

# Chapter 4



## DECISION ANALYSIS

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Decision analysis can be used to determine an optimal strategy when a decision maker is faced with several decision alternatives and an uncertain or risk-filled pattern of future events. For example, a global manufacturer might be interested in determining the best location for a new plant. Suppose that the manufacturer has identified five decision alternatives corresponding to five plant locations in different countries. Making the plant location decision is complicated by factors such as the world economy, demand in various regions of the world, labor availability, raw material costs, transportation costs, and so on. In such a problem, several scenarios could be developed to describe how the various factors combine to form the possible uncertain future events. Then probabilities can be assigned to the events. Using profit or cost as a measure of the consequence for each decision alternative and each future event combination, the best plant location can be selected.

Even when a careful decision analysis has been conducted, the uncertain future events make the final consequence uncertain. In some cases, the selected decision alternative may provide good or excellent results. In other cases, a relatively unlikely future event may occur causing the selected decision alternative to provide only fair or even poor results. The risk associated with any decision alternative is a direct result of the uncertainty associated with the final consequence. A good decision analysis

includes risk analysis. Through risk analysis the decision maker is provided with probability information about the favorable as well as the unfavorable consequences that may occur.

We begin the study of decision analysis by considering problems having reasonably few decision alternatives and reasonably few possible future events. Influence diagrams and payoff tables are introduced to provide a structure for the decision problem and to illustrate the fundamentals of decision analysis. We then introduce decision trees to show the sequential nature of decision problems. Decision trees are used to analyze more complex problems and to identify an optimal sequence of decisions, referred to as an optimal decision strategy. Sensitivity analysis shows how changes in various aspects of the problem affect the recommended decision alternative.

## 4.1 PROBLEM FORMULATION

The first step in the decision analysis process is problem formulation. We begin with a verbal statement of the problem. We then identify the decision alternatives, the uncertain future events, referred to as **chance events**, and the **consequences** associated with each decision alternative and each chance event outcome. Let us begin by considering a construction project of the Pittsburgh Development Corporation.

Pittsburgh Development Corporation (PDC) has purchased land, which will be the site of a new luxury condominium complex. The location provides a spectacular view of downtown Pittsburgh and the Golden Triangle where the Allegheny and Monongahela rivers meet to form the Ohio River. PDC plans to price the individual condominium units between \$300,000 and \$1,400,000.

PDC has preliminary architectural drawings for three different-sized projects: one with 30 condominiums, one with 60 condominiums, and one with 90 condominiums. The financial success of the project depends upon the size of the condominium complex and the chance event concerning the demand for the condominiums. The statement of the PDC decision problem is to select the size of the new luxury condominium project that will lead to the largest profit given the uncertainty concerning the demand for the condominiums.

Given the statement of the problem, it is clear that the decision is to select the best size for the condominium complex. PDC has the following three decision alternatives:

$d_1$  = a small complex with 30 condominiums

$d_2$  = a medium complex with 60 condominiums

$d_3$  = a large complex with 90 condominiums

A factor in selecting the best decision alternative is the uncertainty associated with the chance event concerning the demand for the condominiums. When asked about the possible demand for the condominiums, PDC's president acknowledged a wide range of possibilities, but decided that it would be adequate to consider two possible chance event outcomes: a strong demand and a weak demand.

In decision analysis, the possible outcomes for a chance event are referred to as the **states of nature**. The states of nature are defined so that one and only one of the possible states of nature will occur. For the PDC problem, the chance event concerning the demand for the condominiums has two states of nature:

$s_1$  = strong demand for the condominiums

$s_2$  = weak demand for the condominiums

Thus, management must first select a decision alternative (complex size), then a state of nature follows (demand for the condominiums), and finally a consequence will occur. In this case, the consequence is the PDC's profit.

## Influence Diagrams

An **influence diagram** is a graphical device that shows the relationships among the decisions, the chance events, and the consequences for a decision problem. The **nodes** in an influence diagram are used to represent the decisions, chance events, and consequences. Rectangles or squares are used to depict **decision nodes**, circles or ovals are used to depict **chance nodes**, and diamonds are used to depict consequence nodes. The lines connecting the nodes, referred to as **arcs**, show the direction of influence that the nodes have on one another. Figure 4.1 shows the influence diagram for the PDC problem. The complex size is the decision node, demand is the chance node, and profit is the consequence node. The arcs connecting the nodes show that both the complex size and the demand influence PDC's profit.

## Payoff Tables

Given the three decision alternatives and the two states of nature, which complex size should PDC choose? To answer this question, PDC will need to know the consequence associated with each decision alternative and each state of nature. In decision analysis, we refer to the consequence resulting from a specific combination of a decision alternative and a state of nature as a **payoff**. A table showing payoffs for all combinations of decision alternatives and states of nature is a **payoff table**.

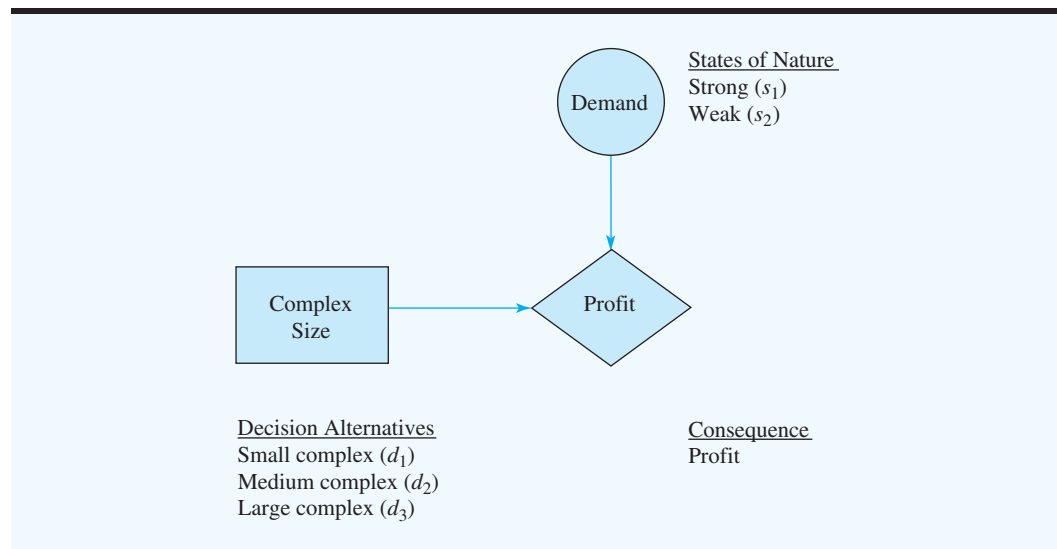
*Payoffs can be expressed in terms of profit, cost, time, distance, or any other measure appropriate for the decision problem being analyzed.*

Because PDC wants to select the complex size that provides the largest profit, profit is used as the consequence. The payoff table with profits expressed in millions of dollars is shown in Table 4.1. Note, for example, that if a medium complex is built and demand turns out to be strong, a profit of \$14 million will be realized. We will use the notation  $V_{ij}$  to denote the payoff associated with decision alternative  $i$  and state of nature  $j$ . Using Table 4.1,  $V_{31} = 20$  indicates a payoff of \$20 million occurs if the decision is to build a large complex ( $d_3$ ) and the strong demand state of nature ( $s_1$ ) occurs. Similarly,  $V_{32} = -9$  indicates a loss of \$9 million if the decision is to build a large complex ( $d_3$ ) and the weak demand state of nature ( $s_2$ ) occurs.

## Decision Trees

A **decision tree** provides a graphical representation of the decision-making process. Figure 4.2 presents a decision tree for the PDC problem. Note that the decision tree shows the natu-

**FIGURE 4.1** INFLUENCE DIAGRAM FOR THE PDC PROBLEM



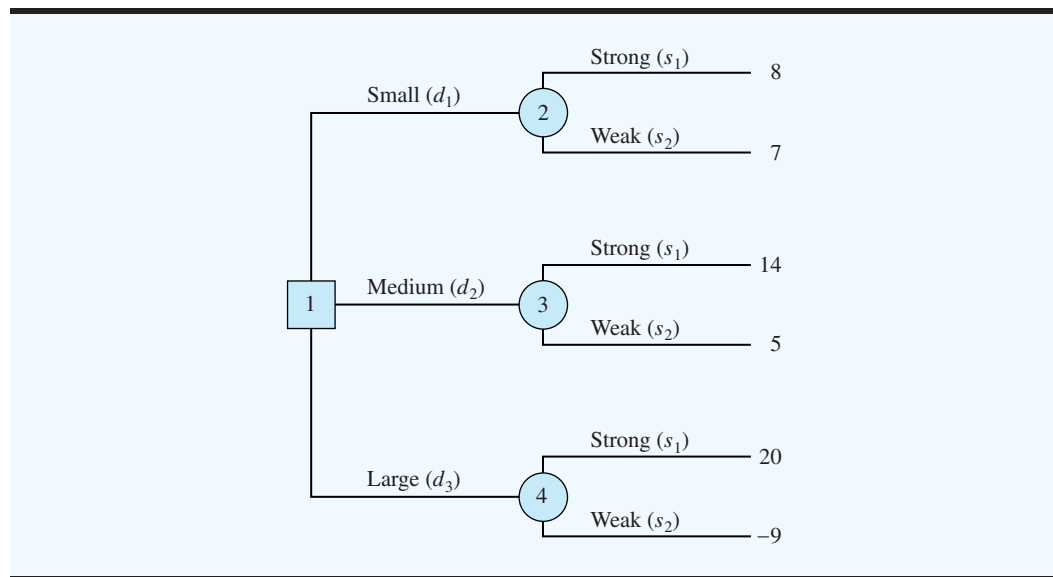
**TABLE 4.1** PAYOFF TABLE FOR THE PDC CONDOMINIUM PROJECT  
(PAYOFFS IN \$ MILLION)

Decision Alternative	State of Nature	
	Strong Demand $s_1$	Weak Demand $s_2$
Small complex, $d_1$	8	7
Medium complex, $d_2$	14	5
Large complex, $d_3$	20	-9

ral or logical progression that will occur over time. First, PDC must make a decision regarding the size of the condominium complex ( $d_1$ ,  $d_2$ , or  $d_3$ ). Then, after the decision is implemented, either state of nature  $s_1$  or  $s_2$  will occur. The number at each end point of the tree indicates the payoff associated with a particular sequence. For example the topmost payoff of 8 indicates that an \$8 million profit is anticipated if PDC constructs a small condominium complex ( $d_1$ ) and demand turns out to be strong ( $s_1$ ). The next payoff of 7 indicates an anticipated profit of \$7 million if PDC constructs a small condominium complex ( $d_1$ ) and demand turns out to be weak ( $s_2$ ). Thus, the decision tree shows graphically the sequences of decision alternatives and states of nature that provide the six possible payoffs for PDC.

*If you have a payoff table, you can develop a decision tree. Try Problem 1(a).*

The decision tree in Figure 4.2 has four nodes, numbered 1–4. Squares are used to depict decision nodes and circles are used to depict chance nodes. Thus, node 1 is a decision node, and nodes 2, 3, and 4 are chance nodes. The branches, which connect the nodes, leaving the decision node correspond to the decision alternatives. The branches leaving each chance node correspond to the states of nature. The payoffs are shown at the end of the states-of-nature branches. We now turn to the question: How can the decision maker use the information in the payoff table or the decision tree to select the best decision alternative? Several approaches may be used.

**FIGURE 4.2** DECISION TREE FOR THE PDC CONDOMINIUM PROJECT  
(PAYOFFS IN \$ MILLION)

## NOTES AND COMMENTS

1. Experts in problem solving agree that the first step in solving a complex problem is to decompose it into a series of smaller subproblems. Decision trees provide a useful way to show how a problem can be decomposed and the sequential nature of the decision process.
2. People often view the same problem from different perspectives. Thus, the discussion regarding the development of a decision tree may provide additional insight about the problem.

## 4.2 DECISION MAKING WITHOUT PROBABILITIES

*Many people think of a good decision as one in which the consequence is good. However, in some instances, a good, well-thought-out decision may still lead to a bad or undesirable consequence.*

In this section we consider approaches to decision making that do not require knowledge of the probabilities of the states of nature. These approaches are appropriate in situations in which the decision maker has little confidence in his or her ability to assess the probabilities, or in which a simple best-case and worst-case analysis is desirable. Because different approaches sometimes lead to different decision recommendations, the decision maker needs to understand the approaches available and then select the specific approach that, according to the decision maker's judgment, is the most appropriate.

## Optimistic Approach

The **optimistic approach** evaluates each decision alternative in terms of the *best* payoff that can occur. The decision alternative that is recommended is the one that provides the best possible payoff. For a problem in which maximum profit is desired, as in the PDC problem, the optimistic approach would lead the decision maker to choose the alternative corresponding to the largest profit. For problems involving minimization, this approach leads to choosing the alternative with the smallest payoff.

To illustrate the optimistic approach, we use it to develop a recommendation for the PDC problem. First, we determine the maximum payoff for each decision alternative; then we select the decision alternative that provides the overall maximum payoff. These steps systematically identify the decision alternative that provides the largest possible profit. Table 4.2 illustrates these steps.

Because 20, corresponding to  $d_3$ , is the largest payoff, the decision to construct the large condominium complex is the recommended decision alternative using the optimistic approach.

## Conservative Approach

The **conservative approach** evaluates each decision alternative in terms of the *worst* payoff that can occur. The decision alternative recommended is the one that provides the best of the worst possible payoffs. For a problem in which the output measure is profit, as in the PDC prob-

*For a maximization problem, the optimistic approach often is referred to as the maximax approach; for a minimization problem, the corresponding terminology is minimin.*

TABLE 4.2 MAXIMUM PAYOFF FOR EACH PDC DECISION ALTERNATIVE

Decision Alternative	Maximum Payoff	
Small complex, $d_1$	8	
Medium complex, $d_2$	14	
Large complex, $d_3$	20	← Maximum of the maximum payoff values

*For a maximization problem, the conservative approach is often referred to as the maximin approach; for a minimization problem, the corresponding terminology is minimax.*

lem, the conservative approach would lead the decision maker to choose the alternative that maximizes the minimum possible profit that could be obtained. For problems involving minimization, this approach identifies the alternative that will minimize the maximum payoff.

To illustrate the conservative approach, we use it to develop a recommendation for the PDC problem. First, we identify the minimum payoff for each of the decision alternatives; then we select the decision alternative that maximizes the minimum payoff. Table 4.3 illustrates these steps for the PDC problem.

Because 7, corresponding to  $d_1$ , yields the maximum of the minimum payoffs, the decision alternative of a small condominium complex is recommended. This decision approach is considered conservative because it identifies the worst possible payoffs and then recommends the decision alternative that avoids the possibility of extremely “bad” payoffs. In the conservative approach, PDC is guaranteed a profit of at least \$7 million. Although PDC may make more, it *cannot* make less than \$7 million.

### Minimax Regret Approach

**Minimax regret** is an approach to decision making that is neither purely optimistic nor purely conservative. Let us illustrate the minimax regret approach by showing how it can be used to select a decision alternative for the PDC problem.

Suppose that the PDC constructs a small condominium complex ( $d_1$ ) and demand turns out to be strong ( $s_1$ ). Table 4.1 shows that the resulting profit for PDC would be \$8 million. However, given that the strong demand state of nature ( $s_1$ ) has occurred, we realize that the decision to construct a large condominium complex ( $d_3$ ), yielding a profit of \$20 million, would have been the best decision. The difference between the payoff for the best decision alternative (\$20 million) and the payoff for the decision to construct a small condominium complex (\$8 million) is the **opportunity loss**, or **regret**, associated with decision alternative  $d_1$  when state of nature  $s_1$  occurs; thus, for this case, the opportunity loss or regret is \$20 million – \$8 million = \$12 million. Similarly, if PDC makes the decision to construct a medium condominium complex ( $d_2$ ) and the strong demand state of nature ( $s_1$ ) occurs, the opportunity loss, or regret, associated with  $d_2$  would be \$20 million – \$14 million = \$6 million.

In general the following expression represents the opportunity loss, or regret.

$$R_{ij} = |V_j^* - V_{ij}| \quad (4.1)$$

where

$R_{ij}$  = the regret associated with decision alternative  $d_i$  and state of nature  $s_j$

$V_j^*$  = the payoff value<sup>1</sup> corresponding to the best decision for the state of nature  $s_j$

$V_{ij}$  = the payoff corresponding to decision alternative  $d_i$  and state of nature  $s_j$

**TABLE 4.3** MINIMUM PAYOFF FOR EACH PDC DECISION ALTERNATIVE

Decision Alternative	Minimum Payoff	
Small complex, $d_1$	7	← Maximum of the minimum payoff values
Medium complex, $d_2$	5	
Large complex, $d_3$	–9	

<sup>1</sup>In maximization problems,  $V_j^*$  will be the largest entry in column  $j$  of the payoff table. In minimization problems,  $V_j^*$  will be the smallest entry in column  $j$  of the payoff table.

**TABLE 4.4** OPPORTUNITY LOSS, OR REGRET, TABLE FOR THE PDC CONDOMINIUM PROJECT (\$ MILLION)

Decision Alternative	State of Nature	
	Strong Demand $s_1$	Weak Demand $s_2$
Small complex, $d_1$	12	0
Medium complex, $d_2$	6	2
Large complex, $d_3$	0	16

**TABLE 4.5** MAXIMUM REGRET FOR EACH PDC DECISION ALTERNATIVE

Decision Alternative	Maximum Regret	
Small complex, $d_1$	12	
Medium complex, $d_2$	6	← Minimum of the maximum regret
Large complex, $d_3$	16	

Note the role of the absolute value in equation (4.1). That is, for minimization problems, the best payoff,  $V_j^*$ , is the smallest entry in column  $j$ . Because this value always is less than or equal to  $V_{ij}$ , the absolute value of the difference between  $V_j^*$  and  $V_{ij}$  ensures that the regret is always the magnitude of the difference.

Using equation (4.1) and the payoffs in Table 4.1, we can compute the regret associated with each combination of decision alternative  $d_i$  and state of nature  $s_j$ . Because the PDC problem is a maximization problem,  $V_j^*$  will be the largest entry in column  $j$  of the payoff table. Thus, to compute the regret, we simply subtract each entry in a column from the largest entry in the column. Table 4.4 shows the opportunity loss, or regret, table for the PDC problem.

