Darcy's Law

- Last time
  - Groundwater flow is in response to gradients of mechanical energy
  - Three types
    - Potential
    - Kinetic
      - Kinetic energy is usually not important in groundwater
    - Elastic (compressional)
  - Fluid Potential, Φ
    - Energy per unit mass
  - Hydraulic Head, h
    - Energy per unit weight
    - Composed of
      - » Pressure head
      - » Elevation head

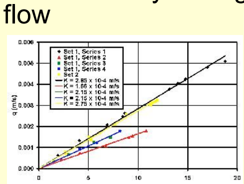
- Today
  - Darcy's Law
  - Hydraulic Conductivity
  - Specific Discharge
  - Seepage Velocity
    - Effective porosity

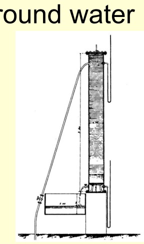
http://biosystems.okstate.edu/darcy/index.htm

Darcy's Law

Henry Darcy, a French hydraulic engineer interested in purifying water supplies using sand filters, conducted experiments to determine the flow rate of water through the filters.

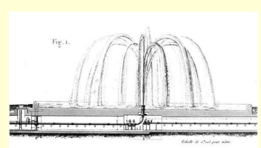
Published in 1856, his conclusions have served as the basis for all modern analysis of ground water flow





A FEW CAREER HIGHLIGHTS:

- In 1828, Darcy was assigned to a deep well drilling project that found water for the city of Dijon, in France, but could not provide an adequate supply for the town. However, under his own initiative, Henry set out to provide a clean, dependable water supply to the city from more conventional spring water sources. That effort eventually produced a system that delivered 8 m^3/min from the Rosoir Spring through 12.7 km of covered aqueduct.

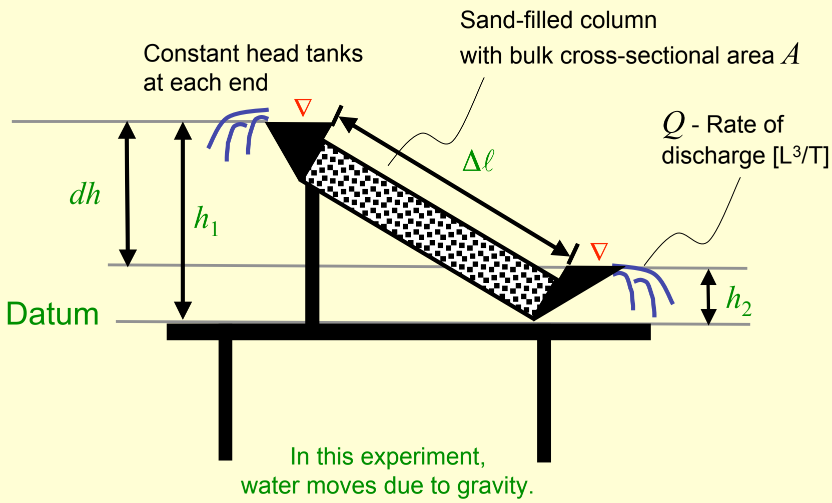


- In 1848 he became Chief Director for Water and Pavements, Paris. In Paris he carried out significant research on the flow and friction losses in pipes, which forms the basis for the Darcy-Weisbach equation for pipe flow.
- He retired to Dijon and, in 1855 and 1856, he conducted the column experiments that established Darcy's law for flow in sands.

Freeze, R. Allen. "Henry Darcy and the Fountains of Dijon." *Ground Water* 32, no.1(1994): 23–30.

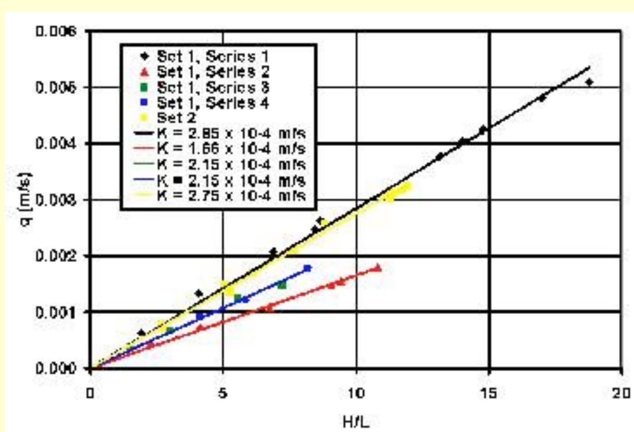
Darcy's Law

Cartoon of a Darcy experiment:



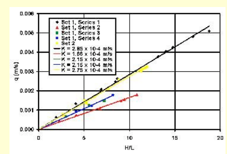
What is the relationship between discharge (flux) Q and other variables?

