Investigating Transformations of Quadratics

<table>
<tr><th>Relation</th><th>Linear</th><th>Quadratic</th></tr>
<tr><td>Equation</td><td>$y = mx + b$</td><td>$y = a(x-h)^2 + k$</td></tr>
<tr><td>No Transformations Equation
PARENT</td><td>$y = x$</td><td>$y = x^2$</td></tr>
<tr><td>Key Points</td><td>
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>-2</td><td>-2</td></tr>
<tr><td>-1</td><td>-1</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>2</td></tr>
</table>
</td><td>
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>-2</td><td>$(-2)^2 = 4$</td></tr>
<tr><td>-1</td><td>$(-1)^2 = 1$</td></tr>
<tr><td>0</td><td>$(0)^2 = 0$</td></tr>
<tr><td>1</td><td>$(1)^2 = 1$</td></tr>
<tr><td>2</td><td>$(2)^2 = 4$</td></tr>
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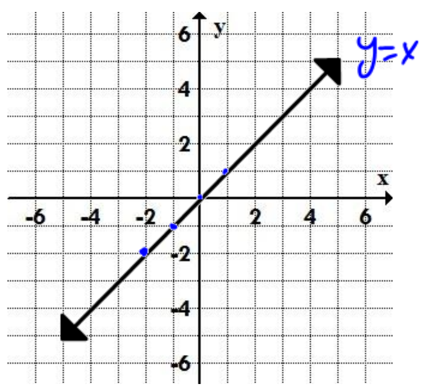
<tr><td>Graph</td><td>


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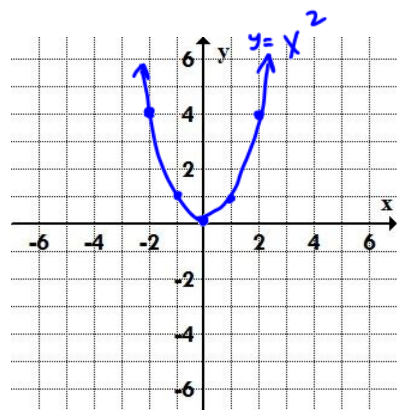

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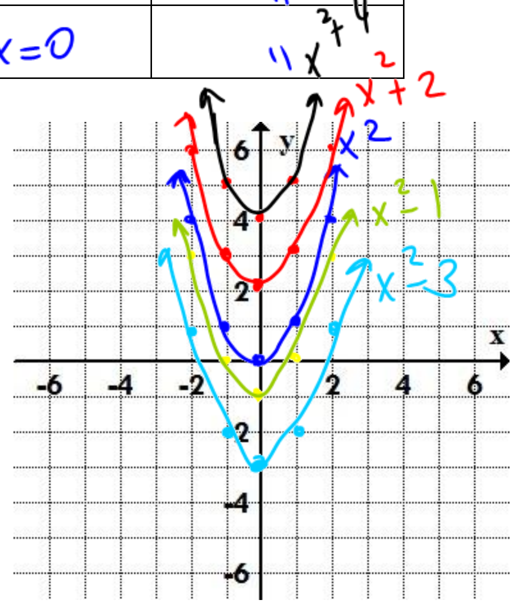
Investigation A: Compare $y = x^2$ and $y = x^2 + k$

Use a graphing calculator to graph the quadratic functions on the same set of axis and complete the following table.

Function	Value of k in $y = x^2 + k$	Direction of opening	Vertex	Axis of symmetry	Congruent to $y = x^2$?
$y = x^2$	0	up	(0,0)	$x = 0$	Yes
$y = x^2 + 2$	2	up	(0,2)	$x=0$	YES
$y = x^2 + 4$	4	up	(0,4)	$x=0$	〃
$y = x^2 - 1$	-1	up	(0,-1)	$x=0$	〃
$y = x^2 - 3$	-3	up	(0,-3)	$x=0$	〃

a. Sketch the graphs on the same axes. Label each parabola with its equation.



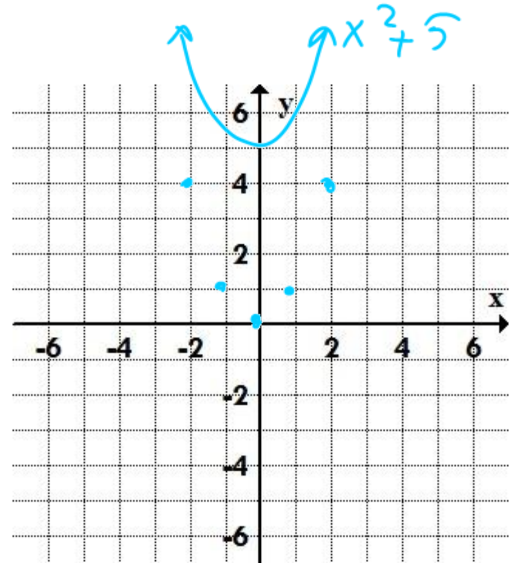
b. Describe how the value of *k* in $y = x^2 + k$ changes the graph of $y = x^2$.