The next step in applying the minimax regret approach is to list the maximum regret for each decision alternative; Table 4.5 shows the results for the PDC problem. Selecting the decision alternative with the *minimum* of the *maximum* regret values—hence, the name *minimax regret*—yields the minimax regret decision. For the PDC problem, the alternative to construct the medium condominium complex, with a corresponding maximum regret of \$6 million, is the recommended minimax regret decision.

Note that the three approaches discussed in this section provide different recommendations, which in itself isn't bad. It simply reflects the difference in decision-making philosophies that underlie the various approaches. Ultimately, the decision maker will have to choose the most appropriate approach and then make the final decision accordingly. The main criticism of the approaches discussed in this section is that they do not consider any information about the probabilities of the various states of nature. In the next section we discuss an approach that utilizes probability information in selecting a decision alternative.

*For practice in developing a decision recommendation using the optimistic, conservative, and minimax regret approaches, try Problem 1(b).*

### 4.3 DECISION MAKING WITH PROBABILITIES

In many decision-making situations, we can obtain probability assessments for the states of nature. When such probabilities are available, we can use the **expected value approach** to



identify the best decision alternative. Let us first define the expected value of a decision alternative and then apply it to the PDC problem.

Let

$N$  = the number of states of nature

$P(s_j)$  = the probability of state of nature  $s_j$

Because one and only one of the  $N$  states of nature can occur, the probabilities must satisfy two conditions:

$$P(s_j) \geq 0 \quad \text{for all states of nature} \quad (4.2)$$

$$\sum_{j=1}^N P(s_j) = P(s_1) + P(s_2) + \cdots + P(s_N) = 1 \quad (4.3)$$

The **expected value (EV)** of decision alternative  $d_i$  is defined as follows.

$$EV(d_i) = \sum_{j=1}^N P(s_j)V_{ij} \quad (4.4)$$

In words, the expected value of a decision alternative is the sum of weighted payoffs for the decision alternative. The weight for a payoff is the probability of the associated state of nature and therefore the probability that the payoff will occur. Let us return to the PDC problem to see how the expected value approach can be applied.

PDC is optimistic about the potential for the luxury high-rise condominium complex. Suppose that this optimism leads to an initial subjective probability assessment of 0.8 that demand will be strong ( $s_1$ ) and a corresponding probability of 0.2 that demand will be weak ( $s_2$ ). Thus,  $P(s_1) = 0.8$  and  $P(s_2) = 0.2$ . Using the payoff values in Table 4.1 and equation (4.4), we compute the expected value for each of the three decision alternatives as follows:

$$EV(d_1) = 0.8(8) + 0.2(7) = 7.8$$

$$EV(d_2) = 0.8(14) + 0.2(5) = 12.2$$

$$EV(d_3) = 0.8(20) + 0.2(-9) = 14.2$$

Thus, using the expected value approach, we find that the large condominium complex, with an expected value of \$14.2 million, is the recommended decision.

*Can you now use the expected value approach to develop a decision recommendation? Try Problem 5.*

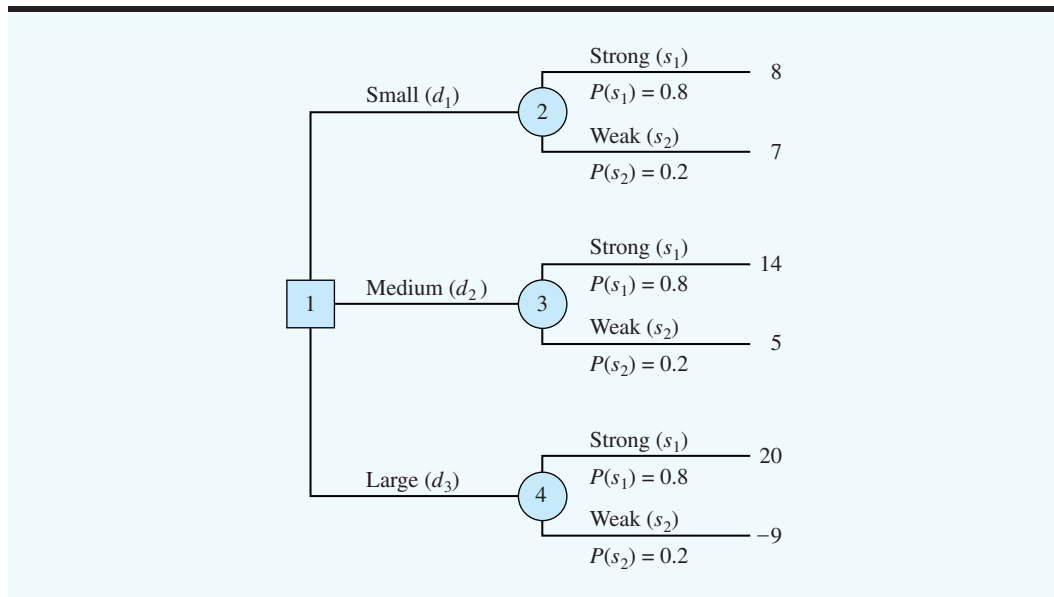
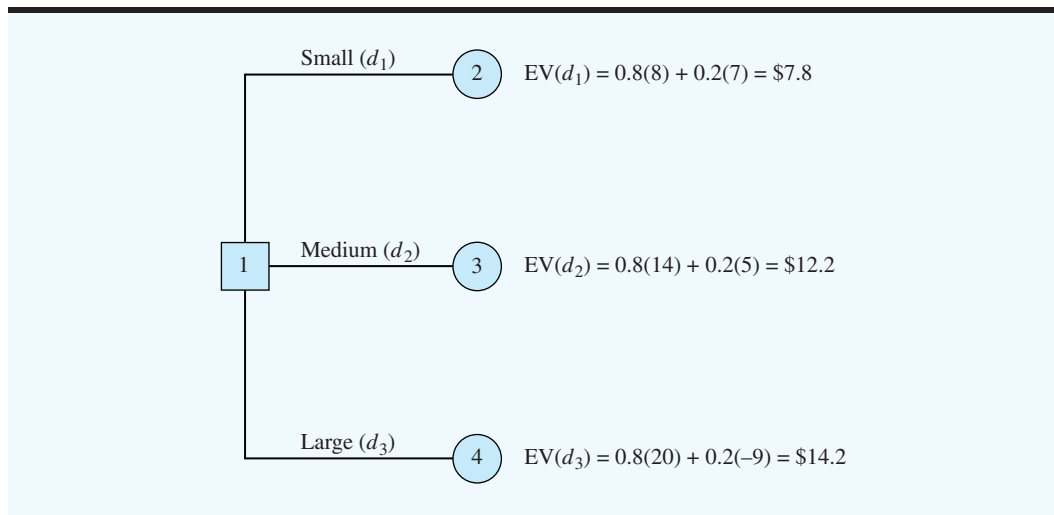
The calculations required to identify the decision alternative with the best expected value can be conveniently carried out on a decision tree. Figure 4.3 shows the decision tree for the PDC problem with state-of-nature branch probabilities. Working backward through the decision tree, we first compute the expected value at each chance node. That is, at each chance node, we weight each possible payoff by its probability of occurrence. By doing so, we obtain the expected values for nodes 2, 3, and 4, as shown in Figure 4.4.

Because the decision maker controls the branch leaving decision node 1 and because we are trying to maximize the expected profit, the best decision alternative at node 1 is  $d_3$ . Thus, the decision tree analysis leads to a recommendation of  $d_3$  with an expected value of \$14.2 million. Note that this recommendation is also obtained with the expected value approach in conjunction with the payoff table.

*Computer software packages are available to help in constructing more complex decision trees.*

Other decision problems may be substantially more complex than the PDC problem, but if a reasonable number of decision alternatives and states of nature are present, you can use the decision tree approach outlined here. First, draw a decision tree consisting of decision nodes, chance nodes, and branches that describe the sequential nature of the problem. If you use the expected value approach, the next step is to determine the probabilities for



**FIGURE 4.3** PDC DECISION TREE WITH STATE-OF-NATURE BRANCH PROBABILITIES**FIGURE 4.4** APPLYING THE EXPECTED VALUE APPROACH USING DECISION TREES

each of the states of nature and compute the expected value at each chance node. Then select the decision branch leading to the chance node with the best expected value. The decision alternative associated with this branch is the recommended decision.

### Expected Value of Perfect Information

Suppose that PDC has the opportunity to conduct a market research study that would help evaluate buyer interest in the condominium project and provide information that management could use to improve the probability assessments for the states of nature. To determine the potential value of this information, we begin by supposing that the study could provide *perfect information* regarding the states of nature; that is, we assume for the mo-

ment that PDC could determine with certainty, prior to making a decision, which state of nature is going to occur. To make use of this perfect information, we will develop a decision strategy that PDC should follow once it knows which state of nature will occur. A decision strategy is simply a decision rule that specifies the decision alternative to be selected after new information becomes available.

To help determine the decision strategy for PDC, we have reproduced PDC's payoff table as Table 4.6. Note that, if PDC knew for sure that state of nature  $s_1$  would occur, the best decision alternative would be  $d_3$ , with a payoff of \$20 million. Similarly, if PDC knew for sure that state of nature  $s_2$  would occur, the best decision alternative would be  $d_1$ , with a payoff of \$7 million. Thus, we can state PDC's optimal decision strategy when the perfect information becomes available as follows:

If  $s_1$ , select  $d_3$  and receive a payoff of \$20 million.

If  $s_2$ , select  $d_1$  and receive a payoff of \$7 million.

What is the expected value for this decision strategy? To compute the expected value with perfect information, we return to the original probabilities for the states of nature:  $P(s_1) = 0.8$ , and  $P(s_2) = 0.2$ . Thus, there is a 0.8 probability that the perfect information will indicate state of nature  $s_1$  and the resulting decision alternative  $d_3$  will provide a \$20 million profit. Similarly, with a 0.2 probability for state of nature  $s_2$ , the optimal decision alternative  $d_1$  will provide a \$7 million profit. Thus, from equation (4.4), the expected value of the decision strategy that uses perfect information is

$$0.8(20) + 0.2(7) = 17.4$$

We refer to the expected value of \$17.4 million as the *expected value with perfect information* (EVwPI).

Earlier in this section we showed that the recommended decision using the expected value approach is decision alternative  $d_3$ , with an expected value of \$14.2 million. Because this decision recommendation and expected value computation were made without the benefit of perfect information, \$14.2 million is referred to as the *expected value without perfect information* (EVwoPI).

The expected value with perfect information is \$17.4 million, and the expected value without perfect information is \$14.2; therefore, the expected value of the perfect information (EVPI) is  $\$17.4 - \$14.2 = \$3.2$  million. In other words, \$3.2 million represents the additional expected value that can be obtained if perfect information were available about the states of nature.

Generally speaking, a market research study will not provide "perfect" information; however, if the market research study is a good one, the information gathered might be worth a sizable portion of the \$3.2 million. Given the EVPI of \$3.2 million, PDC should seriously consider the market survey as a way to obtain more information about the states of nature.

*It would be worth \$3.2 million for PDC to learn the level of market acceptance before selecting a decision alternative.*

**TABLE 4.6** PAYOFF TABLE FOR THE PDC CONDOMINIUM PROJECT (\$ MILLION)

Decision Alternative	State of Nature	
	Strong Demand $s_1$	Weak Demand $s_2$
Small complex, $d_1$	8	7
Medium complex, $d_2$	14	5
Large complex, $d_3$	20	-9

In general, the **expected value of perfect information** is computed as follows:

$$EVPI = |EVwPI - EVwoPI| \quad (4.5)$$

where

EVPI = expected value of perfect information

EVwPI = expected value *with* perfect information about the states of nature

EVwoPI = expected value *without* perfect information about the states of nature

*For practice in determining the expected value of perfect information, try Problem 14.*

Note the role of the absolute value in equation (4.5). For minimization problems the expected value with perfect information is always less than or equal to the expected value without perfect information. In this case, EVPI is the magnitude of the difference between EVwPI and EVwoPI, or the absolute value of the difference as shown in equation (4.5).

### NOTES AND COMMENTS

We restate the *opportunity loss*, or *regret*, table for the PDC problem (see Table 4.4) as follows.

	State of Nature	
	Strong Demand	Weak Demand
Decision Alternative	$s_1$	$s_2$
Small complex, $d_1$	12	0
Medium complex, $d_2$	6	2
Large complex, $d_3$	0	16

Using  $P(s_1)$ ,  $P(s_2)$ , and the opportunity loss values, we can compute the *expected opportunity loss* (EOL) for each decision alternative. With  $P(s_1) =$

0.8 and  $P(s_2) = 0.2$ , the expected opportunity loss for each of the three decision alternatives is

$$EOL(d_1) = 0.8(12) + 0.2(0) = 9.6$$

$$EOL(d_2) = 0.8(6) + 0.2(2) = 5.2$$

$$EOL(d_3) = 0.8(0) + 0.2(16) = 3.2$$

Regardless of whether the decision analysis involves maximization or minimization, the *minimum* expected opportunity loss always provides the best decision alternative. Thus, with  $EOL(d_3) = 3.2$ ,  $d_3$  is the recommended decision. In addition, the minimum expected opportunity loss always is *equal to the expected value of perfect information*. That is,  $EOL(\text{best decision}) = EVPI$ ; for the PDC problem, this value is \$3.2 million.

## 4.4 RISK ANALYSIS AND SENSITIVITY ANALYSIS

In this section, we introduce risk analysis and sensitivity analysis. **Risk analysis** can be used to provide probabilities for the payoffs associated with a decision alternative. As a result, risk analysis helps the decision maker recognize the difference between the expected value of a decision alternative and the payoff that may actually occur. **Sensitivity analysis** also helps the decision maker by describing how changes in the state-of-nature probabilities and/or changes in the payoffs affect the recommended decision alternative.

### Risk Analysis

A decision alternative and a state of nature combine to generate the payoff associated with a decision. The **risk profile** for a decision alternative shows the possible payoffs along with their associated probabilities.

Let us demonstrate risk analysis and the construction of a risk profile by returning to the PDC condominium construction project. Using the expected value approach, we identified the large condominium complex ( $d_3$ ) as the best decision alternative. The expected

value of \$14.2 million for  $d_3$  is based on a 0.8 probability of obtaining a \$20 million profit and a 0.2 probability of obtaining a \$9 million loss. The 0.8 probability for the \$20 million payoff and the 0.2 probability for the -\$9 million payoff provide the risk profile for the large complex decision alternative. This risk profile is shown graphically in Figure 4.5.

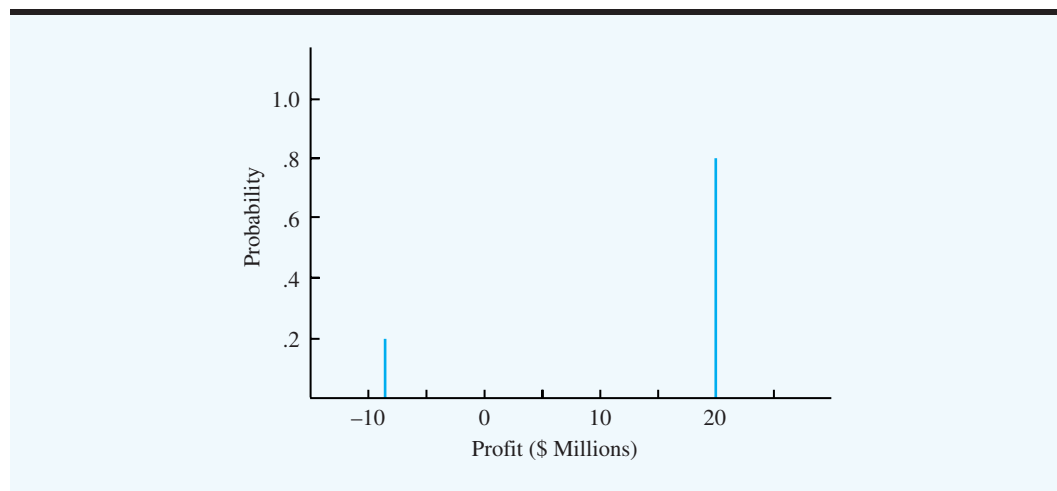
Sometimes a review of the risk profile associated with an optimal decision alternative may cause the decision maker to choose another decision alternative even though the expected value of the other decision alternative is not as good. For example, the risk profile for the medium complex decision alternative ( $d_2$ ) shows a 0.8 probability for a \$14 million payoff and 0.2 probability for a \$5 million payoff. Because no probability of a loss is associated with decision alternative  $d_2$ , the medium complex decision alternative would be judged less risky than the large complex decision alternative. As a result, a decision maker might prefer the less-risky medium complex decision alternative even though it has an expected value of \$2 million less than the large complex decision alternative.

### Sensitivity Analysis

Sensitivity analysis can be used to determine how changes in the probabilities for the states of nature and/or changes in the payoffs affect the recommended decision alternative. In many cases, the probabilities for the states of nature and the payoffs are based on subjective assessments. Sensitivity analysis helps the decision maker understand which of these inputs are critical to the choice of the best decision alternative. If a small change in the value of one of the inputs causes a change in the recommended decision alternative, the solution to the decision analysis problem is sensitive to that particular input. Extra effort and care should be taken to make sure the input value is as accurate as possible. On the other hand, if a modest to large change in the value of one of the inputs does not cause a change in the recommended decision alternative, the solution to the decision analysis problem is not sensitive to that particular input. No extra time or effort would be needed to refine the estimated input value.

One approach to sensitivity analysis is to select different values for the probabilities of the states of nature and/or the payoffs and then resolve the decision analysis problem. If the recommended decision alternative changes, we know that the solution is sensitive to the changes made. For example, suppose that in the PDC problem the probability for a strong demand is revised to 0.2 and the probability for a weak demand is revised to 0.8. Would the

**FIGURE 4.5** RISK PROFILE FOR THE LARGE COMPLEX DECISION ALTERNATIVE FOR THE PDC CONDOMINIUM PROJECT



recommended decision alternative change? Using  $P(s_1) = 0.2$ ,  $P(s_2) = 0.8$ , and equation (4.4), the revised expected values for the three decision alternatives are

$$\begin{aligned}EV(d_1) &= 0.2(8) + 0.8(7) = 7.2 \\EV(d_2) &= 0.2(14) + 0.8(5) = 6.8 \\EV(d_3) &= 0.2(20) + 0.8(-9) = -3.2\end{aligned}$$

With these probability assessments the recommended decision alternative is to construct a small condominium complex ( $d_1$ ), with an expected value of \$7.2 million. The probability of strong demand is only 0.2, so constructing the large condominium complex ( $d_3$ ) is the least preferred alternative, with an expected value of  $-\$3.2$  million (a loss).

