CS738: Advanced Compiler Optimizations

Data Flow Analysis

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Agenda

- Static analysis and compile-time optimizations

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- For the next few lectures

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- *Intraprocedural* Data Flow Analysis

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  - Classical Examples

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  - Classical Examples
  - Components

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- Input in 3-address format

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  L: . . .
- Arrays, Pointers and Functions to be added later when needed

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  - Typically we use "maximal" basic block (maximal sequence of such instructions)

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- E = set of edges

CFG Edges

- Edge $B_1 \rightarrow B_2 \in E$ if control can transfer from B_1 to B_2

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  - Single procedure, single flow graph for now.

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    - Program point after a stmt is same as the program point before the next stmt

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  - For B_1 and B_2:
    - if there is an edge from B_1 to B_2 in CFG, then the program point *after* the last stmt of B_1 *may be* followed immediately by the program point *before* the first stmt of B_2.

Data Flow Abstraction: Execution Paths

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- Infinite number of possible execution paths in practical programs.
- Paths having no finite upper bound on the length.
- Need to *summarize* the information at a program point with a finite set of facts.

Data Flow Schema

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- Different domains for different analyses/optimizations

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  - Why not exact solution?

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- f_s depends on the statement and the analysis

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- $\text{IN}[s_1]$, $\text{OUT}[s_n]$ to come later

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- $f \circ g$: Composition of functions f and g
- $\bigoplus$: An abstract operator denoting some way of combining facts present in a set .

Data Flow Constraints: Basic Blocks

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- **Forward**
  - For B consisting of $s_1, s_2, \ldots, s_n$

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  - Control flow constraints

$$\mathrm{IN}[B] = \bigoplus_{P \in \mathrm{PRED}(B)} \mathrm{OUT}[P]$$

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- **Backward**

$$f_B = f_{s_1} \circ f_{s_2} \circ \ldots \circ f_{s_n}$$

$$\mathrm{IN}[B] = f_B(OUT[B])$$

$$\mathrm{OUT}[B] = \bigoplus_{S \in \mathrm{SUCC}(B)} IN[S]$$

Data Flow Equations

- Typical Equation

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- Example:

```
a = b*c // generates expression b * c
c = 5   // kills expression b*c
d = b*c // is b*c redundant here?
```

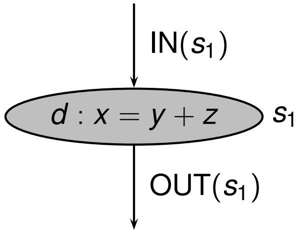
Example Data Flow Analysis

- Reaching Definitions Analysis
- Definition of a variable x: $x = \dots$something$\dots$
- Could be more complex (e.g. through pointers, references, implicit)

Reaching Definitions Analysis

- A definition d reaches a point p if
  - there is a path from the point *immediately following* d to p
  - d is not “killed” along that path
  - “Kill” means redefinition of the left hand side (x in the earlier example)

RD Analysis of a Structured Program



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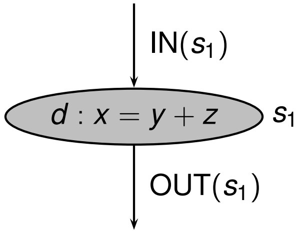

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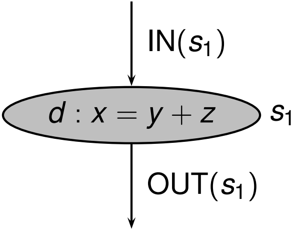

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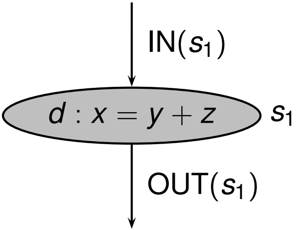

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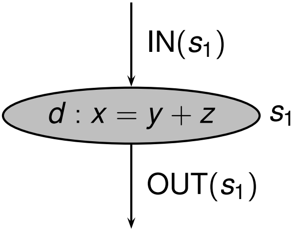

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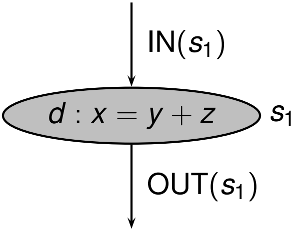

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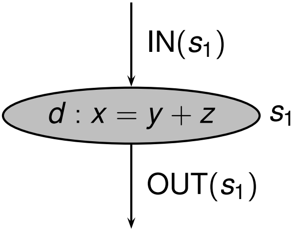

