the firing of the cannon? Explain.
(b) What is the recoil velocity of the cannon after it is fired?

Solution: Let's sketch the cannon and ball before and after they are fired:



(a) The total momentum is not conserved in this case, since the horizontal rail provides an external force acting on the cannon in the vertical direction. Since there is negligible friction acting horizontally on the cannon, we can say that momentum is conserved in the horizontal direction, but not in the vertical direction.

(b) Since momentum is conserved in the horizontal direction, we can set the momentum of the system (cannon and ball) before the collision, which is zero, equal to the momentum of the system after the collision.

$$\mathbf{p}_0 = \mathbf{p}_{f1x} + \mathbf{p}_{f2x}$$

$$0 = m_1\mathbf{v}_{f1x} + m_2\mathbf{v}_{f2x}$$

$$0 = m_1 v_{f1x} + m_2 v_{f2}\cos 60$$

Solving for the horizontal recoil velocity of the cannon, we get

$$v_{f1x} = \frac{-m_2 v_{f2}\cos 60}{m_1} = \frac{-(0.025kg)(40\frac{m}{s})\cos 60}{2kg} = -0.25\frac{m}{s}$$

7.3 Collisions in One Dimension

Example 4

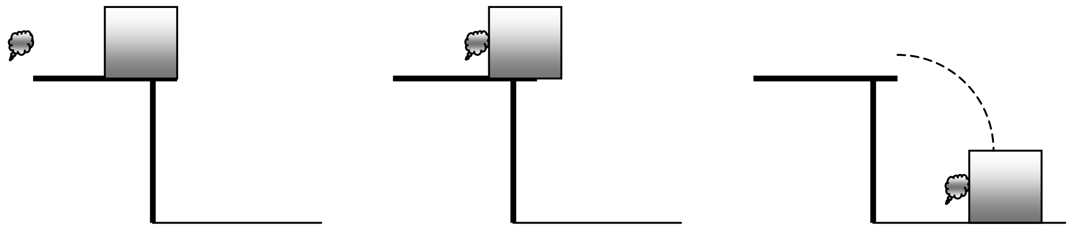
A lump of clay (m_1 = 0.2 kg) moving horizontally with a speed v_{o1} = 16 m/s strikes and sticks to a wood block (m_2 = 3 kg) which is initially at rest on the edge of a horizontal table of height h = 1.5 m. Neglecting friction, find

(a) the horizontal distance x from the edge of the table at which the clay and block strike the floor, and

(b) the total momentum of the clay and block just before they strike the floor.

Solution: The sketches below represent the system before, during, and after the collision.

Before *During* *After*



(a) Before we can find the horizontal distance the clay and block travel we need to find their speed v_f as they leave the edge of the table. Momentum is conserved in this inelastic collision:

$$\mathbf{p}_o = \mathbf{p}_f$$

$$m_1 \mathbf{v}_{o1} = (m_1 + m_2)\mathbf{v}_f$$

$$v_f = \frac{m_1 v_{o1}}{(m_1 + m_2)} = \frac{(0.2\,kg)(16\frac{m}{s})}{(0.2\,kg + 3.0\,kg)} = 1\frac{m}{s}$$

Now the clay and block have become a projectile which is launched horizontally. We can find the time of flight by using the height:

$$h = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1.5\,m)}{10\frac{m}{s^2}}} = 0.55\,s$$

Then the horizontal distance traveled is $x = v_f t = (1\frac{m}{s})(0.55\,s) = 0.55\,m$

(b) The momentum of the clay and block just before it strikes the ground can be found by finding the horizontal and vertical components of the momentum:

$$p_x = mv_x = (3.2\,kg)(1\frac{m}{s}) = 3.2\frac{kg\,m}{s}$$

$$p_y = mv_y = m(gt) = (3.2\,kg)(10\frac{m}{s^2})(0.55\,s) = 17.6\frac{kg\,m}{s}$$

