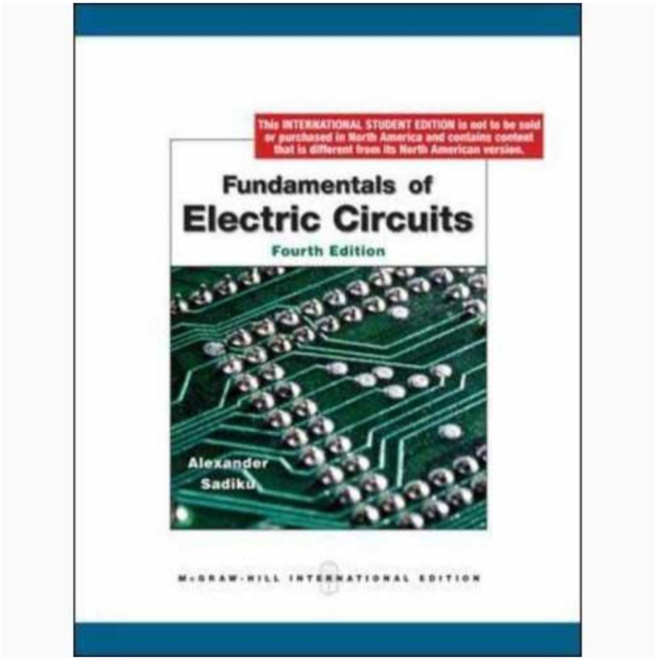


DC Circuits:

Circuit Theorems

Hasan Demirel

Circuit Theorems

- Introduction
- Linearity Property
- Superposition
- Source Transformations
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer

Introduction

A large complex circuits

Simplify circuit analysis

Circuit Theorems

- Thevenin's theorem
- Circuit linearity
- Source transformation
- Norton theorem
- Superposition
- Max. power transfer

Linearity Property

- **Homogeneity property (Scaling)**

$$i \rightarrow v = iR$$

$$ki \rightarrow kv = kiR$$

- **Additivity property**

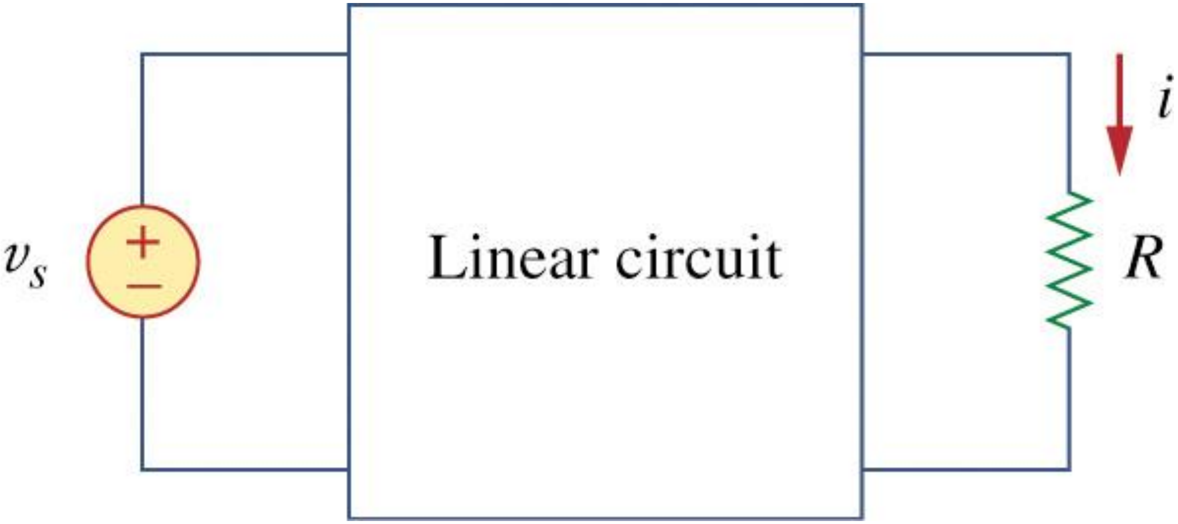
$$i_1 \rightarrow \quad v_1 = i_1 R$$

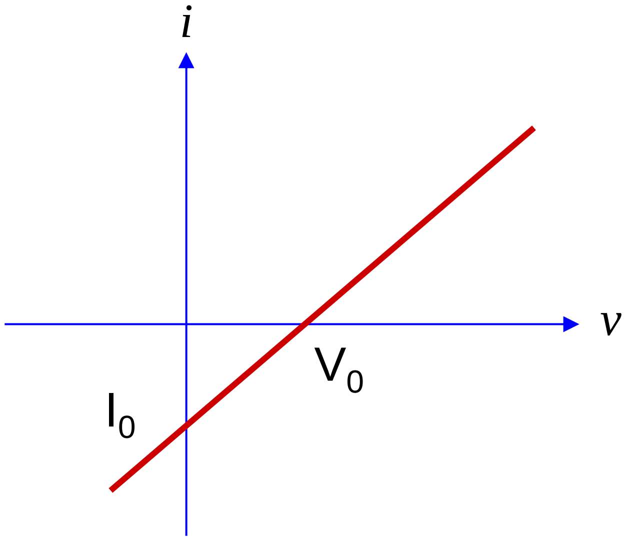
$$i_2 \rightarrow v_2 = i_2 R$$

$$i_1 + i_2 \rightarrow (i_1 + i_2)R = i_1 R + i_2 R = v_1 + v_2$$

Linearity Property

- A **linear circuit** is one whose output is linearly related (directly proportional) to its input.





Linearity Property

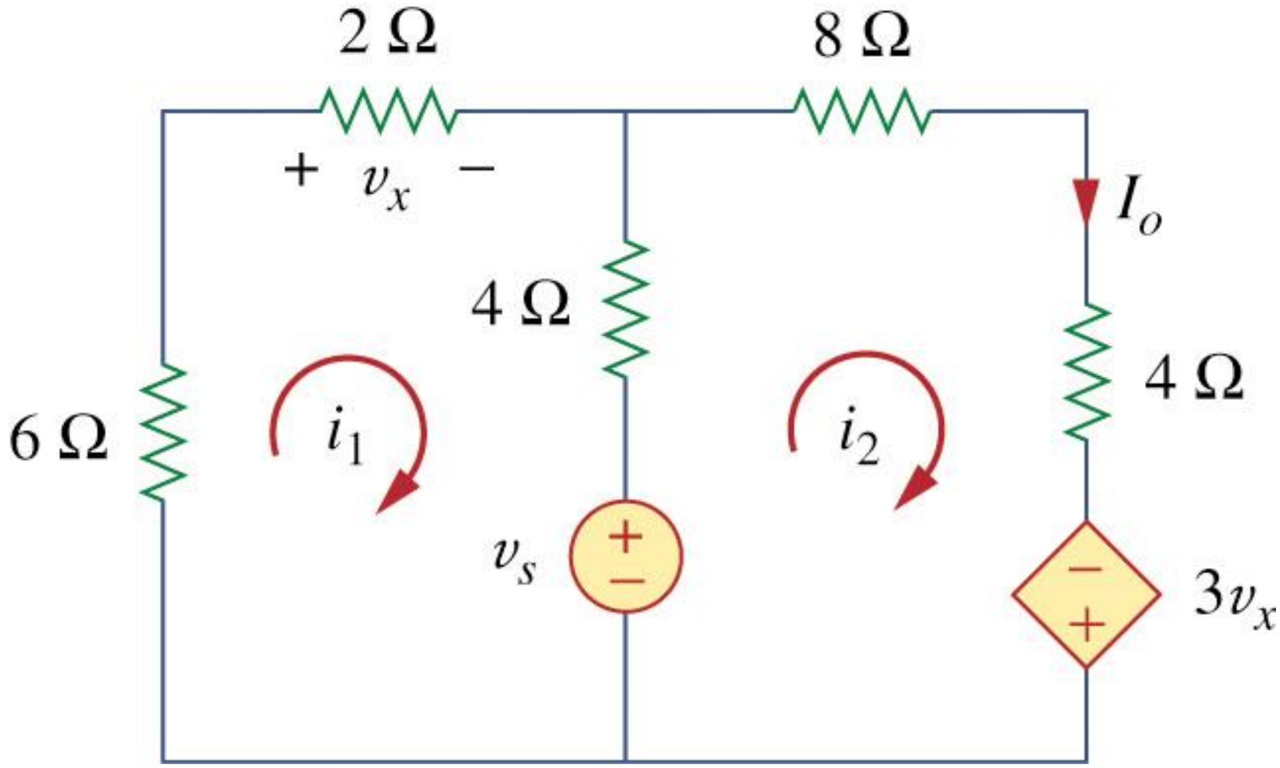
- A **linear circuit** consist of :
  - linear elements (i.e. R=5 Ω)
  - linear dependent sources (i.e. $v_s=6I_o$ V)
  - independent sources (i.e. v_s=12 V)

$$v_s = 10\text{V} \rightarrow i = 2\text{A}$$
$$v_s = 1\text{V} \rightarrow i = 0.2\text{A}$$
$$v_s = 5\text{mV} \leftarrow i = 1\text{mA}$$

$$p = i^2 R = \frac{v^2}{R} \; : nonlinear$$

Linearity Property

Example 4.1: For the circuit in below find I_o when v_s=12V and v_s =24V.



Linearity Property

Example 4.1: For the circuit in below find I_o when v_s=12V and v_s =24V.

KVL

$$12i_1 - 4i_2 + v_s = 0 \qquad (1)$$

$$-4i_1 + 16i_2 - 3v_x - v_s = 0 \qquad (2)$$

$$v_x = 2i_1$$

Eq(2) becomes

$$-10i_1 + 16i_2 - v_s = 0 \qquad (3)$$

Using **Eqs(1)** and **(3)** we get

$$2i_1 + 12i_2 = 0 \rightarrow i_1 = -6i_2$$

Linearity Property

Example 4.1: For the circuit in below find I_o when v_s=12V and v_s =24V.

From Eq(1) we get: $-76i_2 + v_s = 0 \quad \Rightarrow \quad i_2 = \frac{v_s}{76}$

when $v_s = 12\text{V}$

$$I_0 = i_2 = \frac{12}{76}\text{A}$$

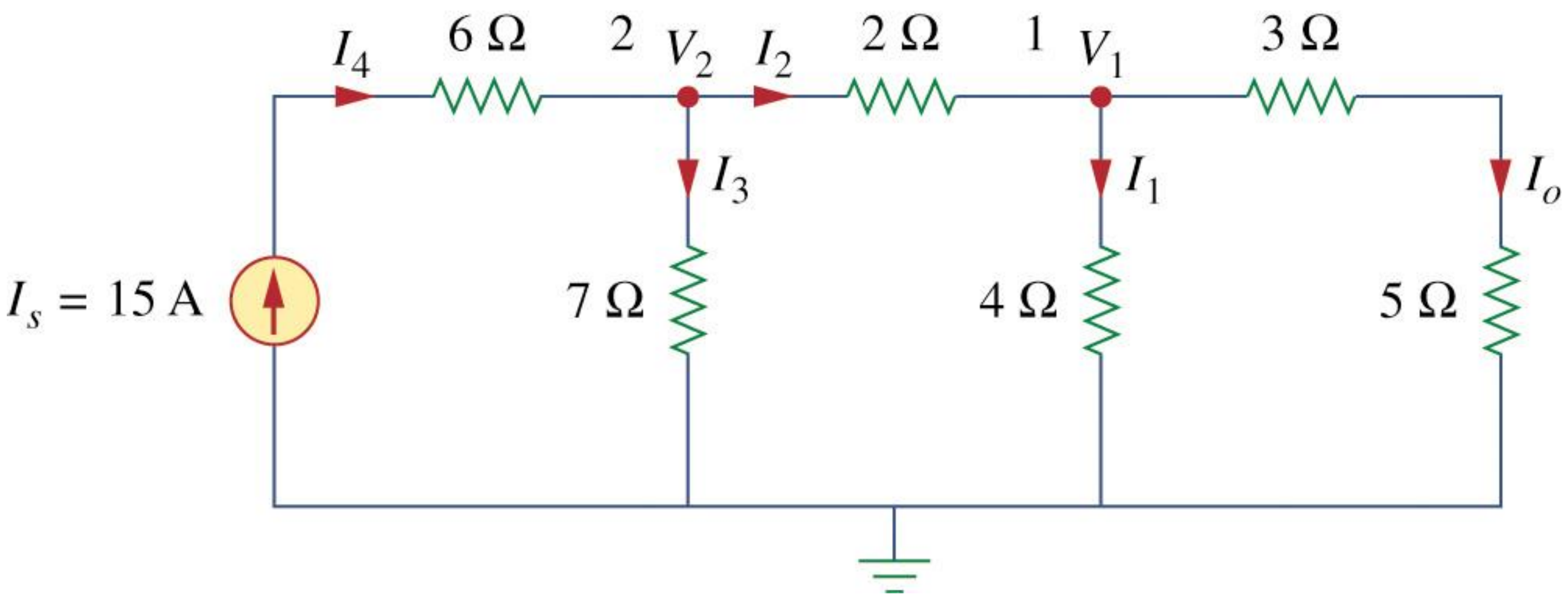
when $v_s = 24\text{V}$

$$I_0 = i_2 = \frac{24}{76}\text{A}$$

Showing that when the **source value is doubled, I_o doubles**.

Linearity Property

Example 4.2: Assume $I_o = 1\text{A}$ and use linearity to find the actual value of I_o in the following circuit.



Linearity Property

Example 4.2: Assume $I_o = 1\text{A}$ and use linearity to find the actual value of I_o in the following circuit.

$$\text{If } I_0 = 1\text{A, then } v_1 = (3+5)I_0 = 8\text{V}$$

$$I_1 = v_1/4 = 2\text{A}, \quad I_2 = I_1 + I_0 = 3\text{A}$$

$$V_2 = V_1 + 2I_2 = 8 + 6 = 14\text{V}, I_3 = \frac{V_2}{7} = 2\text{A}$$

$$I_4 = I_3 + I_2 = 5\text{A} \Rightarrow \quad I_S = 5\text{A}$$

$$I_0 = 1A \rightarrow I_S = 5\text{A}$$

$$\boxed{I_0 = 3\text{A} \leftarrow I_S = 15\text{A}}$$

Superposition

- The **superposition principle** states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each **independent** source acting alone.
- **Turn off, kill, inactive source(s):**
  - independent voltage source: **0 V (short circuit)**
  - independent current source: **0 A (open circuit)**
- **Dependent sources are left intact.**

Superposition

- **Steps to apply superposition principle:**

1. **Turn off** all **independent sources** except **one source**. Find the output (voltage or current) due to that active source using nodal or mesh analysis.

2. **Repeat** step 1 for each of the other independent sources.

3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

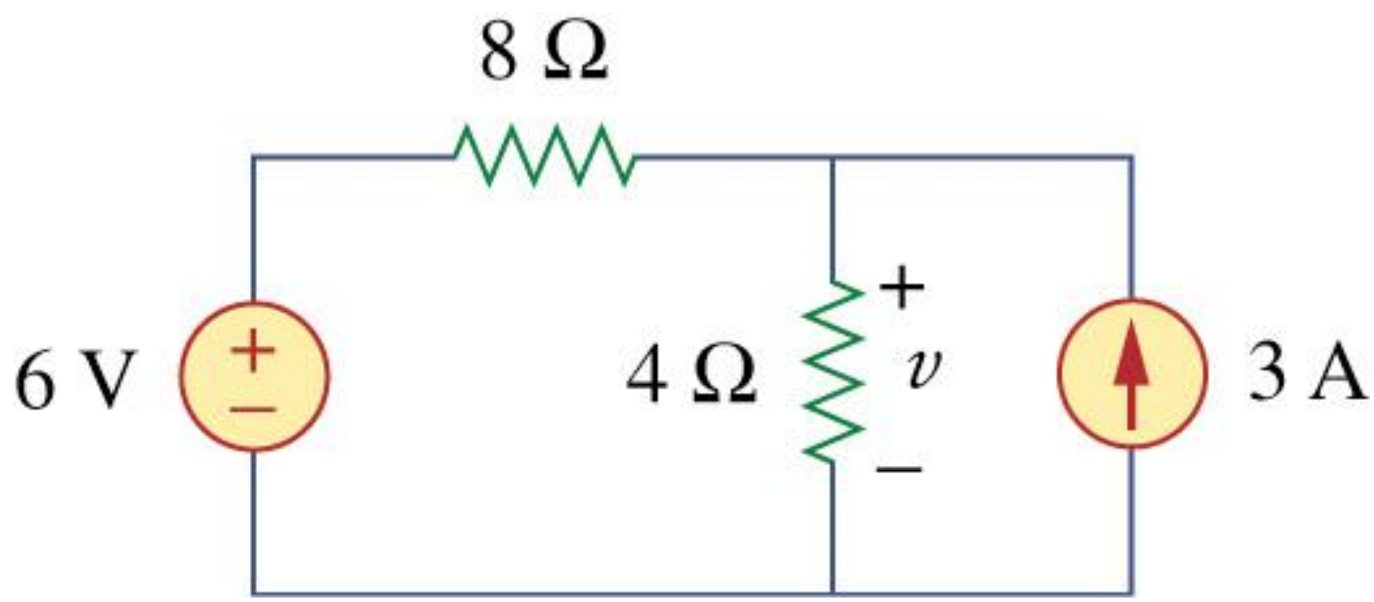
Superposition

- **How to turn off independent sources:**
  - **Turn off voltages sources = short voltage sources; make it equal to zero voltage**
  - **Turn off current sources = open current sources; make it equal to zero current**

- **Superposition** involves **more work** but **simpler circuits.**
- **Superposition** is **not applicable** to the effect on **power.**

Superposition

- **Example 4.3:** Use the superposition theorem to find v in the circuit below.



