DAU: user manual

Harun Šiljak

November 9, 2011

If you want to make a quick start with Dau, don't read the entire manual - simply read the help file (i.e. part II of the manual). The manual is here to tell you more about the game, the characters, history and mathematics.

Figure 1: In case you were interested: the license plates are from the Estonian SSR

Contents

Part I
The History

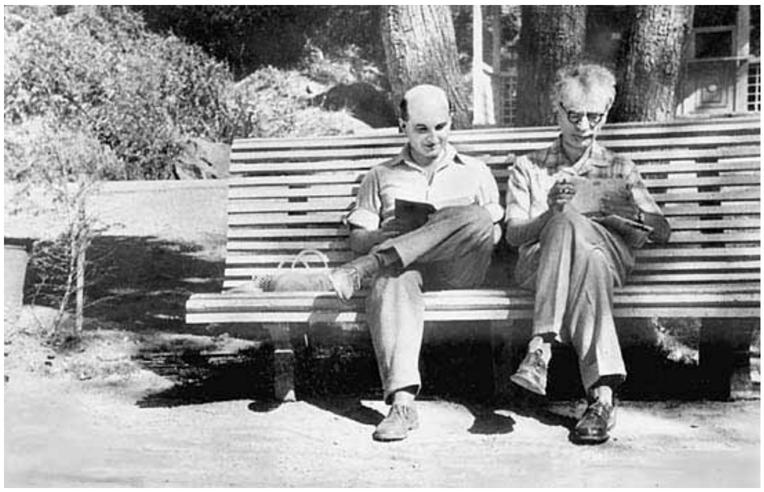

Figure 2: Lifshitz (left) and Landau (right) on a car trip in the Caucasus region

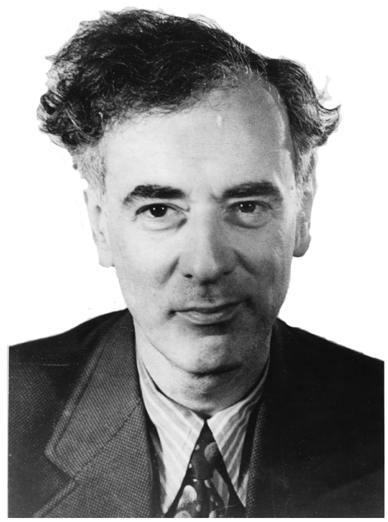

Figure 3: Lev Davidovich Landau

1 Characters involved

Three curious characters appear in DAU, and it would be fair to introduce them here. Each one will be presented with a photo, brief biography and an anecdote. Of course, it was hard to select only one anecdote per person (especially in case of Lev Landau).

1.1 Lev Davidovich Landau

Lev Davidovich Landau (Baku 1908 - Moscow 1968), Soviet physicist, Academician, Hero of Socialist Labour, Stalin, Lenin and Nobel prize laureate. Friends called him simply Dau. He won the Nobel prize in 1962 for his work on superfluids. He is well known for his work in quantum mechanics, theory of phase transitions, quantum electrodynamics... Erudite and active thinker in every meaning of that word. His school of theoretical physics was a great place for making of brilliant scientists, his comprehensive exams in all branches of physics called "The theoretical minimum" - shorter, Teorminimum. Only 43 men ever passed all the exams, as you can see in the list in figure 4.

Landau once attended a session of the Russian Academy of Sciences at which the notorious agronomist Trofim Lysenko (the founder of "creative Darwinism") gave a lecture on the so-called inheritance of acquired traits. When the talk was over, Landau asked a penetrating question: "You argue that if we will cut off the ear of a cow, and the ear of its offspring, and so on, sooner or later the earless cows will start to be born?" "Yes, that's right," Lysenko replied. "Then," Landau continued, "how do you explain the virgins that are still being born?"

Figure 4: Landau keeping track of Teorminimum students

1.2 Evgeny Mikhailovich Lifshitz

Evgeny Mikhailovich Lifshitz (Kharkiv 1915 - Moscow 1985), Soviet physicist, Academician, Stalin and Lenin prize laureate. His brother was Ilya Mikhailovich Lifshitz, another well known physicist. Coauthored the Course of Theoretical Physics with Landau, one of the first people who passed Landau's Teorminimum, made important contributions to general relativity, phase transitions and other branches of theoretical physics.

