Chapter 1 **Rates of Change**

Chapter 1 Prerequisite Skills

Chapter 1 Prerequisite Skills **Question 1 Page 2**

a)

x	y	First Differences
–4	**9**	
		–4
–3	**5**	
		–2
–2	**3**	
		0
–1	**3**	
		2
0	**5**	
		4
1	**9**	
		6
2	**15**	

Answers may vary. For example:
The first differences are not equal, but they progress by an equal amount.

b) Answers may vary. For example:
Yes. Since the first differences are not equal, the function is not linear. Also, since the first differences increase by the same amount each time, the curve is quadratic.

Chapter 1 Prerequisite Skills **Question 2 Page 2**

a) $$\text{slope} = \frac{3-1}{-2-4} = -\frac{2}{6} \text{ or } -\frac{1}{3}$$

b) $$\text{slope} = \frac{-7-(-1)}{3-0} = -\frac{6}{3} \text{ or } -2$$

c) $$\text{slope} = \frac{1-0}{5-0} = \frac{1}{5}$$

d) $$\text{slope} = \frac{4-4}{0-(-9)} = 0$$

Chapter 1 Prerequisite Skills **Question 3 Page 2**

a) $y = \frac{1}{2}x - \frac{7}{4}$; slope: $\frac{1}{2}$; y-intercept: $-\frac{7}{4}$

b) $y = -\frac{5}{3}x + \frac{1}{3}$; slope: $-\frac{5}{3}$; y-intercept: $\frac{1}{3}$

c) $y = -2x - \frac{10}{9}$; slope: −2; y-intercept: $-\frac{10}{9}$

d) $y = \frac{7}{5}x + \frac{2}{5}$; slope: $\frac{7}{5}$; y-intercept: $\frac{2}{5}$

Chapter 1 Prerequisite Skills **Question 4 Page 2**

a) $y = 5x + 3$

b)
$$\begin{aligned}\text{slope} &= \frac{3-1}{-5-1}\\ &= -\frac{2}{6} \text{ or } -\frac{1}{3}\end{aligned}$$

Substitute the point (1, 1) into $y = -\frac{1}{3}x + b$ to find b.

$$1 = -\frac{1}{3}(1) + b$$

$$b = \frac{4}{3}$$

The equation is $y = -\frac{1}{3}x + \frac{4}{3}$.

c) Substitute the point (4, 7) into $y = -2x + b$ to find b.

$$7 = -2(4) + b$$

$$b = 15$$

The equation is $y = -2x + 15$.

d)
$$\begin{aligned}\text{slope} &= \frac{0-(-1)}{3-2}\\ &= 1\end{aligned}$$

Substitute the point (3, 0) into $y = x + b$ to find b.

$$0 = (3) + b$$

$$b = -3$$

The equation is $y = x - 3$.

Chapter 1 Prerequisite Skills   Question 5 Page 2

a) $(a+b)^2 = a^2 + 2ab + b^2$

b) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

c) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

d) $$\begin{aligned}(a+b)^4 &= (a^2 + 2ab + b^2)^2 \\ &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4\end{aligned}$$

e) $$\begin{aligned}(a-b)^5 &= (a-b)^2(a-b)^3 \\ &= a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5\end{aligned}$$

f) $$\begin{aligned}(a+b)^5 &= (a+b)^2(a+b)^3 \\ &= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5\end{aligned}$$

Chapter 1 Prerequisite Skills   Question 6 Page 2

a) $2x^2 - x - 1 = (2x+1)(x-1)$

b) $$\begin{aligned}6x^2 + 17x + 5 &= 6x^2 + 15x + 2x + 5 \\ &= 3x(2x+5) + (2x+5) \\ &= (2x+5)(3x+1)\end{aligned}$$

c) $x^3 - 1 = (x-1)(x^2 + x + 1)$

d) $$\begin{aligned}2x^4 + 7x^3 + 3x^2 &= x^2(2x^2 + 7x + 3) \\ &= x^2[2x(x+3) + (x+3)] \\ &= x^2(2x+1)(x+3)\end{aligned}$$

e) $$\begin{aligned}x^4 - x^3 - x + 1 &= x^3(x-1) \ -1(x-1) \\ &= (x-1)(x^3 - 1) \\ &= (x-1)(x-1)(x^2 + x + 1) \\ &= (x-1)^2(x^2 + x + 1)\end{aligned}$$

f) $$\begin{aligned}t^3 + 2t^2 - 3t &= t(t^2 + 2t - 3) \\ &= t(t-1)(t+3)\end{aligned}$$

Chapter 1 Prerequisite Skills   Question 7 Page 2

b) $(a-b)(a+b)$

c) $a^3 - b^3$

d) $(a-b)(a^3 + a^2b + ab^2 + b^3)$

e) $(a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

f) $(x+h-x)((x+h)^{n-1} + x(x+h)^{n-2} + x^2(x+h)^{n-3} + \cdots + x^{n-3}(x+h)^2 + x^{n-2}(x+h) + x^{n-1})$
$= h((x+h)^{n-1} + x(x+h)^{n-2} + x^2(x+h)^{n-3} + \cdots + x^{n-3}(x+h)^2 + x^{n-2}(x+h) + x^{n-1})$

Chapter 1 Prerequisite Skills Question 8 Page 2

a)
$$\begin{aligned}\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}+\sqrt{2}\right) &= \left(\sqrt{x}\right)^2 + \sqrt{2x} - \sqrt{2x} - \left(\sqrt{2}\right)^2 \\ &= x-2\end{aligned}$$

b)
$$\begin{aligned}\left(\sqrt{x+1}-\sqrt{x}\right)\left(\sqrt{x+1}+\sqrt{x}\right) &= \left(\sqrt{x+1}\right)^2 + \sqrt{x(x+1)} - \sqrt{x(x+1)} - \left(\sqrt{x}\right)^2 \\ &= x+1-x \\ &= 1\end{aligned}$$

c)
$$\begin{aligned}\left(\sqrt{x+1}-\sqrt{x-1}\right)\left(\sqrt{x+1}+\sqrt{x-1}\right) &= \left(\sqrt{x+1}\right)^2 + \sqrt{(x+1)(x-1)} - \sqrt{(x+1)(x-1)} - \left(\sqrt{x-1}\right)^2 \\ &= x+1-(x-1) \\ &= 2\end{aligned}$$

d)
$$\begin{aligned}\left(\sqrt{3(x+h)}-\sqrt{3x}\right)\left(\sqrt{3(x+h)}+\sqrt{3x}\right) &= \left(\sqrt{3(x+h)}\right)^2 + \sqrt{9x(x+h)} - \sqrt{9x(x+h)} - \left(\sqrt{3x}\right)^2 \\ &= 3(x+h)-3x \\ &= 3h\end{aligned}$$

Chapter 1 Prerequisite Skills Question 9 Page 2

a)
$$\begin{aligned}\frac{1}{2+h}-\frac{1}{2} &= \frac{2-(2+h)}{2(2+h)} \\ &= \frac{-h}{2(2+h)}\end{aligned}$$

b)
$$\begin{aligned}\frac{1}{x+h}-\frac{1}{x} &= \frac{x-(x+h)}{x(x+h)} \\ &= \frac{-h}{x(x+h)}\end{aligned}$$

c) $\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2} = \dfrac{x^2 - (x+h)^2}{x^2(x+h)^2}$

$= \dfrac{-2xh - h^2}{x^2(x+h)^2}$

$= -\dfrac{h(2x+h)}{x^2(x+h)^2}$

d) $\dfrac{\dfrac{1}{x+h} - \dfrac{1}{x}}{h} = \dfrac{\dfrac{x-(x+h)}{x(x+h)}}{h}$

$= \dfrac{-h}{xh(x+h)}$

$= \dfrac{-1}{x(x+h)}$

Chapter 1 Prerequisite Skills Question 10 Page 3

a) $f(-2) = 3(-2) + 12$ $\quad$ $f(3) = 3(3) + 12$

$= 6$ $\quad$ $= 21$

The points are (–2, 6) and (3, 21).

b) $f(-2) = -5(-2)^2 + 2(-2) + 1$ $\quad$ $f(3) = -5(3)^2 + 2(3) + 1$

$= -23$ $\quad$ $= -38$

The points are (–2, –23) and (3, –38).

c) $f(-2) = 2(-2)^3 - 7(-2)^2 + 3$ $\quad$ $f(3) = 2(3)^3 - 7(3)^2 + 3$

$= -41$ $\quad$ $= -6$

The points are (–2, –41) and (3, –6).

Chapter 1 Prerequisite Skills Question 11 Page 3

a) $f(3+h) = 6(3+h) - 2$

$= 16 + 6h$

b) $f(3+h) = 3(3+h)^2 + 5(3+h)$

$= 3(h^2 + 6h + 9) + 15 + 5h$

$= 3h^2 + 23h + 42$

c) $f(3+h) = 2(3+h)^3 - 7(3+h)^2$

$= 2(h^3 + 3h^2x + 3hx^2 + x^3) - 7(h^2 + 6h + 9)$

$= 2h^3 + 11h^2 + 12h - 9$

Chapter 1 Prerequisite Skills **Question 12 Page 3**

a) $$\frac{f(2+h)-f(2)}{h}=\frac{6(2+h)-6(2)}{h}$$
$$=6$$

b) $$\frac{f(2+h)-f(2)}{h}=\frac{2(2+h)^3-2(2)^3}{h}$$
$$=\frac{2(2^3+3(2^2)h+3(2)h^2+h^3)-16}{h}$$
$$=2h^2+12h+24$$

c) $$\frac{f(2+h)-f(2)}{h}=\frac{\frac{1}{(2+h)}-\frac{1}{2}}{h}$$
$$=\frac{\frac{-h}{2(2+h)}}{h}$$
$$=-\frac{1}{2(2+h)}$$

d) $$\frac{f(2+h)-f(2)}{h}=\frac{-\frac{4}{(2+h)}-\left(-\frac{4}{2}\right)}{h}$$
$$=\frac{\frac{-8+4(2+h)}{2(2+h)}}{h}$$
$$=\frac{4}{2(2+h)}$$

Chapter 1 Prerequisite Skills **Question 13 Page 3**

a) $\{x \mid x\in\mathbb{R}\}$

b) $\{x \mid x\neq 8,\ x\in\mathbb{R}\}$

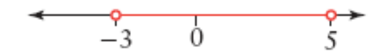
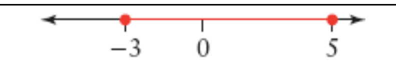
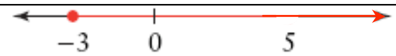
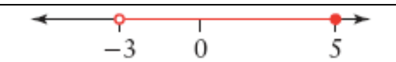
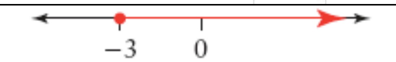
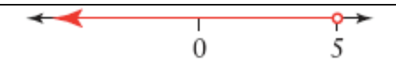
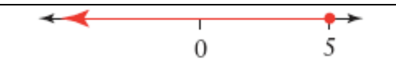
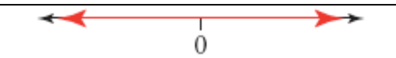
c) $\{x \mid x\in\mathbb{R}\}$

d) $\{x \mid x\geq 0;\ x\in\mathbb{R}\}$

e) $\{x \mid x\neq -3 \text{ and } x\neq 2;\ x\in\mathbb{R}\}$

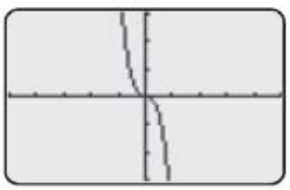
f) $\{x \mid 0\leq x\leq 9,\ x\in\mathbb{R}\}$

Chapter 1 Prerequisite Skills Question 14 Page 3

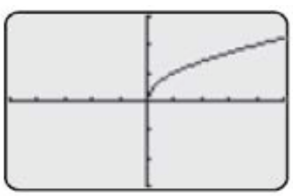
Interval Notation	Inequality	Number Line
(–3, 5)	$-3 < x < 5$	
[–3, 5]	$-3 \le x \le 5$	
[–3, 5)	$-3 \le x < 5$	
(–3, 5]	$-3 < x \le 5$	
(–3, ∞)	$x > -3$	
[–3, ∞)	$x \ge -3$	
(–∞, 5)	$x < 5$	
(–∞, 5]	$x \le 5$	
(–∞, ∞)	$\mathbb{R}$	

Chapter 1 Prerequisite Skills Question 15 Page 3

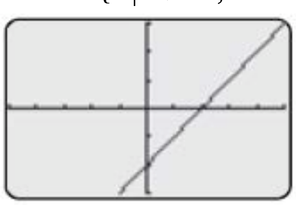
a) domain: $\{x \mid x \in R\}$; range: $\{y \mid y \in \mathbb{R}\}$



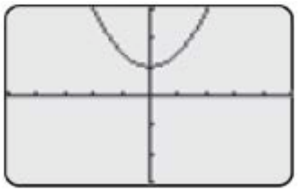
b) domain: $\{x \mid x \ge 0,\ x \in R\}$; range: $\{y \mid y \ge 0,\ y \in \mathbb{R}\}$



c) domain: $\{x \mid x \ne -2,\ x \in R\}$; range: $\{y \mid y \ne -4,\ y \in \mathbb{R}\}$



d) domain: $\{x \mid x \in R\}$; range: $\{y \mid y \geq 1,\ y \in \mathbb{R}\}$



Chapter 1 Section 1 **Rates of Change and the Slope of a Curve**

Chapter 1 Section 1 **Question 1 Page 9**

a) average rate of change $= \frac{6-(-1)}{2-(-4)}$
$= \frac{7}{6}$

b) average rate of change $= \frac{17-(-6.7)}{-5-3.2}$
$= \frac{23.7}{-8.2}$
$= -\frac{237}{82}$

c) average rate of change $= \frac{\frac{3}{4}-\left(-\frac{4}{5}\right)}{-1\frac{1}{2}-\left(\frac{2}{3}\right)}$
$= \frac{\frac{31}{20}}{-\frac{13}{6}}$
$= -\frac{93}{130}$

Chapter 1 Section 1 **Question 2 Page 9**

a) i) average rate of change $= \frac{-3-5}{1-(-3)}$
$= -2$

ii) average rate of change $= \frac{5-5}{3-(-3)}$
$= 0$

iii) average rate of change $= \frac{45-(-3)}{7-1}$
$= 8$

iv) average rate of change $= \dfrac{6-(-5)}{5-(-1)}$

$$= \frac{11}{6}$$

b) Answers may vary. For example: Sketch the graph (scatter plot) to estimate the instantaneous rates by choosing small intervals and using the formula for the average rate of change as in a).

i) –1

ii) 5

iii) 0.5

iv) 10

Chapter 1 Section 1 **Question 3 Page 9**

a) Choosing the points (2, 9) and (1, 7): $\dfrac{7-9}{1-2} = 2$

b) Choosing the points (–2, 2) and (10, 14): $\dfrac{14-2}{10-(-2)} = 1$

c) Choosing the points (1, 0) and (5, 7): $\dfrac{7-0}{5-1} = \dfrac{7}{4}$

Chapter 1 Section 1 **Question 4 Page 9**

Answers may vary. For example:

a) i) They are all zero. B and F are local minima and D is a local maximum.

ii) They have the same magnitude but are opposite in sign. The instantaneous rate of change is negative at A and positive at G.

iii) They are both positive since the function is increasing at both points.

iv) They are both negative since the function is decreasing at both points.

b) i) The instantaneous rate of change is negative at B and positive at C.

ii) They are all negative since the function is decreasing at all three points.

iii) They are both positive since the function is increasing at both points.

iv) The instantaneous rate of change is negative at A and positive at C.

Chapter 1 Section 1 **Question 5 Page 10**

a) The dependent variable is surface area in square centimetres and the independent variable is time in seconds.
The rate of change of surface area over time is expressed in square centimetres per second.

b) i) average rate of change $= \dfrac{324.0 - 10.0}{10 - 0}$

$= 31.4$

The average rate of change during the first 10 s is 31.4 cm^2/s.

ii) average rate of change $= \dfrac{2836.0 - 1266.0}{30 - 20}$

$= 157$

The average rate of change between 20 s and 30 s is 157 cm^2/s.

iii) average rate of change $= \dfrac{2836.0 - 1818.6}{30 - 24}$

$\doteq 169.57$

The average rate of change during the last 6 s is about 169.57 cm^2/s.

c) i) instantaneous rate of change $= \dfrac{60.24 - 10.0}{4 - 0}$

$\doteq 13$

The instantaneous rate of change at $t = 2$ s is an estimated 13 cm^2/s.

ii) instantaneous rate of change $= \dfrac{813.8 - 462.16}{16 - 12}$

$\doteq 88$

The instantaneous rate of change at $t = 14$ s is an estimated 88 cm^2/s.

iii) instantaneous rate of change $= \dfrac{2836.0 - 2132.6}{30 - 26}$

$\doteq 176$

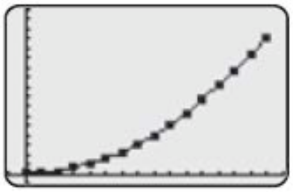
The instantaneous rate of change at $t = 28$ s is an estimated 176 cm^2/s.

d) Answers may vary. For example:

i) 38 cm^2/s

ii) 100 cm^2/s

iii) 163 cm^2/s



e) Answers may vary. For example:

The instantaneous rate of change is increasing rapidly as the time is increasing. The values I found in part d) agree with this statement.

Chapter 1 Section 1 **Question 6 Page 10**

C, since it is the smallest interval.

Chapter 1 Section 1 **Question 7 Page 10**

a) i)

x	*y*	**First Differences**	**Average Rate of Change**
–3	–50		
		38	**38**
–2	–12		
		14	**14**
–1	2		
		2	**2**
0	4		
		2	**2**
1	6		
		14	**14**
2	20		

ii)

x	*y*	**First Differences**	**Average Rate of Change**
–6	–26		
		52	**26**
–4	26		
		–4	**–2**
–2	22		
		–12	**–6**
0	10		
		28	**14**
2	38		
		116	**58**
4	154		

Parts **b)–d)**: Answers may vary. For example:

b) The average rates of change are equal to the first differences in part i), and they are half the value of the first differences in part ii).

c) The values of the calculated first differences and the average rates of change of *y* are the not the same in part ii) because the difference between successive *x*-values is two. In this case, the first differences must be divided by two to calculate the average rate of change.

d) The first differences and average rates of change for a function will be equal if the difference between successive x-values is equal to one.

Chapter 1 Section 1 **Question 8 Page 10**

Explanations may vary. For example:

a) Instantaneous rate of change: The rate of change occurs at the specific instant when the radius is 4 cm.

b) Average rate of change: The rate of change refers to a distance over an interval of 5 h.

c) Instantaneous rate of change: The rate of change occurs at the specific instant when the time is 1 P.M.

d) Average rate of change: The rate of change refers to the stock price over an interval of time of one week.

e) Average rate of change: The rate of change refers to the water level of a lake over an interval from the beginning of March to the end of May.

Chapter 1 Section 1 **Question 9 Page 10**

Answers may vary. For example:

a) The initial temperature of the water was 10° C; After 3 min the water reached its boiling point.

b) The graph shows that the rise in temperature of water is rapid during the first 40 s or so, slowing further until it reaches its boiling point at t = 180 s. After 180 s, the curve is flat, and the instantaneous rate of change is zero after this point.

At t = 60 s, the instantaneous rate of change is about $\frac{87-57}{90-30}$ or 0.5°C/s.

