Demonstrating Reservoir Routing in the Classroom: Physical and Mathematical Modeling

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1.0 Introduction

Reservoir routing is a fundamental topic in engineering hydrology, with applications to flood routing and detention basin design, among others. It is an excellent vehicle for demonstrating basic principles of mass balance, and topics in applied hydraulics (e.g., stage-discharge relationships). In the context of mass balance, reservoir routing provides a focus on the fluid itself, without the complications of multiple transport mechanisms that may control the fluxes of dissolved or suspended chemical constituents. Students often have difficulty with mass balance concepts – relating mass accumulation in a system to inflow and outflow. In reservoir design applications, it is necessary to visualize the relationships among the inflow hydrograph, the outflow hydrograph (as governed by changes in the water surface elevation over time and the hydraulic characteristics of the chosen control structure), and reservoir storage volume (as governed by the water surface elevation and reservoir surface area). A physical bench-scale in-class demonstration using a model reservoir was designed to help students visualize these relationships and develop a deeper understanding of mass balance principles. The model was used to separately demonstrate how to measure elevation-storage and stage-discharge relationships. The scale of the model makes it suitable for real-time, in-class demonstrations and experiments. All required equipment fits on a standard laboratory cart, and can be easily transported to the classroom.

A second objective of the model reservoir was to provide a system of sufficient simplicity to allow mathematical modeling. If a step function is used for the inflow hydrograph, and a vessel having a regular shape is used for the reservoir (e.g., cylinder), the differential mass balance equations can be integrated directly. It has been the author's experience that students find the process of mathematically modeling physical systems quite challenging. One reason is that students seem to have few opportunities to apply the concepts learned in calculus to systems that are real, but simple enough to allow successful application of undergraduate-level mathematics. A second reason stems from the fact that students often have difficulty visualizing the system being modeled. The bench scale system developed here provides an opportunity for such visualization, and facilitates generation of data appropriate for subsequent modeling using either numerical or analytical approaches to solve the governing differential equations.

2.0 Experimental Apparatus and Procedure

The model reservoir, and the equipment required to carry out the classroom demonstration, are shown in Fig. 1. All the required equipment can be transported using a laboratory cart. The model reservoir itself (also shown in detail in Figure 2) is a one-liter glass solvent bottle fitted with a discharge orifice made from a short piece of glass tubing (6-mm outside diameter, 4-mm inside diameter). The discharge is located near the bottom of the bottle. A peristaltic pump with a flow controller was used to provide inflow to the reservoir, and outflow was collected in a plastic tub. While in principle any inflow hydrograph shape could be delivered, step changes in flowrate, resulting in "square wave" hydrographs, are most readily accomplished. Water for the experiment (feed solution) was stored in one-gallon bottles. Water volumes were measured using either a 100-mL or 250-mL graduated cylinder, and flow rates were calculated from the time required to fill a known volume, using a stopwatch and a graduated cylinder. Water surface elevations were measured using a ruler or engineers scale.

Three different experiments were conducted using the model reservoir. First, reservoir storage was related to the water surface elevation. Then, the stage-discharge relationship – outflow as a function of water surface elevation – was measured. Finally, the water surface elevation was measured as a function of time for a specified inflow hydrograph. The results from this final experiment were then predicted using the elevation-storage and stage-discharge relationships.

