DEAD RECKONING NAVIGATION (DR NAVIGATION)

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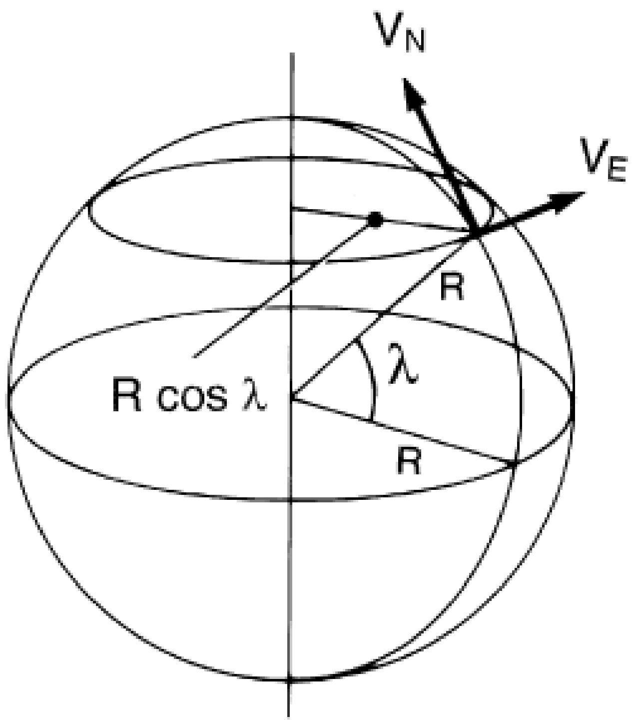
- A Navigation which uses a Inertial sensor (motion & rotational sensor) (or) air data sensor (or) Doppler radar to continuously calculate position, orientation and velocity of a moving object without need of any external references is said to be Dead reckoning Navigation (DR).

- Types of DR Navigation Type
  - By using Inertial Measurement unit
  - By using Doppler Data measurement.
  - By using Air data Measurement

- Two important parameters are needed to perform DR Navigation
  - Initial Position of Latitude and Longitude
  - Northerly and Easterly velocity components of an aircraft, V_N and V_E.



Derivation of rates of change of latitude and longitude.

The rate of change of latitude $\dot{\lambda} = \dfrac{V_N}{R}$

The rate of change of longitude $\dot{\mu} = \dfrac{V_E}{R\cos\lambda} = \dfrac{V_E}{R} sec\lambda$

- *The change of latitude over time with initial latitude* (λ_0) *is given by*

$$\lambda = \lambda_0 + \frac{1}{R}\int_0^t V_N \, dt$$

- *The change of longitude over time with initial longitude* (μ_0) *is given by*

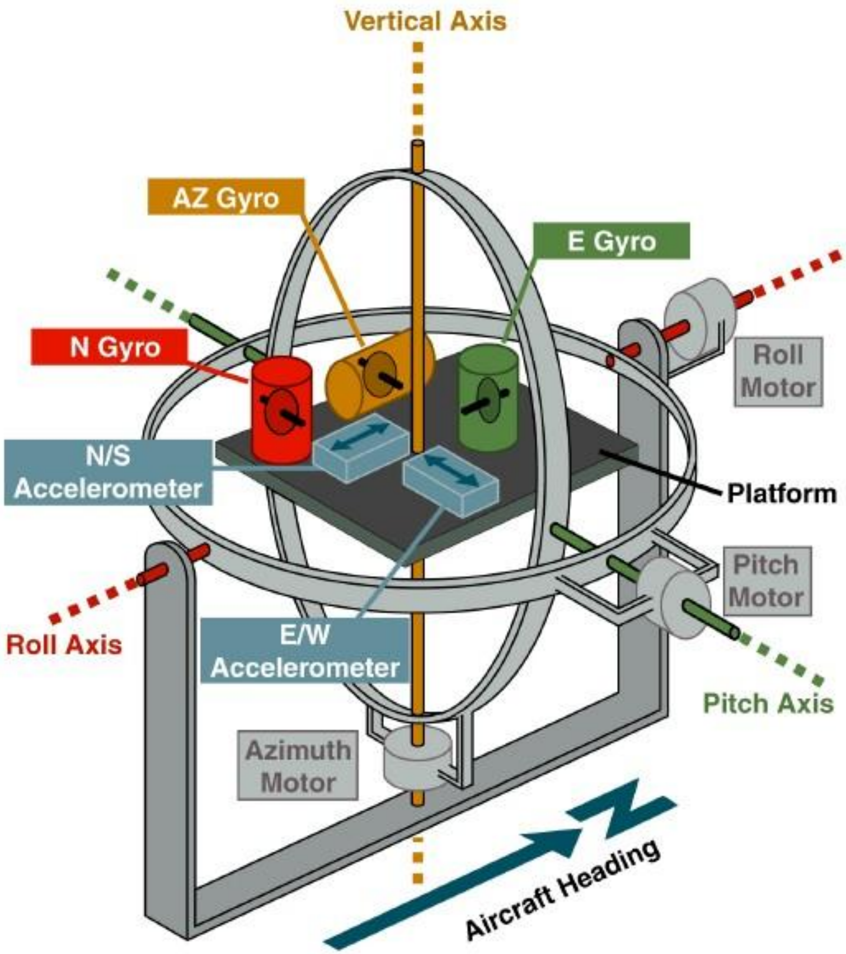
$$\mu = \mu_0 + \frac{1}{R}\int_0^t V_E \sec\lambda \, dt$$