At t = 120 s, the instantaneous rate of change is about $\frac{97-87}{90-30}$ or 0.17°C/s.

At t = 120 s, the instantaneous rate of change is 0°C/s.

Chapter 1 Section 1 **Question 10 Page 11**

a) i) The average rate of change is $\frac{32\ 299\ 496-23\ 143\ 192}{2005-1975}$ or approximately 305 210 people per year.

ii) The average rate of change is $\frac{27\ 697\ 530-24\ 516\ 071}{1990-1980}$ or approximately 318 146 people per year.

iii) From 1975 to 1985: $\frac{25\ 842\ 736-23\ 143\ 192}{1985-1975}$ or approximately 269 954 people per year

From 1985 to 1995: $\frac{29\ 302\ 091-25\ 842\ 736}{1995-1985}$ or approximately 345 936 people per year

From 1995 to 2005: $\frac{32\ 299\ 496-29\ 302\ 091}{2005-1995}$ or approximately 299 741 people per year

Parts **b)–e)**: Answers may vary. For example:

b) While the population has steadily increased, the rate at which it has increased varies.

The estimated instantaneous rate of change in 1983 is:

$\frac{25\ 607\ 651 - 25\ 117\ 442}{1984 - 1982}$ or approximately 245 105 people per year

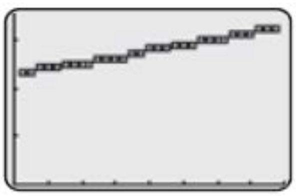
The estimated instantaneous rate of change in 1993 is:

$\frac{28\ 999\ 006 - 28\ 366\ 737}{1994 - 1992}$ or approximately 316 135 people per year

The estimated instantaneous rate of change in 2003 is:

$\frac{31\ 989\ 454 - 31\ 372\ 587}{2004 - 2002}$ or approximately 308 434 people per year

c)



The rate of change of Canada's population has increased steadily since 1975.

d) Canada's population is increasing with respect to time.

e) Q: What do you predict will be Canada's population in the year 2015? Explain.

A: The average rate of change of Canada's population between 1975 and 2005 was 305 210 people per year. Therefore, the estimated population in 2015 is 32 299 496 + (10)(305 210) = 35 351 596.

Q: What do you predict will be the instantaneous rate of change of Canada's population in the year 2015? Explain.

A: The prediction assumes a change equal to the average rate of change over the previous 30 years, which is 305 210 people per year. Therefore, the estimated instantaneous rate of change will be the same figure.

Chapter 1 Section 1 **Question 11 Page 11**

Solutions to the Achievement Checks are shown in the Teacher's Resource.

Chapter 1 Section 1 **Question 12 Page 11**

Answers may vary. For example:

a) The resistance increases as the voltage increases since the slope of the graph increases.

b) Use the points (0.8, 40) and (1.2, 85).

R = instantaneous rate of change of V

$$= \frac{85-40}{1.2-0.8}$$

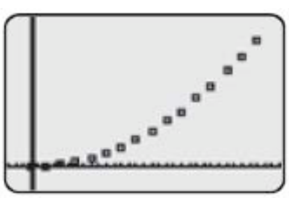
$$\doteq 113$$

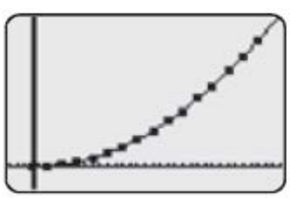
The estimated instantaneous rate of change of V when V = 60 V is 113 V/A or Ω.

Chapter 1 Section 1 **Question 13 Page 12**

a)

Time (min)	**Radius (m)**	**Area (m^2)**
0	0	0
2	4	50.3
4	8	201.1
6	12	452.4
8	16	804.2
10	20	1256.6
12	24	1809.6
14	28	2463.0
16	32	3217.0
18	36	4071.5
20	40	5026.5
22	44	6082.1
24	48	7238.2
26	52	8494.9
28	56	9852.0
30	60	11 309.7





b) i) The average rate of change during the first 4 min is $\frac{201.1-0}{4-0}$ or approximately 50.3 m^2/min.

ii) The average rate of change during the next 10 min is $\frac{2463.0-201.1}{14-4}$ or approximately 226.2 m^2/min.

iii) The average rate of change during the entire 30 min is $\frac{11\,309.7-0}{30-0}$ or approximately 377.0 m^2/min.

c) Answers may vary. For example:

The instantaneous rate of change at t = 5 min is $\frac{452.4-201.1}{6-4}$ or approximately 125.7 m^2/min.

The instantaneous rate of change at t = 25 min is $\frac{8494.9-7238.2}{26-24}$ or approximately 628.4 m^2/min.

d) Answers may vary. For example: The information might be useful to determine when the oil spill will reach shore in order to protect the birds, animals and the environment.

Chapter 1 Section 1 Question 14 Page 12

a) Since x represents t in this case: at t = 0, y = 2; at t = 5 y = 12; at t = 10, y = 22; at t = 15, y = 12; at t = 20, y = 2

From a hand sketch of these points, it is clear that the function can be expressed using either a sine or cosine function.

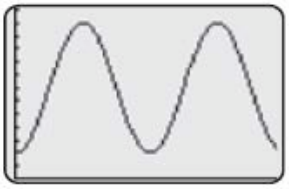
Use a cosine function of the form $y = a\cos(b(x + c)) + d$. Since the function appears upside down compared to cos x, use a negative sign in front of the cosine. The diameter of the windmill is 10 m, so the amplitude of the function is half the diameter, which is 5.
Thus, a = –5. It takes 20 s to complete one full revolution, so the period is 20.

Thus, b is equal to $\frac{2\pi}{20}=\frac{\pi}{10}$. There does not need to be a horizontal shift so c = 0. So far, the function is $y=-5\cos\left(\frac{\pi}{10}t\right)+d$. At time t = 0 s, the height of the ladybug is 2 m, so substitute the point (0, 2) into the function to find d.

$$2=-5\cos\left(\frac{\pi}{10}(0)\right)+d$$

$$d=7$$

Graph $y=-5\cos\left(\frac{\pi}{10}t\right)+7$ using radians or $y=-5\cos(18t)+7$ using degrees.



b) Answers may vary. For example:

No. The rate of change of the ladybug's height will not be constant because the rate of change of the height is affected by the position of the blade.

c) Yes. The rate of change of the height of the blade is constantly changing since the slope of the graph is constantly changing.

Chapter 1 Section 1 **Question 15 Page 12**

Answers will vary. For example:

a) If the wind speed increased the blades would turn faster and the period of the function would decrease. The rate of change of the height of the ladybug would increase since the slope of the graph would be steeper.

If the wind speed decreased the blades would turn more slowly and the period of the function would increase. The rate of change of the height of the ladybug would decrease since the slope of the graph would be less steep.

b) The amplitude of the function representing the motion of the ladybug would be reduced from 10 m to 8 m. Therefore the rate of change of the height would decrease since the graph is less steep.

Chapter 1 Section 1 **Question 16 Page 12**

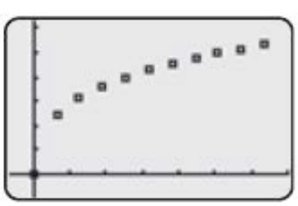
a) i) first 3 s: $\dfrac{3.58-0}{3-0}$ or approximately 1.19 cm/s; last 3 s: $\dfrac{5.35-4.75}{10-7}$ or 0.20 cm/s

ii) at 3 s: $\dfrac{3.94-3.13}{4-2}$ or approximately 0.41 cm/s; at 9 s: $\dfrac{5.35-4.96}{10-8}$ or approximately 0.2 cm/s

b) i) Use the slopes of the secants from (0,0) to (3, 3.58) and from (7, 4.75) to (10, 5.35).

ii) Use the slopes of the secants from (2, 3.13) to (4, 3.94) and from (8, 4.96) to (10, 5.35).

c)



If the cup was a cylinder, the graph would be a straight line.

d)

t (s)	*H* (cm)	*r* (cm)	*V* (cm^3)
0	0	0	**0**
1	2.48	1.24	**4**
2	3.13	1.565	**8**
3	3.58	1.79	**12**
4	3.94	1.97	**16**
5	4.24	2.12	**20**
6	4.51	2.255	**24**
7	4.75	2.375	**28**
8	4.96	2.48	**32**
9	5.16	2.58	**36**
10	5.35	2.675	**40**

The water is being poured at a constant rate.

Chapter 1 Section 1 **Question 17 Page 12**

$$\begin{aligned}\frac{1}{x}+\frac{1}{y}&=\frac{x+y}{xy}\\&=\frac{8}{12}\\&=\frac{2}{3}\end{aligned}$$

Chapter 1 Section 1 **Question 18 Page 12**

$$\begin{aligned}\log_9 3^{\sqrt{g}}&=\log_9 5\\ \sqrt{g}\log_9 3&=\log_9 5\\ \sqrt{g}\left(\frac{1}{2}\right)&=\log_9 5\\ \sqrt{g}&=2\log_9 5\\ \sqrt{g}&=\log_9 5^2\\ \log_9\sqrt{g}&=\log_9(\log_9 5^2)\\ \log_9 g^{\frac{1}{2}}&=\log_9(2\log_9 5)\\ \frac{1}{2}\log_9 g&=\log_9(2\log_9 5)\\ \log_9 g&=2\log_9(2\log_9 5)\end{aligned}$$

Chapter 1 Section 2 **Rates of Change Using Equations**

Chapter 1 Section 2 **Question 1 Page 20**

a) average rate of change: $\frac{4-1}{4-1}=1$

b) average rate of change: $\frac{4^2-1^2}{4-1}=5$

c) average rate of change: $\frac{4^3-1^3}{4-1}=21$

d) average rate of change: $\frac{7-7}{4-1}=0$

Chapter 1 Section 2 **Question 2 Page 20**

a)
$$\begin{aligned}\frac{f(2+h)-f(2)}{h}&=\frac{(2+h)-2}{h}\\&=1\end{aligned}$$

The instantaneous rate of change is 1.

b)
$$\begin{aligned}\frac{f(2+h)-f(2)}{h}&=\frac{(2+h)^2-2^2}{h}\\&=\frac{h^2+4h+4-4}{h}\\&=h+4\end{aligned}$$

The instantaneous rate of change is 4.

c)
$$\begin{aligned}\frac{f(2+h)-f(2)}{h}&=\frac{(2+h)^3-2^3}{h}\\&=\frac{h^3+3(2)h^2+3(2)^2h+(2)^3-8}{h}\\&=h^2+6h+12\end{aligned}$$

The instantaneous rate of change is 12.

d)
$$\begin{aligned}\frac{f(2+h)-f(2)}{h}&=\frac{7-7}{h}\\&=0\end{aligned}$$

The instantaneous rate of change is 0.

Chapter 1 Section 2 **Question 3 Page 20**

$$\frac{f(4+h)-f(4)}{h}$$

Chapter 1 Section 2 **Question 4 Page 20**

$$\frac{(-3+h)^2-(-3)^2}{h}$$

Chapter 1 Section 2 **Question 5 Page 20**

$$\frac{(5+h)^3-(5)^3}{h}$$

Chapter 1 Section 2 **Question 6 Page 20**

$$\frac{(-1+h)^3-(-1)^3}{h}$$

Chapter 1 Section 2 **Question 7 Page 20**

a) True. For $f(x)=4x^3$, $\frac{f(1+h)-f(1)}{h}=\frac{4(1+h)^3-4}{h}$

b) False. The tangent point occurs at $x = 1$.

c) True. For $f(x)=4x^3-4$, $\frac{f(1+h)-f(1)}{h}=\frac{(4(1+h)^3-4)-(4(1)^3-4)}{h}$

$$=\frac{4(1+h)^3-4}{h}$$

d) True. The difference quotient is not defined for $h = 0$.

Chapter 1 Section 2 **Question 8 Page 20**

a) $h = 0.1$: $\frac{(-3+0.1)^2-(-3)^2}{0.1}=-5.9$

$h = 0.01$: $\frac{(-3+0.01)^2-(-3)^2}{0.01}=-5.99$

$h = 0.001$: $\frac{(-3+0.001)^2-(-3)^2}{0.001}=-5.999$

b) $\dfrac{(-3+h)^2-(-3)^2}{h}=\dfrac{9-6h+h^2-9}{h}$
$$=h-6$$

$h = 0.1$: $0.1 - 6 = -5.9$
$h = 0.01$: $0.01 - 6 = -5.99$
$h = 0.001$: $0.001 - 6 = -5.999$

c) Answers may vary. For example: The answers from part a) and part b) are the same. This makes sense since the expression that is used in part b) is a simplified form of the expression in part a). As the interval h is decreased, the calculated result for the difference quotient is getting closer to −6. The final estimate of the instantaneous rate of change is −6.

Chapter 1 Section 2 Question 9 Page 20

$$\frac{(4+h)^4-4^4}{h}=\frac{4^4+4(4)^3h+6(4)^2h^2+4(4)h^3+h^4-4^4}{h}$$
$$=256+96h+16h^2+h^3$$

$h = 0.1$: $256 + 96(0.1) + 16(0.1)^2 + (0.1)^3$
$= 265.761$
$h = 0.01$: $256 + 96(0.01) + 16(0.01)^2 + (0.01)^3$
$= 256.961\ 601$
$h = 0.001$: $256 + 96(0.001) + 16(0.001)^2 + (0.001)^3$
$= 256.096\ 016$

The final estimate of the slope at $x = 4$ is 256.

Chapter 1 Section 2 Question 10 Page 20

a) average rate of change: $\dfrac{(-3)^2+3(-3)-(2^2+3(2))}{-3-2}=\dfrac{-10}{-5}$
$$=2$$

b) average rate of change: $\dfrac{2(-3)-1-(2(2)-1)}{-3-2}=\dfrac{-10}{-5}$
$$=2$$

c) average rate of change: $\dfrac{7(-3)^2-(-3)^4-(7(2)^2-2^4)}{-3-2}=\dfrac{-30}{-5}$
$$=6$$

d) average rate of change: $\dfrac{-3-2(-3)^3-(2-2(2)^3)}{-3-2}=\dfrac{65}{-5}$
$$=-13$$

Chapter 1 Section 2 **Question 11 Page 20**

a) $$\frac{(2+h)^2+3(2+h)-(2^2+3(2))}{h}=\frac{4+4h+h^2+6+3h-4-6}{h}$$
$$=7+h$$

The instantaneous rate of change at $x = 2$ is 7.

b) $$\frac{2(2+h)-1-(2(2)-1)}{h}=\frac{4+2h-1-3}{h}$$
$$=2$$

The instantaneous rate of change at $x = 2$ is 2.

c) $$\frac{7(2+h)^2-(2+h)^4-(7(2)^2-(2)^4)}{h}=\frac{7(h^2+4h+4)-(h^4+4(2)h^3+6(2)^2h^2+4(2)^3h+2^4)-12}{h}$$
$$=-h^3-8h^2-17h-4$$

The instantaneous rate of change at $x = 2$ is −4.

d) $$\frac{(2+h)-2(2+h)^3-(2-2(2)^3)}{h}=\frac{2+h-2(h^3+3h^2(2)+3h(2)^2+2^3)-(-14)}{h}$$
$$=-23-12h-2h^2$$

The instantaneous rate of change at $x = 2$ is −23.

Chapter 1 Section 2 **Question 12 Page 20**

a) i) $$\frac{2(a+h)^2-2a^2}{h}=\frac{2a^2+4ah+2h^2-2a^2}{h}$$
$$=4a+2h$$

$4(-3) + 2(0.01) = -11.98$

ii) $$\frac{(a+h)^3-a^3}{h}=\frac{a^3+3a^2h+3ah^2+h^3-a^3}{h}$$
$$=3a^2+3ah+h^2$$

$3(-3)^2 + 3(-3)(0.01) + (0.01)^2 = 26.9101$

iii) $$\frac{(a+h)^4-a^4}{h}=\frac{a^4+4a^3h+6a^2h^2+4ah^3+h^4-a^4}{h}$$
$$=4a^3+6a^2h+4ah^2+h^3$$

$4(-3)^3 + 6(-3)^2(0.01) + 4(-3)(0.01)^2 + (0.01)^3 = -107.461\,199$

b) Answers may vary. For example:

Each answer represents the estimate of the slope of the tangent line to the function at the point where $x = -3$.

Chapter 1 Section 2 **Question 13 Page 20**

a) **i)** $f(x) = x^2$

ii) $a = 4$

iii) $h = 0.01$

iv) $(a, f(a)) = (4, 16)$

b) **i)** $f(x) = x^3$

ii) $a = 6$

iii) $h = 0.0001$

iv) $(a, f(a)) = (6, 216)$

c) **i)** $f(x) = 3x^4$

ii) $a = -1$

iii) $h = 0.1$

iv) $(a, f(a)) = (-1, 3)$

d) **i)** $f(x) = -2x$

ii) $a = 8$

iii) $h = 0.1$

iv) $(a, f(a)) = (8, -16)$

Chapter 1 Section 2 **Question 14 Page 21**

a) $$\frac{-4.9(1+h)^2 + 15(1+h) + 1 - (-4.9(1)^2 + 15(1) + 1)}{h}$$

b) Answers may vary. For example:
The expression is not valid for $h = 0$. Division by zero is not defined in the real number system.

c) i) $\dfrac{-4.9(1+0.1)^2+15(1+0.1)+1-11.1}{0.1}=4.71$

ii) $\dfrac{-4.9(1+0.01)^2+15(1+0.01)+1-11.1}{0.01}=5.151$

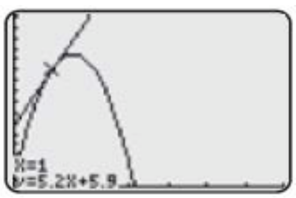
iii) $\dfrac{-4.9(1+0.001)^2+15(1+0.001)+1-11.1}{0.001}=5.1951$

iv) $\dfrac{-4.9(1+0.0001)^2+15(1+0.0001)+1-11.1}{0.0001}=5.199\ 51$

d) Answers may vary. For example:
The instantaneous rate of change of the height of the soccer ball after 1 s is 5.2 m/s because as *h* gets smaller, the rate of change of the height gets closer to 5.2 m/s.

e) Answers may vary. For example:
At time *t* = 1 s, the ball is moving upwards at a speed of 5.2 m/s.

f)



Chapter 1 Section 2 **Question 15 Page 21**

a) first 10 min: $\dfrac{60(25-10)^2-60(25-0)^2}{10-0}=-2400$

The average rate of change of volume during the first 10 min is –2400 L/min.

last 10 min: $\dfrac{60(25-25)^2-60(25-15)^2}{25-15}=-600$

The average rate of change of volume during the last 10 min is –600 L/min.

Both rates of change are negative, but the rate of change of volume during the first 10 min is much more negative since the oil is draining more quickly during this time.

b) **i)** $$\frac{60(25-(5+h))^2-60(25-5)^2}{h}=\frac{60(h^2-40h+400)-60(400)}{h}$$
$$=60h-2400$$

The instantaneous rate of change of volume at t = 5 min is –2400 L/min.

ii) $$\frac{60(25-(10+h))^2-60(25-10)^2}{h}=\frac{60(h^2-30h+225)-60(225)}{h}$$
$$=60h-1800$$

The instantaneous rate of change of volume at t = 10 min is –1800 L/min.

iii) $$\frac{60(25-(15+h))^2-60(25-15)^2}{h}=\frac{60(h^2-20h+100)-60(100)}{h}$$
$$=60h-1200$$

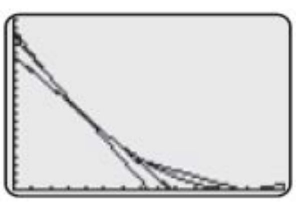
The instantaneous rate of change of volume at t = 15 min is –1200 L/min.

iv) $$\frac{60(25-(20+h))^2-60(25-20)^2}{h}=\frac{60(h^2-10h+25)-60(25)}{h}$$
$$=60h-600$$

The instantaneous rate of change of volume at t = 20 min is –600 L/min.