Superposition

- **Example 4.3:** Use the superposition theorem to find v in the circuit below.

Since there are two sources,

Let $v = v_1 + v_2$

Voltage division to get

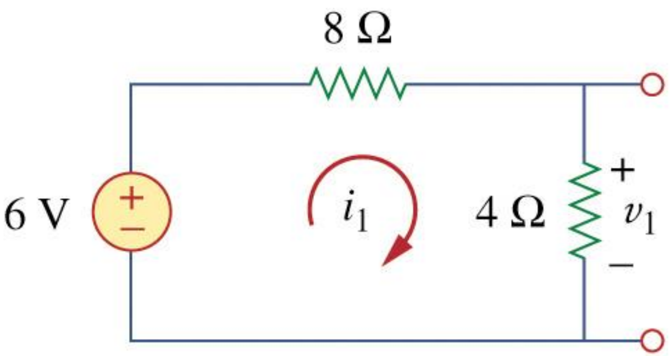
$$v_1 = \frac{4}{4+8}(6) = 2\text{V}$$

Current division, to get

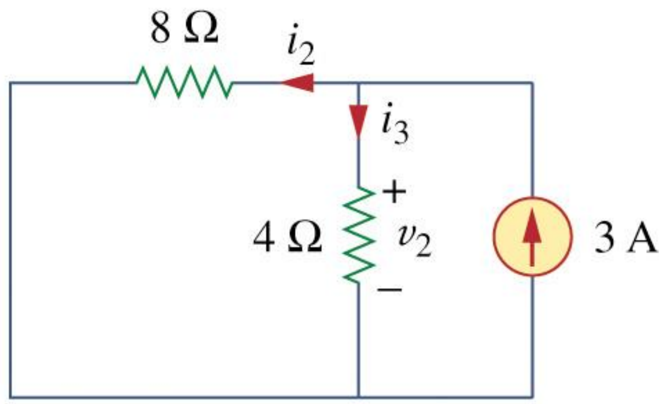
$$i_3 = \frac{8}{4+8}(3) = 2\text{A}$$

Hence $v_2 = 4i_3 = 8\text{V}$

And we find $v = v_1 + v_2 = 2 + 8 = 10\text{V}$



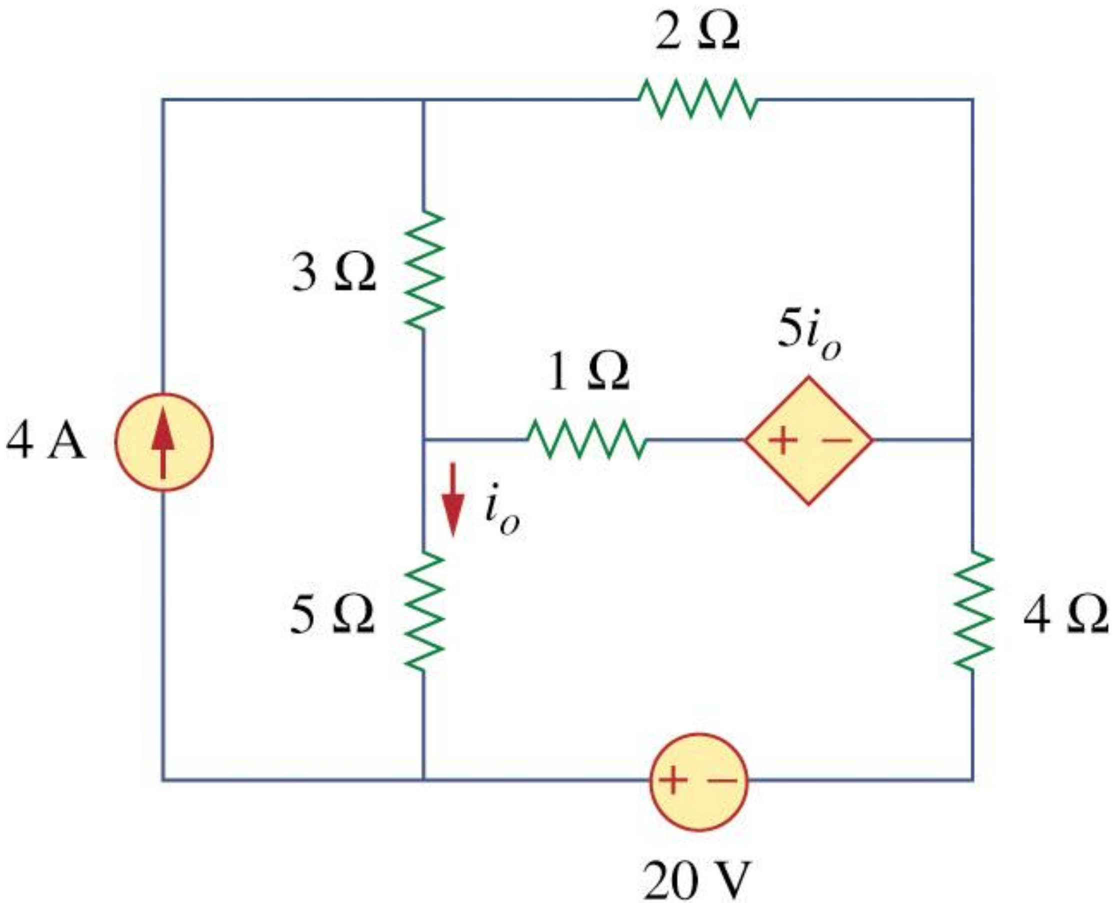
(a)



(b)

Superposition

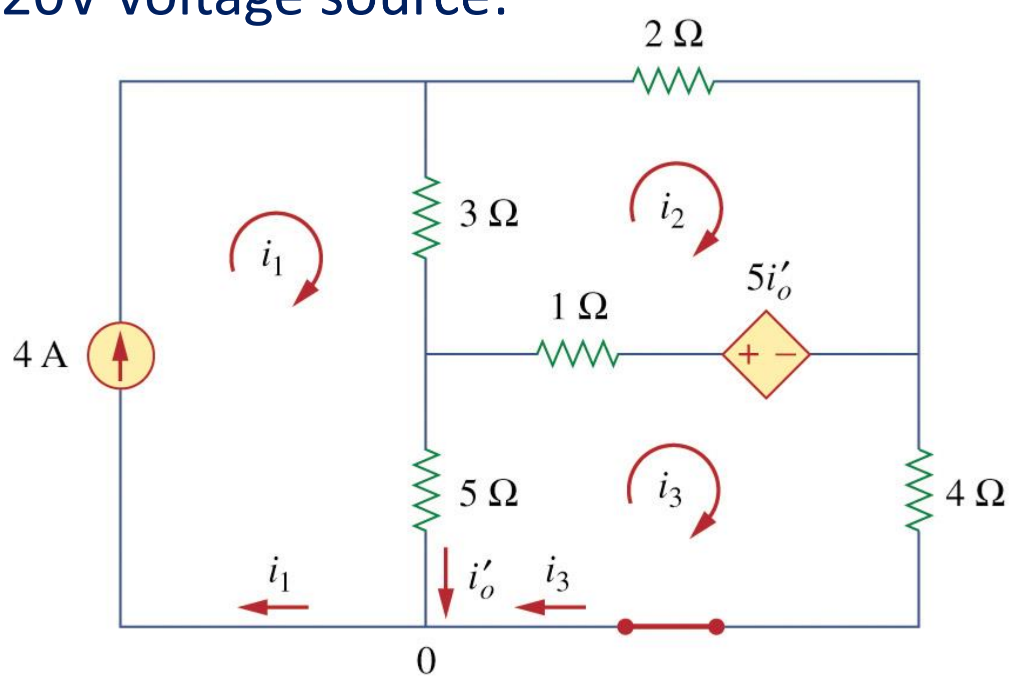
- **Example 4.4:** Find I_o in the circuit below using superposition.



Superposition

- **Example 4.4:** Find I_o in the circuit below using superposition.

Turn off 20V voltage source:

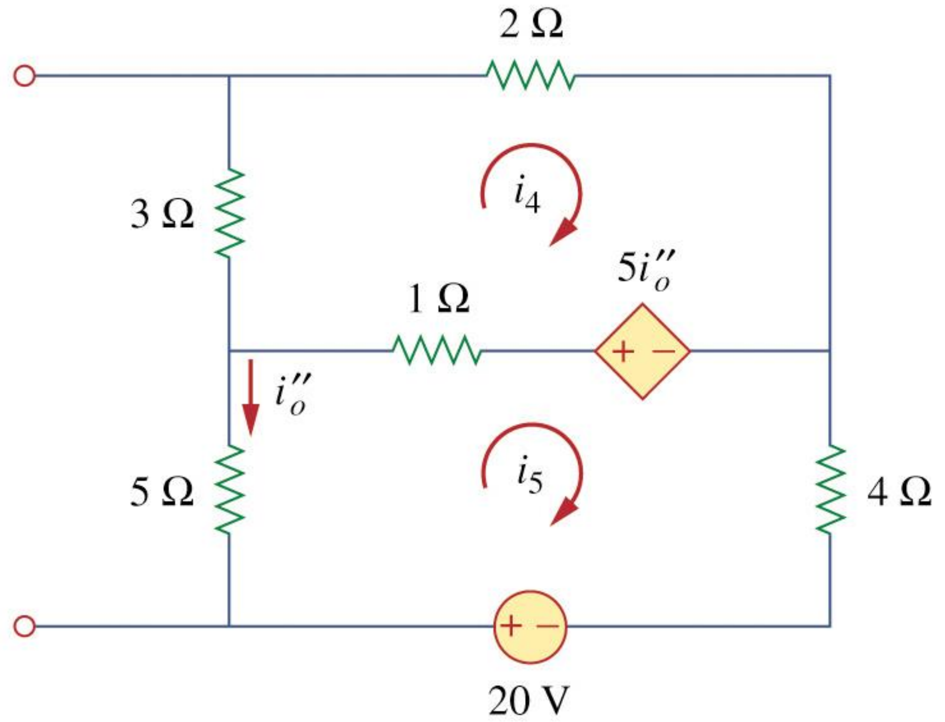


(a)

Superposition

- **Example 4.4:** Find I_o in the circuit below using superposition.

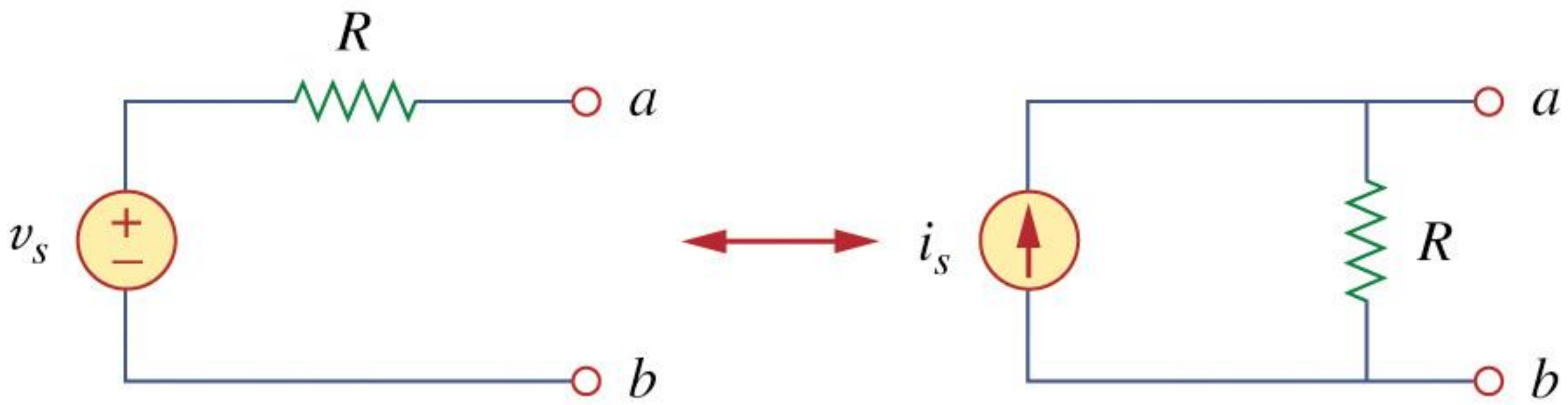
Turn off 4A current source:



(b)

Source Transformation

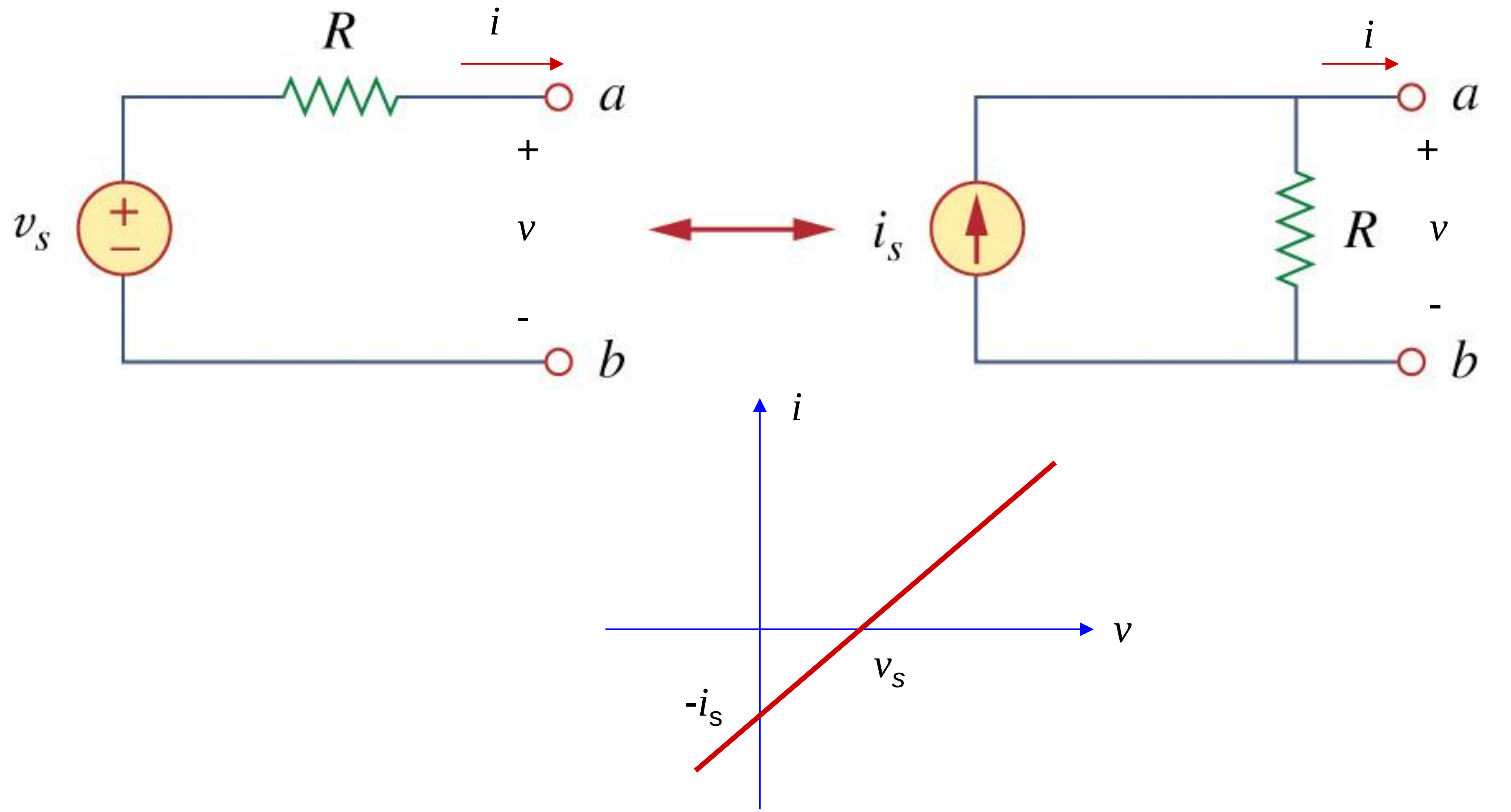
- A **source transformation** is the process of replacing a **voltage source** v_s **in series with a resistor** R by a **current source** i_s **in parallel with a resistor** R, or vice versa



$$v_s = i_s R \quad \text{or} \quad i_s = \frac{v_s}{R}$$

Source Transformation

- **Equivalent Circuits:**



Source Transformation

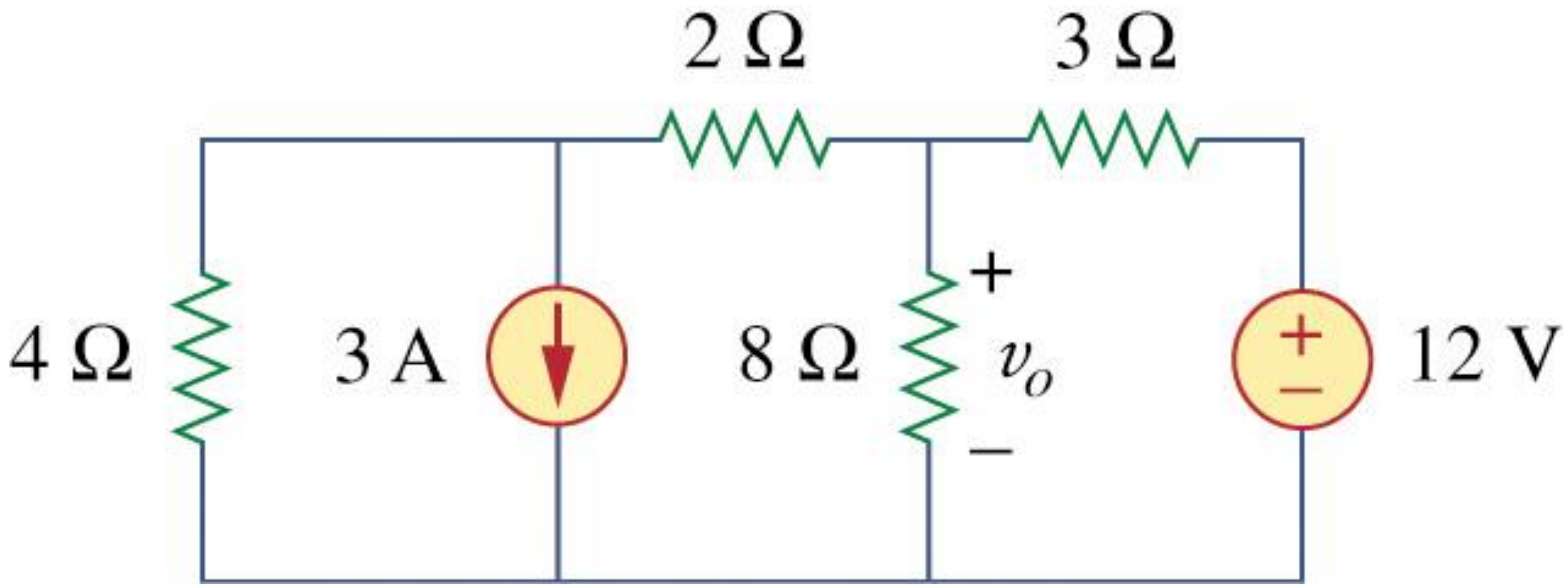
- **Source transformation:** Important points

1. Arrow of the current source is directed toward the positive terminal of the voltage source.

2. Transformation is not possible when:
   - **ideal voltage source (R = 0)**
   - **ideal current source (R=∞)**

Source Transformation

- **Example 4.6:** Use source transformation to find v_o in the circuit below.