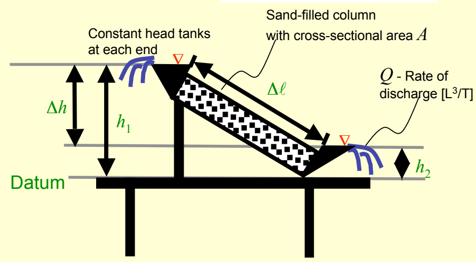
A plot of Darcy's actual data:



http://biosystems.okstate.edu/darcy/index.htm

What is the relationship between discharge (flux) Q and other variables?





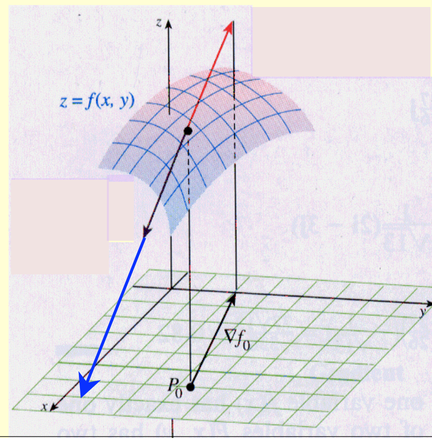
Darcy's Law

Darcy also found that if he used different kinds of sands in the column, discharge Q changed, but for a particular sand, regardless of Q:

Darcy's Law

$$Q = -KA\frac{dh}{dl}$$

Why is there a minus sign in Darcy's Law?



(Bradley and Smith, 1995)

Darcy's Law

Invert Darcy's Law to express conductivity in terms of discharge, area, and gradient:

$$K = \frac{Q}{A}\left[\frac{-1}{dh/d\ell}\right]$$

This is how we measure conductivity:

*Imagine trying to measure gradient in a complex geology with three-dimensional flow and few observation points.

Darcy's Law

What are the dimensions of K? Dimensional analysis:

$$K = -\frac{Q\,d\ell}{A\,dh} = \left[\frac{(L^3T^{-1})(L)}{(L^2)(L)}\right] = \left[\frac{L}{T}\right]$$

Darcy's Law

$$Q = -KA\frac{dh}{dl}$$

This expresses Darcy's Law in terms of discharge. We can also express it in terms of "Darcy velocity" or "specific discharge", that is, discharge per unit bulk area, A:

Darcy's Law

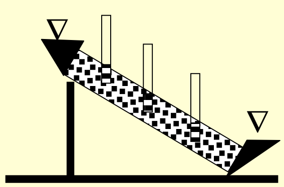
$$Q = -KA\frac{dh}{dl}$$

We will look at three major topics important to Darcy's Law:

- Hydraulic Head Gradient
- Bulk cross-sectional area of flow
- Hydraulic Conductivity (next time)

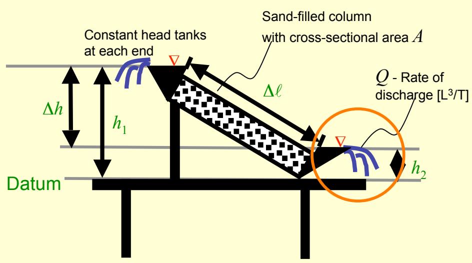
Hydraulic Head

- Head is a measure of the total mechanical energy per unit weight.
- If K, Q and A don't change with distance, then



Hydraulic Head

- In Darcy's experiment, do the drops falling from the constant head tanks have constant velocity?



