Chapter 1

A Review of Functions

Section 1.1: An Introduction to Functions

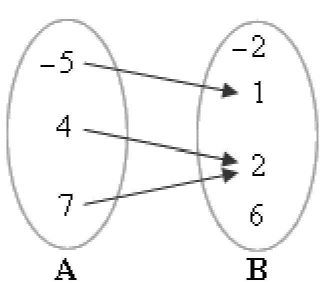
- Definition of a Function and Evaluating a Function
- Domain and Range of a Function

Definition of a Function and Evaluating a Function

Definition:

A function is a rule that assigns to each element in a set A exactly one element in a set B. The set A is called the domain of the function. The range of the function is the set of elements in B that are assigned by this rule.

The following mapping diagram is an example of a function.



Each element of the set $A = \{-5, 4, 7\}$ is assigned to exactly one element of the set $B = \{-2, 1, 2, 6\}$:

-5 is assigned to 1.
4 is assigned to 2.
7 is assigned to 2.

The domain of the function is $\{-5, 4, 7\}$ and the range of the function is $\{1, 2\}$.

Defining a Function by an Equation in the Variables *x* and *y*:

The equation $y = 3x + 1$ defines y as a function of x since for each real number x, the expression $3x + 1$ is a unique real number.

However, not every equation in the variables x and y defines a function. The equation $y^2 = x - 1$ does not define y as function of x. Note that if $x = 5$, then $y^2 = 5 - 1 = 4$. This means that y could be either 2 or -2 since $2^2 = (-2)^2 = 4$. Thus, more than one y value is assigned to $x = 5$.

Example:

Decide whether or not each equation defines y as a function of x.

(a) $2x^2 + 3y^2 = 15$

(b) $y = 3x^2 + 2x + 1$

Solution:

(a) If $x = 0$, then $3y^2 = 15$. Solving this equation for y, we have $y = \pm\sqrt{5}$. This shows that more than one y value is assigned to $x = 0$. Therefore, the equation $2x^2 + 3y^2 = 15$ does not define y as a function of x.

(b) For each real number x, the expression $3x^2 + 2x + 1$ is a unique real number. Thus, for each x, there corresponds exactly one value of y. Therefore, the equation $y = 3x^2 + 2x + 1$ defines y as a function of x.

The Function Notation:

A special notation is reserved for denoting functions. If we know that y is a function of x, we can use the function notation: $y = f(x)$. The symbol $f(x)$ is read as

"f of x." From the example shown above, we know that the equation $y = 3x^2 + 2x + 1$ defines y as a function of x. We can give this function a name, say f, and use the function notation to write $f(x) = 3x^2 + 2x + 1$.

Evaluating a Function:

If f is a function of x, then the variable x represents elements in the domain of f. For each x in the domain of f, the number $f(x)$ is the value of f at x and represents the corresponding element in the range of f.

The function f given by $f(x) = x^4$ assigns to each real number x the fourth power of x.

The domain of f is the set $\mathbb{R}$ of real numbers. Since $f(x) = x^4 \geq 0$ for all real numbers x, the range of f is the set $[0, \infty)$.

To evaluate f at the numbers −3, 0, and 5, substitute those numbers for x in $f(x) = x^4$.

$f(-3) = (-3)^4 = 81$	The value of f at -3 is 81.
$f(0) = (0)^4 = 0$	The value of f at 0 is 0.
$f(5) = (5)^4 = 625$	The value of f at 5 is 625.

Example:

Evaluate the function $f(x) = 2x^2 + 3x - 4$ at the numbers $-2, -\frac{1}{2}$, 2, and $\sqrt{2}$.

Solution:

$$\begin{aligned} f(-2) &= 2(-2)^2 + 3(-2) - 4 \\ &= 2(4) - 6 - 4 \\ &= 8 - 6 - 4 \\ &= -2 \end{aligned}$$

$$\begin{aligned} f\left(-\frac{1}{2}\right) &= 2\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) - 4 \\ &= 2\left(\frac{1}{4}\right) - \frac{3}{2} - 4 \end{aligned}$$

$$
\begin{aligned}
&= \frac{2}{4} - \frac{3}{2} - 4 \\
&= \frac{1}{2} - \frac{3}{2} - \frac{8}{2} \\
&= -\frac{10}{2} \\
&= -5
\end{aligned}
$$

$$
\begin{aligned}
f(2) &= 2(2)^2 + 3(2) - 4 \\
&= 2(4) + 6 - 4 \\
&= 8 + 6 - 4 \\
&= 10
\end{aligned}
$$

$$
\begin{aligned}
f\left(\sqrt{2}\right) &= 2\left(\sqrt{2}\right)^2 + 3\left(\sqrt{2}\right) - 4 \\
&= 2(2) + 3\sqrt{2} - 4 \\
&= 4 + 3\sqrt{2} - 4 \\
&= 3\sqrt{2}
\end{aligned}
$$