Source Transformation

- **Example 4.6:** Use source transformation to find v_o in the circuit below.

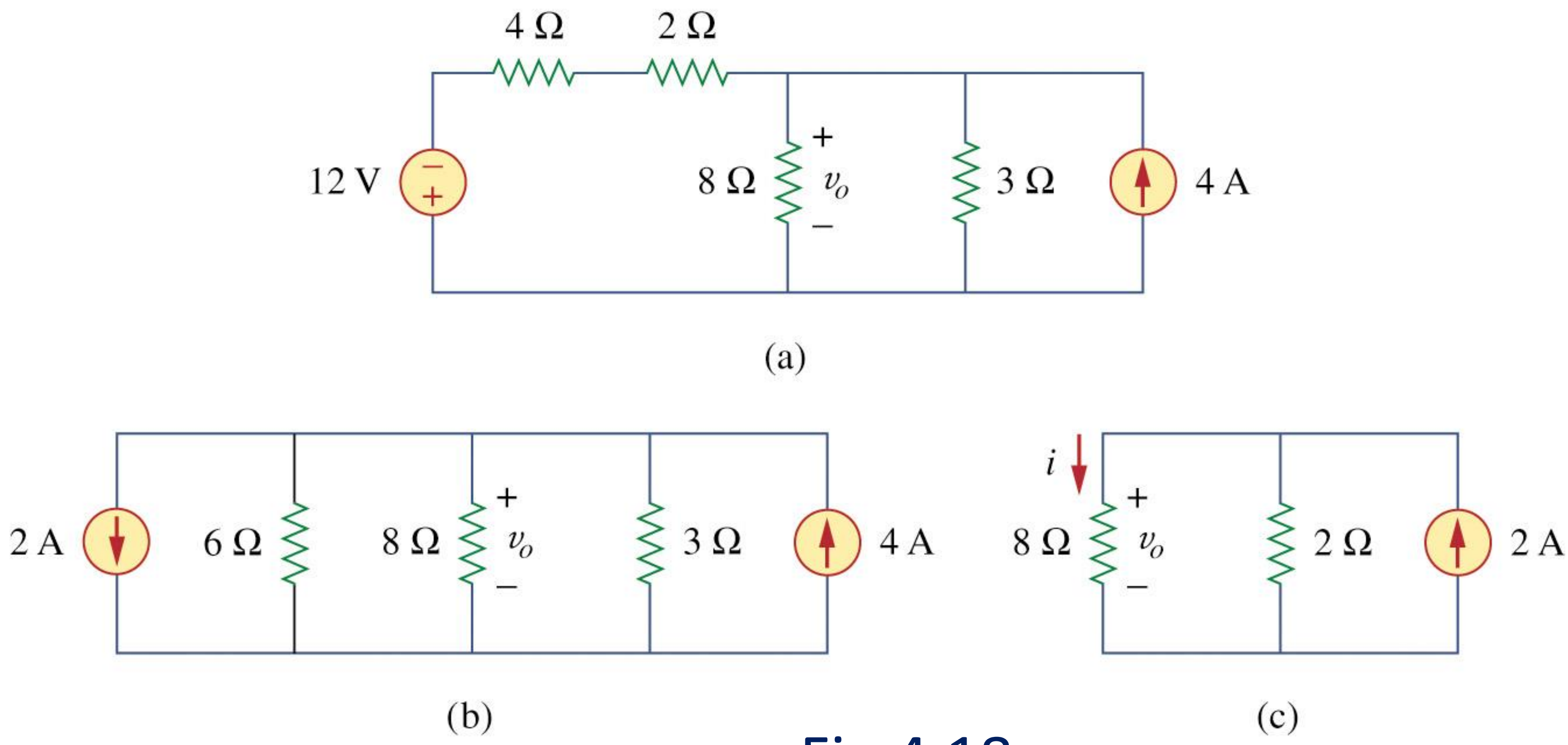


Fig.4.18

Source Transformation

- **Example 4.6:** Use source transformation to find v_o in the circuit below.

we use current division in Fig.4.18(c) to get

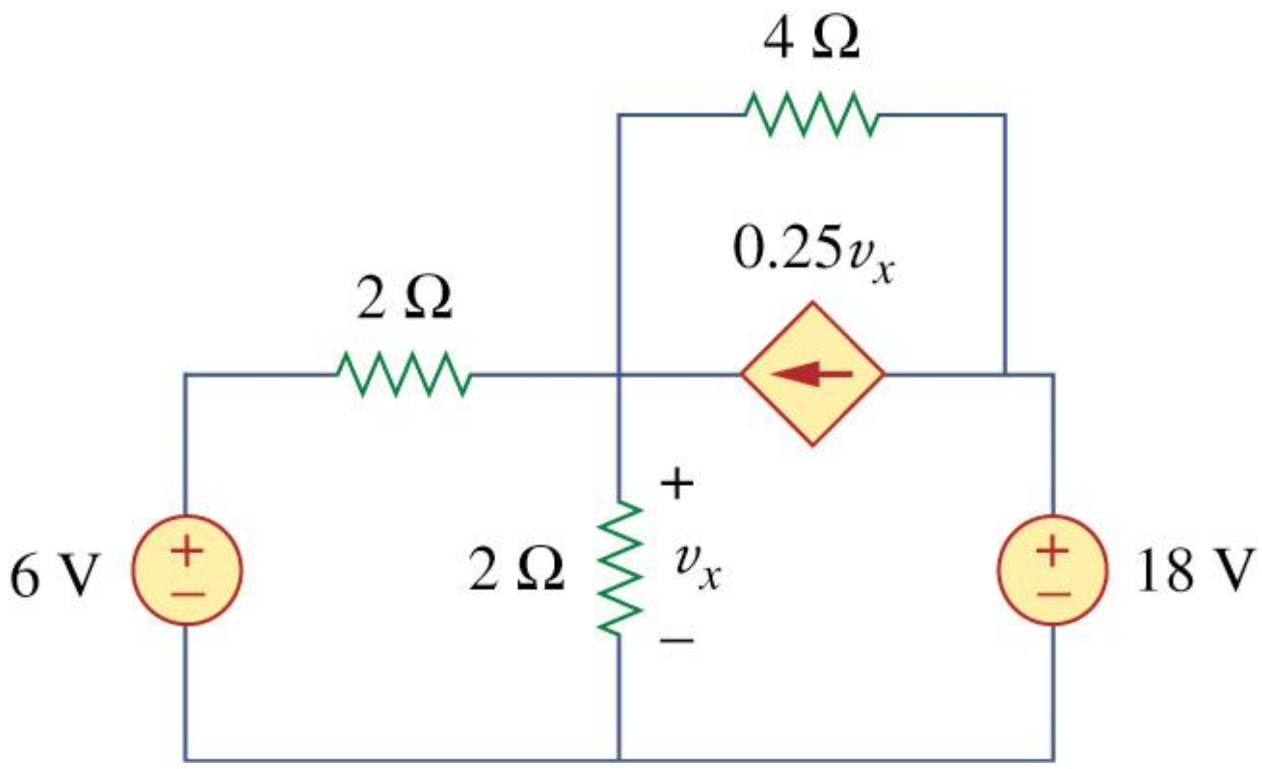
$$i = \frac{2}{2+8}(2) = 0.4\text{A}$$

and

$$v_o = 8i = 8(0.4) = 3.2\text{V}$$

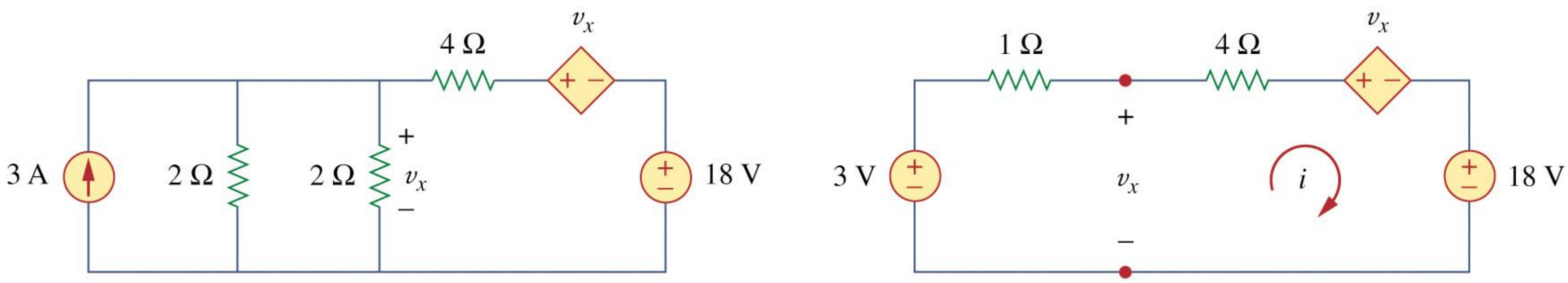
Source Transformation

- **Example 4.7:** Find v_x in the circuit below using source transformation.



Source Transformation

- **Example 4.7:** Find v_x in the circuit below using source transformation.



(a) (b)

KVL around the loop in Fig (b)

$$-3+5i+v_x+18=0 \qquad (1)$$

Appling KVL to the loop containing only the 3V voltage source, the 1Ω resistor, and v_x yields:

$$-3+1i+v_x=0 \Rightarrow v_x=3-i \qquad (2)$$

Source Transformation

- **Example 4.7:** Find v_x in the circuit below using source transformation.

Substituting Eq.(2) into Eq.(1), we obtain

$$15+5i+3=0 \Rightarrow i=-4.5\text{A}$$

Alternatively

$$-v_x+4i+v_x+18=0 \Rightarrow i=-4.5\text{A}$$

thus

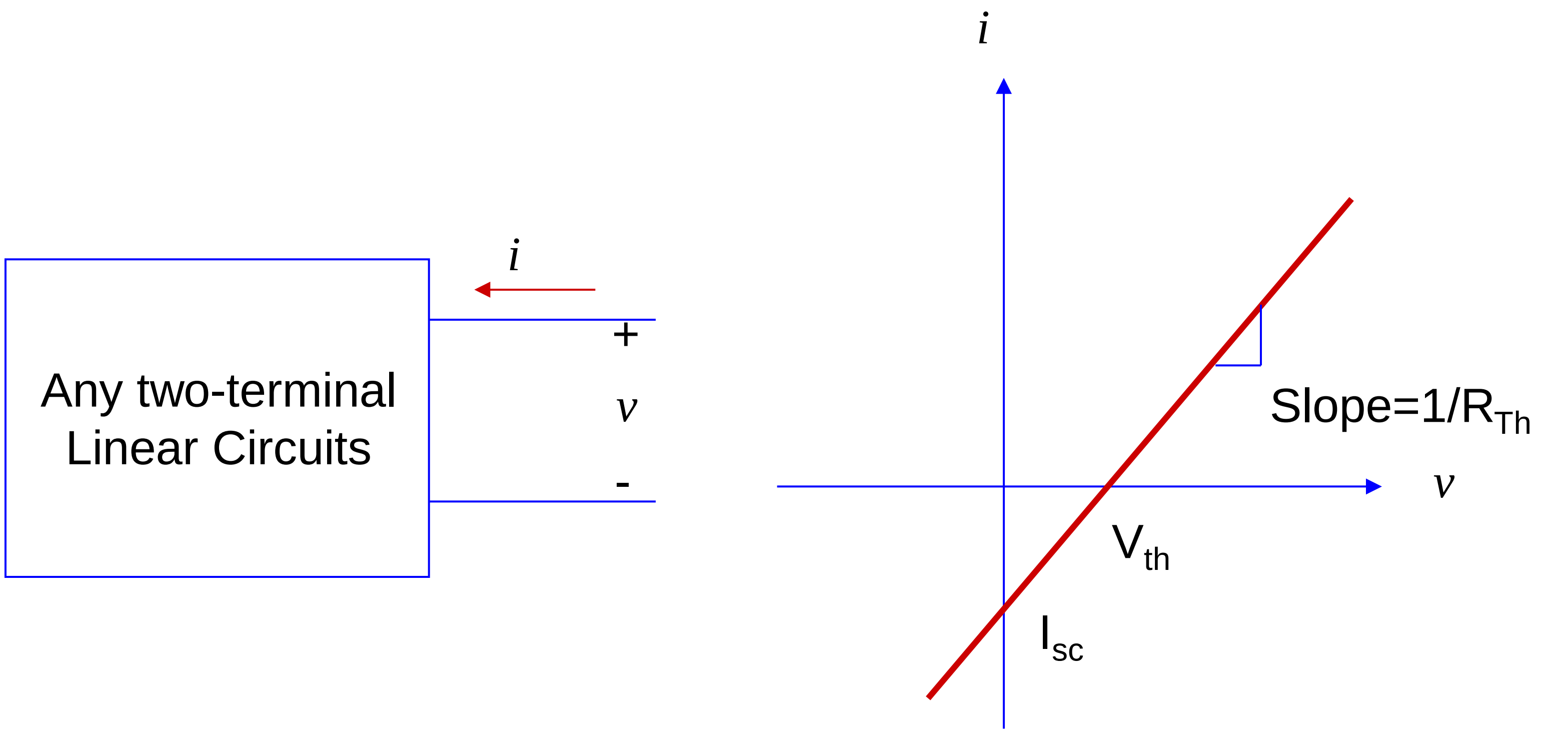
$$v_x=3-i=7.5\text{V}$$

Thevenin's Theorem

- **Thevenin's theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a **voltage source V_{Th} in series with a resistor R_{Th}** where V_{Th} is the open circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the **independent sources are turned off.**

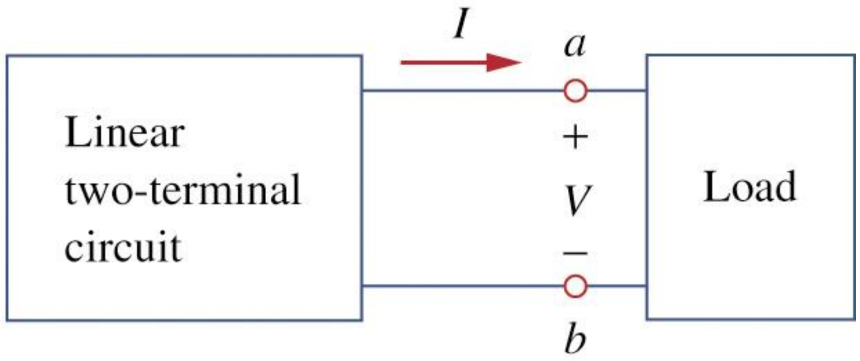
Thevenin’s Theorem

- **Property of Linear Circuits**

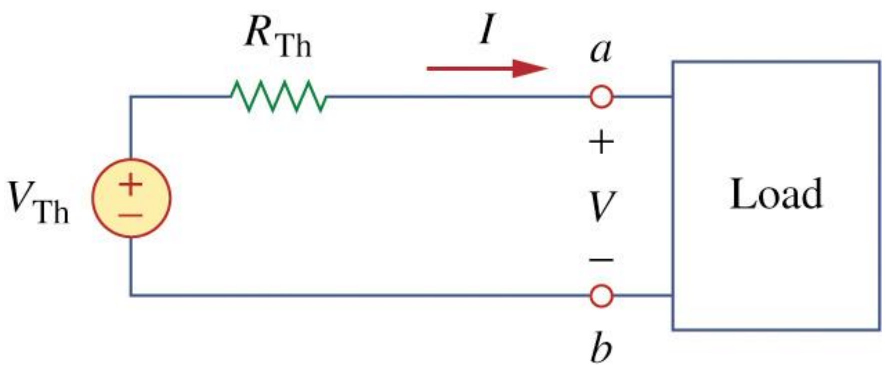


Thevenin's Theorem

- Replacing a **linear two-terminal circuit (a)** by its **Thevenin equivalent circuit (b).**



