Damage detection in a sandwich composite beam using wavelet transforms

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ABSTRACT

There is a growing interest in developing non-destructive damage detection methods for damage assessment of composite structures, especially in the aerospace and marine industries. Although damage detection of composite laminates has been widely investigated, little work has been carried out on sandwich composite configurations. A technique using the Lipschitz exponent, which is estimated by wavelet transforms, as a damage sensitive signal feature is outlined here to identify damage in sandwich composites. It is based on the fact that damage causes singularities to appear in the structure's dynamic response which can be identified, and its severity estimated, using the Lipschitz exponent. In order to demonstrate this technique, damage in cantilevered fibre reinforced plastic (FRP) sandwich beams is investigated both numerically and experimentally.

Keywords: Damage detection, Lipschitz exponent, Wavelet transform, Mode shapes

1. INTRODUTION

Damage is often observed in many engineering structures during their service life and at the time of manufacture. It can occur internally or externally and may be caused by a number of factors such as overloading, long term accumulative crack growth or impact by a foreign object. This could cause the mechanical properties of the structure to degrade severely and can significantly reduce its residual strength or stiffness. This is important in the case of structures constructed from composite materials, since a defect such as a delamination between layers, if allowed to progress unchecked, could eventually reach an advanced stage and lead to the structure collapsing.

Therefore in order to avert a critical loss in structural integrity it is important that the structure be monitored periodically throughout its service life. Since it is not practical to take the structure apart in order to identify damage, non-destructive damage evaluation (NDE) methods are needed. Current NDE methods available to identify damage, such as visual inspections and acoustic or ultrasonic measurement techniques, can be time consuming and costly. In most cases these techniques are not practical, almost all of them require that the general location of the damage be known in advance and that the portion of the structure being inspected is readily accessible.

The limitations of current NDE methods can be overcome by using an approach based on measuring structural vibration characteristics, especially modal parameters, to identify damage. This is due to the fact that modal parameters are functions of mass, stiffness and damping which are physical properties of the structure and hence sensitive to damage. The main advantage of this approach lies in the fact that modal information is relatively cheap and easy to obtain. However, it is not possible to use this approach directly to extract local information that relates to small defects in the structure as it is only effective in characterizing the structure's global behaviour.

Extensive research has been conducted on developing structural damage detection techniques based on the analysis of a wide variety of modal parameters, although the analysis of natural frequencies and mode shapes have proved to be the most popular. These approaches attempt to highlight an association between damage and the specific modal parameter concerned.

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Cawley and Adams[1] provided insight into using vibration data for non-destructive evaluation of a structure's integrity by proposing a method of using changes in natural frequency. They noted that damage is usually accompanied by a local reduction in stiffness and an increase in damping ratio. These local or distributed changes in stiffness produce changes in natural frequencies which affect each mode differently depending on the damage location. By comparing the advantages and disadvantages of using natural frequencies as a structural diagnostic parameter, Salawu[2] concluded in his review that the low sensitivity of frequency shifts to damage requires either very precise measurements or large levels of damage. Diverging away from using natural frequencies, Yuen[3] proposed a method to utilize eigen frequencies and eigen modes by using a finite element model of a cantilever beam.

The use of mode shapes as a damage indicator is based on the assumption that there would be a detectable change in the mode shape after a damage event. Modal assurance criteria (MAC) and co-ordinate modal assurance (COMAC) criteria are some of the methods that have been used by West[4] and Fox[5] to compare mode shapes in order to identify and locate damage. An alternative method was demonstrated by Pandey[6] to identify damage in finite element model (FEM) beams based on curvature information obtained from the finite difference of displacement mode shapes. Maia[7] extended the curvature method and proposed the use of frequency response functions of curvature derived from the frequency response functions of displacement. A method which only uses the Laplacian of the damaged mode shape was demonstrated by Ratcliffe[8] to successfully identify small defects in a beam. However, when the measured data is corrupted by noise, the evaluation of the Laplacian is prone to large errors.

Recently, a great deal of attention has been focused on the use of wavelet transforms for structural damage detection. This is due to the wavelet transforms unique zooming ability or localization property enabling the examination of a signal at different scales which would highlight transient features in the signal that are otherwise undetectable. Although most of the research into wavelet transform based structural damage detection has focused on the analysis of high frequency signals[9,10,11] there have been some instances where this approach has been applied to structural deflection profiles such as mode shapes to identify damage.