Example:

If $g(x) = \dfrac{1-x}{1+x}$, evaluate $g(2)$, $g(-2)$, and $g(a)$.

Solution:

$$g(2) = \frac{1-2}{1+2} = \frac{-1}{3} = -\frac{1}{3}$$

$$g(-2) = \frac{1-(-2)}{1+(-2)} = \frac{3}{-1} = -3$$

$$g(a) = \frac{1-a}{1+a}$$

Difference Quotients:

For a given function f, an expression of the form

$$\frac{f(x+h)-f(x)}{h} \quad \text{or} \quad \frac{f(x)-f(a)}{x-a}$$

is called a difference quotient.

Example:

What is the difference quotient, $\dfrac{f(x+h)-f(x)}{h}$, for $f(x)=-5x+2$?

Solution:

$$\frac{f(x+h)-f(x)}{h}=\frac{-5(x+h)+2-(-5x+2)}{h}$$ To find $f(x+h)$, substitute $x+h$ for x in the expression $-5x+2$.

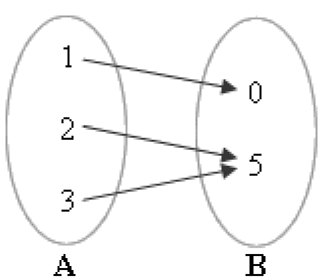
$$=\frac{-5x-5h+2+5x-2}{h}$$ Use the distributive property in the numerator.

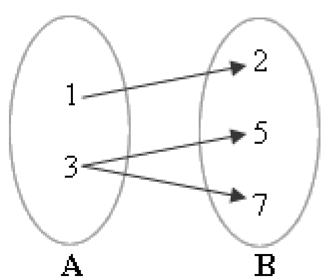
$$=\frac{-5\cancel{h}}{\cancel{h}}$$ Simplify in the numerator.

$$=-5$$ Simplify.

Additional Example 1:

State whether or not each of the mappings shown below represents a function.





Solution:

Describe the first diagram.

1 is assigned to 0.

2 is assigned to 5.

3 is assigned to 5.

The first mapping represents a function since each element of $A = \{1, 2, 3\}$ is assigned to exactly one element of $B = \{0, 5\}$.

Describe the second diagram.

1 is assigned to 2.
3 is assigned to both 5 and 7.

The second mapping does not represent a function since not every element of $A = \{1, 3\}$ is assigned to exactly one element of $B = \{2, 5, 7\}$. 3 in A is assigned to two elements, 5 and 7, in B.

Additional Example 2:

Decide whether or not each equation defines y as a function of x.

(a) $y = |x^2 - 9|$

(b) $y^2 = x - 2$

Solution:

(a) For each real number x, the expression $|x^2 - 9|$ represents a unique real number.

Therefore, the equation does define a function y in terms of x since there is assigned exactly one value y for each x.

(b) Substitute 3 for x in the equation $y^2 = x - 2$. Then $y^2 = 3 - 2 = 1$. Solving this equation for y, we get $y = \pm 1$.

Therefore, the equation does not define a function y in terms of x since $x = 3$ leads to more than one value of y.

Additional Example 3:

Express the rule "multiply by 5, subtract 12, then square" in function notation.

(For example, "subtract 3, then take the square root" would be written as $f(x) = \sqrt{x - 3}$.)

Solution:

For the first step, begin with x and multiply by 5. $5x$

Next subtract 12 from $5x$. $5x-12$

For the last step square $5x-12$. $(5x-12)^2$

Now use function notation. $f(x)=(5x-12)^2$

Additional Example 4:

For the function f given by $f(x)=5x-2$, evaluate $f(-2)$, $f(0)$, and $f(3)$.

