CHAPTER EIGHT

DC-AC CONVERSION: INVERTERS

8.1 INTRODUCTION

Inverter is an electrical device which converts d.c. voltages (or currents) to a.c. voltages (or currents). This device has several important applications in industry such as induction and synchronous motor drives, uninterruptible power supplies (UPSs'), standby power supplies, induction heating, traveling, camping, and HVDC systems. Ideally, the input signal to the inverter is d.c. signal from a battery or an output of a controlled rectifier and the output signal is a.c. which can be square wave, quasi-square wave or nearly sinusoidal wave.

Inverters can be classified as voltage source inverters or current source inverters. Inverter converting voltage is called VOLTAGE SOURCE INVERTER (VSI), whereas inverter converting current is called CURRENT SOURCE INVERTER (CSI). The output of the inverter could be at any desired frequency, voltage or current.

Power transistor such as BJT, MOSFET and IGBT, are widely used in low and medium power inverters. Thyristors (SCRs) or GTOs inverters are used for high power inverter. So there are many types of inverters available in industry today, and each of them is designed to suit particular application or to meet designed performance requirements.However, For VSI inverter, the following types are commonly used in industry:

1- Parallel inverter.
2- Single-phase bridge-type inverter.
   (a) Single-phase half-Bridge inverter.
   (b) Single-phase full-Bridge inverter.
3- Single-phase series inverter.
4- Three-phase inverters.

8.2 PARALLEL INVERTER

The simplest type of voltage source inverter used in low power applications and UPS system for computers is the parallel inverter. Depending on the switching device used , there are two types of parallel inverter namely, transistor parallel inverter and thyristor parallel inverter which is also called parallel capacitor inverter.

8.2.1 Transistor Parallel Inverter

Figure 8.1 shows the diagram of the basic single-phase transistor parallel inverter circuit which consists of two transistors Tr_1 and Tr_2 that are alternatively turned ON and OFF. The transistors employed in the circuit Tr_1 and Tr_2 carry the current in the positive and negative half cycles.

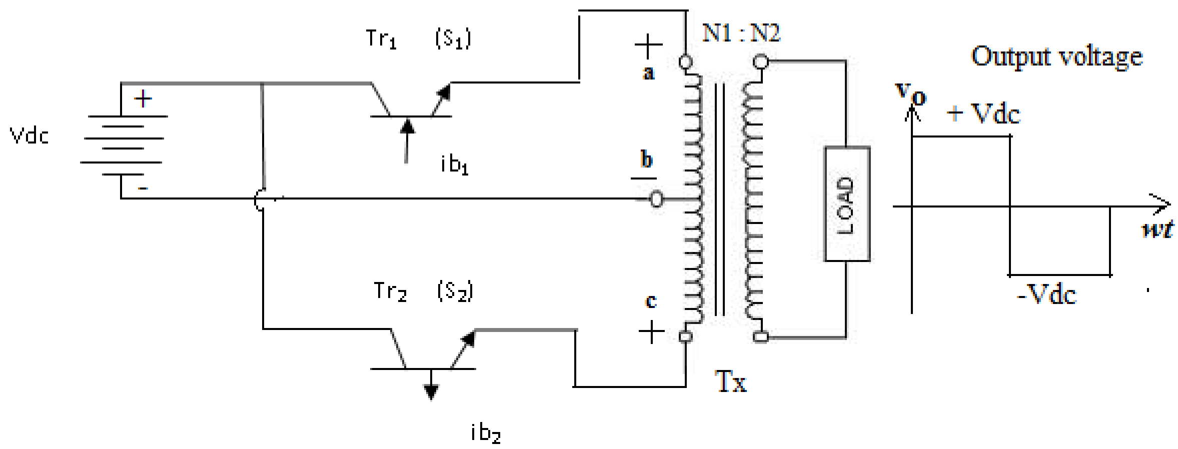


Fig.8.1 Transistor parallel inverter circuit.

Principle of operation

Assuming T_{r1} conducts and Tr_2 OFF, Tr_1 current flows through the primary winding (a-b) of the output transformer Tx and produces an induced voltage on the secondary winding. Hence, supplies a load with positive voltage $+V_{dc}$. When T_{r1} switched off ($i_{b1} \rightarrow 0$) and T_{r2} is switched on by applying current i_{b2} in its base, current flows through the primary winding (c-b) of the output transformer T_x and produce $-V_{dc}$ at the secondary of T_x. Hence a square wave of amplitude V_{dc} is generated. This square or rectangular wave can be approximated to sine wave by using additional filter. Waveforms of this inverter are shown in Fig.8.2 for pure resistive load.

The advantage of the parallel inverter is that, one can use low voltage d.c. source and obtain any output voltage by using appropriate output transformer with proper turns ratio.

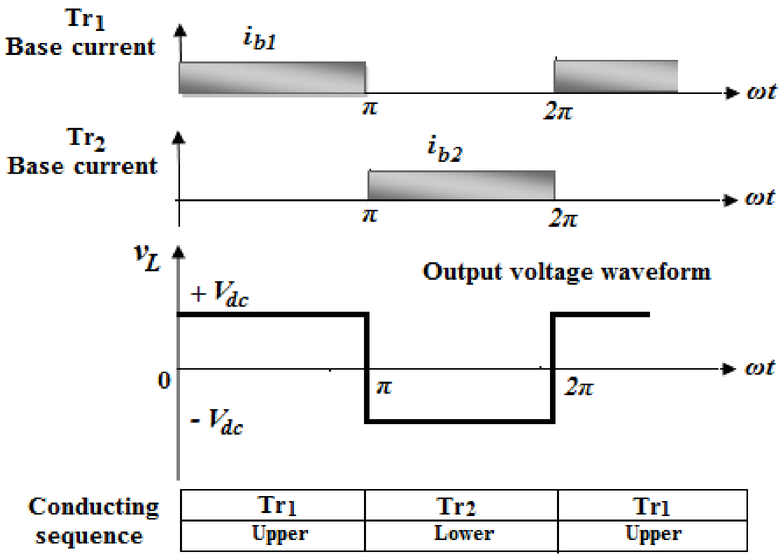


Fig.8.2 Transistor base current and utput voltage waveforms for the parallel transistor inverter.

8.2.2 Thyristor Parallel Inverter

The basic single-phase parallel inverter circuit consists of two SCRs: SCR $_1$ and SCR $_2$, an inductor L, an output transformer, and commutating capacitor C as depicted in Fig.8.3. The output voltage and current are v_o and i_o respectively. The function of L is to make the source current constant and smooth. During the working of this inverter, capacitor C comes in parallel with the load via the transformer and helps to commutate the two thyristors, so it is called a ***parallel capacitor inverter***. The operation of this inverter can be explained in the following modes.

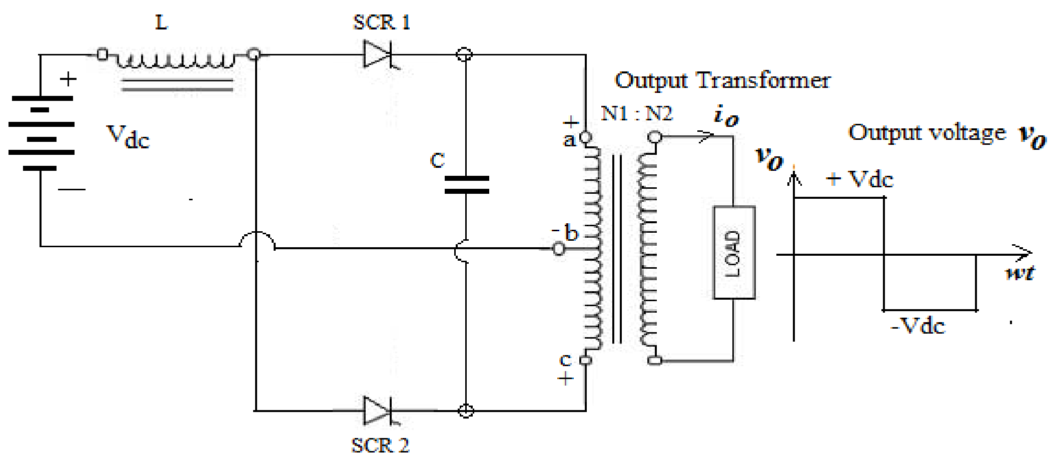


Fig.8.3 Thyristor parallel capacitor inverter circuit.

Mode I

In this mode, SCR_1 is conducting while SCR_2 is OFF. In this case current flows in the upper half of the primary winding (section a-b) of the output transformer. As a result an induced voltage on the secondary is produced and supplied to the load. At this time the voltage between a and c terminals, V_{ac}, is equal two times the d.c. supply voltage. In other words total voltage across primary winding is $2V_{dc}$. Now the capacitor C charges to a voltage of $2V_{dc}$ with upper plate as positive.

Mode II

When SCR_2 is turned ON, by applying a trigger pulse to its gate, at this time (t = 0), capacitor voltage $2V_{dc}$ appears as a reverse bias across SCR_1, it is therefore turned OFF. A current i_o begins to flow through SCR_2 and lower half of primary winding. Now the capacitor has charged (upper plate as negative) from $+2V_{dc}$ to $-2V_{dc}$ at time $t = t_1$. Load voltage also changes from V_{dc} at t = 0 to $-V_{dc}$ at $t = t_1$.

Mode III

When capacitor has charged to $-V_{dc}$, SCR_1 may be tuned ON at any time. When SCR_1 is triggered, capacitor voltage $2V_{dc}$ applies a reverse bias across SCR_2, it is therefore turned OFF. After SCR_2 is OFF, capacitor starts discharging, and charged to the opposite direction, the upper plate as positive. These procedures (Modes) proceed and repeat.
The inductor L in Fig.8.3 is used also to limit the commutating current during commutation process. In the absence of L, The capacitor will charged and discharged very rapidly during each of SCR s' conduction. The result is that the SCR may have no time to turn off. The waveforms of the SCR and capacitor voltages and currents are shown in Fig.8.4 (a) and (b) respectively.

The *rms* Value of the Output Voltage

The output voltage waveform of the inverter is a square wave as shown in Fig.8.2, the *rms* value of the square wave can be evaluated as follows:
Any function $v_o(\omega t)$ that is periodic in 2π radians has a root mean square *rms* or effective value defined by:

$$V_{o(rms)} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v_o^2\,(\omega t) d\omega t} \tag{8.1}$$

The square wave is defined as,

$$\left.\begin{aligned} f(\omega t) = v_o(\omega t) &= V_{dc} \qquad && 0 \le \omega t \le \pi \\ &= -V_{dc} && \pi \le \omega t \le 2\pi \end{aligned}\right\} \tag{8.2}$$

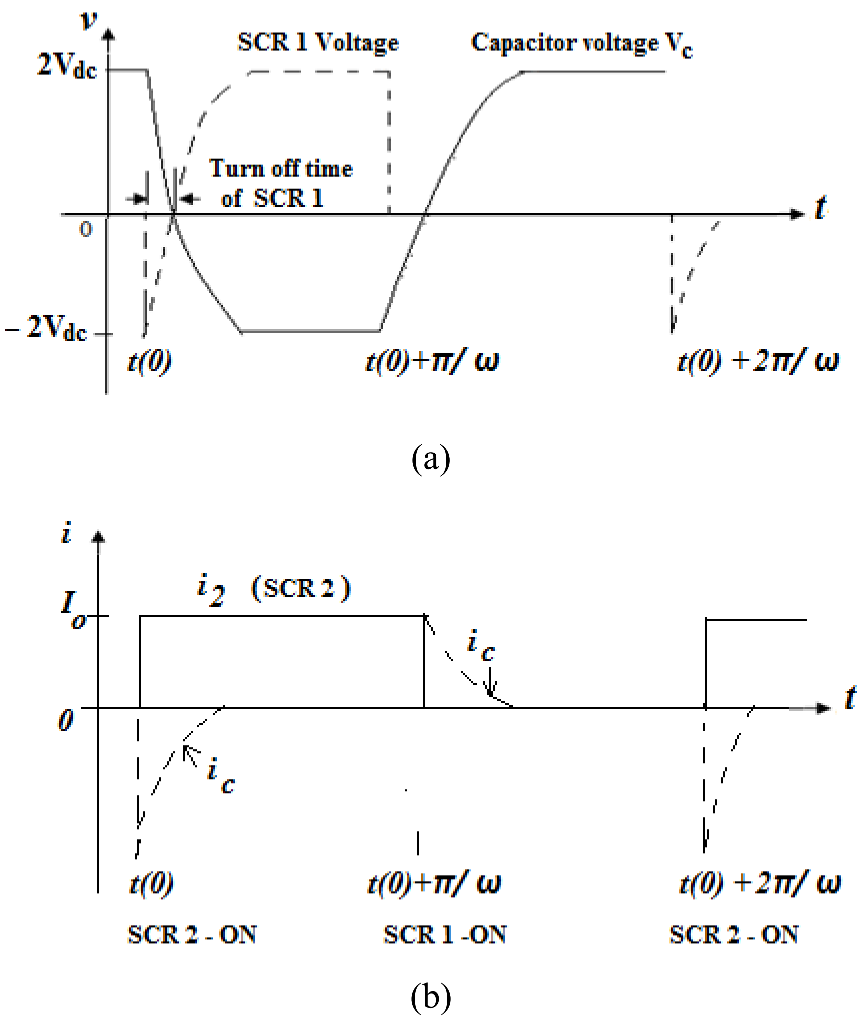


Fig.8.4 Waveforms of the SCR and capacitor (a) voltage, and (b) current for the parallel inverter.

In the present case,

$$V_{o(rms)} = \sqrt{\frac{1}{\pi}\int_0^{\pi}(V_{dc})^2\, d\omega t} = \sqrt{\frac{V_{dc}^2}{\pi}\int_0^{\pi} d\omega t} = V_{dc} \qquad (8.3)$$

Therefore, the *rms* value of the output voltahe waveform is equal to the d.c. input voltage source. The main disadvantages of the parallel capacitor inverter are the heavy transformer required to carry the load current and the large energy trapped in the commutating capacitor which need to be removed by additional feedback large ratings diodes.

Harmonic Analysis of the Output Voltage Waveform

The output voltage waveform of the inverter is a square wave (Fig.8.2) which is non-sinusoidal. Since this wave is periodic, its harmonic content can be found using Fourier series as follows:

The Fourier coefficients are,

$$\left.\begin{aligned} \frac{a_o}{2} &= \frac{1}{2\pi}\int_0^{2\pi} f(\omega t)d\omega t \\ a_n &= \frac{1}{\pi}\int_0^{2\pi} f(\omega t)cosn\omega t\ d\omega t \\ b_n &= \frac{1}{\pi}\int_0^{2\pi} f(\omega t)sinn\omega t\ d\omega t \end{aligned}\right\} \qquad (8.4)$$

Referring to the square wave defined in Eq.(8.2),

The d.c. component is:

$$\frac{a_o}{2} = \frac{1}{2\pi}\left[\int_0^{\pi} V_{dc}\ d\omega t + \int_{\pi}^{2\pi} (-V_{dc})d\omega t\right] = 0 \qquad (8.5)$$

The coefficiants a_n and b_n of the n[th] order harmonic are,

$$a_n = \frac{V_{dc}}{\pi}\left[\int_0^{\pi} cosn\omega t\ d\omega t - \int_{\pi}^{2\pi} cosn\omega t\ d\omega t\right] = 0 \qquad (8.6)$$

$$b_n = \frac{V_{dc}}{\pi}\left[\int_0^{\pi} sinn\omega t\ d\omega t - \int_{\pi}^{2\pi} sinn\omega t\ d\omega t\right]$$

$$= \frac{V_{dc}}{n\pi}\left[- cosn\omega t\ \Big|_0^{\pi} + cosn\omega t\ \Big|_{\pi}^{2\pi}\right]$$

$$= \frac{V_{dc}}{n\pi}\left[(coso - cosn\pi) + (1 - cosn\pi)\right]$$

$$\therefore b_n = \frac{2V_{dc}}{n\pi}(1 - cosn\pi)$$

- When n is even (2, 4, 6…) , $cosn\pi = 1$ $\qquad \therefore b_n = 0$.
- When n is odd (3, 5, 7…) , $cosn\pi = -1$

$$\therefore \quad b_n = \frac{4\,V_{dc}}{n\pi} \qquad (8.7)$$

The Fourier series is given by:

$$f(\omega t) = \frac{a_o}{2} + \sum_{n=1}^{\infty}(a_n\, cosn\omega t + \, b_n sinn\omega t)$$

Hence the output (load voltage) can be represented by Fourier series as,

$$v_o(\omega t) = \sum_{n=1}^{\infty} \frac{4V_{dc}}{n\pi} sinn\omega t \qquad \text{(since: } \frac{a_o}{2} \text{ and } a_n \text{ are zero)} \quad (8.8)$$

From the above equation (8.8), the instantaneous value of the fundam-ental component (first harmonic, n=1) is

$$v_{o1}(\omega t) = \frac{4}{\pi} V_{dc} sin\omega t \qquad (8.9)$$

The fundamental component peak value is,

$$V_{o1p} = c_1 = \frac{4}{\pi} V_{dc} \qquad (8.10a)$$

The *rms* value of the fundamental component is,

$$V_{o1(rms)} = \frac{c_1}{\sqrt{2}} = \frac{\frac{4}{\pi}V_{dc}}{\sqrt{2}} = \frac{4V_{dc}}{\sqrt{2}\pi} \qquad (8.10b)$$

Similarly, the instantaneous value of the third harmonic component (n=3) is,

$$v_{o3p}(\omega t) = \frac{4}{3\pi} V_{dc} sin3\omega t \qquad (8.11)$$

The peak amplitude of third harmonic , n=3 is

$$V_{o3p} = \frac{4}{3\pi} V_{dc} \qquad (8.12a)$$

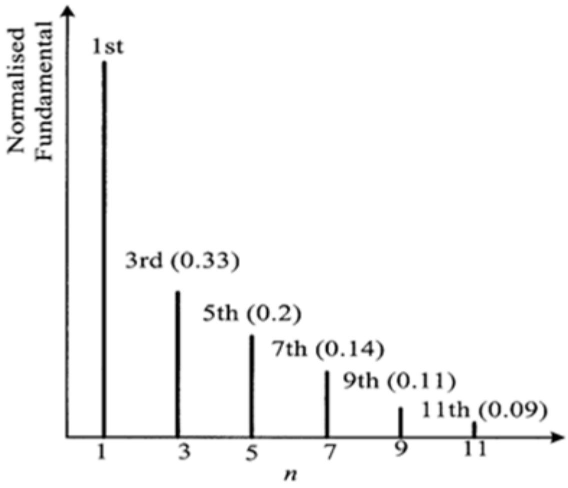
and the *rms* value of the third harmonic is,

$$V_{o3(rms)} = \frac{c_3}{\sqrt{2}} = \frac{\frac{4}{\pi}V_{dc}}{3\sqrt{2}} = \frac{4V_{dc}}{3\sqrt{2}\pi} \qquad (8.12b)$$

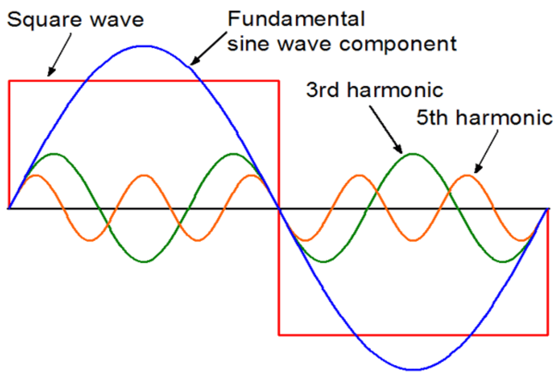
and so on, the *rms* values for the 5th, 7th, …. odd harmonics can be evaluate. The harmonic spectrum of the square wave is as shown in Fig.8.5(a). It is clear from this figure that:

- Harmonic amplitudes decrease as n increases.
- No even harmonics.