On one of Bohr's visits to the USSR (figure 6) he was asked how he had succeeded in making an excellent school of theoretical physics. His answer was that he was never afraid to tell his students that he's a fool. Lifshitz was the official translator and mistranslated it, saying "He was never afraid to tell his students that they are fools". Only after the academicians listening the lecture started laughing, Lifshitz realized that he made a mistake and apologized. Pyotr Kapitsa commented: "No, Lifshitz, you didn't make a mistake. Exactly that is the difference between Bohr's and Landau's school of physics!"

1.3 Igor Yevgenyevich Tamm

Igor Yevgenyevich Tamm (Vladivostok 1895 - Moscow 1971), Soviet physicist, Academician, Hero of Socialist Labour, Stalin and Nobel prize laureate. Together with Pavel Cherenkov and Ilya Frank, Tamm won the Nobel prize in 1958 for the discovery of Cherenkov radiation. He worked on problems of thermonuclear reactors (plasma preservation, tokamak).

Tamm has nothing to do with Landau's game. It's just that I loved that photo shown in figure 7 - both Lifshitz and Landau are there, Tamm explains something to them - and there's even a car in the picture! How convenient, you

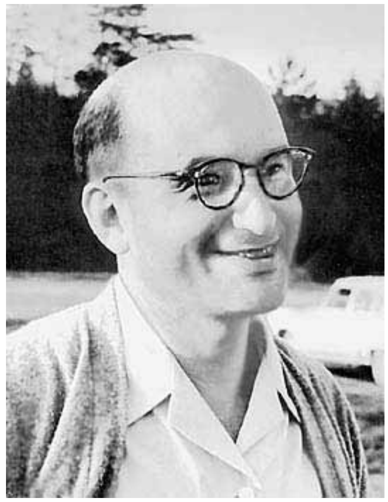

Figure 5: Evgeny Mikhailovich Lifshitz

Figure 6: Lev Landau and Niels Bohr

Figure 7: Igor Yevgenyevich Tamm

must admit.

My favourite anecdote about Tamm: During the Russian revolution, he was a physics professor at the University of Odessa in the Ukraine. Food was in short supply, so he made a trip to a nearby village in search of food. While he was in the village, a bunch of anti-communist bandits surrounded the town.

The leader was suspicious of Tamm, who was dressed in city clothes. He demanded to know what Tamm did for a living. He explained that he was a university professor looking for food. "What subject?," the bandit leader asked. Tamm replied "I teach mathematics."

"Mathematics?" said the leader. "OK. Then give me an estimate of the error one makes by cutting off a Maclaurin series expansion at the nth term. Do this and you will go free. Fail, and I will shoot you."

Tamm was not just a little astonished. At gunpoint, he managed to work out the answer. He showed it to the bandit leader, who perused it and then declared "Correct! Go home." Tamm never discovered the name of the bandit.

Figure 8: Landau, Lifshitz and Tamm

2 Origin of the game

There is not much to say about the game origins. Both Kaganov and Gorobets in [2] and [1] tell roughly the same story.

2.1 The game is born

Kaganov writes in [2] about the game - Landau invented it, played it and told others about it, encouraging them to play. Rules are simple - in those days, Soviet license plates looked like one in figure 1 - four digits were combined in two pairs, separated by a hyphen. Landau applied elementary mathematical operations and functions on both sides using only digits given, in order they're given so the hyphen could be replaced with a sign of equality. For instance, if the license plates were $12 - 34$, potential solutions would be $1 - 2 = 3 - 4$ or $1 \times 2 = 3! - 4$ or $1^2 = 3 - \sqrt{4}$... Kaganov wondered if this game always had a solution - Landau's answer was negative - not because he proved that general solution doesn't exist, but because he didn't solve all the license plates he saw!

So, do general solutions exist? Answer is in the part III of the manual.