When $k>0$ graph moves up
$k<0$ graph moves down

c. What happens to the x-coordinates of all points on $y = x^2$ when the function is changed to $y = x^2 + k$? What happens to the y-coordinates?

They move up or down

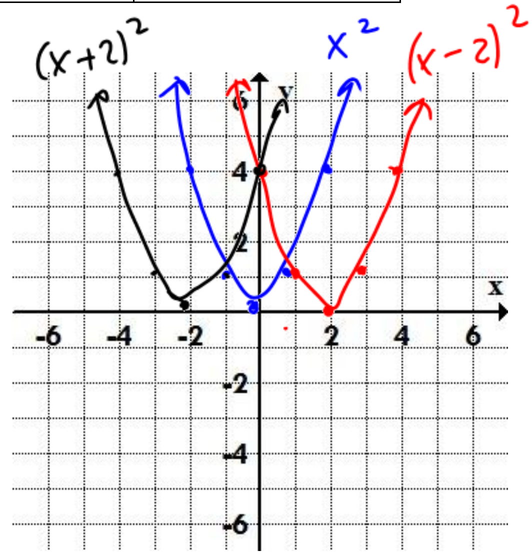
d. Without using a graphing calculator, sketch the graph of $y = x^2 + \cancel{8}$ 5



Investigation B: Compare $y = x^2$ and $y = (x-h)^2$

Clear all previous equations from your calculator. Repeat part **A** for the following:

Function	Value of h in $y=(x-h)^2$	Direction of opening	Vertex	Axis of symmetry	Congruent to $y = x^2$?
$y = x^2$	0	up	(0,0)	$x = 0$	Yes
$y=(x-2)^2$	2	UP	(2,0)	x= 2	YES
$y=(x-4)^2$	4	UP	(4,0)	x=4	YES
$y=(x+2)^2$	-2	UP	(-2,0)	x=-2	YES
$y=(x+3)^2$	-3	UP	(-3,0)	x=-3	YES



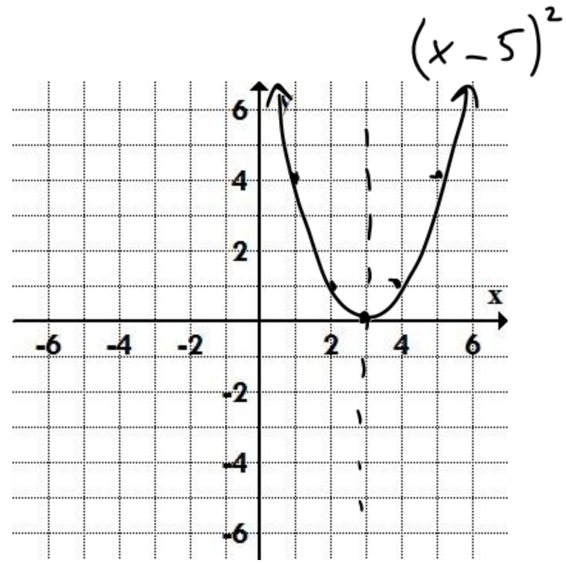
a. Sketch the graphs on the same axes. Label each parabola with its equation.

b. Describe how the value of h in $y=(x-h)^2$ changes the graph of $y = x^2$.

when $h>0$ it moves right
$h<0$ it moves left

c. What happens to the x-coordinates of all points on $y = x^2$ when the function is changed to $y=(x-h)^2$? What happens to the y-coordinates?

* y-coordinate remain the same



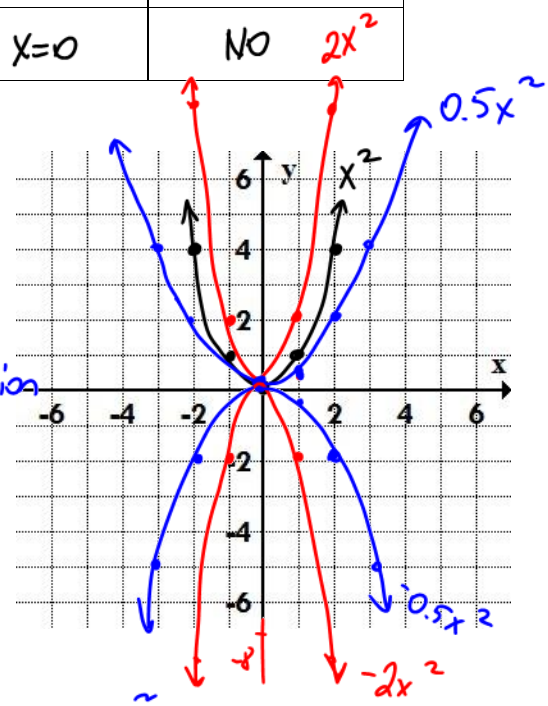
d. Without using a graphing calculator, sketch the graph of $y=(x-5)^2$

$\boxed{x=5}$

Investigation C: Compare $y = x^2$ and $y = ax^2$

Clear all previous equations from your calculator. Repeat part **A** for the following:

Function	Value of a in $y = ax^2$	Direction of opening	Vertex	Axis of symmetry	Congruent to $y = x^2$?
$y = x^2$	1	up	(0,0)	$x = 0$	Yes
$y = 2x^2$	2	↑	(0,0)	x=0	NO
$y = 0.5x^2$	0.5	↑	(0,0)	x=0	NO
$y = -2x^2$	-2	↓	(0,0)	x=0	NO
$y = -0.5x^2$	-0.5	↓	(0,0)	x=0	NO



a. Sketch the graphs on the same axes. Label each parabola with its equation.