Chung-Jen Lu[12] used a string to simulate the original undamaged structure and several point masses, including a spring, to simulate localised defects. Using the discrete wavelet transform to decompose the string's vibration response at different resolutions, the author observed that the maximum value of the deviation region, the area where the difference in wavelet coefficients between the damaged and undamaged case is significant, is in close proximity to the location of the defect. Quan Wang[13] also used the same approach to locate transverse and through thickness cracks from numerically simulated deflection responses of a beam and plate respectively. The author also noticed that the perturbation in the wavelet coefficients resulting from the crack was sensitive to the spatial resolution or the number of measurement points.

Although these methods demonstrated a wavelet based approach to identify damage from mode shapes, no attempt was made to quantify the damage extent in a single parameter. Hong[14] addressed this issue by applying the wavelet transform to the fundamental mode shape of a beam in order to estimate the Lipschitz exponent so that the extent of damage in the beam could be numerically characterised. In this approach it is envisaged that damage in the beam would cause perturbations in the mode shape at the damage location. These local perturbations manifest themselves as singularities and, although not apparent from the total measured response, can be identified from transient features in the wavelet transformed mode shape response. Using numerical and experimental results the author observed that the damage size is well correlated with the magnitude of the Lipschitz exponent which is a measure of the signals regularity.

The work presented here consists of an extension of Hong's work to the application of damage detection in cantilevered FRP sandwich composite beams. The relationship between the estimated Lipschitz exponent and the damage extent in the FRP sandwich beam is investigated numerically and experimentally using the fundamental flexural beam mode shape. In both cases the continuous wavelet transform (CWT) utilising a second order Gaussian wavelet is applied to the fundamental mode shape. In the experimental case damage is introduced to the FRP sandwich composite beam in the form of a delamination between the core and the laminated skin. Fibre optic Bragg grating strain sensors are embedded at specific points along the centre line of the beam between the core and the laminated skin to measure strain.

The ultimate goal of this investigation is to evaluate the feasibility of using the Lipschitz exponent as a damage sensitive signal feature so that it could be used as an input into a neural network to perform real-time structural health monitoring.

2. WAVELET TRANSFORM

2.1 Continuous wavelet transform

Current Fourier methods such as the short term Fourier transform are unable to provide a high degree of resolution in the time and frequency domain simultaneously. Wavelet transforms on the other hand are able to achieve good time-frequency resolution simultaneously due to its multi-resolution analysis property whereby a signal can be analysed at different resolutions or scales simultaneously using a wavelet. Wavelets are a set of mathematical functions of zero average that decompose a signal into its constituent parts using a set of wavelet basis functions. The family of basis functions used for wavelet analysis is created by both dilating (scaling) and translating (in time) a mother wavelet, thereby providing simultaneous time and frequency information about the analysed signal.

The CWT is an ideal tool to effectively analyse and identify local signal features, such as singularities or transients. As defined by Mallat[15], the CWT for any square-integrable signal $f(x)$ is

$$Wf(u,s) = \int_{-\infty}^{+\infty} f(x)\psi^*_{u,s}(x)dx = \int_{-\infty}^{+\infty} f(x)\frac{1}{\sqrt{s}}\psi^*\left(\frac{x-u}{s}\right)dx . \qquad (1)$$

where $\psi^*(x)$ is the conjugate of $\psi(x)$. The function $\psi_{u,s}(x)$ is dilated by the scaling parameter s and translated by the translation parameter u of the mother wavelet $\psi(x)$. Equation (1) holds true as long as the mother wavelet $\psi(x)$ satisfies the admissibility condition

$$C_\psi = \int_0^{+\infty} \frac{\left|\hat{\psi}(\omega)\right|^2}{|\omega|} d\omega < +\infty . \qquad (2)$$

where $\hat{\psi}(\omega)$ is the Fourier transform of $\psi(x)$. In order to guarantee that the integral in equation (2) is finite, it must be ensured that

$$\hat{\psi}(0) = 0 . \qquad (3)$$

2.2 Lipschitz exponent and singularity measurement

By measuring the signal's local regularity, the CWT is highly adept at identifying discontinuities in a signal. The Lipschitz exponent α is often used as a measure of a signals local regularity and can be defined as follows.

A function f is pointwise Lipschitz $\alpha \geq 0$ at $x = v$ if there exists $K > 0$ and a polynomial p_v of degree m (where m is the largest integer satisfying $m \leq \alpha$) such that

$$\left| f(x) - p_v(x)\right| \leq K\left|x - v\right|^\alpha . \qquad (4)$$

A function that is bounded but discontinuous at v is Lipschitz 0 at v. If the Lipschitz regularity is $\alpha < 1$ at v, then f is not differentiable at v and α characterises the singularity type at v. Therefore it can be inferred that if f is not differentiable at $x = v$, $0 \leq \alpha < 1$.