2.2 What can be used?

Landau made it a rule: functions and operations used in the game should be elementary and known to high school students. It's fairly obvious that differentiation would trivialize the game, reducing every combination to $0 = 0$ - but since that's not elementary, it's not allowed. Similarly, integer part of a number, denoted by [ ] and fractional part, denoted by { } also trivialize the game - so, even if the high school students are aware of these functions, they would spoil the fun - so, we're not going to use them.

That leaves us with basic trigonometry, inverse trigonometric functions, logarithms, square root, taking number to a power, factorial... Are they powerful enough to solve any license plate? We'll see.

Part II
The Implementation

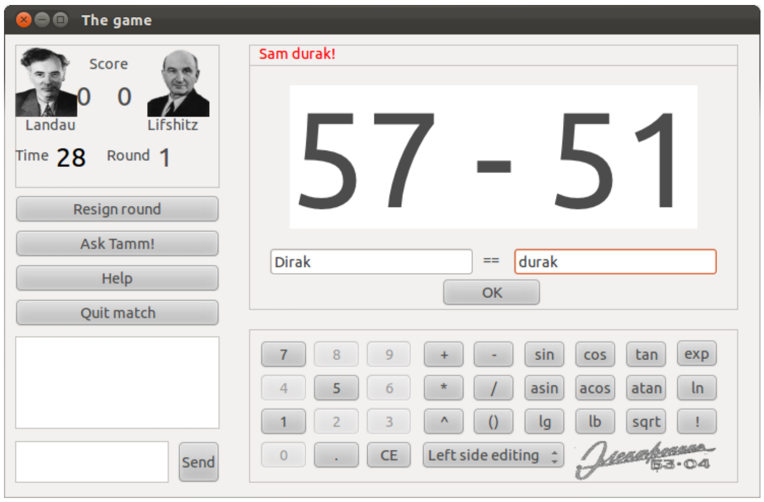

Figure 9: There is an Easter egg in the game - and I couldn't help myself, I had to mention it in this manual. Anecdote says that Landau kept repeating silently during one of Paul Dirac's lectures "Dirac - fool (durak)" until he finished with the lecture. At the end Dirac turned to Dau and said - Sam durak! (You're a fool yourself!)

Figure 10: DAU's main window

3 Interface

In case you skipped the first part of the manual: DAU *is a computerized version of Landau's license plate game. You get two pairs of digits separated with a hyphen (e. g.* 12–34*). Goal is to manipulate digits by using standard elementary functions and mathematical operations to make an equality - for instance* $1-2 = 3-4$ *or* $1 \times 2 = 3! - 4$ *or* $1^2 = 3 - \sqrt{4}$*. All given digits must be used exactly once, no permutations allowed, no additional digits allowed either.*

You could easily play Landau's game walking on the street (don't play and drive, though!), but I wanted to make a PC game out of it - so you can play it online with your friends or offline in your room without having to find real cars. Choice is yours, as shown in the figure 10.

As you may see, it is pretty self-explainable: The story of DAU button opens this manual, while the Help button opens just a part of it - namely, this part. About button informs you about the author and technology used, while Quit game button takes you back to the real world. As far as Single player game and Multiplayer game buttons are concerned - explanation follows.

3.1 Playing a single player game

Choosing single player game in the main menu takes you to the options screen, shown in figure 11. Four options are available for single player game - rest of them are disabled since they are related to the multiplayer game.

- *Number of rounds:* how many license plates do you want to solve in the game - options are 1, 5, 10, 20 or unlimited (default). Unlimited means the game ends when you click the Quit game button, there is no limit on the number of rounds.
- *Time per round (sec):* how much time will you have for each license plate - options are 10, 20, 30, 60 or unlimited (default). Unlimited means the round ends if you skip it clicking on an appropriate button (adds nothing to your score) or give a correct answer (adds one point to your score). If 10, 20, 30 or 60 is chosen, round also may end if the time runs out (adds nothing to your score).
- *Level of difficulty:* how hard do you want the game to be - options are Easy (default), Hard or Teorminimum. Levels are described in detail in section 4. For now we'll just say that easy level gives you the biggest set of functions and operations for use on the digits, while that set is reduced in other two levels.
- *Your character:* since DAU is designed as a two player game, you may choose what character will you be - Landau (default) or Lifshitz. It makes no difference in the single player game - just tells the computer whose score should it increase after every correct answer.