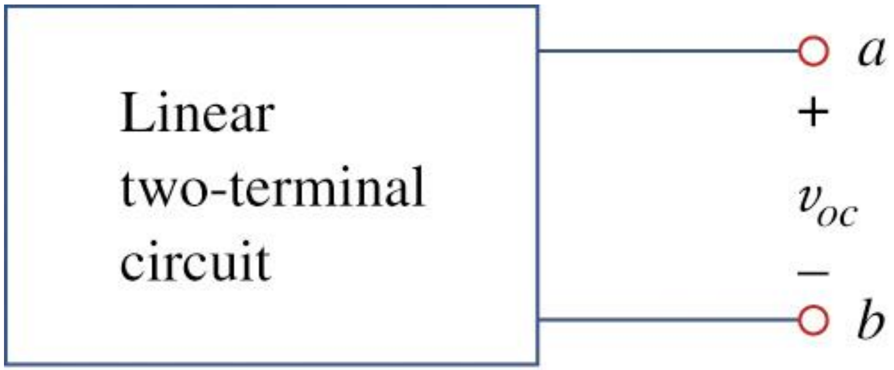
(a)



(b)

Thevenin's Theorem

- **How to find Thevenin Voltage?**
  - Equivalent circuit: same voltage-current relation at the terminals.
  - $V_{\text{Th}} = v_{oc}$:   open circuit voltage at $a - b$



$V_{\text{Th}} = v_{oc}$

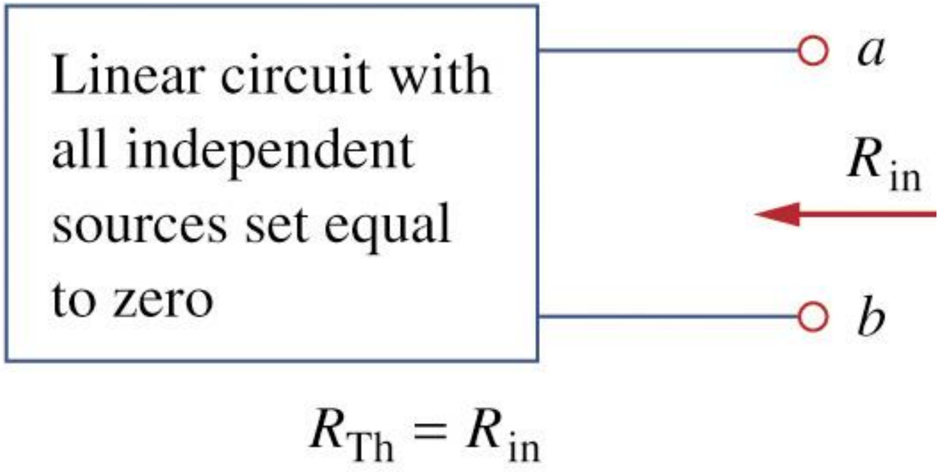
(a)

Thevenin's Theorem

- **How to find Thevenin Resistance?**

$$R_{Th} = R_{in} \; : \; \text{input} - \text{resistance of the dead circuit at } a - b.$$

- $a - b$ open circuited
- Turn off all independent sources



(b)

Thevenin's Theorem

- **How to find Thevenin Resistance?**
  - There are **two cases** in finding Thevenin Resistance R_{Th}.

CASE 1

- If the network has **no dependent sources:**
  - **Turn off all independent sources.**
  - **R_{Th}: can be obtained via simplification of either parallel or series connection seen from a-b**

Thevenin's Theorem

- **How to find Thevenin Resistance?**
  - There are **two cases** in finding Thevenin Resistance R_{Th}.

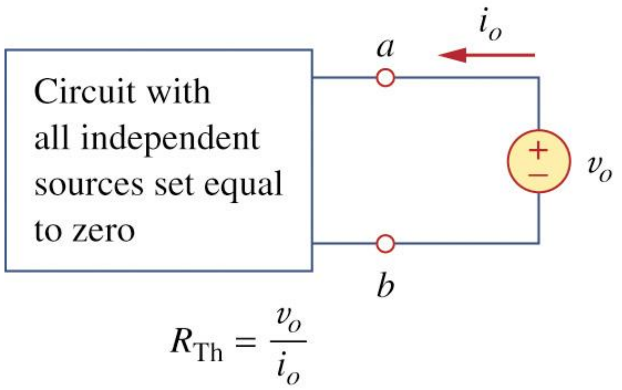
CASE 2

- If the network has **dependent sources:**
  - **Turn off all independent sources.**
  - **Apply a voltage source v_o at a-b**

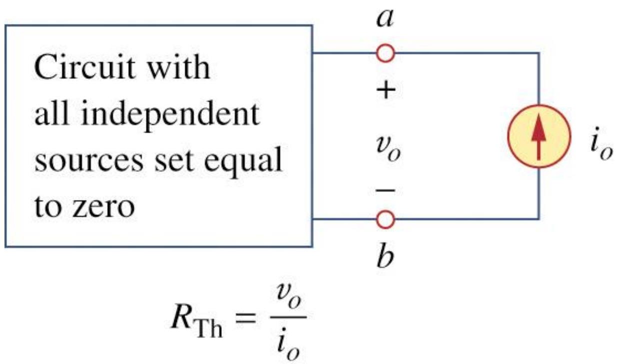
$$R_{\mathrm{Th}} = \frac{v_o}{i_o}$$

  - **Alternatively, apply a current source i_o at a-b**

$$R_{\mathrm{Th}} = \frac{v_o}{i_o}$$



(a)



(b)

Thevenin's Theorem

- **How to find Thevenin Resistance?**
  - **The Thevenin resistance, R_{Th} , may be negative, indicating that the circuit has ability providing (supplying) power.**

Thevenin's Theorem

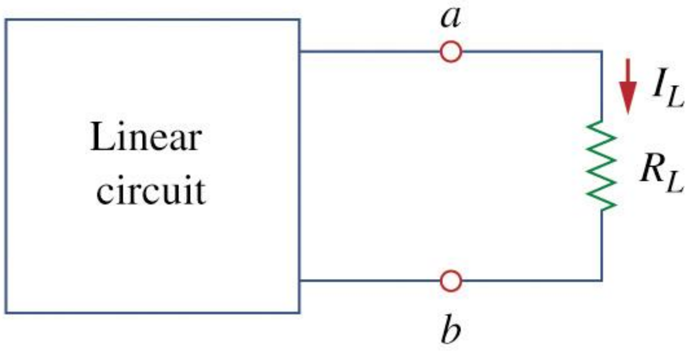
- After the **Thevenin Equivalent** is obtained, **the simplified circuit** can be used to calculate I_L and V_L easily.

Simplified circuit

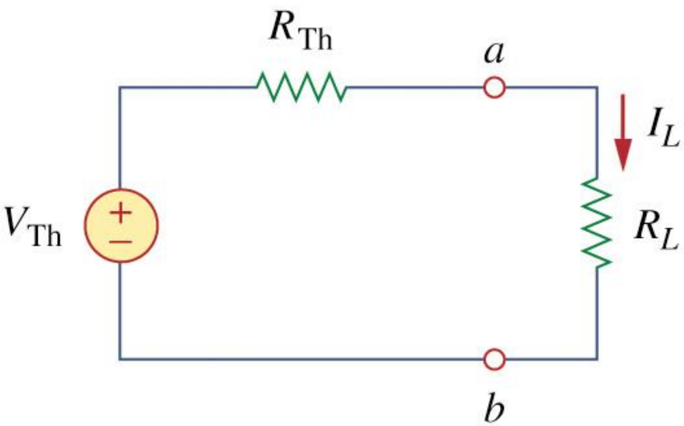
$$I_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{\text{Th}} + R_L} V_{\text{Th}}$$

Voltage divider



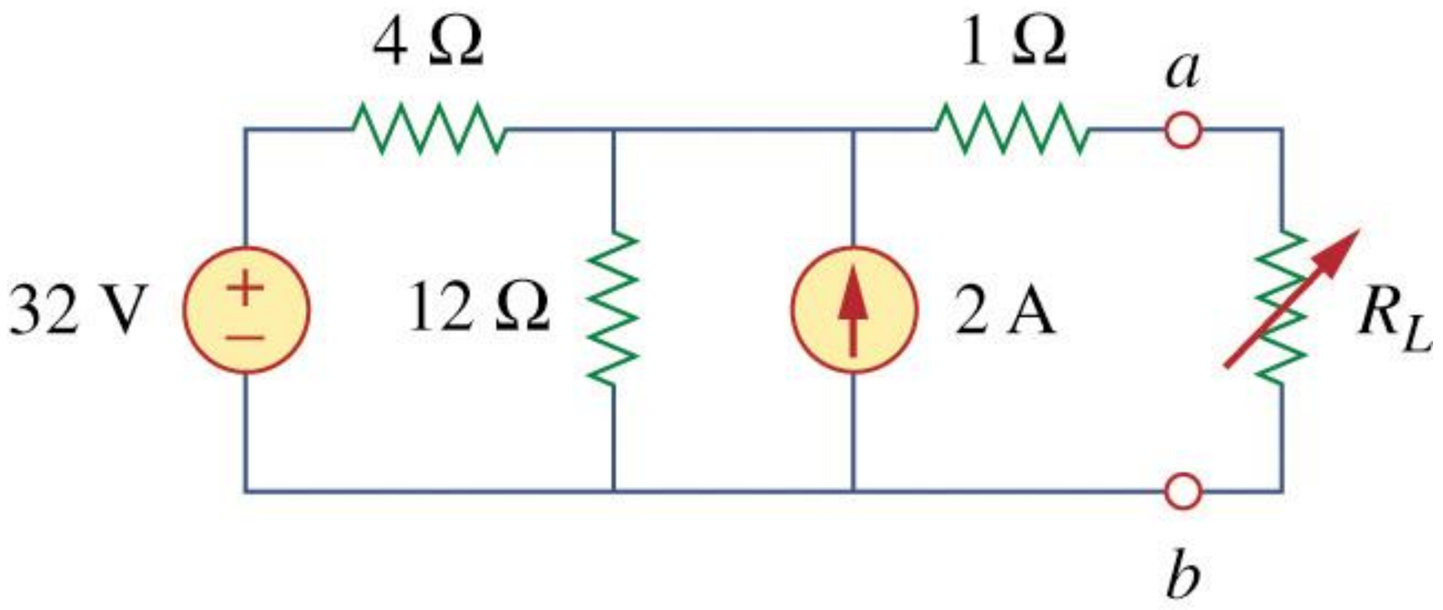
(a)



(b)

Thevenin's Theorem

- **Example 4.8:** Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals *a-b*. Then find the current through $R_L = 6, 16$, and $36\ \Omega$.



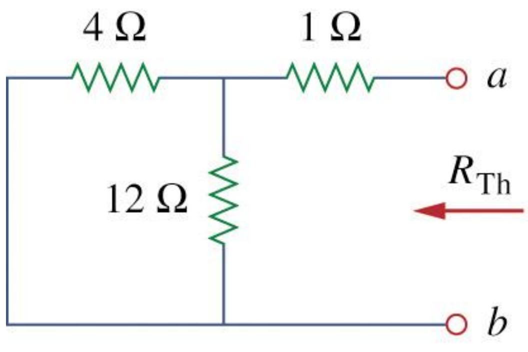
Thevenin's Theorem

- **Example 4.8:** Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals *a*-*b*. Then find the current through $R_L = 6, 16$, and $36\ \Omega$.

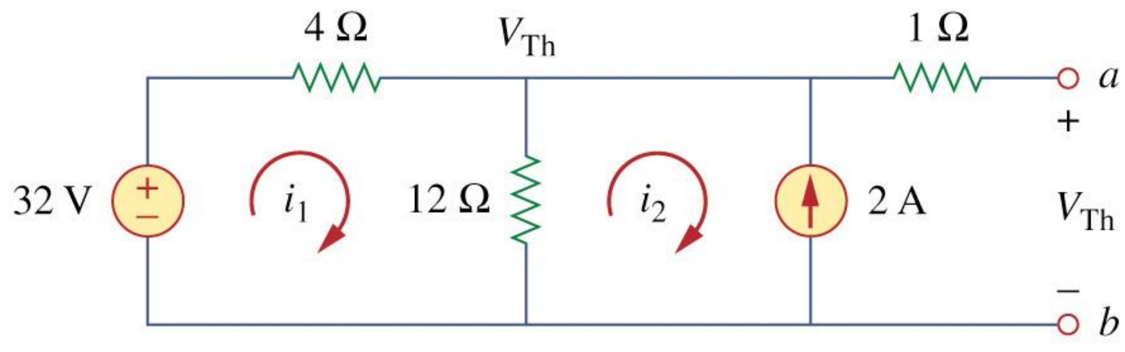
Find R_{Th}:

$$R_{Th}: \text{32V voltage source} \rightarrow \text{short}$$
$$\text{2A current source} \rightarrow \text{open}$$

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\Omega$$



(a)



(b)

Thevenin's Theorem

- **Example 4.8:** Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals *a*-*b*. Then find the current through $R_L = 6, 16$, and $36\ \Omega$.

Find V_{Th}:

V_{Th} :

(1) Mesh analysis

$$-32 + 4i_1 + 12(i_1 - i_2) = 0\ ,\ \ i_2 = -2\text{A}$$

$$\therefore i_1 = 0.5\text{A}$$

$$V_{\text{Th}} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\text{V}$$

(2) Alternatively, Nodal Analysis

$$(32 - V_{\text{Th}})/4 + 2 = V_{\text{Th}}/12$$

$$\boxed{\therefore V_{\text{Th}} = 30\text{V}}$$

Thevenin's Theorem

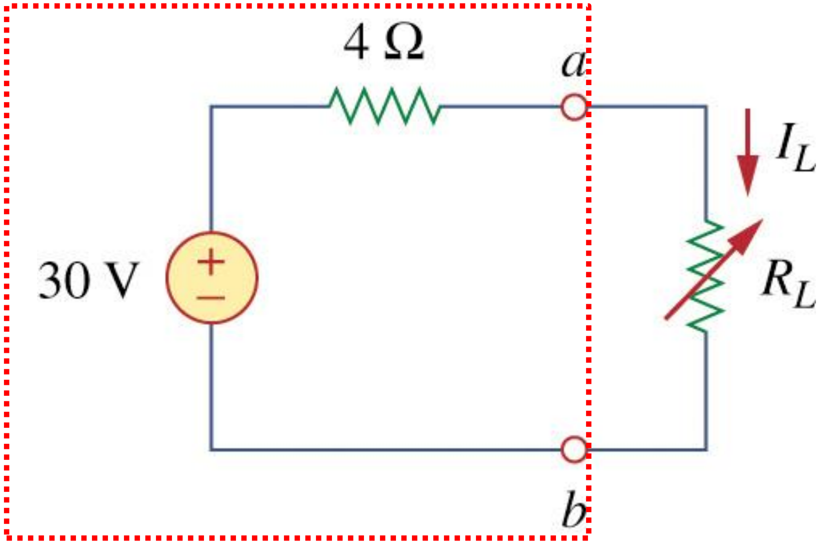
- **Example 4.8:** Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals *a*-*b*. Then find the current through $R_L = 6, 16$, and $36\ \Omega$.

Find V_{Th}: (3) Alternatively, source transform

$$\frac{32 - V_{TH}}{4} + 2 = \frac{V_{TH}}{12}$$

$$96 - 3V_{TH} + 24 = V_{TH} \Rightarrow V_{TH} = 30\text{V}$$

Thevenin Equivalent **circuit:**

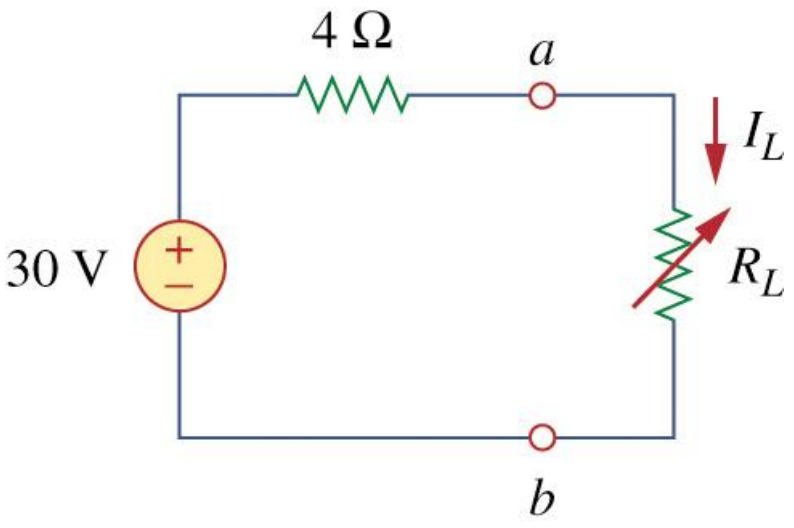


Thevenin's Theorem

- **Example 4.8:** Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals *a*-*b*. Then find the current through R_L = 6,16, and 36 Ω.

