Stress and Strain

Introduction

Strain gages are important to any device under stress or strain. Strain gages provide solutions to many real world systems and testing. In this lab, you will apply various loads to the cantilever beam and measure the strain using a pre-written LabVIEW program.

Objective

- To use strain gages for measurement of strain.

- To determine the modulus of elasticity of a cantilever beam.

- To use Hooke's Law to determine the modulus of elasticity of an unknown material.

Design of Strain Gages

Strain gages are used as sensing elements in displacement or load measurement systems. They can be used to measure extremely small deformations. Strain gages are made of fine wire with a certain cross-sectional area (A), initial length (L), and resistivity (ρ). The resistance of the wire changes when the wire is displaced due to stress. The stress is an applied load acting over the cross-sectional area of the wire. The following equation defines the resistance (R) of the strain gage wire:

$$R = \frac{\rho L}{A}$$

It is common to use a gage factor (G) to describe the changes in resistance (ΔR) of the strain gage due to changes in length (ΔL) of the wire. The gage factor is also used to compare various strain-gage materials:

$$\text{gage factor} = G = \frac{\Delta R / R}{\Delta L / L} \quad \text{where } \varepsilon = \frac{\Delta L}{L} \text{ is defined as strain}$$

So, strain is the change in length over the original length. The units are typically mm/mm or in/in. Sometimes the term μstrain (micro-strain) is used, meaning 10^{-6} in/in.

Stress and Modulus of Elasticity (Young's Modulus)

By mounting strain gages to the surface (top and/or bottom) of a cantilever beam, you can measure the axial deformation of the beam (the deformation along the length of the beam) when a transverse load is applied to the end of the beam. The amount of deformation can be related to the strain using the previous gage factor equation. Figure 2 shows a strain gage bonded to the top surface of the cantilever beam.
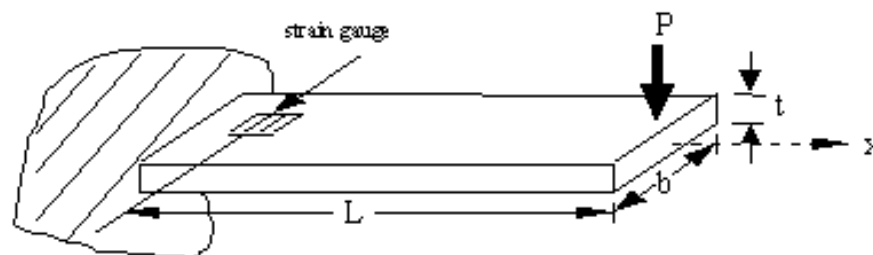

Figure 2. Cantilever Beam with a Strain Gage Mounted on the Top

When a load (P) is applied at the end of the beam, the tensile stress (σ) along the x-axis at the top surface is given as:

$$\sigma = \frac{Mc}{I} \tag{1}$$

where:

- M = bending moment (units of N-m). M is the product of the effective beam length and the force (*P*) applied at the end of the beam.

- c = distance from neutral axis of beam (m), Typically $c = t/2$, where t is the beam thickness.

- I = moment of inertia for a cross section of the beam (units are m^4). The moment of inertia is $I = \frac{bt^3}{12}$.

- P = load (N).

- L = effective beam length = distance between the point where the load is applied and the center of the strain gage (m).

- b = beam width (m).

- t = beam thickness (m).

You can find the stress from the applied force and beam geometry, as given by equation 1, and measure the strain with a strain gage. Conveniently, a relationship exists between the two. The modulus of elasticity (*E*) is an index of the stiffness of the material. When a load (that is, force) is applied to metal such as an aluminum alloy or steel, the strain of the material changes linearly as a function of the stress over a certain, usually small, range. Within this range, the beam exhibits elastic deformation (that is, elastic or reversible strain). When the deformation is not permanent, it is called elastic deformation. That is, when a force is applied, the beam will bend downward and elongate. When the force is removed, the beam will return to its original shape. The modulus of elasticity (*E)* is the ratio of stress to elastic strain and has units of N/m^2:

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} \tag{2}$$

For this lab, strain will be measured by a strain gage bonded to the top surface of a beam. By applying a known load, the stress is calculated using Equation 1. After applying several different loads and measuring the strain directly using a strain gage, you can plot the stress-strain graph. The slope of this graph is the modulus of elasticity as predicted by Equation 2.