- Nearest harmonic is the third, if the fundamental is 50 Hz, then the third harmonic is 150Hz and the fifth harmonic 250 Hz as shown in Fig.8.5 (b).



(a)



(b)

Fig.8.5 (a) Harmonic spectrum of the square wave, (b) The square wave and its odd harmonics up to the fifth order.

8.2.3 Inverter Performance Parameters

It is obvious from the previous analysis that the output voltage waveform of the inverter is a square wave (or quasi-square wave) which is rich of harmonics. The harmonic components are harmful for the load and the system, because they increase the I^2R losses, generate heat and produce mechanical vibrations when applied to a.c. motors. However, the performance parameters used to assess the amount of distortion produced by the harmonic components in the output voltage waveform of the inverter. These parameters are as follow:

1. The distortion actor *DF*

The distortion factor is defined as

$$DF = \frac{1}{V_1}\left[\sum_{n=2}^{\infty}\left(\frac{V_n}{n^2}\right)^2\right]^{1/2} \tag{8.13}$$

where V_n is the *rms* value of the nth harmonic component.
V_1 is the *rms* value of the fundamental component.

Which is a factor for measuring the harmonic distortion remains in a particular waveform after filtering. The distortion factor of the n^{th} order harmonic component may be calculated as

$$DF_n = \frac{1}{V_1} \times \frac{V_n}{n^2} \tag{8.14}$$

2. The harmonic factor HF_n

The harmonic factor of n[th] order harmonic is defined as the ratio of the rms value of the n[th] harmonic component to the *rms* value of the fundamental component.

$$HF_n = \frac{V_n}{V_1} \tag{8.15}$$

This factor represents the contribution of each harmonic component to the harmonic distortion and to evaluate the dominant harmonic component in the output waveform.

3. The total harmonic distortion factor *THD*

This factor gives the ratio of the *rms* values of all harmonic components to the *rms* value of the fundamental component, which is calculated as,

$$THD = \sqrt{\frac{\sum_{n=2,3,..}^{\infty} V_n^2}{V_1^2}} \tag{8.16}$$

The total harmonic distortion factor is very important factor that indicates the amount of distortion in the waveform caused by the harmonic components. It is also represents the measure of closeness of the waveform to pure sine wave. The *THD* can also be written in more general form as:

$$THD = \sqrt{\frac{V^2 - V_1^2}{V_1^2}} \tag{8.17}$$

where V is the *rms* value of the output waveform.
V_1 is the *rms* value of the fundamental component.

4. The lowest order harmonic factor *LOH*

Finally there is another factor which is the lowest order harmonic (*LOH*) that indicates the harmonic of frequency nearest to the fundamental and has amplitude within 3% of the fundamental.

Example 8.1

A single-phase MOSFET parallel inverter has a supply d.c. voltage of 100V supplying a resistive load with R =10 Ω via a center-tap transformer with 1:1 ratio. The output frequency is 50 Hz.

(a) Draw the circuit diagram and the output voltage waveform of the inverter.
(b) Determine the *rms* value of the output voltage waveform.

(c) Determine the amplitude of the Fourier series terms for the square output voltage waveform up to 9th order harmonics.
(d) Calculate the *rms* value of the output voltage in terms of harmonic components that obtained in (b).
(e) Determine the power absorbed by the load consider up to 9th order harmonic.
(f) Draw the frequency spectra of the output voltage waveform.
(g) Calculate the total harmonic distortion factor *THD*.

Solution

(a) The circuit and the output voltage waveform as shown in Fig.8.6.

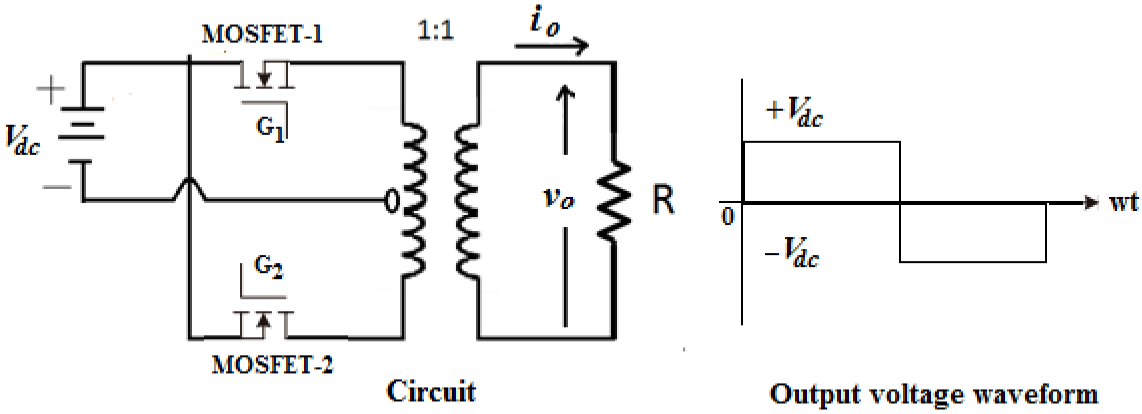


Fig. 8.6.

(b) The *rms* value of the output voltage is found as follows:
Referring to Fig. 8.6,

$$V_{o(rms)} = \sqrt{\frac{1}{T/2}\int_0^{T/2} V_{dc}\,dt} = V_{dc} = 100 \text{ V}$$

(c) From Eq.(8.5), the Fourier series of the output voltage is

$$v_o(\omega t) = \sum_{n=1,3,5,..}^{\infty} \frac{4V_{dc}}{n\pi} sinn\omega t = \sum_{n=1,3,5,..}^{\infty} c_n \, sinn\omega t$$

The amplitude c_n of the n th order harmonic is:

$$c_n = \frac{4V_{dc}}{n\pi} = \frac{4 \times 100}{n\pi} = \frac{127.3}{n}$$

$$v_o(\omega t) = c_1 \sin \omega t + c_3 \sin 3\omega t + c_5 \sin 5\omega t + c_7 \sin 7\omega t + c_9 \sin 9\omega t$$

$$= \frac{127.3}{1} \sin \omega t + \frac{127.3}{3} \sin 3\omega t + \frac{127.3}{5} \sin 5\omega t + \frac{127.3}{7} \sin 7\omega t + \frac{127.3}{9} \sin 9\omega t$$

Hence the output voltage Fourier representation is,

$$v_o(\omega t) = 127.3 \sin \omega t + 42.4 \sin 3\omega t + 25.5 \sin 5\omega t + 18.2 \sin 7\omega t + 14.1 \sin 9\omega t$$

(d) In terms of the harmonics , the *rms* value of the output voltage is

$$V_{o(rms)}^{h} = \sqrt{\left(\frac{c_1}{\sqrt{2}}\right)^2 + \left(\frac{c_3}{\sqrt{2}}\right)^2 + \left(\frac{c_5}{\sqrt{2}}\right)^2 + \left(\frac{c_7}{\sqrt{2}}\right)^2 + \left(\frac{c_9}{\sqrt{2}}\right)^2} = 97.94\ V$$

This value is less than $V_{o(rms)}$ since we calculate up to 9^{th} order harmonics only.

(e) To calculate the power we most calculate the *rms* value of the current for each harmonic the amplitude of the n^{th} harmonic current

$$I_n = \frac{c_n}{Z_n}$$

where $$Z_n = \sqrt{R^2 + (n\omega L)^2} = R$$

$$P_n = I_{n(rms)}^2 R = \left(\frac{I_n}{\sqrt{2}}\right)^2 R$$

n	f_n (Hz)	c_n (V)	Z_n (Ω)	I_n (A)	P_n (W)
1	50	127.3	10	12.73	810
3	150	42.4	10	4.24	89.8
5	250	25.5	10	2.55	32.5
7	350	18.2	10	1.82	16.5
9	450	14.1	10	1.41	9.99

The total power is $\sum P_n = P_1 + P_3 + P_5 + P_7 + P_9 = 958.8$ W (*f*) The frequency spectrum is given in Fig.8.7 below:

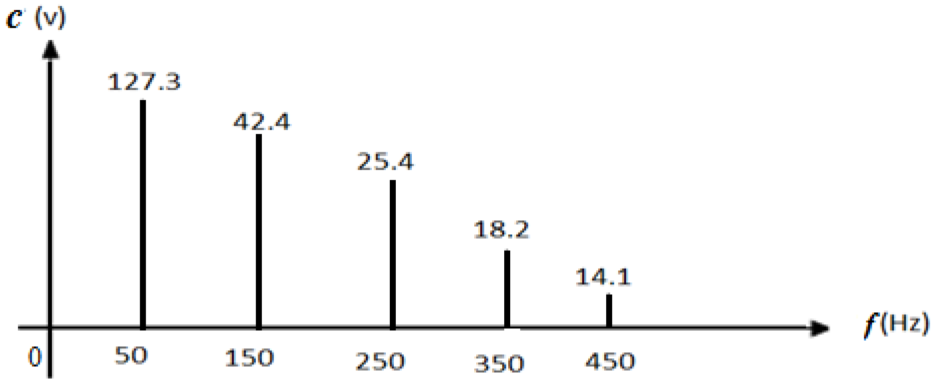


Fig. 8.7 The frequency spectrum of the output voltage waveform.

(*g*) The total harmonic distortion factor

$$THD = \sqrt{\frac{V_{o(rms)}^2 - V_{o1(rms)}^2}{V_{o1(rms)}^2}}$$

$V_{o(rms)} = 100\text{V}$

$V_{o1(rms)} = c_1/\sqrt{2} = 127.3/\sqrt{2} = 90\text{ V}$

$$THD = \sqrt{\frac{100^2 - 90^2}{90^2}} = 0.4843 \rightarrow 48.43\%$$

This is very high *THD* , the practical value of *THD* is about (3-10)% hence we need to use low-pass filter at the output to filter out most of the undesirable harmonic component and to produce nearly sinusoidal output waveform.

8.3 SINGLE-PHASE BRIDGE-TYPE INVERTERS

As it has been stated earlier, the parallel inverter discussed previously has disadvantages in that it uses heavy transformer to carry the load current. Also in the parallel capacitor inverter large amount of energy is trapped in the commutating capacitor which is need to be removed with additional circuit components. These disadvantages may be overcome by using bridge-type inverter which eliminates the need of the magnetic components such as the center-tap transformer or large capacitor. Only semiconductor component are used in this type of inverter.

8.3.1 Single-Phase Half-Bridge Inverter

This type of inverter is very simple in construction. It does not need output transformer like parallel inverter. It sometimes called center-tapped source inverter. The basic configuration of this inverter is shown in Fig 8.8.

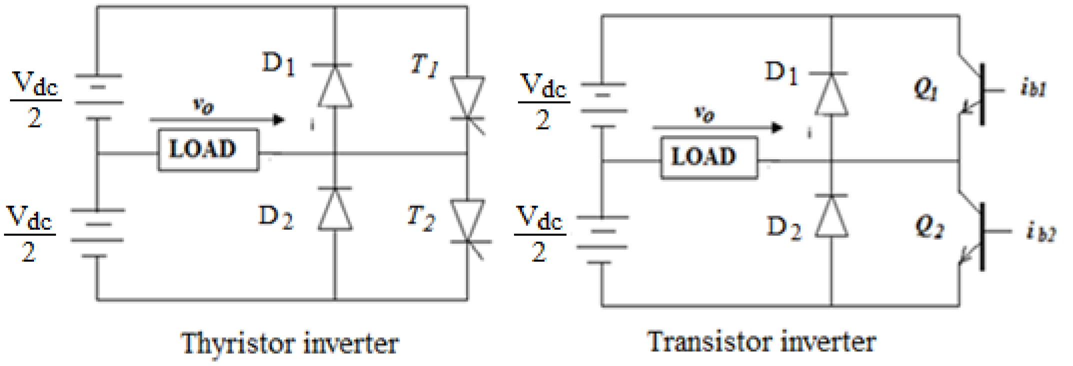


Fig.8.8 Single-phase inverter half-bridge circuit.

The top and bottom switch has to be "complementary" i.e., if the top switch is closed (ON), the bottom must be OFF, and vice-versa. The output voltage waveform is a square wave as that indicated in Fig.8.2. In practice, a dead time between ON and OFF (t_d) for Q_1 & Q_2 is required to avoid short circuit or "shoot-through" faults. This leads to produce quasi-square wave output voltage (Fig.8.9).

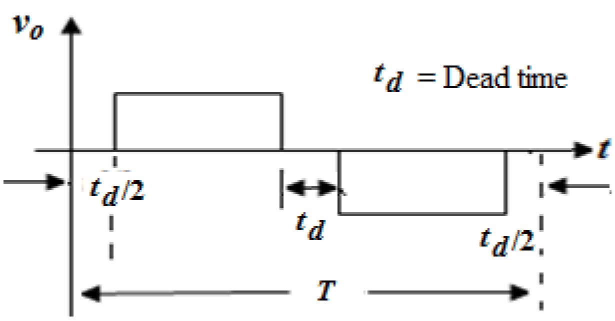


Fig.8.9 Quasi-square output voltage waveform.

Performance of half-bridge inverter with series resistive-inductive loads

If the load is resistive, the output current waveform will be a copy of the voltage waveform as shown in Fig.8.10(a). The output voltage is a square (or quasi-square) wave. However with a series *R-L* load Fig.8.10 (b), the load current *i* is delayed although the output voltage wave is still a square.

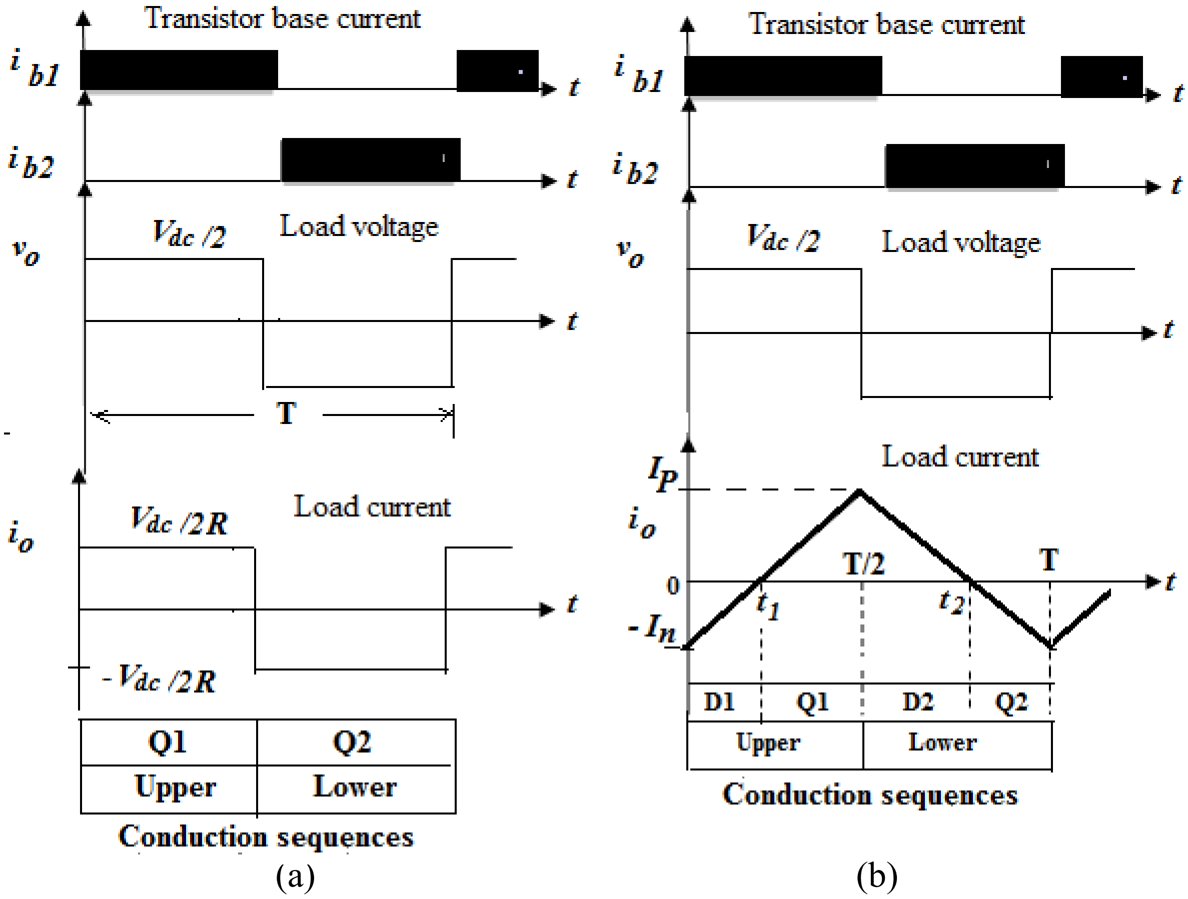


Fig.8.10 Output voltage and current for single-phase half-bridge inverter: (a) Operation with resistive load, (b) Operation with *R-L* load.

In Fig. 8.10(b), Q_1 is applied the driving current i_{b1} from 0 to $T/2$. Since the current is still negative, this cause diode D_1 to conduct from 0 to t_1. The output current i_o increases from $-I_n$ toward zero causing Q_1 to be reversed biased and it does not conduct till D_1 stop conducting at t_1. When Q_1 conducts from t_1 to $T/2$, the output current increases from zero to I_p. Hence, the current i_o will grow exponentially during the positive half-cycle from $-I_n$ to I_p according to the following equation:

$$+\frac{V_{dc}}{2} = Ri + L\frac{di}{dt} \tag{8.18}$$

- Through D_1 [load returning power to the upper half of the source].
- Through Q_1 [load absorbing power from the upper half of the source] until $t = T/2$, whereby $i = I_p$.

At $t = T/2$, Q_1 is turned off and base current drive is applied to Q_2. Negative half cycle starts by conduction of Q_2, but at this instant the current is still positive which cause Q_2 to be reversed bias and D_2 to be forward bias. Current starts to change direction from I_p to zero through D_2 while Q_2 remains off until the it becomes zero at t_2. At this instant Q_2

start to conduct from t_2 to T until i_o = -I_n. The change of current from I_p through D_2 and then to – I_n through Q_2 is governed by the following equation:

$$-\frac{V_{dc}}{2} = Ri + L\frac{di}{dt} \tag{8.19}$$

The *rms* value of the load voltage is

$$V_{o(rms)} = \sqrt{\frac{1}{T}\int_0^T V_o^2 dt} = \left[\frac{1}{T}\left(\frac{V_{dc}}{2}\right)^2 T\right]^{\frac{1}{2}} = \frac{V_{dc}}{2} \tag{8.20}$$

The load voltage $v_o\ (\omega t)$ can be expressed in terms of harmonics by Fourier series as:

$$v_o(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4}{n\pi}\left(\frac{V_{dc}}{2}\right)\sin n\,\omega t \tag{8.21}$$

Here $\quad v_o(\omega t) = 0 \quad$ for n =2,4,6…

where: $\omega = 2\pi f_0$ is the frequency of the output voltage in (rad /sec).