By the Pythagorean theorem, the magnitude of the momentum can be found by

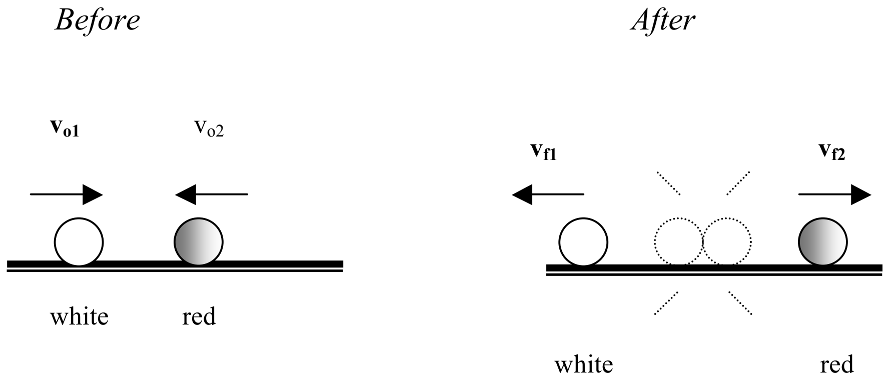
$$p = \sqrt{p_x^{\,2} + p_y^{\,2}} = \sqrt{(3.2\frac{kg\,m}{s})^2 + (17.6\frac{kg\,m}{s})^2} = 17.9\frac{kg\,m}{s}$$

The angle at which the momentum is directed is

$$\theta = \tan^{-1}\left[\frac{p_y}{p_x}\right] = \tan^{-1}\left[\frac{17.6}{3.2}\right] = 80^\circ \text{ below the horizontal.}$$

Example 5
A white pool ball (m_1 = 0.3 kg) moving at a speed of v_{o1} = +3 m/s collides head-on with a red pool ball (m_2 = 0.4 kg) initially moving at a speed of v_{o2} = - 2 m/s. Neglecting friction and assuming the collision is perfectly elastic, what is the velocity of each ball after the collision?

Solution: The sketches below represent the system before and after the collision.



Since momentum is conserved in this collision, we can set the total momentum of the system before the collision equal to the total momentum after the collision:

$$\mathbf{p}_{01} + \mathbf{p}_{02} = \mathbf{p}_{f1} + \mathbf{p}_{f2}$$
$$m_1\mathbf{v}_{01} + m_2\mathbf{v}_{02} = m_1\mathbf{v}_{f1} + m_2\mathbf{v}_{f2}$$

Solving for $\mathbf{v}_{f1}$ we get

$$v_{f1} = \frac{m_1 v_{01} + m_2 v_{02} - m_2 v_{f2}}{m_1}$$

But here we have two unknowns, v_{f1} and v_{f2}, and only one equation. We can generate another equation containing the same variables if we remember that both momentum and kinetic energy are conserved in an elastic collision.

$$KE_{01} + KE_{02} = KE_{f1} + KE_{f2}$$
$$\frac{1}{2}m_1 v_{01}{}^2 + \frac{1}{2}m_2 v_{02}{}^2 = \frac{1}{2}m_1 v_{f1}{}^2 + \frac{1}{2}m_2 v_{f2}{}^2$$

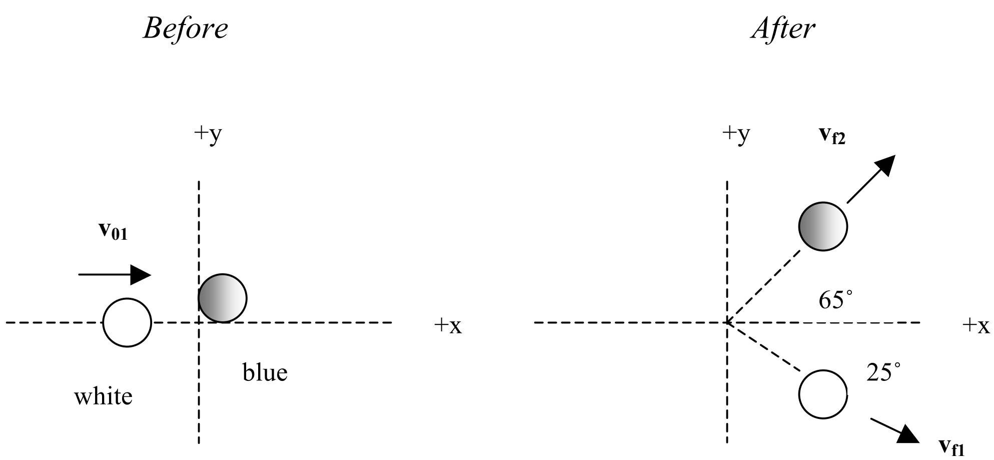
Substituting the equation solved above for v_{f1} into the equation for conservation of kinetic energy, we get one equation with only one unknown, namely v_{f2}. Substituting the known values into this equation and solving for v_{f2} gives v_{f2} = +2.3 m/s. Solving for v_{f1} in the equation above gives v_{f1} = - 2.7 m/s.