*Computer software packages for decision analysis, such as Precision Tree, make it easy to calculate these revised scenarios.*

Thus, when the probability of strong demand is large, PDC should build the large complex; when the probability of strong demand is small, PDC should build the small complex. Obviously, we could continue to modify the probabilities of the states of nature and learn even more about how changes in the probabilities affect the recommended decision alternative. The drawback to this approach is the numerous calculations required to evaluate the effect of several possible changes in the state-of-nature probabilities.

For the special case of two states of nature, a graphical procedure can be used to determine how changes for the probabilities of the states of nature affect the recommended decision alternative. To demonstrate this procedure, we let  $p$  denote the probability of state of nature  $s_1$ ; that is,  $P(s_1) = p$ . With only two states of nature in the PDC problem, the probability of state of nature  $s_2$  is

$$P(s_2) = 1 - P(s_1) = 1 - p$$

Using equation (4.4) and the payoff values in Table 4.1, we determine the expected value for decision alternative  $d_1$  as follows:

$$\begin{aligned}EV(d_1) &= P(s_1)(8) + P(s_2)(7) \\&= p(8) + (1 - p)(7) \\&= 8p + 7 - 7p = p + 7\end{aligned}\tag{4.6}$$

Repeating the expected value computations for decision alternatives  $d_2$  and  $d_3$ , we obtain expressions for the expected value of each decision alternative as a function of  $p$ :

$$EV(d_2) = 9p + 5\tag{4.7}$$

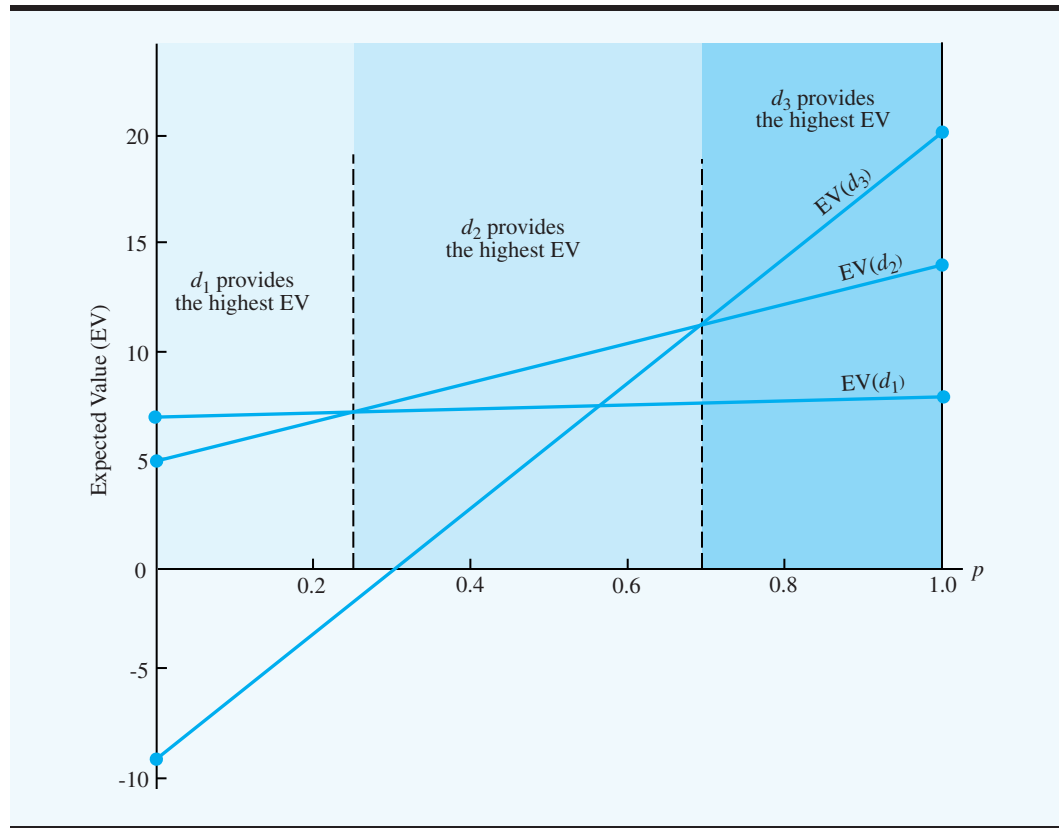
$$EV(d_3) = 29p - 9\tag{4.8}$$

Thus, we have developed three equations that show the expected value of the three decision alternatives as a function of the probability of state of nature  $s_1$ .

We continue by developing a graph with values of  $p$  on the horizontal axis and the associated EVs on the vertical axis. Because equations (4.6), (4.7), and (4.8) are linear equations, the graph of each equation is a straight line. For each equation, then, we can obtain the line by identifying two points that satisfy the equation and drawing a line through the points. For instance, if we let  $p = 0$  in equation (4.6),  $EV(d_1) = 7$ . Then, letting  $p = 1$ ,  $EV(d_1) = 8$ . Connecting these two points, (0, 7) and (1, 8), provides the line labeled  $EV(d_1)$  in Figure 4.6. Similarly, we obtain the lines labeled  $EV(d_2)$  and  $EV(d_3)$ ; these lines are the graphs of equations (4.7) and (4.8), respectively.

Figure 4.6 shows how the recommended decision changes as  $p$ , the probability of the strong demand state of nature ( $s_1$ ), changes. Note that for small values of  $p$ , decision al-

**FIGURE 4.6** EXPECTED VALUE FOR THE PDC DECISION ALTERNATIVES AS A FUNCTION OF  $p$



ternative  $d_1$  (small complex) provides the largest expected value and is thus the recommended decision. When the value of  $p$  increases to a certain point, decision alternative  $d_2$  (medium complex) provides the largest expected value and is the recommended decision. Finally, for large values of  $p$ , decision alternative  $d_3$  (large complex) becomes the recommended decision.

The value of  $p$  for which the expected values of  $d_1$  and  $d_2$  are equal is the value of  $p$  corresponding to the intersection of the  $EV(d_1)$  and the  $EV(d_2)$  lines. To determine this value, we set  $EV(d_1) = EV(d_2)$  and solve for the value of  $p$ :

$$\begin{aligned} p + 7 &= 9p + 5 \\ 8p &= 2 \\ p &= \frac{2}{8} = 0.25 \end{aligned}$$

Hence, when  $p = 0.25$ , decision alternatives  $d_1$  and  $d_2$  provide the same expected value. Repeating this calculation for the value of  $p$  corresponding to the intersection of the  $EV(d_2)$  and  $EV(d_3)$  lines we obtain  $p = 0.70$ .

Using Figure 4.6, we can conclude that decision alternative  $d_1$  provides the largest expected value for  $p \leq 0.25$ , decision alternative  $d_2$  provides the largest expected value for  $0.25 \leq p \leq 0.70$ , and decision alternative  $d_3$  provides the largest expected value for  $p \geq 0.70$ . Because  $p$  is the probability of state of nature  $s_1$  and  $(1 - p)$  is the probability of state

*Graphical sensitivity analysis shows how changes in the probabilities for the states of nature affect the recommended decision alternative. Try Problem 8.*

of nature  $s_2$ , we now have the sensitivity analysis information that tells us how changes in the state-of-nature probabilities affect the recommended decision alternative.

Sensitivity analysis calculations can also be made for the values of the payoffs. In the original PDC problem, the expected values for the three decision alternatives were as follows:  $EV(d_1) = 7.8$ ,  $EV(d_2) = 12.2$ , and  $EV(d_3) = 14.2$ . Decision alternative  $d_3$  (large complex) was recommended. Note that decision alternative  $d_2$  with  $EV(d_2) = 12.2$  was the second best decision alternative. Decision alternative  $d_3$  will remain the optimal decision alternative as long as  $EV(d_3)$  is greater than or equal to the expected value of the second best decision alternative. Thus, decision alternative  $d_3$  will remain the optimal decision alternative as long as

$$EV(d_3) \geq 12.2 \quad (4.9)$$

Let

$S$  = the payoff of decision alternative  $d_3$  when demand is strong

$W$  = the payoff of decision alternative  $d_3$  when demand is weak

Using  $P(s_1) = 0.8$  and  $P(s_2) = 0.2$ , the general expression for  $EV(d_3)$  is

$$EV(d_3) = 0.8S + 0.2W \quad (4.10)$$

Assuming that the payoff for  $d_3$  stays at its original value of  $-\$9$  million when demand is weak, the large complex decision alternative will remain the optimal as long as

$$EV(d_3) = 0.8S + 0.2(-9) \geq 12.2 \quad (4.11)$$

Solving for  $S$ , we have

$$0.8S - 1.8 \geq 12.2$$

$$0.8S \geq 14$$

$$S \geq 17.5$$

Recall that when demand is strong, decision alternative  $d_3$  has an estimated payoff of \$20 million. The preceding calculation shows that decision alternative  $d_3$  will remain optimal as long as the payoff for  $d_3$  when demand is strong is at least \$17.5 million.

Assuming that the payoff for  $d_3$  stays at its original value of \$20 million, we can make a similar calculation to learn how sensitive the optimal solution is with regard to the payoff for  $d_3$  when demand is weak. Returning to the expected value calculation of equation (4.10), we know that the large complex decision alternative will remain optimal as long as

$$EV(d_3) = 0.8(20) + 0.2W \geq 12.2 \quad (4.12)$$

Solving for  $W$ , we have

$$16 + 0.2W \geq 12.2$$

$$0.2W \geq -3.8$$

$$W \geq -19$$

Recall that when demand is weak, decision alternative  $d_3$  has an estimated payoff of  $-\$9$  million. The preceding calculation shows that decision alternative  $d_3$  will remain optimal as long as the payoff for  $d_3$  when demand is weak is at least  $-\$19$  million.



*Sensitivity analysis can assist management in deciding whether more time and effort should be spent obtaining better estimates of payoffs and/or probabilities.*

Based on this sensitivity analysis, we conclude that the payoffs for the large complex decision alternative ( $d_3$ ) could vary considerably and  $d_3$  would remain the recommended decision alternative. Thus, we conclude that the optimal solution for the PDC decision problem is not particularly sensitive to the payoffs for the large complex decision alternative. We note, however, that this sensitivity analysis has been conducted based on only one change at a time. That is, only one payoff was changed and the probabilities for the states of nature remained  $P(s_1) = 0.8$  and  $P(s_2) = 0.2$ . Note that similar sensitivity analysis calculations can be made for the payoffs associated with the small complex decision alternative  $d_1$  and the medium complex decision alternative  $d_2$ . However, in these cases, decision alternative  $d_3$  remains optimal only if the changes in the payoffs for decision alternatives  $d_1$  and  $d_2$  meet the requirements that  $EV(d_1) \leq 14.2$  and  $EV(d_2) \leq 14.2$ .

### NOTES AND COMMENTS

1. Some decision analysis software automatically provide the risk profiles for the optimal decision alternative. These packages also allow the user to obtain the risk profiles for other decision alternatives. After comparing the risk profiles, a decision maker may decide to select a decision alternative with a good risk profile even though the expected value of the decision alternative is not as good as the optimal decision alternative.
2. A *tornado diagram*, a graphical display, is particularly helpful when several inputs combine to determine the value of the optimal solution. By varying each input over its range of values, we obtain information about how each input affects the value of the optimal solution. To display this information, a bar is constructed for the input with the width of the bar showing how the input affects the value of the optimal solution. The widest bar corresponds to the input that is most sensitive. The bars are arranged in a graph with the widest bar at the top, resulting in a graph that has the appearance of a tornado.

## 4.5 DECISION ANALYSIS WITH SAMPLE INFORMATION

In applying the expected value approach, we have shown how probability information about the states of nature affects the expected value calculations and thus the decision recommendation. Frequently, decision makers have preliminary or **prior probability** assessments for the states of nature that are the best probability values available at that time. However, to make the best possible decision, the decision maker may want to seek additional information about the states of nature. This new information can be used to revise or update the prior probabilities so that the final decision is based on more accurate probabilities for the states of nature. Most often, additional information is obtained through experiments designed to provide **sample information** about the states of nature. Raw material sampling, product testing, and market research studies are examples of experiments (or studies) that may enable management to revise or update the state-of-nature probabilities. These revised probabilities are called **posterior probabilities**.

Let us return to the PDC problem and assume that management is considering a six-month market research study designed to learn more about potential market acceptance of the PDC condominium project. Management anticipates that the market research study will provide one of the following two results:

1. Favorable report: A significant number of the individuals contacted express interest in purchasing a PDC condominium.
2. Unfavorable report: Very few of the individuals contacted express interest in purchasing a PDC condominium.

### An Influence Diagram

By introducing the possibility of conducting a market research study, the PDC problem becomes more complex. The influence diagram for the expanded PDC problem is shown in Figure 4.7. Note that the two decision nodes correspond to the research study and the complex-size decisions. The two chance nodes correspond to the research study results and demand for the condominiums. Finally, the consequence node is the profit. From the arcs of the influence diagram, we see that demand influences both the research study results and profit. Although demand is currently unknown to PDC, some level of demand for the condominiums already exists in the Pittsburgh area. If existing demand is strong, the research study is likely to find a significant number of individuals who express an interest in purchasing a condominium. However, if the existing demand is weak, the research study is more likely to find a significant number of individuals who express little interest in purchasing a condominium. In this sense, existing demand for the condominiums will influence the research study results. And clearly, demand will have an influence upon PDC's profit.

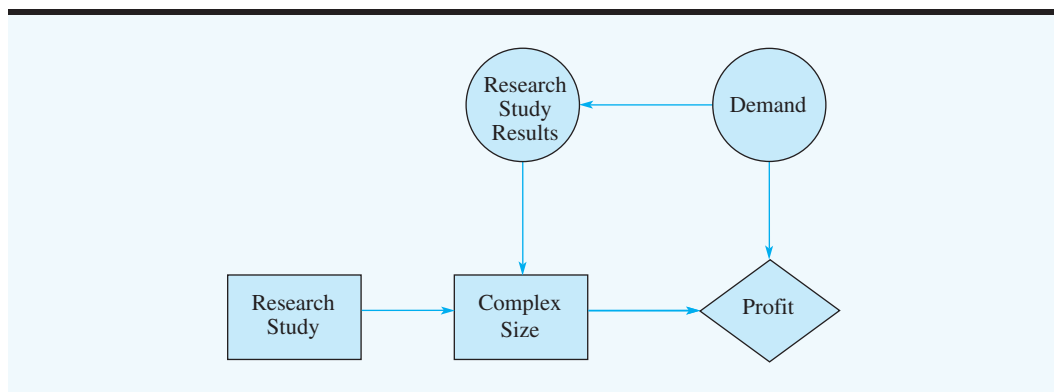
The arc from the research study decision node to the complex-size decision node indicates that the research study decision precedes the complex-size decision. No arc spans from the research study decision node to the research study results node, because the decision to conduct the research study does not actually influence the research study results. The decision to conduct the research study makes the research study results available, but it does not influence the results of the research study. Finally, the complex-size node and the demand node both influence profit. Note that if there were a stated cost to conduct the research study, the decision to conduct the research study would also influence profit. In such a case, we would need to add an arc from the research study decision node to the profit node to show the influence that the research study cost would have on profit.

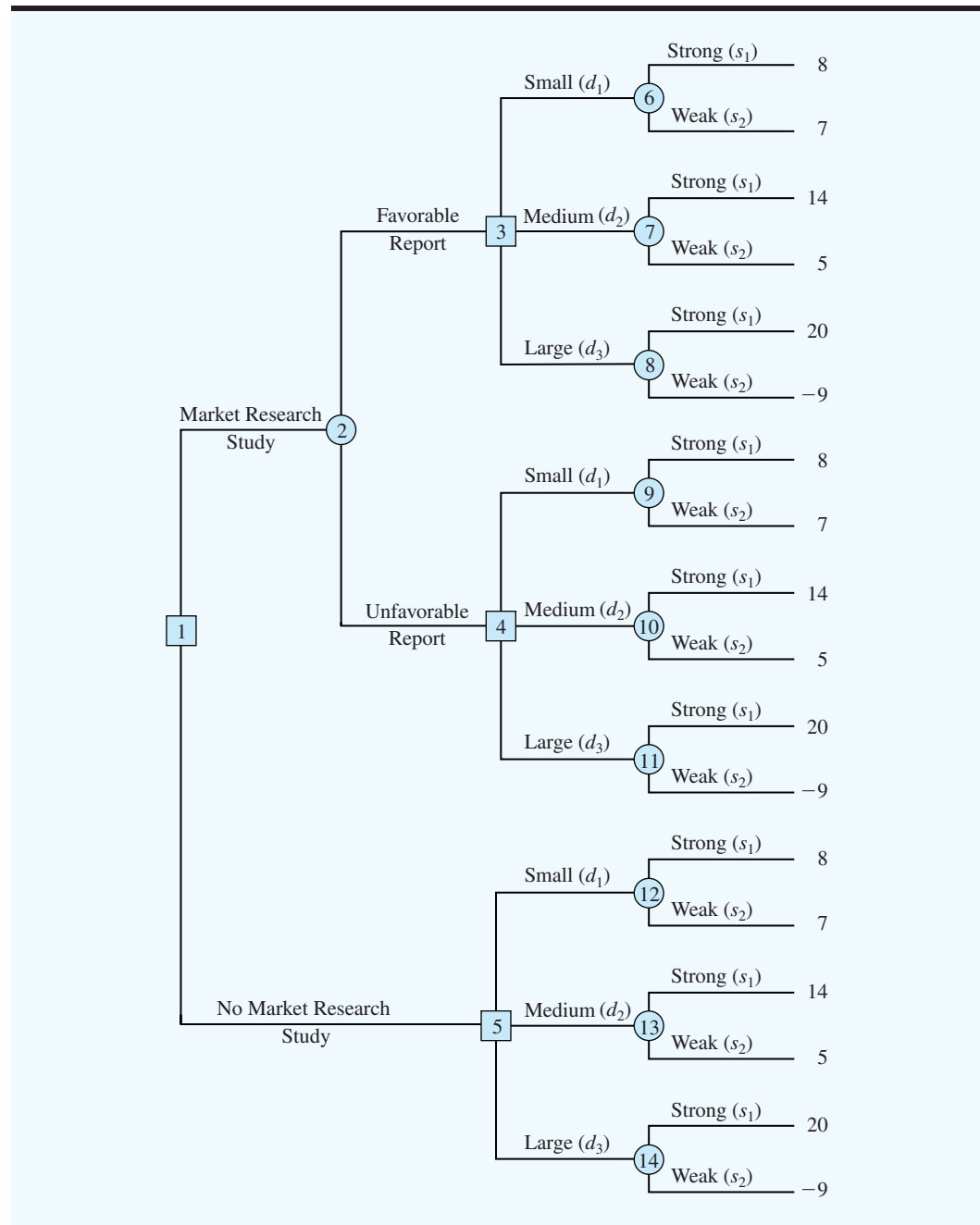
### A Decision Tree

The decision tree for the PDC problem with sample information shows the logical sequence for the decisions and the chance events.