Answers may vary. For example:
The rate of change in the flow of oil may be slowing because of the shape of the tank and a lessening of pressure.

c)



Chapter 1 Section 2 **Question 16 Page 21**

a) S.A.: $\frac{4\pi(25^2)-4\pi(20^2)}{25-20}$ or approximately 565.5 cm^2/cm

V: $\frac{\frac{4}{3}\pi(25^3)-\frac{4}{3}\pi(20^3)}{25-20}$ or approximatley 6387.9 cm^3/cm

Since the surface area and volume are decreasing, the average rate of change of the surface area is –565.5 cm^2/cm and the average rate of change of the volume is –6387.9 cm^3/cm when r decreases from 25 cm to 20 cm.

b) S.A.: $\dfrac{4\pi(10+h)^2 - 4\pi(10)^2}{h} = \dfrac{40\pi h^2 + 80\pi h + 400\pi - 400\pi}{h}$

$= 40\pi h + 80\pi$

V: $\dfrac{\frac{4}{3}\pi(10+h)^3 - \frac{4}{3}\pi(10)^3}{h} = \dfrac{\frac{4}{3}\pi(h^3 + 3h^2(10) + 3h(10)^2 + 10^3) - \frac{4000}{3}\pi}{h}$

$= \frac{4}{3}\pi h^2 + 40\pi h + 400\pi$

Since the surface area and volume are decreasing, the instantaneous rate of change of the surface area is $-80\pi = -251.3$ cm^2/cm and the instantaneous rate of change of the volume is $-400\pi = -1256.6$ cm^3/cm when $r = 10$ cm.

c) Answers may vary. For example:
The rate of change of the volume is greater than the rate of change of the surface area because it is a larger quantity.

Chapter 1 Section 2 Question 17 Page 21

a) $\dfrac{80 - 5(3)^2 - (80 - 5(0)^2)}{3 - 0} = -15$

The average rate of change of the height of the branch from $t = 0$ s to $t = 3$ s is –15 m/s.
This value represents the average velocity over the time interval [0, 3].

b) $\dfrac{80 - 5(a+h)^2 - (80 - 5(a)^2)}{h} = \dfrac{80 - 5a^2 - 10ah - 5h^2 - 80 + 5a^2}{h}$

$= -10a - 5h$

i) $-10(0.5) - 5(0.001)$ or -5.005 m/s

ii) $-10(1) - 5(0.001)$ or -10.005 m/s

iii) $-10(1.5) - 5(0.001)$ or -15.005 m/s

iv) $-10(2) - 5(0.001)$ or -20.005 m/s

v) $-10(2.5) - 5(0.001)$ or -25.005 m/s

vi) $-10(3) - 5(0.001)$ or -30.005 m/s

c) Answers may vary. For example:
The values found in part b) represent the rate of change of the height of the branch at different moments in time during the time that it is falling.

Chapter 1 Section 2 **Question 18 Page 22**

a)

Tangent Point $(a, f(a))$	**Side Length Increment, *h***	**Second Point** $(a+h, f(a+h))$	**Slope of Secant** $\frac{f(a+h)-f(a)}{h}$
(4, −4)	1	**(5, −10)**	**−6**
(4, −4)	0.1	**(4.1, −4.51)**	**−5.1**
(4, −4)	0.01	**(4.01, −4.0501)**	**−5.01**
(4, −4)	0.001	**(4.001, −4.005 001)**	**−5.001**
(4, −4)	0.0001	**(4.0001, −4.000 500 01)**	**−5.0001**

b) Answers may vary. For example:
The values in the last column indicate that the slope of the tangent line to the function $f(x) = 3x - x^2$ at the point $x = 4$ is −5.

Chapter 1 Section 2 **Question 19 Page 22**

a)
$$\frac{-(12)^2+16(12)+3-(-(4)^2+16(4)+3)}{12-4} = \frac{51-51}{8}$$
$$= 0$$

The average rate of change of the price is $0 per year.

b)
$$\frac{-(a+h)^2+16(a+h)+3-(-a^2+16a+3)}{h} = \frac{-a^2-2ah-h^2+16a+16h+3+a^2-16a-3}{h}$$
$$= -2a-h+16$$

i) h = 0.1: –2(2) – 0.1 + 16 or $11.90 per year
h = 0.01: –2(2) – 0.01 + 16 or $11.99 per year
h = 0.001: –2(2) – 0.001 + 16 or $11.999 per year

The instantaneous rate of change of the price at t = 2 years is $12 per year.

ii) h = 0.1: –2(5) – 0.1 + 16 or $5.90 per year
h = 0.01: –2(5) – 0.01 + 16 or $5.99 per year
h = 0.001: –2(5) – 0.001 + 16 or $5.999 per year

The instantaneous rate of change of the price at t = 5 years is $6 per year.

iii) h = 0.1: –2(10) – 0.1 + 16 or –$4.10 per year
h = 0.01: –2(10) – 0.01 + 16 or –$4.01 per year
h = 0.001: –2(10) – 0.001 + 16 or –$4.011 per year

The instantaneous rate of change of the price at t = 10 years is –$4.00 per year.

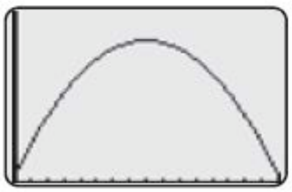
iv) h = 0.1: –2(13) – 0.1 + 16 or –\$10.10 per year
h = 0.01: –2(13) – 0.01 + 16 or –\$10.01 per year
h = 0.001: –2(13) – 0.001 + 16 or –\$10.011 per year

The instantaneous rate of change of the price at t = 13 years is –\$10.00 per year.

v) h = 0.1: –2(15) – 0.1 + 16 or –\$14.10 per year
h = 0.01: –2(15) – 0.01 + 16 or –\$14.01 per year
h = 0.001: –2(15) – 0.001 + 16 or –\$14.011 per year

The instantaneous rate of change of the price at t = 15 years is –\$14.00 per year.

c)



Chapter 1 Section 2 **Question 20 Page 22**

a) The slope of the secant PQ is

$$\frac{(2x-x^2)-1}{x-1}=\frac{-(x^2-2x+1)}{x-1}$$
$$=\frac{-(x-1)^2}{x-1}$$
$$=1-x$$

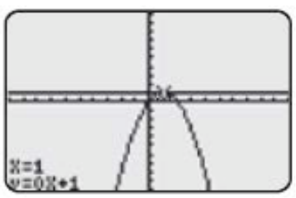
b) x = 1.1: 1 – 1.1 = –0.1
x = 1.01: 1 – 1.01 = −0.01
x = 1.001: 1 – 1.001 = −0.001

x = 0.9: 1 – 0.9 = 0.1
x = 0.99: 1 – 0.99 = 0.01
x = 0.999: 1 – 0.999 = 0.001

c) The slope of the tangent at P is 0.

d) The equation of the tangent is y = 1.

e)



Chapter 1 Section **Question 21 Page 22**

a) The slope of the secant lines at a are:

$a = 5.9$: $\dfrac{\sqrt{6}-\sqrt{5.9}}{6-5.9} \doteq 0.204\ 98$

$a = 5.99$: $\dfrac{\sqrt{6}-\sqrt{5.99}}{6-5.99} \doteq 0.204\ 21$

$a = 5.999$: $\dfrac{\sqrt{6}-\sqrt{5.999}}{6-5.999} \doteq 0.204\ 13$

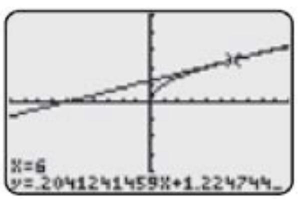
$a = 6.1$: $\dfrac{\sqrt{6}-\sqrt{6.1}}{6-6.1} \doteq 0.203\ 28$

$a = 6.01$: $\dfrac{\sqrt{6}-\sqrt{6.01}}{6-6.01} \doteq 0.204\ 04$

$a = 6.001$: $\dfrac{\sqrt{6}-\sqrt{6.001}}{6-6.001} \doteq 0.204\ 12$

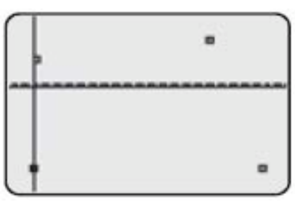
The instantaneous rate of change at $x = 6$ is 0.2.

b)



Chapter 1 Section 2 **Question 22 Page 22**

a)



b) Answers may vary. For example:
From 7:30 A.M. to 7:32 A.M., the rate of change of temperature was 13.50°C/min. From 7:32 A.M. to 9:00 A.M., the rate of change of temperature was 0.06°C/min. From 9:00 A.M. to 9:27 A.M., the rate of change of temperature was −1.19°C/min.

c) average rate of change: $\dfrac{-20-(-20)}{117} = 0$

The average rate of change of temperature over the entire period is 0°C/min.

d) Equation of the line of best fit for the data using quadratic regression:
$y = -0.0127x^2 + 1.383x - 8.098$

e) $\dfrac{f(a+h) - f(a)}{h} = \dfrac{-0.0254ah - 0.0127h^2 + 1.383h}{h}$
$= -0.0254a - 0.0127h + 1.383$

f) To find the instantaneous rates, let $h = 0$ in the above formula.

i) $a = 2$: –0.0254(2) + 1.383 or 1.3322°C/min

ii) $a = 30$: –0.0254(30) + 1.383 or 0.621°C/min

iii) $a = 75$: –0.0254(75) + 1.383 or –0.522°C/min

iv) $a = 105$: –0.0254(105) + 1.383 or −1.284°C/min

g) Answers may vary. For example:
The values from part f) best represent the impact of the Chinook wind. The average rate of change over the entire interval is 0°C/minute. The instantaneous rates of change in part f) approximate the instantaneous rates of change and show the rapid increases and decreases in temperature.

Chapter 1 Section 2 **Question 23 Page 23**

a) first 60 min: $\dfrac{V(60) - V(0)}{60 - 0} = \dfrac{0.1(150 - 60)^2 - 0.1(150 - 0)^2}{60}$
$= -24$

last 30 min: $\dfrac{V(150) - V(120)}{150 - 120} = \dfrac{0.1(150 - 150)^2 - 0.1(150 - 120)^2}{30}$
$= -3$

The average rate of change of the volume of the water during the first 60 min is –24 L/min and during the last 30 min is –3 L/min.

b) Find an expression for the slope of the secant line from t = 75 to t = 75 + h.

$$\begin{aligned}\frac{V(75+h)-V(75)}{h} &= \frac{0.1(150-(75+h))^2-0.1(150-75)^2}{h}\\ &= \frac{0.1(h^2-150h+5625)-562.5}{h}\\ &= -15+0.1h\end{aligned}$$

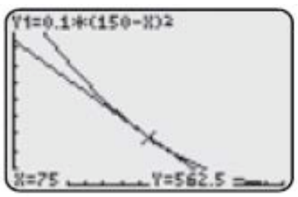
When h is close to zero, the instantaneous rate of change is –15 L/min.

Alternately, find the average rate of change over a small interval containing t = 75 min.

$$\begin{aligned}\frac{V(75.1)-V(74.9)}{75.1-74.9} &= \frac{0.1(150-75.1)^2-0.1(150-74.9)^2}{0.2}\\ &= \frac{561.001-564.001}{0.2}\\ &= -15\end{aligned}$$

The instantaneous rate of change when t = 75 min is –15 L/min.

c)



Chapter 1 Section 2 **Question 24 Page 23**

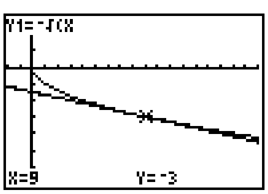
For part ii, use the interval $8.9 \le x \le 9.1$ to estimate the instantaneous rate of change at x = 9.

a) i) average rate of change: $\dfrac{-\sqrt{16}-\left(-\sqrt{9}\right)}{16-9} = -\dfrac{1}{7}$

ii) instantaneous rate of change: $\dfrac{-\sqrt{9.1}-(-\sqrt{8.9})}{9.1-8.9} \doteq -0.1667$

$$\doteq -\frac{1}{6}$$

iii)

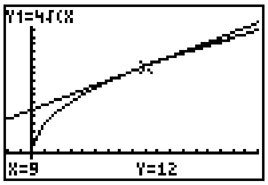


b) i) average rate of change: $\dfrac{4\sqrt{16}-4\sqrt{9}}{16-9}=\dfrac{4}{7}$

ii) instantaneous rate of change: $\dfrac{4\sqrt{9.1}-4\sqrt{8.9}}{9.1-8.9}\doteq 0.6667$

$$\doteq \frac{2}{3}$$

iii)

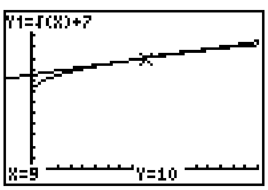


c) i) average rate of change: $\dfrac{\sqrt{16}+7-\left(\sqrt{9}+7\right)}{16-9}=\dfrac{1}{7}$

ii) instantaneous rate of change: $\dfrac{\sqrt{9.1}+7-\left(\sqrt{8.9}+7\right)}{9.1-8.9}\doteq 0.1667$

$$\doteq \frac{1}{6}$$

iii)

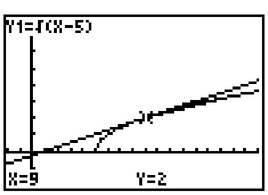


d) i) average rate of change: $\dfrac{\sqrt{16-5}-\sqrt{9-5}}{16-9}=\dfrac{\sqrt{11}-2}{7}$

ii) instantaneous rate of change: $\dfrac{\sqrt{9.1-5}-\sqrt{8.9-5}}{9.1-8.9}\doteq 0.2500$

$$\doteq \frac{1}{4}$$

iii)



Chapter 1 Section 2 **Question 25 Page 23**

For part ii), use the interval $6.9 \le x \le 7.1$ to estimate the instantaneous rate of change at $x = 7$.

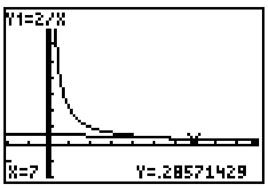
a) i) average rate of change: $\dfrac{\frac{2}{8}-\frac{2}{5}}{8-5} = \dfrac{\frac{10}{40}-\frac{16}{40}}{3}$

$= -\dfrac{1}{20}$

ii) instantaneous rate of change: $\dfrac{\frac{2}{7.1}-\frac{2}{6.9}}{7.1-6.9} = \dfrac{\frac{13.8-14.2}{48.99}}{0.2}$

$\doteq -\dfrac{2}{49}$

iii)



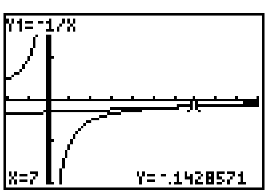
b) i) average rate of change: $\dfrac{-\frac{1}{8}-\left(-\frac{1}{5}\right)}{8-5} = \dfrac{-\frac{5}{40}+\frac{8}{40}}{3}$

$= \dfrac{1}{40}$

ii) instantaneous rate of change: $\dfrac{-\frac{1}{7.1}-\left(-\frac{1}{6.9}\right)}{7.1-6.9} = \dfrac{\frac{-6.9+7.1}{48.99}}{0.2}$

$\doteq \dfrac{1}{49}$

iii)



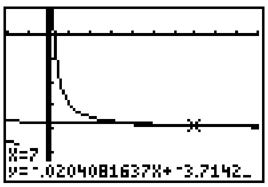
c) i) average rate of change: $\dfrac{\frac{1}{8}-4-\left(\frac{1}{5}-4\right)}{8-5}=\dfrac{\frac{5}{40}-\frac{8}{40}}{3}$

$$=-\frac{1}{40}$$

ii) instantaneous rate of change: $\dfrac{\frac{1}{7.1}-4-\left(\frac{1}{6.9}-4\right)}{7.1-6.9}=\dfrac{\frac{6.9-7.1}{48.99}}{0.2}$

$$\doteq -\frac{1}{49}$$

iii)



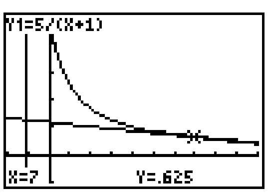
d) i) average rate of change: $\dfrac{\frac{5}{8+1}-\frac{5}{5+1}}{8-5}=\dfrac{\frac{30-45}{54}}{3}$

$$=-\frac{5}{54}$$

ii) instantaneous rate of change: $\dfrac{\frac{5}{7.1+1}-\frac{5}{6.9+1}}{7.1-6.9}=\dfrac{\frac{39.5-40.5}{63.99}}{0.2}$

$$\doteq -\frac{5}{64}$$

iii)



Chapter 1 Section 2 **Question 26 Page 23**

For part ii, use the interval $\frac{\pi}{4} - 0.1 \le \theta \le \frac{\pi}{4} + 0.1$ to estimate the instantaneous rate of change at $\theta = \frac{\pi}{4}$.

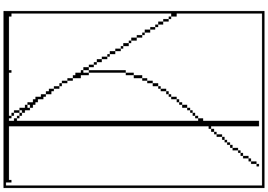
a) i) average rate of change: $\dfrac{\sin\frac{\pi}{3} - \sin\frac{\pi}{6}}{\frac{\pi}{3} - \frac{\pi}{6}} = \dfrac{\frac{\sqrt{3}}{2} - \frac{1}{2}}{\frac{\pi}{6}}$

$$= \frac{3\left(\sqrt{3} - 1\right)}{\pi}$$

ii) instantaneous rate of change: $\dfrac{\sin\left(\frac{\pi}{4} + 0.1\right) - \sin\left(\frac{\pi}{4} - 0.1\right)}{\frac{\pi}{4} + 0.1 - \left(\frac{\pi}{4} - 0.1\right)} \doteq 0.706$

$$\doteq \frac{1}{\sqrt{2}}$$

iii)



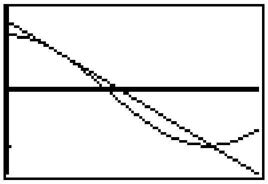
b) i) average rate of change: $\dfrac{\cos\frac{\pi}{3} - \cos\frac{\pi}{6}}{\frac{\pi}{3} - \frac{\pi}{6}} = \dfrac{\frac{1}{2} - \frac{\sqrt{3}}{2}}{\frac{\pi}{6}}$

$$= \frac{3\left(1 - \sqrt{3}\right)}{\pi}$$

ii) instantaneous rate of change: $\dfrac{\cos\left(\frac{\pi}{4} + 0.1\right) - \cos\left(\frac{\pi}{4} - 0.1\right)}{\frac{\pi}{4} + 0.1 - \left(\frac{\pi}{4} - 0.1\right)} \doteq -0.706$

$$\doteq -\frac{1}{\sqrt{2}}$$

iii)



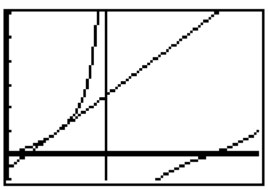
c) i) average rate of change: $\dfrac{\tan\frac{\pi}{3}-\tan\frac{\pi}{6}}{\frac{\pi}{3}-\frac{\pi}{6}} = \dfrac{\sqrt{3}-\frac{1}{\sqrt{3}}}{\frac{\pi}{6}}$

$$= \frac{4\sqrt{3}}{\pi}$$

ii) instantaneous rate of change: $\dfrac{\tan\left(\frac{\pi}{4}+0.1\right)-\tan\left(\frac{\pi}{4}-0.1\right)}{\frac{\pi}{4}+0.1-\left(\frac{\pi}{4}-0.1\right)} \doteq 2.027$

$$\doteq 2$$

iii)



Chapter 1 Section 2 **Question 27 Page 23**

a) *f* is a constant function, so the average rate of change should be 0.

b) Answers may vary. For example:

Using the function $f(x)=4$, the average rate of change of *f* over the interval $a \le x \le b$ is $\dfrac{4-4}{b-a}=0$.

c) $\dfrac{f(b)-f(a)}{b-a} = \dfrac{c-c}{b-a}$

$$= \frac{0}{b-a}$$
$$= 0$$

d) *f* is a constant function, so the instantaneous rate of change should be 0.

e) Answers may vary. For example:
A function $f(x)=c$, for any constant *c*, is a horizontal line. The slope or instantaneous rate of change at any point on the function must be 0.