Note: The location and position of the strain gage is critical. You must know if the position of the gage will result in axial, shear, torsional, or bending strains, or any combination.

Bridge Circuits

Usually, strain gages are connected in a Wheatstone bridge circuit as shown in Figure 3. The change in resistance of the strain gage due to an applied force can be measured as the output voltage of the Wheatstone bridge circuit. The bridge circuit shown has a single strain gage and three fixed resistors and is called a quarter-bridge circuit.
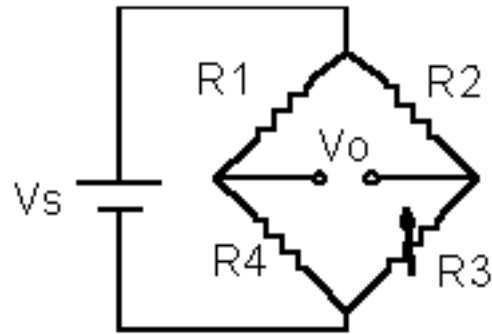
Figure 3. Wheatstone Bridge with Strain Gage, R3

R3 is the strain gage resistance. Therefore,

$$V_0 = \left(\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4} \right) V_s$$

When $V_0 = 0$, the bridge is said to be balanced. The only way to balance the bridge is to set:

$$\frac{R_3}{R_2 + R_3} - \frac{R_4}{R_1 + R_4} = 0$$

or

$$R_1 R_3 = R_2 R_4$$

R_4 is typically a variable resistor used to balance the bridge. In this experiment, the offset potentiometer on the front panel of the LabVIEW VI is used to balance the bridge. Assuming the bridge is initially balanced, then when the strain gage is strained, $R_3 = R_3 + \Delta R$. The output of the bridge circuit becomes:

$$V_0 = \frac{R_1 \Delta R}{\left(R_2 + R_3 + \Delta R \right)\left(R_1 + R_4 \right)} V_s$$

Because ΔR is usually small compared to R_3, it can be neglected from the denominator of the equation. Then, V_0 becomes:

$$V_0 = \frac{R_1 \Delta R}{\left(R_2 + R_3 \right)\left(R_1 + R_4 \right)} V_s$$

Note that V_0 is a linear function of ΔR. Now you can relate strain with the output voltage based on the following derivation.

Because the gage factor (G) is equal to $(\Delta R/R))/(\Delta L/L)$, and strain ($\varepsilon$) is equal to $\Delta L/L = (\Delta R/R)/G$, the conversion from strain to output voltage is:

$$\varepsilon = \frac{\left(R_2 + R_3 \right)\left(R_2 + R_4 \right)}{R_1 R_3 G} \frac{V_0}{V_s}$$

You must know the gage factor (G) to get meaningful results.

Pre-Lab Preparation

- Read through the theory section from this experiment to understand how strain gages are used to measure deformations.

- Find a reference and read about the modulus of elasticity.

- Find a table with the modulus of elasticity for common metals. You will need this table to complete the lab.

- Bring a ruler and a flash drive to the lab.

Workstation Details

Your workstation should have the following items:

- A computer with National Instruments LabVIEW software
- National Instruments DAQ board.
- Strain gage module.
- Clamp.
- Masses for hanging on the cantilever beam.

Procedure

1. Measure the beam's width, thickness, and effective beam length.

2. Open **Measurement & Automation** from your desktop.

   a. Under "My System," pull down **Devices and Interfaces**

   b. Pull down **NI cDAQ-9174 'cDAQ1'.** Click on **NI9235 'cDAQ1Mod1'.**

   c. In the main window, click **Self Test.**

   d. After the self-test passes, click on **Test Panels.** In the dialogue box change **only** the following inputs:

      i. Channel Name: **ai1**

      ii. Mode: **Continuous**

   e. Press **Start.** Manipulate the beam and check the Amplitude vs. Samples Chart to see that the Test Panel responds.

   f. Press **Stop**

   g. **Close** the window.