The presence of a defect in a structure causes its vibration mode shape to change. This change is dependent on the defects location and severity. Small defects are difficult to identify using this approach, but introduce singularities into the vibration mode shape. These singularities can then be characterised using the Lipschitz exponent. Therefore, in order to identify small structural defects, an effective tool is needed to estimate the Lipschitz exponent from the vibration mode shape. In this study the CWT will be used to estimate the Lipschitz exponent.

In order to correctly apply the wavelet transform for Lipschitz exponent estimation, the concept of vanishing moments needs to be introduced. The Lipschitz property in (4) approximates f with a polynomial p_v in the neighbourhood of v

$$f(t) = p_v(x) + \varepsilon_v(x) \text{ with } |\varepsilon_v(x)| \le K\,|x-v|^{\alpha} . \tag{5}$$

The CWT estimates the exponent α by ignoring the polynomial p_v. For this purpose a wavelet $\psi(x)$ having $n > \alpha$ vanishing moments defined by equation (6) is used.

$$\int_{-\infty}^{+\infty} x^k \psi(x)dx = 0 \text{ for } 0 \le k < n . \tag{6}$$

It can be seen from equation (6) that a wavelet with n vanishing moments is orthogonal to polynomials of degree $n-1$. Therefore since $\alpha < n$, the polynomial p_v has degree of at most $n-1$ which results in $Wp_v(u,s) = 0$. Thus the CWT of (5) becomes

$$Wf(u,s) = W\varepsilon_v(u,s) . \tag{7}$$

Hence the vanishing moment property of the wavelet (with sufficiently large n such that $n \ge \alpha$) allows the CWT to focus on the singular part of the function f [14].

Mallat[15] showed that if $f(x)$ is Lipschitz $\alpha \le n$ at $x = v$, then the asymptotic behaviour of $Wf(u,s)$ near $x = v$ becomes

$$|Wf(u,s)| \le A' s^{\alpha+(1/2)} \left(1 + \left|\frac{u-v}{s}\right|^{\alpha}\right) \quad (A' > 0) . \tag{8}$$

If u is in the cone of influence of v, (8) reduces to

$$|Wf(u,s)| \le A s^{\alpha+(1/2)} . \tag{9}$$

High amplitude wavelet coefficients are therefore located within the cone of influence of the singularity. A more convenient form of (9) to estimate the Lipschitz exponent α is

$$\log_2 |Wf(u,s)| \le \log_2 A + (\alpha + \frac{1}{2})\log_2 s . \tag{10}$$

In order to apply (9), singularities need to be identified by following the locus of wavelet transform modulus maxima that converge towards fine scales.

3. DAMAGE DETECTION OF COMPOSITE BEAM STRUCTURES

3.1 Wavelet selection

A representation of a FRP sandwich composite beam is shown in Fig. 1, where a defect is represented as a reduction in stiffness. By applying the theory outlined above, the CWT is used to estimate the Lipschitz exponent from the displacement mode shape of a sandwich composite beam in order to identify and quantify damage. The approach of Hong[14] is adopted to identify the range of the Lipschitz exponent which is derived using simple Euler beam theory. Using this estimation a suitable wavelet is chosen in order to evaluate the Lipschitz exponent.

If $x = v$ is a point on the beam where there is an abrupt change in stiffness as a result of a defect, Hong[14] used Euler beam theory to define a continuity condition across the defect involving displacement, rotation, bending moment and shear force.

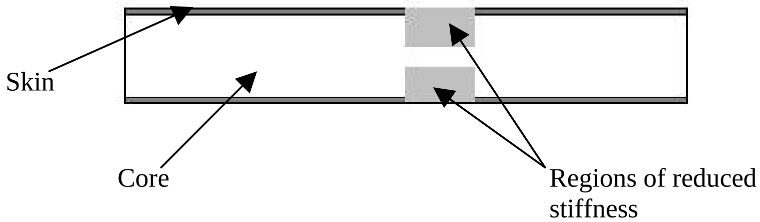


Figure 1: A model of a damaged sandwich composite beam.