Chapter 1 Section 2 **Question 28 Page 23**

a) *f* has slope *m*, so the average rate of change should be *m*.

b) Answers may vary. For example:
Using the function $f(x) = 2x + 1$, the average rate of change of *f* over the interval $1 \le x \le 3$ is

$$\begin{aligned}\frac{f(3) - f(1)}{3 - 1} &= \frac{7 - 3}{2} \\ &= 2\end{aligned}$$

c) For any linear function $f(x) = mx + b$ over the interval $c \le x \le d$, the average rate of change is

$$\begin{aligned}\frac{f(d) - f(c)}{d - c} &= \frac{m(d) + b - \left(m(c) + b\right)}{d - c} \\ &= \frac{md - mc}{d - c} \\ &= \frac{m(d - c)}{d - c} \\ &= m\end{aligned}$$

d) *f* has slope *m*, so the instantaneous rate of change should be *m*.

e) Answers may vary. For example:
A function $y = mx + b$, for any interval $a \le x \le b$ is a straight line with slope *m*. The slope or instantaneous rate of change at any point on the function must be *m*.

Chapter 1 Section 2 **Question 29 Page 23**

First, estimate the instantaneous rate of change of *y* at $x = -2$ over the interval $-2.01 \le x \le -1.99$.

$$\begin{aligned}\frac{f(-1.99) - f(-2.01)}{-1.99 - (-2.01)} &\doteq \frac{-31.208 - (-32.808)}{0.02} \\ &\doteq 80\end{aligned}$$

The slope of the tangent is 80, so the slope perpendicular to that is $-\frac{1}{80}$.

When $x = -2$, $y = -32$.

Using the slope $m = -\frac{1}{80}$ and the point (–2, –32), find *b* in the equation of the perpendicular line, $y = mx + b$.

$$\begin{aligned}-32 &= -\frac{1}{80}(-2) + b \\ b &= \frac{-1281}{40}\end{aligned}$$

The equation of the line is $y = -\frac{1}{80}x - \frac{1281}{40}$.

Chapter 1 Section 2 **Question 30 Page 23**

$$
\begin{aligned}
4-|x| &= \sqrt{x^2+4} \\
\left(4-|x|\right)^2 &= x^2+4 \\
16-8|x|+|x||x| &= x^2+4 \\
12-8|x|+x^2 &= x^2 \\
-8|x| &= -12 \\
|x| &= \frac{3}{2} \\
x &= \pm\frac{3}{2}
\end{aligned}
$$

Chapter 1 Section 2 **Question 31 Page 23**

$$
\begin{aligned}
(a-b)^2 &= 135 \\
a^2-2ab+b^2 &= 135 \\
\log_3(a)+\log_3(b) &= \log_3(ab) \\
\log_3(ab) &= 3 \\
ab &= 3^3 \\
ab &= 27 \\
a^2+b^2 &= 135+2(27) \\
&= 189 \\
(a+b)^2 &= a^2+2ab+b^2 \\
&= 189+54 \\
&= 243 \\
\log_3(a+b)^2 &= \log_3 243 \\
&= 5 \\
2\log_3(a+b) &= 5 \\
\log_3(a+b) &= \frac{5}{2} \\
(a+b) &= 3^{\frac{5}{2}} \\
&= \left(\sqrt{3}\right)^5 \\
\log_{\sqrt{3}}(a+b) &= 5
\end{aligned}
$$

Chapter 1 Section 3 Limits

Chapter 1 Section 3 Question 1 Page 29

Explanations may vary. For example:

a) The limit does not exist.
The sequence continues alternating between 1 and –1, so the limit does not approach any value.

b) The limit is 6.
The sequence approaches 6, so the limit of the sequence is 6.

c) The limit is 0.
Every odd term is half the size of the previous odd term so they eventually will approach 0. Also, all the even terms are 0 so the sequence approaches 0.

d) The limit is 3.
The sequence approaches 3, so the limit of the sequence is 3.

e) The limit is –3.
The even terms get closer to –3 and all odd terms are equal to –3. Thus, the sequence approaches –3.

Chapter 1 Section 3 Question 2 Page 30

a) The limit is 4.

b) Explanations may vary. For example:
The limit does not exist.
The odd terms of the sequence begin to grow by the same amount so they will approach infinity. Similarly, the even terms will approach negative infinity, so the sequence does not approach any value.

c) The limit is 3.

Chapter 1 Section 3 Question 3 Page 30

$\lim_{x \to 1} f(x)$ does not exist since $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$.

Chapter 1 Section 3 Question 4 Page 30

$\lim_{x \to -3} f(x) = 1$

Chapter 1 Section 3 Question 5 Page 30

$\lim_{x \to 2} f(x) = 0$

Chapter 1 Section 3 **Question 6 Page 30**

Explanations may vary. For example:

a) False; $\lim_{x\to 3} f(x) \neq f(3)$

b) False; This function is discontinuous, but the right-hand and left-hand limits are equal, so $\lim_{x\to 3} f(x) = 2$

c) True; $\lim_{x\to 3^-} f(x) = 2$

d) False; $\lim_{x\to 3^+} f(x) = 2$

e) False; $f(3) = -1$

Chapter 1 Section 3 **Question 7 Page 30**

Answers may vary. For example:

a) The graph of $y = h(x)$ is continuous at $x = -1$.

b) The graph of $y = h(x)$ is not continuous at $x = -1$ since $\lim_{x\to -1} h(x)$ does not exist.

Chapter 1 Section 3 **Question 8 Page 30**

a) $t_1 = \frac{2}{3}$; $t_2 = \frac{2}{9}$; $t_3 = \frac{2}{27}$; $t_4 = \frac{2}{81}$; $t_5 = \frac{2}{243}$; $t_6 = \frac{2}{729}$

b) Answers may vary. For example:
The sequence is a convergent sequence. The values of the terms in the sequence get smaller and approach 0 as n gets large.

Chapter 1 Section 3 **Question 9 Page 30**

a) $t_1 = 0$; $t_2 = 4$; $t_3 = 18$; $t_4 = 48$; $t_5 = 100$; $t_6 = 180$

b) Answers may vary. For example:
The sequence is a divergent sequence. The values of the terms in the sequence get larger and approach ∞ as n gets large.

Chapter 1 Section 3 **Question 10 Page 30**

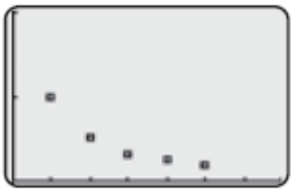
π

Chapter 1 Section 3 **Question 11 Page 30**

$\frac{1}{3}$

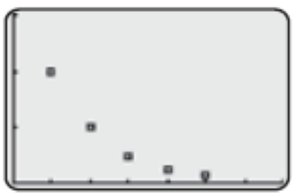
Chapter 1 Section 3 **Question 12 Page 30**

a) i) 0



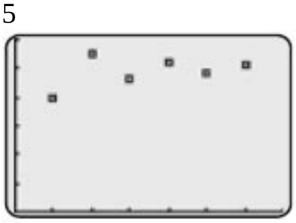
ii) $\lim_{n\to\infty}\left(\frac{1}{n}\right)=0,\ n\in\mathbb{Z}$

b) i) 0



ii) $\lim_{n\to\infty}2^{2-n}=0,\ n\in\mathbb{Z}$

c) i) 5



ii) $\lim_{n\to\infty}\left(5+\frac{(-1)^n}{n}\right)=5,\ n\in\mathbb{Z}$

Chapter 1 Section 3 **Question 13 Page 31**

a) $\{x \mid x \in \mathbb{R}\}$

b) i) $\lim_{x \to -2^-} (-x^3 + 4x) = 0$

ii) $\lim_{x \to -2^+} (-x^3 + 4x) = 0$

iii) $\lim_{x \to -2} (-x^3 + 4x) = 0$

iv) $f(-2) = 0$

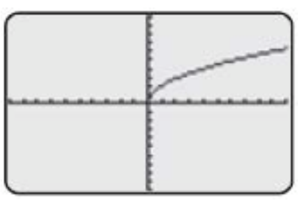
c) Answers may vary. For example:
Since $\lim_{x \to -2} f(x) = f(-2)$, the graph is continuous at $x = -2$.

Chapter 1 Section 3 **Question 14 Page 31**

a) $\lim_{l \to -0^+} 2\sqrt{l} = 0$

b) Answers may vary. For example:
$\lim_{l \to -0^+} 2\sqrt{l}$ exists for $l > 0$; $\lim_{l \to -0^-} 2\sqrt{l}$ does not exist, as the domain of the function $T(l) = 2\sqrt{l}$ is $l \geq 0$.

c)



Answers may vary. For example:
The domain of the function $T(l) = 2\sqrt{l}$ is $l \geq 0$.

Chapter 1 Section 3 **Question 15 Page 31**

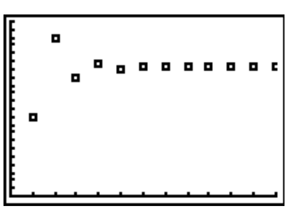
a)

n	f_n	$\frac{f_n}{f_{n-1}}$	**Decimal**
1	1		
2	1	$\frac{1}{1}$	1.000 000
3	2	$\frac{2}{1}$	2.000 000
4	3	$\frac{3}{2}$	1.500 000
5	**5**	$\frac{5}{3}$	1.666 667
6	**8**	$\frac{8}{5}$	1.600 000
7	**13**	$\frac{13}{8}$	1.625 000
8	**21**	$\frac{21}{13}$	1.615 385
9	**34**	$\frac{34}{21}$	1.619 048
10	**55**	$\frac{55}{34}$	1.617 647

b) The ratios approach the Golden Ratio, which is $\frac{1+\sqrt{5}}{2} \doteq 1.618$.

c) $\frac{89}{55} \doteq 1.618\,182$; $\frac{144}{89} \doteq 1.617\,978$; $\frac{233}{144} \doteq 1.618\,056$

d) Answers may vary. For example:
The values of points plotted on the graph approach the value 1.618.



e) $\lim_{n\to\infty}\left(\frac{f_n}{f_{n-1}}\right) = \frac{1+\sqrt{5}}{2}$

Chapter 1 Section 3 **Question 16 Page 32**

a) i)

x	y
3.9	**0.316 23**
3.99	**0.1**
3.999	**0.031 623**
3.9999	**0.01**

$\lim_{x\to 4^-}\sqrt{4-x}=0$

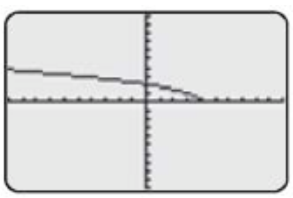
ii)

x	y
4.1	**no value**
4.01	**no value**
4.001	**no value**
4.0001	**no value**

$\lim_{x\to 4^+}\sqrt{4-x}$ does not exist

iii) Since $\lim_{x\to 4^+}\sqrt{4-x}$ does not exist, $\lim_{x\to 4}\sqrt{4-x}$ does not exist.

b) Answers may vary. For example:
The domain of the graph is $x\le 4$, so this shows that the function has no limit at $x = 4$.



Chapter 1 Section 3 **Question 17 Page 32**

a) i)

x	y
–2.1	**no value**
–2.01	**no value**
–2.001	**no value**
–2.0001	**no value**

$\lim_{x\to -2^-}\sqrt{x+2}$ does not exist

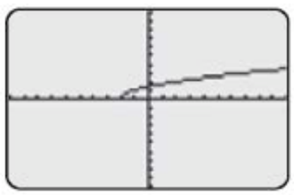
ii)

x	y
–1.9	**0.31623**
–1.99	**0.1**
–1.999	**0.031623**
–1.9999	**0.01**

$\lim_{x \to -2^+} \sqrt{x+2} = 0$

iii) Since $\lim_{x \to -2^-} \sqrt{x+2}$ does not exist, $\lim_{x \to -2} \sqrt{x+2}$ does not exist.

b) Answers may vary. For example:
The domain of the graph is $x \geq -2$, so this shows that the function has no limit at $x = -2$.



Chapter 1 Section 3 **Question 18 Page 32**

a) i) \$2.00

ii) $\$1(1+0.5)^2 = \$2.25$

iii) $\$1(1+0.8333)^{12} = \$2.61$

iv) $\$1(1+0.002\,74)^{365} = \$2.71$

v) $\$1(1+0.000\,001\,9)^{525\,600} = \$2.72$

vi) $\$1(1+0.000\,000\,032)^{315\,360\,0} = \$2.72$

b) Answers may vary. For example:
The compounding periods are examples of the sequence for various values of n: 1, 12, 365, …

c) Answers may vary. For example:

The limit of the sequence is Euler's Number: $\lim_{n \to \infty} \left(1+\frac{1}{n}\right)^n = 2.72$ for $n \in \mathbb{Z}$.

Chapter 1 Section 3 **Question 19 Page 32**

Answers may vary. For example:

Successive values of the sequence for the continued fraction are: 1, 2, $\frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \ldots$

From question 15, this is the Fibonacci sequence and $\lim_{n\to\infty}\left(1+\frac{1}{t_n}\right)=\frac{1+\sqrt{5}}{2}$.

The limit represents the golden ratio or the golden mean.

Chapter 1 Section 3 **Question 20 Page 32**

$$\sqrt{3\sqrt{3}}=\left(3\cdot 3^{\frac{1}{2}}\right)^{\frac{1}{2}}$$
$$=3\frac{1}{4}$$

Successive values of the sequence are:

$\sqrt{3},\ \sqrt{3^{\frac{3}{2}}},\ \sqrt{3^{\frac{7}{8}}},\ \sqrt{3^{\frac{15}{8}}},\ \sqrt{3^{\frac{31}{16}}},\ \ldots\ \sqrt{3^{\frac{2^{n+1}-1}{2^n}}},\ \ldots$

So, the nth term of the sequence is $3^{\frac{2^{n+1}-1}{2^{n+1}}}$.

The limit of the sequence is $\lim_{n\to\infty}\left(3^{\frac{2^{n+1}-1}{2^{n+1}}}\right)=3$.

Chapter 1 Section 3 **Question 21 Page 32**

Let d be the common difference between terms, where the terms are $t_1 = cat,\ t_2 = nut,\ \text{etc}$.

$$t_6 - t_1 = bat - cat$$
$$= 5d$$

Since the last two digits are the same $bat - cat$ must be a multiple of 100. But the digits "n" and "a" occur between *cat* and *bat*, so $bat - cat \geq 300$.

Therefore, since $bat - cat = 5d$ then $d = 60,\ 80,\ 100,\ 120,\ \ldots$

But *nut* and *not* are separated by d, so $d \leq 100$ means $d = 60$ or 80.

The digits A, U, O, C, R must contain all the odd digits or all the even digits.

For example, A = 3, then, for d = 60, (A, U, O, C, R) = (3, 9, 5, 1, 7) and for d = 80, (A, U, O, C, R) = (3, 1, 9, 7, 5).

So the digits A, U, O, C, R must be a permutation of all 5 odd digits or all 5 even digits.

Note that C, N, A, B must be consecutive digits, in increasing order. So A – C must be 2.

Also from the above, d must be 60. (if d = 80, then C- A = 4).

But $d = not - nut$ and $d = art - act$, so O – U = 6 and R – C = 6.

The only solutions are (A, U, O, C, R) = (4, 0, 6, 2, 8) and (A, U, O, C, R) = (5, 1, 7, 3, 9).

In the first case, N = 3 and B = 5

In the second case, N = 4, and B = 6.

Therefore, there are only two sequences that work:

24T, 30T, 36T, 42T, 48T, 54T, 60T
35T, 41T, 47T, 53T, 59T, 65T, 71T

In both cases, the final term of the sequence corresponds to OUT, which is the unique solution.

Chapter 1 Section 3 **Question 22 Page 32**

If (3, *k*) is a point on the curve $x^2y - y^2x = -30$, then

$$\begin{aligned} -30 &= 9k - 3k^2 \\ 0 &= -3k^2 + 9k + 30 \\ 0 &= -3(k^2 - 3k - 10) \\ 0 &= -3(k-5)(k+2) \end{aligned}$$

$k = 5$ or $k = -2$

Chapter 1 Section 4 **Limits and Continuity**

Chapter 1 Section 4 **Question 1 Page 44**

The function has a removable discontinuity at $x = 3$.

Chapter 1 Section 4 **Question 2 Page 44**

The function has an infinite discontinuity at $x = 6$.

Chapter 1 Section 4 **Question 3 Page 44**

a) The function is discontinuous at $x = -2$.

b) The function is discontinuous at $x = -3$ and $x = 1$.

Chapter 1 Section 4 **Question 4 Page 44**

a) Jump discontinuity at $x = -2$: $\lim_{x\to-2^-} f(x) = 1$, $\lim_{x\to-2^+} f(x) = 3$ so $\lim_{x\to-2^-} f(x) \neq \lim_{x\to-2^+} f(x)$.

b) Infinite discontinuity at $x = -6$ and $x = 2$:
$\lim_{x\to-6^-} f(x) = \infty$, $\lim_{x\to-6^+} f(x) = -\infty$; $\lim_{x\to2^-} f(x) = -\infty$, $\lim_{x\to2^+} f(x) = \infty$

c) Removable discontinuity at $x = -2$: $f(-2)$ does not exist

Chapter 1 Section 4 **Question 5 Page 44**

a) $\{x \in \mathbb{R} \mid x \neq -1\}$

b) **i)** $\lim_{x\to-1^+} \dfrac{x^2}{x+1} = \infty$

ii) $\lim_{x\to-1^-} \dfrac{x^2}{x+1} = -\infty$

iii) $\lim_{x\to-1} \dfrac{x^2}{x+1}$ does not exist

iv) $f(-1)$ does not exist

c) Answers may vary. For example:
The graph is discontinuous since it has an infinite discontinuity at $x = -1$. The graph becomes large and negative as x approaches -1 from the left and becomes large and positive as x approaches -1 from the right.

Chapter 1 Section 4 **Question 6 Page 45**

a) $\{x \in \mathbb{R} \mid x \neq 0\}$

b) i) $\lim_{x \to 0^+} \frac{x^2 + 4x - 2}{x^2} = -\infty$

ii) $\lim_{x \to 0^-} \frac{x^2 + 4x - 2}{x^2} = -\infty$

iii) $\lim_{x \to 0} \frac{x^2 + 4x - 2}{x^2} = -\infty$

iv) $f(0)$ does not exist

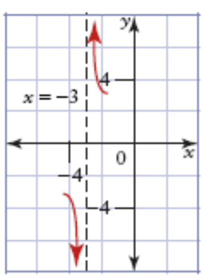
c) Answers may vary. For example:
The graph is discontinuous since it has an infinite discontinuity at $x = 0$. The graph becomes large and negative as x approaches –1 from the left and from the right, but is not continuous since $f(0)$ does not exist.