Calculate I_L:

$$i_L = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{30}{4 + R_L}$$



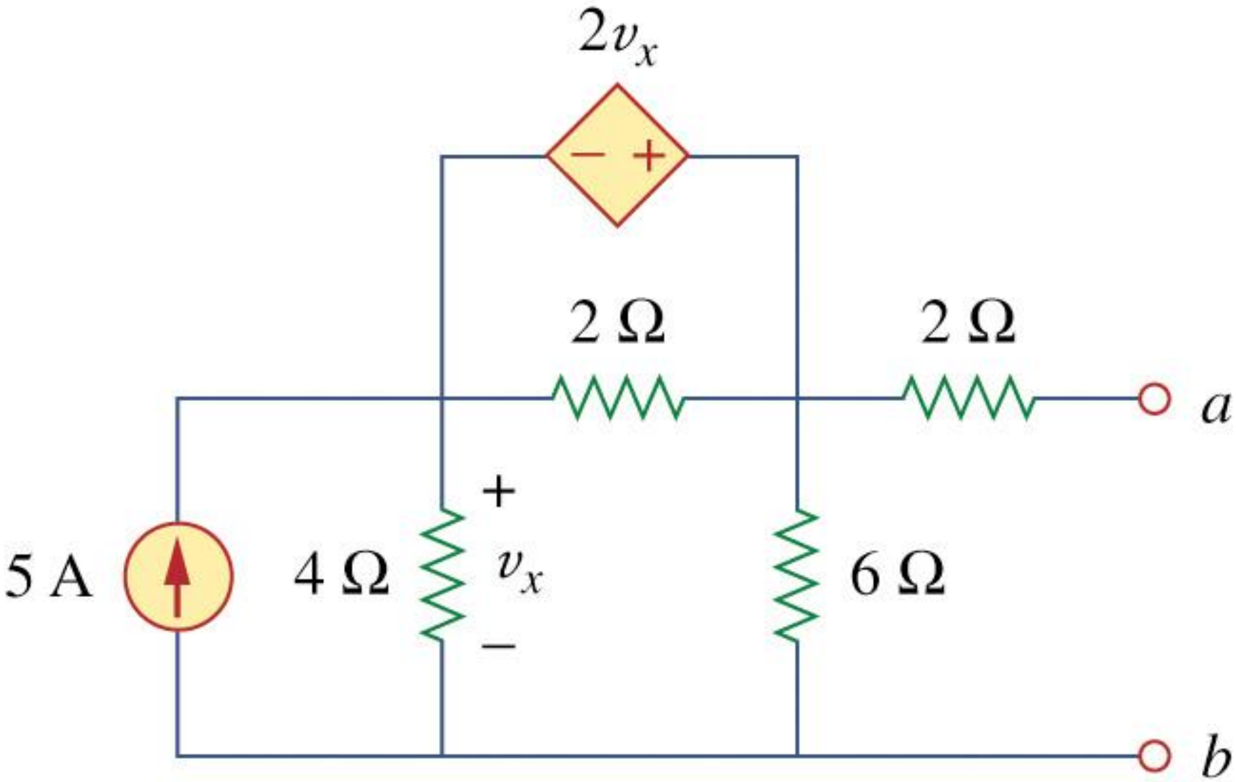
$\boxed{R_L = 6} \rightarrow \boxed{I_L = 30/10 = 3\text{A}}$

$\boxed{R_L = 16} \rightarrow \boxed{I_L = 30/20 = 1.5\text{A}}$

$\boxed{R_L = 36} \rightarrow \boxed{I_L = 30/40 = 0.75\text{A}}$

Thevenin's Theorem

- **Example 4.9:** Find the Thevenin equivalent of the circuit below at terminals *a*-*b*.



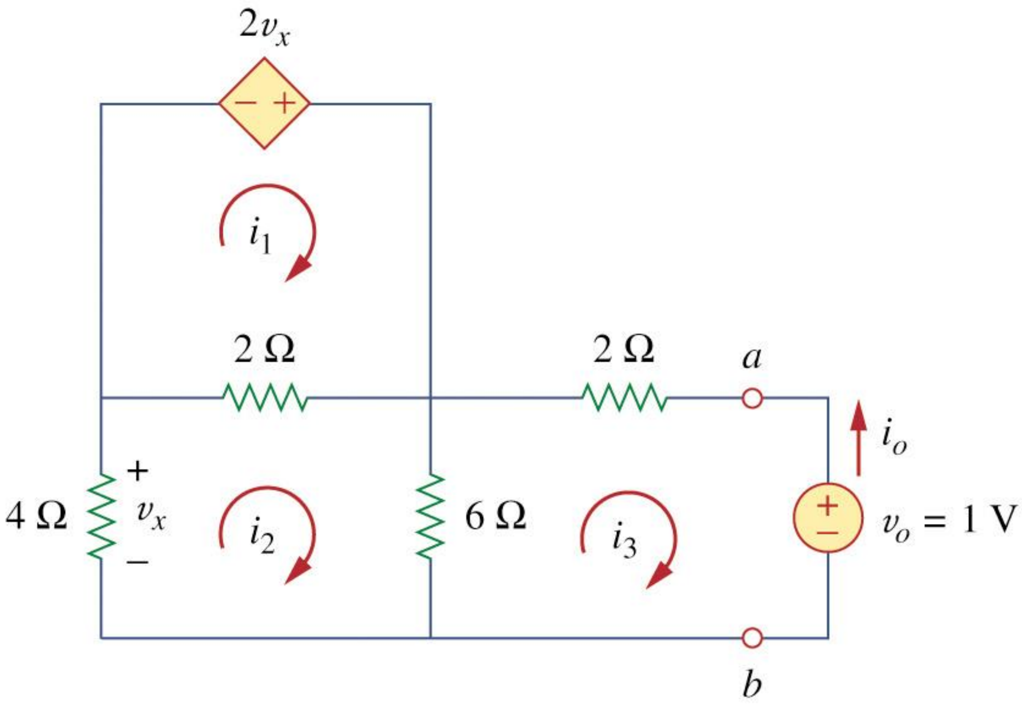
Thevenin's Theorem

- **Example 4.9:** Find the Thevenin equivalent of the circuit below at terminals *a*-*b*.
  - **(independent + dependent sources case)**

Find R_{Th}: Use Fig (a):

$$\text{independent source} \rightarrow 0$$
$$\text{dependent source} \rightarrow \text{intact}$$

$$v_o = 1\text{V}, \quad R_{Th} = \frac{v_o}{i_o} = \frac{1}{i_o}$$



(a)

Thevenin's Theorem

- **Example 4.9:** Find the Thevenin equivalent of the circuit below at terminals *a*-*b*.

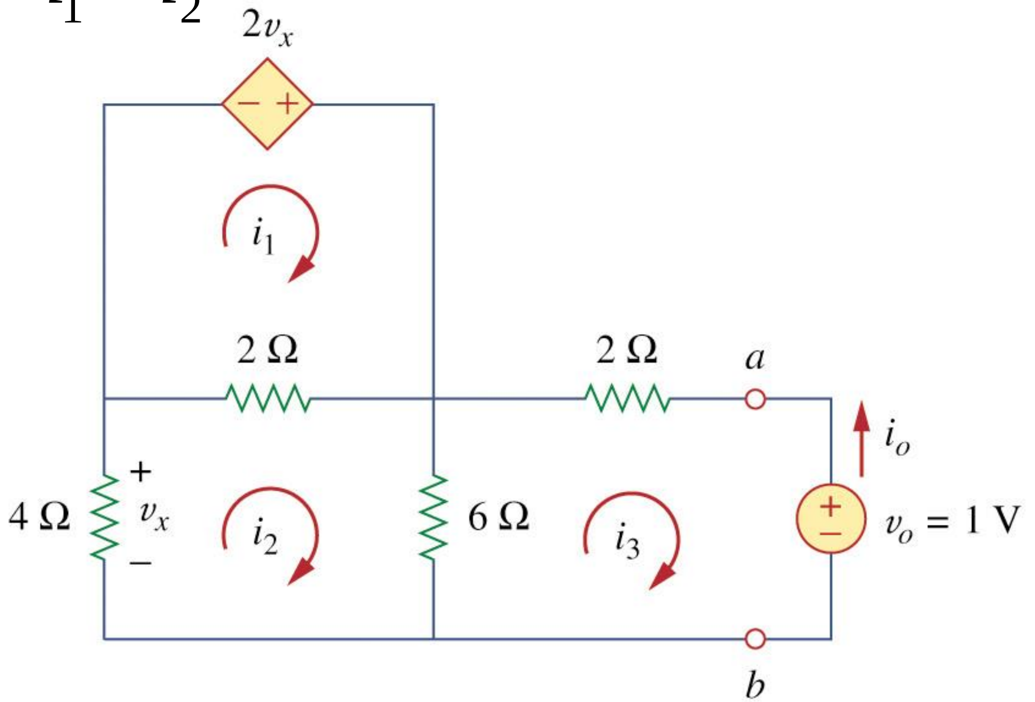
Find R_{Th}:

For **Loop 1:**

$$-2v_x + 2(i_1 - i_2) = 0 \text{ or } v_x = i_1 - i_2$$

$$\text{But } v_x = -4i_2 = i_1 - i_2$$

$$\therefore i_1 = -3i_2$$



(a)

Thevenin's Theorem

- **Example 4.9:** Find the Thevenin equivalent of the circuit below at terminals *a*-*b*.

Find R_{Th}: For **Loops 2 and 3:**

$$4i_2 + 2(i_2 - i_1) + 6(i_2 - i_3) = 0$$

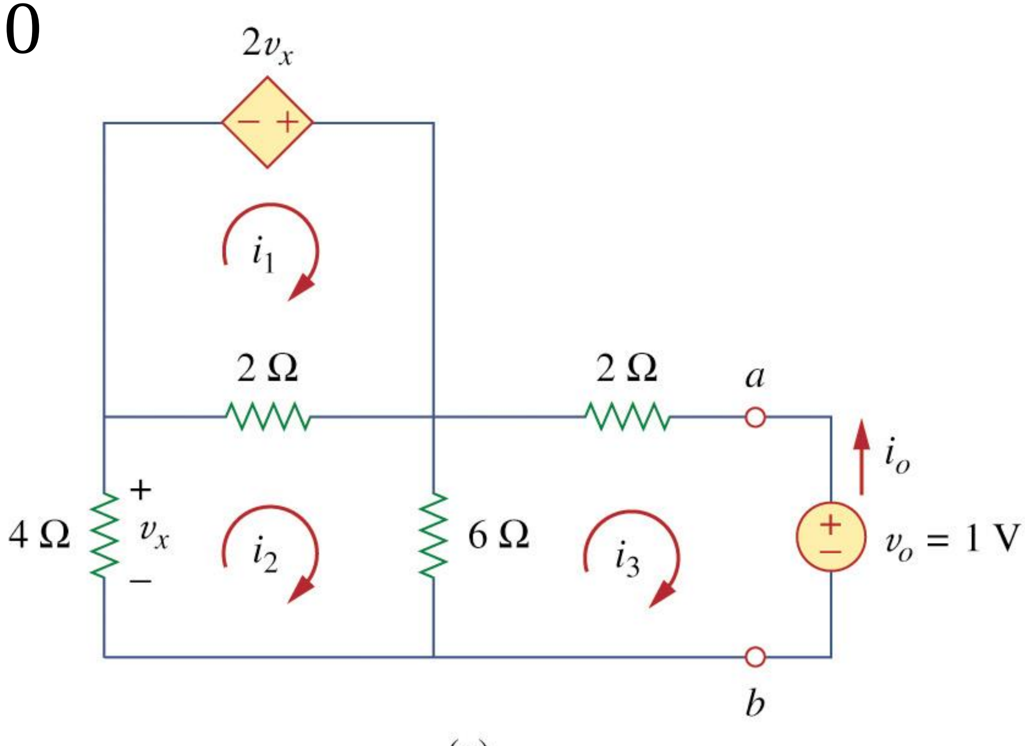
$$6(i_3 - i_2) + 2i_3 + 1 = 0$$

Solving these equations gives

$$i_3 = -1/6\text{A}.$$

$$\text{But } i_o = -i_3 = \frac{1}{6}\text{A}$$

$$\therefore R_{\text{Th}} = \frac{1V}{i_o} = 6\Omega$$

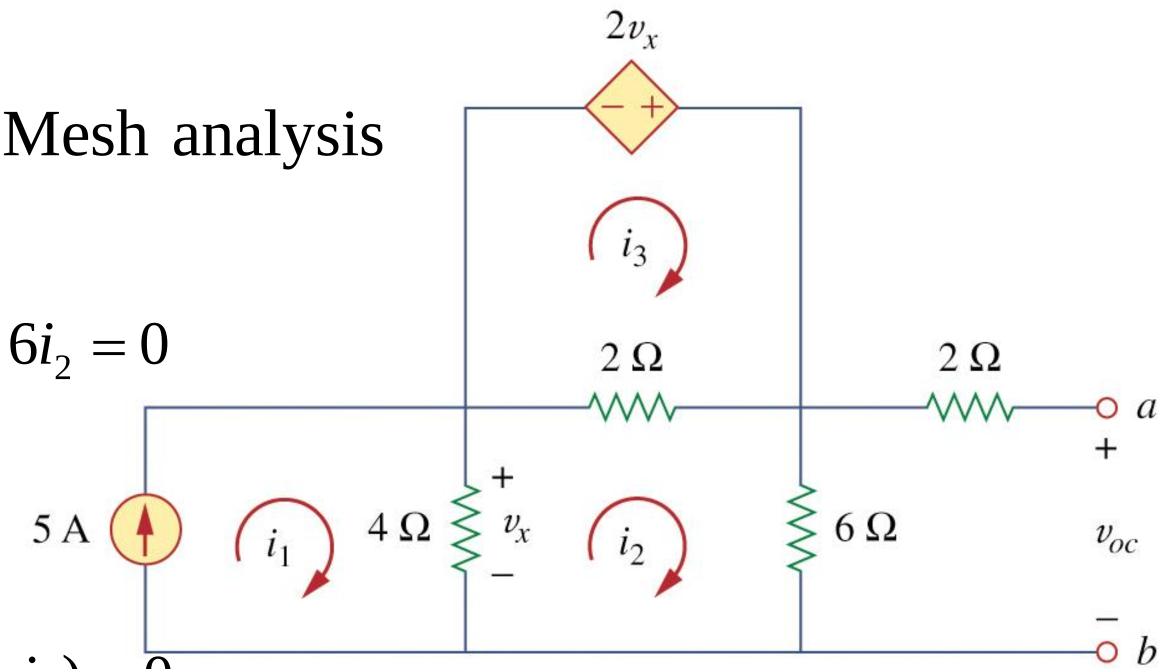


(a)

Thevenin's Theorem

- **Example 4.9:** Find the Thevenin equivalent of the circuit below at terminals *a*-*b*.

Find V_{Th}: Use Fig (b) : Mesh analysis



(b)

Loop 1: $i_1 = 5\,\text{A}$

Loop 2: $4(i_2 - i_1) + 2(i_2 - i_3) + 6i_2 = 0$

$12i_2 - 2i_3 = 20$

Loop 3: $-2v_x + 2(i_3 - i_2) = 0$

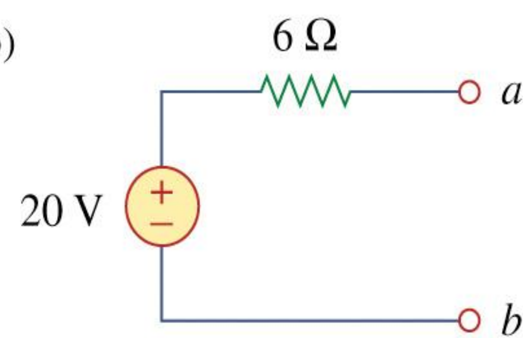
$-2(4(i_1 - i_2)) + 2(i_3 - i_2) = 0$

$6i_2 + 2i_3 = 40$

$\therefore i_2 = 60/18$

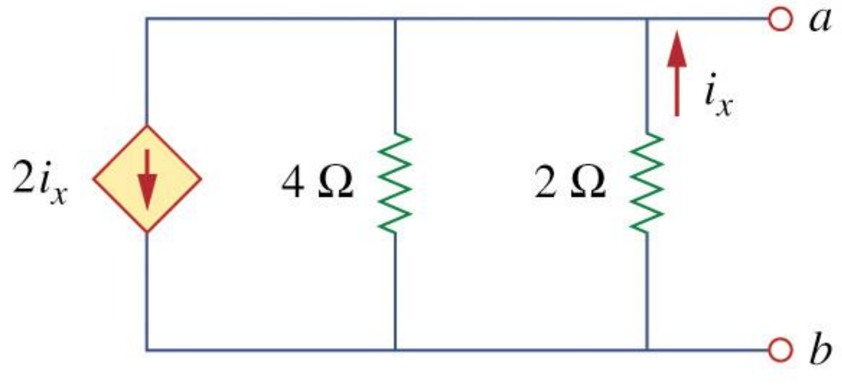
$V_{\text{Th}} = v_{oc} = 6i_2 = 20\text{V}$

Thevenin Equivalent **circuit:**



Thevenin's Theorem

- **Example 4.10:** Determine the Thevenin equivalent circuit in Fig (a).