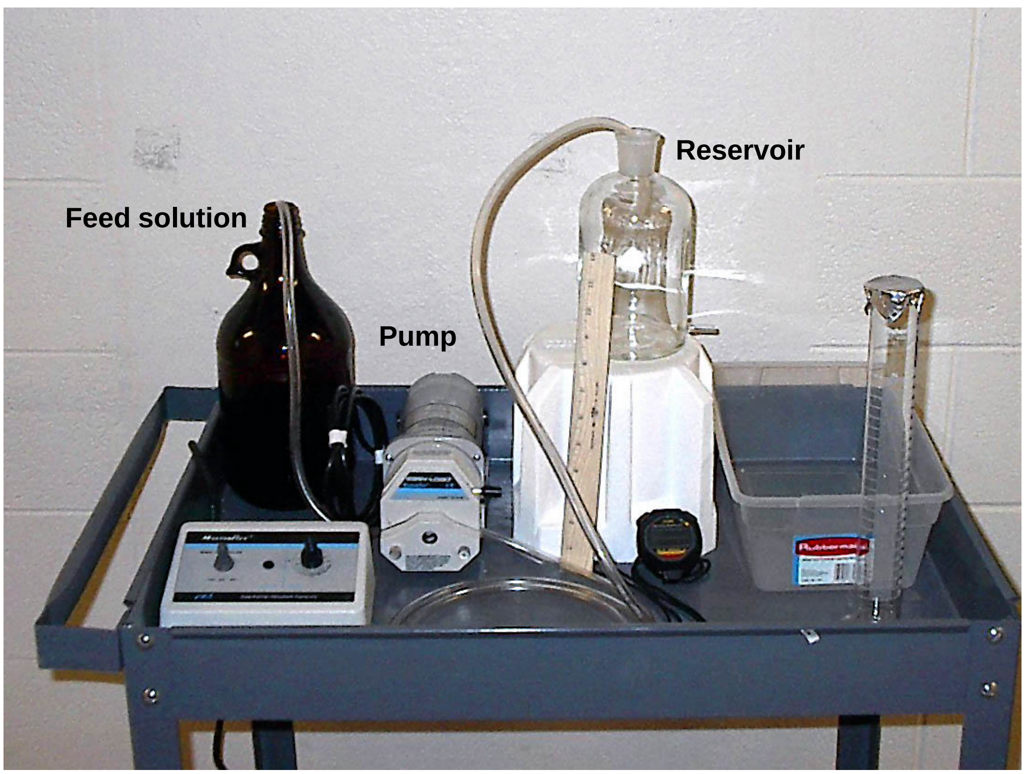

Figure 1. The model reservoir and experimental apparatus.

2.1 *Elevation-Storage Relationship*

Knowledge of the elevation storage relationship is necessary to perform the mass balance calculations required to route a hydrograph through a reservoir. Developing such a relationship for a site with known topography typically involves measuring planimetric area as a function of elevation, e.g., the area corresponding to each contour line is measured and volume is then calculated as the product of an average area times the contour interval. For the model reservoir, this relationship was measured using the following procedure. First, the reservoir was filled partially, and allowed to drain; the resulting water level, corresponding to a "no-flow" condition, was designated as the datum (Fig. 2). Note that this is slightly higher than the centerline of the orifice, due to surface tension effects in the outlet. Then, the outlet was plugged with a rubber septum designed to fit over the 6-mm OD glass tube. The reservoir was then filled in 100 ml increments, using a graduated cylinder, and the elevation above the datum was recorded after each increment.

Figure 2. The model reservoir showing the no flow condition used as an elevation datum .

The storage-elevation curve, well approximated by a linear relationship, is shown in Figure 3. The slope of this plot is the reservoir surface area, in this case equal to about 71 cm^2, independent of elevation. The fact that the reservoir surface area is a constant simplifies subsequent mathematical modeling.

2.2 *Stage-Discharge Relationship*

The next step in the experimental protocol was measurement of the stage discharge relationship. Outflow was measured for several different water surface elevations using a 100 mL graduated cylinder and a stopwatch. First, inflow was initiated with the peristaltic pump operated at a low setting. When the water surface elevation stopped increasing, indicating steady-state conditions (mass inflow = mass outflow; mass accumulation rate = 0), a flow measurement was taken. This was repeated several times, each time after incrementally increasing the inflow rate, to yield stage-discharge data for a range of water surface elevations. Data (symbols) are shown in Figure 4, which also shows a flow model based on the orifice equation (line); the development of this model will be described in detail below.