- *It can be noticed that a mathematical singularity approaches* $\lambda = 90$ *degree and* $Sec\lambda$ *approaches infinity. This method of Computing latitude and longitude of DR position is limited to latitude below* 80 *degree. So a different method is needed to deal with high latitudes.*

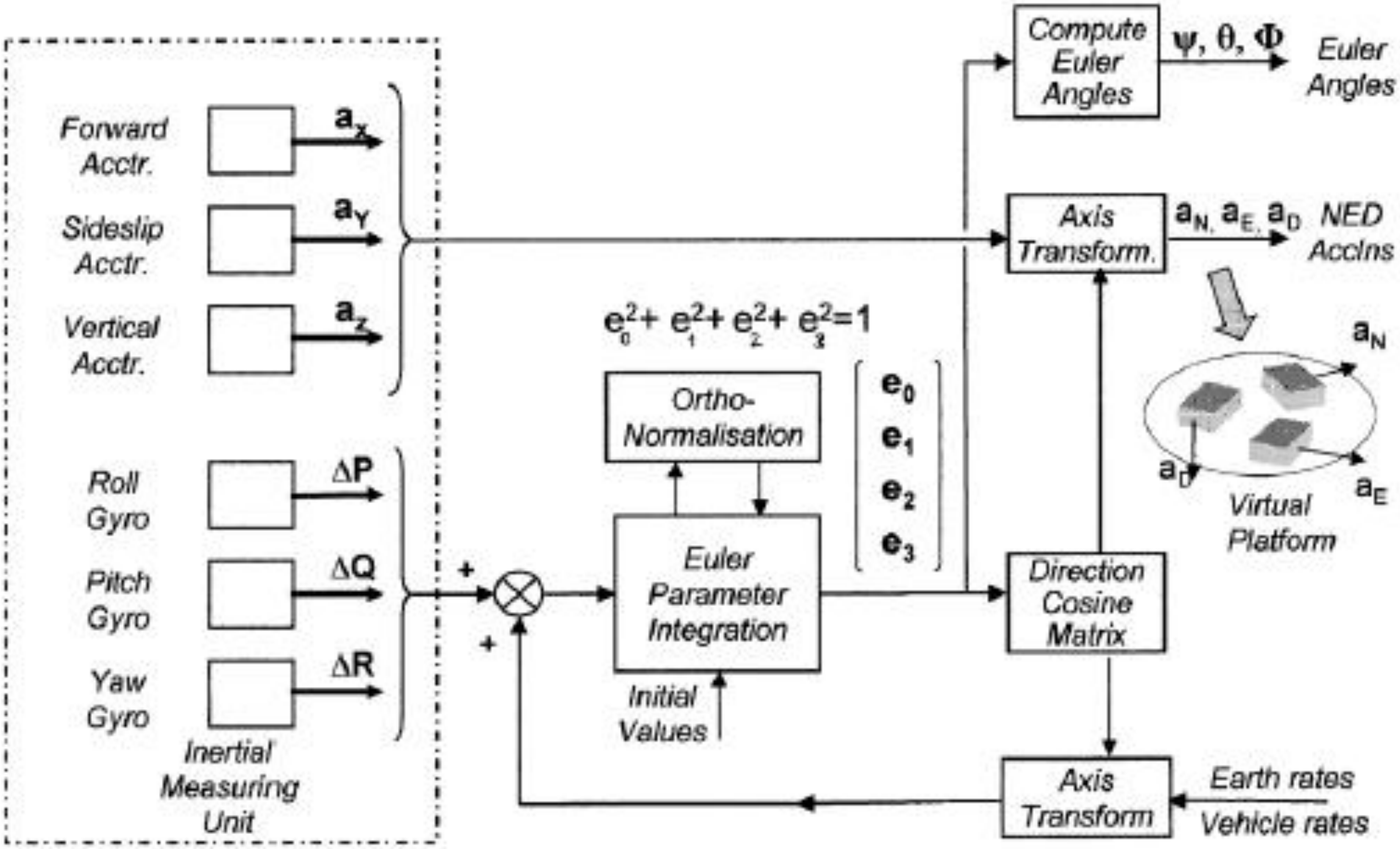
- Some of the method to over singularity problems are
  - A spherical coordinate system with the poles removed to some other regions.
  - A Unipolar system
  - Directional Cosines
- The spherical coordinate system with transferred poles gives additional complication to ellipsoidal correction because of earth is an ellipse.
- Singularity also still exits in spherical coordinate system.
- Space does not permits for Unipolar system because of its length.
- Direction cosine system is a system in which a coordinate of a point is represent by three directional cosine of its radius vector w.r.t to ECEF reference frame.

