PROJECTILES – MECHANICS 2

(WJEC)

The main ideas are

- **Finding speed and direction of motion of projectile at any point on its path**
- **Finding the greatest height**
- **Finding the horizontal range**

Finding the greatest height of a projectile

Example

A ball is kicked with a speed of $15\,\text{ms}^{-1}$ over level ground at an angle of 40° to the horizontal. What is the maximum height reached?

Solution

We are concerned with the vertical component of the ball's motion.

$u_y = 15\sin 40^\circ$

$a_y = g = -9.8$

$v_y = 0$ (at maximum height)

$y = ?$ (this is the maximum height we wish to find)

Choose the appropriate equation of motion, based on the information you have and what you need to calculate:

$v^2 = u^2 + 2as$

$$y = \frac{v_y^2 - u_y^2}{2a_y}$$

$$= \frac{0 - \left(15\sin 40^\circ\right)^2}{2 \times -9.8}$$

$= 4.74\text{m}$ (3 s.f.)

so the maximum height of the ball is 4.74m (3 s.f.)

Before the exam you should know:

- You **must** be completely familiar and fluent with all of the constant acceleration equations, especially:

$$s = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

- You **must** be fluent with the use of vectors and resolving into horizontal and vertical components.
- The only force which acts on a projectile is gravity and we assume:
  - a projectile is a particle
  - it is not powered
  - the air has no effect on its motion
- A projectile experiences a constant acceleration of $g = 9.8\text{ms}^{-2}$ vertically downwards
- The horizontal component of acceleration is 0 for a projectile, so its horizontal component of velocity is **constant**.
- If a projectile has an initial speed u, at an angle of θ to the horizontal, its initial velocity is $\mathbf{u} = \begin{pmatrix} u\cos\theta \\ u\sin\theta \end{pmatrix}$ and its acceleration is $\begin{pmatrix} 0 \\ -g \end{pmatrix}$
- At maximum height, the vertical component of a projectile's velocity, v_y, is 0.
- Know how to derive the equation of the path of a projectile: $y = x\tan\theta - \frac{gx^2}{2u^2}\left(1 + \tan^2\theta\right)$

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Finding the range of a projectile

Continuing the example used for maximum height, if the ball is kicked over level ground, the point where it lands will have vertical displacement, y, of 0. Its range is its horizontal displacement, x, from its starting point at the point where it lands. To calculate the ball's range, you can calculate the time it takes to return to the ground and then use this time to calculate the horizontal displacement at that time.

Question: What is the range of the ball?

Solution

Considering the vertical motion:

$u_y = 15\sin 40^\circ$

$a_y = -9.8$

$y = 0$ (when the ball returns to the ground)

$t = ?$ (when the ball returns to the ground)

Choose the appropriate equation of motion, based on the information you have and what you need to calculate:

$$s = ut + \frac{1}{2}at^2 \qquad s = t\left(u + \frac{1}{2}at\right)$$

$$y = t\left(u_y + \frac{1}{2}a_y t\right) \Rightarrow 0 = t\left(15\sin 40^\circ - 4.9t\right)$$

$$\Rightarrow t = 0 \text{ or } t = \frac{15\sin 40^\circ}{4.9} = 1.968 \text{ seconds (4 s.f.)}$$

$t = 0$ is when the ball left the ground, so it lands when $t = 1.968$ seconds (4s.f.)

Now considering the horizontal motion:

$u_x = 15\cos 40^\circ$

$a_x = 0$

$x = ?$ (the range)

$t = 1.968$

(when the ball returns to the ground, calculated above)

Choose the appropriate equation of motion, based on the information you have and what you need to calculate:

$$s = ut + \frac{1}{2}at^2$$

$$x = t\left(u_x + \frac{1}{2}a_x t\right) \Rightarrow x = 1.968\left(15\cos 40^\circ + 0\right) = 22.6\text{m (3s.f.)}$$

The range of the ball is 22.6m (3 s.f.)

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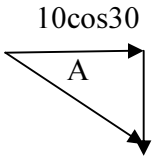
Finding the speed and direction of a projectile

Example:
A ball is projected from the top of a cliff with a velocity 10 ms^{-1} at an angle of 30° above the horizontal. Find the magnitude and direction of the velocity after 2 seconds.

Solution:
Horizontal motion: $v_y = u_y = 10\cos 30$

Vertical motion $v = u + at$ $v_x = 10\sin 30 - 9.8 \times 2 = -14.6$



$$\text{Speed} = \sqrt{(10\cos 30)^2 + 14.6^2} = 16.98 ms^{-1} \text{ (4s.f)}$$

$$TanA = \frac{14.6}{10\cos 30} = 1.6859 \Rightarrow A = 59.3^o \text{ (3s.f.)}$$

Velocity is 16.98ms^{-1} at 59.3° below the horizontal.

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TAFLEGRYN – MECANEG M2 (CBAC)

(CBAC)

Y prif syniadau yw:

- **Darganfod buanedd a chyfeiriad taflegryn ar unrhyw bwynt ar ei lwybr.**
- **Darganfod yr uchder mwyaf**
- **Darganfod y cyrhaeddiad llorweddol**

Darganfod yr uchder mwyaf ar gyfer y taflegryn

Enghraifft

Mae pêl yn cael ei gicio â buanedd $15\,\text{ms}^{-1}$ ar draws dir llorweddol ar ongl 40° i'r llorwedd. Beth yw'r uchder mwyaf a gyrheiddir gan y bêl?