Solution:

To find $f(-2)$, substitute -2 for x.

$$\begin{aligned} f(x)&=5x-2\\ f(-2)&=5(-2)-2\\ &=-10-2\\ &=-12 \end{aligned}$$

To find $f(0)$, substitute 0 for x.

$$\begin{aligned} f(x)&=5x-2\\ f(0)&=5(0)-2\\ &=0-2\\ &=-2 \end{aligned}$$

To find $f(3)$, substitute 3 for x.

$$\begin{aligned} f(x)&=5x-2\\ f(3)&=5(3)-2\\ &=15-2\\ &=13 \end{aligned}$$

Additional Example 5:

For the function f given by $f(x) = (x-1)^2 - 3$, evaluate $f(-1)$, $f(4)$, and $f\left(\frac{1}{2}\right)$.

Solution:

To find $f(-1)$, substitute -1 for x.

$$\begin{aligned} f(x) &= (x-1)^2 - 3 \\ f(-1) &= (-1-1)^2 - 3 \\ &= (-2)^2 - 3 \\ &= 4 - 3 \\ &= 1 \end{aligned}$$

To find $f(4)$, substitute 4 for x.

$$\begin{aligned} f(x) &= (x-1)^2 - 3 \\ f(4) &= (4-1)^2 - 3 \\ &= 3^2 - 3 \\ &= 9 - 3 \\ &= 6 \end{aligned}$$

To find $f\left(\frac{1}{2}\right)$, substitute $\frac{1}{2}$ for x.

$$\begin{aligned} f(x) &= (x-1)^2 - 3 \\ f\left(\frac{1}{2}\right) &= \left(\frac{1}{2} - 1\right)^2 - 3 \\ &= \left(-\frac{1}{2}\right)^2 - 3 \\ &= \frac{1}{4} - 3 \\ &= \frac{1}{4} - \frac{12}{4} \\ &= -\frac{11}{4} \end{aligned}$$

Additional Example 6:

For the function f given by $f(x)=\begin{cases}-3x^2 & \text{if } x<0\\ 6x+1 & \text{if } x\geq 0\end{cases}$, evaluate $f(-4)$, $f(-2)$, $f(0)$, and $f(6)$.

Solution:

If $x<0$, then the value of f at x is given by $-3x^2$. Since $-4<0$, to find $f(-4)$, substitute -4 for x in the expresssion $-3x^2$.

$$\begin{aligned} f(-4) &= -3(-4)^2\\ &= -3(16)\\ &= -48 \end{aligned}$$

Since $-2<0$, to find $f(-2)$, substitute -2 for x in the expresssion $-3x^2$.

$$\begin{aligned} f(-2) &= -3(-2)^2\\ &= -3(4)\\ &= -12 \end{aligned}$$

If $x\geq 0$, then the value of f at x is given by $6x+1$. Since $0\geq 0$, to find $f(0)$, substitute 0 for x in the expresssion $6x+1$.

$$\begin{aligned} f(0) &= 6(0)+1\\ &= 0+1\\ &= 1 \end{aligned}$$

Since $6\geq 0$, to find $f(6)$, substitute 6 for x in the expresssion $6x+1$.

$$\begin{aligned} f(6) &= 6(6)+1\\ &= 36+1\\ &= 37 \end{aligned}$$

Additional Example 7:

Find the difference quotient, $\frac{f(x+h)-f(x)}{h}$ for $f(x)=3x^2+5x-4$.

Solution:

$\frac{f(x+h)-f(x)}{h}$

$= \frac{3(x+h)^2+5(x+h)-4-\left(3x^2+5x-4\right)}{h}$ To find $f(x+h)$, substitute $x+h$ for x in the expression $3x^2+5x-4$.

$= \frac{3\left(x^2+2xh+h^2\right)+5(x+h)-4-\left(3x^2+5x-4\right)}{h}$ Expand $(x+h)^2$.

$= \frac{3x^2+6xh+3h^2+5x+5h-4-3x^2-5x+4}{h}$ Use the distributive property in the numerator.

$= \frac{6xh+3h^2+5h}{h}$ Simplify in the numerator.