After confirmation of preferred options, game screen appears. Depending on the rules chosen it may look different (figures 12, 13, 14). Left corner box tracks score, shows time elapsed in current round and what is the current round. Buttons under it are the in-game menu.

- *Resign round:* skips the current round, you don't get points for the round skipped. New round starts if it wasn't the last round, and if it was - the game ends.
- *Ask Tamm!:* gives you a solution for current round, using binary logarithm (a window appears, as shown in figure 15). This option is enabled only on easy level in single player, since it is considered a warm-up mode of play.
- *Help:* opens this part of user manual, just like the Help button in the main menu.
- *Quit match:* ends the game, final score displays in a window (figure 16). Same window appears when the game ends in its natural course (i.e. when the last round ends).

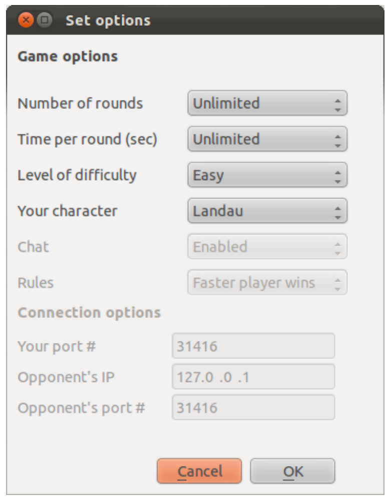

Figure 11: Single player options

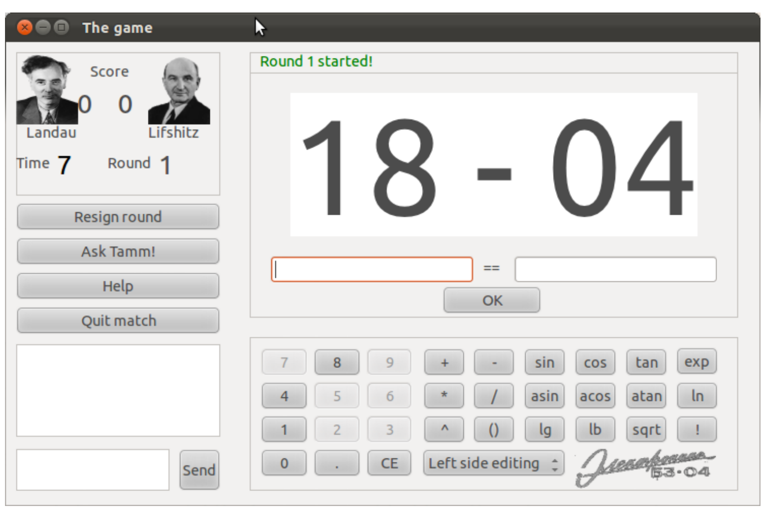

Figure 12: Game screen - easy level

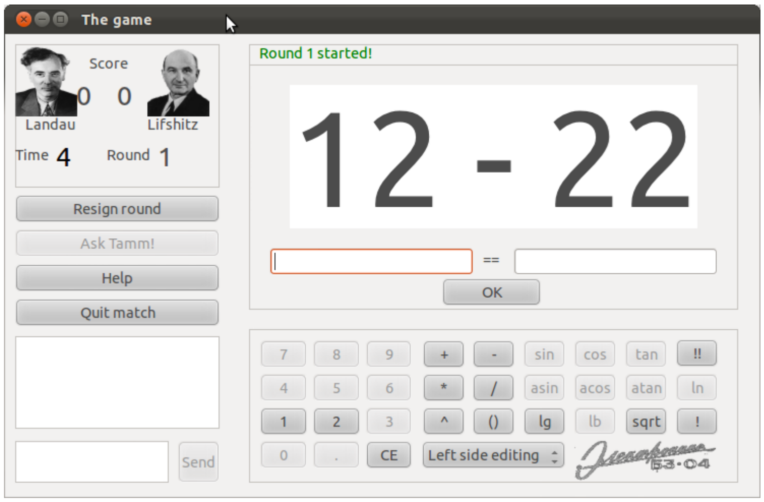

Figure 13: Game screen - hard level

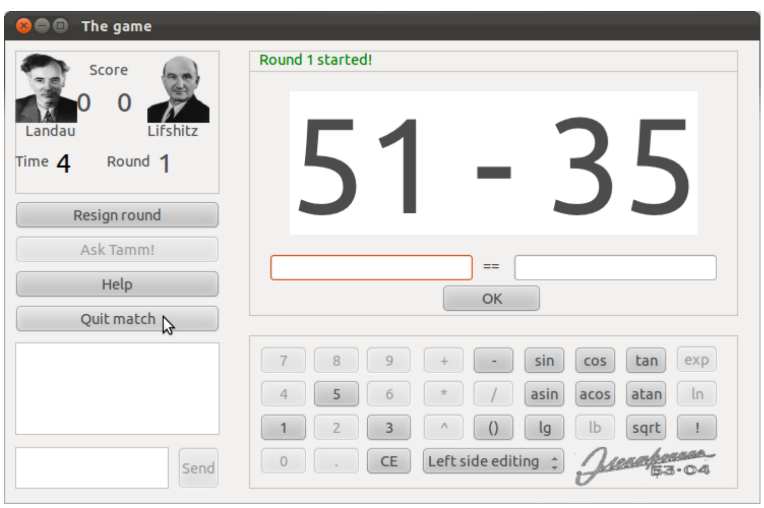

Figure 14: Game screen - Teorminimum level

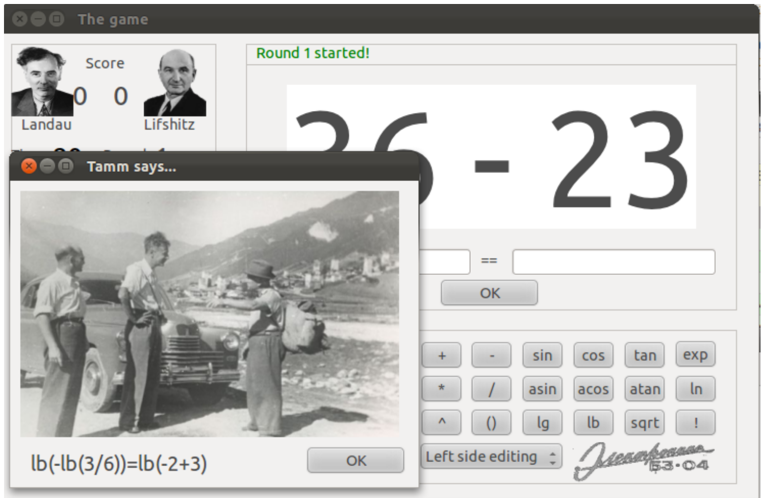

Figure 15: Tamm answers!

Under the in-game menu is chat box - disabled in single player game. On the right we have (top to bottom) a status bar, display of current license plate, boxes for entering mathematical expressions, button for confirmation of entry and a calculator pad with a double meaning - one is possibility of entering digits, functions and operations via that pad in boxes, and the other one is showing what can be used in the round (digits not appearing on the plate are automatically disabled, and so are the functions/operations which are not allowed at certain difficulty level). Mathematical expressions can also be entered in boxes using the real keyboard (most players will probably do so, since it's faster).

If you enter a valid solution, a green message of approval appears in the status bar and new round starts (or the game ends, if it was the last round). If solution is invalid, you will get a red message describing the type of mistake (whether you used additional digits, illegal functions, changed digit order, or simply made a typing error).

3.2 Playing a multiplayer game

As you might have expected, multiplayer game has a lot of in common with single player game. Options dialog is the same as before - but additional options are available, as you can see in figure 17. It's worth mentioning that in a multiplayer game, computer of the player who chooses Landau as the playing character takes the role of game server, while Lifshitz takes the role of game client (that's just a technical note, without any practical effect on the game itself).