Chapter 1 Section 4 **Question 7 Page 45**

a) i) There is an infinite discontinuity at $x = -3$.

ii) $\lim_{x \to -3^-} f(x) = -\infty$ and $\lim_{x \to -3^+} f(x) = +\infty$

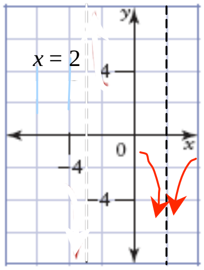
iii)



b) i) There is an infinite discontinuity at $x = 2$.

ii) $\lim_{x \to 2^-} f(x) = +\infty$ and $\lim_{x \to 2^+} f(x) = -\infty$

iii)



Chapter 1 Section 4 **Question 8 Page 45**

a) $\lim_{x\to -2^-} g(x) = 3$

b) $\lim_{x\to -2^+} g(x) = 3$

c) $\lim_{x\to -2} g(x) = 3$

d) $g(-2) = 1$

e) $\lim_{x\to 1^-} g(x) = 0$

f) $\lim_{x\to 1^+} g(x) = 0$

g) $\lim_{x\to 1} g(x) = 0$

h) $g(1) = 0$

Chapter 1 Section 4 **Question 9 Page 45**

a) $\lim_{x\to -1^-} h(x) = 1$

b) $\lim_{x\to -1^+} h(x) = -2$

c) $\lim_{x\to -1} h(x)$ does not exist, since the left- and right-hand limits are not equal.

d) $h(-1) = 1$

e) $\lim_{x\to 3^-} h(x) = 2$

f) $\lim_{x\to 3^+} h(x) = 3$

g) $\lim_{x \to 3} h(x)$ does not exist, since the left- and right-hand limits are not equal.

h) $h(3) = 2$

Chapter 1 Section 4 **Question 10 Page 45**

Refer to page 35 for the list of limit properties.

a) $\lim_{x \to 6} 8 = 8$

Use property 1.

b)
$$\begin{aligned}\lim_{x \to -3} \frac{x^2 - 9}{x + 5} &= \frac{\left(\lim_{x \to -3} x\right)^2 - \lim_{x \to -3} 9}{\lim_{x \to -3} x + \lim_{x \to -3} 5} \\ &= \frac{(-3)^2 - 9}{-3 + 5} \\ &= 0\end{aligned}$$

Use properties 1, 2, 3, 4, 7 and 8.

c)
$$\begin{aligned}\lim_{x \to -5} \frac{6x + 2}{x + 5} &= \frac{6\left(\lim_{x \to -5} x\right) + \lim_{x \to -5} 2}{\lim_{x \to -5} x + \lim_{x \to -5} 5} \\ &= \frac{6(-5) + 2}{-5 + 5} \\ &= \frac{-28}{0}\end{aligned}$$

The limit does not exist. Use properties 1, 2, 3, 5 and 7.

d)
$$\begin{aligned}\lim_{x \to 0} \sqrt[3]{8 - x} &= \sqrt[3]{\lim_{x \to 0} 8 - \lim_{x \to 0} x} \\ &= \sqrt[3]{8 - 0} \\ &= 2\end{aligned}$$

Use properties 1, 2, 4 and 9.

Chapter 1 Section 4 **Question 11 Page 46**

a)
$$\begin{aligned}\lim_{x \to 2} \frac{(2 + x)^2 - 16}{x - 2} &= \lim_{x \to 2} \frac{x^2 + 4x - 12}{x - 2} \\ &= \lim_{x \to 2} \frac{(x - 2)(x + 6)}{x - 2} \\ &= \lim_{x \to 2} x + 6 \\ &= 8\end{aligned}$$

b) $\lim_{x\to 6}\frac{(3-x)^2-9}{x-6}=\lim_{x\to 6}\frac{x^2-6x}{x-6}$

$=\lim_{x\to 6}\frac{x(x-6)}{x-6}$

$=\lim_{x\to 6}x$

$=6$

c) $\lim_{x\to 2}\frac{49-(5+x)^2}{x-2}=\lim_{x\to 2}\frac{-(x^2+10x-24)}{x-2}$

$=\lim_{x\to 2}\frac{-(x-2)(x+12)}{x-2}$

$=\lim_{x\to 2}-(x+12)$

$=-14$

d) $\lim_{x\to 3}\frac{\frac{1}{3}-\frac{1}{x}}{x-3}=\lim_{x\to 3}\frac{\frac{x-3}{3x}}{x-3}$

$=\lim_{x\to 3}\frac{1}{3x}$

$=\frac{1}{9}$

e) $\lim_{x\to -2}\frac{x^4-16}{x+2}=\lim_{x\to -2}\frac{(x^2+4)(x^2-4)}{x+2}$

$=\lim_{x\to -2}\frac{(x^2+4)(x+2)(x-2)}{x+2}$

$=\lim_{x\to -2}(x^2+4)(x-2)$

$=-32$

f) $\lim_{x\to 1}\frac{x^2-1}{x^3-x^2-3x+3}=\lim_{x\to 1}\frac{(x+1)(x-1)}{(x-1)(x^2-3)}$

$=\lim_{x\to 1}\frac{(x+1)}{(x^2-3)}$

$=-1$

Chapter 1 Section 4 **Question 12 Page 46**

a) $\lim_{x\to 0}\frac{\sqrt{9+x}-3}{x}=\lim_{x\to 0}\frac{\left(\sqrt{9+x}-3\right)\left(\sqrt{9+x}+3\right)}{x\left(\sqrt{9+x}+3\right)}$

$=\lim_{x\to 0}\frac{9+x-9}{x\left(\sqrt{9+x}+3\right)}$

$=\lim_{x\to 0}\frac{1}{\sqrt{9+x}+3}$

$=\frac{1}{6}$

b) $\lim_{x\to 25}\frac{5-\sqrt{x}}{x-25}=\lim_{x\to 25}\frac{5-\sqrt{x}}{\left(\sqrt{x}-5\right)\left(\sqrt{x}+5\right)}$

$=\lim_{x\to 25}\frac{-1}{\sqrt{x}+5}$

$=-\frac{1}{10}$

c) $\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=\lim_{x\to 4}\frac{\left(\sqrt{x}-2\right)\left(\sqrt{x}+2\right)}{\sqrt{x}-2}$

$=\lim_{x\to 4}\sqrt{x}+2$

$=4$

d) $\lim_{x\to 0}\frac{\sqrt{1-x}-1}{3x}=\lim_{x\to 0}\frac{\left(\sqrt{1-x}-1\right)\left(\sqrt{1-x}+1\right)}{3x\left(\sqrt{1-x}+1\right)}$

$=\lim_{x\to 0}\frac{1-x-1}{3x\left(\sqrt{1-x}+1\right)}$

$=\lim_{x\to 0}\frac{-1}{3\left(\sqrt{1-x}+1\right)}$

$=-\frac{1}{6}$

e) $\lim_{x\to 0}\frac{\sqrt{3-x}-\sqrt{x+3}}{x}=\lim_{x\to 0}\frac{\left(\sqrt{3-x}-\sqrt{x+3}\right)\left(\sqrt{3-x}+\sqrt{x+3}\right)}{x\left(\sqrt{3-x}+\sqrt{x+3}\right)}$

$$=\lim_{x\to 0}\frac{3-x-(x+3)}{x\left(\sqrt{3-x}+\sqrt{x+3}\right)}$$

$$=\lim_{x\to 0}\frac{-2}{\sqrt{3-x}+\sqrt{x+3}}$$

$$=-\frac{1}{\sqrt{3}}$$

Chapter 1 Section 4 **Question 13 Page 46**

a) $\lim_{x\to -2}\frac{x^2-4}{x+2}=\lim_{x\to -2}\frac{(x-2)(x+2)}{x+2}$

$$=\lim_{x\to -2}x-2$$

$$=-4$$

b) $\lim_{x\to 0}\frac{3x^2-x}{x+5x^2}=\lim_{x\to 0}\frac{x(3x-1)}{x(1+5x)}$

$$=\lim_{x\to 0}\frac{3x-1}{1+5x}$$

$$=-1$$

c) $\lim_{x\to -3}\frac{x^2-9}{x+3}=\lim_{x\to -3}\frac{(x+3)(x-3)}{x+3}$

$$=\lim_{x\to -3}x-3$$

$$=-6$$

d) $\lim_{x\to 0}\frac{-2x}{x^2-4x}=\lim_{x\to 0}\frac{-2x}{x(x-4)}=\frac{1}{9}$

$$=\lim_{x\to 0}\frac{-2}{x-4}$$

$$=\frac{1}{2}$$

e) $\lim_{x\to-5}\frac{x^2+4x-5}{25-x^2}=\lim_{x\to-5}\frac{(x-1)(x+5)}{(5+x)(5-x)}$

$$=\lim_{x\to-5}\frac{x-1}{5-x}$$

$$=\frac{-6}{10}$$

$$=-\frac{3}{5}$$

f) $\lim_{x\to3}\frac{2x^2-5x-3}{x^2-x-6}=\lim_{x\to3}\frac{(x-3)(2x+1)}{(x-3)(x+2)}=-1$

$$=\lim_{x\to3}\frac{2x+1}{x+2}$$

$$=\frac{7}{5}$$

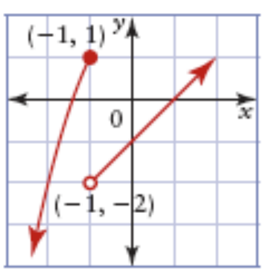
g) $\lim_{x\to-4}\frac{3x^2+11x-4}{x^2+3x-4}=\lim_{x\to-4}\frac{(x+4)(3x-1)}{(x+4)(x-1)}$

$$=\lim_{x\to-4}\frac{3x-1}{x-1}$$

$$=\frac{-13}{-5}$$

$$=\frac{13}{5}$$

Chapter 1 Section 4 **Question 14 Page 46**



a) $\lim_{x\to-1^+} f(x)=-2$

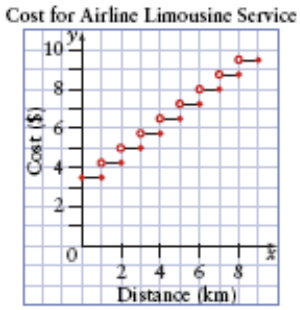
b) $\lim_{x\to-1^-} f(x)=1$

c) $\lim_{x\to-1} f(x)$ does not exist since the left and right limits are not equal.

d) $\lim_{x\to0} f(x)=-1$

Chapter 1 Section 4 **Question 15 Page 46**

a)



b) Answers may vary. For example: The function is a piecewise linear function.

c) The graph is discontinuous at the following distances: 1 km, 2 km, 3 km, 4 km… In general, the graph is discontinuous for all integer values of the distance. The graph has jump discontinuities.

Chapter 1 Section 4 **Question 16 Page 46**

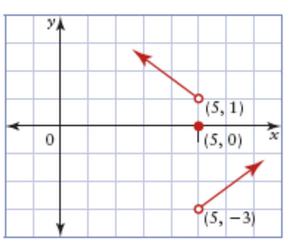
a)

b) Answers may vary. For example:
The function is a piecewise linear function.

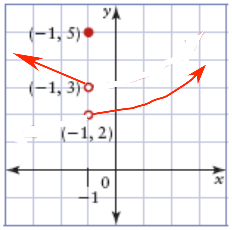
c) The graph is discontinuous for weights of 100 g, 200 g, and 500 g. The graph has jump discontinuities.

Chapter 1 Section 4 **Question 17 Page 46**

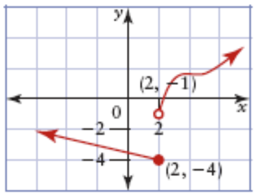
a)



b)

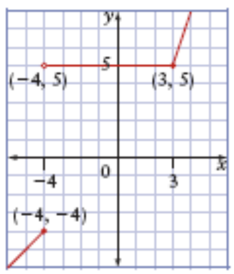


c)



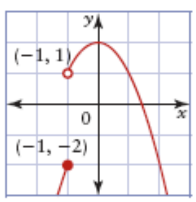
Chapter 1 Section 4 **Question 18 Page 46**

a)



Explanations may vary. For example:
The function is discontinuous at $x = -4$ since $\lim_{x\to -4^-} f(x) \neq \lim_{x\to -4^+} f(x)$.

b)



Explanations may vary. For example:
The function is discontinuous at $x = -1$ since $\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x)$.

Chapter 1 Section 4 **Question 19 Page 47**

a) $\lim_{x\to 0}[4f(x)-1] = 4\lim_{x\to 0} f(x) - 1$
$= 4(-1) - 1$
$= -5$

b) $\lim_{x\to 0}[f(x)]^3 = [\lim_{x\to 0} f(x)]^3$

$$= (-1)^3$$
$$= -1$$

c) $\lim_{x\to 0}\dfrac{[f(x)]^2}{\sqrt{3-f(x)}} = \dfrac{[\lim_{x\to 0} f(x)]^2}{\sqrt{3-\lim_{x\to 0} f(x)}}$

$$= \frac{(-1)^2}{\sqrt{3-(-1)}}$$
$$= \frac{1}{2}$$

Chapter 1 Section 4 **Question 20 Page 47**

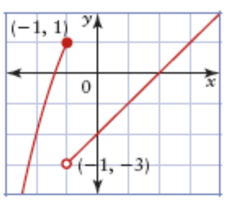
a) Answers may vary. For example:

$\lim_{x\to -1^-} f(x) = a-(-1)^2 = a-1;\ \lim_{x\to -1^+} f(x) = -1-b$

Setting $a - 1 = -1 - b$ gives $a = -b$. However, f is discontinuous when the left and right limits are not equal so when $a \neq -b$.

For instance, the values $a = 2$ and $b = 2$ will make the function discontinuous at $x = -1$.

b) Answers may vary. For example:



c) i) $\lim_{x\to -1^+} f(x) = -1-2$

$$= -3$$

ii) $\lim_{x\to -1^-} f(x) = 2-(-1)^2$

$$= 1$$

iii) $\lim_{x\to -1} f(x)$ does not exist since $\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x)$.

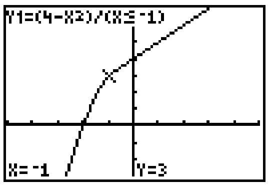
iv) $f(-1) = 2-(-1)^2$

$$= 1$$

Chapter 1 Section 4 **Question 21 Page 47**

a) Answers may vary. For example:
From the previous question, f was found to be discontinuous when $a \neq -b$, thus f is continuous when $a = -b$.
For instance, the values $a = 4$ and $b = -4$ will make the function continuous at $x = -1$.

b)



c) i) $\lim_{x\to -1^+} f(x) = -1 + 4$

$= 3$

ii) $\lim_{x\to -1^-} f(x) = 4 - (-1)^2$

$= 3$

iii) $\lim_{x\to -1} f(x) = 3$

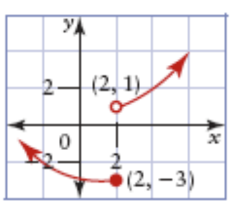
iv) $f(-1) = 4 - (-1)^2$

$= 3$

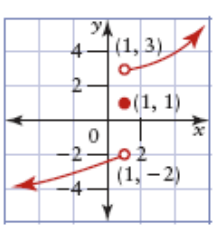
Chapter 1 Section 4 **Question 22 Page 47**

Answers may vary. For example:

a)

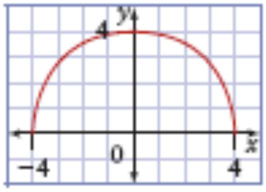


b)



Chapter 1 Section 4 **Question 23 Page 47**

a)



b) $\lim_{x \to 4^-} \sqrt{16 - x^2} = \sqrt{16 - 4^2}$

$= 0$

c) Answers may vary. For example:
The function is not defined in the real number system for x-values that are greater than 4.
Therefore the $\lim_{x \to 4^+} \sqrt{16 - x^2}$ does not exist.

d) $\lim_{x \to 4} \sqrt{16 - x^2}$ does not exist since $\lim_{x \to 4^+} \sqrt{16 - x^2}$ does not exist.

Chapter 1 Section 4 **Question 24 Page 47**

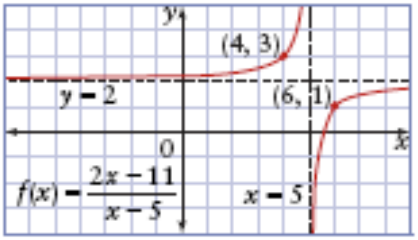
a) $$\lim_{x \to 8} \frac{2 - \sqrt[3]{x}}{8 - x} = \lim_{x \to 8} \frac{2 - \sqrt[3]{x}}{\left(2 - \sqrt[3]{x}\right)\left(4 + 2\sqrt[3]{x} + \sqrt[3]{x^2}\right)}$$
$$= \lim_{x \to 8} \frac{1}{(4 + 2\sqrt[3]{x} + \sqrt[3]{x^2})}$$
$$= \frac{1}{12}$$

b) $$\lim_{x \to 2} \frac{x^5 - 32}{x - 2} = \lim_{x \to 2} \frac{(x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16)}{x - 2}$$
$$= \lim_{x \to 2} x^4 + 2x^3 + 4x^2 + 8x + 16$$
$$= 80$$

c) $$\lim_{x \to 2} \frac{6x^3 - 13x^2 + x + 2}{x - 2} = \lim_{x \to 2} \frac{(x - 2)(2x - 1)(3x + 1)}{x - 2}$$
$$= \lim_{x \to 2} (2x - 1)(3x + 1)$$
$$= 21$$

Chapter 1 Section 4 **Question 25 Page 47**

a)



b) Answers may vary. For example:

A possible equation for this function is $f(x)=\frac{2x-11}{x-5}$. The vertical asymptote is $x = 5$, the horizontal asymptote is $y = 2$, $f(4)=3$, and $f(6)=1$.

The function satisfies all of the required conditions when graphed using a graphing calculator.

Chapter 1 Section 4 **Question 26 Page 47**

$$\begin{aligned}6^x(6-1)&=3^x(3^4-1)\\6^x(5)&=3^x(80)\\6^x&=3^x(16)\\2^x3^x&=3^x(16)\\2^x&=16\\x&=4\end{aligned}$$

Chapter 1 Section 4 **Question 27 Page 47**

$$(6x^2-3x)(2x^2-13x-7)(3x+5)=(2x^2-15x+7)(6x^2+13x+5)$$

$$\text{L.S.}=3x(2x-1)(2x+1)(x-7)(3x+5)$$
$$\text{R.S.}=(2x-1)(2x+1)(x-7)(3x+5)$$

Both sides of the equations are zero when $x=\frac{1}{2}$, $x=-\frac{1}{2}$, $x=7$, and $x=-\frac{5}{3}$.