Dead Reckoning by IMU

- Dead Reckoning by IMU is classified into two types
  - Stable platform system.
  - Strap down system.
- Three important corrections to be done in gyroscope the case of DR by IMU
  - Schuler's Corrections
  - Angular Rate Corrections
    - Angular Rate corrections due to earth rotation.
    - Vehicle Rate correction due to aircraft flies over surface of the Earth.
  - Acceleration Correction terms
    - Coriolis acceleration
    - Centrifugal acceleration
    - Gravitational acceleration



Stable platform



Strap down platform

Angular Rate Corrections due to Earth Rotation.

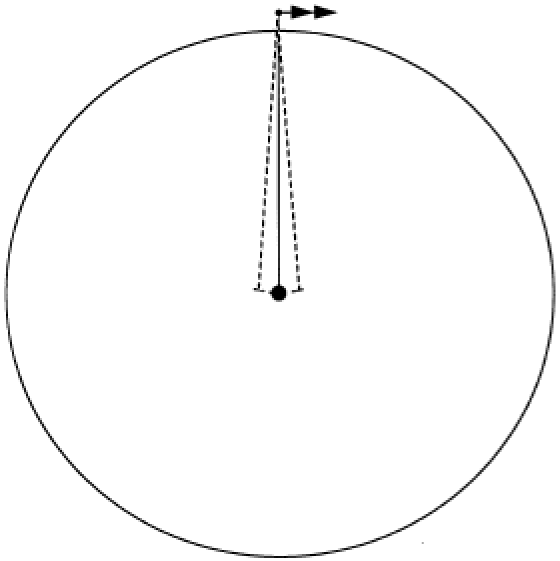
- The earth angular velocity can be resolved into two components
  - *Component about north pointing axis* $= \Omega \cos \lambda$

    This correction must be applied to the vertical gyroscope to precess it about the north pointing axis .
  - *Component about Local Vertical axis* $= \Omega \sin \lambda$

    This correction must be applied to the Azimuth gyroscope to maintain the platform facing north .

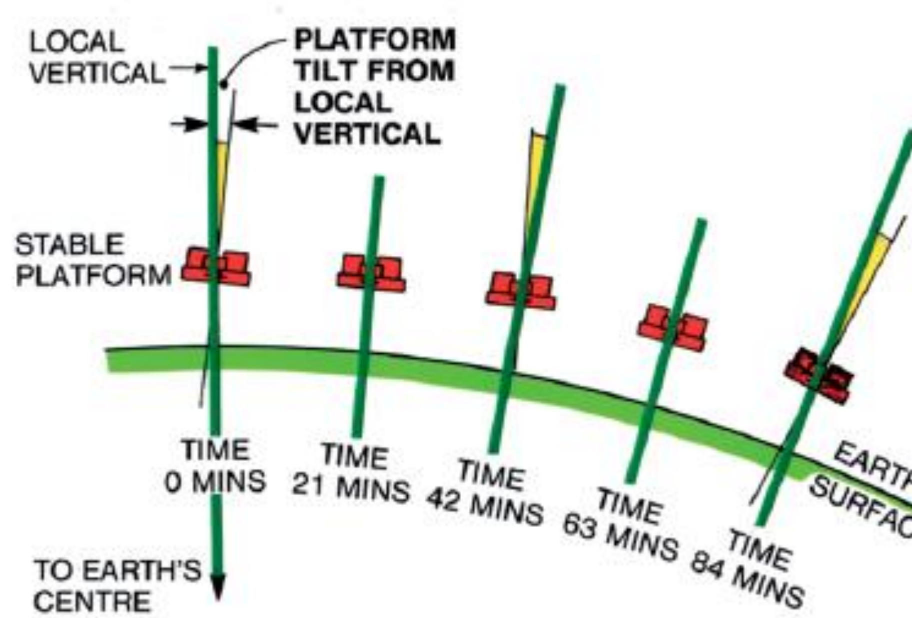
*****It is noted that ,there is zero component of the earth angular velocity about the East pointing axis and this forms the basics of Gyro compassing alignment Technique.