3. Set up the software:

   a. Launch **LabVIEW**.

   b. On the desktop, open **Lab 3**.

   c. On the front panel, set the following parameters. *Note, some parameters must be set according to strain gage specifications:

      i. Channel Parameters
         1. Physical channels: **cDAQ1Mod1/ai1**
         2. Minimum Value = **- .001**
         3. Maximum Value = **+ .001**
         4. Filter Setting: **Default**
         5. Lowpass Cutoff Frequency = **10.0**

      ii. Timing Parameters
         1. Sampling Rate = **1.0**
         2. # of Points Saved = **10**

      iii. Strain Gage Parameters
- 1. Gage factor = ***2.16**
- 2. Gage resistance = ***120.0**
- 3. Poisson Ratio = **0.3**

      iv. Bridge Parameters
- 1. Strain configuration: ***Quarter Bridge I**
- 2. Initial Bridge Voltage = **0.0**
- 3. Lead Wire Resistance = **0.0**
- 4. Excitation Source: **Internal**
- 5. Excitation Value = **2.0**

      v. Scale = 1.0

      vi. Offset = 0*

Example:
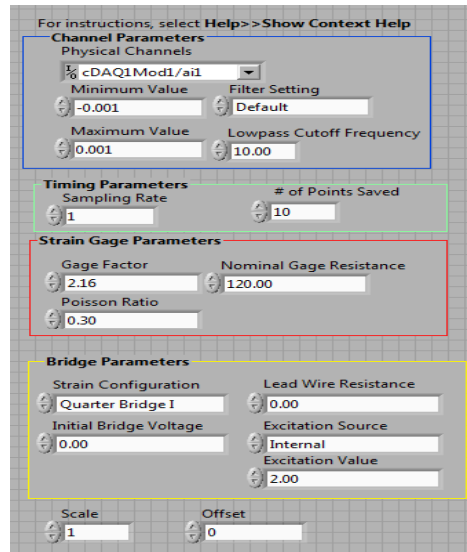
4. Measuring strain:

  a. Calibrate the beam

    i. **Run** the program with no mass on the clip.

    ii. Wait at least 10 seconds for the beam to stabilize.

    iii. Click the **STOP** button.

    iv. Record the number given in the **Average Strain Output** in the **Offset** control as a **Negative** number.

  b. Measure the strain induced on the beam using the loads shown in the table below:

    i. Hook each mass and the holder on the end of the cantilever beam.

    ii. When the mass is steady, **Run** the program and wait for the beam to stabilize.

    iii. Click the **Stop** button and record the Strain.

    iv. You may add more points and save the data as a .dat (MatLab) or .xls (Excel) by switching the button on **Save to File?**.

v. Repeat steps **i-iv** to complete the table below.

Load	Strain
0	
100	
200	
300	
400	
300	
200	
100	
0	

Lab Report

You should submit a lab report. Your lab report should include the following information:

- Type all data into Excel and format it into tables.

- Calculate the stress for each load applied and enter it into the Excel table you create.

- Graph the stress vs. strain and perform a linear regression on the data.

- Determine the modulus of elasticity from your graph and linear regression.

- From the value of the modulus of elasticity, determine the beam material.

- Make sure you label *units* for each variable. **If** the units are not consistent, you will not get the right modulus of elasticity. Show all your calculations, including conversion of units. Work in *SI units*.

- Staple your data sheet to the back of your report.

Data Sheet

1. Record the beam dimensions:
   - Effective length:_____
   - Width:_____
   - Thickness:_____

2. Give two applications of strain gages.

3. Why do you need to set the offset potentiometer until output strain is zero when there is no weight on the beam? What function does it provide?

4. Demonstration of strain measurement.

5. Perform a sample calculation of the modulus of elasticity using one set of your strain and load measurement values. Show units throughout the calculation.

6. Plot stress vs. strain.

7. From the plot of stress vs. strain, determine the modulus of elasticity.

8. Based on the modulus of elasticity and physical appearance of the beam, what material is the beam of?