First, PDC's management must decide whether the market research should be conducted. If it is conducted, PDC's management must be prepared to make a decision about the size of the condominium project if the market research report is favorable and, possibly, a different decision about the size of the condominium project if the market research report is unfavorable. The decision tree in Figure 4.8 shows this PDC decision problem. The squares are de-

**FIGURE 4.7** INFLUENCE DIAGRAM FOR THE PDC PROBLEM WITH SAMPLE INFORMATION



**FIGURE 4.8** THE PDC DECISION TREE INCLUDING THE MARKET RESEARCH STUDY

cision nodes and the circles are chance nodes. At each decision node, the branch of the tree that is taken is based on the decision made. At each chance node, the branch of the tree that is taken is based on probability or chance. For example, decision node 1 shows that PDC must first make the decision of whether to conduct the market research study. If the market research study is undertaken, chance node 2 indicates that both the favorable report branch and the unfavorable report branch are not under PDC's control and will be determined by chance. Node 3 is a decision node, indicating that PDC must make the decision to construct the small,

medium, or large complex if the market research report is favorable. Node 4 is a decision node showing that PDC must make the decision to construct the small, medium, or large complex if the market research report is unfavorable. Node 5 is a decision node indicating that PDC must make the decision to construct the small, medium, or large complex if the market research is not undertaken. Nodes 6 to 14 are chance nodes indicating that the strong demand or weak demand state-of-nature branches will be determined by chance.

*We explain in Section 4.6 how these probabilities can be developed.*

Analysis of the decision tree and the choice of an optimal strategy requires that we know the branch probabilities corresponding to all chance nodes. PDC has developed the following branch probabilities.

If the market research study is undertaken

$$P(\text{Favorable report}) = 0.77$$

$$P(\text{Unfavorable report}) = 0.23$$

If the market research report is favorable

$$P(\text{Strong demand given a Favorable report}) = 0.94$$

$$P(\text{Weak demand given a Favorable report}) = 0.06$$

If the market research report is unfavorable

$$P(\text{Strong demand given an Unfavorable report}) = 0.35$$

$$P(\text{Weak demand given an Unfavorable report}) = 0.65$$

If the market research report is not undertaken the prior probabilities are applicable.

$$P(\text{Strong demand}) = 0.80$$

$$P(\text{Weak demand}) = 0.20$$

The branch probabilities are shown on the decision tree in Figure 4.9.

## Decision Strategy

A **decision strategy** is a sequence of decisions and chance outcomes where the decisions chosen depend on the yet to be determined outcomes of chance events.

The approach used to determine the optimal **decision strategy** is based on a backward pass through the decision tree using the following steps:

1. At chance nodes, compute the expected value by multiplying the payoff at the end of each branch by the corresponding branch probabilities.
2. At decision nodes, select the decision branch that leads to the best expected value. This expected value becomes the expected value at the decision node.

Starting the backward pass calculations by computing the expected values at chance nodes 6 to 14 provides the following results.

$$\text{EV}(\text{Node 6}) = 0.94(8) + 0.06(7) = 7.94$$

$$\text{EV}(\text{Node 7}) = 0.94(14) + 0.06(5) = 13.46$$

$$\text{EV}(\text{Node 8}) = 0.94(20) + 0.06(-9) = 18.26$$

$$\text{EV}(\text{Node 9}) = 0.35(8) + 0.65(7) = 7.35$$

$$\text{EV}(\text{Node 10}) = 0.35(14) + 0.65(5) = 8.15$$

$$\text{EV}(\text{Node 11}) = 0.35(20) + 0.65(-9) = 1.15$$

$$\text{EV}(\text{Node 12}) = 0.80(8) + 0.20(7) = 7.80$$

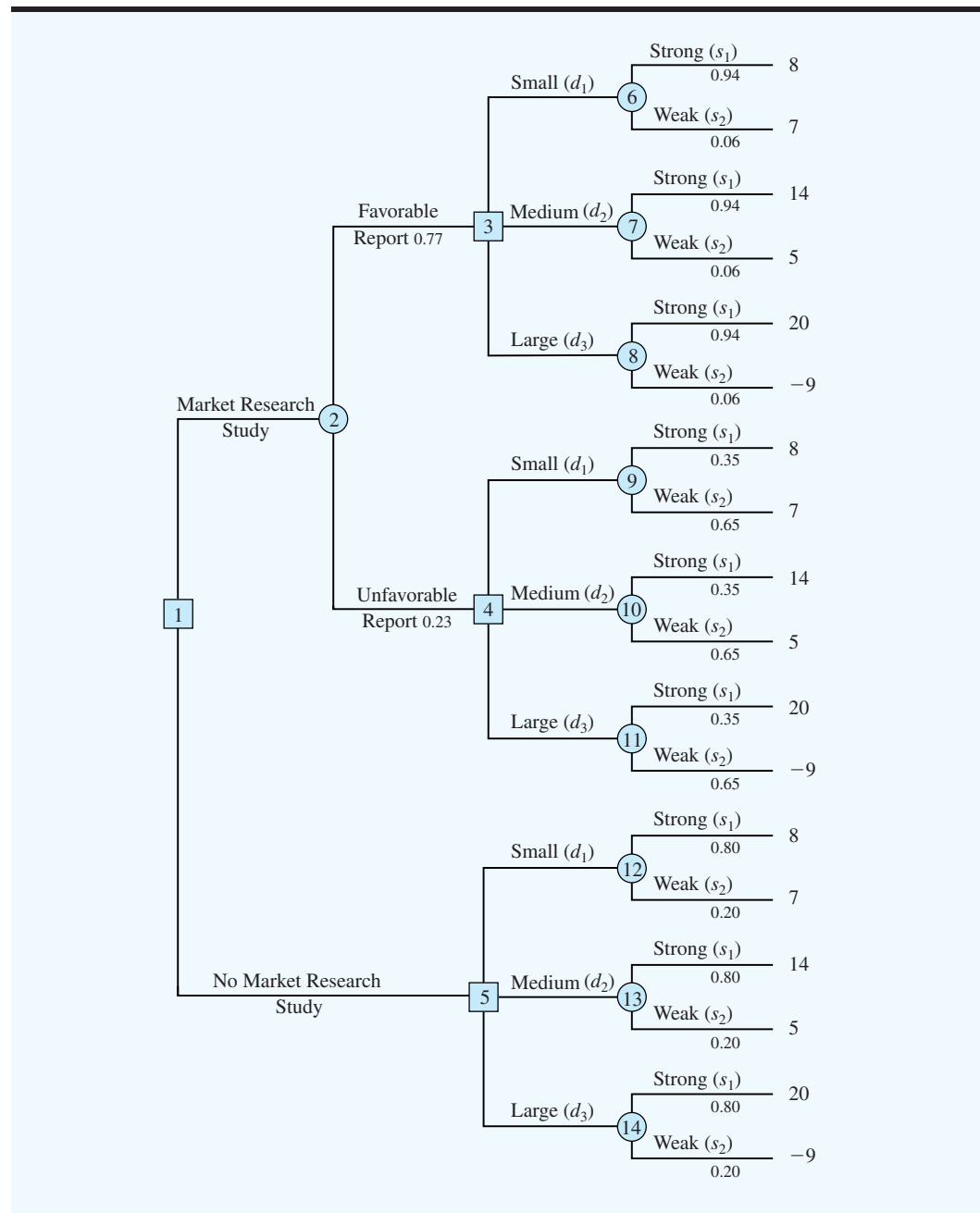
$$EV(\text{Node 13}) = 0.80(14) + 0.20(5) = 12.20$$

$$EV(\text{Node 14}) = 0.80(20) + 0.20(-9) = 14.20$$

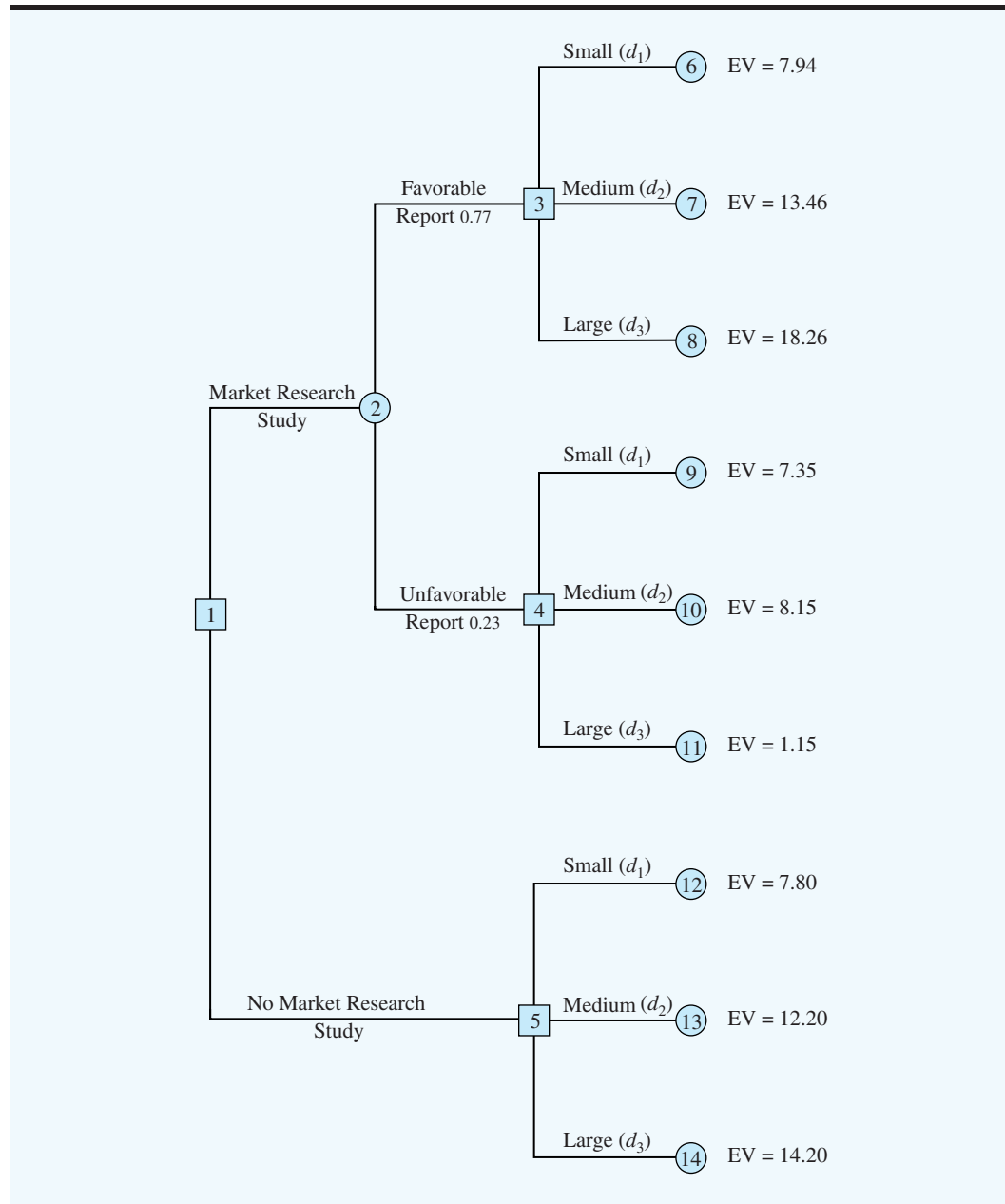
Figure 4.10 shows the reduced decision tree after computing expected values at these chance nodes.

Next move to decision nodes 3, 4, and 5. For each of these nodes, we select the decision alternative branch that leads to the best expected value. For example, at node 3 we have the

**FIGURE 4.9** THE PDC DECISION TREE WITH BRANCH PROBABILITIES



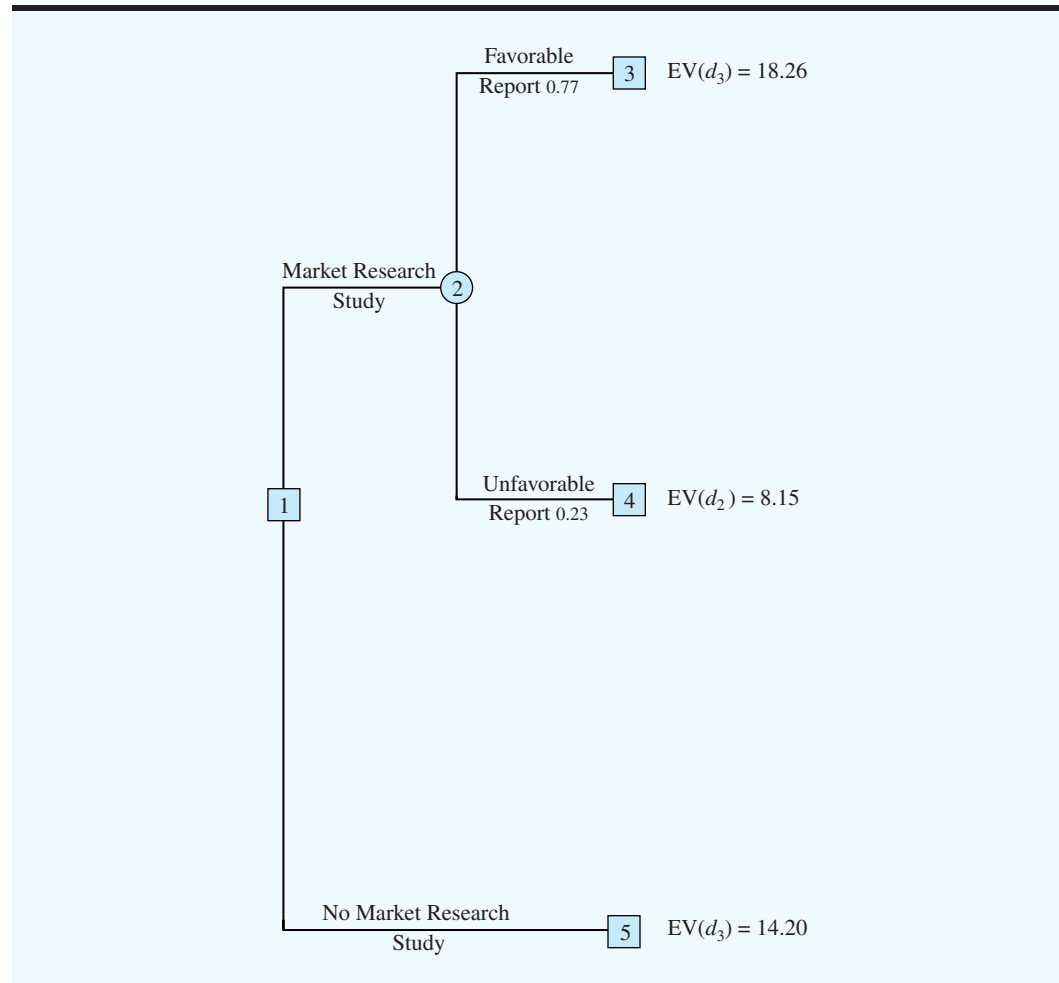
**FIGURE 4.10** PDC DECISION TREE AFTER COMPUTING EXPECTED VALUES AT CHANCE NODES 6 TO 14



choice of the small complex branch with  $EV(\text{Node } 6) = 7.94$ , the medium complex branch with  $EV(\text{Node } 7) = 13.46$ , and the large complex branch with  $EV(\text{Node } 8) = 18.26$ . Thus, we select the large complex decision alternative branch and the expected value at node 3 becomes  $EV(\text{Node } 3) = 18.26$ .

For node 4, we select the best expected value from nodes 9, 10, and 11. The best decision alternative is the medium complex branch that provides  $EV(\text{Node } 4) = 8.15$ . For node 5, we select the best expected value from nodes 12, 13, and 14. The best decision alterna-

**FIGURE 4.11** PDC DECISION TREE AFTER CHOOSING BEST DECISIONS AT NODES 3, 4, AND 5



tive is the large complex branch which provides  $EV(\text{Node } 5) = 14.20$ . Figure 4.11 shows the reduced decision tree after choosing the best decisions at nodes 3, 4, and 5.

The expected value at chance node 2 can now be computed as follows:

$$\begin{aligned} EV(\text{Node } 2) &= 0.77EV(\text{Node } 3) + 0.23EV(\text{Node } 4) \\ &= 0.77(18.26) + 0.23(8.15) = 15.93 \end{aligned}$$

This reduces the decision tree to one involving only the 2 decision branches from node 1 (see Figure 4.12).

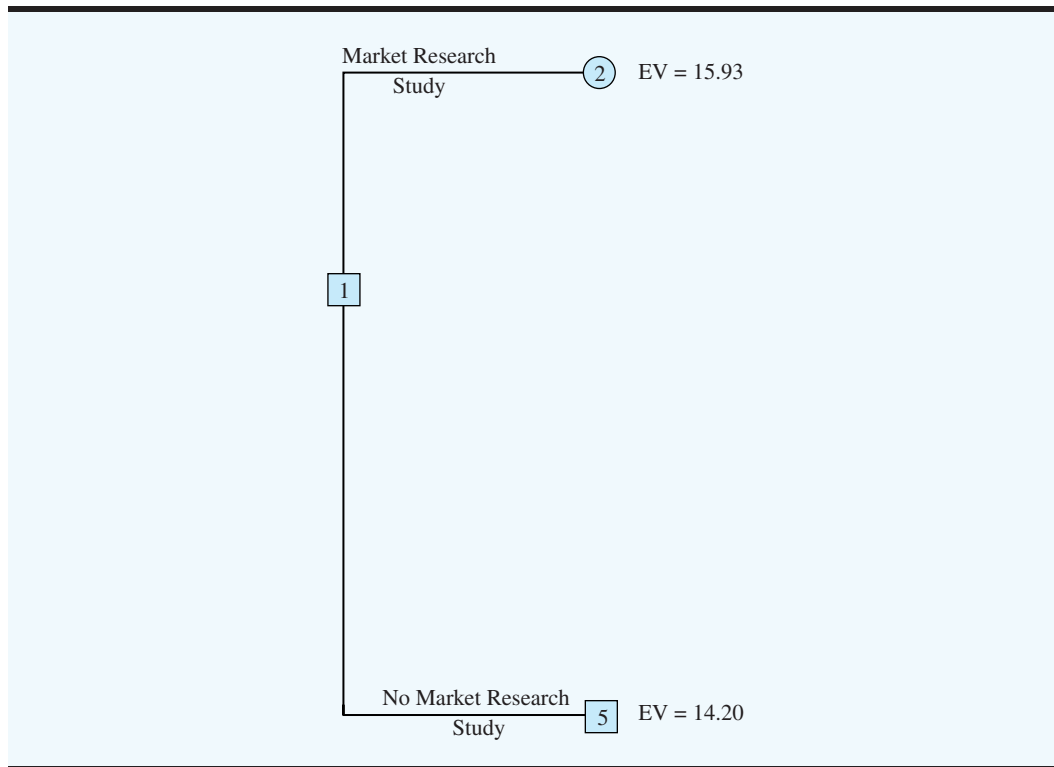
Finally, the decision can be made at decision node 1 by selecting the best expected values from nodes 2 and 5. This action leads to the decision alternative to conduct the market research study, which provides an overall expected value of 15.93.