It was observed that a change in stiffness would cause the bending moment and shear force to be discontinuous at $x = v$ as they are functions of stiffness. Therefore it was reasoned that only the lateral displacement and its first derivative are continuous because the bending moment and shear force are also functions of the second and third derivative of the lateral displacement. Therefore the Lipschitz exponent α extracted from the damaged beam mode shapes should lie between

$$1 < \alpha < 2 . \tag{11}$$

Equation (11) places the condition that in order to estimate the Lipschitz exponent correctly using the wavelet transform, the required minimum number of vanishing moments for the mother wavelet should be

$$n \geq 2 . \tag{12}$$

Hong[15] noted that wavelets with large numbers of vanishing moments have longer support sizes, so a compromise between vanishing moments and support sizes needs to be considered. Therefore in order to obtain the best localization property for this application a wavelet with $n = 2$ vanishing moments is selected.

Also when considering wavelet selection the continuity of the modulus maxima of $Wf(u,s)$ needs to be taken into account. If a wavelet $\psi(x)$ can be represented as the nth derivative of a Gaussian $\theta(x)$ such that $\psi(x) = (-1)^n (d^n\theta(x)/dx^n)$, the modulus maximum of $Wf(u,s)$ is then said to belong to a connected curve that is never interrupted for fine scales (Mallat[15]). In addition the wavelet $\psi(x)$ is said to have n vanishing moments. Based on these criteria the following wavelet is used as the mother wavelet in subsequent sections to perform CWT

$$\psi(x) = \frac{d^2\theta(x)}{dx^2} . \tag{13}$$

This wavelet is known as the second order Gaussian wavelet.

$$\psi(x) = C_p e^{-x^2} \ . \tag{14}$$

A graph of $\psi(x)$ is shown in Fig. 2.

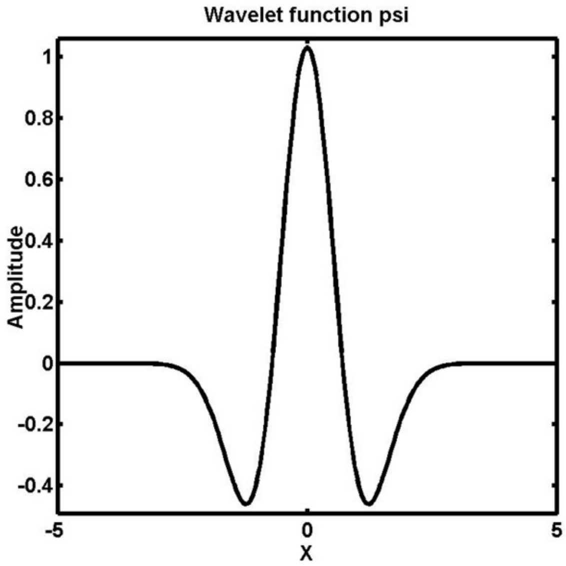
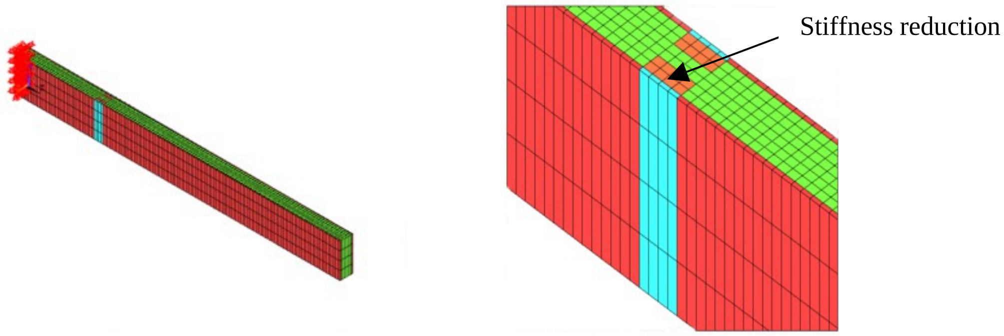


Figure 2: Gaussian wavelet of order two.

3.2 Lipschitz exponent estimation using numerical simulations

Numerical simulations of the sandwich beam's mode shape provide a way of assessing the viability of using the Lipschitz exponent as a signal feature for damage detection before it is applied to experimental mode shapes. The ANSYS finite element (FE) simulation tool is used to obtain the fundamental mode shape of undamaged and damaged FRP sandwich composite beams in order to identify the damage location and assess its severity using the Lipschitz exponent.

The dimension of the beam used in the simulation has a length of 360mm and a width of 33mm. The total thickness of the beam is 14mm which includes a core thickness of 12mm and a laminate skin thickness of 1mm on either side of the core. The laminated skin consists of four equal layers with an orientation of *0º / 90º / 90º / 0º*.



(a) (b)

Figure 3: FE model of composite beam (a) and a zoomed view of the damage area (b).