(a)

Find V_{Th}: $V_{Th} = 0$

Find R_{Th}: Use Fig (b):

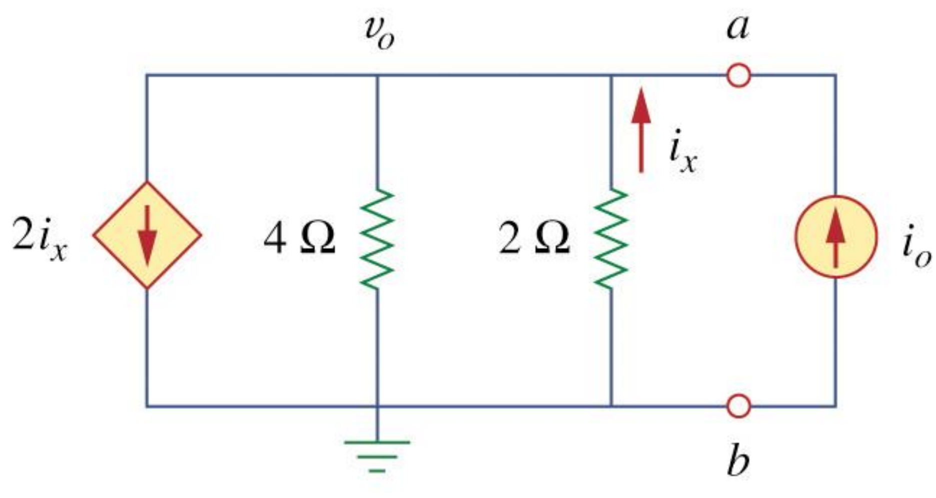
(dependent source only case)

Apply a **current source** i_o at a-b

$$R_{Th} = \frac{v_o}{i_o}$$

Nodal anaysis :

$$i_o + i_x = 2i_x + v_o / 4$$

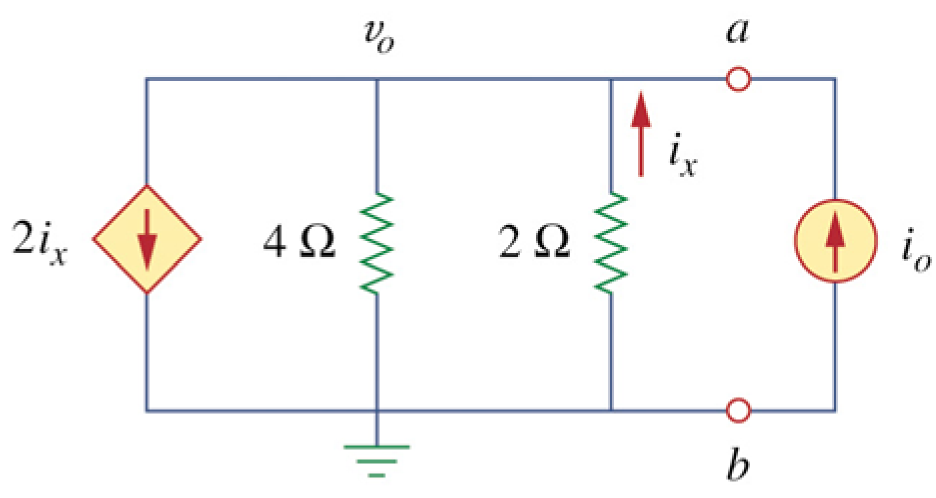


(b)

Thevenin's Theorem

- **Example 4.10:** Determine the Thevenin equivalent circuit in Fig (a).

Find R_{Th}: Use Fig (b):



(b)

But $$i_x = \frac{0 - v_o}{2} = -\frac{v_o}{2}$$

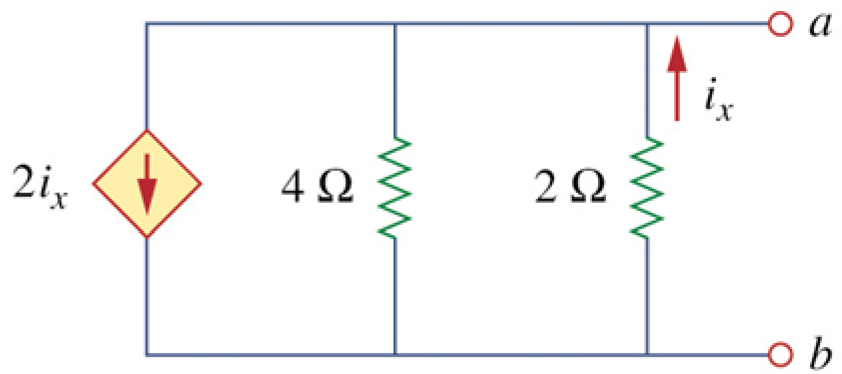
$$i_o = i_x + \frac{v_o}{4} = -\frac{v_o}{2} + \frac{v_o}{4} = -\frac{v_o}{4} \qquad \text{or } v_o = -4i_o$$

$$\text{Thus } R_{Th} = \frac{v_o}{i_o} = -4\Omega \quad \text{(Supplying power)}$$

Thevenin's Theorem

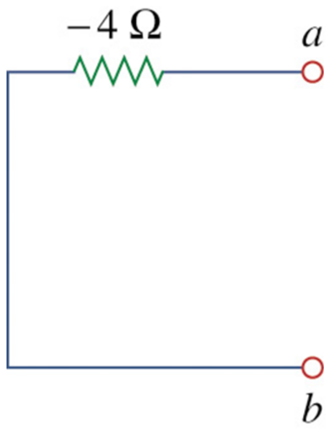
- **Example 4.10:** Determine the Thevenin equivalent circuit in Fig (a).

Input **circuit:**



(a)

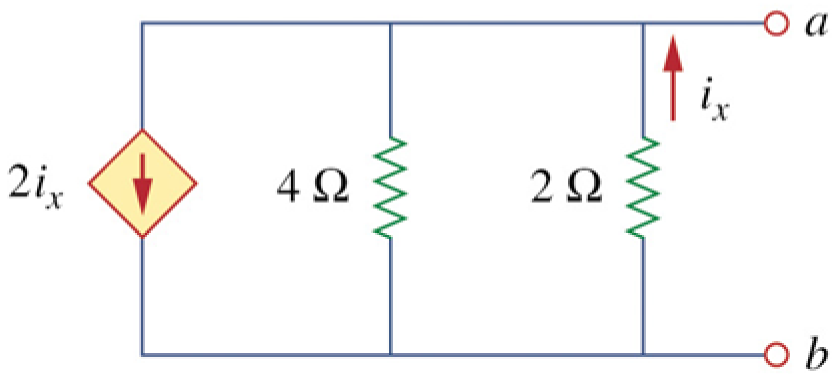
Thevenin Equivalent **circuit:**

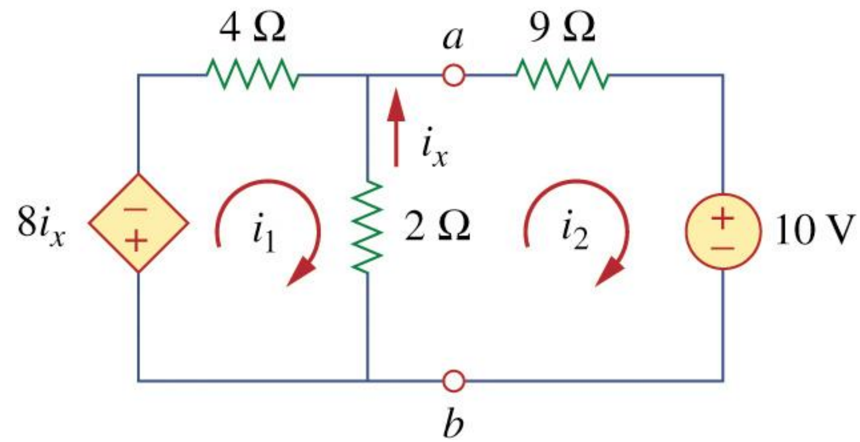


- **When R_{Th} is negative, you must evaluate (check) if the two circuits are equivalent or not!**

Thevenin's Theorem

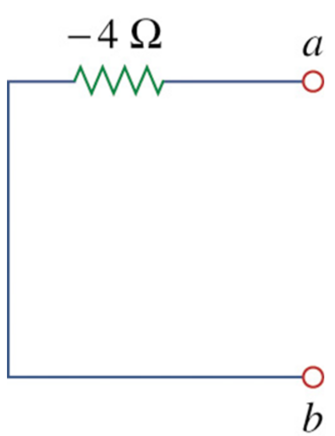
- **Example 4.10:** Determine the Thevenin equivalent circuit in Fig (a).

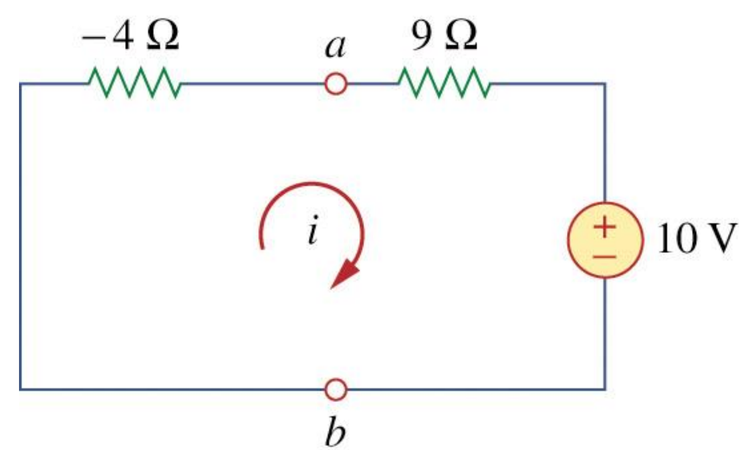




Input **circuit:** (a)

Check with R_L (9Ω) and voltage source (10V)





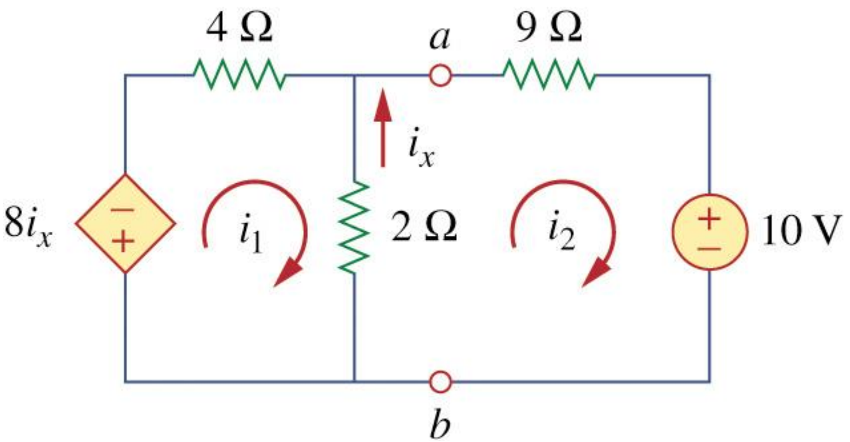
Thevenin Equivalent **circuit:**

Check with R_L (9Ω) and voltage source (10V)

Thevenin's Theorem

- **Example 4.10:** Determine the Thevenin equivalent circuit in Fig (a).

Input **circuit:**



$$8i_x + 4i_1 + 2(i_1 - i_2) = 0 \quad , \quad i_x = i_2 - i_1$$

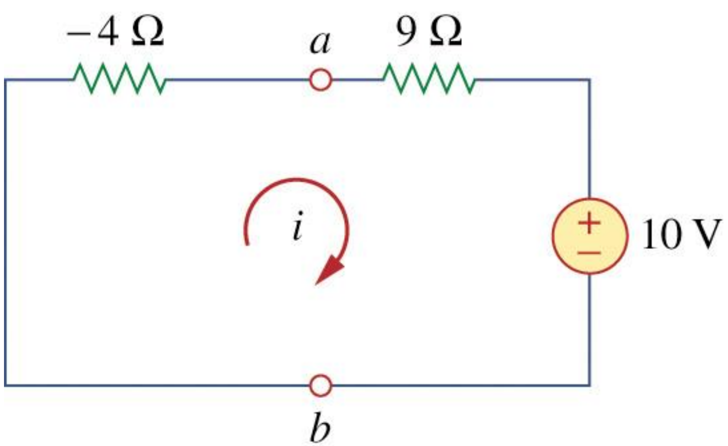
$$-2i_1 + 6i_2 = 0 \quad , \quad i_1 = 3i_2 \quad (1)$$

$$2(i_2 - i_1) + 9i_2 + 10 = 0$$

$$5i_2 = -10 \quad => \quad i_2 = -2\text{A}$$

Check with R_L (9Ω) and voltage source (10V)

Thevenin Equivalent **circuit:**



$$-4i + 9i + 10 = 0$$

$$5i = -10 \quad => \quad i = -2\text{A}$$

Load current in both circuits are equal.

***So the Thevenin Equivalent* circuit is OK.**

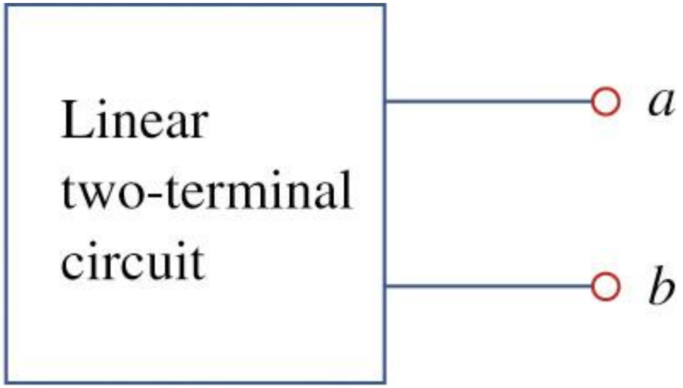
Check with R_L (9Ω) and voltage source (10V)

Norton's Theorem

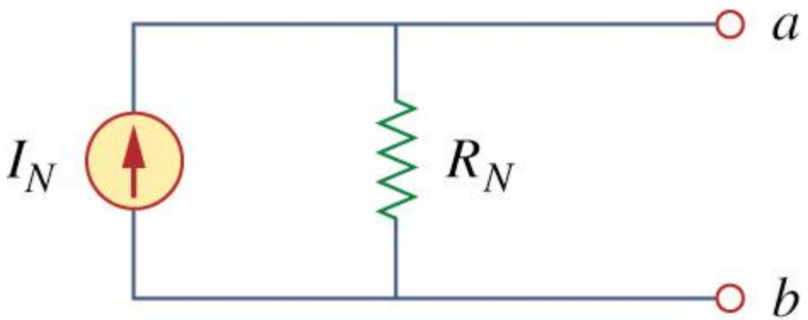
- **Norton's theorem** states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a **current source I_N in parallel** with a **resistor R_N** .
- I_N is the short-circuit current through the terminals and
- R_N is the input or equivalent resistance at the terminals when the **independent source are turn off**.