When the factors are cancelled, the equation remaining is $3x = 1$, so $x=\frac{1}{3}$ is also a solution.

$$\begin{aligned}
\log_{(2\cos x)} 6 + \log_{(2\cos x)} \sin x &= 2 \\
\log_{(2\cos x)} 6\sin x &= 2 \\
(2\cos x)^2 &= 6\sin x \\
4\cos^2 x &= 6\sin x \\
4(1-\sin^2 x) &= 6\sin x \\
4\sin^2 x + 6\sin x - 4 &= 0 \\
(2\sin x - 1)(2\sin x + 4) &= 0 \\
\sin x &= \frac{1}{2} \quad (\sin x \neq -2) \\
x &= \frac{\pi}{6} \text{ or } 30^\circ
\end{aligned}$$

Chapter 1 Section 5 — Introduction to Derivatives

Chapter 1 Section 5 — Question 1 Page 58

a) The derivative is C, since y is a quadratic function and the slope goes from negative to positive.

b) The derivative is A, since y is a cubic function and the slope is not negative anywhere.

c) The derivative is B, since y is a linear function and the slope of the graph is negative and constant.

Chapter 1 Section 5 — Question 2 Page 58

a) $f'(x) = 3x^2$

b) i) $$\begin{aligned} f'(-6) &= 3(-6)^2 \\ &= 108 \end{aligned}$$

ii) $$\begin{aligned} f'(-0.5) &= 3(-0.5)^2 \\ &= 0.75 \end{aligned}$$

iii) $$\begin{aligned} f'\left(\frac{2}{3}\right) &= 3\left(\frac{4}{9}\right) \\ &= \frac{4}{3} \end{aligned}$$

iv) $$\begin{aligned} f'(2) &= 3(2)^2 \\ &= 12 \end{aligned}$$

c) i) When $x = -6$, $f(-6) = -216$

Use the point (–6, –216) and $m = 108$ in the equation of the tangent $y = mx + b$ to find b.

$$\begin{aligned} -216 &= 108(-6) + b \\ b &= 432 \end{aligned}$$

The equation of the tangent is $y = 108x + 432$.

ii) When $x = -0.5$, $f(-0.5) = -0.125$

Use the point (–0.5, –0.125) and $m = 0.75$ in the equation of the tangent $y = mx + b$ to find b.

$$\begin{aligned} -0.125 &= 0.75(-0.5) + b \\ b &= 0.25 \end{aligned}$$

The equation of the tangent is $y = 0.75x + 0.25$.

iii) When $x = \frac{2}{3}$, $f\left(\frac{2}{3}\right) = \frac{8}{27}$

Use the point $\left(\frac{2}{3}, \frac{8}{27}\right)$ and $m = \frac{4}{3}$ in the equation of the tangent $y = mx + b$ to find b.

$$\frac{8}{27} = \frac{4}{3}\left(\frac{2}{3}\right) + b$$

$$b = -\frac{16}{27}$$

The equation of the tangent is $y = \frac{4}{3}x - \frac{16}{27}$.

iv) When $x = 2$, $f(2) = 8$

Use the point (2, 8) and $m = 12$ in the equation of the tangent $y = mx + b$ to find b.

$8 = 12(2) + b$

$b = -16$

The equation of the tangent is $y = 12x - 16$.

Chapter 1 Section 5 **Question 3 Page 58**

Answers may vary. For example:
The function is not differentiable at the given x-value, possibly because it is discontinuous or the function makes an abrupt change.

Chapter 1 Section 5 **Question 4 Page 58**

a) $f'(x) = 1$

b) **i)** $f'(-6) = 1$

ii) $f'(-0.5) = 1$

iii) $f'\left(\frac{2}{3}\right) = 1$

iv) $f'(2) = 1$

Chapter 1 Section 5 **Question 5 Page 58**

a) $f(x) = 3x$

b) $f(x) = x^2$

c) $f(x) = 4x^3$

d) $f(x) = -6x^2$

e) $f(x) = \frac{5}{x}$

f) $f(x) = \sqrt{x}$

Chapter 1 Section 5 **Question 6 Page 58**

a) $f'(x) = -\frac{1}{x^2}$

b) i) $f'(-6) = -\frac{1}{36}$

ii) $f'(-0.5) = -4$

iii) $f'\left(\frac{2}{3}\right) = -\frac{9}{4}$

iv) $f'(2) = -\frac{1}{4}$

c) i) When $x = -6$, $f(-6) = -\frac{1}{6}$.

Use the point $\left(-6, -\frac{1}{6}\right)$ and $m = -\frac{1}{36}$ in the equation of the tangent $y = mx + b$ to find b.

$$-\frac{1}{6} = -\frac{1}{36}(-6) + b$$
$$b = -\frac{1}{3}$$

The equation of the tangent is $y = -\frac{1}{36}x - \frac{1}{3}$.

ii) When $x = -0.5$, $f(-0.5) = -2$.

Use the point (–0.5, –2) and $m = -4$ in the equation of the tangent $y = mx + b$ to find b.

$-2 = -4(-0.5) + b$

$b = -4$

The equation of the tangent is $y = -4x - 4$.

iii) When $x = \frac{2}{3}$, $f\left(\frac{2}{3}\right) = \frac{3}{2}$.

Use the point $\left(\frac{2}{3}, \frac{3}{2}\right)$ and $m = -\frac{9}{4}$ in the equation of the tangent $y = mx + b$ to find b.

$\frac{3}{2} = -\frac{9}{4}\left(\frac{2}{3}\right) + b$

$b = 3$

The equation of the tangent is $y = -\frac{9}{4}x + 3$.

iv) When $x = 2$, $f(2) = \frac{1}{2}$.

Use the point $\left(2, \frac{1}{2}\right)$ and $m = -\frac{1}{4}$ in the equation of the tangent $y = mx + b$ to find b.

$\frac{1}{2} = -\frac{1}{4}(2) + b$

$b = 1$

The equation of the tangent is $y = -\frac{1}{4}x + 1$.

Chapter 1 Section 5 **Question 7 Page 59**

Explanations may vary. For example:

a) $x \in (-\infty, -1)$ or $(-1, \infty)$

The function is differentiable for all values of x except at $x = -1$ since there is an abrupt change in slope at $x = -1$.

b) $x \in (-\infty, \infty)$

The function is differentiable for all values of x since there are no discontinuities or abrupt changes.

c) $x \in (3, \infty)$

The function is differentiable for all values of $x > 3$ since the function is not defined for $x < 3$ and $x = 3$ is an endpoint.

d) $x \in (-\infty, -1)$ or $(-1, \infty)$
The function is differentiable for all values of x except at $x = -1$ since this point is an infinite discontinuity.

Chapter 1 Section 5 Question 8 Page 59

Explanations may vary.

a) Linear; The graph of the derivative of a linear function with non-zero slope is a non-zero constant function, in this case 2.

b) Cubic; The graph of the derivative of a cubic function is a quadratic function.

c) Constant; The graph of the derivative of a constant is zero.

d) Quadratic; The graph of the derivative of a quadratic function is a linear function.

Chapter 1 Section 5 Question 9 Page 59

a)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{(x+h)^2-x^2}{h}\\ &= \lim_{h\to 0}\frac{2xh+h^2}{h}\\ &= \lim_{h\to 0}(2x+h)\\ &= 2x\end{aligned}$$

b) domain for y: $\{x \mid x \in \mathbb{R}\}$; domain for $\frac{dy}{dx}$: $\{x \mid x \in \mathbb{R}\}$

c) Answers may vary. For example: The derivative at a point x represents the slope of the tangent to the original function at x.

Chapter 1 Section 5 Question 10 Page 59

a) i)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{-3(x+h)^2+3x^2}{h}\\ &= \lim_{h\to 0}\frac{-6xh-3h^2}{h}\\ &= \lim_{h\to 0}(-6x-3h)\\ &= -6x\end{aligned}$$

ii) $\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{4(x+h)^2 - 4x^2}{h}$

$= \lim_{h\to 0} \frac{8xh + 4h^2}{h}$

$= \lim_{h\to 0}(8x + 4h)$

$= 8x$

b) Answers may vary. For example:

If $y = ax^2$, where a is a constant, then the derivative of y is $\frac{dy}{dx} = 2ax$.

c) i) $\frac{dy}{dx} = -4x$

ii) $\frac{dy}{dx} = 10x$

d) i) $\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{-2(x+h)^2 + 2x^2}{h}$

$= \lim_{h\to 0} \frac{-4xh - 2h^2}{h}$

$= \lim_{h\to 0}(-4x - 2h)$

$= -4x$

ii) $\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h\to 0} \frac{5(x+h)^2 - 5x^2}{h}$

$= \lim_{h\to 0} \frac{10xh + 5h^2}{h}$

$= \lim_{h\to 0}(10x + 5h)$

$= 10x$

Chapter 1 Section 5 **Question 11 Page 59**

a) $$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{-4-(-4)}{h}\\ &= \lim_{h\to 0}\frac{0}{h}\\ &= 0\end{aligned}$$

b) Answers may vary. For example:
Yes. The slope of a horizontal line is always 0.

c) $$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{c-c}{h}\\ &= \lim_{h\to 0}\frac{0}{h}\\ &= 0\end{aligned}$$

Chapter 1 Section 5 **Question 12 Page 59**

a) $(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

b) i) $$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{2(x^3+3x^2h+3xh^2+h^3)-2x^3}{h}\\ &= \lim_{h\to 0}\frac{6x^2h+6xh^2+2h^3}{h}\\ &= \lim_{h\to 0}(6x^2+6xh+2h^2)\\ &= 6x^2\end{aligned}$$

ii) $$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{-[x^3+3x^2h+3xh^2+h^3]+x^3}{h}\\ &= \lim_{h\to 0}\frac{-3x^2h-3xh^2-h^3}{h}\\ &= \lim_{h\to 0}(-3x^2-3xh-h^2)\\ &= -3x^2\end{aligned}$$

Chapter 1 Section 5 **Question 13 Page 59**

a) Answers may vary. For example:

If $y = ax^3$, where a is a constant, then the derivative of y is $\frac{dy}{dx} = 3ax^2$.

b) i) $\frac{dy}{dx} = -12x^2$

ii) $\frac{dy}{dx} = \frac{3}{2}x^2$

c) i)

$$\begin{aligned}
\frac{dy}{dx} &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{-4[x^3 + 3x^2h + 3xh^2 + h^3] + 4x^3}{h} \\
&= \lim_{h \to 0} \frac{-12x^2h - 12xh^2 - 4h^3}{h} \\
&= \lim_{h \to 0} (-12x^2 - 12xh - 4h^2) \\
&= -12x^2
\end{aligned}$$

ii)

$$\begin{aligned}
\frac{dy}{dx} &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\frac{1}{2}[x^3 + 3x^2h + 3xh^2 + h^3] - \frac{1}{2}x^3}{h} \\
&= \lim_{h \to 0} \frac{\frac{3}{2}x^2h + \frac{3}{2}xh^2 + \frac{1}{2}h^3}{h} \\
&= \lim_{h \to 0} \left(\frac{3}{2}x^2 + \frac{3}{2}xh + \frac{1}{2}h^2 \right) \\
&= \frac{3}{2}x^2
\end{aligned}$$

Chapter 1 Section 5 **Question 14 Page 60**

a) $$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{8(x+h)-8x}{h}$$
$$= \lim_{h\to 0}\frac{8h}{h}$$
$$= \lim_{h\to 0}(8)$$
$$= 8$$

b) $$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{3(x+h)^2-2(x+h)-(3x^2-2x)}{h}$$
$$= \lim_{h\to 0}\frac{6xh+3h^2-2h}{h}$$
$$= \lim_{h\to 0}(6x+3h-2)$$
$$= 6x-2$$

c) $$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{7-(x+h)^2-(7-x^2)}{h}$$
$$= \lim_{h\to 0}\frac{-2xh-h^2}{h}$$
$$= \lim_{h\to 0}(-2x-h)$$
$$= -2x$$

d) $$y = x(4x+5)$$
$$= 4x^2+5x$$
$$\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{4(x+h)^2+5(x+h)-(4x^2+5x)}{h}$$
$$= \lim_{h\to 0}\frac{8xh+4h^2+5h}{h}$$
$$= \lim_{h\to 0}(8x+4h+5)$$
$$= 8x+5$$

e) $\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{[2(x+h)-1]^2-(2x-1)^2}{h}$

$= \lim_{h\to 0}\frac{4x^2+8xh+4h^2-4x-4h+1-4x^2+4x-1}{h}$

$= \lim_{h\to 0}\frac{8xh+4h^2-4h}{h}$

$= \lim_{h\to 0}(8x+4h-4)$

$= 8x-4$

Chapter 1 Section 5 **Question 15 Page 60**

a) $(x + h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

b) i) $\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{(x^4+4hx^3+6h^2x^2+4h^3x+h^4)-x^4}{h}$

$= \lim_{h\to 0}\frac{4hx^3+6h^2x^2+4h^3x+h^4}{h}$

$= \lim_{h\to 0}(4x^3+6hx^2+4h^2x+h^3)$

$= 4x^3$

ii) $\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{2(x^4+4hx^3+6h^2x^2+4h^3x+h^4)-2x^4}{h}$

$= \lim_{h\to 0}\frac{8hx^3+12h^2x^2+8h^3x+2h^4}{h}$

$= \lim_{h\to 0}(8x^3+12hx^2+8h^2x+2h^3)$

$= 8x^3$

iii) $\frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0}\frac{3(x^4+4hx^3+6h^2x^2+4h^3x+h^4)-3x^4}{h}$

$= \lim_{h\to 0}\frac{12hx^3+18h^2x^2+12h^3x+3h^4}{h}$

$= \lim_{h\to 0}(12x^3+18hx^2+12h^2x+3h^3)$

$= 12x^3$

c) Answers may vary. For example:

If $y = ax^4$, where a is a constant, then the derivative of y is $\frac{dy}{dx} = 4ax^3$.

d) i) $\frac{dy}{dx} = -4x^3$

ii) $\frac{dy}{dx} = 2x^3$

e) i)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{-(x^4+4hx^3+6h^2x^2+4h^3x+h^4)+x^4}{h}\\ &= \lim_{h\to 0}\frac{-4hx^3-6h^2x^2-4h^3x-h^4}{h}\\ &= \lim_{h\to 0}(-4x^3-6hx^2-4h^2x-h^3)\\ &= -4x^3\end{aligned}$$

ii)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{\frac{1}{2}(x^4+4hx^3+6h^2x^2+4h^3x+h^4)-\frac{1}{2}x^4}{h}\\ &= \lim_{h\to 0}\frac{\frac{1}{2}(4hx^3+6h^2x^2+4h^3x+h^4)}{h}\\ &= \lim_{h\to 0}(2x^3+3hx^2+2h^2x+\frac{1}{2}h^3)\\ &= 2x^3\end{aligned}$$

Chapter 1 Section 5 **Question 16 Page 60**

a)
$$\begin{aligned}H'(t) &= \lim_{h\to 0}\frac{H(t+h)-H(t)}{h}\\ &= \lim_{h\to 0}\frac{-4.9(t+h)^2+3.5(t+h)+1-(-4.9t^2+3.5t+1)}{h}\\ &= \lim_{h\to 0}\frac{-4.9t^2-9.8ht-4.9h^2+3.5t+3.5h+1+4.9t^2-3.5t-1}{h}\\ &= \lim_{h\to 0}\frac{-9.8ht-4.9h^2+3.5h}{h}\\ &= \lim_{h\to 0}(-9.8t-4.9h+3.5)\\ &= -9.8t+3.5\end{aligned}$$

b) $H'(0.5) = -9.8(0.5) + 3.5$
$= -1.4$

The rate of change of the height of the soccer ball at 0.5 s is –1.4 m/s.

c) The ball stops when $H'(t) = 0$.
$0 = -9.8t + 3.5$
$t \doteq 0.357$

The ball momentarily stops when $t \doteq 0.357$ s.

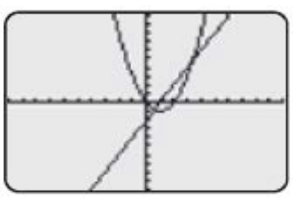
$$H(0.357) = -4.9(0.357)^2 + 3.5(0.357) + 1$$
$$\doteq 1.625$$

The height of the ball at this time is 1.625 m.

Chapter 1 Section 5 **Question 17 Page 60**

a)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\ &= \lim_{h\to 0}\frac{(x+h)^2-2(x+h)-(x^2-2x)}{h}\\ &= \lim_{h\to 0}\frac{2xh+h^2-2h}{h}\\ &= \lim_{h\to 0}(2x+h-2)\\ &= 2x-2\end{aligned}$$

b)



c) The slope of the tangent to the function at $x = -3$ is $\left.\frac{dy}{dx}\right|_{x=-3}$

$$\left.\frac{dy}{dx}\right|_{x=-3} = 2(-3) - 2$$
$$= -8$$

When $x = -3$, $y = 15$.
Use the point (–3, 15) and $m = -8$ in the equation of the tangent $y = mx + b$ to find b.
$15 = -8(-3) + b$
$b = -9$

The equation of the tangent is $y = -8x - 9$.

d)

Chapter 1 Section 5 **Question 18 Page 60**

a) i)
$$\frac{dy}{dx} = \lim_{h\to 0}\frac{\frac{2}{x+h} - \frac{2}{x}}{h}$$
$$= \lim_{h\to 0}\frac{\frac{2x - 2(x+h)}{x(x+h)}}{h}$$
$$= \lim_{h\to 0}\frac{\frac{-2h}{x(x+h)}}{h}$$
$$= \lim_{h\to 0}\frac{-2}{x^2 + xh}$$
$$= -\frac{2}{x^2}$$

ii) $$\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{-1}{x+h} + \frac{1}{x}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{-x+(x+h)}{x(x+h)}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{h}{x(x+h)}}{h}$$
$$= \lim_{h\to 0} \frac{1}{x^2 + xh}$$
$$= \frac{1}{x^2}$$

iii) $$\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{3x-3(x+h)}{x(x+h)}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{-3h}{x(x+h)}}{h}$$
$$= \lim_{h\to 0} \frac{-3}{x^2 + xh}$$
$$= -\frac{3}{x^2}$$

iv) $$\frac{dy}{dx} = \lim_{h\to 0} \frac{-\frac{4}{3(x+h)} + \frac{4}{3x}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{-4x+4(x+h)}{3x(x+h)}}{h}$$
$$= \lim_{h\to 0} \frac{\frac{4h}{3x(x+h)}}{h}$$
$$= \lim_{h\to 0} \frac{4}{3x^2 + 3xh}$$
$$= \frac{4}{3x^2}$$

b) Answers may vary. For example:

If $y = \dfrac{a}{bx}$, where *a* and *b* are constants, then the derivative of *y* is $\dfrac{dy}{dx} = -\dfrac{a}{bx^2}$.

c) i)–iv)

domain of *y*: $\{x \in \mathbb{R} \mid x \neq 0\}$; domain of $\dfrac{dy}{dx}$: $\{x \in \mathbb{R} \mid x \neq 0\}$

Chapter 1 Section 5 **Question 19 Page 60**

a) i) $\dfrac{dy}{dx} = -\dfrac{5}{x^2}$

ii) $\dfrac{dy}{dx} = \dfrac{3}{5x^2}$

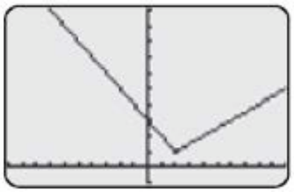
b) i)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{\frac{5}{x+h}-\frac{5}{x}}{h}\\ &= \lim_{h\to 0}\frac{\frac{5x-5(x+h)}{x(x+h)}}{h}\\ &= \lim_{h\to 0}\frac{\frac{-5h}{x(x+h)}}{h}\\ &= \lim_{h\to 0}\frac{-5}{x^2+xh}\\ &= -\frac{5}{x^2}\end{aligned}$$

ii)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{\frac{-3}{5(x+h)}+\frac{3}{5x}}{h}\\ &= \lim_{h\to 0}\frac{\frac{-3x+3(x+h)}{5x(x+h)}}{h}\\ &= \lim_{h\to 0}\frac{\frac{3h}{5x(x+h)}}{h}\\ &= \lim_{h\to 0}\frac{3}{5x^2+5xh}\\ &= \frac{3}{5x^2}\end{aligned}$$

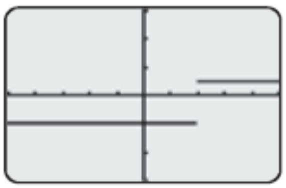
Chapter 1 Section 5 **Question 20 Page 60**

Answers may vary. For example:

a) A piecewise function defined by $y = -x + 3$ if $x \le 2$ and $y = 0.5x$ if $x > 2$



b)



c) For $y = -x + 3$ if $x \le 2$:

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \to 0} \frac{-(x+h)+3+x-3}{h} \\ &= \lim_{h \to 0} \frac{-h}{h} \\ &= -1\end{aligned}$$

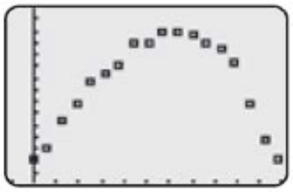
For $y = 0.5x$ if $x > 2$:

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \to 0} \frac{0.5(x+h)-0.5x}{h} \\ &= \lim_{h \to 0} \frac{0.5h}{h} \\ &= 0.5\end{aligned}$$

The derivatives of y as x approaches 2 from the left and right sides are not equal, so y is not differentiable at $x = 2$.