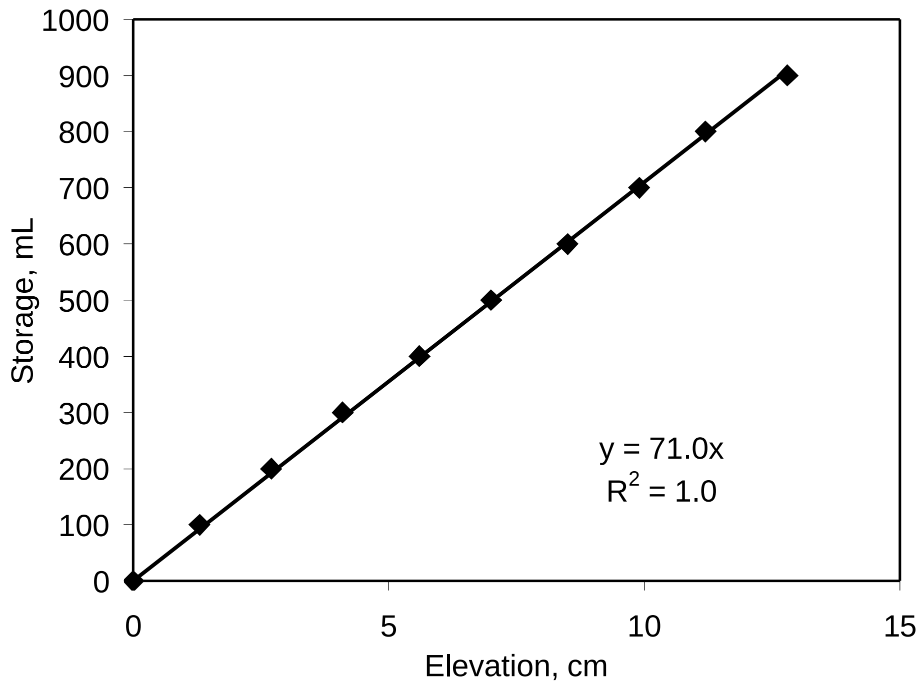

Figure 3. Storage – Elevation curve. Symbols are storage volume data measured with a graduated cylinder; line is a linear regression fit.

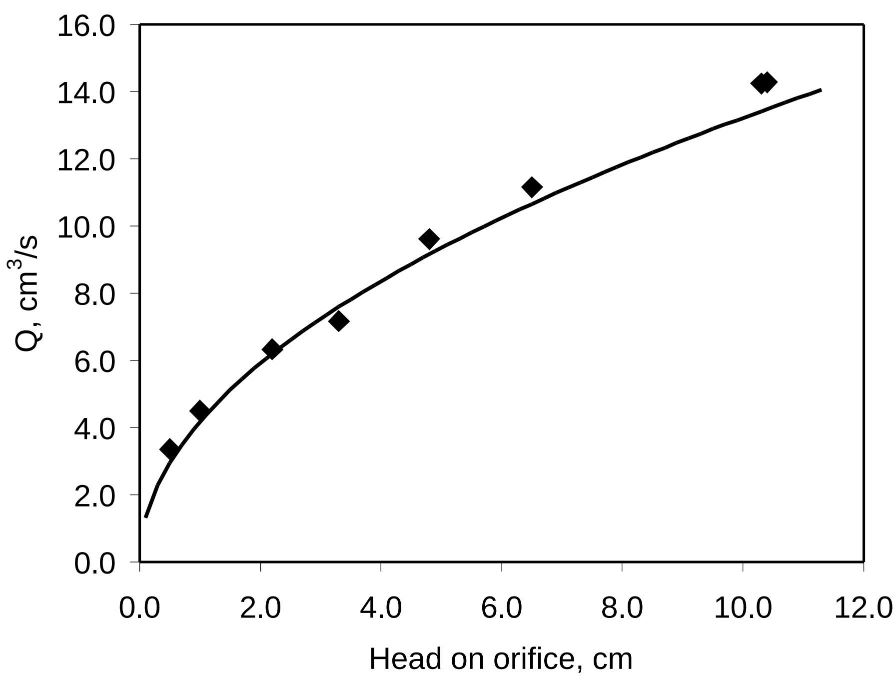

Figure 4. Discharge through the orifice (filled symbols) modeled by the calibrated stage-discharge equation (line).