Modal analysis was performed on the beam in a cantilevered configuration. In order to simulate damage occurring in a sandwich composite beam such as a delamination, an equal percentage reduction in stiffness is introduced to the core and laminated skin 97.5mm away from the clamped end of the beam. The size of the damage introduced was 10mm in length. The one dimensional displacement mode shape was obtained by selecting the nodes in the centre of the beam along the span from the clamped end to the free end.

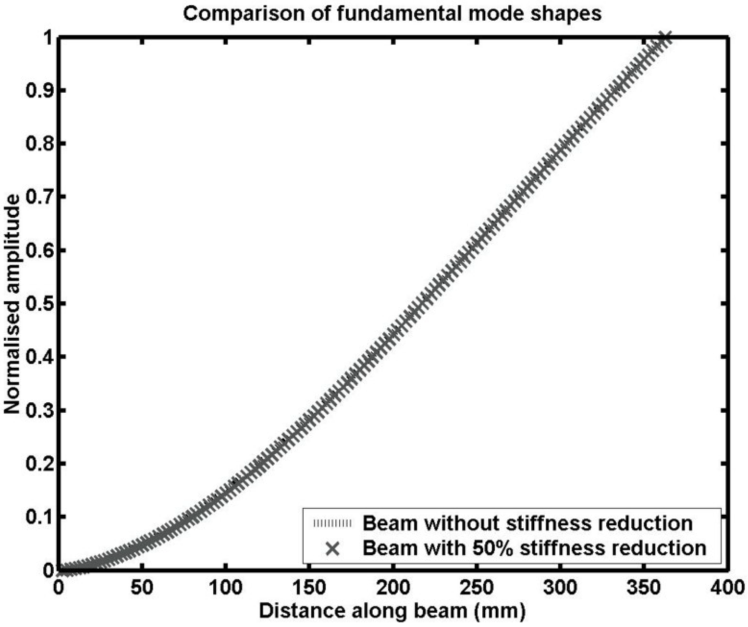


Figure 4: Comparison of fundamental mode shape for an undamaged and damaged beam.

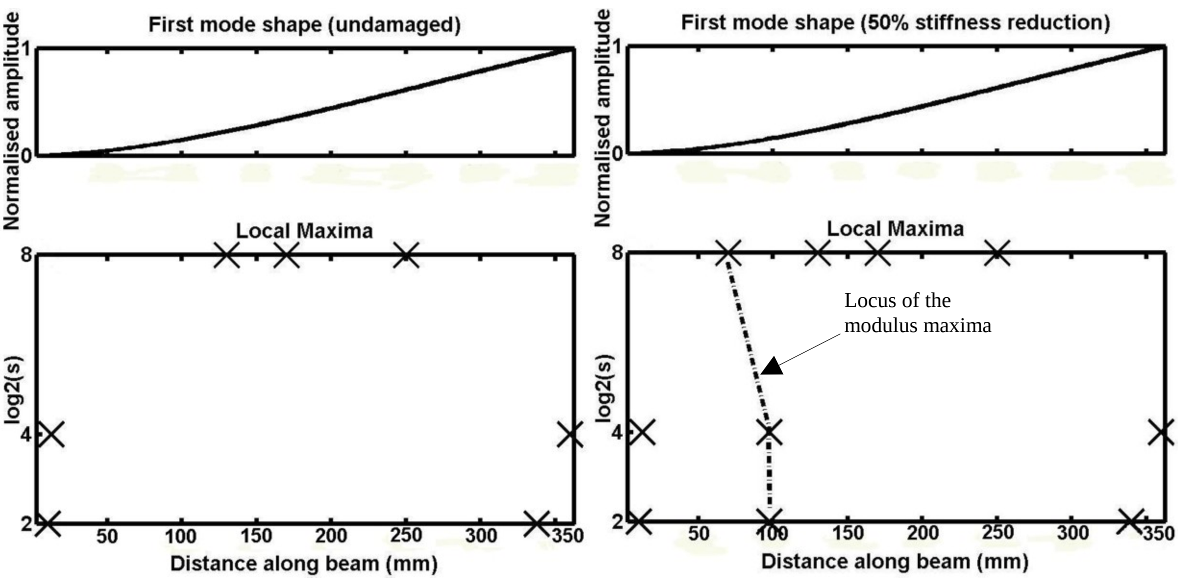


Figure 5: Modulus maxima for the undamaged and damaged beam

Comparing the fundamental mode shape for an undamaged beam and damaged beam with 50% stiffness reduction, no obvious difference can be can be seen in Fig 4. Now the CWT is applied to the mode shapes in Fig 4. using a second order Gaussian wavelet (19). Comparing the modulus maxima of $|Wf(u,s)|$ at fine dyadic scales it can be seen in Fig 5. that, compared with the undamaged beam, the locus of the modulus maxima for the damaged beam's mode shape converge to the position of the damage (97.5mm) at fines scales. This indicates the presence of a singularity or discontinuity at this position which can be attributed to damage which is indeed the case. This observation was verified for a number of damage scenarios ranging from a twenty percent reduction in stiffness to a ninety percent reduction in stiffness.