- *Chat:* in-game chatting in DAU can either be disabled (default) or enabled.

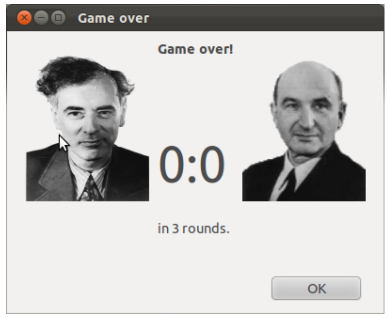

Figure 16: End of game

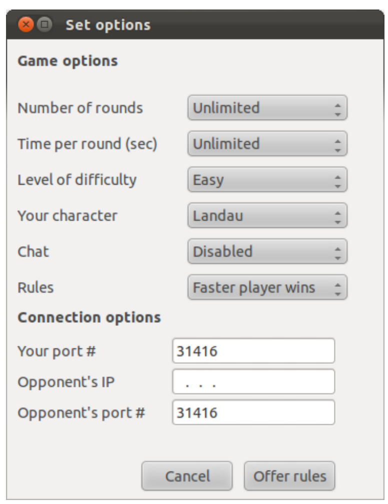

Figure 17: Multiplayer options

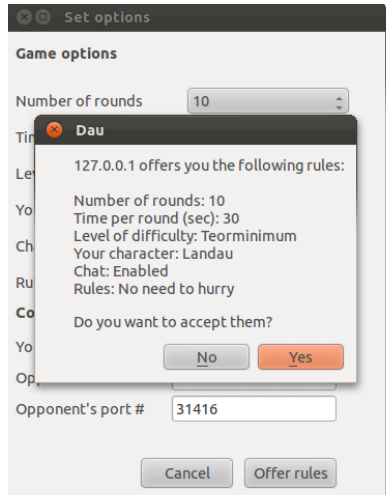

Figure 18: Rules offer

- *Rules:* two types of game rules are possible: Faster player wins (default) and No need to hurry. In the first scenario only one player can take the point in one round, and as soon as one answers correctly, that round ends (or when time expires, or after both players give up the round - then none of the players gets the point). In the second one, round doesn't end before both players give it up, answer correctly (or the time expires), so both can get the point in one round.
- *Your port #, Opponent's IP, Opponent's port #:* data needed for multiplayer connection. Default port number for DAU is 31416. If there is a need to change it, you can always do it in this dialog.

From the moment when player selects Multiplayer game in main menu, port 31416 (or the port (s)he selects) starts waiting for invitations from other players. Such invitations are sent by clicking the Offer rules button in this dialog. As soon as a set of rules is offered to a player, (s)he gets a message like in figure 18, where the other player is identified with IP address (s)he uses. If the offer is accepted, game begins on both computers. If not, the player who offered the rules is informed about the other player's refusal.

In-game messages and interface are same as in single player game (see figures 19 and 20 for an example). It is worth mentioning that you are informed via status bar about every correct answer, round skipping, or match quitting done by your opponent. If one of the players leaves the game, other gets that information and game ends.