The final part of the experiment involved measuring water surface elevation as a function of time for a specified starting water level (t = 0) and desired input hydrograph. The starting water level was chosen as that which corresponded to a "no flow" condition. The peristaltic pump was turned on, and the inflow to the model reservoir was kept constant at 738 mL/min. The water level rises rapidly at first, when inflow is much greater than outflow, then it rises more slowly, until reaching a constant value at about 5 minutes. Once this steady-state condition was reached, the inflow was stopped, and the reservoir was allowed to drain. Water surface elevation data was taken during the filling step and during the draining step (each is described by a different mathematical model). This protocol provides a "square wave" input hydrograph. While this shape is not ideally representative of what occurs in nature, it suffices to demonstrate the principles involved, is readily achieved in the classroom, and facilitates subsequent modeling. It should also be noted that while steady-state conditions can be created under controlled conditions, they are rarely seen in nature, because inflow varies continuously.

Several different flow rates could be employed, within the time constraints of a single class, to demonstrate how the steady-state water surface elevation depends on inflow rate. As the inflow rate is increased, the water surface elevation increases, with increasing utilization of storage volume. The opposite occurs when the inflow rate is decreased. This is also a good opportunity to introduce students to some key elements of the reservoir design process. In the model reservoir system, inflow is varied, while the hydraulic characteristics (stage-discharge relationship) of the outlet structure, and the reservoir surface area, remain constant. In a typical design scenario, the design inflow hydrograph is known, and the hydraulic characteristics of the outlet are varied to evaluate the effects of outlet design on the maximum water surface elevation and discharge. This is done for an assumed reservoir surface area, which may be specified, or may also be a design variable. Typical design constraints are maximum discharge and maximum water surface elevation. The design process commences by selecting trial values for the reservoir surface area and outlet hydraulic characteristics. The relationship between the inflow rate and the discharge determines how much storage volume is required; discharge, in turn, is related to the water surface elevation through the stage-discharge relationship. The change in water surface elevation associated with the required storage volume is inversely related to the reservoir area. Therefore, the maximum discharge depends on both the reservoir surface area and the outlet hydraulics. The design process continues by adjusting the outlet hydraulic characteristics and the reservoir surface area until maximum discharge and maximum water surface elevation criteria are met. For a given outlet design, increasing the surface area reduces the maximum water surface elevation, head on the outlet, and maximum discharge. For a given surface area, reducing outlet capacity reduces maximum discharge, but increases required storage volume and maximum water surface elevation.

3.0 Modeling the Model Reservoir System

Reservoir routing calculations are based on a mass balance equation that relates the time rate of change in storage, S, with the difference between inflow, I and outflow, Q (both functions of time):

$$\frac{dS}{dt} = I(t) - Q(t) \qquad (1)$$

In practice, the inflow hydrograph $I(t)$ is either known or predicted from an appropriate surface water hydrologic model (e.g., those incorporated in the Army Corps of Engineers HEC 1 program). In practice, outflow from the reservoir, $Q(t)$ is regulated by a spillway; these are hydraulic structures that can be designed in many different configurations. Common spillway types include straight drop (weir), overflow (ogee), open channel, drop inlet, culvert and siphon configurations. Orifice flow is often and appropriate model for drop inlet, culvert, and siphon spillways under submerged inlet conditions. An orifice is easily fabricated from glass tubing, therefore, such a control "structure" was chosen for the demonstration system.