The fundamental component of the load voltage has a peak value of,

$$V_{o1p} = c_1 = \frac{4}{\pi}\frac{V_{dc}}{2}$$

and it has *rms* value of

$$V_{o1(rms)} = \frac{c_1}{\sqrt{2}} = \frac{\sqrt{2}}{\pi}V_{dc} = 0.45\,V_{dc} \tag{8.22}$$

For an *R-L* load, the instantaneous load current $i_o(\omega t)$ can be found by dividing the instantaneous output voltage $v_o(\omega t)$ given in Eq.(8.21) by the load harmonic impedance $Z_n = R + jn\omega L$, or

$$|Z_n| = \sqrt{R^2 + (n\omega L)^2} \text{ , thus}$$

$$i_o(\omega t) = \frac{v_o(n\omega t)}{|Z_n|}$$

$$i_o(\omega t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{2V_{dc}}{n\pi\sqrt{R^2 + (n\omega L)^2}}\sin(n\omega t - \psi_n) \tag{8.23}$$

where ,

$$\psi_n = tan^{-1}\frac{n\omega L}{R}$$

or $$i_o(\omega t) = \sum_{n=1,3,5...}^{\infty} c_n \sin(n\omega t - \psi_n)$$

If I_{o1} is the *rms* fundamental load current, $c_1 / \sqrt{2}$, then the fundamental output power (for n = 1) is

$$P_1 = V_{o1} I_{o1} cos\psi_1 = I_{o1}^2 R = \left[\frac{2V_{dc}}{\sqrt{2}\pi\sqrt{R^2 + (\omega L)^2}}\right]^2 \times R \qquad (8.24)$$

Note: In most applications (e.g. electric motor drives) the output power due to the fundamental current is generally the useful power, and the power due to harmonic currents is dissipated as heat and increases the load temperature.

Example 8.2

The single-phase half-bridge transistor inverter shown in Fig.8.8 has a resistive load of R = 3Ω and the d.c. input voltage V_{dc} = 60 V. Determine:

(a) The *rms* value of the output voltage.
(b) The *rms* value of the load voltage at the fundamental frequency $V_{o1(rms)}$.
(c) The output power.
(d) The average and peak current of each transistor.
(e) The peak reverse blocking voltage V_{BR} of each transistor.
(f) The total harmonic distortion factor.

Solution

(a) The *rms* value of the output voltage is

$$V_{o(rms)} = \sqrt{\frac{1}{T}\int_0^T V_o^2 dt} = \left[\frac{1}{T}\left(\frac{V_{dc}}{2}\right)^2 T\right]^{\frac{1}{2}} = \frac{V_{dc}}{2} = \frac{60}{2} = 30 \text{ V}$$

(b) The *rms* value of the load voltage at the fundamental frequency is

$$V_{o1(rms)} = \frac{c_1}{\sqrt{2}} = \frac{\sqrt{2}}{\pi} V_{dc} = 0.45\, V_{dc} = 0.45 \times 60 = 27 \text{ V}$$

(c) The output power is

$$P_o = \frac{{V_{o1(rms)}}^2}{R} = \frac{30^2}{3} = 300 \text{ W}$$

(d) The average and peak current of each transistor are

$$I_P = \frac{V_{o(rms)}}{R} = \frac{30}{3} = 10 \text{ A}$$

Because each transistor conducts for a 50% duty cycle, the average current of each transistor is

$$I_{av} = 10 \times 0.5 = 5 \text{ A}$$

(e) The peak reverse blocking voltage V_{BR} of each transistor is

$$V_{BR} = 2 \times 30 = 60 \text{ V}$$

(f) The total harmonic distortion factor is

$$THD = \sqrt{\frac{V_{o(rms)}^2 - V_{o1(rms)}^2}{V_{o1(rms)}^2}} = \sqrt{\frac{30^2 - 27^2}{27^2}} = 0.4843$$

8.3.2 Single-Phase Full-Bridge Inverter

The single-phase half-bridge Inverter discussed in the previous sub-section although it has the advantage that it uses only two semiconductor switches, but it has the distinct disadvantage in that it needs a center-tap d.c. supply. However, this problem can be solved using full-bridge inverter shown in Fig. 8.11.

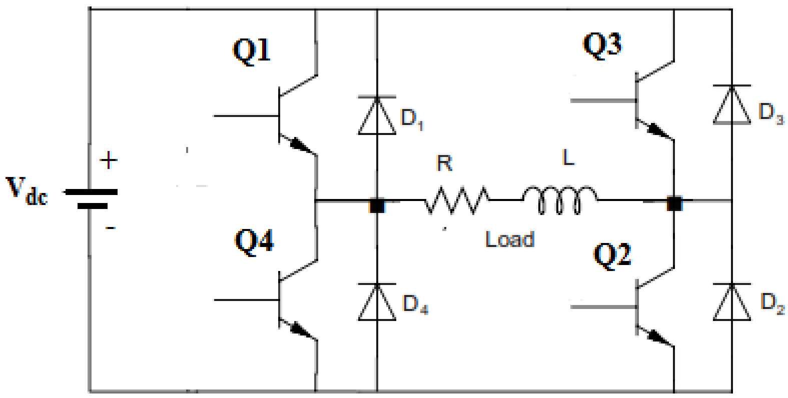


Fig.8.11 Single-phase full-bridge inverter circuit.

The single-phase full-bridge inverter uses four BJT transistors in its operation. Practically, this inverter is constructed from two half-bridge inverter using single d.c. source V_{dc} and the load is connected between the centers of the two legs. The four diodes shown in Fig.8.11 are used for feedback when the load is inductive.

Operation with resistive load

When the load is resistive, the operation of the inverter can be described as follows: Output voltage is a square wave and the current is replica of the voltage shape. In this circuit, Q_1 and Q_2 are triggered simultaneously and so are Q_3 and Q_4. Each device is made to conduct for half-time of the output cycle, the load voltage waveform with the transistor base currents are shown in Fig 8.12. The output voltage is a square-wave and the current is replica of the voltage shape.

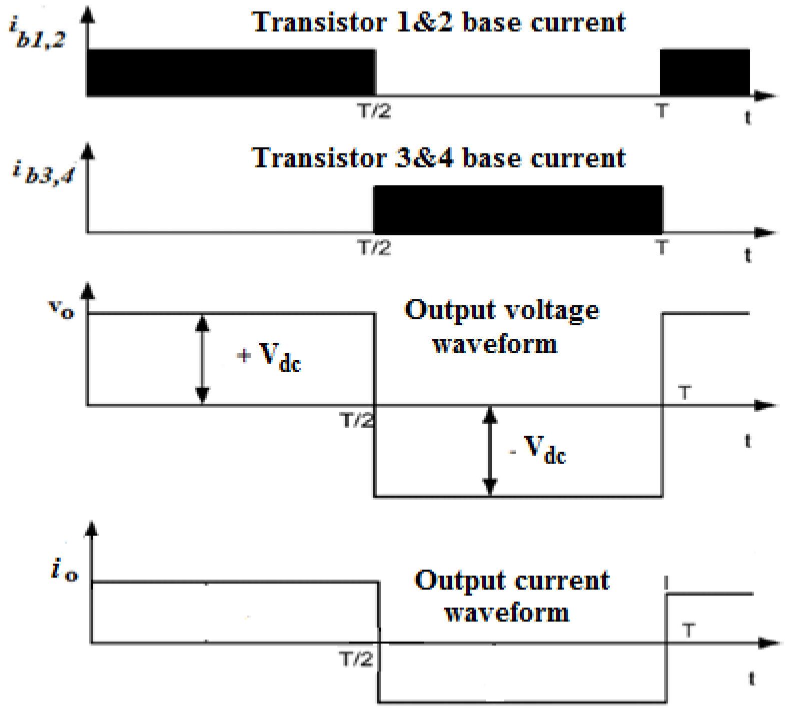


Fig.8.12 Operation of the full-bridge inverter with resistive load.

Operation with resistive-inductive load

With resistive-inductive load, the current i_0 lags the square-wave output voltage v_0 as shown in Fig.8.13. The operation of the inverter in this case can be explained as follows :

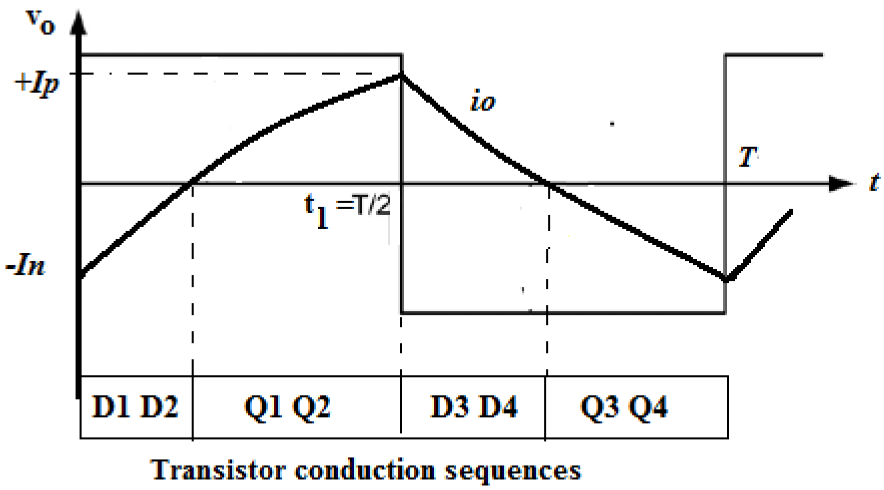


Fig.8.13 Current through a single-phase full-bridge inverter.

- Triggering Q_1 & Q_2 connects the load to V_d . For steady load condition, i_0 grows exponentially through D1& D2 and then through Q_1&Q_2 from $-I_n$ to I_p according to $(V_{dc} = Ri + L\frac{di}{dt})$.
- Triggering Q_1&Q_2 connects the load to V_d. For steady load condition, i_0 grows exponentially through D_1&D_2 and then through Q_1&Q_2 from $-I_n$ to I_p according to $(V_{dc} = Ri + L\frac{di}{dt})$.
- When diodes D_1&D_2 are conducting, the energy fed back to the source, thus they are known as feedback diodes.
- Negative half-cycle starts by triggering Q_3&Q_4 at $t_1 = \frac{T}{2}$, and Q_1&Q_2 goes off when $i_{b1} = i_{b2} = 0$ (base current blocking).
- Load voltage reverses to $-V_{dc}$ and i_o will flow through D_3&D_4 and then through Q_3&Q_4 according to the equation ($-V_d = Ri + L\frac{di}{dt}$). At the end of negative-half cycle $i_o = -I_n$.

Note that:

$$t_1 = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{\omega}$$

Load *rms* voltage is

$$V_{o(rms)} = \left[\frac{1}{t_1}\int_0^{t_1} V_d^2 dt\right]^{\frac{1}{2}} = V_{dc} \qquad (8.25)$$

Load voltage $v_o(\omega t)$ may be expressed as:

$$v_o(\omega t) = \sum_{n=1,3,5,..}^{\infty} \frac{4V_{dc}}{n\pi} sin n\omega t \qquad (8.26)$$

The peak value of the fundamental component (n=1) of the load voltage is,

$$V_{o1p} = \frac{4}{\pi} V_{dc} \qquad (8.27)$$

The *rms* value of the fundamental component is

$$V_{o1(rms)} = \frac{4V_{dc}}{\sqrt{2}\pi} = 0.9\, V_{dc} \qquad (8.28)$$

Load *rms* current and power can be determine from

$$I_{orms} = \left[\frac{2}{T}\int_0^{T/2} i_o^2 dt\right]^{\frac{1}{2}} \qquad (8.29)$$

$$P = \frac{2}{T}\int_0^{T/2} v_o . i_o \, dt \qquad (8.30)$$

where the instantaneous value of the load current i_o for an R-L load is

$$i_o(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \psi_n) \qquad (8.31)$$

The angle ψ_n by which the load current lags the load voltage is

$$\psi_n = tan^{-1}\frac{n\omega L}{R}$$

The total harmonic distortion factor is,

$$THD = \sqrt{\frac{V_{o(rms)}^2 - V_{o1(rms)}^2}{V_{o1(rms)}^2}}$$

Example 8.3

For the single-phase MOSFET bridge inverter circuit shown in Fig. 8.14, the source V_{dc} =125 V, load resistance R =10 Ω and output voltage frequency f_o = 50 Hz.

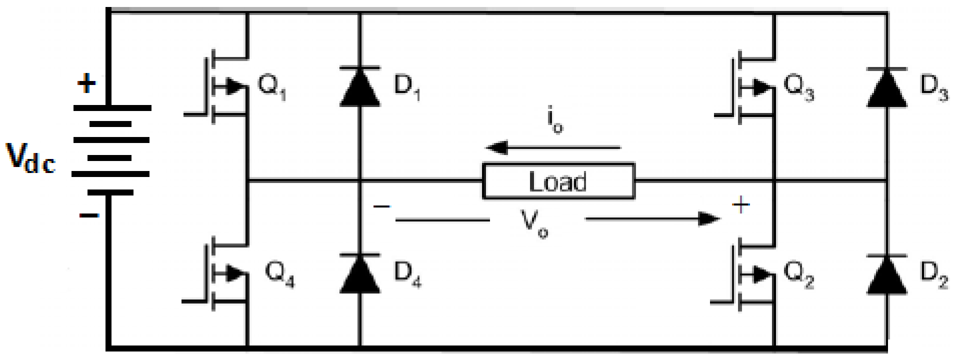


Fig.8.14 Single-phase full-bridge inverter.

(a) Draw the output voltage and load current waveforms.
(b) Derive the *rms* value of the output voltage waveform and hence calculate the output power P_o in terms of the output voltage.
(c) Analyse the amplitude of the Fourier series terms of the output voltage waveform by considering up to the 7th order harmonic. Determine the value of the *rms* output voltage in terms of harmonics *rms* values.
(d) Calculate the average and peak currents of each transistor.
(e) Estimate the total harmonic distortion factor *THD* of the circuit.

Solution

(a) The waveforms of the output voltage and current are depicted in Fig.8.15 as:

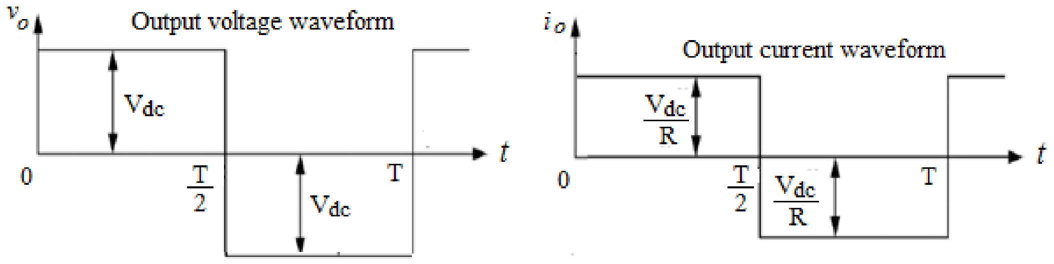


Fig.8.15 Output voltage and current waveforms.

(b) The *rms* value of the output voltage is found from Eq.(8.25) as follows:

$$V_{o(rms)} = \left[\frac{1}{t_1}\int_0^{t_1} V_{dc}^2 dt\right]^{\frac{1}{2}} = V_{dc} = 125 \text{ V}$$

$$P_o = \frac{V_{o(rms)}^2}{R} = \frac{125^2}{10} = 1562.5 \text{ W}$$

(c) The Fourier series of the output voltage is found from Eq.(7.26) as

$$v_o(\omega t) = \sum_{n=1,3,5,..}^{\infty} c_n sin n\omega t$$

The amplitude c_n of the n^{th} harmonic is:

$$c_n = \frac{4V_{dc}}{n\pi} = \frac{4 \times 125}{n\pi} = \frac{159.12}{n}$$

$$v_o(\omega t) = c_1 \sin \omega t + c_3 \sin 3\omega t + c_5 \sin 5\omega t + c_7 \sin 7\omega t$$

$$= \frac{159.12}{1} \sin \omega t + \frac{159.12}{3} \sin 3\omega t + \frac{159.12}{5} \sin 5\omega t + \frac{159.12}{7} \sin 7\omega t$$

Hence the output voltage Fourier representation is,

$$v_o(\omega t) = 159.12 \sin \omega t + 53.04 \sin 3\omega t + 31.82 \sin 5\omega t + 22.73 \sin 7\omega t$$

In terms of the harmonics:

$$V_{o(rms)}^{h} = \sqrt{\left(\frac{c_1}{\sqrt{2}}\right)^2 + \left(\frac{c_3}{\sqrt{2}}\right)^2 + \left(\frac{c_5}{\sqrt{2}}\right)^2 + \left(\frac{c_7}{\sqrt{2}}\right)^2 + \left(\frac{c_9}{\sqrt{2}}\right)^2} = 121.7 \text{ V}$$

This value is less than $V_{o(rms)}$ since we calculate up to 7th order harmonics only.

(d) Since the duty cycle of each transistor is 0.5, the current waveform is as shown in Fig.8.16.

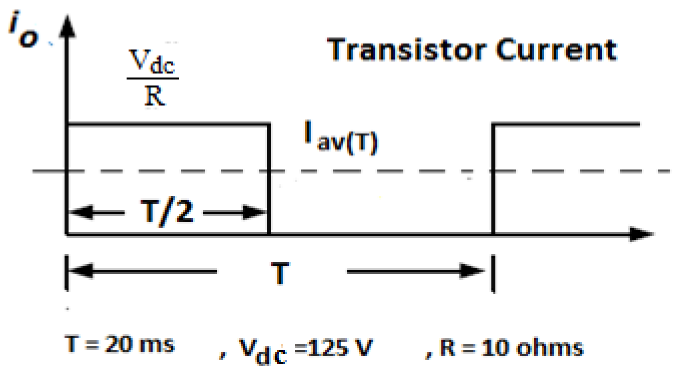


Fig.8.16 Transistor current waveform.

$$I_{av(T)} = \frac{1}{T} \int_{0}^{1/T} \frac{V_{dc}}{R} \, dt = \frac{V_{dc}}{R} \left[\frac{1}{T} \times \frac{T}{2} \right] = \frac{V_{dc}}{2R} = \frac{125}{20} = 6.25 \text{ A}$$

Peak current

$$\frac{V_{dc}}{R} = \frac{125}{10} = 12.5 \text{ A}$$

(e) The total harmonic distortion factor:

$V_{o1(rms)}$ = $c_1/\sqrt{2}$ =159.12 /√2 =112.6 V

$$THD = \sqrt{\frac{V_{o(rms)}^2 - V_{o1(rms)}^2}{V_{o1(rms)}^2}} = \sqrt{\frac{125^2 - 112.6^2}{112.6^2}} = 0.4843$$

Example8.4

For the single-phase full-bridge, transistor inverter shown in Fig.8.14 of the previous example, V_{dc} =100 V, load is a series resistance-inductance with R=10 Ω, and L = 25 mH and the output frequency is 50 Hz. It is required to analyse the circuit by determining: (a)The amplitudes of the Fourier series terms for the output voltage wave up to the 9th order harmonics, (b)The amplitudes of the Fourier series terms for the load current wave up to the 9th order harmonics, (c) The power absorbed by the load in terms of harmonics, (d) Compute the total harmonic distortion factor (THD).

Solution

(a) From Eq.(8.5) ,the Fourier series of the output voltage is

$$v_o(\omega t) = \sum_{n=1,3,5,..}^{\infty} \frac{4V_{dc}}{n\pi} sinn\omega t$$

The amplitude of the nth order harmonic c_n is:

$c_n = 4V_{dc} / 2\pi$ = (4x100) / nπ = 127.3 / n

$v_o(\omega t)$ = c_1 sinωt + c_2 sin 3ωt + c_5 sin 5ωt + c_7 sin 7ωt + c_9 sin 9ωt
= (127.3/1) sinωt + (127.3/3) sin 3ωt + (127.3/5) sin 5ωt
+ (127.3/7) sin 7ωt + (127.3/9) sin 9ωt

Hence the output voltage Fourier representation is,

v_o (ωt) = 127.3 sin ωt + 42.4 sin 3ωt + 25.5 sin 5ωt + 18.2 sin 7ωt
+ 14.1 sin 9ωt.