It can be shown that if we solve the equations for conservation of momentum and conservation of kinetic energy simultaneously, it always turns out that the relative speeds of the two masses remains the same (except for a negative sign) before and after a

perfectly elastic collision regardless of the masses of the two objects. That is,
$\mathbf{v}_{01} - \mathbf{v}_{02} = -(\mathbf{v}_{f1} - \mathbf{v}_{f2})$

7.4 Collisions in Two Dimensions

Example 6
The diagram below shows a collision between a white pool ball (m_1 = 0.3 kg) moving at a speed v_{01} = 5 m/s in the +x direction and a blue pool ball (m_2 = 0.6 kg) which is initially at rest. The collision is not head-on, so the balls bounce off of each other at the angles shown. Find the final speed of each ball after the collision.



Solution: The components of the momenta before and after the collision are conserved.

Writing the x – components of the momentum before and after the collision:

$$\mathbf{p}_{01x} + \mathbf{p}_{02x} = \mathbf{p}_{f1x} + \mathbf{p}_{f2x}$$
$$m_1\mathbf{v}_{01x} + m_2\mathbf{v}_{02x} = m_1\mathbf{v}_{f1x} + m_2\mathbf{v}_{f2x}$$
$$(0.3\,kg)(5\frac{m}{s}) = (0.3\,kg)(v_{f1}\cos 25 + (0.6\,kg)(v_{f2}\cos 65)$$

Here we have two unknowns, v_{f1} and v_{f2}, and only one equation so far. When we write the conservation of momentum equation for the y-components of the momentum of each ball, we see that the total momentum in the y direction is zero before the collision, and thus must be zero after the collision.

$$\mathbf{p}_{01y} + \mathbf{p}_{02y} = \mathbf{p}_{f1y} + \mathbf{p}_{f2y}$$
$$0 = m_1\mathbf{v}_{f1y} + m_2\mathbf{v}_{f2y}$$
$$0 = (0.3\,kg)(v_{f1}\sin 25) + (0.6\,kg)(v_{f2}\sin 65)$$

Solving these two equations simultaneously for the unknown speeds gives $v_{f1} =$ 1.8 m/s and $v_{f2} =$ 4 m/s. In any two-dimensional elastic collision in which one mass is at rest, the angle between the two objects after the collision will be 90°.

7.5 Center of Mass

The *center of mass* of a system of particles can be thought of as the point at which all of the mass of the system can be considered to be concentrated. The equation given to you in your textbook used for calculating the center of mass of two objects on the x – axis is

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{(m_1 + m_2)}$$

This equation is typically not needed on the AP Physics B exam, but the concept of center of mass is important in many applications on the AP exam. For a conservation law to be true, it must hold true in any inertial reference frame, including the reference frame of the center of mass. If we multiply both sides of the center of mass equation above by the total mass $(m_1 + m_2)$, we get

$$(m_1 + m_2) x_{cm} = m_1 x_1 + m_2 x_2$$

If the two objects are moving, each x is changing with respect to time, we can replace each x with Δx, and divide both sides by Δt. The ratio $\frac{\Delta x}{\Delta t}$ is velocity, so our center of mass equation becomes

$$(m_1 + m_2) v_{cm} = m_1 v_1 + m_2 v_2$$

In words, this equation says that the momentum of the center of mass is equal to the sum of the momenta of each of the particles, which is conservation of momentum. If we take the rate of change of this equation, we get

$$(m_1 + m_2) a_{cm} = m_1 a_1 + m_2 a_2$$

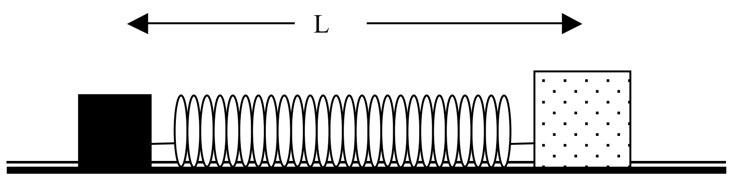
which is an expression of Newton's second law, that is, the net force acting on the center of mass of a system of particles is equal to the sum of the forces acting on the individual particles. Here we have written the scalar form of these equations, since we are assuming all velocities, accelerations, and forces to be directed along the x – axis. As your textbook shows, often we can solve problems from the reference frame of the center of mass.