3.1 *Stage-Discharge Relationship*

Input to a reservoir routing calculation includes relationships between water surface elevation (stage) and storage and between stage and discharge. The development of a stage-discharge model is based on the orifice flow equation. Deriving this equation provides a good opportunity to illustrate the broad utility of Bernoulli's equation. In the simplest development, one point is taken on the reservoir water surface, which specifies $v_1 \approx 0$ and $P_1 = 1$ atm (= 0 gauge pressure). At the orifice discharge, P_2 is also 1 atm (= 0 gauge pressure); therefore, Bernoulli's equation written between these two points is simply:

$$z_1 = \frac{v_2^2}{2g} + z_2 \qquad (2)$$

Combining the continuity relationship, $Q = vA$ yields:

$$Q = v_2 A = A\sqrt{2g(z_1 - z_2)} \qquad (3)$$

Here, the flow area, A, is equal to the orifice area multiplied by a coefficient of contraction, C_c. Letting $z_1 - z_2 = H$, and lumping all constants, this relationship can be simplified to:

$$Q = C_c \frac{\pi d^2}{4}\sqrt{2g}\sqrt{H} = C_d\sqrt{H} \qquad (4)$$

where d is the orifice diameter and C_d can be considered a coefficient of discharge. Calibration of the stage-discharge relationship, Eq. 4, is done by plotting Q versus the square root of the effective head on the orifice (i.e., head above the "no flow" condition). Linearization of the data in this way is shown in Figure 5. The slope of this plot is the discharge coefficient, C_d, here equal to 4.37.

The value of the discharge coefficient can be checked for reasonableness by computing the contraction coefficient:

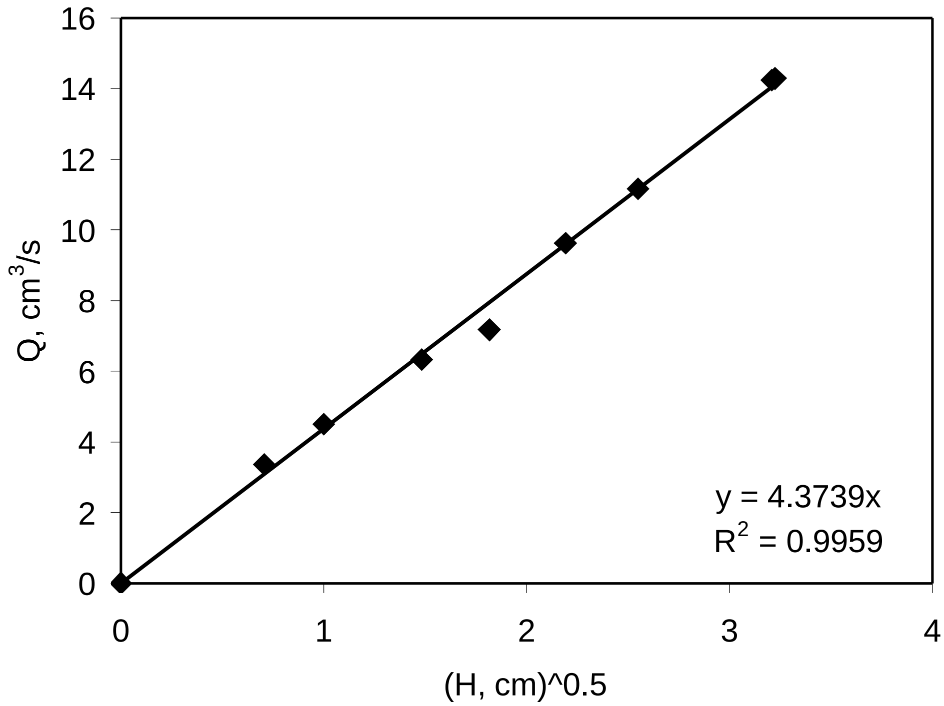

Figure 5. Calibration of the discharge equation. Symbols are flow data; line is a linear regression fit (forced through the intercept). The slope is equal to the discharge coefficient (Eq. 4).