Now that the position of the damage in the beam has been identified, the next step is to estimate the Lipschitz exponent along the modulus maxima line that lies within the cone of influence of the singularity and which converges to it at fine scales. Using equation (10) and applying linear regression, the Lipschitz exponent α for the beam with a fifty percent reduction in stiffness was estimated to be 1.67. This value lies between the range determined for α in (17) for a damaged beam.

In order to investigate the relationship between the estimated Lipschitz exponent and the severity of damage, a number of damage scenarios were simulated at a fixed position on the beam. The results obtained are shown in Fig. 6.

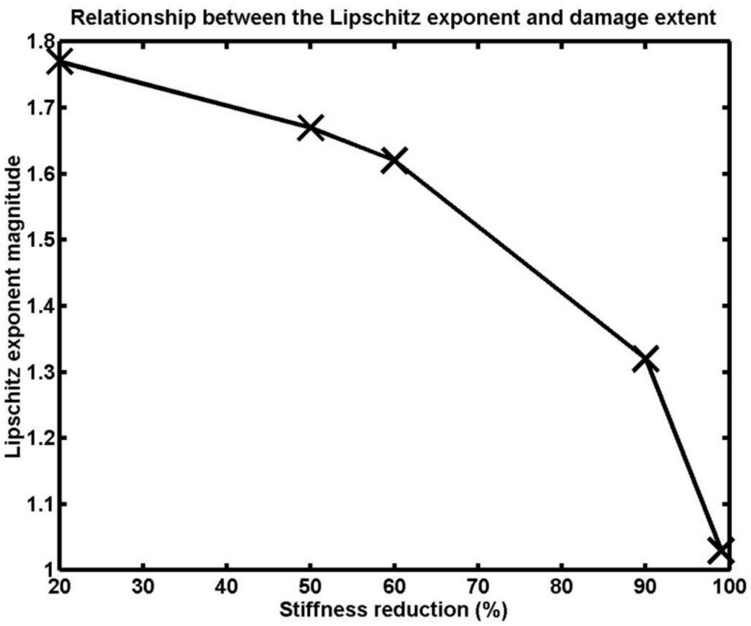


Figure 6: The estimates of the Lipschitz exponent for different degrees of stiffness reduction

It can be observed in Fig. 6 that as the severity of the damage increases the value of the Lipschitz exponent decreases. This is consistent with the physical behaviour of the beam, because increasing the severity of the damage would cause the effect of the singularity to get stronger. Therefore it can be concluded that there is a strong relationship between damage severity in the beam and the Lipschitz exponent.

3.3 Lipschitz exponent estimation using experimental results

In order to validate the use of the Lipschitz exponent as a damage sensitive signal feature experimentally, three FRP sandwich composite beams were manufactured. The beam dimensions were the same as the one used for the numerical simulation. Damage was introduced into two of these specimens in the form of small (width 20mm) and large (width 40mm) debonds between the core and the skin laminate 20mm and 60mm away from the root of the beam respectively.

During manufacture, fibre optic Bragg grating strain sensors were embedded between the core and the laminate at 10mm, 50mm, 145mm and 185mm from the root in order to monitor dynamic strain along the beam.

