

# DC Circuits and Kirchoff's Laws

So far we have learned about Ohm's law and the rules for finding equivalent resistances for resistors in series and parallel. In this lab we will begin by finding the currents flowing through a simple DC (Direct Current- meaning input voltage is constant) circuit that can be evaluated in terms of an equivalent resistance. Then we will build a more complicated circuit, called a Wheatstone bridge. We will learn a more general method of analyzing circuits called Kirchoff's laws. Finally, we will write a program in Mathematica that will help us efficiently solve these two circuits for the unknown currents so that the theoretical values may be compared with our measurements.

Materials – Breadboard with DC voltage source, two variable resistor boxes, a digital multimeter, 330  $\Omega$  resistor, 220  $\Omega$  resistor, 10  $\Omega$  resistor, cables, alligator clips, computer with Mathematica

## Part 1: A simple DC circuit

1. You will be building the circuit in Figure 1. To prepare use the multimeter to measure and record the resistance of each resistor.
2. Construct the circuit shown using values of  $V = 5.0 \text{ V}$ ,  $R_A = 100 \Omega$  (use the variable resistor box),  $R_B = 330 \Omega$ , and  $R_C = 220 \Omega$ .
3. Measure and record the voltage drop in four locations: across the voltage source and across each of the three resistors. To do this, put the multimeter on the DC voltage setting in *parallel* with each element. *Note: The meter reads the voltage at the red terminal minus the voltage at the black terminal, so make sure you record which way you are measuring across the circuit element.*
4. Now measure the current flowing through each of the elements by putting the multimeter on the DC current setting in *series* with the element. *Note: When measuring current, always begin with the red lead in the 10A socket on the multimeter and only move it to the 40mA socket for a more precise reading if the current reads less than 40mA. Sending more current through the meter's circuits than it can handle*

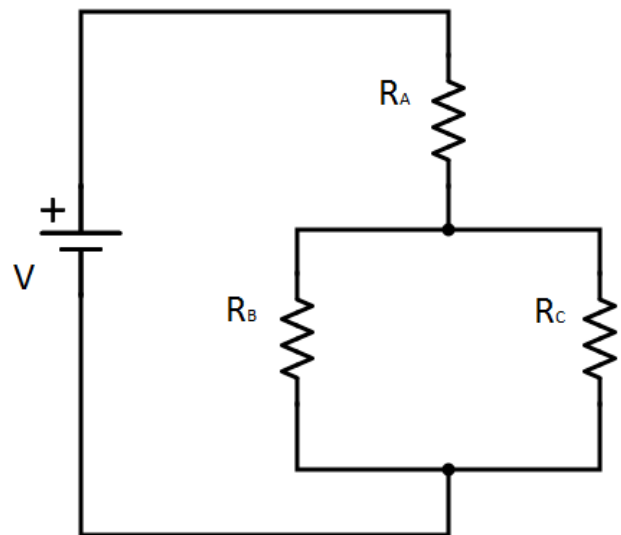


Figure 1 - The first circuit used in this lab. It can be solved using the rules of equivalent resistance for resistors in series and in parallel.

could blow a fuse. You must again record the direction that the multimeter is measuring in (red minus black) since the current can be going one of two directions.

- Now use the rules for resistors in series and in parallel to solve for what the voltages and currents should be if the resistors and the voltage source have their ideal values. How do these compare to your measured values? Quote a %-error as part of your results.

### Part 2: Kirchhoff's laws and the Wheatstone bridge

Now take a look at the circuit shown in Figure 2. This is a Wheatstone bridge, a circuit that is often used to find the resistance of an unknown resistor.

- Discuss and record: **Are the resistors in series, parallel, both, or neither series nor parallel?**
- To solve for the currents in this circuit we must appeal to a new concept called Kirchhoff's laws. Kirchhoff came up with two laws, known as the loop rule and the junction rule. **Justify with words why Kirchhoff's loop rule is true.** Hint: think about dragging a charge all the way around the circuit from the positive terminal back to the positive terminal. Does it change potential energy?