Chapter 1 Section 5 **Question 21 Page 60**

a) Answers may vary.



b) Answers may vary. For example:
Use the QuadReg feature on the graphing calculator, which gives:
$y = -1499x^2 + 26\,808x + 356\,532$

c)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{-1499(x+h)^2 + 26\,808(x+h) + 356\,532 - (-1499x^2 + 26\,808x + 356\,532)}{h}\\ &= \lim_{h\to 0}\frac{-2998xh + 26\,808h - 1499h^2}{h}\\ &= \lim_{h\to 0}(-2998x + 26\,808 - 1499h)\\ &= -2998x + 26\,808\end{aligned}$$

d) i)
$$\begin{aligned}\left.\frac{dy}{dx}\right|_{x=3} &= -2998(3) + 26\,808\\ &= 17\ 814\end{aligned}$$

The instantaneous rate of change after 3 years is 17 814 births per year.

ii)
$$\begin{aligned}\left.\frac{dy}{dx}\right|_{x=7} &= -2998(7) + 26\,808\\ &= 5822\end{aligned}$$

The instantaneous rate of change after 7 years is 5822 births per year.

iii)
$$\begin{aligned}\left.\frac{dy}{dx}\right|_{x=10} &= -2998(10) + 26\,808\\ &= -3172\end{aligned}$$
The instantaneous rate of change after 10 years is –3172 births per year.

iv)
$$\begin{aligned}\left.\frac{dy}{dx}\right|_{x=13} &= -2998(13) + 26\,808\\ &= -12\ 166\end{aligned}$$
The instantaneous rate of change after 13 years is –12 166 births per year.

v) $\left.\dfrac{dy}{dx}\right|_{x=16} = -2998(16) + 26\,808$

$= -21\,160$

The instantaneous rate of change after 16 years is –21 160 births per year.

e) Answers may vary. For example:
Births increased steadily from 1950 until 1959 and started to decline after that as the baby boomers got older.

f)

g) Answers will vary. For example: The equation of the tangent for any given year would give the rate of change of the birth rate (i.e., the slope).

Chapter 1 Section 5 Question 22 Page 61

Solutions to the Achievement Checks are shown in the Teacher's Resource.

Chapter 1 Section 5 Question 23 Page 61

a) i) $\dfrac{dy}{dx} = 2x + 3$

ii) $\dfrac{dy}{dx} = 1 - 6x^2$

iii) $\dfrac{dy}{dx} = 8x^3 - 1$

b) i)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - (x^2 + 3x)}{h} \\ &= \lim_{h \to 0} \frac{2hx + h^2 + 3h}{h} \\ &= \lim_{h \to 0}(2x + h + 3) \\ &= 2x + 3\end{aligned}$$

ii) $\frac{dy}{dx} = \lim_{h\to 0} \frac{(x+h) - 2(x+h)^3 - (x - 2x^3)}{h}$

$= \lim_{h\to 0} \frac{x + h - 2(x^3 + 3hx^2 + 3h^2x + h^3) - x + 2x^3}{h}$

$= \lim_{h\to 0} \frac{h - 6hx^2 - 6h^2x - 2h^3}{h}$

$= \lim_{h\to 0}(1 - 6x^2 - 6hx - 2h^2)$

$= 1 - 6x^2$

iii) $\frac{dy}{dx} = \lim_{h\to 0} \frac{2(x+h)^4 - (x+h) + 5 - (2x^4 - x + 5)}{h}$

$= \lim_{h\to 0} \frac{2(x^4 + 4hx^3 + 6h^2x^2 + 4h^3x + h^4) - x - h + 5 - 2x^4 + x - 5}{h}$

$= \lim_{h\to 0} \frac{8hx^3 + 12h^2x^2 + 8h^3x + 2h^4 - h}{h}$

$= \lim_{h\to 0}(8x^3 + 12hx^2 + 8h^2x + 2h^3 - 1)$

$= 8x^3 - 1$

Chapter 1 Section 5 **Question 24 Page 61**

a) i) $\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

$= \lim_{h\to 0} \frac{\frac{x^2 - (x^2 + 2xh + h^2)}{x^2(x+h)^2}}{h}$

$= \lim_{h\to 0} \frac{\frac{-2xh - h^2}{x^2(x+h)^2}}{h}$

$= \lim_{h\to 0} \frac{-2x - h}{x^2(x+h)^2}$

$= \frac{-2x}{x^2x^2}$

$= -\frac{2}{x^3}$ for $x \neq 0$

ii) $\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$

$= \lim_{h\to 0} \frac{\frac{x^3 - (x^3 + 3x^2h + 3xh^2 + h^3)}{x^3(x+h)^3}}{h}$

$= \lim_{h\to 0} \frac{\frac{-3x^2h - 3xh^2 - h^3}{x^3(x+h)^3}}{h}$

$= \lim_{h\to 0} \frac{-3x^2 - 3xh - h^2}{x^3(x+h)^3}$

$= \frac{-3x^2}{x^3x^3}$

$= -\frac{3}{x^4}$ for $x \neq 0$

iii) $\frac{dy}{dx} = \lim_{h\to 0} \frac{\frac{1}{(x+h)^4} - \frac{1}{x^4}}{h}$

$= \lim_{h\to 0} \frac{\frac{x^4 - (x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4)}{x^4(x+h)^4}}{h}$

$= \lim_{h\to 0} \frac{\frac{-4x^3h - 6x^2h^2 - 4xh^3 - h^4}{x^4(x+h)^4}}{h}$

$= \lim_{h\to 0} \frac{-4x^3 - 6x^2h - 4xh^2 - h^3}{x^4(x+h)^4}$

$= \frac{-4x^3}{x^4x^4}$

$= -\frac{4}{x^5}$ for $x \neq 0$

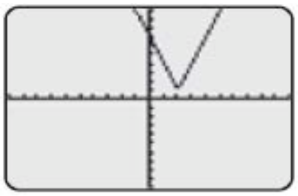
b) i)–iii)

domain of y: $\{x \mid x \neq 0, x \in \mathbb{R}\}$; domain of $\frac{dy}{dx}$: $\{x \mid x \neq 0, x \in \mathbb{R}\}$

c) Answers may vary. For example:
If the degree of the function is –*n*, then the degree of the derivative function is –*n* – 1.

Chapter 1 Section 5 **Question 25 Page 61**

a)



The function is non-differentiable at $x = 2$.

b) Answers may vary. For example:

If $x \le 2$, then $|x - 2| = -(x - 2)$ so $y = -3(x - 2) + 1$

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{-3[(x+h)-2]+1-[-3(x-2)+1]}{h}\\ &= \lim_{h\to 0}\frac{-3x-3h+6+1+3x-6-1}{h}\\ &= \lim_{h\to 0}\frac{-3h}{h}\\ &= -3\end{aligned}$$

If $x > 2$, then $|x - 2| = x - 2$ so $y = 3(x - 2) + 1$

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{3[(x+h)-2]+1-[3(x-2)+1]}{h}\\ &= \lim_{h\to 0}\frac{3x+3h-6+1-3x+6-1}{h}\\ &= \lim_{h\to 0}\frac{3h}{h}\\ &= 3\end{aligned}$$

Since the derivatives of y as x approaches 2 from the left and right sides are not equal, y is non-differentiable at $x = 2$.

Chapter 1 Section 5 **Question 26 Page 62**

a) i) $\frac{dy}{dx} = 0$

ii) $\frac{dy}{dx} = 1$

iii) $\frac{dy}{dx} = 2x$

iv) $\frac{dy}{dx} = 3x^2$

v) $\dfrac{dy}{dx} = 4x^3$

b) Answers may vary. For example: The derivatives are equal to the value in the exponent multiplied by the variable, with the variable to the power of one less than the exponent.

c) i) $\dfrac{dy}{dx} = 5x^4$

ii) $\dfrac{dy}{dx} = 6x^5$

d) i)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{(x+h)^5 - x^5}{h}\\ &= \lim_{h\to 0}\frac{x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5 - x^5}{h}\\ &= \lim_{h\to 0}\left(5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4\right)\\ &= 5x^4\end{aligned}$$

ii)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{(x+h)^6 - x^6}{h}\\ &= \lim_{h\to 0}\frac{x^6 + 6x^5h + 15x^4h^2 + 20x^3h^3 + 15x^2h^4 + 6xh^5 - x^6}{h}\\ &= \lim_{h\to 0}\left(6x^5 + 15x^4h + 20x^3h^2 + 15x^2h^3 + 6xh^4\right)\\ &= 6x^5\end{aligned}$$

e) $\dfrac{dy}{dx} = nx^{n-1}$ for $n \in \mathbb{N}$

f) Answers may vary. For example:
Consider $y = x^8$.

Then $\dfrac{dy}{dx}$ should be equal to $8x^{8-1} = 8x^7$.

Use Pascal's triangle in the first principles definition in order to find the derivative.

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{(x+h)^8 - x^8}{h}\\ &= \lim_{h\to 0}\frac{x^8 + 8x^7h + 28x^6h^2 + 56x^5h^3 + 70x^4h^4 + 56x^3h^5 + 28x^2h^6 + 8xh^7 + h^8 - x^8}{h}\\ &= \lim_{h\to 0}(8x^7 + 28x^6h + 56x^5h^2 + 70x^4h^3 + 56x^3h^4 + 28x^2h^5 + 8xh^6 + h^7)\\ &= 8x^7\end{aligned}$$

a) $$\begin{aligned} f'(a) &= \lim_{x\to a}\frac{f(x)-f(a)}{x-a} \\ &= \lim_{x\to a}\frac{x^2-a^2}{x-a} \\ &= \lim_{x\to a}\frac{(x+a)(x-a)}{x-a} \\ &= \lim_{x\to a}(x+a) \\ &= 2a \end{aligned}$$

Therefore, $\frac{dy}{dx} = 2x$.

It is necessary to factor using the difference of two squares in order to reduce the expression and find the limit.

b) i) $$\begin{aligned} f'(a) &= \lim_{x\to a}\frac{f(x)-f(a)}{x-a} \\ &= \lim_{x\to a}\frac{x^3-a^3}{x-a} \\ &= \lim_{x\to a}\frac{(x^2+ax+a^2)(x-a)}{x-a} \\ &= \lim_{x\to a}(x^2+ax+a^2) \\ &= 3a^2 \end{aligned}$$

Therefore, $\frac{dy}{dx} = 3x^2$.

ii) $$\begin{aligned} f'(a) &= \lim_{x\to a}\frac{f(x)-f(a)}{x-a} \\ &= \lim_{x\to a}\frac{x^4-a^4}{x-a} \\ &= \lim_{x\to a}\frac{(x+a)(x-a)(x^2+a^2)}{x-a} \\ &= \lim_{x\to a}(x+a)(x^2+a^2) \\ &= \lim_{x\to a}(x^3+x^2a+xa^2+a^3) \\ &= 4a^3 \end{aligned}$$

Therefore, $\frac{dy}{dx} = 4x^3$.

iii) $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$$= \lim_{x \to a} \frac{x^5 - a^5}{x - a}$$
$$= \lim_{x \to a} \frac{(x - a)(x^4 + x^3 a + x^2 a^2 + x a^3 + a^4)}{x - a}$$
$$= \lim_{x \to a} (x^4 + x^3 a + x^2 a^2 + x a^3 + a^4)$$
$$= 5a^4$$

Therefore, $\frac{dy}{dx} = 5x^4$.

c) Answers may vary. For example:It is easier to factor than to expand.

Chapter 1 Section 5 **Question 28 Page 62**

a) $f'(x) = \lim_{h \to 0} \frac{\frac{(x+h)+2}{(x+h)-1} - \frac{x+2}{x-1}}{h}$

$$= \lim_{h \to 0} \frac{\frac{(x+h+2)(x-1) - (x+h-1)(x+2)}{(x+h-1)(x-1)}}{h}$$
$$= \lim_{h \to 0} \frac{\frac{(x^2 + hx + x - h - 2) - (x^2 + hx + x + 2h - 2)}{(x+h-1)(x-1)}}{h}$$
$$= \lim_{h \to 0} \frac{\frac{-3h}{(x+h-1)(x-1)}}{h}$$
$$= \lim_{h \to 0} \frac{-3}{(x+h-1)(x-1)}$$
$$= -\frac{3}{(x-1)^2} \quad \text{for } x \neq 1$$

b) $f'(x) = \lim_{h \to 0} \frac{\frac{3(x+h)-1}{(x+h)+4} - \frac{3x-1}{x+4}}{h}$

$$= \lim_{h \to 0} \frac{\frac{(3x+3h-1)(x+4)-(x+h+4)(3x-1)}{(x+h+4)(x+4)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(3x^2+3hx+11x+12h-4)-(3x^2+3hx+11x-h-4)}{(x+h+4)(x+4)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{13h}{(x+h+4)(x+4)}}{h}$$

$$= \lim_{h \to 0} \frac{13}{(x+h+4)(x+4)}$$

$$= \frac{13}{(x+4)^2} \quad \text{for } x \neq -4$$

Chapter 1 Section 5 **Question 29 Page 62**

a) $\frac{dy}{dx} = \lim_{h \to 0} \frac{\sqrt{(x+h)+1} - \sqrt{x+1}}{h}$

$$= \lim_{h \to 0} \frac{\left(\sqrt{x+h+1} - \sqrt{x+1}\right)\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \lim_{h \to 0} \frac{x+h+1-(x+1)}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}}$$

$$= \frac{1}{2\sqrt{x+1}}$$

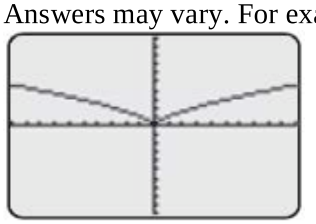
domain of $f(x)$ is: $\{x \mid x \geq -1, x \in R\}$; domain of $f'(x)$ is: $\{x \mid x > -1, x \in R\}$

b) $$\frac{dy}{dx} = \lim_{h\to 0}\frac{\sqrt{2(x+h)-1}-\sqrt{2x-1}}{h}$$
$$= \lim_{h\to 0}\frac{\left(\sqrt{2x+2h-1}-\sqrt{2x-1}\right)\left(\sqrt{2x+2h-1}+\sqrt{2x-1}\right)}{h\left(\sqrt{2x+2h-1}+\sqrt{2x-1}\right)}$$
$$= \lim_{h\to 0}\frac{2x+2h-1-(2x-1)}{h\left(\sqrt{2x+2h-1}+\sqrt{2x-1}\right)}$$
$$= \lim_{h\to 0}\frac{2h}{h\left(\sqrt{2x+2h-1}+\sqrt{2x-1}\right)}$$
$$= \lim_{h\to 0}\frac{2}{\sqrt{2x+2h-1}+\sqrt{2x-1}}$$
$$= \frac{2}{2\sqrt{2x-1}}$$
$$= \frac{1}{\sqrt{2x-1}}$$

domain of $f(x)$ is: $\{x \mid x \geq 0.5, x \in \mathbb{R}\}$; domain of $f'(x)$ is: $\{x \mid x > 0.5, x \in \mathbb{R}\}$

Chapter 1 Section 5 **Question 30 Page 62**

a) Answers may vary. For example:



The function is non-differentiable at (0, 0) since there is an abrupt change in slope at this point.

b) Answers may vary. For example:

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{(x+h)^{\frac{2}{3}} - x^{\frac{2}{3}}}{h} \\
&= \lim_{h\to 0}\frac{\left[(x+h)^{\frac{2}{3}} - x^{\frac{2}{3}}\right]\left[(x+h)^{\frac{4}{3}} + x^{\frac{2}{3}}(x+h)^{\frac{2}{3}} + x^{\frac{4}{3}}\right]}{h\left[(x+h)^{\frac{4}{3}} + x^{\frac{2}{3}}(x+h)^{\frac{2}{3}} + x^{\frac{4}{3}}\right]} \\
&= \lim_{h\to 0}\frac{(x+h)^2 - x^2}{h\left[(x+h)^{\frac{4}{3}} + x^{\frac{2}{3}}(x+h)^{\frac{2}{3}} + x^{\frac{4}{3}}\right]} \\
&= \lim_{h\to 0}\frac{2xh + h^2}{h\left[(x+h)^{\frac{4}{3}} + x^{\frac{2}{3}}(x+h)^{\frac{2}{3}} + x^{\frac{4}{3}}\right]} \\
&= \lim_{h\to 0}\frac{2x + h}{(x+h)^{\frac{4}{3}} + x^{\frac{2}{3}}(x+h)^{\frac{2}{3}} + x^{\frac{4}{3}}} \\
&= \frac{2x}{3x^{\frac{4}{3}}} \\
&= \frac{2}{3x^{\frac{1}{3}}} \quad \text{for } x \neq 0
\end{aligned}
$$

Chapter 1 Section 5 **Question 31 Page 62**

The common differences in an arithmetic sequence are equal.

$3^b - 2^a = 4^c - 3^b$
$3^b - 2^a = (2^2)^c - 3^b$
$3^b - 2^a = 2^{2c} - 3^b$
$2(3^b) = 2^{2c} + 2^a$
$3^b = 2^{2c-1} + 2^{a-1}$

Since 3^b is always odd, the right side of the equation can only be equal when it is odd, so $a = 1$.

$3^b = 2^{2c-1} + 1$

If $c = 1$, $b = 1$ so the ordered triple is (1, 1, 1).
If $c = 2$, $b = 2$ so the ordered triple is (1, 2, 2).

The two solutions are (1, 1, 1) and (1, 2, 2).

Chapter 1 Section 5 **Question 32 Page 62**

Find the area of the first triangle.

$$\begin{aligned}\text{area} &= \frac{1}{2}\left(\sqrt{3}\right)(1)\\ &= \frac{\sqrt{3}}{2}\end{aligned}$$

The area of the first triangle is $\frac{\sqrt{3}}{2}$ cm^2.

The second triangle is equilateral with side length x and all interior angles 60°. The triangle can be split into two right triangles with one leg the unknown height, the other measuring $\frac{x}{2}$ and the hypotenuse x.