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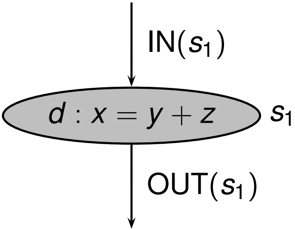

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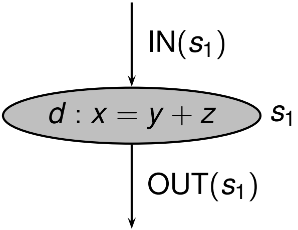

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\text{KILL}(s_1) &= D_x? \ \text{ will also work here} \\
&\qquad\quad \text{but may not work in general}
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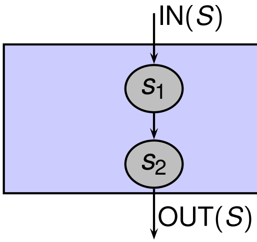

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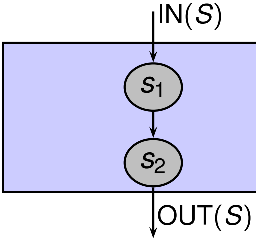

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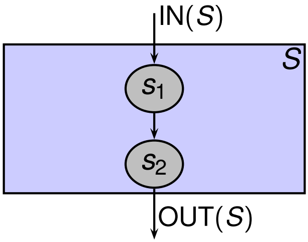
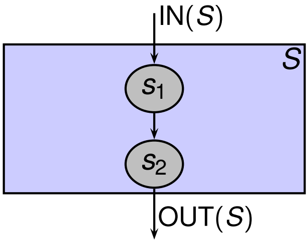


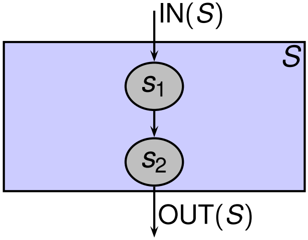
$$\text{GEN}(S) \quad = \quad \text{GEN}(s_1) - \text{KILL}(s_2) \cup \text{GEN}(s_2)$$

RD Analysis of a Structured Program



$$
\begin{aligned}
\text{GEN}(S) &= \text{GEN}(s_1) - \text{KILL}(s_2) \cup \text{GEN}(s_2) \\
\text{KILL}(S) &=
\end{aligned}
$$

RD Analysis of a Structured Program



$$
\begin{aligned}
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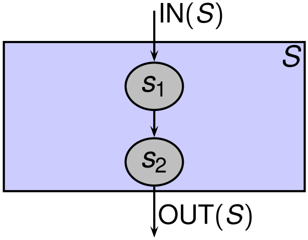
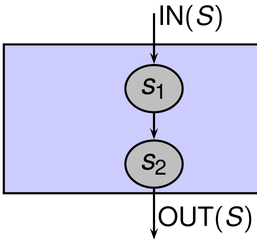
\end{aligned}
$$

RD Analysis of a Structured Program



$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) - \mathrm{KILL}(s_2) \cup \mathrm{GEN}(s_2)\\
\mathrm{KILL}(S) &= \mathrm{KILL}(s_1) - \mathrm{GEN}(s_2) \cup \mathrm{KILL}(s_2)\\
\mathrm{IN}(s_1) &=
\end{aligned}
$$

RD Analysis of a Structured Program



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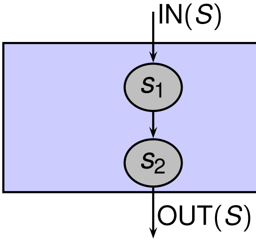
\end{aligned}
$$

RD Analysis of a Structured Program



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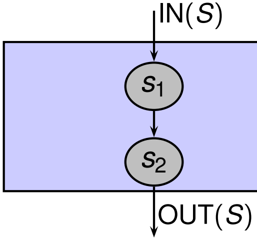
\end{aligned}$$

RD Analysis of a Structured Program



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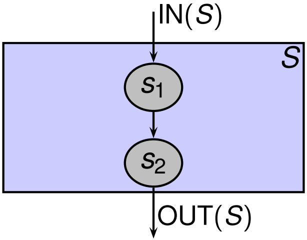
\mathrm{IN}(s_1) &= \mathrm{IN}(S) \\
\mathrm{IN}(s_2) &= \mathrm{OUT}(s_1)
\end{aligned}
$$