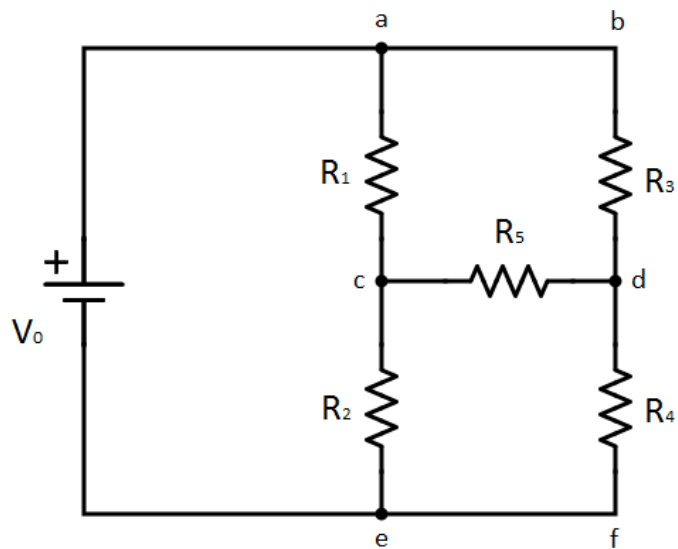


Figure 2 - The Wheatstone bridge, a circuit that must be evaluated using Kirchhoff's laws.

*Kirchhoff's Loop Rule: The sum of all the changes in potential around any loop in a circuit must be zero. Mathematically, this can be expressed as*

$$\sum_{loop} V = 0.$$

- We can assume the change in potential across a resistor to be given by Ohm's law, so the potential either falls or increases by  $V = IR$  depending on whether we are going with or against the current across that resistor. Using this we can add up all the voltage changes around a loop. As an example, taking the circuit from page 1, Figure 1 containing  $V$ ,  $R_A$ , and  $R_B$  and assuming the current is flowing clockwise around the loop, Kirchhoff's loop

rule gives us  $V - I_A R_A - I_B R_B = 0$ , where  $I_A$  and  $I_B$  are the current through resistors A and B respectively.

The direction we evaluate the loop in doesn't matter. If we apply the loop rule in the counterclockwise direction, we will have the same equation multiplied by -1, which is identical to the equation we just arrived at.

**How many “loops” can you identify in Figure 2? A loop is any path where you end in the same location you start.**

4. The loop rule is not enough, however. We need to have additional equations that directly link the different currents together in order to solve for them. This brings us to the other of Kirchhoff's laws: the junction rule.

*Kirchhoff's Junction Rule: The sum of all currents entering and leaving a junction in the circuit must be zero. Mathematically this can be expressed as*

$$\sum_{\text{junction}} I = 0.$$

Let's use the circuit in Figure 1 as an example again. There is a junction between resistors  $R_A$ ,  $R_B$ , and  $R_C$ . If we assume that current  $I_A$  is entering the junction and currents  $I_B$  and  $I_C$  are leaving it we have that Kirchhoff's junction rule gives us  $I_A - I_B - I_C = 0$ .

**Justify with words why Kirchhoff's junction rule is true.**

### **Part 3: Caveats to Kirchhoff's Rules; How to apply them correctly**

Using the loop and junction rules we can write down and solve a system of equations for currents, voltages, and resistances in a DC circuit. We mention three caveats about Kirchhoff's laws.

#### **Caveat 1: Mind your signs.**

If you are going from the negative to the positive side of a battery use +V (a positive charge would gain potential energy). If you are going from the positive to the negative side use -V. If you are going around the loop in the same direction as the local current write  $V = -IR$  (a positive charge is losing energy) for the voltage across the resistor. Conversely, if your loop goes in the opposite direction of the current write  $V = IR$ .

**Caveat 2: Be consistent with current direction signs.** Before writing any equations you must choose a direction for the current in each part of the circuit at the start. If you guess wrong, that's OK! Your solution will simply have a negative current along the segments you guessed incorrectly, indicating that the current is actually flowing the other way.