The optimal decision for PDC is to conduct the market research study and then carry out the following decision strategy:

If the market research is favorable, construct the large condominium complex.

If the market research is unfavorable, construct the medium condominium complex.



**FIGURE 4.12** PDC DECISION TREE REDUCED TO 2 DECISION BRANCHES

*Problem 16 will test your ability to develop an optimal decision strategy.*

The analysis of the PDC decision tree describes the methods that can be used to analyze more complex sequential decision problems. First, draw a decision tree consisting of decision and chance nodes and branches that describe the sequential nature of the problem. Determine the probabilities for all chance outcomes. Then, by working backward through the tree, compute expected values at all chance nodes and select the best decision branch at all decision nodes. The sequence of optimal decision branches determines the optimal decision strategy for the problem. The Q. M. in Action article on drug testing for student athletes describes how Santa Clara University used decision analysis to make a decision regarding whether to implement a drug testing program for student athletes.

### Risk Profile

Figure 4.13 provides a reduced decision tree showing only the sequence of decision alternatives and chance events for the PDC optimal decision strategy. By implementing the optimal decision strategy, PDC will obtain one of the four payoffs shown at the terminal branches of the decision tree. Recall that a risk profile shows the possible payoffs with their associated probabilities. Thus, in order to construct a risk profile for the optimal decision strategy we will need to compute the probability for each of the four payoffs.

Note that each payoff results from a sequence of branches leading from node 1 to the payoff. For instance, the payoff of \$20 million is obtained by following the upper branch from node 1, the upper branch from node 2, the lower branch from node 3 and the upper branch from node 8. The probability of following that sequence of branches can be found by multiplying the probabilities for the branches from the chance nodes in the sequence.

## Q.M. IN ACTION

## DECISION ANALYSIS AND DRUG TESTING FOR STUDENT ATHLETES\*

The athletic governing board of Santa Clara University considered whether to implement a drug-testing program for the university's intercollegiate athletes. The decision analysis framework contains two decision alternatives: implement a drug-testing program and do not implement a drug-testing program. Each student athlete is either a drug user or not a drug user, so these two possibilities are considered to be the states of nature for the problem.

If the drug-testing program is implemented, student athletes will be required to take a drug-screening test. Results of the test will be either positive (test indicates a possible drug user) or negative (test does not indicate a possible drug user). The test outcomes are considered to be the sample information in the decision problem. If the test result is negative, no follow-up action will be taken.

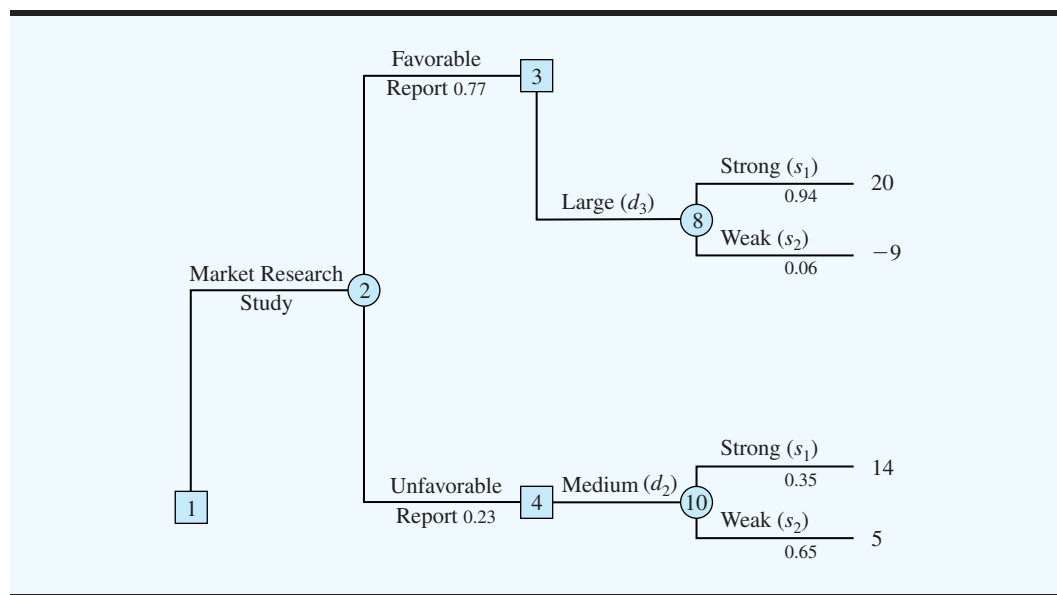
However, if the test result is positive, follow-up action will be taken to determine whether the student athlete actually is a drug user. The payoffs include the cost of not identifying a drug user and the cost of falsely identifying a nonuser.

Decision analysis showed that if the test result is positive, a reasonably high probability still exists that the student athlete is not a drug user. The cost and other problems associated with this type of misleading test result were considered significant. Consequently, the athletic governing board decided not to implement the drug-testing program.

\*Charles D. Feinstein, "Deciding Whether to Test Student Athletes for Drug Use," *Interfaces* 20, no. 3 (May–June 1990): 80–87.

Thus the probability the \$20 million payoff is  $(0.77)(0.94) = 0.72$ . Similarly, the probabilities for each of the other payoffs are obtained by multiplying the probabilities for the branches from the chance nodes leading to the payoffs. Doing so, we find the probability of the  $-\$9$  million payoff is  $(0.77)(0.06) = 0.05$ ; the probability of the \$14 million payoff is  $(0.23)(0.35) = 0.08$ ; and the probability of the \$5 million payoff is  $(0.23)(0.65) = 0.15$ . The following table showing the probability distribution for the payoffs for the PDC

**FIGURE 4.13** PDC DECISION TREE SHOWING ONLY BRANCHES ASSOCIATED WITH OPTIMAL DECISION STRATEGY



optimal decision strategy is the tabular representation of the risk profile for the optimal decision strategy.

Payoff (\$ Million)	Probability
−9	0.05
5	0.15
14	0.08
20	<u>0.72</u>
	1.00

Figure 4.14 provides a graphical representation of the risk profile. Comparing Figures 4.5 and 4.14, we see that the PDC risk profile is changed by the strategy to conduct the market research study. In fact, the use of the market research study has lowered the probability of the \$9 million loss from 0.20 to 0.05. PDC's management would most likely view that change as a significant reduction in the risk associated with the condominium project.

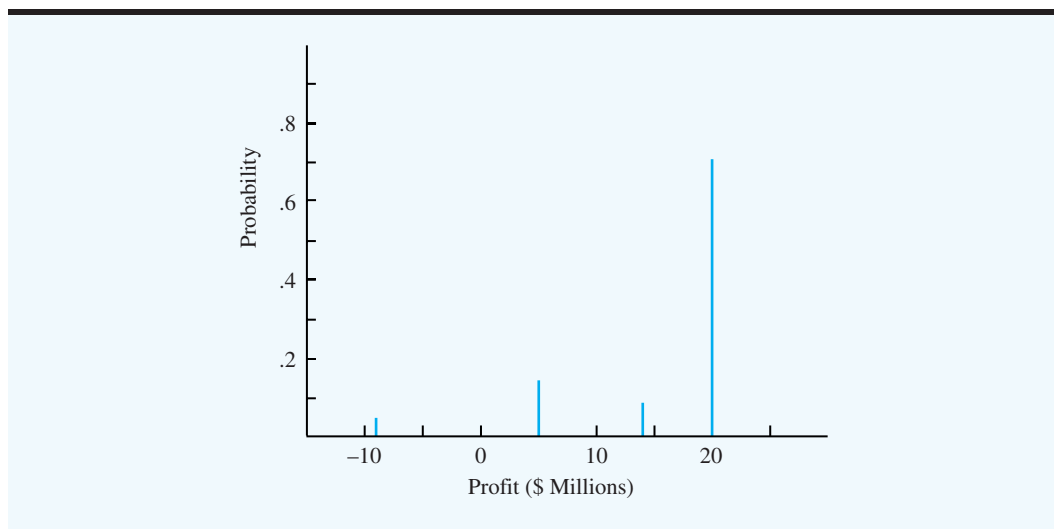
### Expected Value of Sample Information

In the PDC problem, the market research study is the sample information used to determine the optimal decision strategy. The expected value associated with the market research study is \$15.93. In Section 4.3 we showed that the best expected value if the market research study is *not* undertaken is \$14.20. Thus, we can conclude that the difference,  $\$15.93 - \$14.20 = \$1.73$ , is the **expected value of sample information**. In other words, conducting the market research study adds \$1.73 million to the PDC expected value. In general, the expected value of sample information is as follows:

$$EVSI = |EV_{wSI} - EV_{woSI}| \quad (4.13)$$

*The EVSI = \$1.73 million suggests PDC should be willing to pay up to \$1.73 million to conduct the market research study.*

**FIGURE 4.14** RISK PROFILE FOR PDC CONDOMINIUM PROJECT WITH SAMPLE INFORMATION SHOWING PAYOFFS ASSOCIATED WITH OPTIMAL DECISION STRATEGY



where

EVSI = expected value of sample information

EVwSI = expected value *with* sample information about the states of nature

EVwoSI = expected value *without* sample information about the states of nature

Note the role of the absolute value in equation (4.13). For minimization problems the expected value with sample information is always less than or equal to the expected value without sample information. In this case, EVSI is the magnitude of the difference between EVwSI and EVwoSI; thus, by taking the absolute value of the difference as shown in equation (4.13), we can handle both the maximization and minimization cases with one equation.

### Efficiency of Sample Information

In Section 4.3 we showed that the expected value of perfect information (EVPI) for the PDC problem is \$3.2 million. We never anticipated that the market research report would obtain perfect information, but we can use an **efficiency** measure to express the value of the market research information. With perfect information having an efficiency rating of 100%, the efficiency rating  $E$  for sample information is computed as follows.

$$E = \frac{\text{EVSI}}{\text{EVPI}} \times 100 \quad (4.14)$$

For the PDC problem,

$$E = \frac{1.73}{3.2} \times 100 = 54.1\%$$

In other words, the information from the market research study is 54.1% as efficient as perfect information.

Low efficiency ratings for sample information might lead the decision maker to look for other types of information. However, high efficiency ratings indicate that the sample information is almost as good as perfect information and that additional sources of information would not yield significantly better results.

## 4.6 COMPUTING BRANCH PROBABILITIES

In Section 4.5 the branch probabilities for the PDC decision tree chance nodes were specified in the problem description. No computations were required to determine these probabilities. In this section we show how **Bayes Theorem**, a topic covered in Chapter 2, can be used to compute branch probabilities for decision trees.

The PDC decision tree is shown again in Figure 4.15. Let

$F$  = Favorable market research report

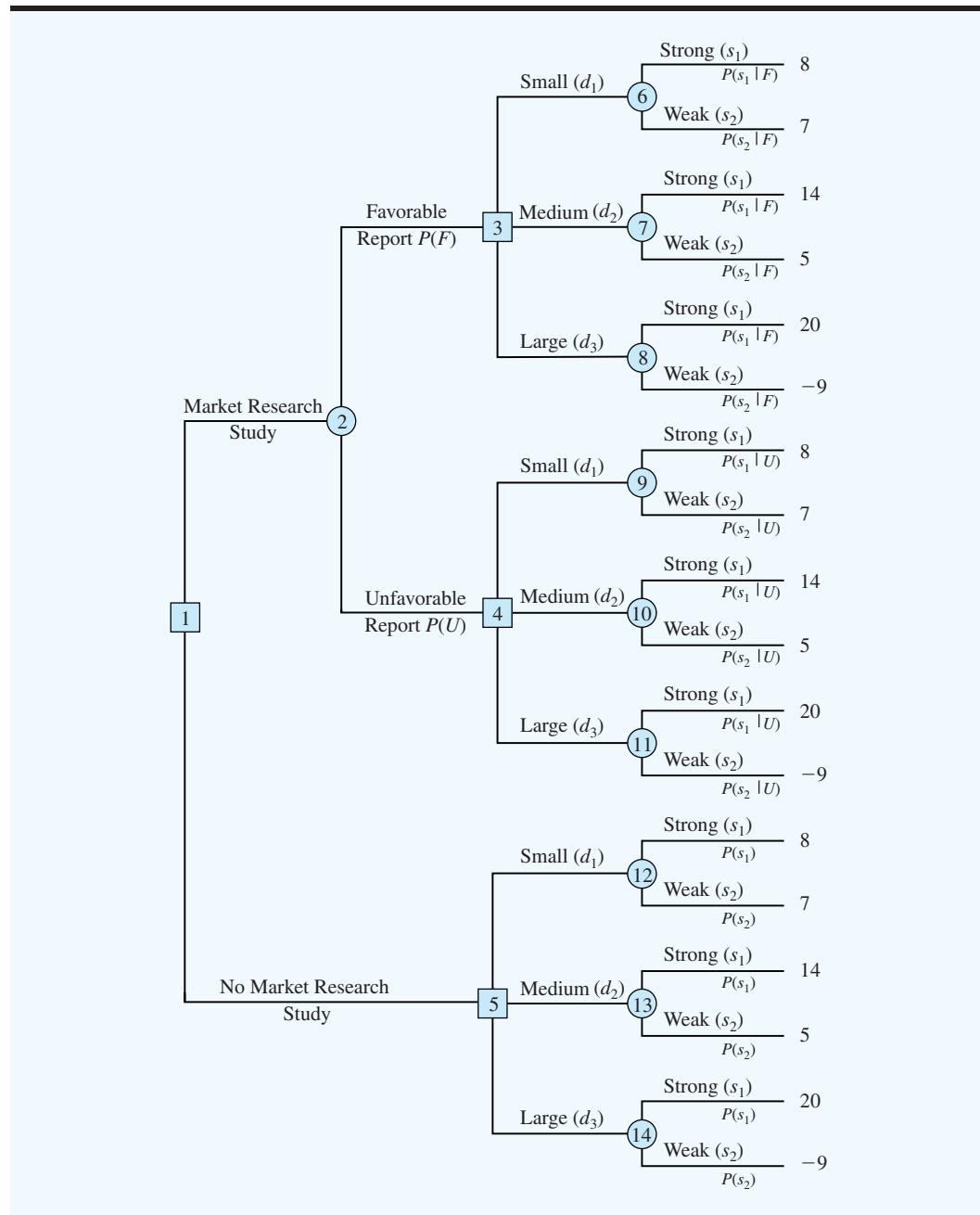
$U$  = Unfavorable market research report

$s_1$  = Strong demand (state of nature 1)

$s_2$  = Weak demand (state of nature 2)

At chance node 2, we need to know the branch probabilities  $P(F)$  and  $P(U)$ . At chance nodes 6, 7, and 8, we need to know the branch probabilities  $P(s_1 | F)$ , the probability of state of nature 1 given a favorable market research report, and  $P(s_2 | F)$ , the probability of state

FIGURE 4.15 THE PDC DECISION TREE



of nature 2 given a favorable market research report.  $P(s_1 | F)$  and  $P(s_2 | F)$  are referred to as **posterior probabilities** because they are conditional probabilities based on the outcome of the sample information. At chance nodes 9, 10, and 11, we need to know the branch probabilities  $P(s_1 | U)$  and  $P(s_2 | U)$ ; note that these are also posterior probabilities, denoting the probabilities of the two states of nature *given* that the market research report is unfavorable. Finally at chance nodes 12, 13, and 14, we need the probabilities for the states of nature,  $P(s_1)$  and  $P(s_2)$ , if the market research study is not undertaken.

In making the probability computations, we need to know PDC's assessment of the probabilities for the two states of nature,  $P(s_1)$  and  $P(s_2)$ ; these are the prior probabilities as discussed earlier. In addition, we must know the **conditional probability** of the market research outcomes (the sample information) *given* each state of nature. For example, we need to know the conditional probability of a favorable market research report given that the state of nature is strong demand for the PDC project; note that this conditional probability of  $F$  given state of nature  $s_1$  is written  $P(F | s_1)$ . To carry out the probability calculations, we will need conditional probabilities for all sample outcomes given all states of nature, that is,  $P(F | s_1)$ ,  $P(F | s_2)$ ,  $P(U | s_1)$  and  $P(U | s_2)$ . In the PDC problem, we assume that the following assessments are available for these conditional probabilities.

State of Nature	Market Research	
	Favorable, $F$	Unfavorable, $U$
Strong demand, $s_1$	$P(F   s_1) = 0.90$	$P(U   s_1) = 0.10$
Weak demand, $s_2$	$P(F   s_2) = 0.25$	$P(U   s_2) = 0.75$

Note that the preceding probability assessments provide a reasonable degree of confidence in the market research study. If the true state of nature is  $s_1$ , the probability of a favorable market research report is 0.90, and the probability of an unfavorable market research report is 0.10. If the true state of nature is  $s_2$ , the probability of a favorable market research report is 0.25, and the probability of an unfavorable market research report is 0.75. The reason for a 0.25 probability of a potentially misleading favorable market research report for state of nature  $s_2$  is that when some potential buyers first hear about the new condominium project, their enthusiasm may lead them to overstate their real interest in it. A potential buyer's initial favorable response can change quickly to a "no thank you" when later faced with the reality of signing a purchase contract and making a down payment.