RD Analysis of a Structured Program



$$\begin{aligned}
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\text{IN}(s_1) &= \text{IN}(S) \\
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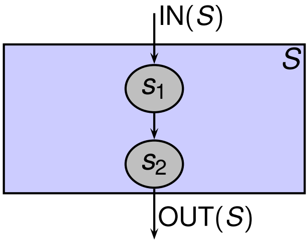
\text{OUT}(S) &=
\end{aligned}$$

RD Analysis of a Structured Program



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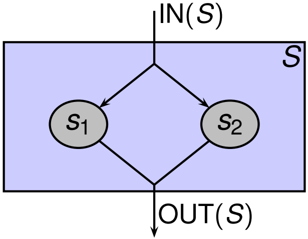
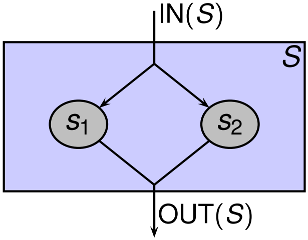
\mathrm{OUT}(S) &= \mathrm{OUT}(s_2)
\end{aligned}
$$

RD Analysis of a Structured Program

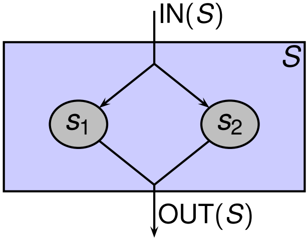


RD Analysis of a Structured Program



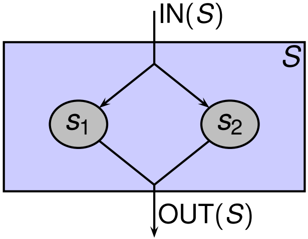
$$\mathrm{GEN}(S) \quad =$$

RD Analysis of a Structured Program



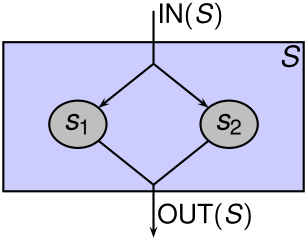
$$\text{GEN}(S) \quad = \quad \text{GEN}(s_1) \cup \text{GEN}(s_2)$$

RD Analysis of a Structured Program



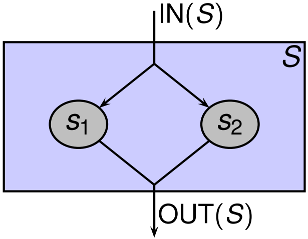
$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \cup \mathrm{GEN}(s_2) \\
\mathrm{KILL}(S) &=
\end{aligned}
$$

RD Analysis of a Structured Program



$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \cup \mathrm{GEN}(s_2) \\
\mathrm{KILL}(S) &= \mathrm{KILL}(s_1) \cap \mathrm{KILL}(s_2)
\end{aligned}
$$

RD Analysis of a Structured Program



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\mathrm{IN}(s_1) &=
\end{aligned}
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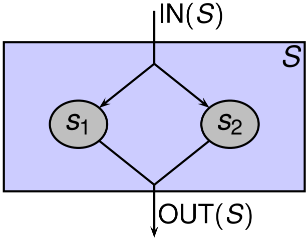

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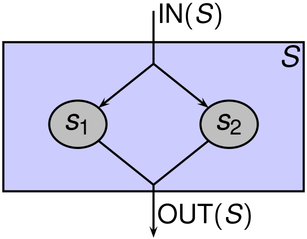
\mathrm{IN}(s_1) &= \mathrm{IN}(s_2) = \mathrm{IN}(S)
\end{aligned}
$$

RD Analysis of a Structured Program



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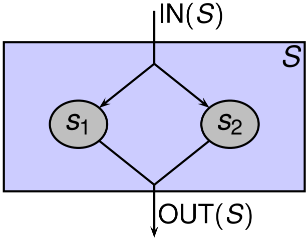
\text{IN}(s_1) &= \text{IN}(s_2) \quad = \quad \text{IN}(S) \\
\text{OUT}(S) &=
\end{aligned}
$$

RD Analysis of a Structured Program



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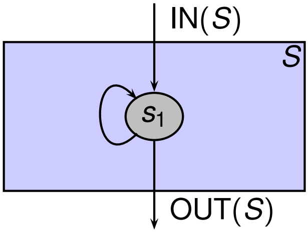
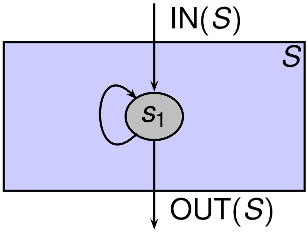
\text{OUT}(S) &= \text{OUT}(s_1) \cup \text{OUT}(s_2)
\end{aligned}
$$