(b) From Eq.(8.31) , the Fourier series for the current is ,

$$i_o(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \psi_n)$$

$$= \sum_{n=1,3,5}^{\infty} \frac{c_n}{Z_n} \sin(n\omega t - \psi_n) = \sum_{n=1,3,5}^{\infty} I_n \sin(n\omega t - \psi_n)$$

where

$$|Z_n| = \sqrt{R^2 + (n\omega L)^2} \quad , \quad I_n = \frac{c_n}{Z_n} \quad , \text{and} \quad \psi_n = tan^{-1}\frac{n\omega L}{R}$$

Calculating I_n from the above equation yields,

i_o (ωt) = 9.29 sin (ωt – 43.25°)+ 1.42 sin (3ωt –70.52°)
+ 0.53 sin(5ωt – 78.02°) + 0.25 sin (7ωt – 81.37°)
+ 0.17 (sin 9ωt – 83.27°).

(c) To calculate the power, the *rms* value of the current for each harmonic must be calculated, hence the power P_n of the nth harmonic is

$$P_n = I_{n(rms)}^2 R = \left(\frac{I_n}{\sqrt{2}}\right)^2 R$$

n	f_n (Hz)	c_n (V)	Z_n (Ω)	I_n (A)	P_n (W)
1	60	127.3	13.73	9.29	431
3	180	42.4	30.0	1.42	10
5	300	25.5	48.2	0.53	1.4
7	420	18.2	66.7	0.25	0.34
9	540	14.1	85.4	0.17	0.14

The total power is $\sum P_n = P_1 + P_3 + P_5 + P_7 + P_9 = 443$ W

(d) The total harmonic distortion factor

$$V_{o(rms)} = \sqrt{\frac{1}{T/2}\int_0^{T/2} V_{dc}\, dt} = V_{dc} = 100 \text{ V}$$

$V_{o1(rms)} = c_1/\sqrt{2} = 127.3/\sqrt{2} = 90$ V

$$\therefore \quad THD = \sqrt{\frac{V_{o(rms)}^2 - V_{o1(rms)}^2}{V_{o1(rms)}^2}} = \sqrt{\frac{100^2 - 90^2}{90^2}} = 0.4843$$

or $\quad THD = 48.43 \%$

8.4 THREE-PHASE INVERTER

In high-power applications, three-phase inverters are normally used. Moreover when three-phase voltage source required from a d.c. source three-phase inverter is needed. However, a three-phase inverter can be constructed using three single-phase parallel inverters which are connected in parallel as shown in Fig.8.17. The gate signals of single-phase inverters should be advanced or delayed by 120^0 with respect to each other in order to obtain three-phase balanced (fundamental) voltages. The transformer secondary windings may be connected in wye or delta whereas the primary windings must be isolated from each other.

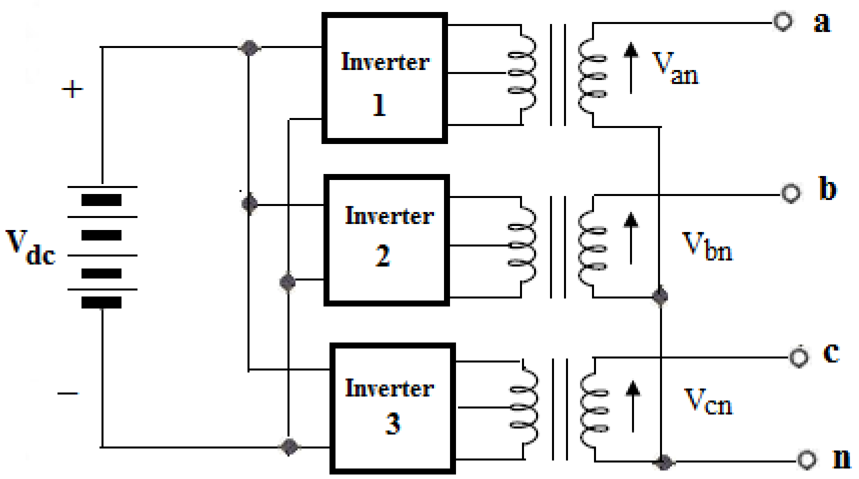


Fig.8.17 Schematic diagram of a three-phase inverter constructed from three single-phase parallel inverters.

Three single-phase half-bridge inverters can be connected as shown in Fig.8.18 to form a configuration of three-phase bridge inverter. In this circuit, there are six BJTs, Q_1, Q_2, Q_3, Q_4, Q_5, and Q_6. Also, there are six diodes connected across each transistor as shown in the figure. Observe that the upper transistors are numbered as Q_1, Q_3, and Q_5. Similarly, the lower transistors are numbered as Q_2, Q_4, and Q_6. Here, Q_1 and Q_4 are connected to phase-a. When Q_1 conducts, phase-a is connected to $+V_{dc}$ and when Q_4 conducts, phase-a is connected to $-V_{dc}$. Similarly, Q_3 and Q_6

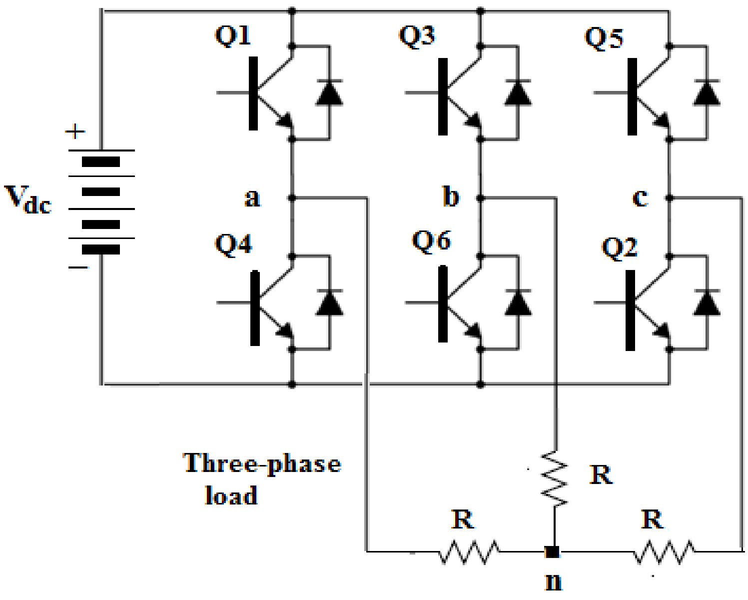


Fig.8.18 Three-phase bridge inverter circuit diagram with star-connected *R*-load.

are connected to phase-b and Q_5 and Q_2 are connected to phase-c.

In practice, for the three-phase bridge inverter there are two types of conduction depending on the control signal used to drive the six transistors with 120˚ or 180˚ conduction. Referring to Fig.8.18 : in 120˚, each BJT transistor conducts for 120˚ and in 180˚, each BJT transistor conducts for 180˚ as it will be described in the following subsection.

8.4.1 120 – Degree Conduction

In this type of control, only two transistors conduct at the same time such that each transistor conducts for 120˚ and remains OFF for 240˚. This means that only two transistors remain "ON" at any instant of time. The per-phase and line-to-line voltage waveforms for this type of operation are shown in Fig.8.19.

The conduction sequence of the transistors is : 1,6-1,2-2,3-3,4-4,5-1,6-1,2-2,3. For star-connected load, three modes for three-phase bridge inverter operation exist. In each case, the effective resistance across the source is $2R$ as shown in Fig.8.20.

- During mode-1 for $0 \le \omega t \le \pi/3$ transistor 1and 6 conduct:

$$v_{an} = +\frac{V_{dc}}{2} \quad , \quad v_{bn} = -\frac{V_{dc}}{2} \quad , \quad v_{cn} = 0 \quad , \quad v_{ab} = V_{dc}$$

- During mode-2 for $\pi/3 \le \omega t \le 2\pi/3$ transistor 1and 2 conduct:

$$v_{an} = +\frac{V_{dc}}{2} \quad , \quad v_{bn} = 0 \quad , \quad v_{cn} = -\frac{V_{dc}}{2}$$

- During mode-3 for $2\pi/3 \leq \omega t \leq \pi$ transistor 3 and 2 conduct:

$$v_{an} = 0 \quad , \quad v_{bn} = +\frac{V_{dc}}{2} \quad , \quad v_{cn} = -\frac{V_{dc}}{2}$$

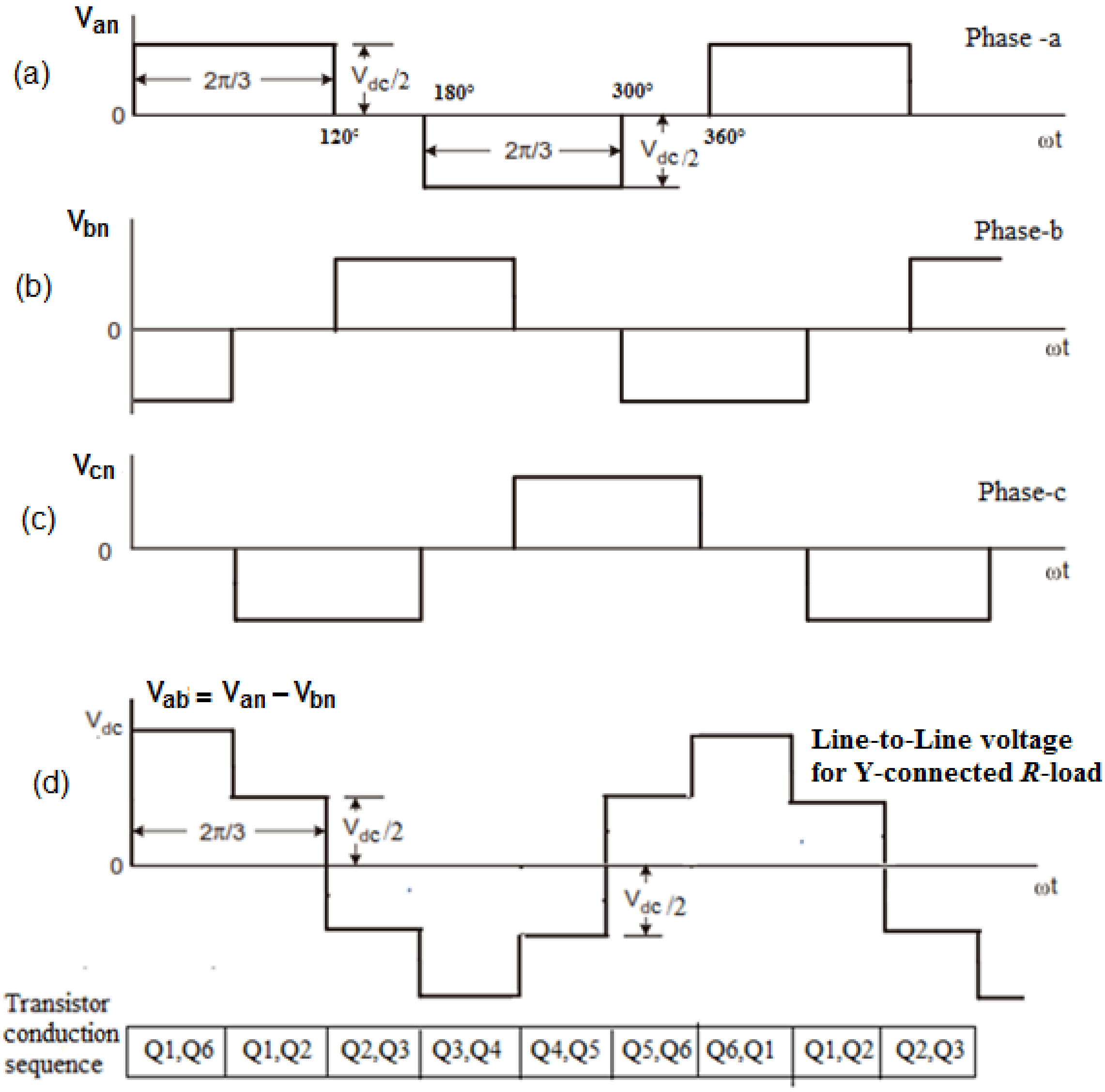


Fig.8.19 Voltage waveforms when two transistor in 120°conduction.

It should be noted from Fig.8.20 that the phase voltage is a quasi-square waveform. It has peak value of $\pm\frac{V_{dc}}{2}$. They are shifted by 120° with respect to each other. The output line to line voltage is a six-step waveform, it has peak value of $\pm V_{dc}$.

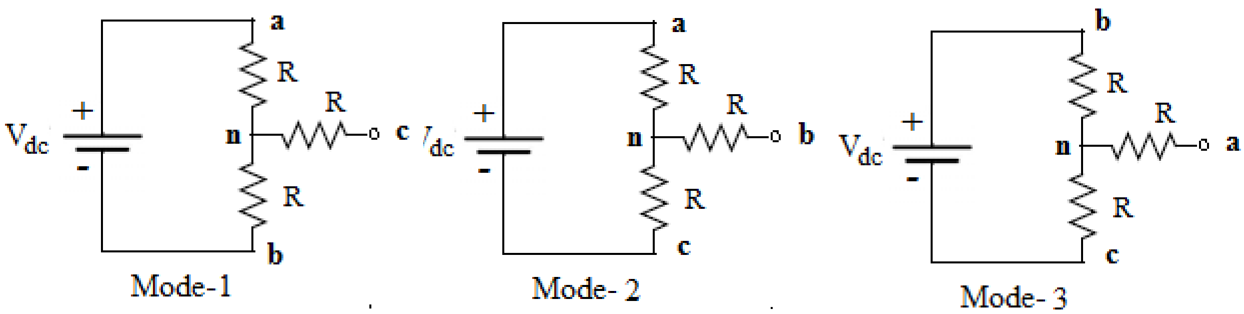


Fig.8.20 Equivalent circuits for the three modes of operation for 120° conduction for the three-phase bridge inverter.

Mathematical analysis of 120° Mode inverter

The *rms* value of the output voltage waveform of 120° mode inverter shown in Fig.8.19 for phase-a can be obtain as,

$$V_{a(rms)} = \frac{1}{2\pi}\int_0^{2\pi} v_a^2 \; d\omega t$$

$$= \frac{1}{2\pi}\sqrt{\int_0^{2\pi/3} (\frac{V_{dc}}{2})^2 \; d\omega t} + \frac{1}{2\pi}\sqrt{\int_{\pi}^{5\pi/3} (-\frac{V_{dc}}{2})^2 \; d\omega t}$$

$$\therefore \quad V_{a(rms)} = \frac{V_{dc}}{2}\sqrt{\frac{2}{3}} = \frac{V_{dc}}{\sqrt{6}} \qquad (8.32)$$

The line-to-line voltages for star-connected load can be found as:

$$V_{L-L} = \sqrt{3}\, V_{phase} = \sqrt{3} \times \frac{V_{dc}}{\sqrt{6}} = \frac{V_{dc}}{\sqrt{2}} \qquad (8.33)$$

The line-to-neutral voltages shown in Fig.8.19 (a) to (c) can be expressed in Fourier series as:

$$V_{an}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2V_{dc}}{n\pi} \sin\frac{n\pi}{2} \sin\frac{n\pi}{3} \sin n\left(\omega t + \frac{\pi}{6}\right) \qquad (8.34a)$$

$$V_{bn}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2V_{dc}}{n\pi} \sin\frac{n\pi}{2} sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{\pi}{2}\right) \qquad (8.34b)$$

$$V_{cn}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2V_{dc}}{n\pi} \sin\frac{n\pi}{2} sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{7\pi}{6}\right) \qquad (8.34c)$$

The peak amplitude of the fundamental component (n=1): for phase a, b or c is:

$$V_{L1P} = \frac{2}{\pi}V_{dc}\sin\frac{\pi}{3} = \sqrt{3}\,\frac{V_{dc}}{\pi} \qquad (8.35)$$

The *rms* value of the fundamental component is

$$V_{L1(rms)} = \frac{V_{1P}}{\sqrt{2}} = \sqrt{\frac{3}{2}} \times \frac{V_{dc}}{\pi} \qquad (8.36)$$

The line-to-line voltage waveform shown in Fig.8.19(d) is $v_{ab} = \sqrt{3}v_{an}$ with phase lead of 30° can be expressed in Fourier series as:

$$V_{ab}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2\sqrt{3}V_{dc}}{n\pi}\sin\frac{n\pi}{2}\sin\frac{n\pi}{3}\sin[n\,(\omega t + \frac{\pi}{6}) \pm \frac{\pi}{6}] \quad (8.37a)$$

Similarly for the other line voltages v_{bc} and v_{ca} , the Fourier representation are

$$V_{bc}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2\sqrt{3}V_{dc}}{n\pi}\sin\frac{n\pi}{2}sin\frac{n\pi}{3}\sin[n\left(\omega t - \frac{\pi}{2}\right) \pm \frac{\pi}{6}] \; (8.37b)$$

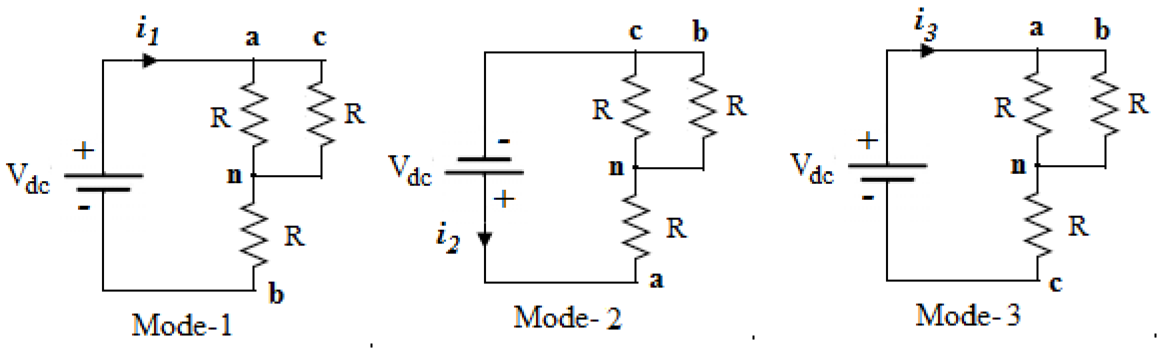
$$V_{ca}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{2\sqrt{3}V_{dc}}{n\pi}\sin\frac{n\pi}{2}sin\frac{n\pi}{3}\sin[n\left(\omega t - \frac{7\pi}{6}\right) \pm \frac{\pi}{6}] \; (8.37c)$$

where $+\frac{\pi}{6}$ for positive sequence harmonic components (n=1,7,13,19,……) and $-\frac{\pi}{6}$ for negative sequence harmonic components (n=5,11,17,23,……).

8.4.2 180 –Degree Conduction

In this mode of operation of the three-phase inverter, three transistors remain on at any instant of time. The base current drive of Q_1, in Fig.8.18, is applied for 180° (half-cycle) and is off for the remaining 180° (next half-cycle). Base current drive of Q_2 is applied with 60° delay with respect to Q_1. Hence for half a cycle, the following modes are presented for star-connected load:

- During mode-1 for $0 \leq \omega t \leq \pi/3$, transistor 1,5 and 6 conduct, the equivalent circuit is shown in Fig.8.21 (a).