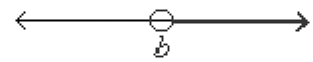
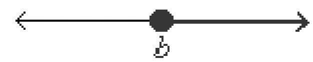
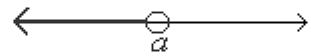
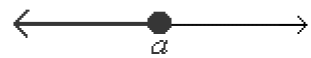
$= \frac{\cancel{h}(6x+3h+5)}{\cancel{h}}$ Factor out an h in the numerator.

$= 6x+3h+5$ Simplify.

Domain and Range of a Function

Review of Interval Notation:

The domain and range for many functions are intervals on the real line. In this case, the domain and range are often specified in interval notation. The chart below gives a summary of interval notation with its corresponding description and graph on the real number line.

Interval Notation	Description of Interval	Graph
(b, ∞)	includes all real numbers x such that x is greater than b $(x > b)$	
$[b, \infty)$	includes all real numbers x such that x is greater than or equal to b $(x \geq b)$	
$(-\infty, a)$	includes all real numbers x such that x is less than a $(x < a)$	
$(-\infty, a]$	includes all real numbers x such that x is less than or equal to a $(x \leq a)$	
$(-\infty, \infty)$	includes all real numbers x	
(a, b)	includes all real numbers x such that x is between a and b $(a < x < b)$	
$[a, b)$	includes all real numbers x such that x is greater than or equal to a and x is less than b $(a \leq x < b)$	
$(a, b]$	includes all real numbers x such that x is greater than a and x is less than or equal to b $(a < x \leq b)$	
$[a, b]$	includes all real numbers x such that x is between and including a and b $(a \leq x \leq b)$	

Finding the Domain of a Function:

If the domain of a function has not been explicitly stated, then by convention the domain is the set of all real numbers for which the expression is defined as a real number.

Example:

Find the domain of the function $f(x) = \sqrt{2x-1}$.

Solution:

For $\sqrt{2x-1}$ to be a real number, $2x-1$ cannot be negative. Solve the inequality $2x-1 \geq 0$ for x to find the domain of f.

$$2x - 1 \geq 0$$
$$2x - 1 + 1 \geq 0 + 1$$
$$2x \geq 1$$
$$\frac{\not{2}x}{\not{2}} \geq \frac{1}{2}$$
$$x \geq \frac{1}{2}$$

The domain in interval notation is $\left[\frac{1}{2}, \infty\right)$.

Example:

Find the domain of the function $f(x) = \dfrac{5}{3x-2}$.

Solution:

Find the values of x where the denominator is 0 by solving the equation $3x - 2 = 0$.

$$3x - 2 = 0$$
$$3x - 2 + 2 = 0 + 2$$
$$3x = 2$$
$$\frac{\not{3}x}{\not{3}} = \frac{2}{3}$$
$$x = \frac{2}{3}$$

The function is defined for all real numbers x except those values of x where the denominator is 0. Thus, the domain is $\left\{x \mid x \neq \frac{2}{3}\right\}$.

The domain in interval notation is $\left(-\infty, \frac{2}{3}\right) \cup \left(\frac{2}{3}, \infty\right)$.

Finding the Range of a Function:

To determine the range of a function $y = f(x)$, find the set of all real numbers y that are obtained by the equation $y = f(x)$ as x varies throughout the domain of f.

Example:

Find the domain and range of the function $y = \sqrt{x} - 3$.

Solution:

For $\sqrt{x} - 3$ to be a real number, x cannot be negative.

Thus, the domain in interval notation is $[0, \infty)$.

To find the range of the function, we must determine the set of all values of y that are obtained by the equation $y = \sqrt{x} - 3$ for nonnegative values of x.

For each $x \geq 0$, we know that $\sqrt{x} \geq 0$. Subtracting 3 from both sides of this inequality, we see that $\sqrt{x} - 3 \geq -3$.

Thus, the range of the function in interval notation is $[-3, \infty)$.

Additional Example 1:

Find the domain of the function $f(x) = \sqrt{8x - 2}$. Express the answer in interval notation.

Solution:

For $\sqrt{8x - 2}$ to be a real number, $8x - 2$ cannot be negative. Solve the inequality $8x - 2 \geq 0$ for x to find the domain of f.

$$8x - 2 \geq 0$$
$$8x - 2 + 2 \geq 0 + 2$$
$$8x \geq 2$$
$$\frac{\cancel{8}x}{\cancel{8}} \geq \frac{2}{8}$$
$$x \geq \frac{1}{4}$$

The domain in interval notation is $\left[\frac{1}{4}, \infty\right)$.