RD Analysis of a Structured Program

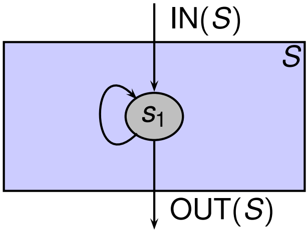


RD Analysis of a Structured Program



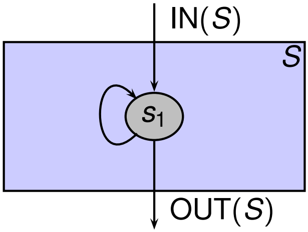
$$\text{GEN}(S) \quad =$$

RD Analysis of a Structured Program



$$\text{GEN}(S) \quad = \quad \text{GEN}(s_1)$$

RD Analysis of a Structured Program



$$
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\mathrm{KILL}(S) &=
\end{aligned}
$$

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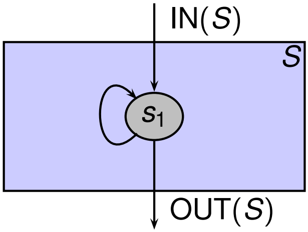

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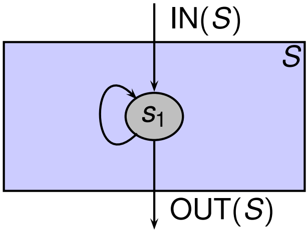

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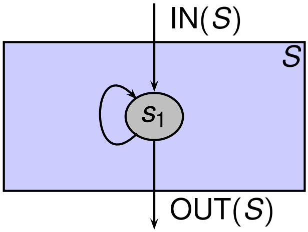

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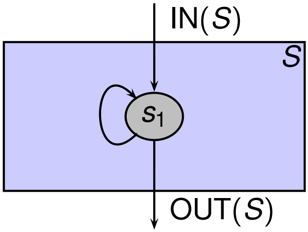
RD Analysis of a Structured Program



$$\begin{aligned}
\text{GEN}(S) &= \text{GEN}(s_1) \\
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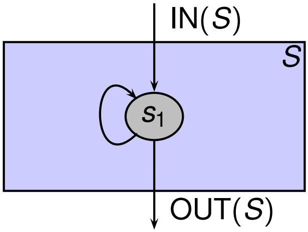
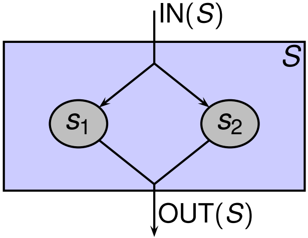
\text{OUT}(S) &= \text{OUT}(s_1) \\
\text{IN}(s_1) &=
\end{aligned}$$

RD Analysis of a Structured Program



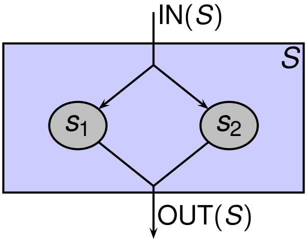
$$
\begin{array}{rcl}
\mathrm{GEN}(S) & = & \mathrm{GEN}(s_1) \\
\mathrm{KILL}(S) & = & \mathrm{KILL}(s_1) \\
\mathrm{OUT}(S) & = & \mathrm{OUT}(s_1) \\
\mathrm{IN}(s_1) & = & \mathrm{IN}(S) \cup \mathrm{GEN}(s_1)
\end{array}
$$

RD Analysis is Approximate



- Assumption: All paths are feasible.

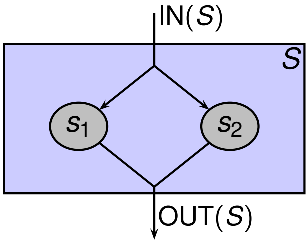
RD Analysis is Approximate



- Assumption: All paths are feasible.
- Example:

```
if (true) s1;
else      s2;
```

RD Analysis is Approximate



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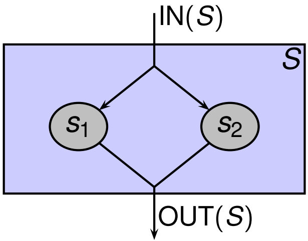

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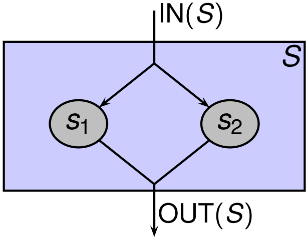
Fact		Computed		Actual
GEN(S)	$=$	GEN(s_1) $\cup$ GEN(s_2)	$\supseteq$	GEN(s_1)