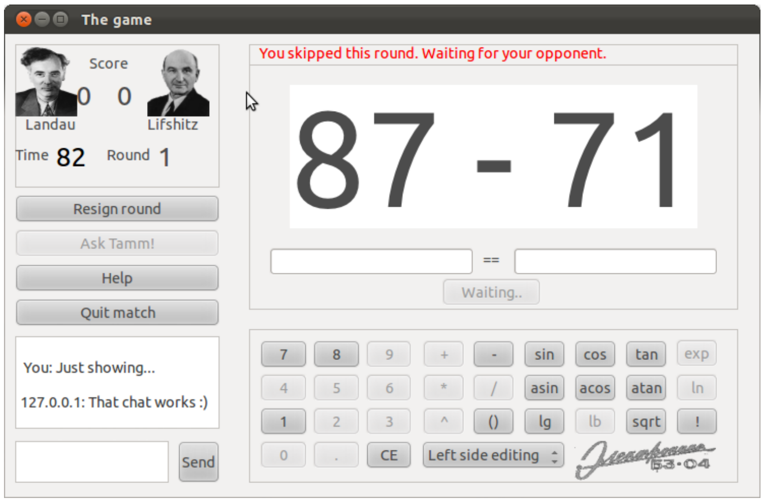

Figure 19: One player's screen in a multiplayer game

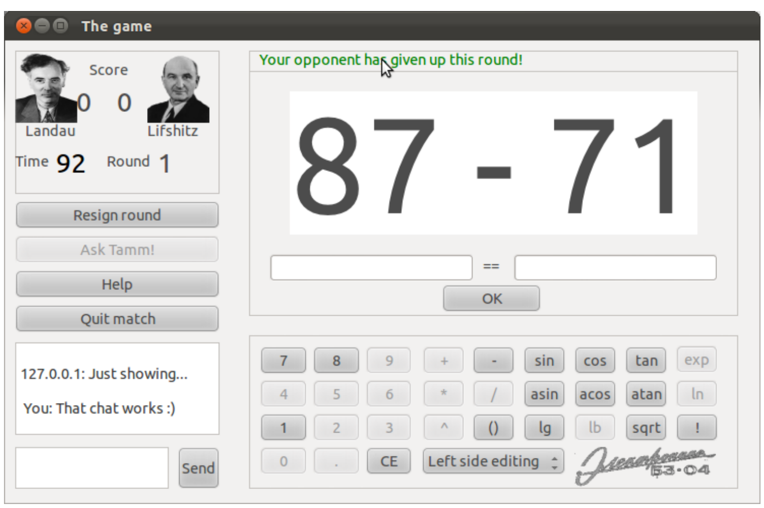

Figure 20: Other player's screen in the same game

4 Difficulty levels

Most games have certain difficulty levels. Dau has three - easy, hard, and teorminimum. Teorminimum could be considered hardest, although some may claim that hard level is somewhat more difficult than teorminimum.

4.1 Easy level

Basically all functions which we discussed in section 2.2 of this manual are allowed. muParser is used for parsing of expressions - all used functions except for factorial are implemented in muParser (factorial and later double factorial are implemented in in Dau separately). As you can see in figure 12, this means you can use addition, substraction, multiplication, division, take x to power y, put decimal point between digits, place brackets, take square root, exponentiate e^x, take logarithm base 10 (lg), base e (ln) and base 2 (lb), sine, cosine, tangent, inverse sine, inverse cosine, inverse tangent, factorial.

4.2 Hard Level

In the hard level, trigonometry is not allowed. Exponentiation e^x, logarithms other than lg, as well as decimal point placing also cannot be used. On the other hand, use of double factorial is allowed (figure 13).

4.3 Teorminimum

Add trigonometry to hard level, but take away double factorial, addition, multiplication, division, rising to power - and you get Teorminimum (figure 14).

Part III

The Mathematics

Figure 21: In case you wondered what was the logo that appears in the game, on the calculator pad - it is a tribute to the first Soviet pocket calculator Elektronika B3-04

5 General solutions

As we pointed out before, there are general solutions, which you can use to solve any license plate combination. Three of those will be presented here. What is the point of such solutions? On one hand, they ruin the fun of the game, since you don't have to use your imagination anymore - while on the other hand looking for new general solutions gives a totally new dimension to the game.

5.1 Iterated trigonometry

Knowing that $\tan^2 x + 1 = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$, it is easy to make a function $f(n) = n + 1 = \sec^2 \arctan \sqrt{n}$. Taking square roots in this equality yields

$$\sqrt{n+1} = \sec \arctan \sqrt{n}$$

Iterated use of this formula makes it possible to transform the smaller number on the plates into the larger one, making the necessary equality hold. This was the solution Kaganov presented to Landau (after one of the Kharkiv mathematicians found it, according to [2]).