Hydraulic Head

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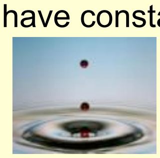
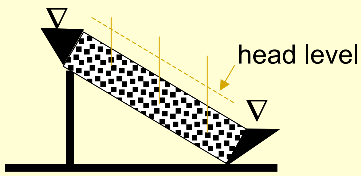


Hydraulic Head

But water through our column has constant velocity—why?



In this experiment, with constant K, Q and A, the head drops linearly with distance, and the specific discharge is constant.

$$q = -K\frac{dh}{dl} = \text{constant}$$

Darcy's Law

- Could we use Darcy's Law to model the falling drops?

- Darcy's Law works because the driving forces (gravity and pressure) in the fluid are balanced by the viscous resistance of the medium.

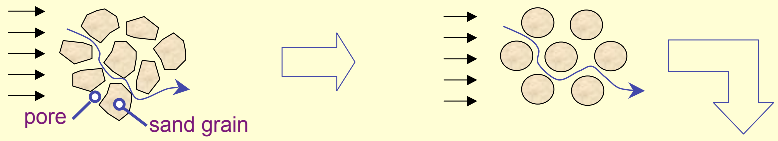
Darcy’s Law

- What happens if the head gradient is too steep?
  - The fluid will have enough energy to accelerate in spite of the resistance of the grains, and inertial forces become important.
  - In this case potential energy (head) is not dissipated linearly with distance and Darcy’s Law does not apply.
- How can we tell when this occurs?

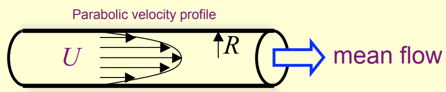
Reynolds Number

•How can we tell when this occurs?

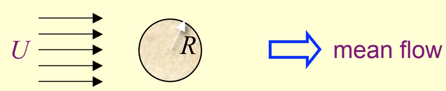
Conceptual Model



- There are two standard, simple models used to explain Darcy's Law, and thus to explore the Reynolds number:
  - Flow in a tube,

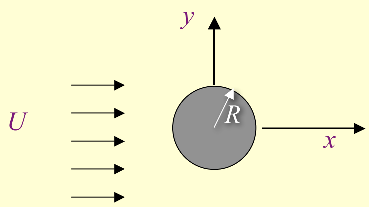


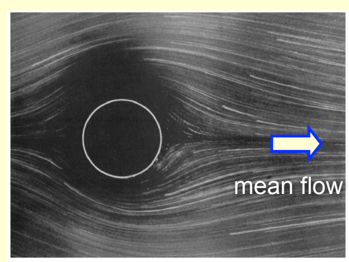
  - Flow around an object, usually a cylinder or sphere,



    - say, representing a grain of sand.

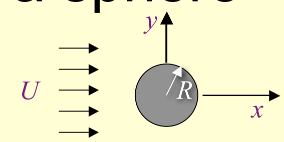
Flow in the vicinity of a sphere





Flow visualization (see slide 32)

Flow in the vicinity of a sphere



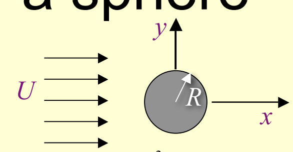
ρ = density [M/L^3]
u = local fluid velocity [L/T]
U = mean approach velocity [L/T]
μ = fluid dynamic viscosity [M/LT]
υ = fluid kinematic viscosity [L^2/T]
x, y = Cartesian coordinates [L],
x in the direction of free stream velocity, U

- Inertial force per unit volume at any location:
  Rate of change of momentum
- Viscous force per unit volume:
- Dynamic similarity: Re =

Re = = constant for similitude

Don't worry about where the expressions for forces come from. H503 students, see Furbish, p. 126

Flow in the vicinity of a sphere



ρ = density [M/L^3]
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U = mean approach velocity [L/T]
μ = fluid dynamic viscosity [M/LT]
υ = fluid kinematic viscosity [L^2/T]
x, y = Cartesian coordinates [L],
x in the direction of free stream velocity, U

Using a dimensional analysis approach, assume that functions vary with characteristic quantities U and R, thus