RD Analysis is Approximate



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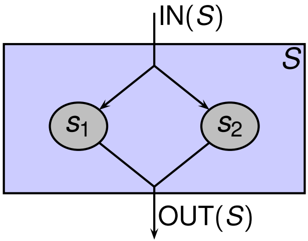
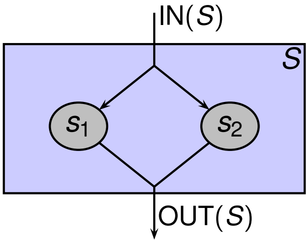
Fact		Computed		Actual
GEN(S)	$=$	GEN(s_1) $\cup$ GEN(s_2)	$\supseteq$	GEN(s_1)
KILL(S)	$=$	KILL(s_1) $\cap$ KILL(s_2)	$\subseteq$	KILL(s_1)

RD Analysis is Approximate



- Thus,

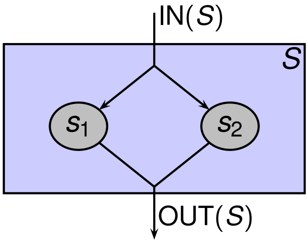
RD Analysis is Approximate



- Thus,

  true GEN(S) $\subseteq$ analysis GEN(S)

RD Analysis is Approximate



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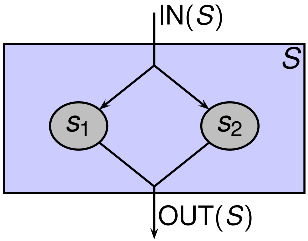
RD Analysis is Approximate



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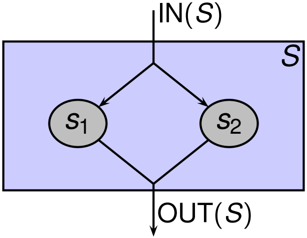
- More definitions computed to be reaching than actually do!

RD Analysis is Approximate



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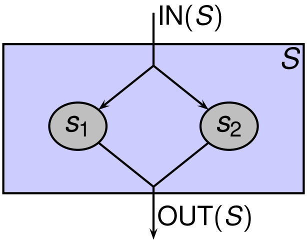
RD Analysis is Approximate



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- More definitions computed to be reaching than actually do!
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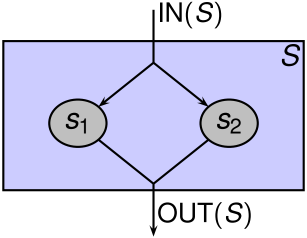
  - prevents optimizations

RD Analysis is Approximate



- Thus,

  true GEN(S) $\subseteq$ analysis GEN(S)
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- More definitions computed to be reaching than actually do!
- Later we shall see that this is SAFE approximation
  - prevents optimizations
  - but NO wrong optimization

RD at BB level

- A definition d can reach the start of a block from any of its predecessor

$$\text{IN}(B) = \bigcup_{P \in \text{PRED}(B)} \text{OUT}(P)$$

RD at BB level

- A definition d can reach the start of a block from any of its predecessor
  - if it reaches the end of some predecessor

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RD at BB level

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$$\text{IN}(B) = \bigcup_{P \in \text{PRED}(B)} \text{OUT}(P)$$

- A definition d reaches the end of a block if

$$\text{OUT}(B) = \text{IN}(B) - \text{KILL}(B) \cup \text{GEN}(B)$$

RD at BB level

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  - either it is generated in the block

$$\mathrm{OUT}(B) = \mathrm{IN}(B) - \mathrm{KILL}(B) \cup \mathrm{GEN}(B)$$

RD at BB level

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  - if it reaches the end of some predecessor

$$\text{IN}(B) = \bigcup_{P \in \text{PRED}(B)} \text{OUT}(P)$$

- A definition d reaches the end of a block if
  - either it is generated in the block
  - or it reaches block and not killed

$$\text{OUT}(B) = \text{IN}(B) - \text{KILL}(B) \cup \text{GEN}(B)$$

Solving RD Constraints

- KILL & GEN known for each BB.