(a) (b) (c)

Fig. 8.21 Equivalent circuits for the three modes of operation for 180° conduction.

$$R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$$

$$v_{an} = v_{cn} = +\frac{V_{dc}}{3}$$

$$i_1 = \frac{V_{dc}}{R_{eq}} = \frac{2V_{dc}}{3R}$$

$$v_{bn} = -i_1 R = -\frac{2V_{dc}}{3}$$

- During mode-2 for $\pi/3 \leq \omega t \leq 2\pi/3$, transistor 1,2 and 6 conduct: Equivalent circuit is shown in Fig.8.21 (b).

$$R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$$

$$i_2 = \frac{V_{dc}}{R_{eq}} = \frac{2V_{dc}}{3R}$$

$$v_{an} = i_2 R = \frac{2V_{dc}}{3}$$

$$v_{bn} = v_{cn} = \frac{-i_1 R}{2} = -\frac{V_{dc}}{3}$$

- During mode-3 for $2\pi/3 \leq \omega t \leq \pi$, transistor 1, 3 and 2 conduct:

  Equivalent circuit is shown in Fig.8.21 (c).

$$R_{eq} = R + \frac{R}{2} = \frac{3R}{2} \quad \Rightarrow \quad i_3 = \frac{V_{dc}}{R_{eq}} = \frac{2V_{dc}}{3R}$$

$$v_{an} = v_{bn} = i_3 \times \frac{R}{2} = \frac{V_{dc}}{3} \quad \text{and} \quad v_{cn} = -i_3 R = -\frac{2V_{dc}}{3}$$

Waveforms for the transistors base currents and load phase voltage and current for *R*-load are shown in Fig.8.22 for 180-degree conduction. The line-to-neural voltage and line current waveform, in case of resistive load, will be replica of the phase voltage waveform as shown in Fig.8.22(e). However, if the load is inductive, the phase current through each load branch would be delayed to its phase voltage as shown in Fig.8.22(f).

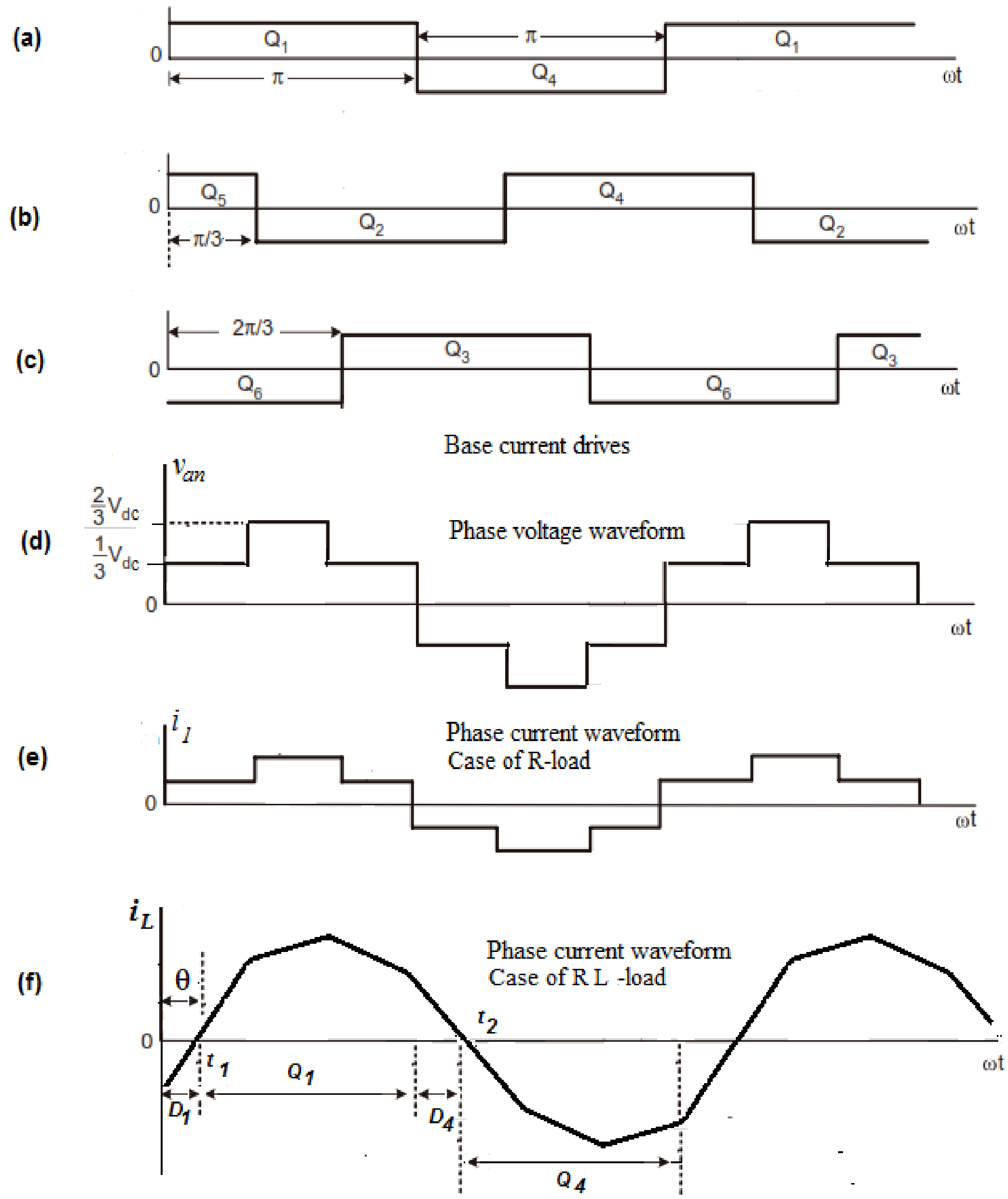


Fig.8.22 Line-to-neural voltage and line current waveforms for 180° conduction, case of *R*-load and phase current, (f) in case of *R*-*L* load.

Fourier representation of the phase and line voltages

The line-to-neutral voltage waveform shown in Fig.8.22(d) can be expressed in Fourier series, which is given in many reference and text books for the three phases a, b and c as,

$$v_{an}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{\sqrt{3}n\pi} \sin\frac{n\pi}{2} \sin\frac{n\pi}{3} \sin[n\left(\omega t + \frac{\pi}{6}\right) \mp \frac{\pi}{6}] \quad (8.38a)$$

$$v_{bn}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{\sqrt{3}n\pi} \sin\frac{n\pi}{2} sin\frac{n\pi}{3} \sin[n\left(\omega t - \frac{\pi}{2}\right) \mp \frac{\pi}{6}] \quad (8.38b)$$

$$v_{cn}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{\sqrt{3}n\pi} \sin\frac{n\pi}{2} sin\frac{n\pi}{3} \sin[n\left(\omega t - \frac{7\pi}{6}\right) \mp \frac{\pi}{6}] \quad (8.38c)$$

where : $-\frac{\pi}{6}$ for positive sequence harmonic components (n=1,7,13,19,...) and $+\frac{\pi}{6}$ for negative sequence harmonic components (n=5,11,17,23,...) with respect to the line to line voltage v_{ab} .
For Y-connected load, the line to line voltage is $v_{ab} = \sqrt{3}\ \ v_{an}$ without $\mp\frac{\pi}{6}$ delay angle, hence the three line-to-line voltages are given by

$$v_{ab}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{n\pi} \sin\frac{n\pi}{2} \sin\frac{n\pi}{3} \sin n\left(\omega t + \frac{\pi}{6}\right) \quad (8.39a)$$

$$v_{bc}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{n\pi} \sin\frac{n\pi}{2} sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{\pi}{2}\right) \quad (8.39b)$$

$$v_{ca}\,(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{n\pi} \sin\frac{n\pi}{2} sin\frac{n\pi}{3} \sin n\left(\omega t - \frac{7\pi}{6}\right) \quad (8.39c)$$

The line current i_L in phase-a for an *R-L* load is obtained by dividing the phase voltage v_{an} in by the load impedance, *Z=R+jnωL*

$$i_a\,(\omega t) = \sum_{n=1,3,5}^{\infty} I_n \sin[n\left(\omega t + \frac{\pi}{6}\right) \mp \frac{\pi}{6} - \theta_n] \quad (8.40)$$

where

$$I_n = \frac{4V_{dc}}{\sqrt{3}n\pi\sqrt{R^2 + (n\omega L)^2}} \sin\frac{n\pi}{2} \sin\frac{n\pi}{3}$$

and $\theta_n = \tan^{-1} n\omega L/R$, the impedance angle for the n[th] order harmonic.

The line-to-line *rms* voltage is

$$V_{L(rms)} = \sqrt{\frac{2}{2\pi}\int_0^{2\pi/3} V_{dc}^2 \; d\omega t} = \sqrt{\frac{2}{3}}\; V_{dc} \qquad (8.41)$$

From (8.38a), the *rms* value of the nth harmonic component of the phase voltage is

$$V_{Ln} = \frac{4V_{dc}}{\sqrt{6}n\pi}\sin\frac{n\pi}{2}\sin\frac{n\pi}{3} \qquad (8.42)$$

From which, the *rms* value of the fundamental component (n =1) of the output phase voltage is

$$V_{L1} = \frac{4V_{dc}}{\sqrt{6}\pi}\sin\frac{\pi}{3} = \frac{2\sqrt{2}}{\pi}\; V_{dc} \qquad (8.43)$$

Example 8.5

The three-phase inverter in Fig.8.18 used to feed a Y-connected resistive load with R =15 Ω per-phase. The d.c. input to the inverter V_{dc} = 300 V and the output frequency is 50 Hz. If the inverter is operating with 120˚ conduction mode, calculate : (a) The peak and *rms* value of the load current I_L, (b) The output power, and the average and *rms* values of the current of each transistor.

Solution

(a) For 120° conduction mode, at any time the load resistances of two phases are connected in series, hence, peak value of load current is

$$I_p = \frac{V_{dc}}{2R} = \frac{300}{2 \times 15} = 10 \text{ A}$$

From Eq.(8.32), the *rms* value of the phase voltage is

$$V_{a(rms)} = \frac{V_{dc}}{\sqrt{6}} = \frac{300}{\sqrt{6}} = 122.44\text{V}$$

Hence the *rms* value of the load current is

$$I_{a(rms)} = \frac{V_{a(rms)}}{R} = \frac{122.44}{15} = 8.16 \text{ A}$$

(b) The load power is

$$P_o = 3\, {I_{a(rms)}}^2 R = 3 \times 8.16^2 \times 15 = 3000 \text{ W}$$

(c) For 120° conduction mode, each transistor carries current for (1/3)rd of a cycle, hence the average transistor current is

$$I_{T(av)} = \frac{I_p}{3} = \frac{10}{3} = 3.33 \text{ A}$$

The *rms* value of the thyristor current is

$$I_{T(rms)} = \sqrt{(\frac{I_p}{3})^2} = \sqrt{(\frac{10}{3})^2} = 3.33 \text{ A}$$

8.5 INVERTER OUTPUT FREQUENCY AND VOLTAGE CONTROL

In many applications of the inverter require means of controlling the output voltage as well as its output frequency. In drive systems, the ratio of voltage to frequency (*V/f*) need to be maintained constant in order not to drive the machine into the saturation region of its magnetic circuit. Similarly, in UPS the output voltage of the inverter should be regulated. The output frequency can be easily controlled by controlling the triggering instants of the power switch electronically. However, the control of the voltage in most of these applications is usually required in order to provide stepless adjustment of the inverter output voltage. The methods of voltage control can be grouped into three broad categories:

1. Control of voltage supplies to the inverter.
2. Control of voltage delivered by the inverter.
3. Control of voltage within the inverter.

Control of voltage supplies to the inverter can be achieved by several methods including the use of d.c. choppers and phase-controlled rectifiers. The principal disadvantage of these methods is that the power delivered by the inverter is handed twice, once by the d.c. or a.c. voltage control and once by the inverter. In the second category, the output may be controlled by means of auto transformer.

In the third category, control of the inverter output voltage may be achieved by incorporating time-ratio controls within the inverter circuit. A more common method of controlling the voltage within an inverter involves the use of pulse width modulation (PWM) techniques. With this technique the inverter output voltage is controlled by varying the duration of the output voltage pulses. Moreover, the output frequency can also be controlled by this technique.

Principle of pulse width modulation (PWM) technique

PWM is simply the variation (modulation) of the duty of a square pulse to produce a controlled average voltage. In its simple form, PWM is obtained by comparing a reference signal, A_r (also called the modulating wave) with a triangular carrier wave, A_c as shown in Fig.8.23.

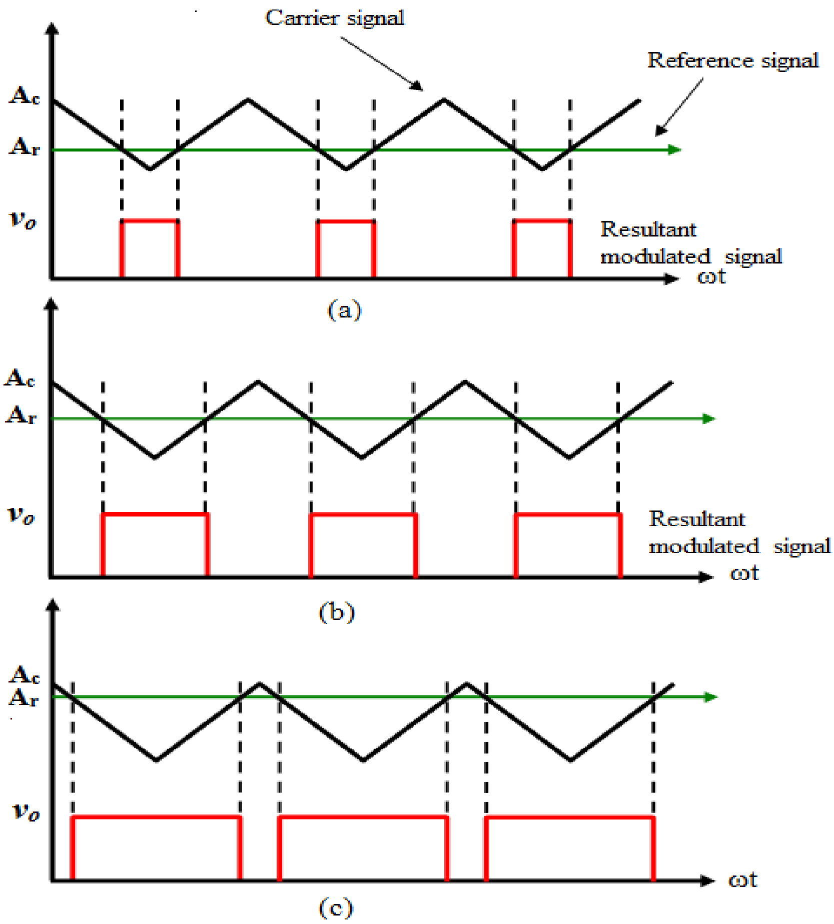


Fig.8.23 PWM technique: (a) low value of modulation index M, (b) moderate value of M, (c) high value of M.

By varying A_r from 0 to A_c , the pulse width δ can be varied from 0° to 180°. Thus, the voltage and hence, the amount of power sent to the load is controlled. The modulation index is defined as,

$$\text{Modulation index} = M = \frac{A_r}{A_c} \tag{8.44}$$

In power electronics, the modulating signal is produced by controlling the turn-on and turn-off of the power semiconductor devices such as thyristors, BJTs, MOSFETs, GTOs, etc. The duty of the positive and negative pulses (*rms* value) depends upon switching period which is controlled using PWM in the gate signals in the electronic switches. By the vision of Power Electronic applications, the quality of the PWM technique is measured by its powerful to eliminate the output harmonics or at least to eliminate the low order harmonics (*LOH*) which are difficult to filtered out.

There are many types of PWM, however, the commonly used techniques for controlling output voltage of a single-phase inverter are:

1- Single-pulse width modulation
2- Multiple-pulse-width modulation
3- Sinusoidal pulse-width modulation
4- Modified sinusoidal pulse-width modulation
5- Phase displacement control

Sinusoidal pulse-width modulation (SPWM) is the most commonly used among all these techniques. However, this type of modulation has some drawbacks that will be discussed later on in this chapter. Advanced modulation methods are also used recently such as staircase modulation, harmonic injection modulation, trapezoidal modulation and delta modulation. These methods will not be discussed in this book due to their complex theory. However in all above techniques the aim is to generate a sinusoidal output voltage, but they differ from each other in the harmonic content in their respective output voltage.

8.5.1 Single Pulse Width Modulation

The simplest form of pulse width modulation technique is the single pulse width modulation. This type of modulation gives quasi – square wave output as shown in Fig.8.24(c). According to this figure one can observe that, there is a single pulse of output voltage during each half-cycle and the width of the pulse is varied to control the output voltage. A carrier signal of frequency f_c with amplitude A_c is modulated by another signal or reference signal of amplitude A_r as shown in Fig.8.24 (a).

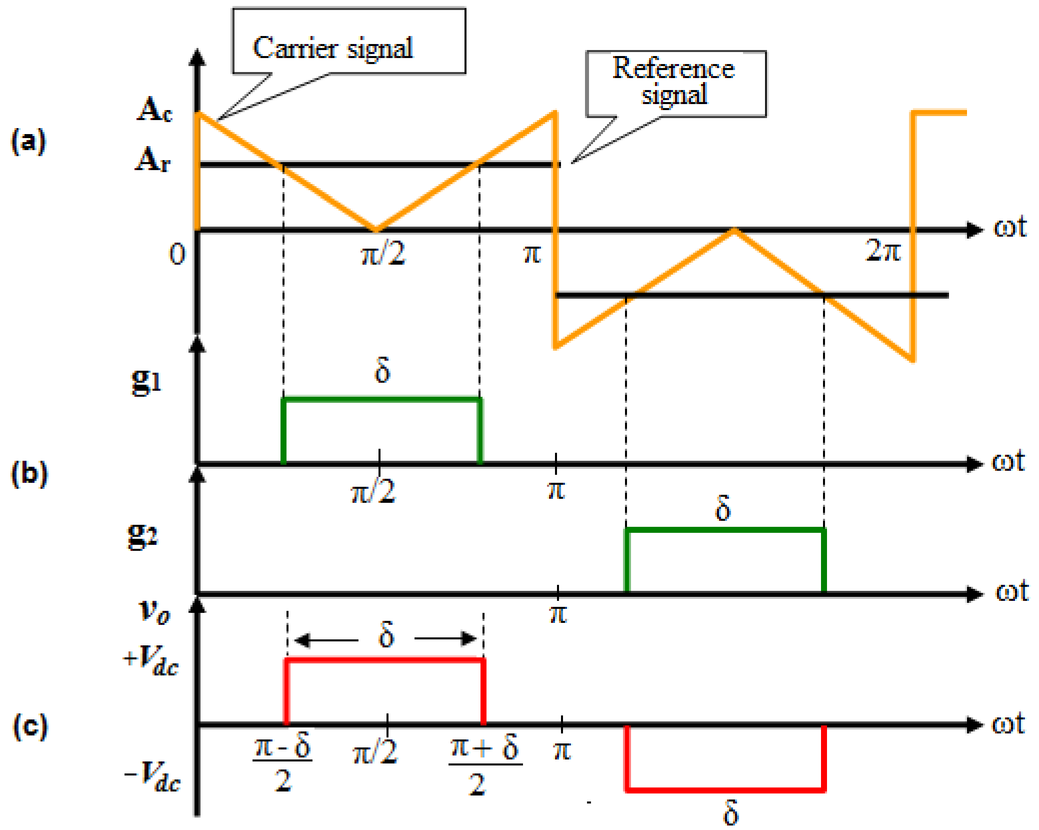


Fig.8.24 Single-pulse width modulation (SPM): (a) Reference signal, (b) Gate signals , (c) Output voltage waveform.

The main objective of using modulation process in power electronics engineering is to generate the gate signal to the power switches and thereby determine the output frequency of the inverter. As mentioned before, the output voltage can be varied by varying the pulse width δ by varying the amplitude A_r from 0 to A_c. i.e by varying the modulation index M from 0 to 1.