Example 7

Two masses m and $2m$ are connected by a spring and rest on a horizontal table on which friction can be neglected. The two masses are pulled apart to a length L and released from rest. Describe

(a) the location of the center of mass before the masses are released, and

(b) the motion of the center of mass after the masses are released from rest.



Solution:
(a) The center of mass will be located closer to the greater mass. In fact, the center of mass will be located at a distance of $\frac{1}{3}L$ from *2m* and $\frac{2}{3}L$ from *m*.

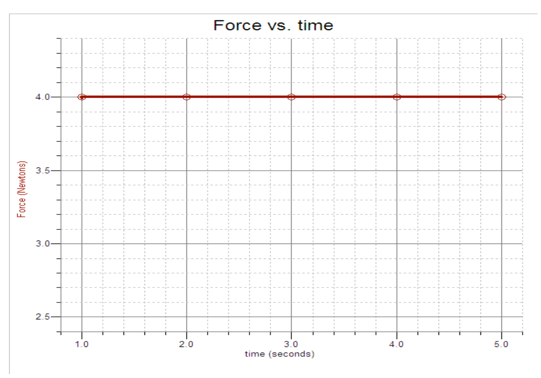
(b) Before the masses are released, the center of mass is at rest. After the masses are released, there are no external forces acting on the center of mass, and therefore the momentum of the center of mass does not change, remaining at rest while the two masses oscillate about it. It also follows that the momentum of block *m* is equal and opposite to block *2m* at all times during the oscillation of the masses.

CHAPTER 7 REVIEW QUESTIONS

For each multiple choice question below, choose the BEST answer.

1. A 0.2-kg hockey puck is sliding along the ice with an initial speed of 12 m/s when a player strikes it with his stick, causing it to reverse its direction and giving it a speed of 23 m/s. The impulse the stick applies to the puck is most nearly
(A) - 2 N s
(B) - 6 N s
(C) - 7 N s
(D) - 70 N s
(E) - 120 N s

Questions 2 – 3: A net force is applied to a block of mass 4 kg according to the *Force vs. time* graph below.



2. The impulse given to the mass between 1 and 5 seconds is most nearly
(A) 20 N s
(B) 16 N s
(C) 12 N s
(D) 10 N s
(E) 4 N s

3. If the mass starts from rest at t = 1 s, the speed of the mass at t = 5 s is most nearly

(A) 20 $\frac{m}{s}$

(B) 16 $\frac{m}{s}$

(C) 12 $\frac{m}{s}$

(D) 8 $\frac{m}{s}$

(E) 4 $\frac{m}{s}$

4. An astronaut floating at rest in space throws a wrench in one direction and subsequently recoils back with a velocity in the opposite direction. Which of the following statements is/are true?

I. The velocity of the wrench is equal and opposite to the velocity of the astronaut.

II. The momentum of the wrench is equal and opposite to the momentum of the astronaut.

III. The impulse applied to the wrench is equal and opposite to the impulse applied to the astronaut.

(A) I only
(B) II only
(C) I and II only
(D) II and III only
(E) I, II, and III

5. A block of mass m slides with a speed v_o on a frictionless surface and collides with another mass M which is initially at rest. The two blocks stick together and move with a speed of $\frac{v_o}{3}$. In terms of m, mass M is most nearly

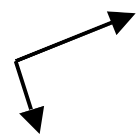
(A) $\frac{m}{4}$

(B) $\frac{m}{3}$

(C) $\frac{m}{2}$

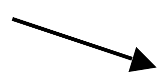
(D) $2m$

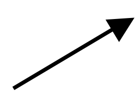
(E) $3m$



6. The vector diagram above represents the momenta of two objects after they collide. One of the objects is initially at rest. Which of the following vectors may represent the initial momentum of the other object before the collision?