$$C_d = C_c \frac{\pi d^2}{4}\sqrt{2g} = C_c \frac{3.14(0.4\,\text{cm})^2}{4}\sqrt{2(989\,\text{cm/s}^2)} = 5.59 C_c$$

This yields a value for the contraction coefficient of C_c = 4.37/5.59 = 0.78, which is quite reasonable [2]. The discharge model, Eq. 4, is plotted on arithmetic coordinates as a line in Fig. 4. Agreement between the model and the data is satisfactory; however, it can be noted that the agreement is best at smaller values of head. This occurs as a result of the data transformation used to estimate the discharge coefficient, shown in Fig. 5. Taking the square root of the head has the effect of weighting smaller values more heavily. This could be avoided by using a non-linear regression approach.

3.2 *Water Surface Elevation*

A primary objective of any routing exercise is to predict the water surface elevation as a function of time. This is accomplished using a hydrologic routing procedure. One such procedure for reservoirs is the storage-indication (modified Puls) method. This method is based on a numerical discretization of the mass-balance relationship [1]:

$$\frac{dS}{dt} \cong \frac{S_2 - S_1}{\Delta t} = \frac{I_1 + I_2}{2} - \frac{Q_1 + Q_2}{2} \qquad (5)$$

Usually, the inflow hydrograph and initial conditions are given; therefore, I_1, I_2, S_1 and Q_1 are known. Re-arranging to solve for the unknown quantities,

$$\frac{2S_2}{\Delta t} + Q_2 = I_1 + I_2 + 2\frac{S_1}{\Delta t} - Q_1 \qquad (6)$$

Because storage and outflow are known independently as a function of elevation, the quantity $2S/\Delta t + Q$ on the left hand side is known as a function of water surface elevation, $2S/\Delta t + Q = f(\text{stage})$. A numerical value for this quantity can be computed from the known quantities on the right hand side of Eq. 6. Using the computed value for $2S/\Delta t + Q$, the water surface elevation corresponding to the end of the time step is then computed. From this value, S and Q are then computed.

Two important features of the experimental protocol proposed here are that the inflow can be kept constant, and by employing a cylindrical reservoir, the reservoir area does not depend on reservoir stage. Therefore, the mass balance equation can be integrated directly:

$$\frac{dS}{dt} = A\frac{dH}{dt} = I - C_d\sqrt{H} \qquad (7)$$

Separating variables and integrating from H_o, the water surface elevation at $t = 0$:

$$\int_{H_o}^{H} \frac{dH}{I - C_d\sqrt{H}} = \frac{1}{A}\int_{o}^{t} dt \qquad (8)$$

Using the substitution $u = I - C_d\sqrt{H}$ facilitates the integration. The solution is implicit in head but explicit in time:

$$t = \frac{2A}{C_d}\left[\left(H_o^{1/2} - H^{1/2}\right) + \frac{I}{C_d}\ln\left(\frac{I - C_d H_o^{1/2}}{I - C_d H^{1/2}}\right)\right] \qquad (9)$$

After the hydrograph passes (i.e., the flow is stopped), the inflow is zero, and the mass balance equation is written:

$$A\frac{dH}{dt} = -C_d\sqrt{H} \qquad (10)$$

Separating variables and integrating from $H(t)$, the water surface elevation at the time the flow was stopped:

$$\int_{H(t)}^{H} \frac{dH}{\sqrt{H}} = -\frac{C_d}{A} \int_{o}^{t} dt \qquad (11)$$

The solution is:

$$H = \left(\sqrt{H_o} - \frac{C_d t}{2A} \right)^2 \qquad (12)$$

The routing model predictions, Eq. 9 (corresponding to I > 0 for $0 < t < 300$ s) and Eq. 12 (corresponding to I = 0 for $t > 300$ s), are depicted in Figure 6 (lines) with data collected in the laboratory (symbols). Water surface elevation increases while $I > Q$ (filled symbols modeled by Eq. 9) and decreases while $I < Q$ (open symbols modeled by Eq. 12) until the reservoir drains completely (i.e., to the no-flow condition). The agreement between the model and the data is quite satisfactory; however, these results are likely to be somewhat better than can be achieved in the classroom, depending on the extent to which the demonstration can be practiced.