What is important is that you consistently apply Kirchhoff's laws consistently using the current directions you select at the outset. Otherwise your signs may be mixed up in separate equations and your solution incorrect.

**Caveat 3: Not all of the equations for loops and junctions are independent of each other.**

In Figure 1 you could write the loop rule for each of the three possible loops and the junction rule for the two possible junctions.

1. Try it for yourself: write down the equations from the junction rule for the two junctions in Figure 1.

We see that the same three currents are involved in both junctions, so the junction rule provides only one independent equation.

Similarly, the equations for any two of the three loops can be substituted into each other to yield the equation for the third loop, so only two of the loops yield independent equations. You must make sure that you've found enough independent equations to solve the system.

2. Apply Kirchhoff's rules to solve for the current in the circuit in Figure 1. You know the values of the resistors and the voltage source. Solve without using "equivalent resistance". You should get the same result as before.

#### **Part 4: Build and Analyze the Wheatstone Bridge**

1. Now construct the Wheatstone bridge as shown in Figure 2. Use values of  $V_0 = 5 \text{ V}$ ,  $R_1 = 300 \text{ } \Omega$  (resistor box),  $R_2 = 220 \text{ } \Omega$ ,  $R_3 = 100 \text{ } \Omega$  (resistor box),  $R_4 = 330 \text{ } \Omega$ , and  $R_5 = 10 \text{ } \Omega$ .
2. Measure and record the voltage across each of the five resistors and the voltage source using the multimeter as you did with the previous circuit.
3. Then measure and record the current through each of the six elements using the multimeter. Again, you must pay attention to the orientation of the multimeter in the circuit when measuring.

- Use your values to show that Kirchhoff's loop rule applies to the voltage changes around the three loops  $abdca$ ,  $cdfec$ , and  $abdfeca$ . Use your measurements to show that Kirchhoff's junction rule applies to the currents at junctions  $c$  and  $d$ . Explain any discrepancies in your measurements with Kirchhoff's laws.

### Part 5: Solving DC circuits using Mathematica

For both of the circuits above, we'd like to solve for the unknown currents assuming the resistors and voltage source are known to be ideal (so we can compare our measurements to theoretically expected values). Solving the circuit to find the currents is not difficult in principle, but it can be tedious and mistake-prone when there are many loops and junctions. Every application of either the loop rule or junction rule provides a new equation which contains several variables. In the case of the Wheatstone bridge, there are six unknown currents, so we are required to solve a system of six equations in six variables. Fortunately, we can write a short program in Mathematica that will do all of the algebra for us.

Open Mathematica and start a new file (called a "notebook" by the program). Let us solve the first circuit that we constructed above. Enter the code given below into your notebook. The indentation inside the `Simplify` and `Solve` functions should occur automatically. You can enter subscript mode by holding `Ctrl+_` and exit it by holding `Ctrl+Space`. When you are done, hit `Shift+Enter` to execute it.

```
(* Solving DC Circuits using Mathematica *)

(* Simple example *)
Simplify[
  Solve[
    {V - iA RA - iB RB == 0 &&
      iC RC - iB RB == 0 &&
      iA == iB + iC},
    {iA, iB, iC}
  ]
]
```

You should now see output beneath your code that looks like the following:

$$\left\{ \left\{ i_A \rightarrow \frac{V(R_B+R_C)}{R_A(R_B+R_C)+R_B R_C}, i_B \rightarrow \frac{V R_C}{R_A(R_B+R_C)+R_B R_C}, i_C \rightarrow \frac{V R_B}{R_A(R_B+R_C)+R_B R_C} \right\} \right\}.$$

Let's analyze what we did here. The first two lines are comments with `(* *)` acting as the comment operator. Mathematica ignores these lines when running the code. The next two

commands are the functions `Simplify` and `Solve`, and the code nested inside each set of square brackets contains the arguments that each function operates on. `Simplify` takes the expression enclosed and performs algebraic manipulations on it to give it a more elegant form (e.g. it cancels double minus signs, creates common denominators, factors, reduces fractions, etc.). Sometimes this is just aesthetic, but for particularly complicated expressions that may not be easily simplified by eye, this capability is very useful.