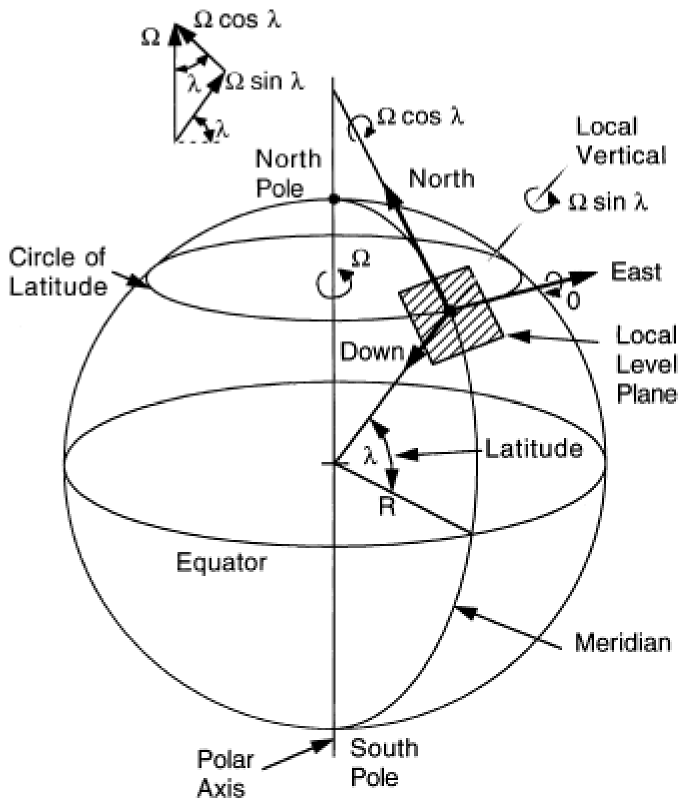
SCHULER'S OSCILLATIONS



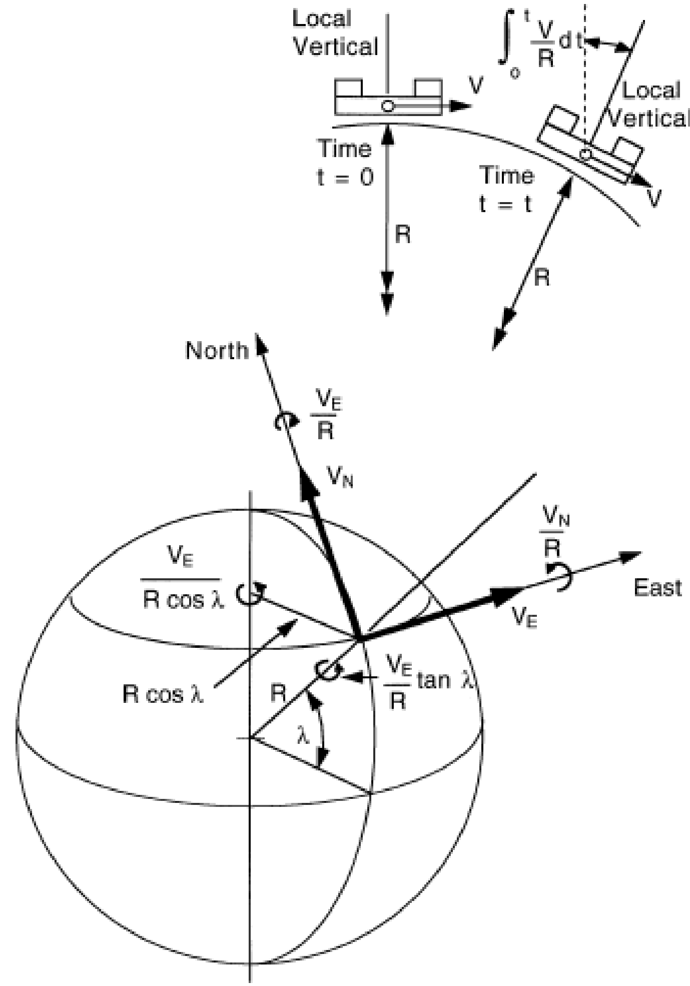
$$\text{Period} = 2\pi\sqrt{\frac{R}{g}}$$

$$= 84.4\text{mins}$$





Earth's rotation rate components



Vehicle rates

Vehicle Rate Corrections due to Aircraft Flies over the Surface of Earth.