Referring to Fig.8.24, the *rms* of the output voltage is given by

$$V_{o(rms)} = \sqrt{\frac{2}{2\pi}\int_{\pi-\delta/2}^{\pi+\delta/2} V_{dc}^2 \; d\omega t} = V_{dc}\sqrt{\frac{\delta}{\pi}} \tag{8.45}$$

The output voltage waveform has the Fourier series

$$v_o(\omega t) = \sum_{n=1,3,5,..}^{\infty} \frac{4V_{dc}}{n\pi}\sin\frac{n\delta}{2}\,sinn\omega t \tag{8.46}$$

The peak value of the fundamental component is

$$V_1 = \frac{4V_{dc}}{\pi} \sin\frac{\delta}{2} \tag{8.47}$$

Pulse width δ has a maximum value of π radians at which the fundamental term in Eq.(8.47) is a maximum. The n[th] order harmonic in Eq.(8.46) is seen to have peak value,

$$V_1 = \frac{4V_{dc}}{n\pi} \sin\frac{n\delta}{2} \tag{8.48}$$

The distortion factor of the single-pulse waveform is therefore

$$\text{Distortion factor} = \frac{\frac{V_1}{\sqrt{2}}}{V_{orms}} = \frac{2\sqrt{2}}{\sqrt{\pi\delta}} \sin\frac{\delta}{2} \tag{8.49}$$

which has a maximum value of 0.9 when δ = π.

8.5.2 Multiple-PulseWidth Modulation

An alternative waveform consisting *m* symmetrical spaced pulses per half-cycle can be obtained by control the output voltage of the inverter such that it can be switched on and off rapidly several times during each half-cycle to produce a train of constant magnitude pulses. Fig.8.25 shows the idea of multiple pulse-width modulation.

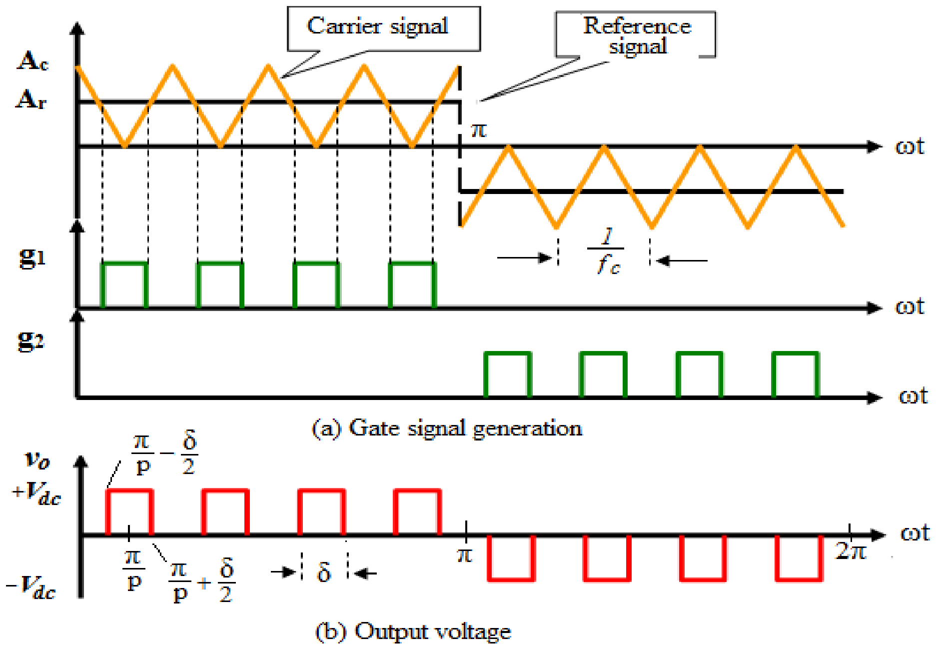


Fig.8.25 Multiple pulse width modulation.

The output voltage waveform consists of p pulses for each half-cycle of the required output voltage. Minimum pulse width is zero and maximum pulse width will be π/p.

Let f_o be the output frequency of the inverter, and $T = 1/f_o = 2\pi$. In this period there are 2p pulses of equal width. The first pulse is located at π/p, then its width is $\frac{\pi}{p}-\frac{\delta}{2}$ to $\frac{\pi}{p}+\frac{\delta}{2}$ as depicted in Fig.8.25. In this case the *rms* value of the output voltage will be:

$$V_{o(rms)} = \sqrt{\frac{2p}{2\pi}\int_{(\pi/p)-(\delta/2)}^{(\pi/p)+(\delta/2)} V_{dc}^2 \; d\omega t}$$

$$V_{o(rms)} = V_{dc}\sqrt{p[\frac{\pi}{p}+\frac{\delta}{2}-\frac{\pi}{p}+\frac{\delta}{2}]} = V_{dc}\sqrt{\frac{p\,\delta}{\pi}} \qquad (8.50)$$

The frequency of the pulses is $f_p = 2\ p\ f_o$ which must be the same frequency of the carrier signal f_c that is always greater than the output frequency f_o. Thus, the number of equal symmetrical pulses p per half-cycle can be calculated as

$$p = \frac{f_c}{2f_o} = \frac{m_f}{2} = \text{integer} \qquad (8.51)$$

where $m_f = f_c/f_o$ is a factor defined as the *frequency modulation ratio.*

The Fourier series of the output voltage is found to be

$$v_o(\omega t) = \sum_{n=1,3,5,..}^{\infty} B_n sinn\omega t \qquad (8.52)$$

where

$$B_n = \sum_{n=1,3,5,..}^{p} \frac{4V_{dc}}{n\pi}\sin\frac{n\delta}{4}[sin\, n\left(\alpha_m+\frac{3\delta}{4}\right)$$
$$-\ sin\, n\left(\pi+\alpha_m+\frac{\delta}{4}\right) \qquad (8.53)$$

here the angle α_m (m = 1,2,3,4,……) is the switching angle of each individual pulse in the positive half-cycle of the output voltage wave as depicted in Fig.8.26.

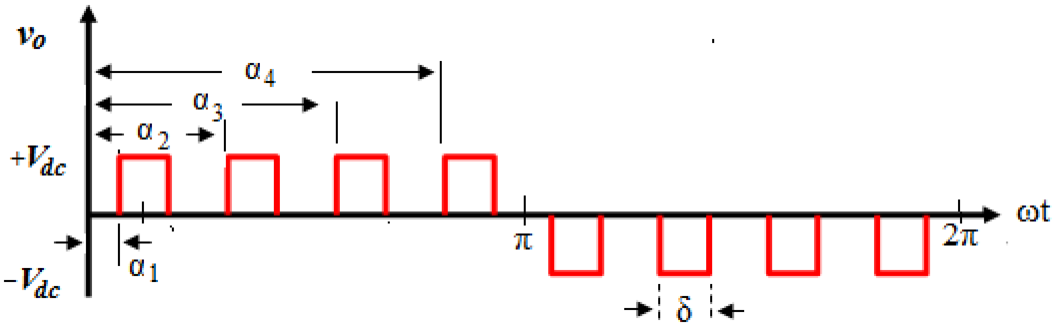


Fig.8.26 The output voltage wave and the meaning of the angle α_m.

The normalized amplitude of harmonics and the percentage distortion factor variation with the modulation index M are shown in Fig.8.27 for p = 5 of the multiple pulse modulation.

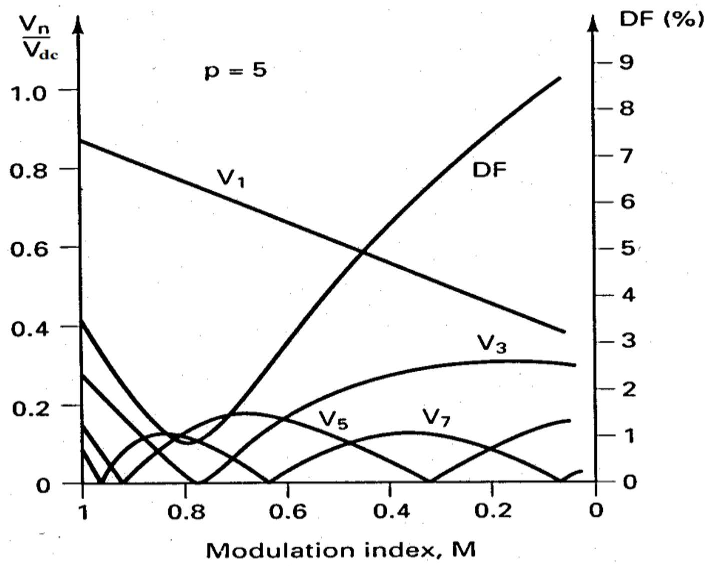


Fig.8.27 Normalized harmonic amplitude and distortion factor variations with the modulation index for p = 5.

8.5.3 Sinusoidal Pulse-Width Modulation (SPWM)

Using sinusoidal reference signal will produce varied width pulses that proportional to the amplitude of the sine wave as shown in Fig.8.28. In this technique, the lower order harmonics of the modulated voltage wave are greatly reduce. The *rms* value of the output voltage of the inverter depends on the widths of the pulses (δ_m). These widths depend on the mdulation index *M* which controls the output voltage of the inverter.

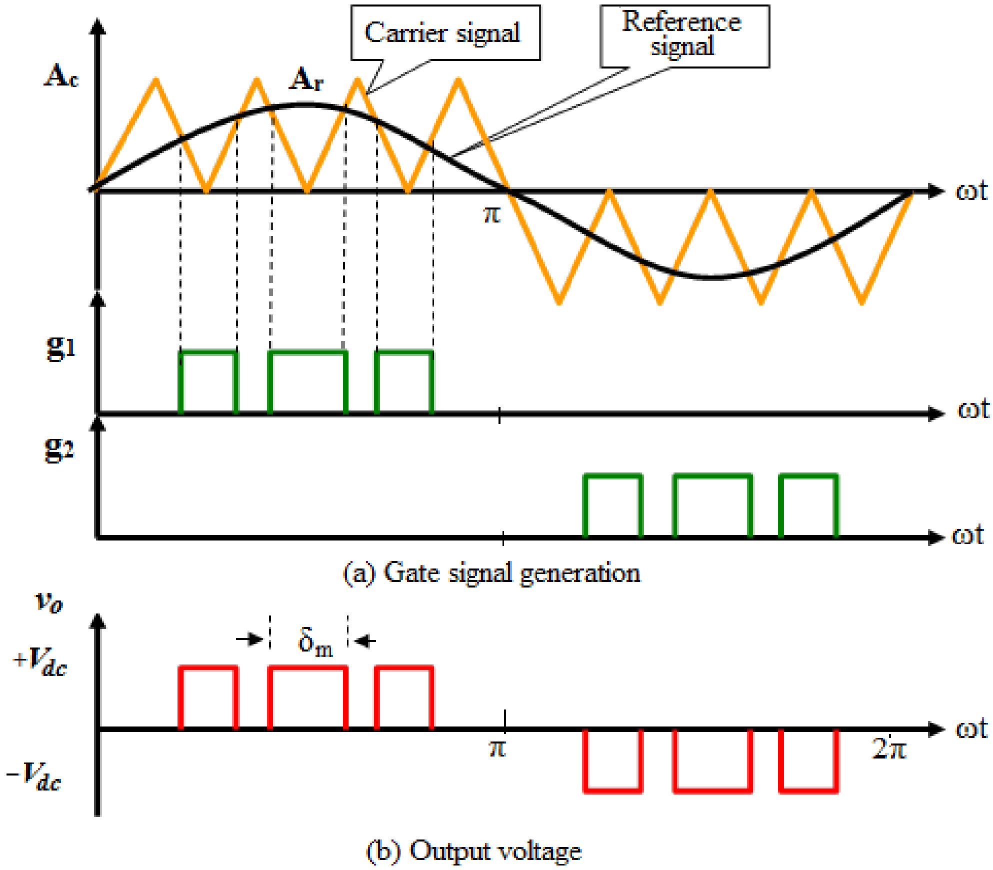


Fig.8.28 Sinusoidal pulse width modulation.

The magnitude of the fundamental component of the output voltage is clearly proportional to the modulation index *Ar /Ac* . But the highest practical value of *M* is unity. If *Ar > Ac* the output voltage waveform $v_o(\omega t)$ approaches a rectangular form and undesirable low frequency harmonics such as the third , fifth and seventh harmonics are introduced and intensified.

The *rms* output voltage is

$$V_o = V_{dc}\sqrt{\sum_{m=1}^{p}\frac{\delta_m}{\pi}} \qquad (8.54)$$

and the coefficient (B_n) of the Fourier series of the output voltage will be

$$B_n = \sum_{n=1.}^{p} \frac{4V_{dc}}{n\pi} \sin\frac{n\delta}{4} [sin\, n\left(\alpha_m + \frac{3\delta_m}{4}\right) - sin\, n\left(\pi + \alpha_m + \frac{\delta_m}{4}\right)] \qquad (8.55)$$

Another type of sinusoidal modulation can obtained by using two anti-phase sinusoidal reference (modulating) signal as shown in Fig.8.29. This technique is called double-sided triangular carrier wave modulation. The reference signal v_{ra} produces the resultant modulated wave v_a , Fig.8.29 (b), whereas the reference signal v_{rb} produces the resultant modulated wave v_b , Fig.8.29(c). The corresponding line voltage $v_{ab} = v_a - v_b$ has a fundamental component as shown in Fig.8.29(d).

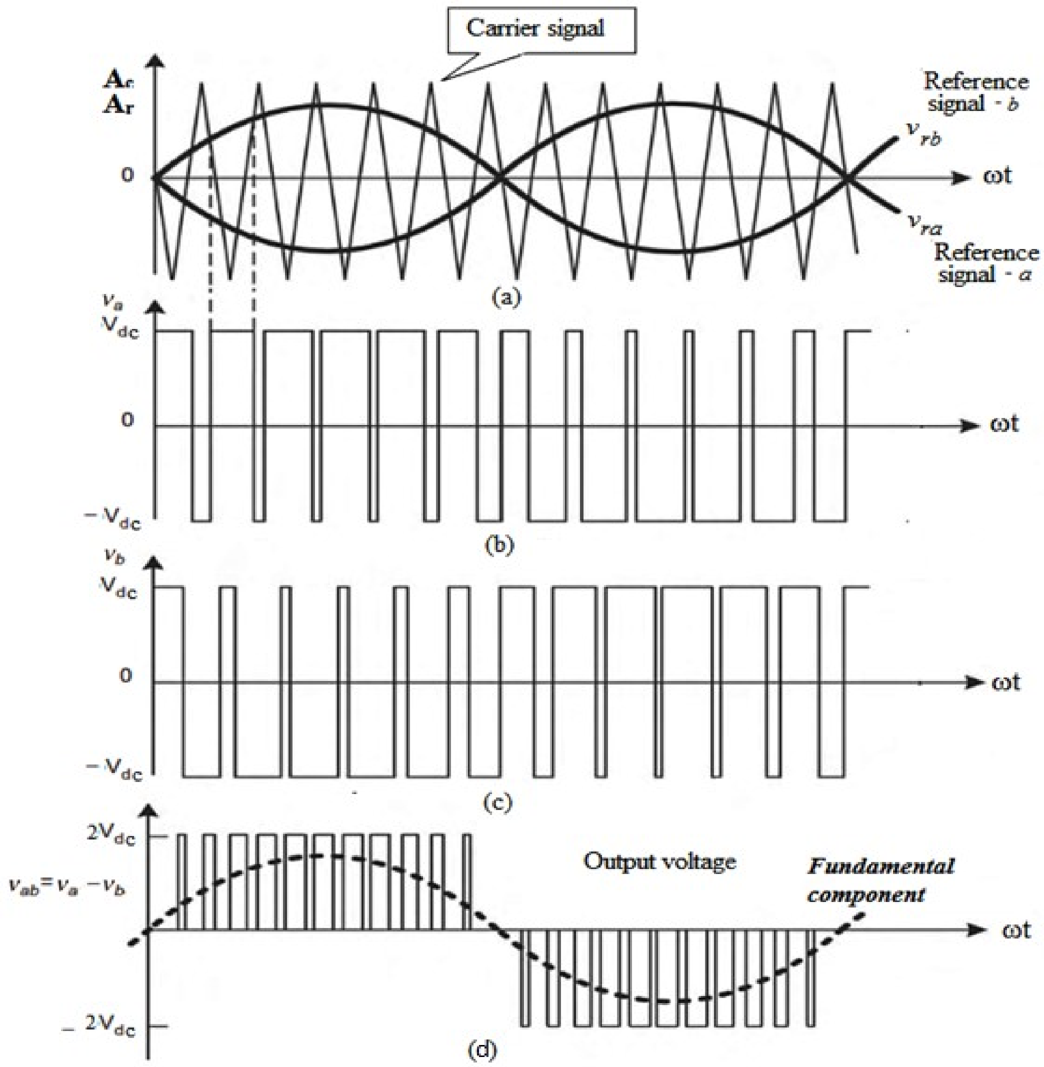


Fig.8.29 PWM voltage waveforms obtained by sinusoidal modulation of double-sided triangular carrier wave.

8.5.4 Modified Sinusoidal PulseWidth Modulation

Here the carrier signal is not symmetry triangle wave. The carrier wave is generated so the widths of the pulses that are near to the peak of the sine wave not change much when modulation index changed. Such scheme is shown in Fig.8.30 and known as MSPWM. Note that the triangular wave is present for the period of first 60˚ of the half cycle of sine wave. The MSPWM increase the fundamental component and improve the harmonic characteristic. This technique reduces the number of switching of power devices and also reduces switching losses.

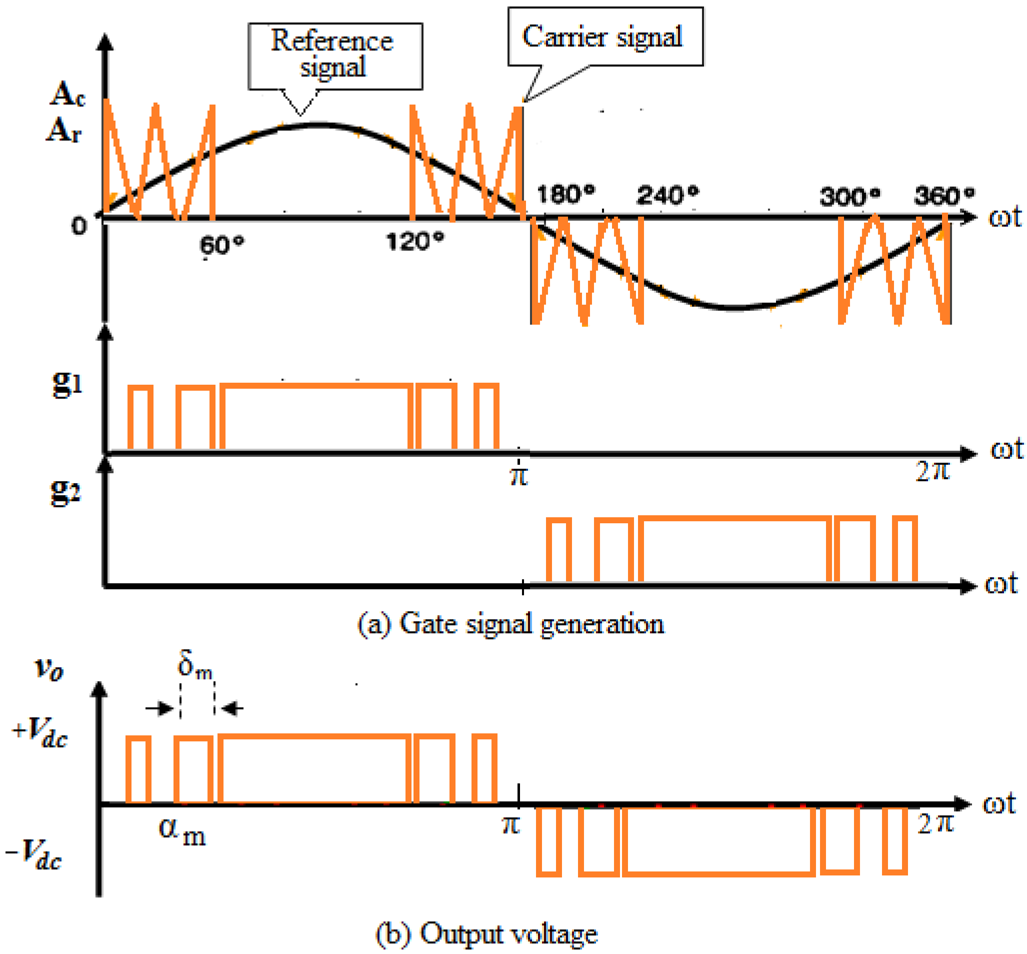


Fig.8.30 Modifide PWM voltage waveforms.