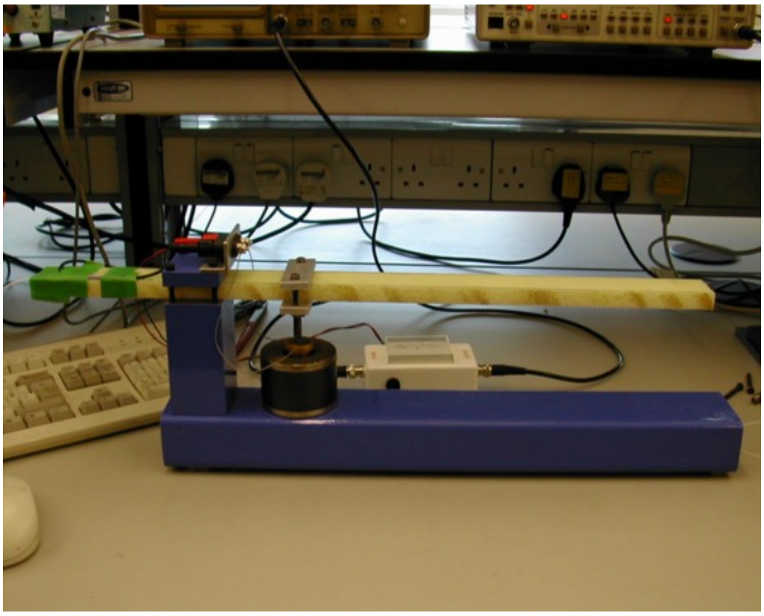


Figure 7: Experimental rig

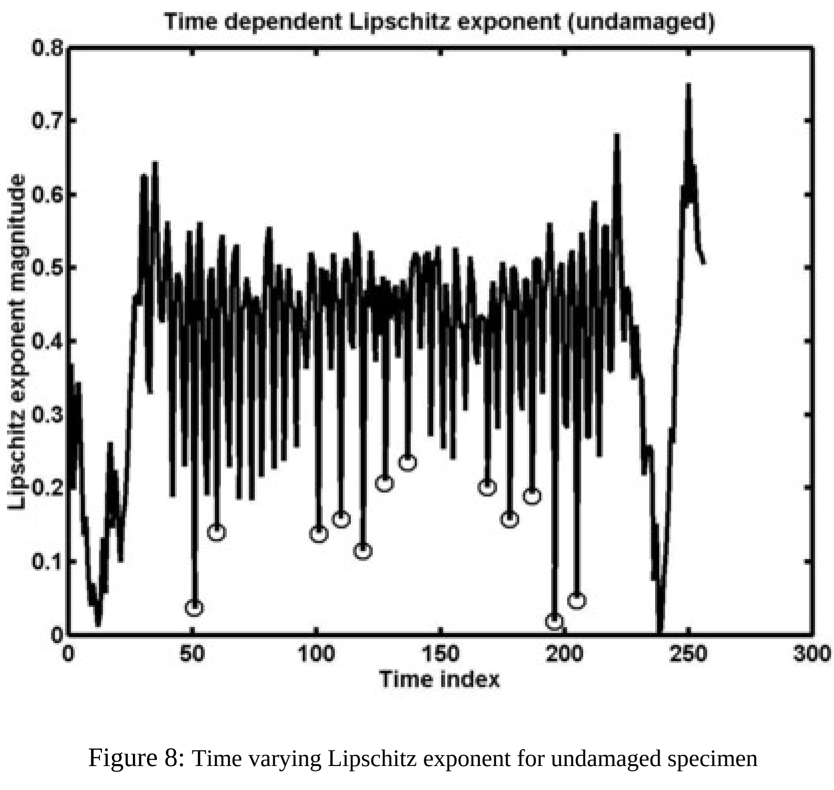


Figure 8: Time varying Lipschitz exponent for undamaged specimen

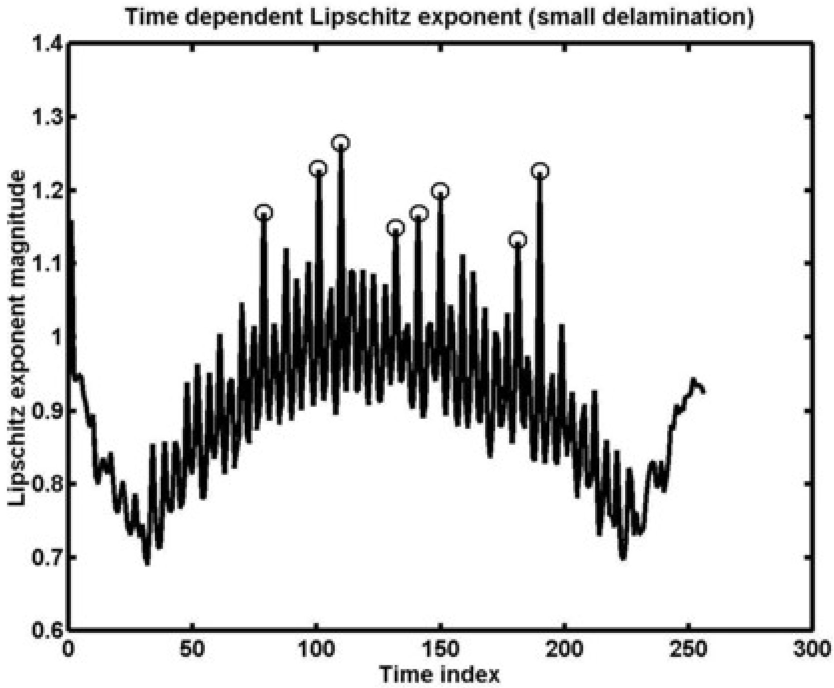


Figure 9: Time varying Lipschitz exponent for specimen with small delamination

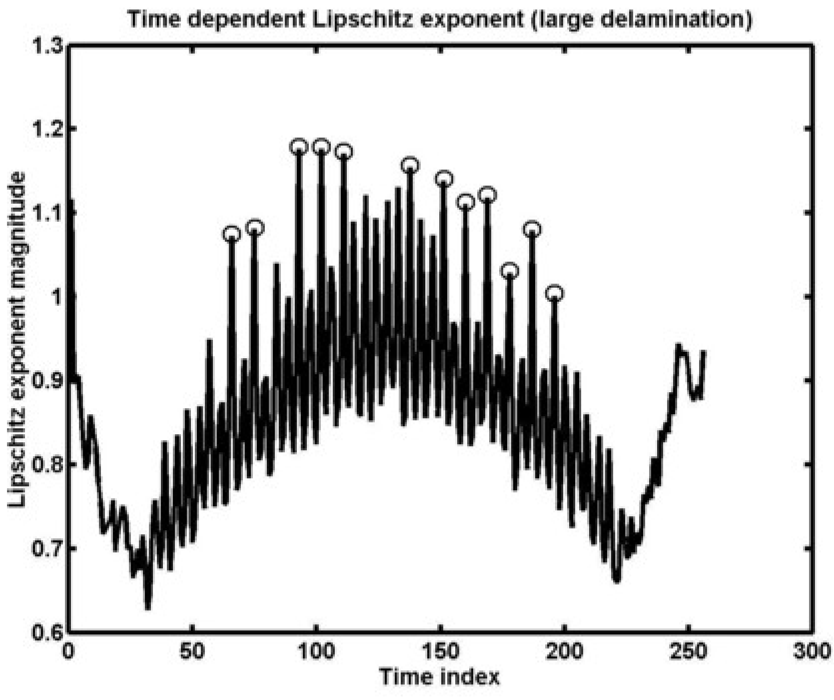


Figure 10: Time varying Lipschitz exponent for specimen with large delamination

The beams were excited in the first mode using a laboratory shaker in a cantilevered configuration. The experimental arrangement is shown in Fig 7. Due to the insufficient number of sensing points the mode shape of the beam could not be obtained. However, it was envisaged that the sensor closest to the damage position would pick up irregularities in the beam vibration due to the effect of delamination close to the sensor.

It is proposed that these irregularities or singularities can be identified as sharp peaks in the variation of the Lipschitz exponent with respect to time and can be quantified using the Lipschitz exponent since the dynamic strain measured is related to the lateral displacement.

The CWT using a second order Gaussian wavelet was applied to samples of the vibration signal, taken from the sensor closest to the damage location, for all three cases and their time dependent Lipschitz exponent calculated and plotted. It can be seen in Fig. 8 that for the undamaged case the sharp peaks, marked with circles, in the Lipschitz exponent do not lie within the range quoted in (11), but in the two damaged cases (Fig. 9 and Fig. 10) the peaks lie within the quoted range for α. Also it can be seen that for the large delamination (Fig. 10), the peaks in the Lipschitz exponent have a slightly lower average value compared with the small delamination (Fig. 9). This confirms the relationship between the Lipschitz exponent and the damage severity predicted in the numerical study.

4. CONCLUSION

The intention of this investigation was to highlight the feasibility of using the Lipschitz exponent, estimated using the continuous wavelet transform, as a damage sensitive signal feature for real-time structural health monitoring of FRP sandwich composites. Taking a sandwich composite beam as an example, we deduced that damage in the beam would cause the magnitude of the Lipschitz exponent to lie between 1 and 2. This result was initially verified using numerical simulations of the first mode shape for composite beams with varying degrees of stiffness reduction at a particular position. A strong relationship between the damage severity and the magnitude of the Lipschitz exponent was also observed. Although the first mode shape couldn’t be obtained experimentally, the dynamic strain information recorded by the sensor closest to the damage location was used to verify the numerical result. As a result of this study further work is underway to implement this technique using digital signal processing hardware for real-time processing of sensor information.

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