In the following discussion, we present a tabular approach as a convenient method for carrying out the probability computations. The computations for the PDC problem based on a favorable market research report ( $F$ ) are summarized in Table 4.7. The steps used to develop this table are as follows.

- Step 1.** In column 1 enter the states of nature. In column 2 enter the *prior probabilities* for the states of nature. In column 3 enter the *conditional probabilities* of a favorable market research report ( $F$ ) given each state of nature.
- Step 2.** In column 4 compute the **joint probabilities** by multiplying the prior probability values in column 2 by the corresponding conditional probability values in column 3.
- Step 3.** Sum the joint probabilities in column 4 to obtain the probability of a favorable market research report,  $P(F)$ .
- Step 4.** Divide each joint probability in column 4 by  $P(F) = 0.77$  to obtain the revised or *posterior probabilities*,  $P(s_1 | F)$  and  $P(s_2 | F)$ .

Table 4.7 shows that the probability of obtaining a favorable market research report is  $P(F) = 0.77$ . In addition,  $P(s_1 | F) = 0.94$  and  $P(s_2 | F) = 0.06$ . In particular, note that a favorable market research report will prompt a revised or posterior probability of 0.94 that the market demand of the condominium will be strong,  $s_1$ .

The tabular probability computation procedure must be repeated for each possible sample information outcome. Thus, Table 4.8 shows the computations of the branch probabilities of the PDC problem based on an unfavorable market research report. Note that the probability of obtaining an unfavorable market research report is  $P(U) = 0.23$ . If an

**TABLE 4.7** BRANCH PROBABILITIES FOR THE PDC CONDOMINIUM PROJECT BASED ON A FAVORABLE MARKET RESEARCH REPORT

States of Nature	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilities
$s_j$	$P(s_j)$	$P(F   s_j)$	$P(F \cap s_j)$	$P(s_j   F)$
$s_1$	0.8	0.90	0.72	0.94
$s_2$	0.2	0.25	0.05	0.06
	1.0		$P(F) = 0.77$	1.00

**TABLE 4.8** BRANCH PROBABILITIES FOR THE PDC CONDOMINIUM PROJECT BASED ON AN UNFAVORABLE MARKET RESEARCH REPORT

States of Nature	Prior Probabilities	Conditional Probabilities	Joint Probabilities	Posterior Probabilities
$s_j$	$P(s_j)$	$P(U   s_j)$	$P(U \cap s_j)$	$P(s_j   U)$
$s_1$	0.8	0.10	0.08	0.35
$s_2$	0.2	0.75	0.15	0.65
	1.0		$P(U) = 0.23$	1.00

unfavorable report is obtained, the posterior probability of a strong market demand,  $s_1$ , is 0.35 and of a weak market demand,  $s_2$ , is 0.65. The branch probabilities from Tables 4.7 and 4.8 were shown on the PDC decision tree in Figure 4.9.

*Problem 22 asks you to compute the posterior probabilities.*

The discussion in this section shows an underlying relationship between the probabilities on the various branches in a decision tree. To assume different prior probabilities,  $P(s_1)$  and  $P(s_2)$ , without determining how these changes would alter  $P(F)$  and  $P(U)$ , as well as the posterior probabilities  $P(s_1 | F)$ ,  $P(s_2 | F)$ ,  $P(s_1 | U)$  and  $P(s_2 | U)$  would be inappropriate.

## SUMMARY

Decision analysis can be used to determine a recommended decision alternative or an optimal decision strategy when a decision maker is faced with an uncertain and risk-filled pattern of future events. The goal of decision analysis is to identify the best decision alternative or the optimal decision strategy given information about the uncertain events and the possible consequences or payoffs. The uncertain future events are called chance events and the outcomes of the chance events are called states of nature.

We showed how influence diagrams, payoff tables, and decision trees could be used to structure a decision problem and describe the relationships among the decisions, the chance events, and the consequences. We presented three approaches to decision making without probabilities: the optimistic approach, the conservative approach, and the minimax regret approach. When probability assessments are provided for the states of nature, the expected value approach can be used to identify the recommended decision alternative or decision strategy.

In cases where sample information about the chance events is available, a sequence of decisions has to be made. First we must decide whether to obtain the sample information.



If the answer to this decision is yes, an optimal decision strategy based on the specific sample information must be developed. In this situation, decision trees and the expected value approach can be used to determine the optimal decision strategy.

Even though the expected value approach can be used to obtain a recommended decision alternative or optimal decision strategy, the payoff that actually occurs will usually have a value different from the expected value. A risk profile provides a probability distribution for the possible payoffs and can assist the decision maker in assessing the risks associated with different decision alternatives. Finally, sensitivity analysis can be conducted to determine the effect changes in the probabilities for the states of nature and changes in the values of the payoffs have on the recommended decision alternative.

Decision analysis has been widely used in practice. The Q. M. in Action: Investing in a Power Transmission System describes how Oglethorpe Power Corporation used decision analysis to decide whether to invest in a major transmission system between Georgia and Florida. The Quantitative Methods in Practice at the end of the chapter describes how Ohio Edison used decision analysis to select equipment that helped the company meet emission standards.

### Q. M. IN ACTION

#### INVESTING IN A TRANSMISSION SYSTEM

Oglethorpe Power Corporation (OPC) provides wholesale electrical power to consumer-owned cooperatives in the state of Georgia. Florida Power Corporation proposed that OPC join in the building of a major transmission line from Georgia to Florida. Deciding whether to become involved in the building of the transmission line was a major decision for OPC because it would involve the commitment of substantial OPC resources. OPC worked with Applied Decision Analysis, Inc., to conduct a comprehensive decision analysis of the problem.

In the problem formulation step, three decisions were identified: (1) deciding whether to build a transmission line from Georgia to Florida; (2) deciding whether to upgrade existing transmission facilities; and (3) deciding who would control the new facilities. Oglethorpe was faced with five chance events: (1) construction costs, (2) competition, (3) demand in Florida, (4) OPC's share of the operation, and (5) pricing. The consequence or payoff was measured in terms of dollars saved. The influence diagram for the problem had three decision nodes, five chance nodes, a consequence node,

and several intermediate nodes that described intermediate calculations. The decision tree for the problem had more than 8000 paths from the starting node to the terminal branches.

An expected value analysis of the decision tree provided an optimal decision strategy for OPC. However, the risk profile for the optimal decision strategy showed that the recommended strategy was very risky and had a significant probability of increasing OPC's cost rather than providing a savings. The risk analysis led to the conclusion that more information about the competition was needed in order to reduce OPC's risk. Sensitivity analysis involving various probabilities and payoffs showed that the value of the optimal decision strategy was stable over a reasonable range of input values. The final recommendation from the decision analysis was that OPC should begin negotiations with Florida Power Corporation concerning the building of the new transmission line.

Based on Borison, Adam, "Oglethorpe Power Corporation Decides about Investing in a Major Transmission System" *Interfaces*, March–April, 1995, pp. 25–36.

### GLOSSARY

**Chance event** An uncertain future event affecting the consequence, or payoff, associated with a decision.

**States of nature** The possible outcomes for chance events that affect the payoff associated with a decision alternative.

**Influence diagram** A graphical device that shows the relationship among decisions, chance events, and consequences for a decision problem.

**Consequence** The result obtained when a decision alternative is chosen and a chance event occurs. A measure of the consequence is often called a payoff.

**Payoff** A measure of the consequence of a decision such as profit, cost, or time. Each combination of a decision alternative and a state of nature has an associated payoff, (consequence).

**Payoff table** A tabular representation of the payoffs for a decision problem.

**Decision tree** A graphical representation of the decision problem that shows the sequential nature of the decision-making process.

**Node** An intersection or junction point of an influence diagram or a decision tree.

**Decision nodes** Nodes indicating points where a decision is made.

**Chance nodes** Nodes indicating points where an uncertain event will occur.

**Branch** Lines showing the alternatives from decision nodes and the outcomes from chance nodes.

**Optimistic approach** An approach to choosing a decision alternative without using probabilities. For a maximization problem, it leads to choosing the decision alternative corresponding to the largest payoff; for a minimization problem, it leads to choosing the decision alternative corresponding to the smallest payoff.

**Conservative approach** An approach to choosing a decision alternative without using probabilities. For a maximization problem, it leads to choosing the decision alternative that maximizes the minimum payoff; for a minimization problem, it leads to choosing the decision alternative that minimizes the maximum payoff.

**Minimax regret approach** An approach to choosing a decision alternative without using probabilities. For each alternative, the maximum regret is computed, which leads to choosing the decision alternative that minimizes the maximum regret.

**Opportunity loss, or regret** The amount of loss (lower profit or higher cost) from not making the best decision for each state of nature.

**Expected value approach** An approach to choosing a decision alternative that is based on the expected value of each decision alternative. The recommended decision alternative is the one that provides the best expected value.

**Expected value (EV)** For a chance node, it is the weighted average of the payoffs. The weights are the state-of-nature probabilities.

**Expected value of perfect information (EVPI)** The expected value of information that would tell the decision maker exactly which state of nature is going to occur (i.e., perfect information).

**Decision strategy** A strategy involving a sequence of decisions and chance outcomes to provide the optimal solution to a decision problem.

**Risk analysis** The study of the possible payoffs and probabilities associated with a decision alternative or a decision strategy.

**Risk profile** The probability distribution of the possible payoffs associated with a decision alternative or decision strategy.

**Sensitivity analysis** The study of how changes in the probability assessments for the states of nature and/or changes in the payoffs affect the recommended decision alternative.

**Prior probabilities** The probabilities of the states of nature prior to obtaining sample information.

**Sample information** New information obtained through research or experimentation that enables an updating or revision of the state-of-nature probabilities.

**Posterior (revised) probabilities** The probabilities of the states of nature after revising the prior probabilities based on sample information.

**Expected value of sample information (EVSI)** The difference between the expected value of an optimal strategy based on sample information and the “best” expected value without any sample information.

**Efficiency** The ratio of EVSI to EVPI as a percent; perfect information is 100% efficient.

**Bayes theorem** A probability expression that enables the use of sample information to revise prior probabilities.

**Conditional probabilities** The probability of one event given the known outcome of a (possibly) related event.

**Joint probabilities** The probabilities of both sample information and a particular state of nature occurring simultaneously.

## PROBLEMS

### SELFtest

- The following payoff table shows profit for a decision analysis problem with two decision alternatives and three states of nature.

Decision Alternative	State of Nature		
	$s_1$	$s_2$	$s_3$
$d_1$	250	100	25
$d_2$	100	100	75

- Construct a decision tree for this problem.
  - If the decision maker knows nothing about the probabilities of the three states of nature, what is the recommended decision using the optimistic, conservative, and minimax regret approaches?
- Suppose that a decision maker faced with four decision alternatives and four states of nature develops the following profit payoff table.

Decision Alternative	State of Nature			
	$s_1$	$s_2$	$s_3$	$s_4$
$d_1$	14	9	10	5
$d_2$	11	10	8	7
$d_3$	9	10	10	11
$d_4$	8	10	11	13

- If the decision maker knows nothing about the probabilities of the four states of nature, what is the recommended decision using the optimistic, conservative, and minimax regret approaches?
- Which approach do you prefer? Explain. Is establishing the most appropriate approach before analyzing the problem important for the decision maker? Explain.
- Assume that the payoff table provides *cost* rather than profit payoffs. What is the recommended decision using the optimistic, conservative, and minimax regret approaches?

**SELFtest**

3. Southland Corporation's decision to produce a new line of recreational products has resulted in the need to construct either a small plant or a large plant. The best selection of plant size depends on how the marketplace reacts to the new product line. To conduct an analysis, marketing management has decided to view the possible long-run demand as either low, medium, or high. The following payoff table shows the projected profit in millions of dollars:

Plant Size	Long-Run Demand		
	Low	Medium	High
Small	150	200	200
Large	50	200	500

- What is the decision to be made, and what is the chance event for Southland's problem?
  - Construct an influence diagram.
  - Construct a decision tree.
  - Recommend a decision based on the use of the optimistic, conservative, and minimax regret approaches.
4. Amy Lloyd is interested in leasing a new Saab and has contacted three automobile dealers for pricing information. Each dealer has offered Amy a closed-end 36-month lease with no down payment due at the time of signing. Each lease includes a monthly charge and a mileage allowance. Additional miles receive a surcharge on a per-mile basis. The monthly lease cost, the mileage allowance, and the cost for additional miles follow:

Dealer	Monthly Cost	Mileage Allowance	Cost per Additional Mile
Forno Saab	\$299	36,000	\$0.15
Midtown Motors	\$310	45,000	\$0.20
Hopkins Automotive	\$325	54,000	\$0.15

Amy has decided to choose the lease option that will minimize her total 36-month cost. The difficulty is that Amy is not sure how many miles she will drive over the next three years. For purposes of this decision she believes it is reasonable to assume that she will drive 12,000 miles per year, 15,000 miles per year, or 18,000 miles per year. With this assumption Amy has estimated her total costs for the three lease options. For example, she figures that the Forno Saab lease will cost her \$10,764 if she drives 12,000 miles per year, \$12,114 if she drives 15,000 miles per year, or \$13,464 if she drives 18,000 miles per year.

- What is the decision, and what is the chance event?
- Construct a payoff table for Amy's problem.
- If Amy has no idea which of the three mileage assumptions is most appropriate, what is the recommended decision (leasing option) using the optimistic, conservative, and minimax regret approaches?
- Suppose that the probabilities that Amy drives 12,000, 15,000, and 18,000 miles per year are 0.5, 0.4, and 0.1, respectively. What option should Amy choose using the expected value approach?
- Develop a risk profile for the decision selected in Part (d). What is the most likely cost, and what is its probability?

- f. Suppose that after further consideration, Amy concludes that the probabilities that she will drive 12,000, 15,000, and 18,000 miles per year are 0.3, 0.4, and 0.3, respectively. What decision should Amy make using the expected value approach?

**SELFtest**

5. The following profit payoff table was presented in Problem 1. Suppose that the decision maker has obtained the probability assessments:  $P(s_1) = 0.65$ ,  $P(s_2) = 0.15$ , and  $P(s_3) = 0.20$ . Use the expected value approach to determine the optimal decision.

Decision Alternative	State of Nature		
	$s_1$	$s_2$	$s_3$
$d_1$	250	100	25
$d_2$	100	100	75

6. The profit payoff table presented in Problem 2 is repeated here.

Decision Alternative	State of Nature			
	$s_1$	$s_2$	$s_3$	$s_4$
$d_1$	14	9	10	5
$d_2$	11	10	8	7
$d_3$	9	10	10	11
$d_4$	8	10	11	13

Suppose that the decision maker obtains information that enables the following probability assessments to be made:  $P(s_1) = 0.5$ ,  $P(s_2) = 0.2$ ,  $P(s_3) = 0.2$ , and  $P(s_4) = 0.1$ .

- a. Use the expected value approach to determine the optimal decision.  
 b. Now assume that the entries in the payoff table are costs; use the expected value approach to determine the optimal decision.

**SELFtest**

7. Hudson Corporation is considering three options for managing its data processing operation: continuing with its own staff, hiring an outside vendor to do the managing (referred to as *outsourcing*), or using a combination of its own staff and an outside vendor. The cost of the operation depends on future demand. The annual cost of each option (in \$000s) depends on demand as follows.

Staffing Options	Demand		
	High	Medium	Low
Own Staff	650	650	600
Outside Vendor	900	600	300
Combination	800	650	500

- a. If the demand probabilities are 0.2, 0.5, and 0.3, which decision alternative will minimize the expected cost of the data processing operation? What is the expected annual cost associated with that recommendation?  
 b. Construct a risk profile for the optimal decision in part (a). What is the probability of the cost exceeding \$700,000?

**SELFtest**

8. The following payoff table shows the profit for a decision problem with two states of nature and two decision alternatives.

Decision Alternative	State of Nature	
	$s_1$	$s_2$
$d_1$	10	1
$d_2$	4	3

- Use graphical sensitivity analysis to determine the range of probabilities of state of nature  $s_1$  for which each of the decision alternatives has the largest expected value.
  - Suppose  $P(s_1) = 0.2$  and  $P(s_2) = 0.8$ . What is the best decision using the expected value approach?
  - Perform sensitivity analysis on the payoffs for decision alternative  $d_1$ . Assume the probabilities are as given in part (b) and find the range of payoffs under states of nature  $s_1$  and  $s_2$  that will keep the solution found in part (b) optimal. Is the solution more sensitive to the payoff under state of nature  $s_1$  or  $s_2$ ?
9. Myrtle Air Express has decided to offer direct service from Cleveland to Myrtle Beach. Management must decide between a full price service using the company's new fleet of jet aircraft and a discount service using smaller capacity commuter planes. It is clear that the best choice depends on the market reaction to the service Myrtle Air offers. Management has developed estimates of the contribution to profit for each type of service based upon two possible levels of demand for service to Myrtle Beach: strong and weak. The following table shows the estimated quarterly profits in thousands of dollars.

Service	Demand for Service	
	Strong	Weak
Full Price	\$960	−\$490
Discount	\$670	\$320

- What is the decision to be made, what is the chance event, and what is the consequence for this problem? How many decision alternatives are there? How many outcomes are there for the chance event?
  - If nothing is known about the probabilities of the chance outcomes, what is the recommended decision using the optimistic, conservative, and minimax regret approaches?
  - Suppose that management of Myrtle Air Express believes that the probability of strong demand is 0.7 and the probability of weak demand is 0.3. Use the expected value approach to determine an optimal decision.
  - Suppose that the probability of strong demand is 0.8 and the probability of weak demand is 0.2. What is the optimal decision using the expected value approach?
  - Use graphical sensitivity analysis to determine the range of demand probabilities for which each of the decision alternatives has the largest expected value.
10. Political Systems, Inc., is a new firm specializing in information services such as surveys and data analysis for individuals running for political office. The firm is opening its headquarters in Chicago and is considering three office locations, which differ in cost due to square footage and office equipment requirements. The profit projections shown (in thousands of dollars) for each location were based on both strong demand and weak demand states of nature.