8.5.5 Other PWM Methods

Besides SPWM, other PWM methods are proposed for generating three-phase voltages to deliver more power, including third-harmonic PWM, sixty-degree PWM and space vector PWM. All these methods are aimed at making better use of the d.c. bus voltage and thus increasing modulation index. These methods are shortly mentioned here without detailed analysis.

1. Third-harmonic PWM

In this method, the reference signal is not a pure sinusoidal wave, but the sum of the fundamental and the third harmonic. It is shown with curve $f(\omega t)$ in Fig.8.31. Same as the SPWM method, this signal will be compared with the triangle signal to generate PWM signals.

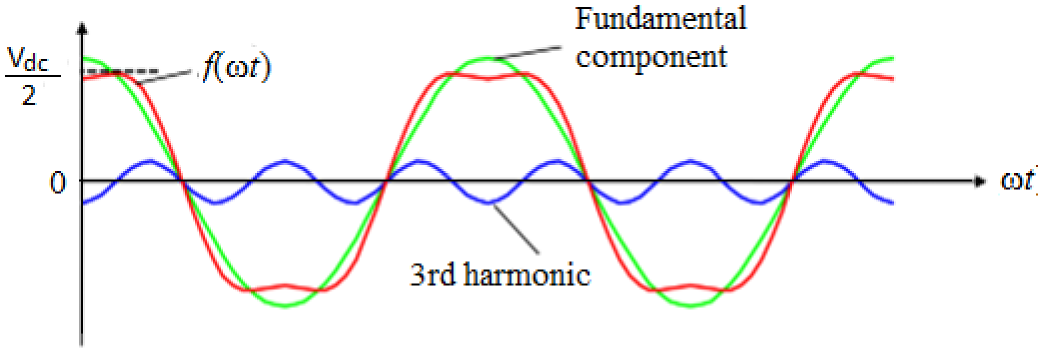


Fig.8.31 Reference signal of third harmonic.

Same as the SPWM method, this signal will be compared with the triangle signal to generate PWM signals. This concept is derived from the nature of three-phase motors that the third harmonic will be filtered out in the windings. Thus only the fundamental part will remain and the modulation index of this method reaches 1.

2. Sixty-degree PWM

The sixty-degree PWM is an extension of third-harmonic PWM. It is based on the nature of the motor that not only third harmonic, but also all none-even triple harmonics are filtered out by the windings. Adding all these harmonics with the fundamental together, a function with flat segments are obtained as shown in Fig.8.32. The period of the flat part covers 60° signal phase. The modulation index of this method can reach 1, too.

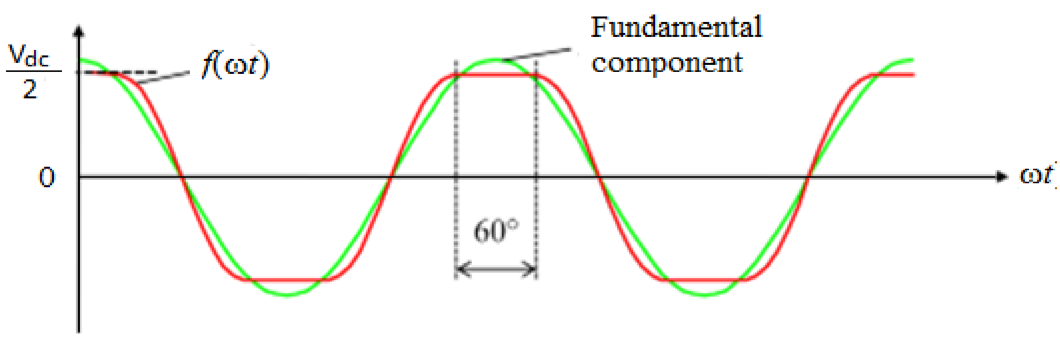


Fig.8.32 Concept of sixty-degree PWM.

Drawbacks of PWM

Pulse width modulation is a simple, flexible process. It is very common in inverter that need to provide adjustable performance and in small battery powered audio amplifiers. However it has a few limitation that preclude certain specialized applications.

- The highest output amplitude is equal to the input.
- Numerous switching operations occur each period. In real switches, a small energy loss is incurred every time a switch turns on or off. In a PMW converter with frequency ratio f_{switch} / f_{out} , the energy lost in switching increases by the same ratio, relative to the loss in a VSI inverter.
- Distortion : Although a low-pass filter should be adequate for recovery modulation function, the inverter output has large harmonics extending to very high frequencies. Even small residual components at megahertz frequencies can interfere with communication equipment, sensors, analog signal processing, and sometime digital logic.

8.6 VOLTAGE CONTROL OF THREE-PHASE INVERTER

For the three-phase inverters, output voltage control can be achieved by the same techniques discussed in the previous section. However, the following methods are mostly used for voltage control in three-phase inverters.

- Sinusoidal pulse width modulation.
- 60° pulse width modulation.
- Third harmonic pulse width modulation.
- Space vector modulation.

In sinusoidal pulse width modulation, three reference sinusoidal signals are used one for each phase. These three reference waves have phase shift of 120° between each other so that they produce three – phase output voltage waves.

8.7 HARMONIC REDUCTIONS IN THE INVERTER OUTPUT VOLTAGE

The output voltage waveform of the inverter is rich of harmonics; the main objective is to obtain a sinusoidal ac output voltage waveform where the fundamental component can be adjusted within a range and the intrinsic harmonics selectively eliminated and to element the unwanted harmonics.

There are several industrial applications, which may allow a harmonic content of 5 % of its fundamental component if, input voltage when inverter is used. Actually, the inverter output voltage may have harmonic content much higher than 5 % of its fundamental component. In order to bring this content to reasonable limit, there are several methods which can be used:

1- Harmonic reduction by PWM.
2- Harmonic reduction by transformer connections.
3- Harmonic reduction by stepped-wave inverter.
4- Harmonic reduction using phase displacement control.

8.7.1 Harmonic Reduction by PWM

With single pulse width modulation discussed in subsection 8.5.1, the amplitude c_n of the nth order harmonic as given from the Fourier series of the output voltage in Eq.(8.46) ,

$$c_n = \frac{4V_{dc}}{n\pi}\sin\frac{n\delta}{2}$$

where δ is the pulse width
If it is required to eliminate the third order harmonic (n=3) from the output voltage waveform , let

$$\sin\frac{3\delta}{2} = 0 \;\rightarrow\; \frac{3\delta}{2} = 180° \qquad \therefore \;\; \delta = 120°$$

To eliminate the fifth order harmonic:

$$\sin\frac{5\delta}{2} = 0 \;\rightarrow\; \frac{5\delta}{2} = 180° \qquad \therefore \;\; \delta = 72°$$

It is obvious that m with this method only single specific order harmonic can be eliminated. However, since the third harmonic is the next most predominant one in the output voltage waveform of the inverter, δ must always be 120˚.

8.7.2 Harmonic Reduction by Transformer Connections

The output of two or more inverters may be connected together using separate transformers to produce one output voltage waveform that has other harmonic characteristics. In order to understand the method let us assume that we want to eliminate the third harmonic as well as the triplen order harmonics (9, 15, 18,…), two inverter outputs v_{o1} andv_{o2} are connected through two transformers as shown in Fig.8.33(a).

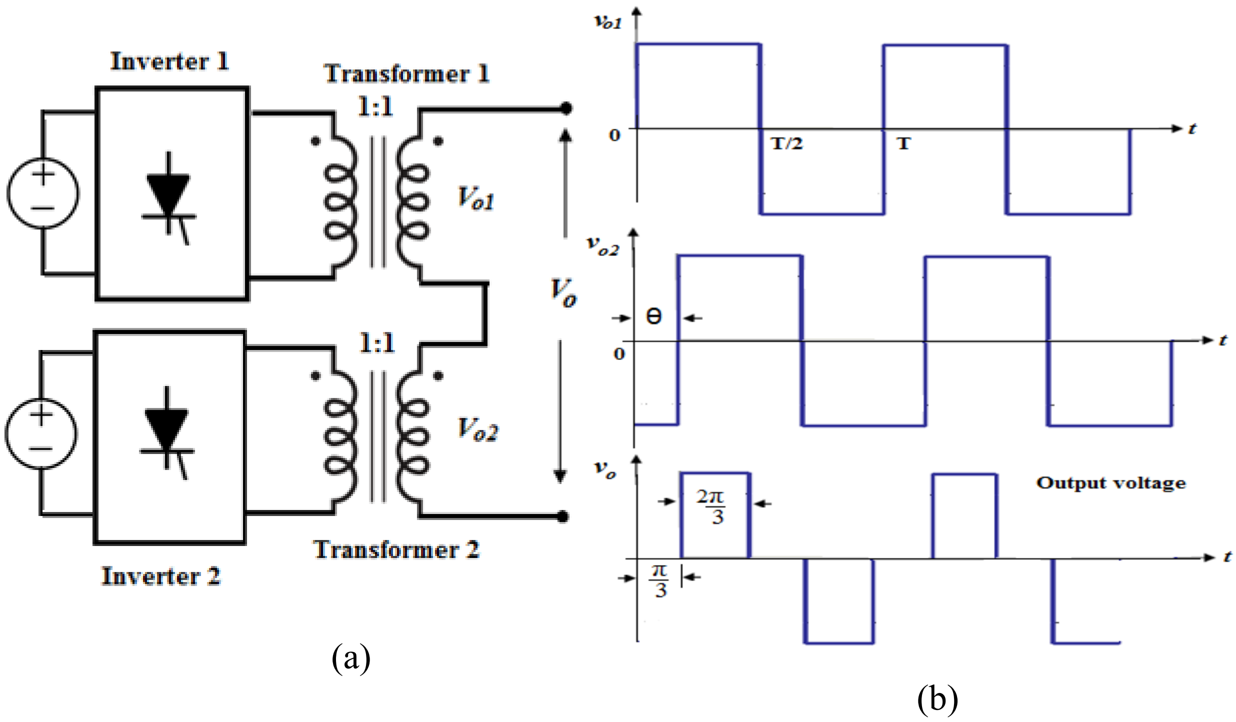


Fig.8.33 Harmonic elimination using transformer connection: (a) circuit, (b) waveforms.

Let the two voltages are expressed in Fourier series as

For inverter 1

$$v_{o1}(\omega t) = V_{m1}\sin\omega t + V_{m3}\sin 3\omega t + V_{m5}\sin 5\omega t + V_{m7}\sin 7\omega t + V_{m9}\sin 9\omega t + V_{m11}\sin 11\omega t + \cdots \quad (8.56)$$

For inverter 2 , the output voltage v_{o2} is made to lag v_{o1} by π/3, thus

$$v_{o2}(\omega t) = V_{m1}\sin\left(\omega t - \frac{\pi}{3}\right) + V_{m3}\sin(3\omega t - \frac{\pi}{3}) + V_{m5}\sin(5\omega t - \frac{\pi}{3} + V_{m7}(\sin 7\omega t - \frac{\pi}{3}) + V_{m9}(\sin 9\omega t - \frac{\pi}{3}) + V_{m11}(\sin 11\omega t - \frac{\pi}{3}) + \cdots \quad (8.57)$$

The resultant output voltage is $v_o(\omega t) = v_{o1}(\omega t) + v_{o2}(\omega t)$, hence

$$v_o(\omega t) = \sqrt{3}\left[V_{m1}\sin\left(\omega t - \frac{\pi}{6}\right) + V_{m5}\sin\left(5\omega t - \frac{\pi}{6}\right) + V_{m7}(\sin 7\omega t - \frac{\pi}{6}) + V_{m11}(\sin 11\omega t - \frac{\pi}{6}) + \cdots\right] \quad (8.58)$$

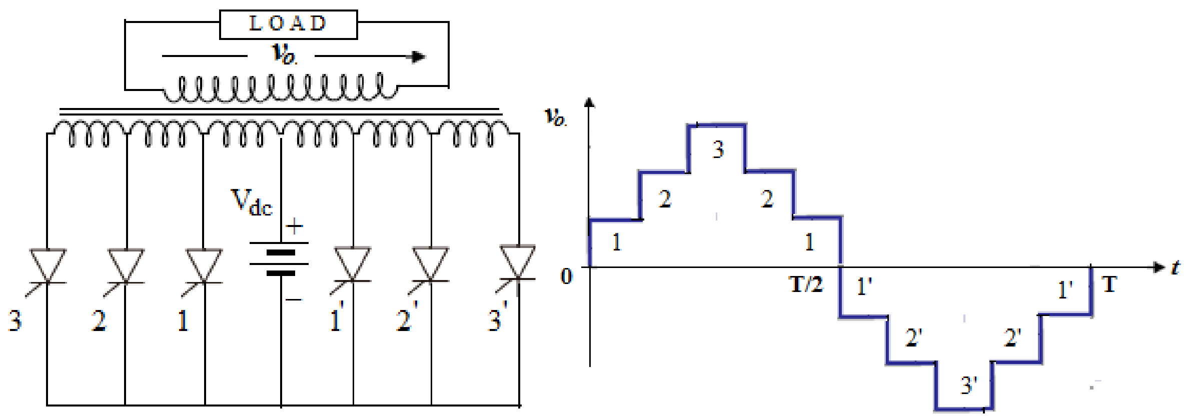
It is obvious from Eq.(8.58) that the third order harmonic as well as all the triplen harmonics (n=9,15,18,…) are entirely eliminated from the output voltage wave. Waveforms of the system are shown in Fig.8.33(b) for clarity.

Alternatively the triplen harmonics can also be suppressed using delta-star transformer at the output of a three-phase inverter, since these harmonic componenets are all in phase and they will cause circulating currents that cancel each other through the delta-connection. Therefore, the output voltage from the star side will not contain any triplen order harmonic voltage.

8.7.3 Harmonic Reduction by Stepped-Wave Inverter

This method of harmonic reduction is similar to the previous one as it also requires two inverters with separate transformers that their secondary windings have different turn ratios and are connected in series as shown in Fig.8.33(a). Transformer 1 has 1:1 turns ratio whereas transformer 1 may has 1:3 turns ratio such that the output resultant waveform is a stepped wave of different amplitudes.

Alternatively, one can use parallel inverter with multi taps on its primary winding as shown in Fig.8.34 (a).When the thyristors on the left side of the battery conduct, the output voltage is positive. When the thyristors on the right side conduct the output voltage is negative. The conduction sequence of the thyristors is 1-2-3-2-1-1'-2'-3'-2'-1'to get 12 steps output voltage waveform.



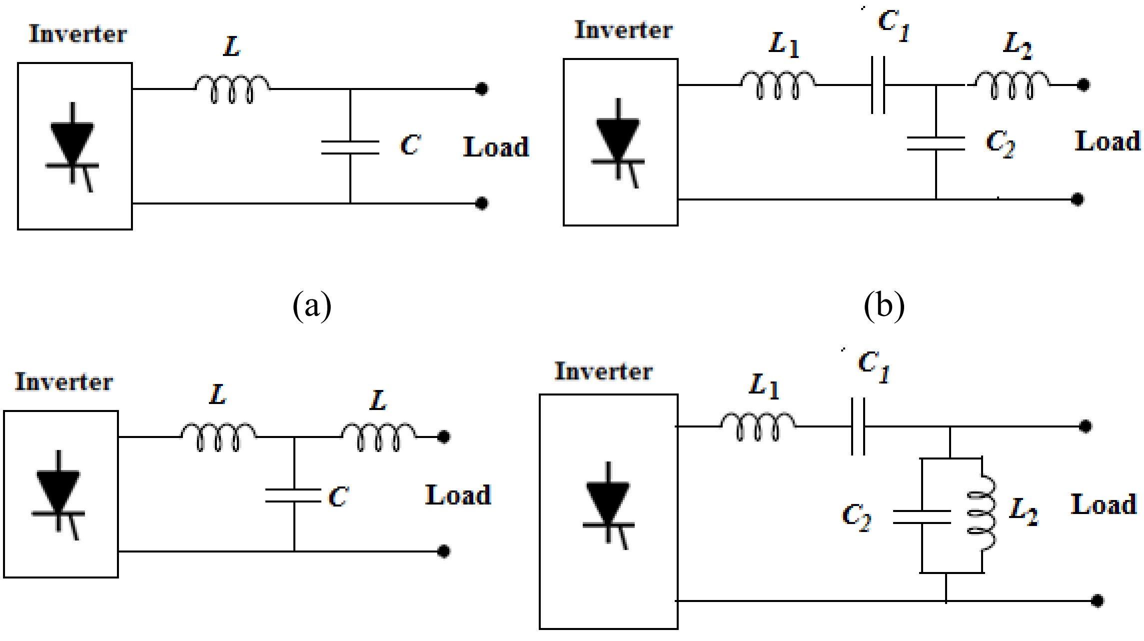
(a) (b)

Fig.8.34 Stepped wave inverter (a) Circuit, (b) output voltage waveform.

8.7.4 Harmonic Reduction Using Filters

Filters of various types such as *LC*, *LCL* or *OTT* are also used to reduce the harmonic contents of the output voltage waveform of an inverter. The reactance ($X_L = 2\pi f L$) of an inductor increases with frequency, therefore the effect of series inductance is to attenuate high frequencies. On the other hand the reactance of a capacitor ($X_C = 1/2\pi f C$) is inversely proportional to frequency, hence a parallel capacitor provides by-pass to high frequencies. These features are used in combination to eliminate higher order harmonics to some extent. The types of filters in common used for inverter circuits are depicted in Fig. 8.35.

Although *LC* filter shown in Fig.35(a) is simple and gives nearly sinusoidal output voltage waveform , *OTT* filter shown in Fig.8.35(b) gives better performance characteristics due to the generation of high quality and very low harmonic content output voltage waveform. The drastic reduction in the higher order harmonics leads to transfer of their energies to the fundamental component of the desired frequency. However *LC* filter does not has resonance problems as it exist in other types of filters.Hoever , for an inductive load that requires a smooth current (e.g. an electrical machine), the machine inductance provides the filtering because it acts as a low pass filter.



(c) (d)

Fig.8.35 Four types of filters commonly used with inverter circuits: (a) Simple *LC* filter, (b) OTT filter, (c) LCL filter, (d) Resonant filter.

8.8 THREE-PHASE NATURALLY COMMUTATED INVERTER

The three-phase full-wave, fully-controlled bridge rectifier, discussed in Chapter Three, can be used as an inverter if the load is replaced by a d.c. supply with reverse polarity voltage as shown in Fig.8.36. The current direction, on the d.c. side is unchanged. If this rectifier bridge is used as inverter, the voltage and frequency as well as the shape of the wave on the a.c. side are set by the bus and cannot be changed.