Additional Example 2:

Find the domain of the function $f(x) = \sqrt{4-3x}$. Express the answer in interval notation.

Solution:

For $\sqrt{4-3x}$ to be a real number, $4-3x$ cannot be negative. Solve the inequality $4-3x \geq 0$ for x to find the domain of f.

$$4-3x \geq 0$$
$$4-3x-4 \geq 0-4$$
$$-3x \geq -4$$
$$\frac{\cancel{-3}x}{\cancel{-3}} \leq \frac{-4}{-3}$$
$$x \leq \frac{4}{3}$$

The domain in interval notation is $\left(-\infty, \frac{4}{3}\right]$.

Additional Example 3:

Find the domain of the function $f(x) = \dfrac{1}{4-9x}$. Express the answer in interval notation.

Solution:

Find the values of x where the denominator is 0 by solving the equation $4-9x=0$.

$$4-9x = 0$$
$$4-9x-4 = 0-4$$
$$-9x = -4$$
$$\frac{\cancel{-9}x}{\cancel{-9}} = \frac{-4}{-9}$$
$$x = \frac{4}{9}$$

The function is defined for each real number x except those values of x where the denominator is 0. Thus, the domain is $\left\{x \mid x \neq \frac{4}{9}\right\}$.

The domain in interval notation is $\left(-\infty, \frac{4}{9}\right) \cup \left(\frac{4}{9}, \infty\right)$.

Additional Example 4:

Find the domain of the function $f(x) = \dfrac{x}{x^2 - 8x + 7}$. Express the answer in interval notation.

Solution:

Find the values of x where the denominator is 0 by solving the equation $x^2 - 8x + 7 = 0$.

$$x^2 - 8x + 7 = 0$$
$$(x-1)(x-7) = 0$$
$$x - 1 = 0 \quad \text{or} \quad x - 7 = 0$$
$$x - 1 + 1 = 0 + 1 \qquad x - 7 + 7 = 0 + 7$$
$$x = 1 \qquad x = 7$$

The function is defined for each real number x except those values of x where the denominator is 0. Thus, the domain is $\{x \mid x \neq 1, x \neq 7\}$.

The domain in interval notation is $(-\infty, 1) \cup (1, 7) \cup (7, \infty)$.

Additional Example 5:

Find the domain of the function $f(x) = \sqrt{x^2 + x - 12}$. Express the answer in interval notation.

Solution:

For $\sqrt{x^2 + x - 12}$ to be a real number, $x^2 + x - 12$ cannot be negative. Solve the inequality $x^2 + x - 12 \geq 0$ to find the domain of f.

$x^2 + x - 12 \geq 0$

$(x+4)(x-3) \geq 0$

Solve the corresponding equation: $(x+4)(x-3) = 0$.

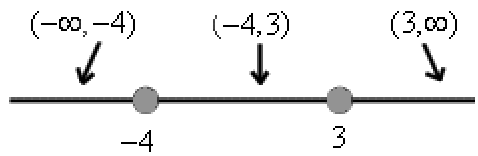
$x+4=0$ or $x-3=0$

$x+4-4=0-4$ $\quad$ $x-3+3=0+3$

$x=-4$ $\quad$ $x=3$

Place -4 and 3 on a number line. The closed circles indicate that the numbers satisfy the inequality $(x+4)(x-3) \geq 0$.

List the intervals determined by -4 and 3.



Check the signs of the factors $x+4$ and $x-3$ on the interval $(-\infty,-4)$ by using a test value.

-5 is in the interval $(-\infty,-4)$.

For $x=-5$,

$x+4=-5+4=-1<0$

and

$x-3=-5-3=-8<0$.

Sign of $x+4$:	$-$
Sign of $x-3$:	$-$
Sign of $(x+4)(x-3)$:	$+$

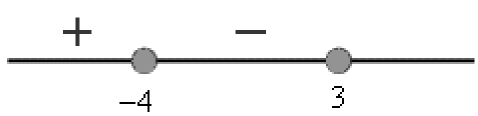
Show the results on the number line.