Norton's Theorem

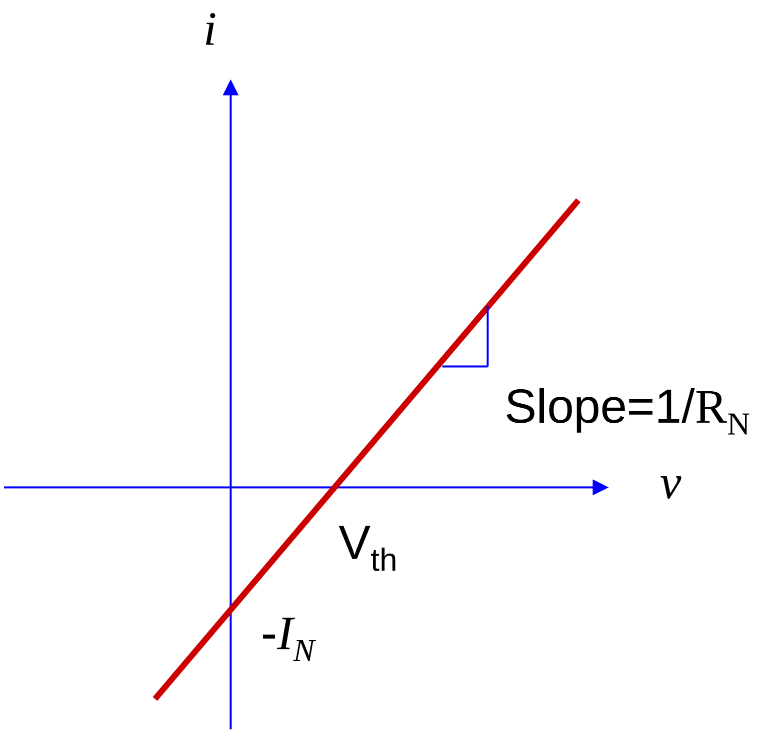
- **Property of Linear Circuits**



(a)



(b)



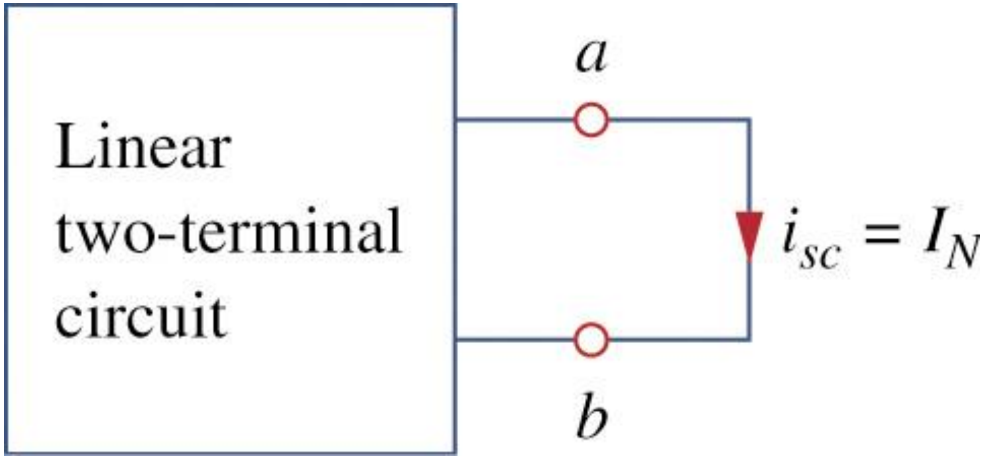
Norton's Theorem

- **How to find Norton Current**
  - **Thevenin** and **Norton** resistances are **equal:**

$$R_N = R_{\text{Th}}$$

  - **Short circuit current** from *a* to *b* gives the Norton current:

$$I_N = i_{sc} = \frac{V_{\text{Th}}}{R_{\text{Th}}}$$



Norton's Theorem

- **Thevenin or Norton equivalent circuit**
  - The **open circuit voltage** v_{oc} across terminals ***a*** and ***b***
  - The **short circuit current** i_{sc} at terminals ***a*** and ***b***
  - The **equivalent** or **input resistance** R_{in} at terminals ***a*** and ***b*** when all **independent source** are **turn off.**

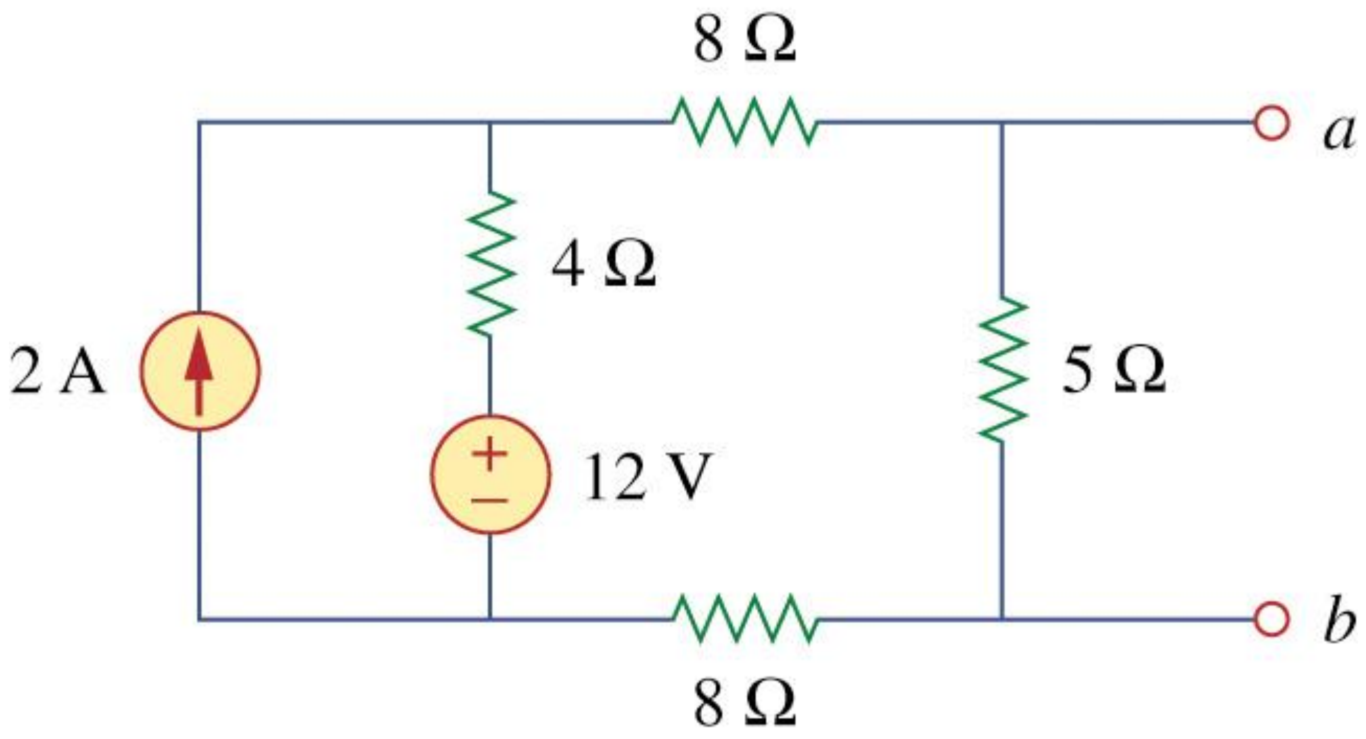
$$V_{Th} = v_{oc}$$

$$I_N = i_{sc}$$

$$R_{Th} = \frac{V_{Th}}{I_N} = R_N$$

Norton's Theorem

- **Example 4.11:** Find the Norton equivalent circuit of the circuit in Fig 4.39.



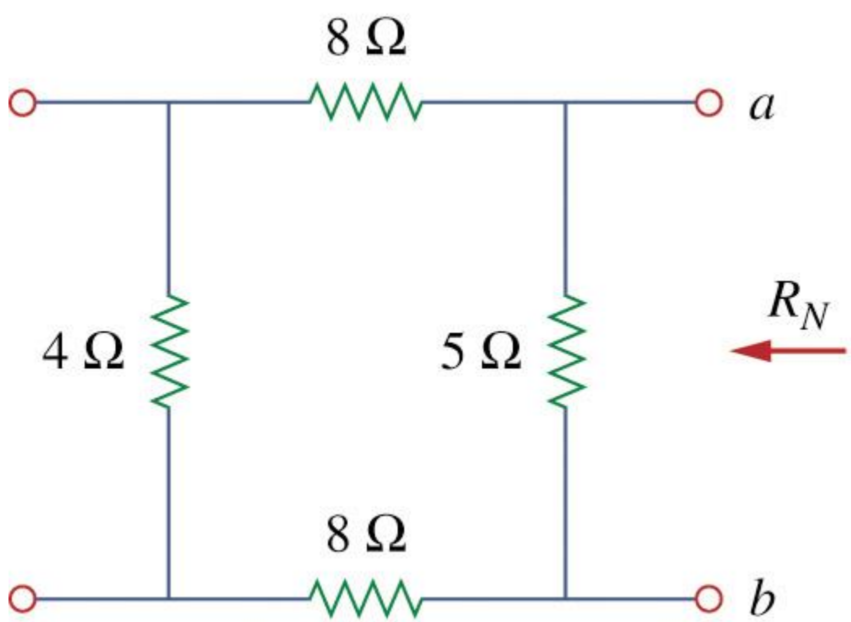
Norton's Theorem

- **Example 4.11:** Find the Norton equivalent circuit of the circuit in Fig 4.39.

Find R_N: Use Fig (a):

$$R_N = 5 \| (8+4+8)$$

$$= 5 \| 20 = \frac{20 \times 5}{25} = 4\Omega$$



(a)

Norton's Theorem

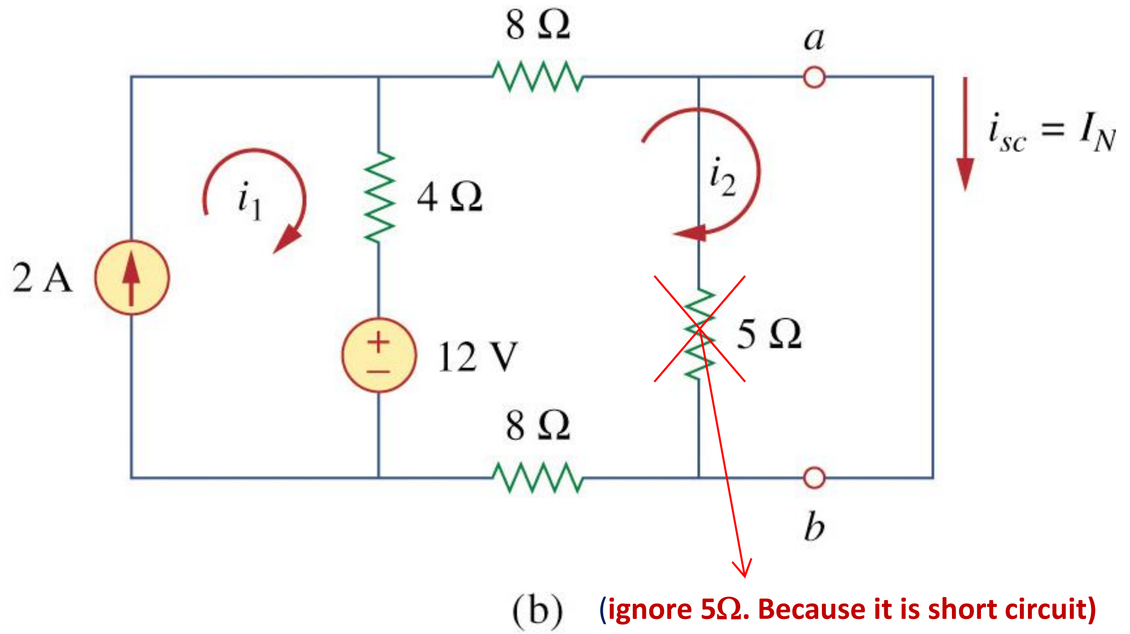
- **Example 4.11:** Find the Norton equivalent circuit of the circuit in Fig 4.39.

Find I_N: Use Fig (b):

(**short circuit** terminals ***a*** and ***b***)

$$\text{Mesh}: \; i_1 = 2A, \quad 20i_2 - 4i_1 - 12 = 0$$

$$i_2 = 1\text{A} = i_{sc} = I_N$$



(b)

Norton's Theorem

- **Example 4.11:** Find the Norton equivalent circuit of the circuit in Fig 4.39.

Find I_N: Use Fig (c): **Alternative Method** $I_N = \dfrac{V_{Th}}{R_{Th}}$

V_{Th}: (**open circuit voltage** accross terminals ***a*** and ***b***)

Mesh analysis:

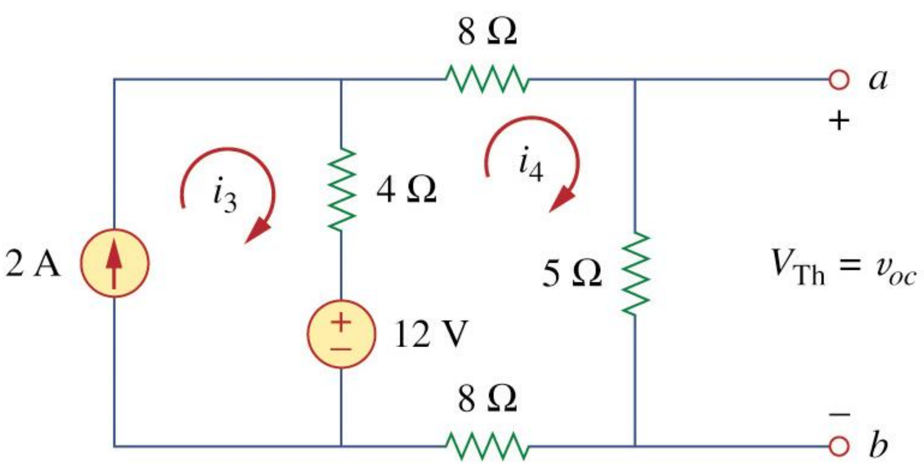
$$i_3 = 2A,\ \ 25i_4 - 4i_3 - 12 = 0$$

$$\therefore\ i_4 = 0.8\text{A}$$

$$\therefore v_{oc} = V_{Th} = 5i_4 = 4\text{V}$$

Hence,

$$I_N = \frac{V_{Th}}{R_{Th}} = 4/4 = 1\text{A}$$

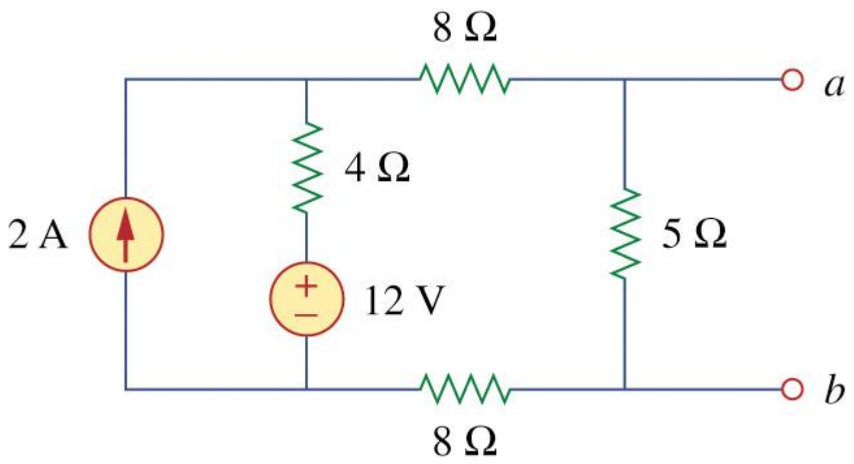


(c)

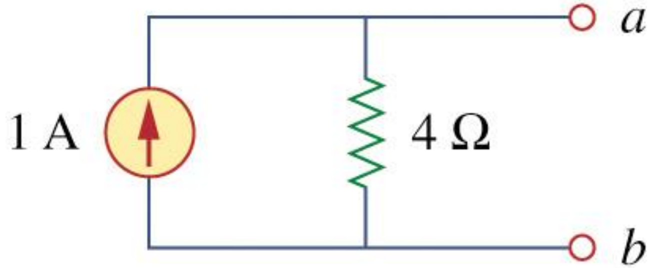
Norton's Theorem

- **Example 4.11:** Find the Norton equivalent circuit of the circuit in Fig 4.39.

Input **circuit:**

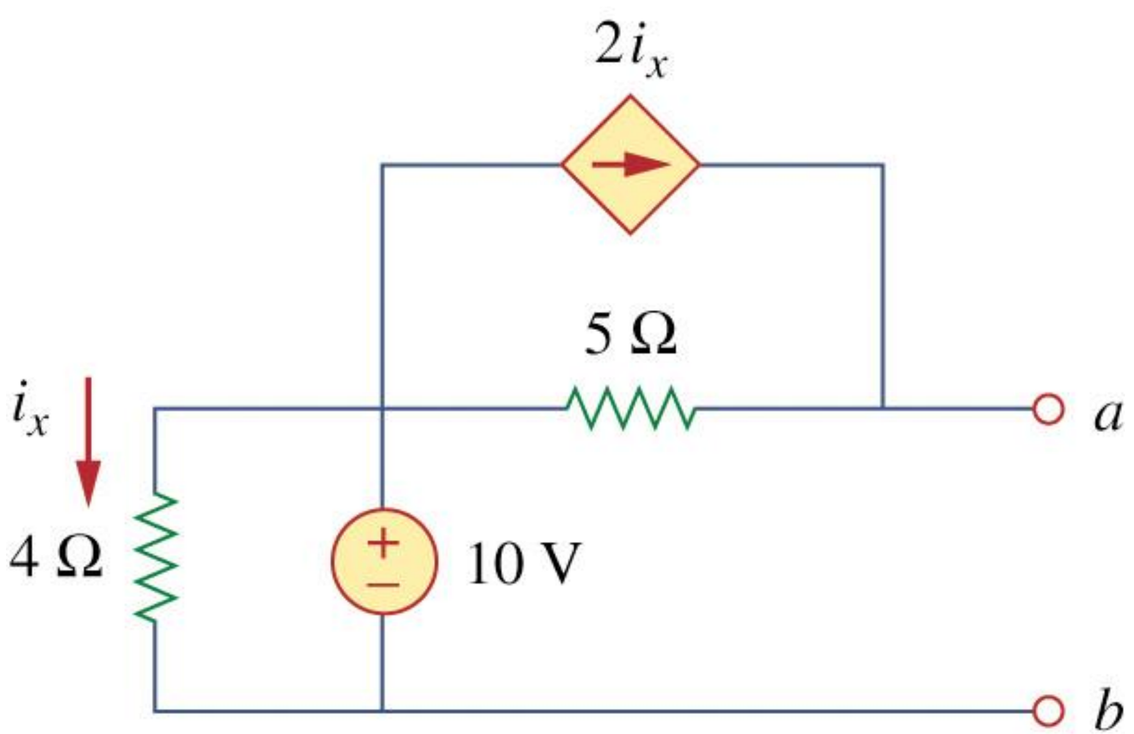


Norton's Equivalent **circuit:**



Norton's Theorem

- **Example 4.12:** Using Norton's theorem, find R_N and I_N of the circuit in Fig 4.43 at terminals *a-b*.



Norton's Theorem

- **Example 4.12:** Using Norton's theorem, find R_N and I_N of the circuit in Fig 4.43 at terminals *a-b*.