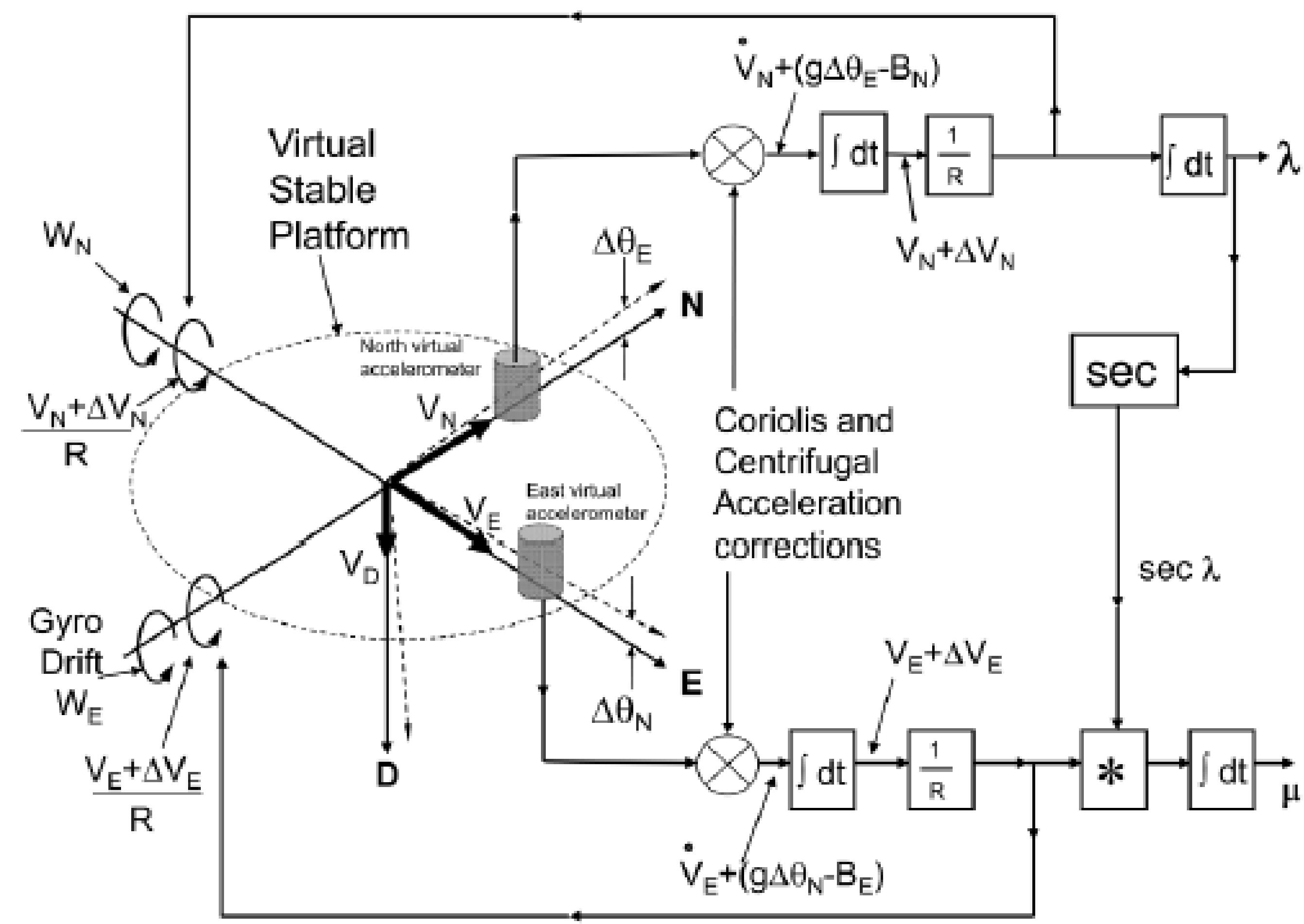
- The local vertical rotates in space as the aircraft flies over the surface of the earth because the earth is spherical.
- The angular rate of rotation of the local vertical with respect to the inertial axis are
  - V_N / R about the East pointing axis
  - V_E / R about the North pointing axis
  - The angular rates are referred as vehicle rate. In order to maintain the platform local level , it is necessary to correct vertical gyro about its east pointing and north point axes at an angular rate of V_N / R and V_E / R respectively.