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- KILL & GEN known for each BB.
- A program with N BBs has $2N$ equations with $2N$ unknowns.
  - Solution is possible.
  - Iterative approach (on the next slide).

for each block B {

```
for each block B {
    OUT(B) = ∅;
```

```
for each block B {
    OUT(B) = ∅;
}
OUT(Entry) = ∅; // note this for later discussion
```

```
for each block B {
    OUT(B) = ∅;
}
OUT(Entry) = ∅; // note this for later discussion
change = true;
while (change) {
    change = false;
```

```
for each block B {
    OUT(B) = ∅;
}
OUT(Entry) = ∅; // note this for later discussion
change = true;
while (change) {
    change = false;
    for each block B other than Entry {
```

```
for each block B {
    OUT(B) = ∅;
}
OUT(Entry) = ∅; // note this for later discussion
change = true;
while (change) {
    change = false;
    for each block B other than Entry {
        IN(B) = ⋃_{P∈PRED(B)} OUT(P);
```

```
for each block B {
    OUT(B) = ∅;
}
OUT(Entry) = ∅; // note this for later discussion
change = true;
while (change) {
    change = false;
    for each block B other than Entry {
        IN(B) = ⋃_{P∈PRED(B)} OUT(P);
        oldOut = OUT(B);
        OUT(B) = IN(B) − KILL(B) ∪ GEN(B);
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for each block B {
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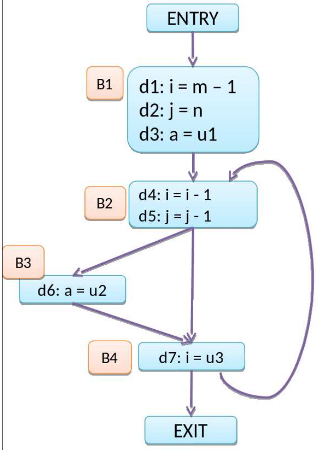
        oldOut = OUT(B);
        OUT(B) = IN(B) − KILL(B) ∪ GEN(B);
        if (OUT(B) ≠ oldOut) then {
            change = true;
        }
    }
}
```

Reaching Definitions: Example



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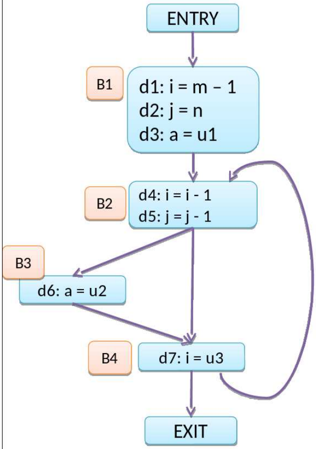

BB	GEN	KILL
B1		
B2		
B3		
B4		

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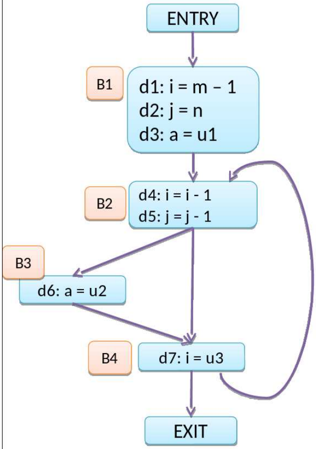

BB	GEN	KILL
B1	{d1, d2, d3}	
B2		
B3		
B4		

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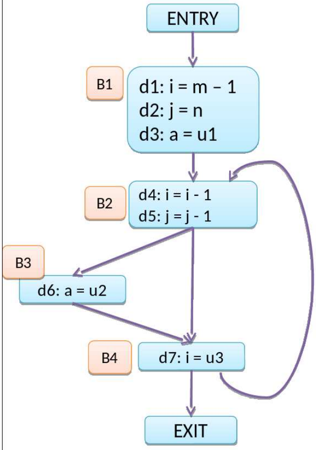

BB	GEN	KILL
B1	{d1, d2, d3}	{d4, d5, d6, d7}
B2		
B3		
B4		

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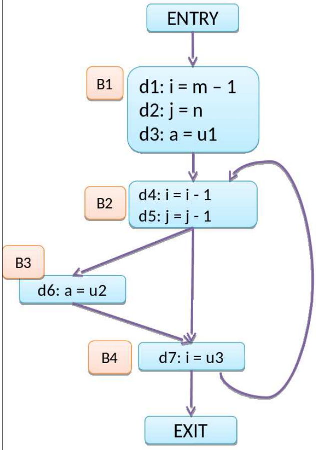

BB	GEN	KILL
B1	{d1, d2, d3}	{d4, d5, d6, d7}
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B3		
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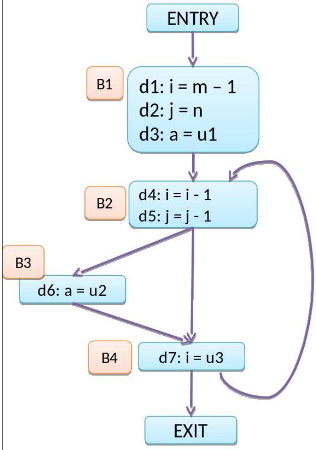