Office Location	Demand	
	Strong	Weak
A	200	-20
B	120	10
C	100	60

- a. Initially, management is uncomfortable stating probabilities for the states of nature. Let  $p$  denote the probability of the strong demand state of nature. What does graphical sensitivity analysis tell management about location preferences? Can any location be dropped from consideration? Why or why not?
  - b. After further review, management estimated the probability of a strong demand at 0.65. Based on the results in part (a), which location should be selected? What is the expected value associated with that decision?
11. For the Pittsburgh Development Corporation problem in Section 4.3, the decision alternative to build the large condominium complex was found to be optimal using the expected value approach. In Section 4.4 we conducted a sensitivity analysis for the payoffs associated with this decision alternative. We found that the large complex remained optimal as long as the payoff for the strong demand was greater than or equal to \$17.5 million and as long as the payoff for the weak demand was greater than or equal to -\$19 million.
  - a. Consider the medium complex decision. How much could the payoff under strong demand increase and still keep decision alternative  $d_3$  the optimal solution?
  - b. Consider the small complex decision. How much could the payoff under strong demand increase and still keep decision alternative  $d_3$  the optimal solution?
12. The distance from Potsdam to larger markets and limited air service have hindered the town in attracting new industry. Air Express, a major overnight delivery service is considering establishing a regional distribution center in Potsdam. But Air Express will not establish the center unless the length of the runway at the local airport is increased. Another candidate for new development is Diagnostic Research, Inc. (DRI), a leading producer of medical testing equipment. DRI is considering building a new manufacturing plant. Increasing the length of the runway is not a requirement for DRI, but the planning commission feels that doing so will help convince DRI to locate their new plant in Potsdam. Assuming that the town lengthens the runway, the Potsdam planning commission believes that the probabilities shown in the following table are applicable.

	New DRI Plant	No DRI Plant
New Air Express Center	.30	.10
No Air Express Center	.40	.20

For instance, the probability that Air Express will establish a new distribution center and DRI will build a new plant is .30.

The estimated annual revenue to the town, after deducting the cost of lengthening the runway, is as follows:

	New DRI Plant	No New Plant
New Air Express Center	\$600,000	\$150,000
No Air Express Center	\$250,000	-\$200,000



If the runway expansion project is not conducted, the planning commission assesses the probability DRI will locate their new plant in Potsdam at 0.6; in this case, the estimated annual revenue to the town will be \$450,000. If the runway expansion project is not conducted and DRI does not locate in Potsdam, the annual revenue will be \$0 since no cost will have been incurred and no revenues will be forthcoming.

- a. What is the decision to be made, what is the chance event, and what is the consequence?
- b. Compute the expected annual revenue associated with the decision alternative to lengthen the runway.
- c. Compute the expected annual revenue associated with the decision alternative to not lengthen the runway.
- d. Should the town elect to lengthen the runway? Explain.
- e. Suppose that the probabilities associated with lengthening the runway were as follows:

	New DRI Plant	No DRI Plant
New Air Express Center	.40	.10
No Air Express Center	.30	.20

What effect, if any, would this change in the probabilities have on the recommended decision?

13. Seneca Hill Winery has recently purchased land for the purpose of establishing a new vineyard. Management is considering two varieties of white grapes for the new vineyard: Chardonnay and Riesling. The Chardonnay grapes would be used to produce a dry Chardonnay wine, and the Riesling grapes would be used to produce a semi-dry Riesling wine. It takes approximately four years from the time of planting before new grapes can be harvested. This length of time creates a great deal of uncertainty concerning future demand and makes the decision concerning the type of grapes to plant difficult. Three possibilities are being considered: Chardonnay grapes only; Riesling grapes only; and both Chardonnay and Riesling grapes. Seneca management decided that for planning purposes it would be adequate to consider only two demand possibilities for each type of wine: strong or weak. With two possibilities for each type of wine it was necessary to assess four probabilities. With the help of some forecasts in industry publications management made the following probability assessments.

Chardonnay Demand	Riesling Demand	
	Weak	Strong
Weak	0.05	0.50
Strong	0.25	0.20

Revenue projections show an annual contribution to profit of \$20,000 if Seneca Hill only plants Chardonnay grapes and demand is weak for Chardonnay wine, and \$70,000 if they only plant Chardonnay grapes and demand is strong for Chardonnay wine. If they only plant Riesling grapes, the annual profit projection is \$25,000 if demand is weak for Riesling grapes and \$45,000 if demand is strong for Riesling grapes. If Seneca plants both types of grapes, the annual profit projections are shown in the following table.

Chardonnay Demand	Riesling Demand	
	Weak	Strong
Weak	\$22,000	\$40,000
Strong	\$26,000	\$60,000

- What is the decision to be made, what is the chance event, and what is the consequence? Identify the alternatives for the decisions and the possible outcomes for the chance events.
- Develop a decision tree.
- Use the expected value approach to recommend which alternative Seneca Hill Winery should follow in order to maximize expected annual profit.
- Suppose management is concerned about the probability assessments when demand for Chardonnay wine is strong. Some believe it is likely for Riesling demand to also be strong in this case. Suppose the probability of strong demand for Chardonnay and weak demand for Riesling is 0.05 and that the probability of strong demand for Chardonnay and strong demand for Riesling is 0.40. How does this change the recommended decision? Assume that the probabilities when Chardonnay demand is weak are still 0.05 and 0.50.
- Other members of the management team expect the Chardonnay market to become saturated at some point in the future causing a fall in prices. Suppose that the annual profit projections fall to \$50,000 when demand for Chardonnay is strong and Chardonnay grapes only are planted. Using the original probability assessments, determine how this change would affect the optimal decision.

**SELFtest**

14. The following profit payoff table was presented in Problems 1 and 5.

Decision Alternative	State of Nature		
	$s_1$	$s_2$	$s_3$
$d_1$	250	100	25
$d_2$	100	100	75

The probabilities for the states of nature are:  $P(s_1) = 0.65$ ,  $P(s_2) = 0.15$ , and  $P(s_3) = 0.20$ .

- What is the optimal decision strategy if perfect information were available?
  - What is the expected value for the decision strategy developed in part (a)?
  - Using the expected value approach, what is the recommended decision without perfect information? What is its expected value?
  - What is the expected value of perfect information?
15. The Lake Placid Town Council has decided to build a new community center to be used for conventions, concerts, and other public events. But, considerable controversy surrounds the appropriate size. Many influential citizens want a large center that would be a showcase for the area. But the mayor feels that if demand does not support such a center, the community will lose a large amount of money. To provide structure for the decision process, the council narrowed the building alternatives to three sizes: small, medium, and large. Everybody agreed that the critical factor in choosing the best size is the number of people who will want to use the new facility. A regional planning consultant provided demand estimates under three scenarios: worst case, base case, and best case. The worst-case scenario corresponds to a situation in which tourism drops significantly; the base-case

scenario corresponds to a situation in which Lake Placid continues to attract visitors at current levels; and the best-case scenario corresponds to a significant increase in tourism. The consultant has provided probability assessments of 0.10, 0.60, and 0.30 for the worst-case, base-case, and best-case scenarios, respectively.

The town council suggested using net cash flow over a five-year planning horizon as the criterion for deciding on the best size. The following projections of net cash flow, in thousands of dollars, for a five-year planning horizon have been developed. All costs, including the consultant's fee, have been included.

Center Size	Demand Scenario		
	Worst Case	Base Case	Best Case
Small	400	500	660
Medium	−250	650	800
Large	−400	580	990

- What decision should Lake Placid make using the expected value approach?
- Construct risk profiles for the medium and large alternatives. Given the mayor's concern over the possibility of losing money and the result of part (a), which alternative would you recommend?
- Compute the expected value of perfect information. Do you think it would be worth trying to obtain additional information concerning which scenario is likely to occur?
- Suppose the probability of the worst-case scenario increases to 0.2, the probability of the base-case scenario decreases to 0.5, and the probability of the best-case scenario remains at 0.3. What effect, if any, would these changes have on the decision recommendation?
- The consultant has suggested that an expenditure of \$150,000 on a promotional campaign over the planning horizon will effectively reduce the probability of the worst-case scenario to zero. If the campaign can be expected to also increase the probability of the best-case scenario to 0.4, is it a good investment?

- SELFtest** 16. Consider a variation of the PDC decision tree shown in Figure 4.9. The company must first decide whether to undertake the market research study. If the market research study is conducted, the outcome will either be favorable ( $F$ ) or unfavorable ( $U$ ). Assume there are only two decision alternatives  $d_1$  and  $d_2$  and two states of nature  $s_1$  and  $s_2$ . The payoff table showing profit is as follows:

Decision Alternative	State of Nature	
	$s_1$	$s_2$
$d_1$	100	300
$d_2$	400	200

- Show the decision tree.
- Using the following probabilities, what is the optimal decision strategy?

$$\begin{array}{llll}
 P(F) = 0.56 & P(s_1 | F) = 0.57 & P(s_1 | U) = 0.18 & P(s_1) = 0.40 \\
 P(U) = 0.44 & P(s_2 | F) = 0.43 & P(s_2 | U) = 0.82 & P(s_2) = 0.60
 \end{array}$$

17. A real estate investor has the opportunity to purchase land currently zoned residential. If the county board approves a request to rezone the property as commercial within the next year, the investor will be able to lease the land to a large discount firm that wants to open a new store on the property. However, if the zoning change is not approved, the investor will have to sell the property at a loss. Profits (in \$000s) are shown in the following payoff table.

Decision Alternative	State of Nature	
	Rezoning Approved	Rezoning Not Approved
	$s_1$	$s_2$
Purchase, $d_1$	600	-200
Do not purchase, $d_2$	0	0

- If the probability that the rezoning will be approved is 0.5, what decision is recommended? What is the expected profit?
- The investor can purchase an option to buy the land. Under the option, the investor maintains the rights to purchase the land anytime during the next 3 months while learning more about possible resistance to the rezoning proposal from area residents. Probabilities are as follows.

Let  $H$  = High resistance to rezoning

$L$  = Low resistance to rezoning

$$\begin{array}{lll}
 P(H) = 0.55 & P(s_1 | H) = 0.18 & P(s_2 | H) = 0.82 \\
 P(L) = 0.45 & P(s_1 | L) = 0.89 & P(s_2 | L) = 0.11
 \end{array}$$

What is the optimal decision strategy if the investor uses the option period to learn more about the resistance from area residents before making the purchase decision?

- If the option will cost the investor an additional \$10,000, should the investor purchase the option? Why or why not? What is the maximum that the investor should be willing to pay for the option?
18. McHuffter Condominiums, Inc., of Pensacola, Florida, recently purchased land near the Gulf of Mexico and is attempting to determine the size of the condominium development it should build. It is considering three sizes of developments: small,  $d_1$ ; medium,  $d_2$ ; and large,  $d_3$ . At the same time, an uncertain economy makes ascertaining the demand for the new condominiums difficult. McHuffter's management realizes that a large development followed by low demand could be very costly to the company. However, if McHuffter makes a conservative small-development decision and then finds a high demand, the firm's profits will be lower than they might have been. With the three levels of demand—low, medium, and high—McHuffter's management has prepared the following profit (in \$000s) payoff table.

Decision Alternatives	State of Nature		
	Low, $s_1$	Medium, $s_2$	High, $s_3$
Small Condo, $d_1$	400	400	400
Medium Condo, $d_2$	100	600	600
Large Condo, $d_3$	-300	300	900

The probabilities for the states of nature are  $P(s_1) = 0.20$ ,  $P(s_2) = 0.35$ , and  $P(s_3) = 0.45$ . Suppose that before making a final decision, McHuffer is considering conducting a survey to help evaluate the demand for the new condominium development. The survey report is anticipated to indicate one of three levels of demand: weak ( $W$ ), average ( $A$ ), or strong ( $S$ ). The relevant probabilities are as follows:

$$\begin{array}{llll} P(W) = 0.30 & P(s_1 | W) = 0.39 & P(s_1 | A) = 0.16 & P(s_1 | S) = 0.06 \\ P(A) = 0.38 & P(s_2 | W) = 0.46 & P(s_2 | A) = 0.37 & P(s_2 | S) = 0.22 \\ P(S) = 0.32 & P(s_3 | W) = 0.15 & P(s_3 | A) = 0.47 & P(s_3 | S) = 0.72 \end{array}$$

- a. Construct a decision tree for this problem.
  - b. What is the recommended decision if the survey is not undertaken? What is the expected value?
  - c. What is the expected value of perfect information?
  - d. What is McHuffer's optimal decision strategy?
  - e. What is the expected value of the survey information?
  - f. What is the efficiency of the survey information?
19. Hale's TV Productions is considering producing a pilot for a comedy series in the hope of selling it to a major television network. The network may decide to reject the series, but it may also decide to purchase the rights to the series for either one or two years. At this point in time, Hale may either produce the pilot and wait for the network's decision or transfer the rights for the pilot and series to a competitor for \$100,000. Hale's decision alternatives and profits (in thousands of dollars) are as follows:

Decision Alternative	State of Nature		
	Reject, $s_1$	1 Year, $s_2$	2 Years, $s_3$
Produce Pilot, $d_1$	-100	50	150
Sell to Competitor, $d_2$	100	100	100

The probabilities for the states of nature are  $P(s_1) = 0.20$ ,  $P(s_2) = 0.30$ , and  $P(s_3) = 0.50$ . For a consulting fee of \$5,000, an agency will review the plans for the comedy series and indicate the overall chances of a favorable network reaction to the series. Assume that the agency review will result in a favorable ( $F$ ) or an unfavorable ( $U$ ) review and that the following probabilities are relevant.

$$\begin{array}{lll} P(F) = 0.69 & P(s_1 | F) = 0.09 & P(s_1 | U) = 0.45 \\ P(U) = 0.31 & P(s_2 | F) = 0.26 & P(s_2 | U) = 0.39 \\ & P(s_3 | F) = 0.65 & P(s_3 | U) = 0.16 \end{array}$$

- a. Construct a decision tree for this problem.
- b. What is the recommended decision if the agency opinion is not used? What is the expected value?
- c. What is the expected value of perfect information?
- d. What is Hale's optimal decision strategy assuming the agency's information is used?
- e. What is the expected value of the agency's information?
- f. Is the agency's information worth the \$5,000 fee? What is the maximum that Hale should be willing to pay for the information?
- g. What is the recommended decision?

20. Martin's Service Station is considering entering the snowplowing business for the coming winter season. Martin can purchase either a snowplow blade attachment for the station's pick-up truck or a new heavy-duty snowplow truck. Martin has analyzed the situation and believes that either alternative would be a profitable investment if the snowfall is heavy. Smaller profits would result if the snowfall is moderate, and losses would result if the snowfall is light. The following profits have been determined.

Decision Alternatives	State of Nature		
	Heavy, $s_1$	Moderate, $s_2$	Light, $s_3$
Blade Attachment, $d_1$	3500	1000	-1500
New Snowplow, $d_2$	7000	2000	-9000

The probabilities for the states of nature are  $P(s_1) = 0.4$ ,  $P(s_2) = 0.3$ , and  $P(s_3) = 0.3$ . Suppose that Martin decides to wait until September before making a final decision. Assessments of the probabilities associated with a normal ( $N$ ) or unseasonably cold ( $U$ ) September are as follows:

$$\begin{array}{lll}
 P(N) = 0.80 & P(s_1 | N) = 0.35 & P(s_1 | U) = 0.62 \\
 P(U) = 0.20 & P(s_2 | N) = 0.30 & P(s_2 | U) = 0.31 \\
 & P(s_3 | N) = 0.35 & P(s_3 | U) = 0.07
 \end{array}$$

- Construct a decision tree for this problem.
  - What is the recommended decision if Martin does not wait until September? What is the expected value?
  - What is the expected value of perfect information?
  - What is Martin's optimal decision strategy if the decision is not made until the September weather is determined? What is the expected value of this decision strategy?
21. Lawson's Department Store faces a buying decision for a seasonal product for which demand can be high, medium, and low. The purchaser for Lawson's can order 1, 2, or 3 lots of the product before the season begins but cannot reorder later. Profit projections (in \$000s) are shown.

Decision Alternative	State of Nature		
	High Demand $s_1$	Medium Demand $s_2$	Low Demand $s_3$
Order 1 lot, $d_1$	60	60	50
Order 2 lots, $d_2$	80	80	30
Order 3 lots, $d_3$	100	70	10

- If the prior probabilities for the three states of nature are 0.3, 0.3, and 0.4, respectively, what is the recommended order quantity?
- At each preseason sales meeting, the vice-president of sales provides a personal opinion regarding potential demand for this product. Because of the vice-president's enthusiasm and optimistic nature, the predictions of market conditions have always been either "excellent" ( $E$ ) or "very good" ( $V$ ). Probabilities are as follows. What is the optimal decision strategy?

$$\begin{array}{lll}
 P(E) = 0.70 & P(s_1 | E) = 0.34 & P(s_1 | V) = 0.20 \\
 P(V) = 0.30 & P(s_2 | E) = 0.32 & P(s_2 | V) = 0.26 \\
 & P(s_3 | E) = 0.34 & P(s_3 | V) = 0.54
 \end{array}$$

- c. Use the efficiency of sample information and discuss whether the firm should consider a consulting expert who could provide independent forecasts of market conditions for the product.
22. Suppose that you are given a decision situation with three possible states of nature:  $s_1$ ,  $s_2$ , and  $s_3$ . The prior probabilities are  $P(s_1) = 0.2$ ,  $P(s_2) = 0.5$ , and  $P(s_3) = 0.3$ . With sample information  $I$ ,  $P(I | s_1) = 0.1$ ,  $P(I | s_2) = 0.05$ , and  $P(I | s_3) = 0.2$ . Compute the revised or posterior probabilities:  $P(s_1 | I)$ ,  $P(s_2 | I)$ , and  $P(s_3 | I)$ .
23. In the following profit payoff table for a decision problem with two states of nature and three decision alternatives, the prior probabilities, for  $s_1$  and  $s_2$  are  $P(s_1) = 0.8$  and  $P(s_2) = 0.2$ .

Decision Alternative	State of Nature	
	$s_1$	$s_2$
$d_1$	15	10
$d_2$	10	12
$d_3$	8	20

- a. What is the optimal decision?
- b. Find the EVPI.
- c. Suppose that sample information  $I$  is obtained, with  $P(I | s_1) = 0.2$  and  $P(I | s_2) = 0.75$ . Find the posterior probabilities  $P(s_1 | I)$  and  $P(s_2 | I)$ . Recommend a decision alternative based on these probabilities.
24. To save on expenses, Rona and Jerry agreed to form a carpool for traveling to and from work. Rona preferred to use the somewhat longer but more consistent Queen City Avenue. Although Jerry preferred the quicker expressway, he agreed with Rona that they should take Queen City Avenue if the expressway had a traffic jam. The following payoff table provides the one-way time estimate in minutes for traveling to or from work.

Decision Alternative	State of Nature	
	Expressway Open	Expressway Jammed
	$s_1$	$s_2$
Queen City Avenue, $d_1$	30	30
Expressway, $d_2$	25	45

Based on their experience with traffic problems, Rona and Jerry agreed on a 0.15 probability that the expressway would be jammed.