- These types of errors propagation are unacceptable except in application where the time of flight is short (short range missile) or system could be frequently corrected by another navigation system(GPS).
- In order to avoid the tilt error the torque motor's gives correction to the INS, so that the INS follows the local vertical irrespective of the vehicle acceleration so that it behave like a Schuler's pendulum.
- Schuler's pendulum is proposed by Austrian Physicist Max Schuler's in 1924. He proposed a idea of a new pendulum whose length was equal to the radius of the earth is unaffected by acceleration.
- The 'plump bob' of such pendulum would always be at the centre and would defines the local vertical irrespective of motion ,or acceleration of the point of suspension of pendulum (moving vehicle).

- Any disturbance to the pendulum bob would causes it oscillate about the local vertical with a period equal to

$$T = 2\pi\sqrt{\frac{L}{g}} \approx 2\pi\sqrt{\frac{6378100}{9.81}} \approx 5066 Seconds \approx 84.4 minutes$$

- A simple pendulum of length equal to the earth radius is not feasible. however a pendulous system could be produced with an exact time period of 84.4 minutes would indicate the local vertical irrespective of the acceleration of vehicle carrying it.
- In 1924 the system is not available to implement in practical because of the un available of computing technology. The first successful airborne demonstration of a Schuler tuned inertial navigation was achieved by Dr.Charles Strak Draper and his team in Massachusetts Institute of Technology in 1952.



Virtual
Stable
Platform
$\dot{V}_N+(g\Delta\theta_E-B_N)$
∫ dt
1/R
∫ dt
λ
W_N
$V_N+\Delta V_N$
$\Delta\theta_E$
N
North virtual accelerometer
$\frac{V_N+\Delta V_N}{R}$
V_N
sec
Coriolis and
Centrifugal
Acceleration
corrections
East virtual accelerometer
V_E
V_D
sec λ
Gyro
Drift
W_E
$V_E+\Delta V_E$
E
$\Delta\theta_N$
D
$\frac{V_E+\Delta V_E}{R}$
∫ dt
1/R
*
∫ dt
μ
$\dot{V}_E+(g\Delta\theta_N-B_E)$

- The X and Y accelerometers are mounted on a stable platform, so that
  - X accelerometer measures aircraft acceleration along East-West axis
  - Y accelerometer measures acceleration along the North-South axis
- The output of X and Y accelerometer after corrected for the **Coriolis and centrifugal acceleration component**, are then integrated with respect to time to gives the easterly and northerly velocity component of the aircraft. From the velocity we can able to find the latitude and longitude of the vehicle.
- A Schuler tuning is achieved by feedback the accelerometer derived vehicle rates VN/R and VE/R to the **vertical gyro** so the platform follows the local vertical as the aircraft moves over the spherical earth.

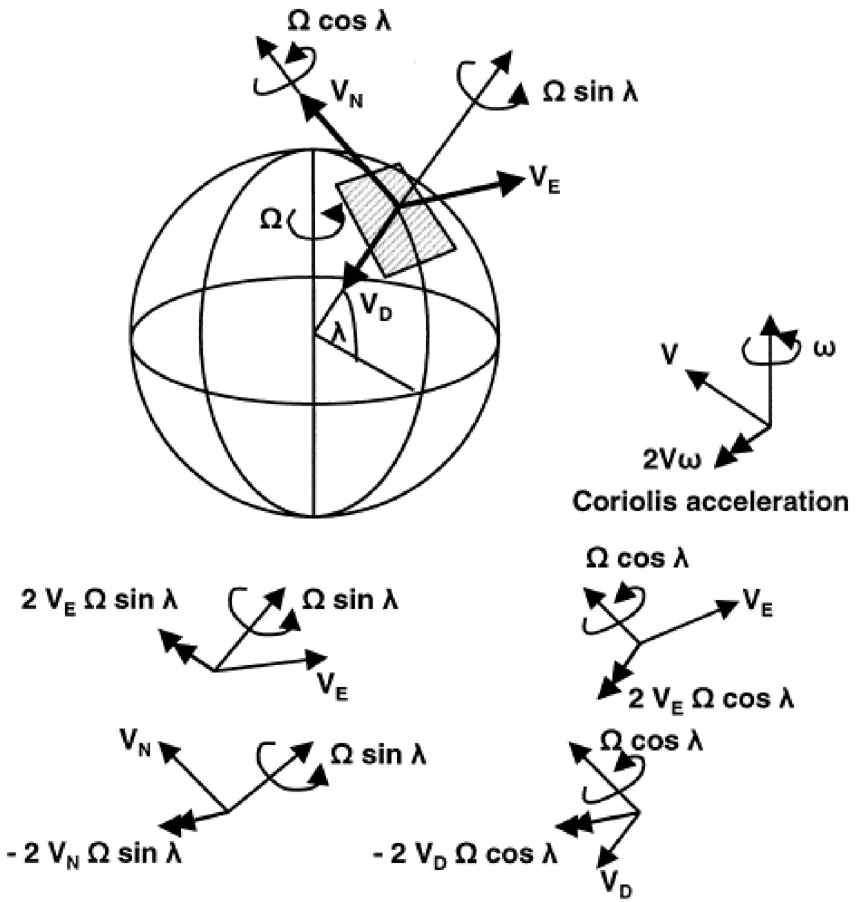
- There is a vehicle rate correction $(V_E / R) \tan\lambda$, about local vertical (down vertical) which must be applied to the azimuth gyro in order to maintain the north reference. The (V_E / R) tan term is derived by resolving the vehicle rate component about earth polar axis about , through the latitude angle.