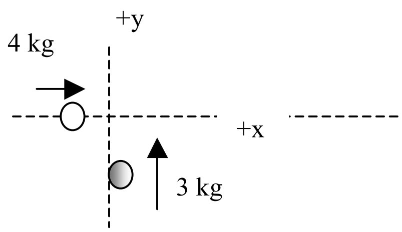
(A) 

(B) 

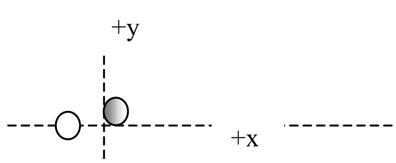
(C) 

(D) 

(E) zero



7. Two objects of mass 4 kg and 3 kg approach each other at a right angle as shown above. The 4-kg mass moves along the +x – axis with an initial speed of 5 m/s, and the 3-kg mass moves in the +y – direction with a speed of 5 m/s. The two masses collide at the origin and stick together. Measured from the +x – axis, the angle of the resulting momentum of the two objects after the collision is most nearly
(A) 30° above the +x – axis
(B) 37° above the +x – axis
(C) 45° above the +x – axis
(D) 53° above the +x – axis
(E) 60° above the +x – axis



Questions 8 – 9: A 0.2-kg billiard ball approaches an identical ball at rest with a speed of 10 m/s along the +x - axis, as shown above. The collision between the balls is perfectly elastic, and after the collision the incident ball moves at an angle of 50° below the x – axis.

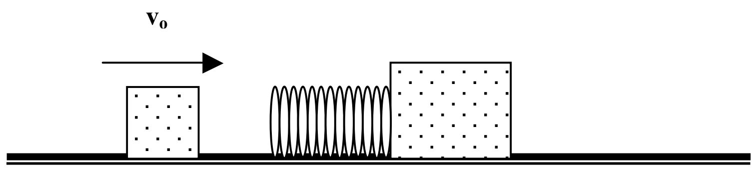
8. The angle at which the target ball moves after the collision above the +x – axis is most nearly
(A) 10°
(B) 40°
(C) 50°
(D) 90°
(E) 140°

9. The total momentum of the two balls after the collision is most nearly
(A) 1 $\frac{kg\ m}{s}$
(B) 2 $\frac{kg\ m}{s}$
(C) 3 $\frac{kg\ m}{s}$
(D) 4 $\frac{kg\ m}{s}$
(E) zero

10. A bullet moving with an initial speed of v_0 strikes and embeds itself in a block of wood which is suspended by a string, causing the bullet and block to rise to a maximum height *h*. Which of the following statements is true of the collision?
(A) The initial kinetic energy of the bullet before the collision is equal to the kinetic energy of the bullet and block immediately after the collision.
(B) The initial kinetic energy of the bullet before the collision is equal to the potential energy of the bullet and block when they reach the maximum height *h*.
(C) The initial momentum of the bullet before the collision is equal to the momentum of the bullet and block at the instant they reach the maximum height *h*.
(D) The initial momentum of the bullet before the collision is equal to the momentum of the bullet immediately after the collision.
(E) The kinetic energy of the bullet and block immediately after the collision is equal to the potential energy of the bullet and block at the instant they reach the maximum height *h*.

Free Response Question

Directions: *Show all work in working the following question. The question is worth 15 points, and the suggested time for answering the question is about 15 minutes. The parts within a question may not have equal weight.*



1. (15 points)
A block of mass *m* is moving on a horizontal frictionless surface with a speed v_o as it approaches a block of mass 2*m* which is at rest and has an ideal spring attached to one side. When the two blocks collide, the spring is completely compressed and the two blocks momentarily move at the same speed, and then separate again, each continuing to move.

(a) Briefly explain why the two blocks have the same speed when the spring is completely compressed.

(b) Determine the speed v_f of the two blocks while the spring is completely compressed.

(c) Determine the kinetic energy of the two blocks as they move together with the same speed.

(d) When the spring expands, the blocks are again separated, and the spring returns its compressed potential energy to kinetic energy in the two blocks. On the axes below, sketch a graph of *kinetic energy vs. time* from the time block *m* approaches block 2*m* until the two blocks are separated after the collision.

(e) Write the equations that could be used to solve for the speed of each block after they have separated. It is not necessary to solve these equations for the two speeds.