Datrysiad

Ystyriwn gydran mudiant fertigol y bêl

$u_y = 15\sin 40^\circ$

$a_y = g = -9.8$

$v_y = 0$ (ar yr uchder fwyaf)

$y = ?$ (hwn yw'r uchder fwyaf rydyn angen darganfod)

Dewiswch yr hafaliad mudiant perthnasol ar gyfer y wybodaeth ar gael ac sydd angen arnoch:

$$v^2 = u^2 + 2as$$

$$y = \frac{v_y^2 - u_y^2}{2a_y}$$

$$= \frac{0 - \left(15\sin 40^\circ\right)^2}{2 \times -9.8}$$

$$= 4.74\text{m (3 s.f.)}$$

Felly uchder fwyaf y bêl yw 4.74m (3 ff.y)

Cyn yr arholiad dylech fod yn gwybod:

- **Rhaid** eich bod yn gwbl gyffyrddus a rhugl gyda'r hafaliadau cyflymiad cyson, yn enwedig:

$$s = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

- Rhaid eich bod yn rhugl gyda defnydd o fectorau a chydrannu i'r cydrannau llorweddol a fertigol.
- Disgyrchiant yw'r unig rym sy'n gweithredu ar daflegryn, ac fe dybiwn:
  - bod y taflegryn yn ronyn
  - does dim pŵer ganddo
  - nid oes gan yr awyr effaith ar y taflegryn
- Mae'r taflegryn yn profi cyflymiad cyson o $g = 9.8\text{ms}^{-2}$ yn fertigol i lawr.
- Y gydran lorweddol ar gyfer y cyflymiad yw 0 am y taflegryn, felly mae ei gydran lorweddol ar gyfer y cyflymder yn **gyson**.
- Os yw buanedd cychwynnol y taflegryn yn u, ar ongl θ i'r llorwedd, ei gyflymder cychwynnol yw $\mathbf{u} = \begin{pmatrix} u\cos\theta \\ u\sin\theta \end{pmatrix}$ a'i gyflymiad yw $\begin{pmatrix} 0 \\ -g \end{pmatrix}$
- Ar yr uchder mwyaf, cydran fertigol cyflymder v_y y taflegryn yw 0.
- Deallwch sut i ddod o hyd i hafaliad llwybr y taflegryn: $y = x\tan\theta - \frac{gx^2}{2u^2}\left(1 + \tan^2\theta\right)$

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Darganfod y cyrhaeddiad llorweddol

Gan barhau gyda'r enghraifft ar gyfer yr uchder mwyaf, os yw'r bêl wedi ei gicio ar draws dir gwastad, bydd gan y pwynt glanio dadleoliad fertigol, y, gwerth 0. Cymerwn mai ei **gyrhaeddiad** yw'r dadleoliad llorweddol, x, o'r man cychwynnol. I gyfrifo'r cyrhaeddiad, gallwn gyfrifo'r amser mae'n cymryd i ddychwelid i'r ddaear a defnyddio'r amser hyn i gyfrifo'r dadleoliad llorweddol.

Cwestiwn: Beth yw cyrhaeddiad y bêl?

Datrysiad

Ystyried y mudiant fertigol:

$u_y = 15\sin 40^\circ$

$a_y = -9.8$

$y = 0$ (pan mae'r bel yn dychwelid i'r ddaear)

$t = ?$ (pan mae'r bel yn dychwelid i'r ddaear)

Dewiswch yr hafaliad mudiant perthnasol ar gyfer y wybodaeth ar gael ac sydd angen arnoch:

$$s = ut + \frac{1}{2}at^2 \qquad s = t\left(u + \frac{1}{2}at\right)$$

$$y = t\left(u_y + \frac{1}{2}a_y t\right) \Rightarrow 0 = t\left(15\sin 40^\circ - 4.9t\right)$$

$$\Rightarrow t = 0 \text{ neu } t = \frac{15\sin 40^\circ}{4.9} = 1.968 \text{ eiliad (4 s.f.)}$$

$t = 0$ yw pan mae'r bêl yn gadael y ddaear, felt mae'n glanio pan mae $t = 1.968$ eiliad (4ff.y)

Gan ystyried y mudiant llorweddol:

$u_x = 15\cos 40^\circ$

$a_x = 0$

$x = ?$ (y cyrhaeddiad)

$t = 1.968$ (pan mae'r bel yn dychwelid i'r ddaear, a gyfrifid uchod)

Dewiswch yr hafaliad mudiant perthnasol ar gyfer y wybodaeth ar gael ac sydd angen arnoch:

$$s = ut + \frac{1}{2}at^2$$

$$x = t\left(u_x + \frac{1}{2}a_x t\right) \Rightarrow x = 1.968\left(15\cos 40^\circ + 0\right) = 22.6\text{m (3ff.y)}$$

Cyrhaeddiad y bêl yw 22.6m (3 ff.y)

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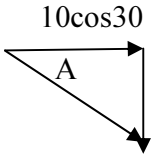
Darganfod buanedd a chyfeiriad taflegryn

Enghraifft:
Teflir pêl o dop clogwyn gyda chyflymder 10 ms^{-1} ar ongl 30° uwchben y llorwedd. Darganfyddwch faint a chyfeiriad y cyflymder ar ôl 2 eiliad.

Datrysiad:
Mudiant llorweddol: $v_y = u_y = 10\cos 30$

Mudiant fertigol $v = u + at$ $v_x = 10\sin 30 - 9.8 \times 2 = -14.6$



$$\text{Buanedd} = \sqrt{(10\cos 30)^2 + 14.6^2} = 16.98 ms^{-1} \text{ (4s.f)}$$

$$TanA = \frac{14.6}{10\cos 30} = 1.6859 \Rightarrow A = 59.3^o \text{ (3s.f.)}$$

Y cyflymder yw 16.98ms^{-1} ar ongl 59.3^o o dan y llorwedd.

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