Check the signs of the factors $x+4$ and $x-3$ on the interval $(-4,3)$ by using a test value.

0 is in the interval $(-4,3)$.
For $x=0$,
$x+4=0+4=4>0$
and
$x-3=0-3=-3<0$.

Sign of $x+4$:	$+$
Sign of $x-3$:	$-$
Sign of $(x+4)(x-3)$:	$-$

Show the results on the number line.

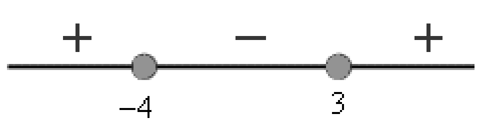


Check the signs of the factors $x+4$ and $x-3$ on the interval $(3,\infty)$ by using a test value.

4 is in the interval $(3,\infty)$.
For $x=4$,
$x+4=4+4=8>0$
and
$x-3=4-3=1>0$.

Sign of $x+4$:	$+$
Sign of $x-3$:	$+$
Sign of $(x+4)(x-3)$:	$+$

Show the results on the number line.



The domain of the function is $\{x \mid (x+4)(x-3) \geq 0\}$. Write the domain in interval notation from the number line analysis.

The domain in interval notation is $(-\infty, -4] \cup [3, \infty)$.

Additional Example 6:

Find the domain and range of the function $y = |x| + 4$. Express each answer in interval notation.

Solution:

For <u>each</u> real number x, the expression $|x| + 4$ is a real number. Thus, the domain of the function in interval notation is $(-\infty, \infty)$.

To find the range of the function, we must determine the set of all values of y that are obtained by the equation $y = |x| + 4$ as x varies throughout the set of real numbers.

For each real number x, we know that $|x| \geq 0$. Adding 4 to both sides of this inequality, we see that $|x| + 4 \geq 4$.

Thus, the range of the function in interval notation is $[4, \infty)$.

Additional Example 7:

Find the domain and range of the function $y = \dfrac{x+4}{x-2}$. Express each answer in interval notation.

Solution:

We first find the domain, the set of all real numbers x for which the expression $\dfrac{x+4}{x-2}$ is a real number. Begin by finding where the denominator is 0 by solving the equation below.

$$\begin{aligned} x - 2 &= 0 \\ x - 2 + 2 &= 0 + 2 \\ x &= 2 \end{aligned}$$

The function is defined for each real number x except those values of x where the denominator is 0.

Therefore, the domain is $\{x \mid x \neq 2\}$. The domain in interval notation is $(-\infty, 2) \cup (2, \infty)$.

We now find the range, which is the set of all real numbers y that are obtained when all real numbers except 2 are substituted for x in the expression $\dfrac{x+4}{x-2}$.
Begin by solving the equation below for x.

$$
\begin{aligned}
y &= \frac{x+4}{x-2} \\
y(x-2) &= \frac{x+4}{x-2}(x-2) \\
y(x-2) &= x+4 \\
yx-2y &= x+4 \\
yx-2y-x &= x+4-x \\
yx-2y-x &= 4 \\
yx-2y-x+2y &= 4+2y \\
yx-x &= 4+2y \\
x(y-1) &= 4+2y \\
\frac{x\cancel{(y-1)}}{\cancel{y-1}} &= \frac{4+2y}{y-1} \\
x &= \frac{4+2y}{y-1}
\end{aligned}
$$

The expression $\dfrac{2y+4}{y-1}$ is a real number when the denominator is not 0. Find the values of y where the denominator is 0 by solving the equation below.

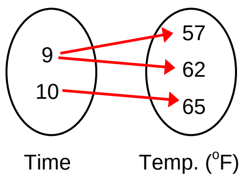
$$
\begin{aligned}
y-1 &= 0 \\
y-1+1 &= 0+1 \\
y &= 1
\end{aligned}
$$

Therefore, the range is $\{y \mid y \neq 1\}$. The range in interval notation is $(-\infty, 1) \cup (1, \infty)$.

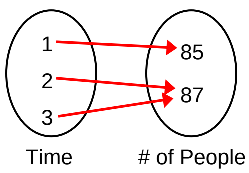
Exercise Set 1.1: An Introduction to Functions

For each of the examples below, determine whether the mapping makes sense within the context of the given situation, and then state whether or not the mapping represents a function.