The really important function here is `Solve`. `Solve` takes two sets of arguments: a set of equations and a set of variables. The three equations enclosed in the curly braces and separated by `&&` are two applications of the loop rule and one use of the junction rule. Note the double equals sign. This means ‘equals’ in a mathematical sense, whereas a single equals sign is used to assign a value to a variable. Also, the choice of `i` rather than `I` is intentional here. Mathematica automatically interprets `I` as the unit imaginary number ‘*i*’ instead of as a variable, which is not what we want. After the list of equations, another set of curly braces tells Mathematica which set of variables to solve for. Here we have chosen the three currents, though we could choose any three of the seven variables to solve for and Mathematica would find a solution in terms of the remaining four. The output text shows the three currents in terms of the other variables, as expected. Go ahead and save your program.

To find a numerical solution, we can define four variables `V`, `RA`, `RB`, and `RC` to have certain values. Add four lines above `Simplify` and below the comments that look like this:

```
V=5  
RA=100  
RB=330  
RC=220
```

and hit Shift+Enter to evaluate. Your output should look like this:

$$\left\{ \left\{ i_A \rightarrow \frac{5}{232}, i_B \rightarrow \frac{1}{116}, i_C \rightarrow \frac{3}{232} \right\} \right\},$$

where these values are in amps because the voltage and resistance were given respectively in volts and ohms. If we had chosen different initial directions for the three currents when applying the loop and junction rules, some of these values would have been negative. This would mean that the current is actually flowing in the opposite direction that we first guessed.

Mathematica remembers the values you assigned each of the four variables `V`, `RA`, `RB`, and `RC` even if you delete the lines of code that assign them. To return to the symbolic solution, you need to clear the variable assignments. To clear the value of the variable `V`, type and execute `V=.` (and do the same for the other variables). You can also assign variables to be identified with other variables.

For example, to solve the circuit in the special case when  $R_B$ , and  $R_C$  have the same resistance, you can type and execute  $R_B=R_C$ . This will produce a symbolic solution only in terms of  $V$ ,  $R_A$ , and  $R_C$ , eliminating  $R_B$ .

Mathematica notebooks execute code in units called cells. The thin blue bracket on the far right-hand side of the notebook shows you what lines are contained in what cells (along with a further breakdown into the inputs and outputs of a cell). Right now we only have one cell, but typing beneath the output of this cell will start a whole second cell. Move the cursor below the output Mathematica generated to start creating this new cell. When the cursor is in the second cell, Shift+Enter will only execute the contents of that cell and ignore the first one.

In the second cell, add code to solve the Wheatstone bridge circuit for each of the six currents. You now need to input six independent equations rather than three into your `Solve` function. Use the format of the code from the first cell to guide you. Find the symbolic solution first. It may surprise you how complicated the expressions appear for what still looks like a fairly simple circuit. Imagine doing it all by hand! Then add the numerical assignments to your variables to get numbers which you can compare to your measurements of the current. If your calculation seems very different from your measurements, that may indicate you've either programmed your equations incorrectly or that you've measured the current in your circuit incorrectly.

Report a %-error between the theoretical values you calculated and the measured values you found. Instead of the ideal values, you can also try using exactly the resistance and voltage source values you measured with the multimeter in your program to see if that yields a solution closer to your measured currents, but you only need to report the %-error when compared to the currents expected from the ideal values. Include your program as part of what you turn in for this week (or, if your TA allows, just include the full solution of the circuit worked out by hand). You may want to use this program to check your solutions to homework problems on DC circuits, but it should not be a substitute for knowing how to do the algebra yourself. On an exam you won't be able to use Mathematica to help you out!