In addition, they agreed that weather seemed to affect the traffic conditions on the expressway. Let

$C$  = clear

$O$  = overcast

$R$  = rain

The following conditional probabilities apply.

$$\begin{array}{lll} P(C | s_1) = 0.8 & P(O | s_1) = 0.2 & P(R | s_1) = 0.0 \\ P(C | s_2) = 0.1 & P(O | s_2) = 0.3 & P(R | s_2) = 0.6 \end{array}$$

- a. Use the Bayes' probability revision procedure to compute the probability of each weather condition and the conditional probability of the expressway open  $s_1$  or jammed  $s_2$  given each weather condition.
  - b. Show the decision tree for this problem.
  - c. What is the optimal decision strategy, and what is the expected travel time?
25. The Gorman Manufacturing Company must decide whether to manufacture a component part at its Milan, Michigan, plant or purchase the component part from a supplier. The resulting profit is dependent upon the demand for the product. The following payoff table shows the projected profit (in \$000s).

Decision Alternative	State of Nature		
	Low Demand $s_1$	Medium Demand $s_2$	High Demand $s_3$
Manufacture, $d_1$	-20	40	100
Purchase, $d_2$	10	45	70

The state-of-nature probabilities are  $P(s_1) = 0.35$ ,  $P(s_2) = 0.35$ , and  $P(s_3) = 0.30$ .

- a. Use a decision tree to recommend a decision.
- b. Use EVPI to determine whether Gorman should attempt to obtain a better estimate of demand.
- c. A test market study of the potential demand for the product is expected to report either a favorable ( $F$ ) or unfavorable ( $U$ ) condition. The relevant conditional probabilities are as follows:

$$\begin{array}{ll} P(F | s_1) = 0.10 & P(U | s_1) = 0.90 \\ P(F | s_2) = 0.40 & P(U | s_2) = 0.60 \\ P(F | s_3) = 0.60 & P(U | s_3) = 0.40 \end{array}$$

What is the probability that the market research report will be favorable?

- d. What is Gorman's optimal decision strategy?
- e. What is the expected value of the market research information?
- f. What is the efficiency of the information?

### Case Problem PROPERTY PURCHASE STRATEGY

Glenn Foreman, president of Oceanview Development Corporation, is considering submitting a bid to purchase property that will be sold by sealed bid at a county tax foreclosure. Glenn's initial judgment is to submit a bid of \$5 million. Based on his experience, Glenn estimates that a bid of \$5 million will have a 0.2 probability of being the highest bid and securing the property for Oceanview. The current date is June 1. Sealed bids for the property must be submitted by August 15. The winning bid will be announced on September 1.

If Oceanview submits the highest bid and obtains the property, the firm plans to build and sell a complex of luxury condominiums. However, a complicating factor is that the



property is currently zoned for single-family residences only. Glenn believes that a referendum could be placed on the voting ballot in time for the November election. Passage of the referendum would change the zoning of the property and permit construction of the condominiums.

The sealed-bid procedure requires the bid to be submitted with a certified check for 10% of the amount bid. If the bid is rejected, the deposit is refunded. If the bid is accepted, the deposit is the down payment for the property. However, if the bid is accepted and the bidder does not follow through with the purchase and meet the remainder of the financial obligation within six months, the deposit will be forfeited. In this case, the county will offer the property to the next highest bidder.

To determine whether Oceanview should submit the \$5 million bid, Glenn has done some preliminary analysis. This preliminary work provided an assessment of 0.3 for the probability that the referendum for a zoning change will be approved and resulted in the following estimates of the costs and revenues that will be incurred if the condominiums are built.

Cost and Revenue Estimates	
Revenue from condominium sales	\$15,000,000
Cost	
Property	\$5,000,000
Construction expenses	\$8,000,000

If Oceanview obtains the property and the zoning change is rejected in November, Glenn believes that the best option would be for the firm not to complete the purchase of the property. In this case, Oceanview would forfeit the 10% deposit that accompanied the bid.

Because the likelihood that the zoning referendum will be approved is such an important factor in the decision process, Glenn has suggested that the firm hire a market research service to conduct a survey of voters. The survey would provide a better estimate of the likelihood that the referendum for a zoning change would be approved. The market research firm that Oceanview Development has worked with in the past has agreed to do the study for \$15,000. The results of the study will be available August 1, so that Oceanview will have this information before the August 15 bid deadline. The results of the survey will be either a prediction that the zoning change will be approved or a prediction that the zoning change will be rejected. After considering the record of the market research service in previous studies conducted for Oceanview, Glenn has developed the following probability estimates concerning the accuracy of the market research information.

$$\begin{aligned} P(A | s_1) &= 0.9 & P(N | s_1) &= 0.1 \\ P(A | s_2) &= 0.2 & P(N | s_2) &= 0.8 \end{aligned}$$

where

$A$  = prediction of zoning change approval

$N$  = prediction that zoning change will not be approved

$s_1$  = the zoning change is approved by the voters

$s_2$  = the zoning change is rejected by the voters

## Managerial Report

Perform an analysis of the problem facing the Oceanview Development Corporation, and prepare a report that summarizes your findings and recommendations. Include the following items in your report:

1. A decision tree that shows the logical sequence of the decision problem
2. A recommendation regarding what Oceanview should do if the market research information is not available
3. A decision strategy that Oceanview should follow if the market research is conducted
4. A recommendation as to whether Oceanview should employ the market research firm, along with the value of the information provided by the market research firm. Include the details of your analysis as an appendix to your report.

## Case Problem LAWSUIT DEFENSE STRATEGY

John Campbell, an employee of Manhattan Construction Company, claims to have injured his back as a result of a fall while repairing the roof at one of the Eastview apartment buildings. He has filed a lawsuit against Doug Reynolds, the owner of Eastview Apartments, asking for damages of \$1,500,000. John claims that the roof had rotten sections and that his fall could have been prevented if Mr. Reynolds had told Manhattan Construction about the problem. Mr. Reynolds has notified his insurance company, Allied Insurance, of the lawsuit. Allied must defend Mr. Reynolds and decide what action to take regarding the lawsuit.

Some depositions have been taken, and a series of discussions have taken place between both sides. As a result, John Campbell has offered to accept a settlement of \$750,000. Thus, one option is for Allied to pay John \$750,000 to settle the claim. Allied is also considering making John a counteroffer of \$400,000 in the hope that he will accept a lesser amount to avoid the time and cost of going to trial. But, Allied's preliminary investigation has shown that John has a strong case; Allied is concerned that John may reject their counteroffer and request a jury trial. Allied's lawyers have spent some time exploring John's likely reaction if they make a counteroffer of \$400,000.

The lawyers have concluded that it is adequate to consider three possible outcomes to represent John's possible reaction to a counteroffer of \$400,000: (1) John will accept the counteroffer and the case will be closed; (2) John will reject the counteroffer and elect to have a jury decide the settlement amount; or (3) John will make a counteroffer to Allied of \$600,000. If John does make a counteroffer, Allied has decided that they will not make additional counteroffers. They will either accept John's counteroffer of \$600,000 or go to trial.

If the case goes to a jury trial, Allied has decided that it should be adequate to consider three possible outcomes: (1) the jury may reject John's claim and Allied will not be required to pay any damages; (2) the jury will find in favor of John and award him \$750,000 in damages; or (3) the jury will conclude that John has a strong case and award him the full amount that he sued for, \$1,500,000.

Key considerations as Allied develops its strategy for disposing of the case are the probabilities associated with John's response to an Allied counteroffer of \$400,000, and the probabilities associated with the three possible trial outcomes. Allied's lawyers believe the probability that John will accept a counteroffer of \$400,000 is 0.10, the probability that John will reject a counteroffer of \$400,000 is 0.40, and the probability that John will, himself, make a counteroffer to Allied of \$600,000 is 0.50. If the case goes to court, they believe that

the probability the jury will award John damages of \$1,500,000 is 0.30, the probability that the jury will award John damages of \$750,000 is 0.50, and the probability that the jury will award John nothing is 0.20.

### Managerial Report

Perform an analysis of the problem facing Allied Insurance and prepare a report that summarizes your findings and recommendations. Be sure to include the following items:

1. A decision tree
2. A recommendation regarding whether Allied should accept John's initial offer to settle the claim for \$750,000
3. A decision strategy that Allied should follow if they decide to make John a counter-offer of \$400,000
4. A risk profile for your recommended strategy

## Appendix 4.1 DECISION ANALYSIS WITH SPREADSHEETS

A spreadsheet provides a convenient way to perform the basic decision analysis computations. A spreadsheet may be designed for any of the decision analysis approaches described in this chapter. We will demonstrate use of the spreadsheet in decision analysis by solving the PDC condominium problem using the expected value approach.

### The Expected Value Approach

This spreadsheet solution is shown in Figure 4.16. The payoff table with appropriate headings is placed into cell A3 through cell C8. In addition, the probabilities for the two states of nature are placed in cells B9 and C9. The Excel formulas that provide the calculations and optimal solution recommendation are as follows:

Cells D6:D8	Compute the expected value for each decision alternative
Cell D6	=B9*B6+C9*C6
Cell D7	=B9*B7+C9*C7
Cell D8	=B9*B8+C9*C8
Cell D11	Compute the maximum expected value =MAX(D6:D8)
Cell E6:E8	Determine which decision alternative is recommended
Cell E6	=IF(D6=D11,A6," ")
Cell E7	=IF(D7=D11,A7," ")
Cell E8	=IF(D8=D11,A8," ")

As Figure 4.16 shows, the expected value approach recommends the large complex decision alternative with a maximum expected value of 14.2.

The only change required to convert the spreadsheet in Figure 4.16 into a minimization analysis is to change the formula in cell D11 to =MIN(D6:D8). With this change, the decision alternative with the minimum expected value will be shown in column E.

### Computation of Branch Probabilities

Spreadsheets can be used to compute the branch probabilities for a decision tree as discussed in Section 4.6. A spreadsheet used to compute the branch probabilities for the PDC problem is shown in Figure 4.17. The prior probabilities are entered into cells B5 and B6.

**FIGURE 4.16** SPREADSHEET SOLUTION FOR THE PDC PROBLEM USING THE EXPECTED VALUE APPROACH

	A	B	C	D	E
1	<b>PDC Problem - Expected Value Approach</b>				
2					
3	<b>Payoff Table</b>				
4		<b>State of Nature</b>		<b>Expected</b>	<b>Recommended</b>
5	<b>Decision Alternative</b>	High acceptance	Low acceptance	<b>Value</b>	<b>Decision</b>
6	Small complex	8	7	7.8	Large complex
7	Medium complex	14	5	12.2	
8	Large complex	20	-9	14.2	
9	<b>Probability</b>	0.8	0.2		
10					
11	<b>Maximum Expected Value</b>			14.2	

**FIGURE 4.17** SPREADSHEET SOLUTION FOR THE PDC PROBLEM PROBABILITY CALCULATIONS

	A	B	C	D	E
1	<b>PDC Problem - Bayes' Probability Calculations</b>				
2					
3		Prior			
4	<b>States of Nature</b>	Probabilities			
5	High Acceptance	0.8			
6	Low Acceptance	0.2			
7					
8	<b>Conditional Probabilities</b>				
9		Market Research			
10	If State of Nature Is	Favorable	Unfavorable		
11	High Acceptance	0.90	0.10		
12	Low Acceptance	0.25	0.75		
13					
14					
15	<b>Market Research Favorable (F)</b>				
16		Prior	Conditional	Joint	Posterior
17	State of Nature	Probabilities	Probabilities	Probabilities	Probabilities
18	High Acceptance	0.8	0.90	0.72	0.94
19	Low Acceptance	0.2	0.25	0.05	0.06
20			<b>P(F) =</b>	0.77	
21					
22					
23	<b>Market Research Unfavorable (U)</b>				
24		Prior	Conditional	Joint	Posterior
25	State of Nature	Probabilities	Probabilities	Probabilities	Probabilities
26	High Acceptance	0.8	0.10	0.08	0.35
27	Low Acceptance	0.2	0.75	0.15	0.65
28			<b>P(U) =</b>	0.23	

The four conditional probabilities are entered into cells B11, B12, C11, and C12. The following cell formulas perform the probability calculations for the PDC problem based on a favorable market research report, shown previously in Table 4.7.

Cells B18 and B19	Enter the prior probabilities =B5 =B6
Cells C18 and C19	Enter the conditional probabilities for a favorable market research report =B11 =B12
Cells D18 and D19	Compute the joint probabilities =B18*C18 =B19*C19
Cell D20	Compute the probability of a favorable market research report =SUM(D18:D19)
Cells E18 and E19	Compute the posterior probabilities for each state of nature =D18/D20 =D19/D20

The same logic was used to perform the probability calculations based on an unfavorable market research report shown in cells B23:E28.

## Quantitative Methods in Practice

### OHIO EDISON COMPANY\*

AKRON, OHIO

Ohio Edison Company is an operating company of FirstEnergy Corporation. Ohio Edison and its subsidiary, Pennsylvania Power Company, provide electrical service to more than one million customers in central and northeastern Ohio and western Pennsylvania. Most of this electricity is generated by coal-fired power plants. To meet evolving air-quality standards, Ohio Edison replaced existing particulate control equipment at most of its generating plants with more efficient equipment. The combination of this program to upgrade air-quality control equipment with the continuing need to construct new generating plants to meet future power requirements resulted in a large capital investment program.

Quantitative methods at Ohio Edison are distributed throughout the company rather than centralized in a specific department, and are more or less evenly divided among the following areas: fossil and nuclear fuel planning, environmental studies, capacity planning, large equipment evaluation, and corporate planning. Applications include decision analysis, optimal ordering strategies, computer modeling, and simulation.

#### A Decision Analysis Application

The flue gas emitted by coal-fired power plants contains small ash particles and sulfur dioxide ( $\text{SO}_2$ ). Federal and state regulatory agencies have established emission limits for both particulates and sulfur dioxide. In the late 1970s, Ohio Edison developed a plan to comply with new air-quality standards at one of its largest power plants. This plant, which consists of seven coal-fired units (most of which were constructed in the 1960s), constitutes about one-third of the generating capacity of Ohio Edison and its subsidiary company. Although all units were initially constructed with

particulate emission control equipment, that equipment was no longer capable of meeting new particulate emission requirements.

A decision had already been made to burn low-sulfur coal in four of the smaller units (units 1–4) at the plant in order to meet  $\text{SO}_2$  emission standards. Fabric filters were to be installed on these units to control particulate emissions. Fabric filters, also known as bag-houses, use thousands of fabric bags to filter out the particulates; they function in much the same way as a household vacuum cleaner.

It was considered likely, although not certain, that the three larger units (units 5–7) at this plant would burn medium- to high-sulfur coal. A method of controlling particulate emissions at these units had not yet been selected. Preliminary studies narrowed the particulate control equipment choice to a decision between fabric filters and electrostatic precipitators (which remove particulates suspended in the flue gas by passing the flue gas through a strong electric field). This decision was affected by a number of uncertainties, including the following:

- Uncertainty in the way some air-quality laws and regulations might be interpreted
- Potential requirements that either low-sulfur coal or high-sulfur Ohio coal (or neither) be burned in units 5–7
- Potential future changes to air-quality laws and regulations
- An overall plant reliability improvement program already under way at this plant
- The outcome of this program itself, which would affect the operating costs of whichever pollution control technology was installed in these units
- Uncertain construction costs of the equipment, particularly because limited space at the plant site made it necessary to install the equipment on

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a massive bridge deck over a four-lane highway immediately adjacent to the power plant

- Uncertain costs associated with replacing the electrical power required to operate the particulate control equipment
- Various other factors, including potential accidents and chronic operating problems that could increase the costs of operating the generating units (the degree to which each of these factors could affect operating costs varied with the choice of technology and with the sulfur content of the coal)

### Particulate Control Decision

The air-quality program involved a choice between two types of particulate control equipment (fabric filters and electrostatic precipitators) for units 5–7. Because of the complexity of the problem, the high degree of uncertainty associated with factors affecting the decision, and the importance (because of potential reliability and cost impact on Ohio Edison) of the choice, decision analysis was used in the selection process.

The decision measure used to evaluate the outcomes of the particulate technology decision analysis was the annual revenue requirements for the three large units over their remaining lifetime. Revenue requirements are the monies that would have to be collected from the utility customers to recover costs resulting from the decision. They include not only direct costs but also the cost of capital and return on investment.

A decision tree was constructed to represent the particulate control decision and its uncertainties and costs. A simplified version of this decision tree is shown in Figure 4.18. The decision and chance nodes are indicated. Note that to conserve space, a type of shorthand notation is used. The coal sulfur content chance node should actually be located at the end of each branch of the capital cost chance node, as the dashed lines indicate. Each chance node actually represents several probabilistic cost models or submodels. The total revenue requirements are the sum of the revenue requirements for capital and operating costs. Costs associated with these models were obtained from engineering calculations or estimates. Probabilities were obtained from existing data or the subjective assessments of knowledgeable persons.

### Results

A decision tree similar to that shown in Figure 4.18 was used to generate cumulative probability distributions for the annual revenue requirements outcomes calculated for each of the two particulate control alternatives. Careful study of these results led to the following conclusions:

- The expected value of annual revenue requirements for the electrostatic precipitator technology was approximately \$1 million lower than that for the fabric filters.
- The fabric filter alternative had a higher upside risk—that is, a higher probability of high revenue requirements—than did the precipitator alternative.
- The precipitator technology had nearly an 80% probability of lower annual revenue requirements than the fabric filters.
- Although the capital cost of the fabric filter equipment (the cost of installing the equipment) was lower than for the precipitator, this cost was more than offset by the higher operating costs associated with the fabric filter.

These results led Ohio Edison to select the electrostatic precipitator technology for the generating units in question. Had the decision analysis not been performed, the particulate control decision might have been based chiefly on capital cost, a decision measure that would have favored the fabric filter equipment. Decision analysis offers a means for effectively analyzing the uncertainties involved in a decision. Because of this analysis, the use of decision analysis methodology in this application resulted in a decision that yielded both lower expected revenue requirements and lower risk.

### Questions

1. Why was decision analysis used in the selection of particulate control equipment for units 5, 6, and 7?
2. List the decision alternatives for the decision analysis problem developed by Ohio Edison.
3. What were the benefits of using decision analysis in this application?

**FIGURE 4.18** SIMPLIFIED PARTICULATE CONTROL EQUIPMENT DECISION TREE