1. Erik conducts a science experiment and maps the temperature outside his kitchen window at various times during the morning.

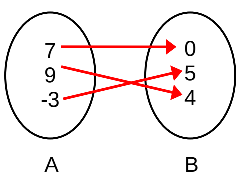


2. Dr. Kim counts the number of people in attendance at various times during his lecture this afternoon.

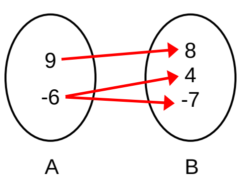


State whether or not each of the following mappings represents a function. If a mapping is a function, then identify its domain and range.

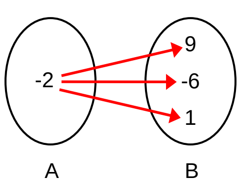
3.



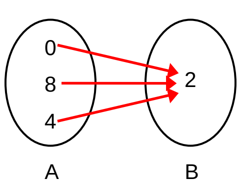
4.



5.



6.



Express each of the following rules in function notation. (For example, "Subtract 3, then square" would be written as $f(x) = (x-3)^2$.)

7. Divide by 7, then add 4

8. Multiply by 2, then square

9. Take the square root, then subtract 6

10. Add 4, square, then subtract 2

Find the domain of each of the following functions. Then express your answer in interval notation.

11. $f(x) = \dfrac{5}{x-3}$

12. $f(x) = \dfrac{x-6}{x+1}$

13. $g(x) = \dfrac{x-4}{x^2-9}$

14. $f(x) = \dfrac{3x+1}{x^2+4}$

15. $f(x) = \dfrac{x^2+6x+5}{x^2-11x+28}$

16. $g(x) = \dfrac{3x+15}{x^2+8x-20}$

17. $f(t) = \sqrt{t}$

18. $h(x) = \sqrt[3]{x}$

19. $g(x) = \sqrt[5]{x}$

20. $h(t) = \sqrt[4]{t}$

21. $f(x) = \sqrt{x-5}$

22. $g(x) = \sqrt{x+7}$

23. $F(x) = \dfrac{\sqrt{3-2x}}{x+4}$

24. $G(x) = \dfrac{\sqrt{x-3}}{x-7}$

25. $f(x)=\sqrt[3]{x-5}$

26. $g(x)=\sqrt[3]{x^2+x-6}$

27. $h(t)=\sqrt[3]{\dfrac{t^2-7t-8}{t+5}}$

28. $f(x)=\sqrt[5]{\dfrac{2x-9}{4x-7}}$

29. $f(t)=\sqrt{t^2-10t+24}$

30. $g(t)=\sqrt{t^2-5t-14}$

Find the domain and range of each of the following functions. Express answers in interval notation.

31. **(a)** $f(x)=\sqrt{x}$
(b) $g(x)=\sqrt{x}-6$
(c) $h(x)=\sqrt{x-6}$
(d) $p(x)=\sqrt{x-6}+3$

32. **(a)** $f(t)=3-t$
(b) $g(t)=3-\sqrt{t}$
(c) $h(t)=\sqrt{3-t}$
(d) $p(t)=\sqrt{3-t}-7$

33. **(a)** $f(x)=x^2-4$
(b) $g(x)=4-x^2$
(c) $h(x)=x^2+4$
(d) $p(x)=\sqrt{x^2-4}$
(e) $q(x)=\sqrt{4-x^2}$
(f) $r(x)=\sqrt{x^2+4}$

34. **(a)** $f(t)=25+t^2$
(b) $g(t)=t^2-25$
(c) $h(t)=25-t^2$
(d) $p(t)=\sqrt{25+t^2}$
(e) $q(t)=\sqrt{t^2-25}$
(f) $r(t)=\sqrt{25-t^2}$

35. **(a)** $f(t)=|t|$
(b) $g(t)=|9-t|$
(c) $h(t)=9-|t|$

36. **(a)** $f(x)=|x|$
(b) $g(x)=|x|+1$
(c) $h(x)=|x+1|$

37. **(a)** $f(x)=|x^2+3|$
(b) $g(x)=|x^2+3|-4$
(c) $h(x)=2|x^2+3|+5$

38. **(a)** $f(t)=|t^2+6|$
(b) $g(t)=|t^2+6|+7$
(c) $h(t)=-\frac{1}{3}|t^2+6|-8$

39. $g(t)=|2x-7|+5$

40. $h(t)=6-|t-1|$

41. $f(x)=\sqrt[3]{5x+6}-4$

42. $g(x)=\sqrt[4]{8-3x}+2$

Find the domain and range of each of the following functions. Express answers in interval notation. ***(Hint: When finding the range, first solve for x.)***