$$\frac{V_E}{R\cos\lambda} * \sin\lambda = \frac{V_E}{R}\tan\lambda$$

It can be noted that the $\frac{V_E}{R}\tan\lambda$ *increases rapidly at high altitudes and become infinite at the poles* $\lambda = 90^\circ$

Acceleration Correction Terms

- The NED frame is rotating with respect to an inertial frame and this introduces further complication in deriving the aircraft rate of change of velocity along the north east axes.
- Aircraft linear motion in a rotational axis frame introduces a acceleration name as Coriolis acceleration.
- Coriolis acceleration introduced by French mathematician who formulated the general principle who formulated the study of moving bodies in a rotational frame.
- Coriolis acceleration experienced by a moving body with a velocity V with respect to an axis frame which is rotating at an angular rate w is equal to 2VW and is mutually right angle to linear velocity and angular velocity vector.



Coriolis accelerations due to Earth's rotation

$$North\ axis = 2V_E\,\Omega\ \sin\lambda$$

$$East\ axis = -2V_N\,\Omega\ \sin\lambda - 2V_D\,\Omega\ \cos\lambda$$

$$Vertical\ axis = -2V_E\,\Omega\ \cos\lambda$$

CENTRIFUGAL ACCELERATION

- The aircraft is also turning in space as it flies over the surface of earth , because of the earth spherical in nature, it introduces a centrifugal acceleration.
- The centrifugal acceleration component along North, East and Down axes are.

$$North\ axis = \frac{\left(V_E \tan\lambda - V_D V_N\right)}{R}$$

$$East\ axis = \frac{-\left(V_N V_E \tan\lambda + V_D V_E\right)}{R}$$

$$Vertical\ axis = \frac{\left(V_N{}^2 + V_E{}^2\right)}{R}$$

GRAVITATIONAL ACCELERATION

- The Down axis (or vertical) axis accelerometer measures the gravitational acceleration of the aircraft, so it is necessary to correct the output for the gravity acceleration g.
- The gravity acceleration reduces as the altitude increases and follows an inverse square law. The value of g at an altitude, H is given by

$$g = \frac{R_0^2}{\left(R_0 + H\right)^2} \cdot g_0$$

g_0 is the gravitational acceleration at the earth surface

- The local value of the gravitational acceleration g0 varies by a small amount with latitude. This is because of centrifugal acceleration created by earth rate rotation .