ANSWERS AND EXPLANATIONS TO CHAPTER 7 REVIEW QUESTIONS

Multiple Choice

1. A

$$F\Delta t = m\left(v_f - v_0\right) = (0.2\,kg)(-23\frac{m}{s} - 12\frac{m}{s}) = -7\,N\,s$$

2. B

The impulse is equal to the area under the graph from 1 s to 5 s = 16 N s.

3. D

Impulse = $16\,Ns = (4\,kg)(v_f - 0)$ gives $v_f = 4\ \frac{m}{s}$.

4. D

Conservation of momentum indicates that the two momenta are equal and opposite, and since they both experience the same force during the same time interval, the impulses must also be equal and opposite. Since the two masses are different, their velocities would not be the same.

5. D

Conservation of momentum for the inelastic collision gives

$mv_0 = (m+M)\left(\frac{v_0}{3}\right)$, implying that $M = 2m$.

6. B

Conservation of momentum states that the momentum vector before the collision must equal the vector sum of the momenta after the collision. Adding the two vectors tip-to-tail gives a resultant which points in the direction of the vector arrow in answer (B).

7. B

After the collision, there is one mass of 7 kg moving upward and to the right. Measuring the angle θ from the +x – axis, the conservation of momentum equations in the horizontal and vertical directions are

$$\mathbf{p}_{0x} = \mathbf{p}_{fx}$$

$(4\,kg)(5\,kg) = (7\,kg)v_f \cos\theta$ and

$$\mathbf{p}_{0y} = \mathbf{p}_{fy}$$

$$(3\,kg)(5\,kg) = (7\,kg)v_f \sin\theta$$

Dividing the y-component equation by the x-component equation yields

$\frac{15}{20} = \tan\theta$ which implies that $\theta = 37°$ from the +x – axis.

8. B

In a perfectly elastic collision, the angle between the two momentum vectors after the collision must add to 90°. So, 90° – 50° = 40° below the +x – axis.

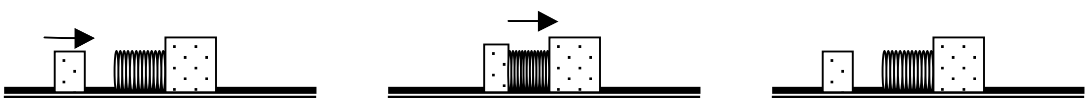
9. B
Conservation of momentum states that the total momentum of the system is equal to the total momentum after the collision. Since the total momentum before the collision is (0.2 kg)(10 m/s) = 2 kg m/s, the total momentum after the collision is also 2 kg m/s.

10. E
During the inelastic collision between the bullet and the block, kinetic energy is lost. But the kinetic energy of the bullet and block immediately after the collision is transformed into potential energy at their maximum height.

Free Response Question Solution

1. We begin by drawing the situation before, during, and after the collision:

Before *During* *After*



(a) 2 points
When the spring is completely compressed, the two blocks are at rest relative to each other and must have the same speed. At this point, it is as if they are stuck together immediately after an inelastic collision.

(b) 3 points
Conservation of momentum:

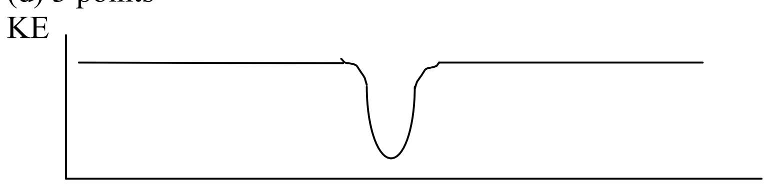
$$mv_0 = (m + 2m)v_f$$

$$v_f = \frac{v_0}{3}$$

(c) 3 points

$$K = \frac{1}{2}(3m)\left(\frac{v_0}{3}\right)^2 = \frac{1}{6}mv_0^{\,2}$$

(d) 3 points



The dip in the graph indicates the time during which the spring is compressed. After the blocks separate, all of the kinetic energy is restored to the system.

(e) 4 points
Since the kinetic energy is the same before the collision and after the blocks have separated, it is as if the blocks have undergone and elastic collision, where both momentum and kinetic energy are conserved. Thus, two equations that could be solved for the speeds after the blocks separate are

$$mv_0 = mv_{f1} + mv_{f2} \text{ and } \frac{1}{2}mv_0^{\,2} = \frac{1}{2}mv_{f1}^{\,2} + \frac{1}{2}mv_{f2}^{\,2}$$