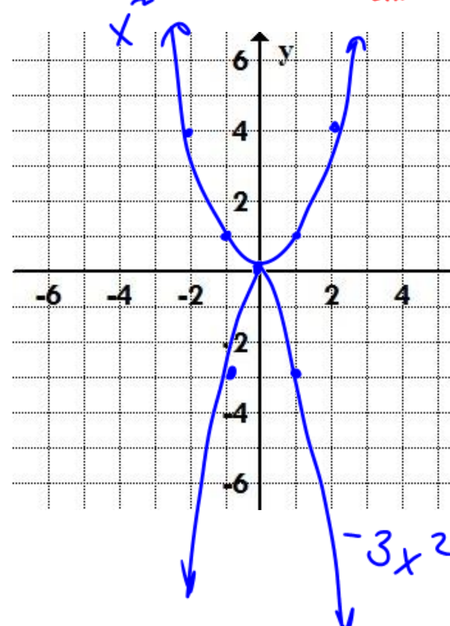
b. Describe how the value of *a* in $y = ax^2$ changes the graph of $y = x^2$.

a>1 vertical stretch
0<a<1 vertical compression
a<0 vertical reflection in the "x" axis

c. What happens to the x-coordinates of all points on $y = x^2$ when the function is changed to $y = ax^2$? What happens to the y-coordinates?

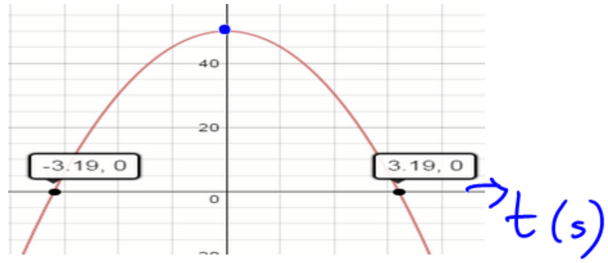
x remains the same, y coordinates get multiplied by a

d. Without using a graphing calculator, sketch the graph of $y = -3x^2$



Example A stone is dropped from the top of a 50-m cliff above a river. Its height, y, in metres, above the water can be estimated using the relation $y = -4.9x^2 + 50$, where x is the time, in seconds.

a. Graph the relation.



b. Find the intercepts. What do they represent?

$x_1 \doteq -3.2$ $x_2 \doteq 3.2$ how long it takes for the stone to reach the river.

c. How would the equation change if the stone were dropped from a 75-m cliff instead of a 50-m cliff?

$y = -4.9x^2 + 75$

d. For what values of x is each equation valid?

x<0 b/c time cannot be negative.

Reflection/Summary: Summarize the effect that *a*, *h, and* k, have on a parabola's:

"x" does the opposite
"y" does what it says

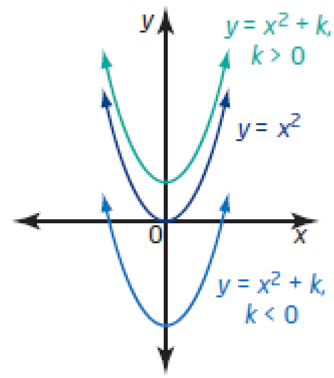
outside
output
"y"

$$y=\pm a\underbrace{(x-h)}^2+k$$

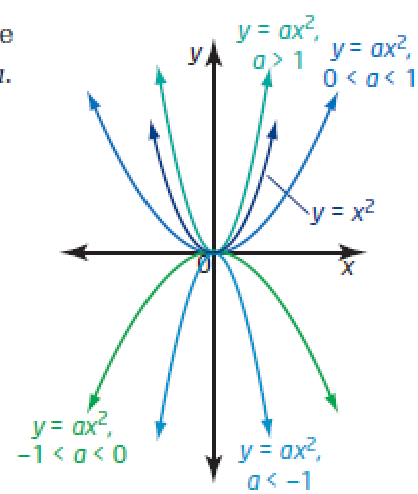
inside
input
"x"

Key Concepts

- To graph $y = x^2 + k$, translate the graph of $y = x^2$ vertically k units.
  – If $k > 0$, then the graph is translated k units upward.
  – If $k < 0$, then the graph is translated k units downward.



- To graph $y = ax^2$, stretch or compress the graph of $y = x^2$ vertically by a factor of a.
  – If $a < 0$, the parabola is reflected in the x-axis.
  – If $a > 1$ or $a < -1$, then the graph is stretched vertically (narrows).
  – If $-1 < a < 0$ or $0 < a < 1$, then the graph is compressed vertically (widens).



- To graph $y = (x - h)^2$, translate the graph of $y = x^2$ horizontally h units.
  – If $h > 0$, then the graph is translated h units to the right.
  – If $h < 0$, then the graph is translated h units to the left.