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B3		
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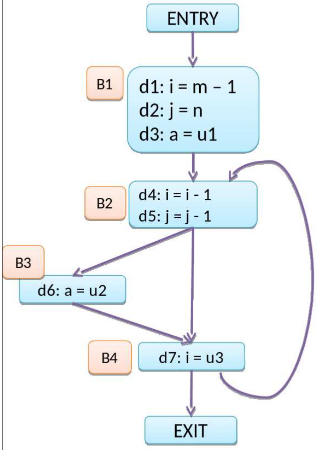

BB	GEN	KILL
B1	{d1, d2, d3}	{d4, d5, d6, d7}
B2	{d4, d5}	{d1, d2, d7}
B3	{d6}	
B4		

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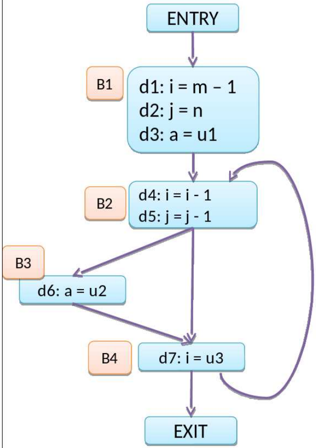

BB	GEN	KILL
B1	{d1, d2, d3}	{d4, d5, d6, d7}
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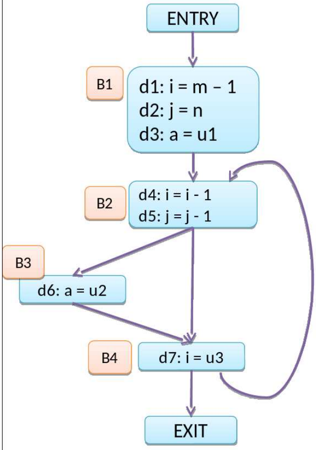

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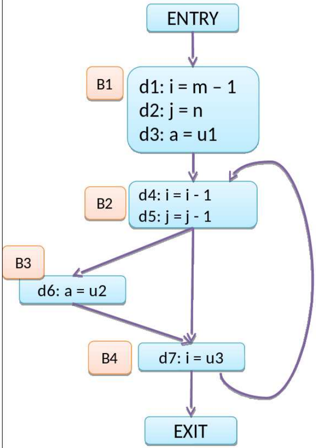

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B4	{d7}	{d1, d4}

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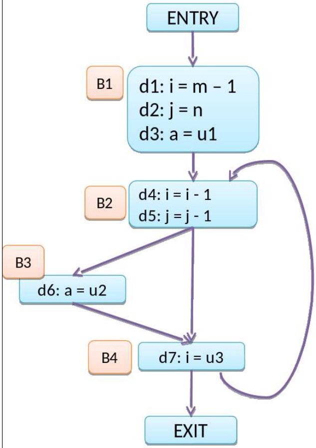
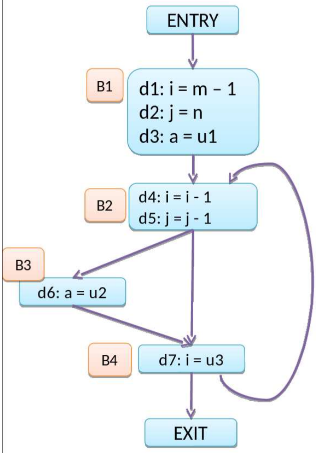


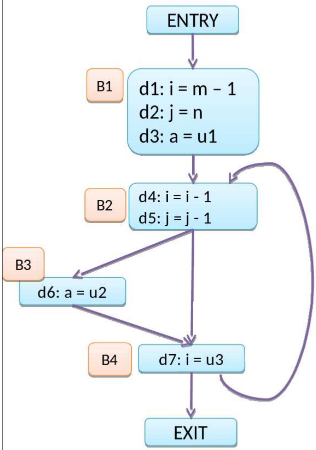
Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

Reaching Definitions: Example



Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	∅	∅	∅	∅
1	IN	∅	d1, d2, d3	d3, d4, d5	d3, d4, d5, d6
	OUT	d1, d2, d3	d3, d4, d5	d4, d5, d6	d3, d5, d6, d7