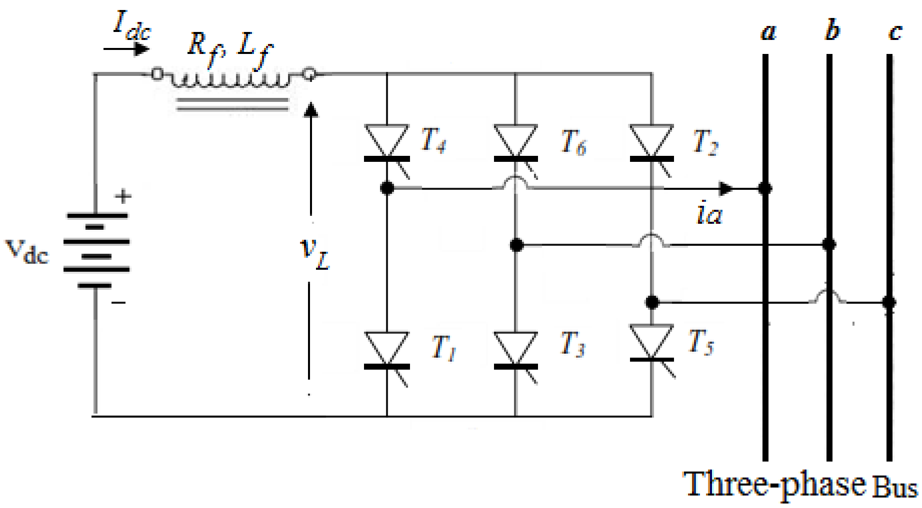


Fig.8.36 Three-phase naturally commutated bridge inverter.

The instantaneous values v_L on the d.c. side of the bridge and the output waveforms are identical to those of rectifier operation with passive load shown in Fig.3.37. For $\alpha < 60^o$, v_L always positive and at $\alpha = 90^o$, $v_L = 0$, whereas for $\alpha > 90^o$, the average value of v_L is negative.
As it was given in Chapter Three , the average value of the bridge voltage is , from Eq.(3.40),

$$V_{Lav} = \frac{3\sqrt{3}V_m}{\pi} cos\alpha \quad = 2.34\, V_{sL}\, cos\alpha \tag{8.59}$$

where $V_m$is the peak phase voltage and V_{sL} is the *rms* phase voltage.
The average value of the current on the d.c. side is, from Fig.3.36,

$$I_{dc} = \frac{V_{dc} + V_{Lav}}{R_f} = \frac{1}{R_f}[\, V_{dc} + 2.34\, V_{sL}\, cos\alpha] \tag{8.60}$$

The power on the d.c. side is

$$P = I_{dc}V_{Lav} \tag{8.61}$$

8.9 CURRENT SOURCE INVERTER

The current source inverter (CSI) is a device that converts the input direct current into an alternating current. It is also called current fed inverter in which the output current is maintained constant irrespective of load on the inverter. This means that, the magnitude and nature of the load current depends on the nature of load impedance. The output voltage of the inverter is independent of the load. The major advantage of current source inverter is its reliability. In the case of current source inverter a commutation failure in the same leg does not occur due to the presence of a large inductance L_d connected in series with the voltage source.

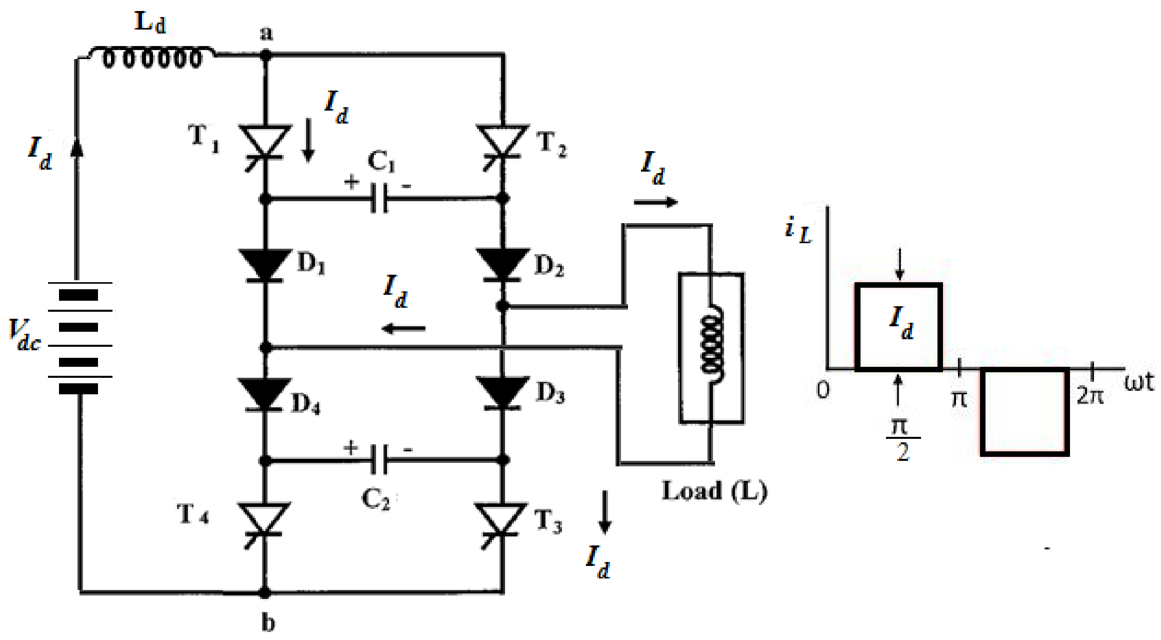
Compared with a VSI system, the output current of a CSI system is not influenced by the supply voltage, so its output current has low THD and high *PF*. Hence, in the 1980s the current source inverters were the main commonly used electric machine feeding devices. The current source inverter was constructed of a thyristor bridge with large inductance and large commutation capacitors. Serious problems in such drive systems were unavoidable overvoltage cases during the thyristor commutation, as the current source inverter current is supplied in a cycle from a dc-link circuit to the machine phase winding. The thyristor CSI has been replaced recently by the transistor reverse blocking IGBT devices (RBIGBT), where the diode is series-connected and placed in one casing with transistor. The power transistors like RBIGBT or Silicon Carbide (SiC) used in the modern CSIs guarantee superior static and dynamic drive characteristics. The use of current sources for the electric machine control ensures better drive properties than in case of voltage sources, where it may be necessary to use an additional passive filter at the inverter output. Therefore, due to its advantages, the current source inverters are used in many industrial applications such as induction heating, static var compensators (SVC) , variable speed a.c. motors etc.

The freewheeling diodes that are used for voltage source inverters become useless if an inverter is supplied from a d.c. current source, this is because the current in any half-leg of the inverter cannot change its polarity, hence it can only flow through the power semiconductor switches. Therefore, absence of the freewheeling diodes reduces the size and weight of the inverter circuit.

In practice CSI is mainly used for a.c. motor control, hence it invariably of three-phase type. However, we shall describe both the single-phase and three-phase types hereafter.

8.9.1 Single-Phase Current Source Inverter

Fig.8.37 shows the circuit of a single-phase current source inverter. A constant current source is assumed here, which may be realized by using a large inductance L_d of suitable value in series with the current limited d.c. voltage source V_{dc}. The thyristors of the inverter circuits are, T_1 , T_3, T_2 and T_4 , are alternatively turned ON to obtain a nearly square wave current waveform. Two commutating capacitors – C_1 in the upper half, and C_2 in the lower half, are used. Four diodes, D_1, D_2, D_3 and D_4 are connected in series with each thyristor to isolate the commutating capacitors from the load and prevent them from discharging through the load.



(a) (b)

Fig. 8.37 Single-phase current source inverter: (a) Circuit, (b) Output current waveform.

The output frequency of the inverter is controlled by varying the half time period, ($T/2$), at which the thyristors in pair are triggered by pulses being fed to the respective gates by the control circuit, to turn them ON. The operation of the circuit can be described as :

In the circuit of Fig.8.37(a), two thyristors must trigger simultaneously to permit current to flow. For example, T_1 and T_3 must be triggered, while in reverse, T_2 and T_4 must trigger at the same time. The output current waveform is shown in Fig.8.37(b) for the case of inductive load.

When T_1 and T_3 conduct, capacitors C_1 and C_2 would be charged with the polarity as shown. When T_2 and T_4 are turned on, thyristors T_1 and T_3 are reversed biased by the capacitors C_1 and C_2 respectively to commutate them. At this instant, the load current flows through $T_2 – C_1 – D_1$ – load – $D_3 – C_2 – T_4$ charging capacitors C_1 and C_2 with opposite polarity and are ready now to commutate T_2 and T_4 , while the current in the load changes its direction and the cycle is repeated.

8.9.2 Three-Phase Current Source Inverter

The circuit of a three-phase current source inverter is shown in Fig.8.38. As in the circuit of a single-phase CSI, the input is also a constant current source. In this circuit, six thyristors, two in each of three arms, are used, as in a three-phase VSI. Also, six diodes D_1, D_2, D_3, D_4, D_5 and D_6, each one in series with the respective thyristor, are needed here, as used for single-phase CSI.

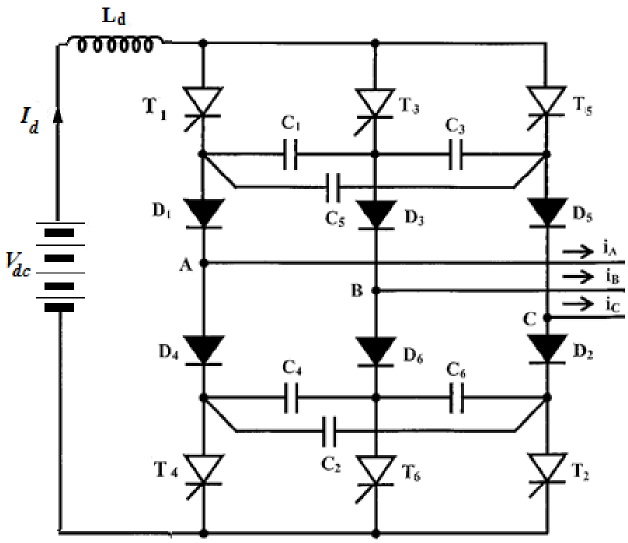


Fig. 8.38 Three-phase current source inverter.

Six capacitors, three each in two (top and bottom) halves, are used for commutation. It may be noted that six capacitors are equal, i.e. C1= C2 = C3.....= C6. The six diodes are needed in CSI, so as to prevent the capacitors from discharging through the load. The numbering scheme for the thyristors and diodes are same, as used in a three-phase VSI, with the thyristors being triggered in sequence as per number assigned in 120° conduction mode or 180° conduction mode discussed in three-phase voltage source inverter in subsections 8.4.1 and 8.4.2 respectively.

PROBLEMS

8. 1 (a) Draw the circuit diagram of a single-phase half-bridge inverter.
(b) Sketch the output voltage waveform and show conducting devices.
(c) Give an expression for the *rms* values of the inverter output (load) voltage.
(d) Sketch one transistor current and give its average value for a purely resistive load.
(e) What is the peak inverse voltage of the transistors?

8.2 A single-phase half-bridge inverter has a resistive load of 2.5 Ω and input voltage of 50 V. Calculate the following:

(a) The *rms* voltage of the fundamental frequency component,
(b) The output power,
(c) The average current and peak current,
(d) Harmonic *rms* voltage, and
(e) The total harmonic distortion factor.

[Ans : (a) 45 V, (b) 1000 W, (c) 10 A , 20 A , (d) 21.8 V , (e) 48 %]

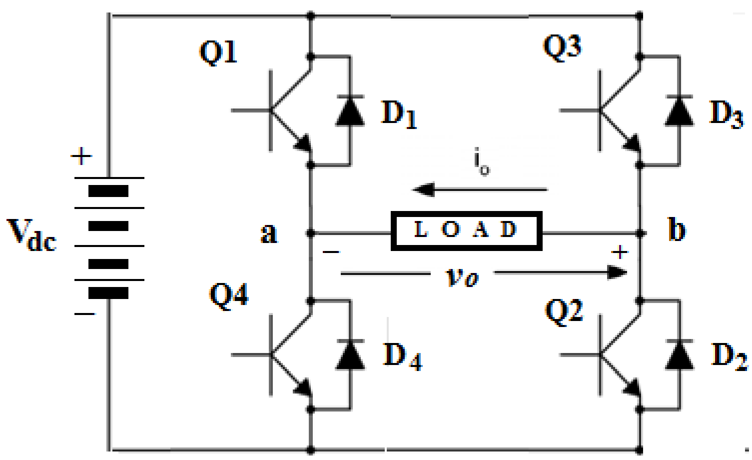
8.3 For a single-phase transistor bridge inverter, the d.c. supply voltage V_{dc} = 300 V, the load consists of resistance R = 20 Ω in series with L = 40 mH and the output voltage frequency f_o = 50 Hz.

(a) Draw the output voltage and load current waveforms for highly inductive load.
(b) Determine the amplitude of the Fourier series terms of the output voltagewaveform up to the 9th order harmonics.
(c) What is the maximum value of the load current (i_{omax})?
(d) Calculate the power absorbed by the load (consider up to 9th order harmonics).

[Ans : (b) c_1=382 V, c_3 = 127.3 V, c_5 = 76.3 V, c_7 = 54.5 V, c_9 = 42.4 V , (c) I_{omax} = 37.5 A , (d) 4316 W]

8.4 For the single-phase bridge inverter circuit shown in Fig.8.39, V_{dc} =125V, load resistance R =10 Ω and output voltage frequency f_o = 50 Hz.

Fig.8.39.



(a) Write the Fourier series terms of the output voltage waveform up to the 7th order harmonic.
(b) Draw the output voltage and load current waveforms.
(c) Determine the value of the *rms* output voltage in terms of

harmonics rms values and the output power P_o (consider up to 7^{th} order harmonics).
(d) Calculate the average and peak currents of each transistor.
(e) Estimate the total harmonic distortion factor *THD* of the circuit.

[Ans : (a) $v_o(\omega t)$ = 159.12 sinωt + 53.04 sin3ωt +31.82 sin 5ωt + 22.73 sin7ωt , (c) 121.78 V, 1483 W (d) 6.25 A, 12.5 A , (e) 48.36 %]

8.5 In a single-phase full-bridge PWM inverter, the control logic for power electronics devices is so adjusted that the output voltage waveform produced is a quasi-square as shown in Fig.8.40. The inverter has a resistive load of 5 Ω and input voltage of 100V. Find an expression for the Fourier coefficient of the output voltage for the first sixth harmonics and their corresponding harmonic currents.

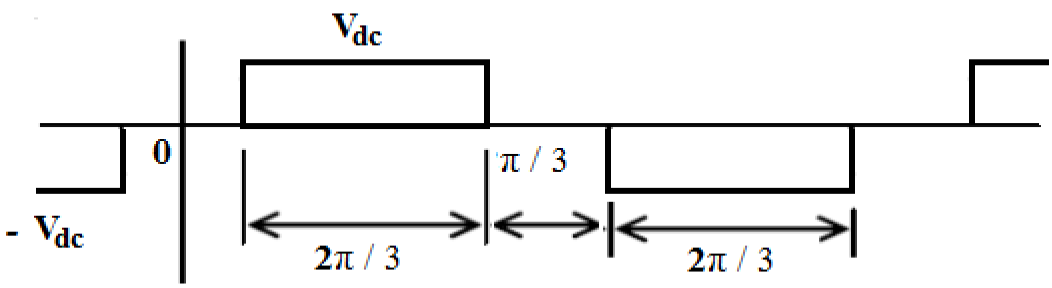


Fig.8.40.

[Ans: The Fourier series is

$$V_{dc} = 0\,, \qquad V_n \; \frac{4V_d}{n\pi\sqrt{2}} \sin\left(\frac{n\pi}{3}\right) \qquad n = 1,3,5,\dots\dots$$

{ Peak harmonics voltage and currents are : V_1 = 77.97 V, V_3 = 0 , V_3 = - 15.95 V, I_{dc} = 0A , I_1 = 15.58 A , I_3 = 0A , I_5 = 3.19 A}

8.6 A three-phase transistor voltage-source inverter supplies a three-phase load, as shown in Fig.8.41. The load consists of star connected resistance of 10 Ω in each phase. The inverter supply voltage is 200V d.c. and each inverter switch conducts for 120˚.
(a) Sketch the switching signals for the six transistors.
(b) Sketch the line-to-neutral voltage for one complete cycle of the output voltage.
(c) Sketch the line-to-line voltage for one complete cycle of the output voltage.

(d) Calculate the *rms* values of the first five harmonics in the line-to-line output voltage, including the fundamental.
(e) Calculate the *rms* values of the first five harmonics in the line-to-neutral output voltage, including the fundamental.

[Ans: (d) $V_{L\text{-}L}$ = 244.95 V, (e) V_{ph} = 141.42 V]

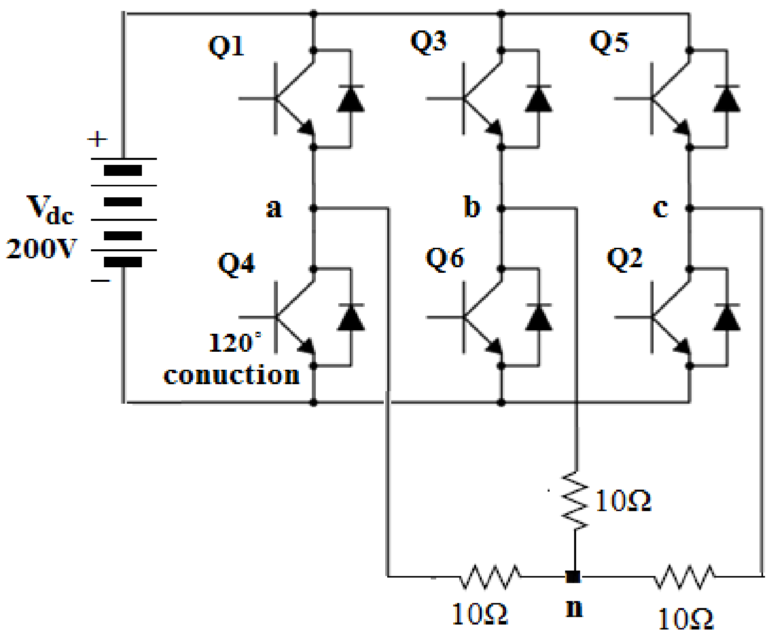


Fig.8.41.

8.7 A single-phase full-bridge voltage source inverter, fed from 200 V d.c. source. The output voltage is controlled by sinusoidal pulse width modulation technique. The carrier and modulating signals are so adjusted that the modulation index produces three pulses per half a cycle of widths, 20°, 60°, and 20°. Assuming the load is purely resistive, find the *rms* value of the output voltage.

[Ans: 149 V]

8.8 For the inverter of problem 8.7, the strategy of output voltage control is changed such that the technique used is multiple pulse width modulation. The width of each pulse is 20° and each half cycle has 6 pulses,

(a) Determine the *rms* value of the output voltage,
(b) Find the pulse width to maintain the output *rms* value in (a) constant if the input voltage increases by 20%,
(c) What is the maximum input voltage if the maximum possible pulse width is $\delta_{max} = 25°$?

[Ans: (a) 163.3 V, (b) δ =13.8°, (c) 179.45 V]

8.9 Repeat problem 8.7 above if single-pulse width modulation technique with $\delta = 120\,^{\circ}$ is used to control the output voltage.

[Ans: 163.3V]

8.10 Calculate the *rms* value of the output voltage waveform shown in Fig.8.19. The d.c. input voltage is 200V and the fundamental frequency component of the output voltage waveform is 100Hz.

[Ans: 94V]

8.11 (a) Explain with the aid of neat sketches of waveforms the voltage control using phase-displacement thchnique.

(b) The d.c. supply voltage of a single pulse width modulated inverter is 120V. Calculate the displacement angle to produce rms output voltages of 50V and 100V.

[Ans : 31.25^{o} , 125^{o}]

8.12 A three-phase natural commutated inverter is used to transfer power from a 300 V battery to a three-phase 230 V, 50 Hz a.c. bus. A large filter inductor with resistance 10Ω is included in the d.c. side. Estimate the power transferred for (a) $\alpha = 120^{o}$, (b) $\alpha = 150^{o}$.

[Hint : Use equations (8.59) – (8.61)]

[Ans: (a) 2248 W, (b) 834 W]