$u \propto$

$\frac{\partial u}{\partial x} \propto$

$\frac{\partial^2 u}{\partial y^2} \propto$

Flow in the vicinity of a sphere

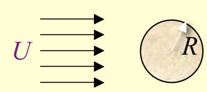
- Inertial force per unit volume at any location: $\rho u \frac{\partial u}{\partial x} \propto$
- Viscous force per unit volume: $\mu \frac{\partial^2 u}{\partial y^2} \propto$
- Dynamic similarity:

$$Re =$$

Reynolds Number

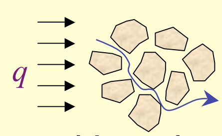
- For a fluid flow past a sphere

$$Re = \frac{\rho UR}{\mu} = \frac{UR}{\nu}$$



- For a flow in porous media?

$$Re =$$



– where characteristic velocity and length are:

q = specific discharge

L = characteristic pore dimension

For sand L usually taken as the mean grain size, d_{50}

Reynolds Number

- When does Darcy’s Law apply in a porous media?
  - For Re < 1 to 10,
  - For Re > 1 to 10,

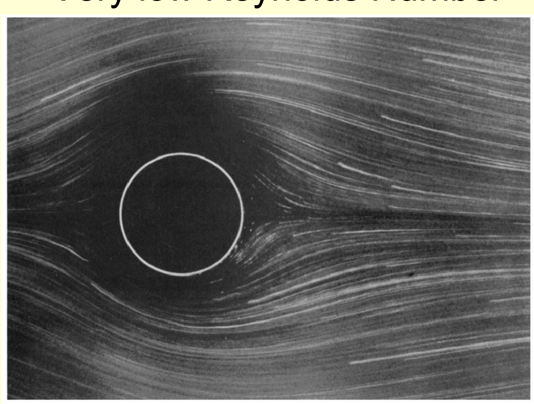
Let’s revisit flow around a sphere to see why this non-linear flow happens.

$$Re = \frac{\rho q d_{50}}{\mu} = \frac{q d_{50}}{\upsilon}$$

Flow in the vicinity of a sphere

Very low Reynolds Number

mean flow



Laminar, linear flow

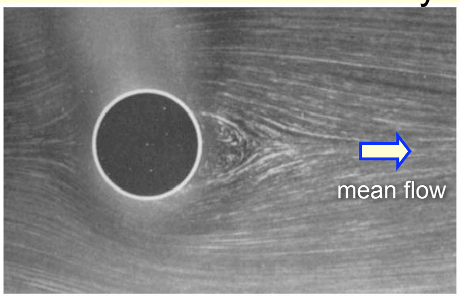
24. Circular cylinder at R=1.54. At this Reynolds number the streamline pattern has clearly lost the fore-and-aft symmetry of figure 6. However, the flow has not yet separated at the rear. That begins at about R=5, though the value is not known accurately. Streamlines are made visible by aluminum powder in water. *Photograph by Sadatoshi Taneda*

$$Re = \frac{\rho U R}{\mu} = \frac{UR}{\upsilon}$$

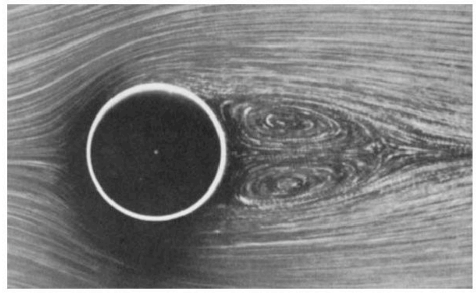
Source: Van Dyke, 1982, An Album of Fluid Motion

Flow in the vicinity of a sphere

Low Reynolds Number



41. Circular cylinder at R**=13.1.** The standing eddies become elongated in the flow direction as the speed increases. Their length is found to increase linearly with Reynolds number until the flow becomes unstable above R=40. *Taneda 1956a*



42. Circular cylinder at R**=26.** The downstream distance to the cores of the eddies also increases linearly with Reynolds number. However, the lateral distance between the cores appears to grow more nearly as the square root. *Photograph by Sadatoshi Taneda*

"In these examples of a flow past a cylinder at low flow velocities a small zone of reversed flow -a separation bubble- is created on the lee side as shown (flow comes from the left)."