Let h represent the height of the triangle.

$$\begin{aligned}\frac{h}{x} &= \sin 60°\\ h &= x\sin 60°\\ h &= \frac{\sqrt{3}x}{2}\end{aligned}$$

Now find x.

$$\begin{aligned}\text{area} &= \frac{1}{2}\left(\frac{\sqrt{3}x}{2}\right)(x)\\ \frac{\sqrt{3}}{2} &= \frac{\sqrt{3}x^2}{4}\\ x^2 &= 2\\ x &= \sqrt{2}\end{aligned}$$

Chapter 1 Section 5 **Question 33 Page 62**

$$\begin{aligned}\frac{x^{12}+3x^{11}+2x^{10}}{x^{11}+2x^{10}} &= \frac{x^{10}(x^2+3x+1)}{x^{10}(x+2)}\\ &= \frac{(x+1)(x+2)}{(x+2)}\\ &= x+1\end{aligned}$$

When x = 2008, the value of the expression equals 2009.

Chapter 1 Review

Chapter 1 Review **Question 1 Page 64**

a) Answers may vary. For example:
Since the slope of the graph is negative and becomes less negative, the rate of drainage slows with time.

b) Answers may vary. For example:

i) $\frac{1250-2150}{1-0}=-900$
The average rate of change of the volume of water remaining during the first hour is –900 L/h.

ii) $\frac{400-520}{4-3}=-120$
The average rate of change of the volume of water remaining during the last hour is –120 L/h.

c) Answers may vary. For example:

i) $\frac{1450-1900}{45-15}=-15$
The instantaneous rate of change of the volume of water remaining at 30 min is –15 L/min or –900 L/h.

ii) $\frac{800-1100}{105-75}=-10$
The instantaneous rate of change of the volume of water remaining at 1.5 h is –10 L/min or –600 L/h.

iii) $\frac{500-575}{195-165}=-2.5$
The instantaneous rate of change of the volume of water remaining at 3 h is –2.5 L/min or –150 L/h.

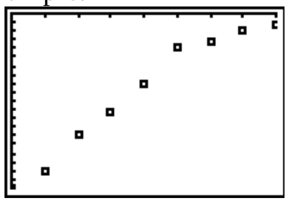
d) i) Answers may vary. For example:
The graph would be steeper because the rate of change of the volume of water would become more negative.

ii) Answers may vary. For example:
The graph would be extended further to the right, since the pool would take longer to drain.

e) Answers may vary. For example:

L1	L2	-----
0.0000	-1000	
30.000	-900.0	
60.000	-700.0	
90.000	-560.0	
120.00	-400.0	
150.00	-200.0	
180.00	-160.0	

L2(1)= -1000



Chapter 1 Review **Question 2 Page 64**

Answers may vary. For example:

a) The volume of gas remaining in a gas tank as a car is driven.

b) The volume of water in a beaker as the beaker is filled with water.

c) The velocity of an airplane the instant it lifts off the runway during takeoff.

d) The velocity of a car the instant that the brakes are applied.

Chapter 1 Review **Question 3 Page 64**

a)
$$\begin{aligned}\text{average rate of change} &= \frac{-4.9(4)^2+35(4)+10-(-4.9(2)^2+35(2)+10)}{4-2}\\ &= \frac{71.6-60.4}{2}\\ &= 5.6\end{aligned}$$

The average rate of change is 5.6 m/s.

b) Using the interval $4 \le t \le 6$:
$$\begin{aligned}\text{instantaneous rate of change} &= \frac{-4.9(6)^2+35(6)+10-(-4.9(4)^2+35(4)+10)}{6-4}\\ &= \frac{43.6-71.6}{2}\\ &= -14\end{aligned}$$

The estimated instantaneous rate of change is –14 m/s.

c) Answers may vary. For example:
Use a smaller interval for the secant. For instance, use the interval $4.9 \le t \le 5.1$.

Chapter 1 Review **Question 4 Page 64**

$$\frac{f(a+h)-f(a)}{h}=\frac{3(a+h)^2+2(a+h)-(3a^2+2a)}{h}$$
$$=\frac{6ah+3h^2+2h}{h}$$
$$=6a+3h+2$$

i) $h = 0.1$: $6(2) + 3(0.1) + 2 = 14.3$
$h = 0.01$: $6(2) + 3(0.01) + 2 = 14.03$
$h = 0.001$: $6(2) + 3(0.001) + 2 = 14.003$

The slope of the tangent is 14.

ii) $h = 0.1$: $6(–3) + 3(0.1) + 2 = –15.7$
$h = 0.01$: $6(–3) + 3(0.01) + 2 = –15.97$
$h = 0.001$: $6(–3) + 3(0.001) + 2 = –15.997$

The slope of the tangent is –16.

b) i) When $x = 2$, $y = 16$. Using the point (2, 16) and the slope $m = 14$, find b in the equation of the tangent $y = mx + b$.

$16 = 14(2) + b$
$b = –12$

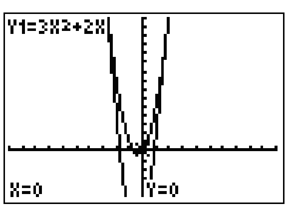
The equation of the tangent at $x = 2$ is $y = 14x – 12$.

ii) When $x = –3$, $y = 21$. Using the point (–3, 21) and the slope $m = –16$, find b in the equation of the tangent $y = mx + b$.

$21 = –16(–3) + b$
$b = –27$

The equation of the tangent at $x = –3$ is $y = –16x – 27$.

c)



Chapter 1 Review **Question 5 Page 64**

a) $t_1 = \frac{4}{3}; t_2 = \frac{1}{6}; t_3 = -\frac{4}{9}; t_4 = -\frac{11}{12};\ t_5 = -\frac{4}{3}$

b) Answers may vary. For example:
No. The sequence does not have a limit as $n \to \infty$. The sequence is divergent.

Chapter 1 Review **Question 6 Page 64**

a) $t_1 = \frac{35}{8}; t_2 = \frac{245}{64}; t_3 = \frac{1715}{512}; t_4 = \frac{12\,005}{4096};\ t_5 = \frac{84\,035}{32\,768}$

b) The sequence is converging to zero as $n \to \infty$.

c) $t_{12} = 1.007\,09;\ t_{13} = 0.881\,20$
Therefore, it takes 13 bounces.

Chapter 1 Review **Question 7 Page 65**

Justifications may vary. For example:

a) Yes. The function is continuous at $x = 3$ since the right and left limits approach the value at $x = 3$.

x	2.9	2.99	3	3.01	3.1
***g*(*x*)**	–0.356	–0.335	–0.333	–0.331	–0.311

b) Yes. The function is discontinuous at $x = –3$ since there is a vertical asymptote at this point.

Chapter 1 Review **Question 8 Page 65**

a) domain: $\{x \mid x \neq 0,\ x \in \mathbb{R}\}$; range: $\{y \mid y \leq 2,\ y \in \mathbb{R}\}$

b) i) $$\lim_{x\to+\infty} \frac{-2x^2 + 8x - 4}{x^2} = \lim_{x\to+\infty}\left(-2 + \frac{8}{x} - \frac{4}{x^2}\right) = -2$$

ii) $$\lim_{x\to-\infty} \frac{-2x^2 + 8x - 4}{x^2} = \lim_{x\to-\infty}\left(-2 + \frac{8}{x} - \frac{4}{x^2}\right) = -2$$

iii) $$\lim_{x\to0^+} \frac{-2x^2 + 8x - 4}{x^2} = \lim_{x\to0^+}\left(-2 + \frac{8x - 4}{x^2}\right) = -\infty$$

iv) $\lim_{x\to 0^-} \frac{-2x^2+8x-4}{x^2} = \lim_{x\to 0^-}\left(-2+\frac{8x-4}{x^2}\right)$
$= -\infty$

c) Answers may vary. For example:
The graph has a discontinuity at $x = 0$, since the limits for 0^+ and 0^- are $-\infty$.

Chapter 1 Review **Question 9 Page 65**

a)
$$\begin{aligned}\lim_{x\to -1} \frac{(-1+x)^2-4}{x+1} &= \lim_{x\to -1}\frac{x^2-2x-3}{x+1}\\ &= \lim_{x\to -1}\frac{(x+1)(x-3)}{x+1}\\ &= \lim_{x\to -1}(x-3)\\ &= -4\end{aligned}$$

b) $\lim_{x\to 3}\frac{-x^2+8x}{2x+1} = \frac{15}{7}$

c)
$$\begin{aligned}\lim_{x\to 0}\frac{\sqrt{x+16}+4}{x} &= \lim_{x\to 0}\frac{\left(\sqrt{x+16}+4\right)\left(\sqrt{x+16}-4\right)}{x\left(\sqrt{x+16}-4\right)}\\ &= \lim_{x\to 0}\frac{(x+16)-16}{x\left(\sqrt{x+16}-4\right)}\\ &= \lim_{x\to 0}\frac{x}{x\left(\sqrt{x+16}-4\right)}\\ &= \lim_{x\to 0}\frac{1}{\sqrt{x+16}-4}\\ &= \frac{1}{0}\end{aligned}$$

The limit does not exist.

d)
$$\begin{aligned}\lim_{x\to -2}\frac{3x^2+5x-2}{x^2-2x-8} &= \lim_{x\to -2}\frac{(3x-1)(x+2)}{(x-4)(x+2)}\\ &= \lim_{x\to -2}\frac{3x-1}{x-4}\\ &= \frac{7}{6}\end{aligned}$$

e) $\lim_{x\to 7}\frac{x^2-49}{x-7}=\lim_{x\to 7}\frac{(x-7)(x+7)}{x-7}$

$=\lim_{x\to 7}(x+7)$

$=14$

Chapter 1 Review **Question 10 Page 65**

Answers may vary. For example:
As x approaches −6 from the left and right, the graph of $y = h(x)$ approaches $y = 3$ so the limit as x approaches –6 exists and equals 3. There is a hole in the graph of $y = h(x)$ at (–6, 3). Since $h(-6)\neq 3$, the function is discontinuous at $x = -6$.

Chapter 1 Review **Question 11 Page 65**

a) $\frac{dy}{dx}=\lim_{h\to 0}\frac{4(x+h)-1-(4x-1)}{h}$

$=\lim_{h\to 0}\frac{4h}{h}$

$=4$

b) $h'(x)=\lim_{h\to 0}\frac{11(x+h)^2+2(x+h)-(11x^2+2x)}{h}$

$=\lim_{h\to 0}\frac{22xh+11h^2+2h}{h}$

$=\lim_{h\to 0}(22x+11h+2)$

$=22x+2$

c) $s'(t)=\lim_{h\to 0}\frac{\frac{1}{3}(t+h)^3-5(t+h)^2-\left(\frac{1}{3}t^3-5t^2\right)}{h}$

$=\lim_{h\to 0}\frac{\frac{1}{3}(t^3+3t^2h+3th^2+h^3)-5(t^2+2th+h^2)-\left(\frac{1}{3}t^3-5t^2\right)}{h}$

$=\lim_{h\to 0}\frac{t^2h+th^2+\frac{1}{3}h^3-10th-5h^2}{h}$

$=\lim_{h\to 0}(t^2+th+\frac{1}{3}h^2-10t-5h)$

$=t^2-10t$

d) $f'(x) = \lim_{h\to 0} \frac{(x+h+3)(x+h-1)-(x+3)(x-1)}{h}$

$= \lim_{h\to 0} \frac{2xh+2h+h^2}{h}$

$= \lim_{h\to 0}(2x+2+h)$

$= 2x+2$

Chapter 1 Review **Question 12 Page 65**

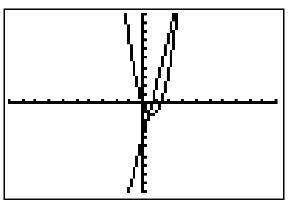
a) $\frac{dy}{dx} = \lim_{h\to 0} \frac{3(x+h)^2-4(x+h)-(3x^2-4x)}{h}$

$= \lim_{h\to 0} \frac{6xh+3h^2-4h}{h}$

$= \lim_{h\to 0}(6x+3h-4)$

$= 6x-4$

b)



c) The slope of the tangent at $x = -2$ is $\left.\frac{dy}{dx}\right|_{x=-2}$

$\left.\frac{dy}{dx}\right|_{x=-2} = 6(-2)-4$

$= -16$

When $x = -2$, $y = 20$.
Use the point (–2, 20) and $m = -16$ to find b in the equation of the tangent, $y = mx + b$.

$20 = -16(-2) + b$
$b = -12$

The equation of the tangent at $x = -2$ is $y = -16x - 12$.

Chapter 1 Practice Test

Chapter 1 Practice Test **Question 1 Page 66**

C; The function is 0 at $x = 2$, but there is a cusp there.

Chapter 1 Practice Test **Question 2 Page 66**

C; Explanations may vary. For example:
This expression gives the slope of the secant for the interval $x \in (a,\ h)$.

Chapter 1 Practice Test **Question 3 Page 66**

a) $\lim_{x \to 9}(4x - 1) = 35$

b) $\lim_{x \to -3}(2x^4 - 3x^2 + 6) = 141$

c)
$$\begin{aligned}\lim_{x \to 5} \frac{x^2 - 3x - 10}{x - 5} &= \lim_{x \to 5} \frac{(x-5)(x+2)}{x-5} \\ &= \lim_{x \to 5}(x+2) \\ &= 7\end{aligned}$$

d)
$$\begin{aligned}\lim_{x \to 0} \frac{9x}{2x^2 - 5x} &= \lim_{x \to 0} \frac{9x}{x(2x-5)} \\ &= \lim_{x \to 0} \frac{9}{(2x-5)} \\ &= -\frac{9}{5}\end{aligned}$$

e)
$$\begin{aligned}\lim_{x \to 7} \frac{x^2 - 49}{x - 7} &= \lim_{x \to 7} \frac{(x-7)(x+7)}{x-7} \\ &= \lim_{x \to 7}(x+7) \\ &= 14\end{aligned}$$

f) $\lim_{x \to \infty} \frac{-1}{2 + x^2} = 0$

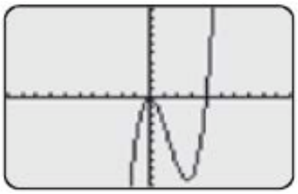
Chapter 1 Practice Test **Question 4 Page 66**

C; Explanations may vary. For example:
A limit can be used to find the instantaneous rate of change at a point, not the average rate of change.

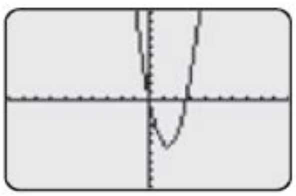
Chapter 1 Practice Test **Question 5 Page 66**

a)
$$\begin{aligned}\frac{dy}{dx} &= \lim_{h\to 0}\frac{(x+h)^3 - 4(x+h)^2 - (x^3 - 4x^2)}{h}\\ &= \lim_{h\to 0}\frac{3x^2h + 3xh^2 + h^3 - 8xh - 4h^2}{h}\\ &= \lim_{h\to 0}(3x^2 + 3xh + h^2 - 8x - 4h)\\ &= 3x^2 - 8x\end{aligned}$$

b) $y = x^3 - 4x^2$



$\frac{dy}{dx} = 3x^2 - 8x$



c) The slope of the tangent at $x = -1$ is $\left.\frac{dy}{dx}\right|_{x=-1}$.

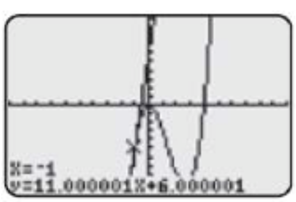
$$\begin{aligned}\left.\frac{dy}{dx}\right|_{x=-1} &= 3(-1)^2 - 8(-1)\\ &= 11\end{aligned}$$

When $x = -1$, $y = -5$.
Use the point (–1, –5) and $m = 11$ to find b in the equation of the tangent, $y = mx + b$.

$-5 = 11(-1) + b$
$b = 6$

The equation of the tangent at $x = -1$ is $y = 11x + 6$.

d)



Chapter 1 Practice Test **Question 6 Page 66**

a) $\lim_{x \to -2} f(x) = -3$

b) $\lim_{x \to 0^-} f(x) = 1$

c) $\lim_{x \to 1^-} f(x) = -2$

d) $\lim_{x \to 1^+} f(x) = -2$

e) $\lim_{x \to -4^-} f(x) = 1$

f) $\lim_{x \to \infty} f(x) = \infty$

Chapter 1 Practice Test **Question 7 Page 67**

a) domain: $\{x \mid x \neq 4,\ x \in \mathbb{R}\}$; range: $\{y \mid y \neq 3,\ y \in \mathbb{R}\}$

b) i) $\lim_{x \to +\infty} \frac{3x}{x-4} = 3$ from above

ii) $\lim_{x \to -\infty} \frac{3x}{x-4} = 3$ from below

iii) $\lim_{x \to 4^+} \frac{3x}{x-4} = +\infty$

iv) $\lim_{x \to 4^-} \frac{3x}{x-4} = -\infty$

v) $\lim_{x \to 6} \frac{3x}{x-4} = 9$

vi) $\lim_{x \to -2} \frac{3x}{x-4} = 1$

c) Explanations may vary. For example:
No. The graph is not continuous. As x approaches 4 from the left, the graph becomes large and negative. As x approaches 4 from the right, the graph becomes large and positive. Therefore, there is a discontinuity at $x = 4$.

Chapter 1 Practice Test **Question 8 Page 67**

a) $V(x) = 4x - \frac{1}{4}x^3$

b) average rate of change: $\frac{V(3) - V(1.5)}{3 - 1.5} = \frac{5.25 - 5.15625}{1.5}$

$$= 0.0625$$

The average rate of change of the volume of the shed is 0.0625 m^3/m.

c) instantaneous rate of change:

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{4x - \frac{x^3}{4} - \left(4a + \frac{a^3}{4}\right)}{x - a}$$

$$= \lim_{x \to a} \frac{4(x - a) - \frac{1}{4}(x^3 - a^3)}{x - a}$$

$$= \lim_{x \to a} \frac{4(x - a) - \frac{1}{4}(x - a)(x^2 + ax + a^2)}{x - a}$$

$$= \lim_{x \to a} \left(4 - \frac{1}{4}(x^2 + ax + a^2)\right)$$

$$= 4 - \frac{3a^2}{4}$$

When a = 3, the instantaneous rate of change of the volume of the shed is –2.75 m^3/m.

Chapter 1 Practice Test **Question 9 Page 67**

a) i) After 1 s, the radius is 0.2 m.

ii) After 3 s, the radius is 0.6 m.

iii) After 5 s, the radius is 1.0 m.

b) The area of a circle is $A = \pi r^2$.

The instantaneous rate of change in the area is $\frac{dA}{dr} = 2\pi r$.

c) i) $\left.\frac{dA}{dr}\right|_{r=0.2} = 2\pi(0.2)$

$\doteq 1.3$

The instantaneous rate of change of the area when r = 0.2 m is 1.3 m^2/m.

ii) $\left.\frac{dA}{dr}\right|_{r=0.6} = 2\pi(0.6)$

$\doteq 3.8$

The instantaneous rate of change of the area when r = 0.6 m is 3.8 m^2/m.

iii) $\left.\frac{dA}{dr}\right|_{r=1} = 2\pi(1)$

$\doteq 6.3$

The instantaneous rate of change of the area when r = 1 m is 6.3 m^2/m.

Chapter 1 Practice Test Question 10 Page 67

a) C

b) A

c) D

d) B