Gorobets in [1] noted that secant isn't in Russian high school curriculum anymore, and therefore the solution isn't valid today. In one of the articles from *Nauka i zhizn* reprinted in [1] S. N. Fedin gives an equivalent formula to the previous one but without secant, which uses the fact that $\tan \operatorname{arccot} x = \frac{1}{x}$:

$$\sqrt{n+1} = \tan \operatorname{arccot} \cos \arctan \sqrt{n}$$

Note, however that neither of these two variants can be used in DAU since cotangent, inverse cotangent and secant are not available (partly because they were not available in muParser, partly because they don't put them on calculators these days!). That doesn't mean the formulae are completely inapplicable: think of another way to make a reciprocal (I'm not going to give away those small tricks, find them yourself!)

I. Dobganchuk has shown that using $+, -, \times, :, \sqrt{}, \lg, \log$ and factorial one can either solve a license plate or reduce it to one of three forms: $0 - \sqrt{3}$, $0 - \sqrt{2}$ or $\sqrt{2} - \sqrt{3}$, as noted in [1].

5.2 Factorials and trigonometry

You may know by heart that $6! = 720$. Since $720 = 2 \cdot 360$, $n!$ is divisible by 360 for all $n \geq 6$. This means that $\sin n!^\circ = 0$ for such n and that gives us a simple idea - we can reduce any two digits down to 0 (if one is zero, we simply multiply them, if none is zero - we use this trigonometric identity) and obtain trivial $0 = 0$ equality. This solution was also introduced by Gorobets.

As you could have guessed, you can't apply this general solution in DAU since implemented trigonometric functions work with radians. I could have implemented alternative functions for angles in degrees, but it would spoil the fun. If anyone wants to use such functions, (s)he can edit the source code.

5.3 Binary logarithm

The following solution is mine - as introduced in [3]. It uses the fact that $\log_2 2 = 1$, $\log_2 1 = 0$, $\log_2 4 = 2$ so it's easy to convert a 4 into a 2, a 2 into a 1, and 1 into a 0. That way one can make trivial $0 = 0$ equalities whenever sum, difference or quotient of a pair of digits is 0, 1, 2 or 4. Only 5 pairs of digits haven't got that property, but they can be reduced to 2, 1, 0 using square root and/or factorial.

How can we use $\log_2$ when we can't use additional digits (and we need a 2 for logarithm's base there)? According to the ISO standard, binary logarithm (logarithm with base two) is special, just like logarithm with base 10 and with base e: all three have special symbols: lg for base 10, ln for base e, lb for base 2. This indicates the importance of these bases (10 and e earned their special place through centuries, while base 2 became important in the second half of the last century, with development of computers and theory of information).

This solution can be used in DAU (only on the easy level, though - since ld is not allowed on upper levels). As you may have noticed, this is the solution you get when you *Ask Tamm!*

6 Minimizing the functions set

Question whether it's easier to play Landau's game with more or with less functions allowed is open for discussion - I chose to implement three levels of difficulty in DAU in form of three sets of functions allowed.

6.1 Easy level of the game

With all functions and operations listed in 4.1 all possible license plates are solvable (as we've seen, general solutions from 5.1 and 5.3 are applicable). Still, some of the functions are almost never used, and some seem redundant - in the hard level, they are disabled.

6.2 Hard level of the game

With functions listed in 4.2 not all license plates are solvable. Double factorial is added to reduce number of unsolvable plates, but I assume there are still more than a few in the game - they make the game harder, since you don't know where you must concentrate and find the answer, and where trying is futile.

6.3 The Teorminimum

This level is named after the famous Landau's exam but has no other connection with it. It forces the player to use logarithms intensively in order to do everything - multiplication, division, exponentiation, powers, roots, making reciprocals - everything can be done using logarithms (here lies the answer to questions where's the $\log_y x$ button, $\sqrt[y]{x}$ button, etc in all levels - manipulate logarithms). Thanks to the fact that trigonometry is allowed in this level, all plates are solvable, using general solution from 5.1. That formula does not have to be iterated too many times in light of Dobganchuk's analysis.

References

[1] B. S. Gorobets, Krug Landau, Letnii Sad, Moskva 2006.

[2] M. I. Kaganov, Landau's license plate game, Quantum, Vol. 3 No. 4 (1983)

[3] H. Šiljak, Landau's game - solution for the 21st century, submitted to Matematičko-fizički list Zagreb, 2011