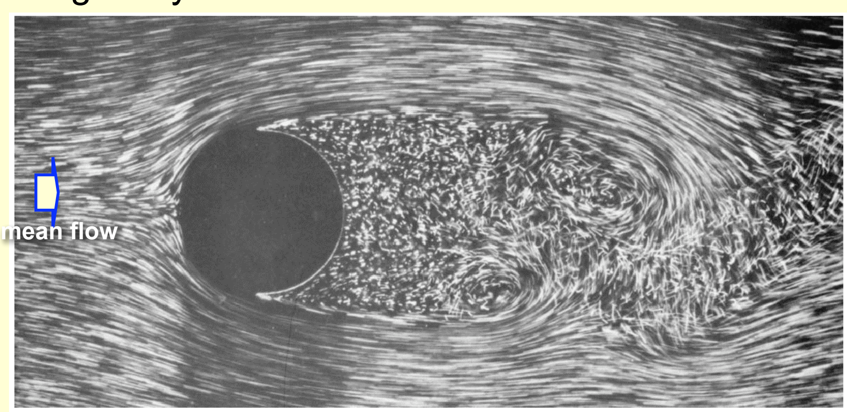
Laminar but non-linear flow. Non-linear because of the separation & eddies

$Re = \frac{\rho UR}{\mu} = \frac{UR}{\upsilon}$; The numerical values of Re for cylinders, are not necessarily comparible to Re for porous media.

Source: Van Dyke, 1982, An Album of Fluid Motion

Flow in the vicinity of a sphere

High Reynolds Number → transition to turbulence



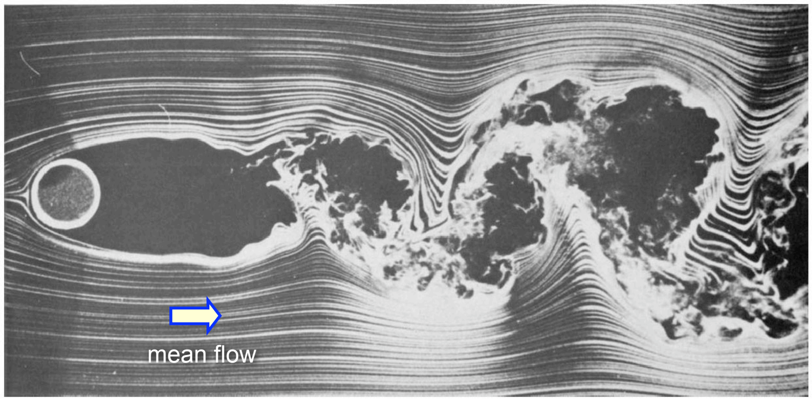
47. Circular cylinder at R**=2000.** At this Reynolds number one may properly speak of a boundary layer. It is laminar over the front, separates, and breaks up into a turbulent wake. The separation points, moving forward as the Reynolds number is increased, have now attained their upstream limit, ahead of maximum thickness. Visualization is by air bubbles in water. *ONERA photograph, Werlé & Gallon 1972*

$Re = \frac{\rho UR}{\mu} = \frac{UR}{\upsilon}$

Source: Van Dyke, 1982, An Album of Fluid Motion

Flow in the vicinity of a sphere

Higher Reynolds Number → Turbulence

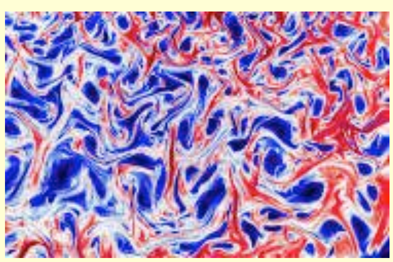


48. Circular cylinder at R**=10,000.** At five times the speed of the photograph at the top of the page, the flow pattern is scarcely changed. The drag coefficient consequently remains almost constant in the range of Reynolds number spanned by these two photographs. It drops later when, as in figure 57, the boundary layer becomes turbulent at separation. *Photograph by Thomas Corke and Hassan Nagib*

$$Re = \frac{\rho U R}{\mu} = \frac{UR}{\upsilon}$$

Source: Van Dyke, 1982, An Album of Fluid Motion

Turbulence



False-color image of (soap film) grid turbulence. Flow direction is from top to bottom, the field of view is 36 by 24 mm. The comb is located immediately above the top edge of the image. Flow velocity is 1.05 m/s.
http://me.unm.edu/~kalmoth/fluids/2dturb-index.html