43. **(a)** $f(x)=\dfrac{3}{x+2}$
(b) $g(x)=\dfrac{x+5}{x-2}$

44. **(a)** $f(x)=\dfrac{4}{x-3}$
(b) $g(x)=\dfrac{5x-2}{x-3}$

Exercise Set 1.1: An Introduction to Functions

Evaluate the following.

45. If $f(x)=5x-4$, find:

$f(3),\ f\left(-\frac{1}{2}\right),\ f(a),\ f(a+3),\ f(a)+3,\ f(a)+f(3)$

46. If $f(x)=3x+1$, find:

$f(5),\ f(-8),\ f\left(-\frac{4}{7}\right),\ f(t)-2,\ f(t-2),\ f(t)-f(2)$

47. If $g(x)=x^2-3x+4$, find:

$g(0),\ g\left(-\frac{1}{4}\right),\ g(x+5),\ g\left(\frac{1}{a}\right),\ g(3a),\ 3g(a)$

48. If $h(t)=t^2+2t-5$, find:

$h(1),\ h\left(\frac{3}{2}\right),\ h(c+6),\ -h(x),\ h(2x),\ 2h(x)$

49. If $f(x)=\dfrac{2+x}{x-3}$, find:

$f(-7),\ f(0),\ f\left(\frac{3}{5}\right),\ f(t),\ f(t^2-3)$

50. If $f(x)=\dfrac{x^2}{x+4}-x$, find:

$f(2),\ f(-5),\ f\left(-\frac{7}{4}\right),\ f(3p),\ f(p^3)+2$

51. If $f(x)=\begin{cases}2x-5, & \text{if } x\geq 4\\ 3-x^2, & \text{if } x<4\end{cases}$, find:

$f(6),\ f(2),\ f(-3),\ f(0),\ f(4),\ f\left(\frac{9}{2}\right)$

52. If $f(x)=\begin{cases}x^2+4x, & \text{if } x<-2\\ 7-2x, & \text{if } x\geq -2\end{cases}$, find:

$f(-5),\ f(3),\ f(0),\ f(1),\ f(-2),\ f\left(-\frac{10}{3}\right)$

53. If $f(x)=\begin{cases}3x^2, & \text{if } x<0\\ 4, & \text{if } 0\leq x<2\\ x+5, & \text{if } x\geq 2\end{cases}$, find:

$f(0),\ f(-6),\ f(2),\ f(1),\ f(4),\ f\left(\frac{3}{2}\right)$

54. If $f(x)=\begin{cases}-4x-7, & \text{if } x\leq -1\\ x^2+6, & \text{if } -1<x<3\\ -7, & \text{if } x\geq 3\end{cases}$, find:

$f(0),\ f(-4),\ f(-1),\ f(3),\ f(6),\ f\left(-\frac{5}{3}\right)$

Determine whether each of the following equations defines *y* as a function of *x*. (Do not graph.)

55. $3x-5y=8$

56. $x+3=y^2$

57. $x^2+y=3$

58. $3x^4-2xy=5x$

59. $7x-y^4=5$

60. $3x^2+y^2=16$

61. $x^3y-2y=6$

62. $\sqrt{x}-3y=5$

63. $x^3=5y$

64. $y^3=-7x$

65. $|y|-2=x$

66. $|x|+3y=4$

Exercise Set 1.1: An Introduction to Functions

For each of the following problems:

(a) **Find** $f(x+h)$.

(b) **Find the difference quotient** $\dfrac{f(x+h)-f(x)}{h}$.

(Assume that $h \neq 0$.**)**

67. $f(x) = 7x - 4$

68. $f(x) = 5 - 3x$

69. $f(x) = x^2 + 5x - 2$

70. $f(x) = x^2 - 3x + 8$

71. $f(x) = -8$

72. $f(x) = 6$

73. $f(x) = \dfrac{1}{x}$

74. $f(x) = \dfrac{1}{x-3}$