Reaching Definitions: Example



Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	∅	∅	∅	∅
1	IN	∅	d1, d2, d3	d3, d4, d5	d3, d4, d5, d6
	OUT	d1, d2, d3	d3, d4, d5	d4, d5, d6	d3, d5, d6, d7
2	IN	∅	d1, d2, d3, d5, d6, d7	d3, d4, d5, d6	d3, d4, d5, d6
	OUT	d1, d2, d3	d3, d4, d5, d6	d4, d5, d6	d3, d5, d6, d7

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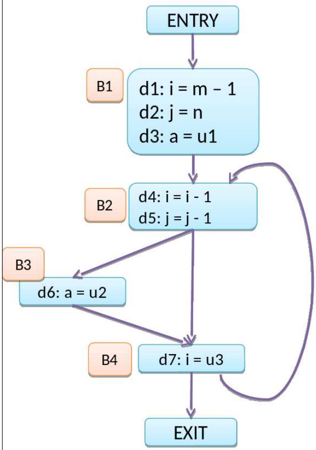
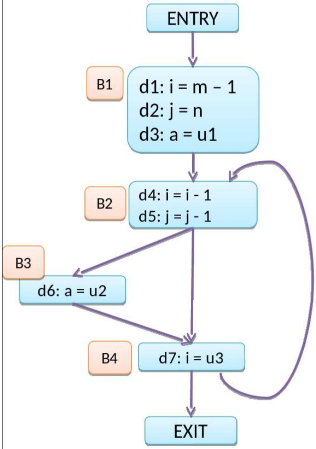


Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	∅	∅	∅	∅
1	IN	∅	d1, d2, d3	d3, d4, d5	d3, d4, d5, d6
	OUT	d1, d2, d3	d3, d4, d5	d4, d5, d6	d3, d5, d6, d7
2	IN	∅	d1, d2, d3, d5, d6, d7	d3, d4, d5, d6	d3, d4, d5, d6
	OUT	d1, d2, d3	d3, d4, d5, d6	d4, d5, d6	d3, d5, d6, d7
3	IN	∅	d1, d2, d3, d5, d6, d7	d3, d4, d5, d6	d3, d4, d5, d6
	OUT	d1, d2, d3	d3, d4, d5, d6	d4, d5, d6	d3, d5, d6, d7

Reaching Definitions: Bitvectors



a bit for each definition:

d1	d2	d3	d4	d5	d6	d7

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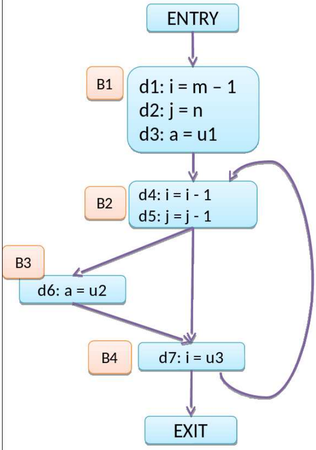

a bit for each definition:

d1	d2	d3	d4	d5	d6	d7

Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	0000000	0000000	0000000	0000000
1	IN	0000000	1110000	0011100	0011110
	OUT	1110000	0011100	0001110	0010111
2	IN	0000000	1110111	0011110	0011110
	OUT	1110000	0011110	0001110	0010111
3	IN	0000000	1110111	0011110	0011110
	OUT	1110000	0011110	0001110	0010111

Reaching Definitions: Bitvectors

- Set-theoretic definitions:

$$\mathrm{IN}(B) = \bigcup_{P \in \mathrm{PRED}(B)} \mathrm{OUT}(P)$$

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- Bitwise $\vee, \wedge, \neg$ operators

Reaching Definitions: Application

Constant Folding

```
while changes occur {
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```

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         that reaches S and is a constant C {
            replace B by C in S;
            if all operands of S are constant {
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Constant Folding

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while changes occur {
   forall the stmts S of the program {
      foreach operand B of S {
         if there is a unique definition of B
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            if all operands of S are constant {
               replace rhs by eval(rhs);
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      foreach operand B of S {
         if there is a unique definition of B
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            replace B by C in S;
            if all operands of S are constant {
               replace rhs by eval(rhs);
               mark definition as constant;
}}}}}
```

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- Safety? Profitability?

Reaching Definitions: Summary

- $\text{GEN}(B) = \left\{ d_x \;\middle|\; \begin{array}{l} d_x \text{ in } B \text{ defines variable } x \text{ and is not} \\ \text{followed by another definition of } x \text{ in } B \end{array} \right\}$

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- meet ($\bigwedge$) operator: The operator to combine information coming along different predecessors is $\cup$
- What about the *Entry* block?

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- Why?