Experimentally observed velocity variation (here in space)

Velocity
temporal covariance
spatial covariance

Highly chaotic

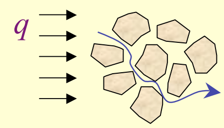
It is difficult to get turbulence to occur in a porous media. Velocity fluctuations are dissipated by viscous interaction with ubiquitous pore walls:

You may find it in flow through bowling ball size sediments, or near the bore of a pumping or injection well in gravel.

Reynolds Number

- When does Darcy's Law apply in a porous media?
  - For Re < 1 to 10,
    - flow is laminar and linear,
    - Darcy's Law applies
  - For Re > 1 to 10,
    - flow is still laminar but no longer linear
    - inertial forces becoming important
      - (e.g., flow separation & eddies)
    - linear Darcy's Law no longer applies

$$Re = \frac{\rho q d_{50}}{\mu} = \frac{q d_{50}}{\upsilon}$$



Notice that Darcy's Law starts to fail when inertial effects are important, even through the flow is still laminar. It is not turbulence, but inertia and that leads to non-linearity. Turbulence comes at much higher velocities.

Specific Discharge, q

$$Q = -KA\frac{dh}{d\ell}$$

Suppose we want to know water "velocity." Divide Q by A to get the volumetric flux density, or <u>specific discharge</u>, often called the Darcy velocity:

$$\frac{Q}{A} = \frac{L^3T^{-1}}{L^2} = \frac{L}{T}$$ → units of velocity

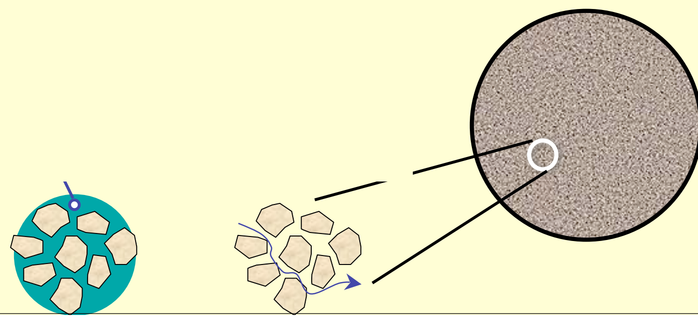
$$\frac{Q}{A} = \boxed{q = -K\frac{dh}{d\ell}}$$

By definition, this is the discharge per unit bulk cross-sectional area.

But if we put dye in one end of our column and measure the time it takes for it to come out the other end, it is much faster suggested by the calculated q – why?

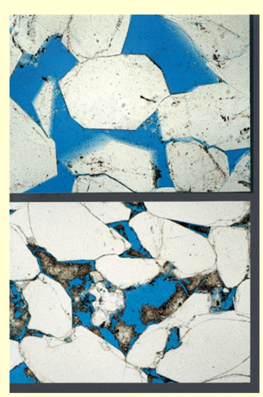
Bulk Cross-section Area, A

$$Q = -KA\frac{dh}{d\ell}$$



Effective Porosity, n_e

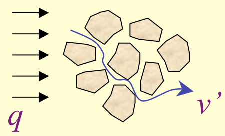
In some materials, some pores may be essentially isolated and unavailable for flow. This brings us to the concept of effective porosity.



$n_e =$

Average or seepage velocity, *v*.

- Actual fluid velocity varies throughout the pore space, due to the connectivity and geometric complexity of that space. This variable velocity can be characterized by its mean or average value.
- The average fluid velocity depends on
  - how much of the cross-sectional area A is made up of pores, and how the pore space is connected
- The typical model for average velocity is:



v = spatial average of local velocity, v'

Darcy's Law

- Review
  - Darcy's Law
    - *Re* restrictions
  - Hydraulic Conductivity
  - Specific Discharge
  - Seepage Velocity
    - Effective porosity

- Next time
  - Hydraulic Conductivity
  - Porosity
  - Aquifer, Aquitard, etc