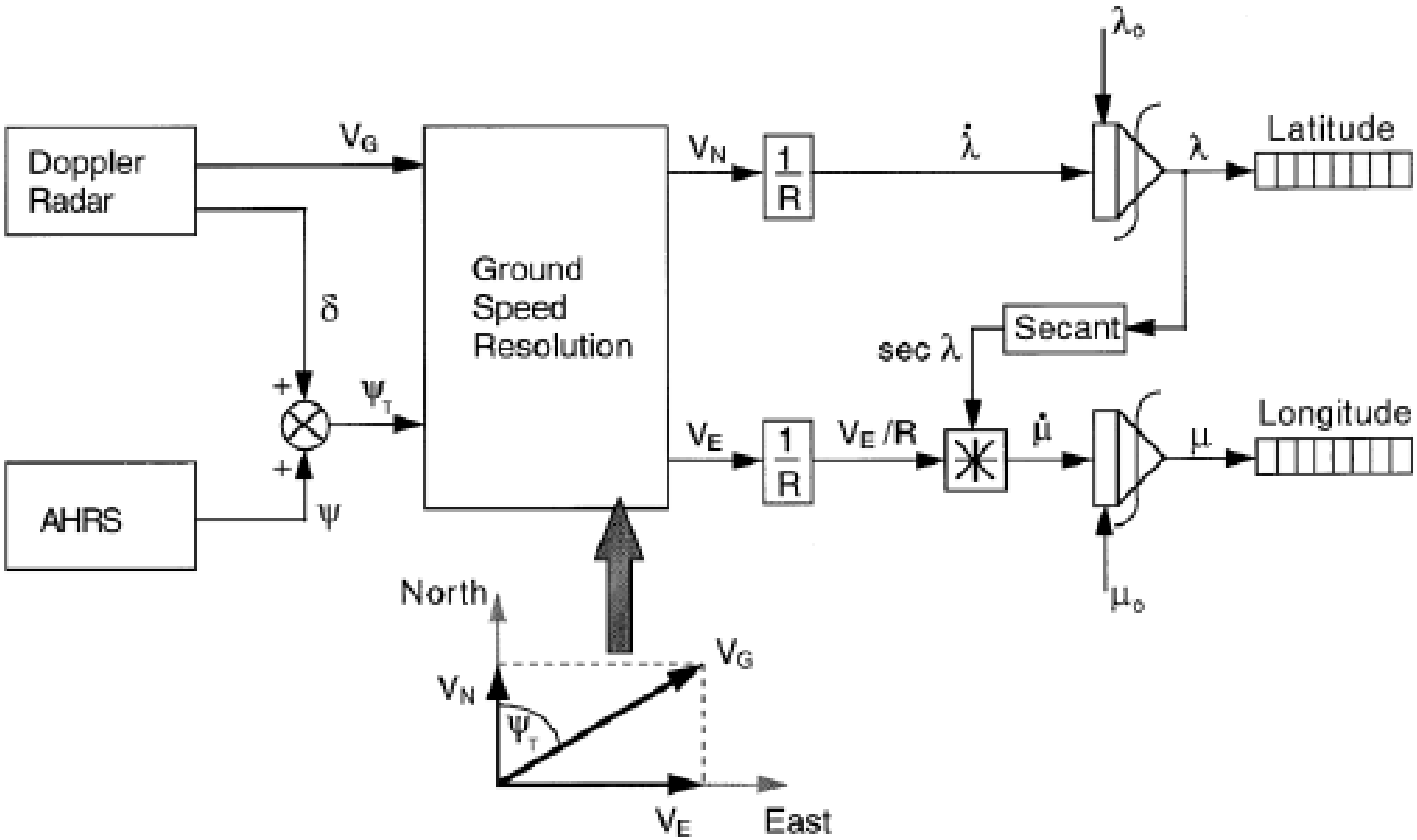
$$g = gequ\left(\frac{1 + k\sin^2\lambda}{\left(1 - e^2\sin^2\lambda\right)^{\frac{1}{2}}}\right)$$

$$gequ = 9.7803267714 m / s^2$$

$$k = 0.00193185138639$$

$$e^2 = 0.00669437999013$$

DEAD RECKONING NAVIGATION BY USING DOPPLER/HEADING REFERENCE SYSTEM



Doppler/heading Reference DR Navigation System

- *The ground speed V_G and drift angle δ are measured directly by doppler radar velocity sensor system*
- *The heading angle ψ measured directly from AHRS*

- The primary function of a Doppler radar is to continuously determine the velocity vector of an aircraft with respect to the ground.
- The measured velocity vector is first subjected to suitable coordinate frame and then to find the velocity vector components about north east and down components.
- The velocity vector is determined by measuring the Doppler shift of microwave signal transmitted from an aircraft at an angle steep angles, backscattered by a surface and received by a Doppler radar receiver.
- The track angle (heading) of an aircraft has been obtained from Inertial Measurement unit.

The track angle ψ_T *can be obtained from*

$$\psi_T = \psi + \delta$$

The northerly velocity component of aircraft V_N *and easterly velocity component* V_E *are driven by resolution of Ground speed* V_G

$$V_N = V_G \cos \psi_T$$

$$V_E = V_G \sin \psi_T$$

DEAD RECKONING NAVIGATION USING AIR DATA SYSTEM


MPXV7002DP

- *The Horizontal velocity component* V_H *of the true airspeed* V_T *is obtained by resolving* V_T *through the aircraft pitch angle* θ

$$V_H = V_T \cos\theta$$

- *The northerly and easterly velocity component of the airspeed are then derived by resolving* V_H *of the true airspeed* V_T *is obtained by resolving* V_T *through the aircraft pitch angle* θ

$$Northerly\ Airspeed = V_H \cos\psi$$
$$Easterly\ Airspeed = V_H \sin\psi$$

- *The forecast(or)estimated wind speed* V_M *and direction* ψ_W *is resolved into northerly and easterly components*

$$\text{Northerly Wind component} = V_W \cos\psi_W$$
$$\text{Easterly Wind component} = V_W \sin\psi_W$$

The northerly and easterly velocity component of the aircraft are

$$V_N = V_H \cos\psi + V_W \cos\psi_W$$
$$V_E = V_H \sin\psi + V_W \sin\psi_W$$

THANK YOU