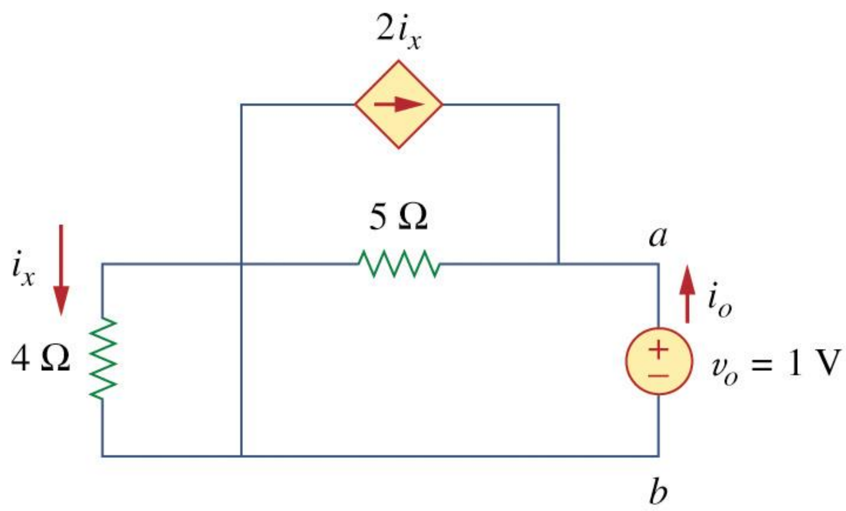
Find R_N: Use Fig (a):

- 4Ω resistor shorted
  $5\Omega \,\|\, v_o \,\|\, 2i_x$: Parallel

Hence,

$$i_x = 0\text{A}, \qquad i_0 = \frac{v_0}{5\Omega} = \frac{1\text{V}}{5\Omega} = 0.2\text{A}$$

$$\therefore R_N = \frac{v_o}{i_o} = \frac{1}{0.2} = 5\Omega$$



(a)

Norton's Theorem

- **Example 4.12:** Using Norton's theorem, find R_N and I_N of the circuit in Fig 4.43 at terminals *a*-*b*.

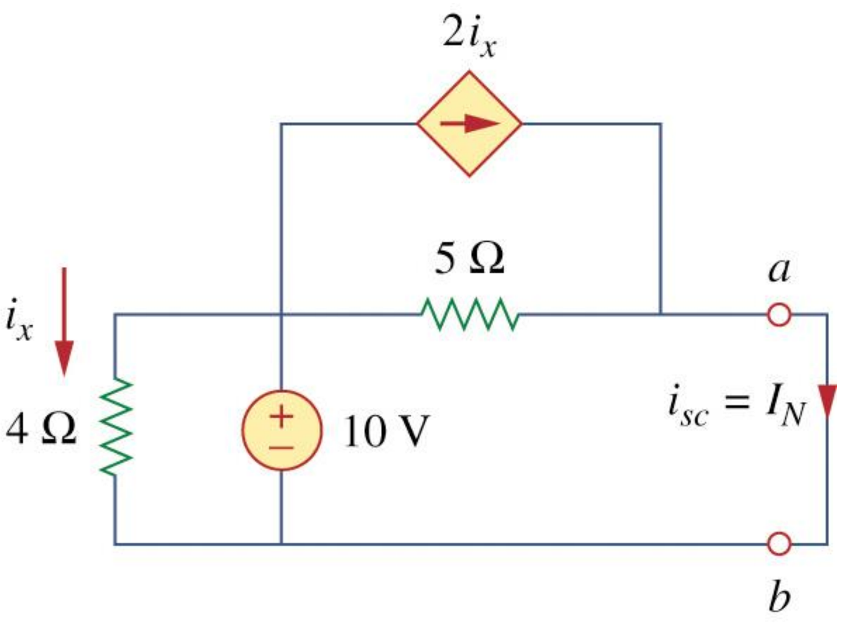
Find I_N:

- $4\Omega \| 10v \| 5\Omega \| 2i_x$:Parallel

$$i_x = \frac{10-0}{4} = 2.5\text{A},$$

$$i_{sc} = i_x + 2i_x = \frac{10}{5} + 2(2.5) = 7\,\text{A}$$

$$\therefore I_N = 7\text{A}$$

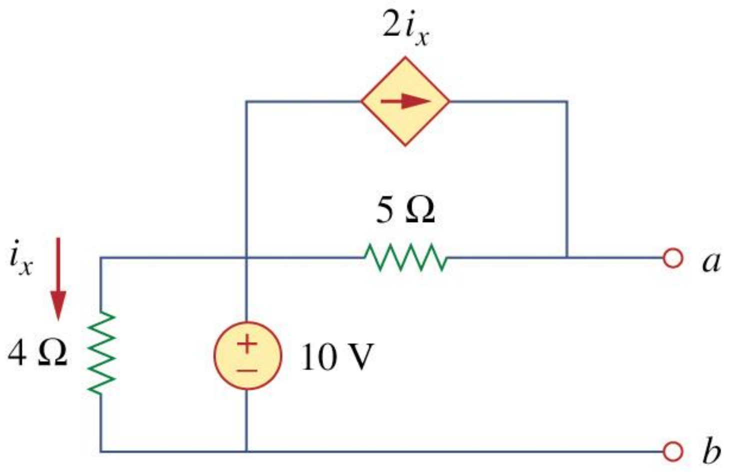


(b)

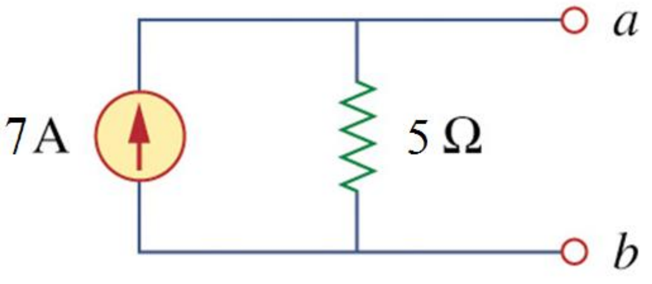
Norton's Theorem

- **Example 4.12:** Using Norton's theorem, find R_N and I_N of the circuit in Fig 4.43 at terminals *a-b*.

Input **circuit:**

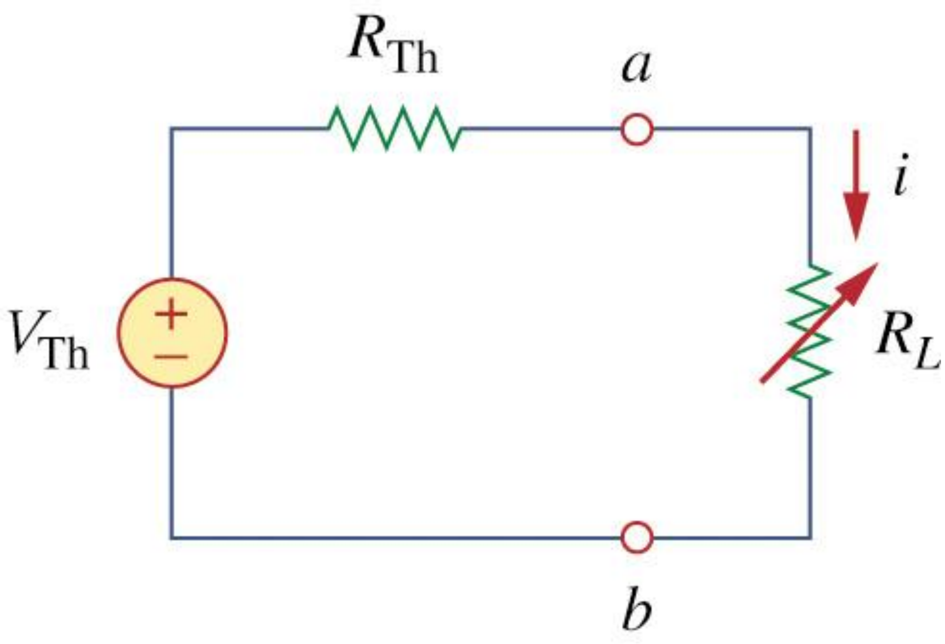


Norton's Equivalent **circuit:**



Maximum Power Transfer

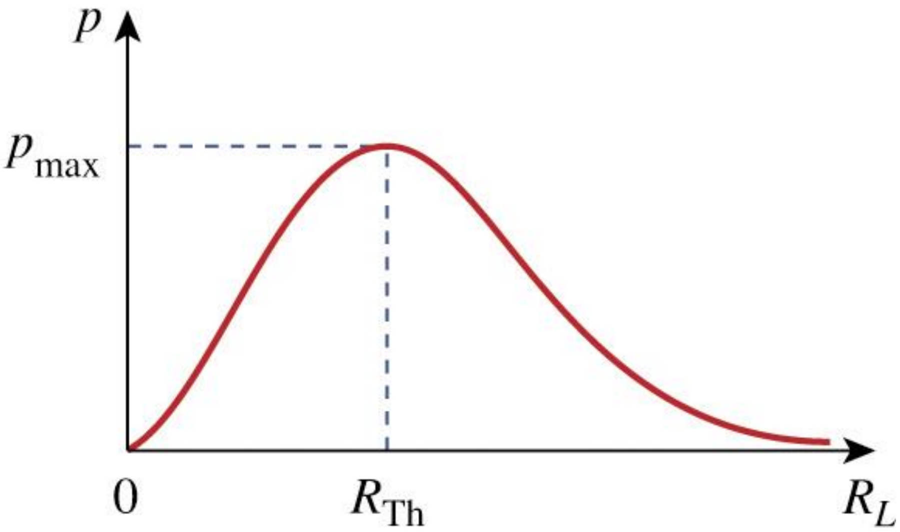
- The Thevenin equivalent is useful in finding the **maximum power** a linear circuit can deliver to a load.



$$p = i^2 R_L = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L$$

Maximum Power Transfer

- **Maximum power** is transferred to the load when the load resistance equals the **Thevenin resistance** as seen from the load **($R_L = R_{TH}$).**



$$p = i^2 R_L = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L$$

Maximum Power Transfer

- $P_{\max}$ can be obtained when:

$$\frac{dp}{dR_L} = 0$$

$$\frac{dp}{dR_L} = V_{TH}^2 \left[\frac{(R_{TH} + R_L)^2 - 2R_L(R_{TH} + R_L)}{((R_{TH} + R_L)^2)^2} \right] = 0$$

$$= V_{TH}^2 \left[\frac{(R_{TH} + R_L)^2 - 2R_L(R_{TH} + R_L)}{(R_{TH} + R_L)^4} \right] = V_{TH}^2 \left[\frac{(R_{TH} + R_L - 2R_L)}{(R_{TH} + R_L)^3} \right] = 0$$

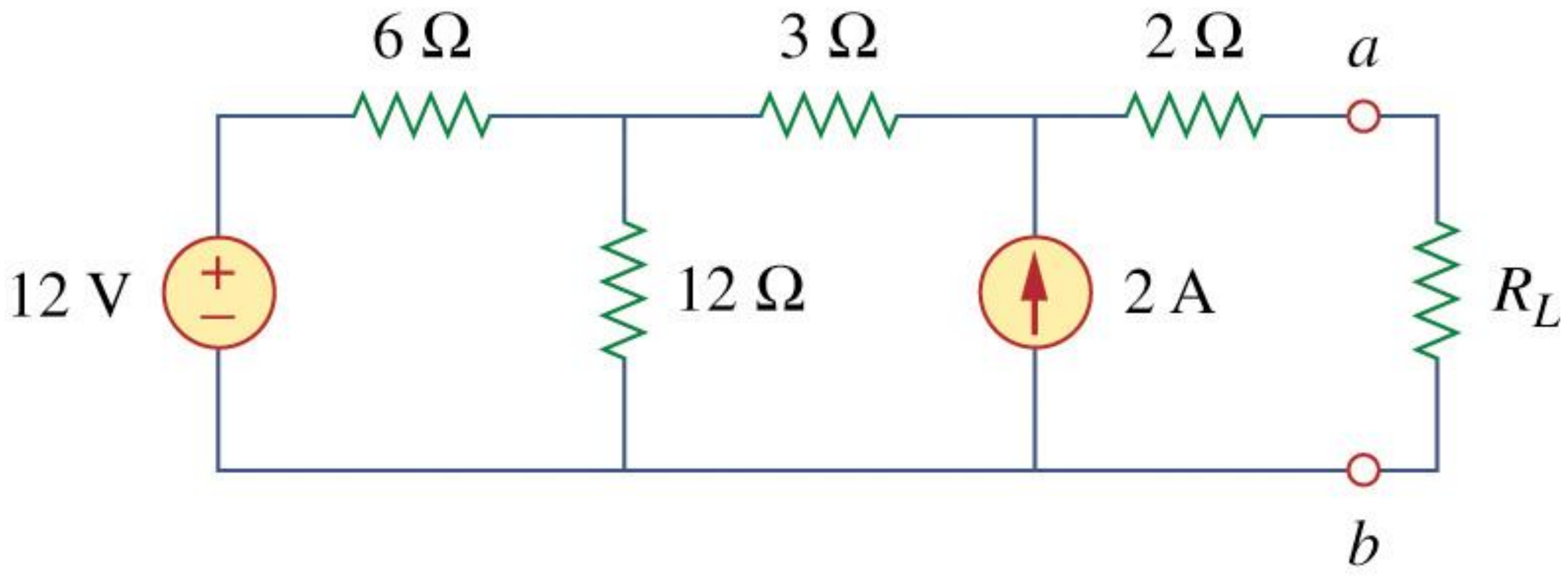
$$\Rightarrow (R_{TH} + R_L - 2R_L) = 0 \qquad \Rightarrow (R_{TH} - R_L) = 0$$

$$R_L = R_{TH}$$

$$p_{\max} = \frac{V_{TH}^2}{4R_{TH}}$$

Maximum Power Transfer

- **Example 4.13:** Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

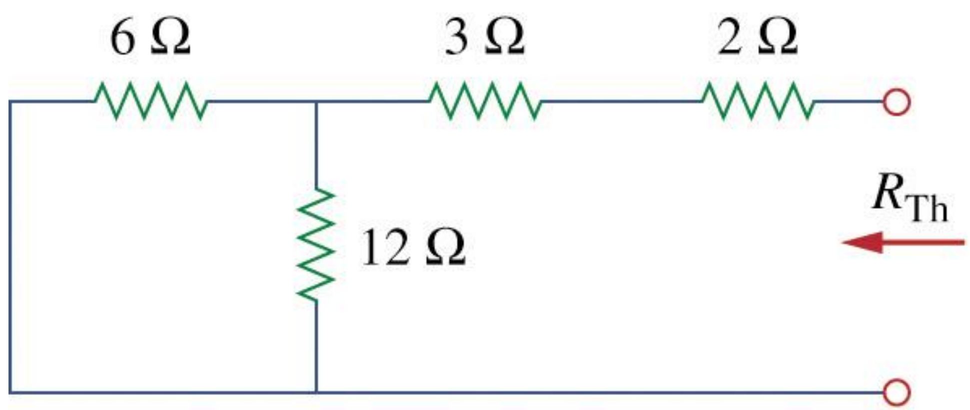


Maximum Power Transfer

- **Example 4.13:** Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

Find R_{Th}:

$$R_{TH} = 2 + 3 + 6 \| 12 = 5 + \frac{6 \times 12}{18} = 9\Omega$$



(a)

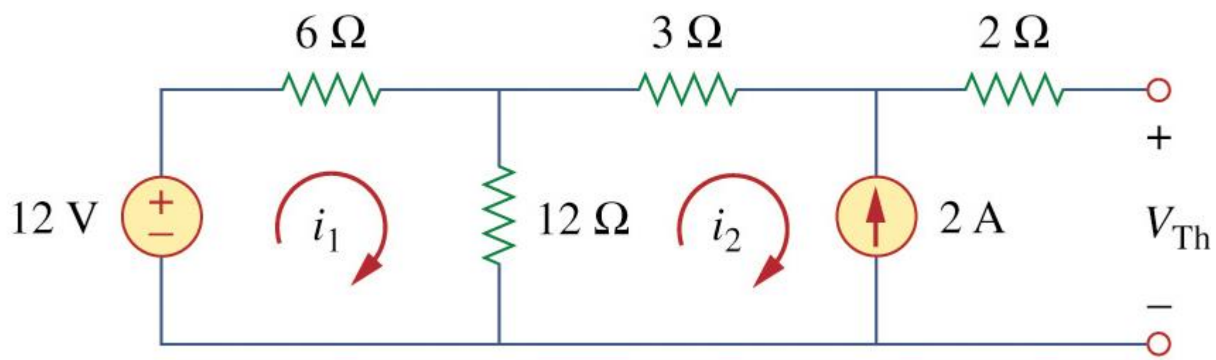
Maximum Power Transfer

- **Example 4.13:** Find the value of R_L for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

Find V_{Th}:

$$-12+18i_1-12i_2, \quad i_2=-2A, \quad i_2=-\frac{2}{3}A$$

KVL around the outer loop: $-12+6i_1+3i_2+2(0)+V_{TH}=0 \Rightarrow V_{TH}=22V$



(b)

$$R_L=R_{TH}=9\Omega$$

$$\Rightarrow \quad p_{max}=\frac{V_{TH}^2}{4R_L}=\frac{22^2}{4\times 9}=13.44\text{W}$$