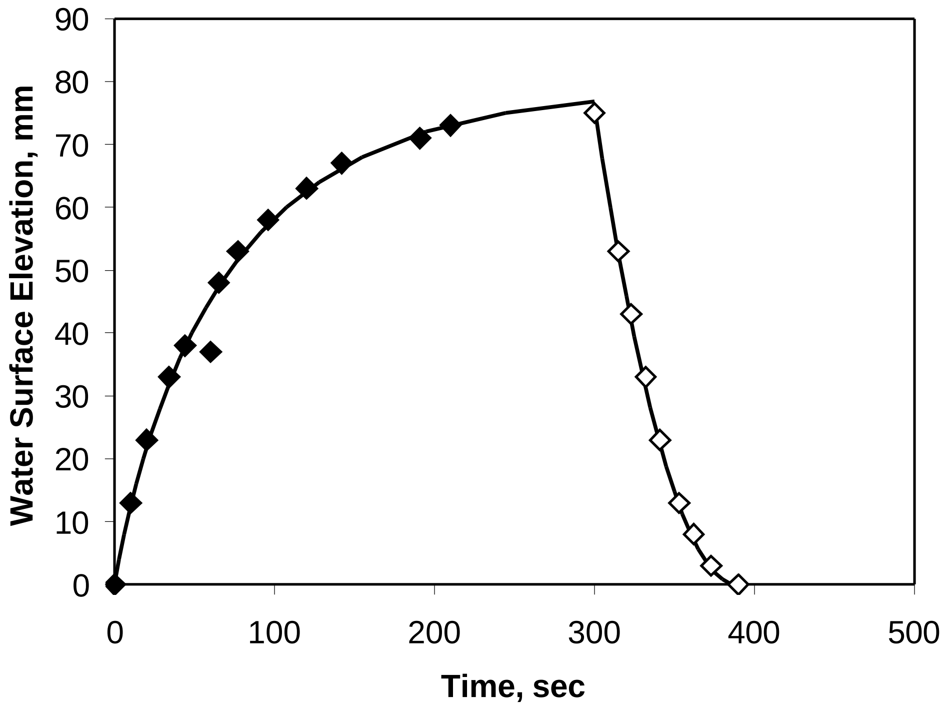

Figure 6. Measured water surface elevations (symbols) and model predictions (lines). Inflow was 12.3 mL/s (738 mL/min) for the first 300 s (5 min), after which time the inflow was stopped.

The time required to measure the stage-discharge and storage-elevation curves is about a half hour, and as shown in Figure 6, the actual routing experiment can be conducted within about five minutes, allowing time to demonstrate the effects of several different flow rates. The entire demonstration with data collection was readily done within an hour and a half class period. An advantage of employing a direct integration approach to solving the differential mass balance equations is that data need not be collected in the classroom using constant time intervals. This is a requirement of the storage-indication method, to facilitate computing the storage-indication function, $2S/\Delta t + Q$.

4.0 Assessment

Limited quantitative assessment data is provided in the form of course and instructor evaluations. Overall instructor evaluations, ability of the instructor to "stimulate interest and motivate learning," and overall course quality all improved after the classroom demonstration was instituted. The improvement seemed significant (initial ratings were 2.8 to 3.1 (out of 4) and later improved to 3.6 to 3.9) but was likely caused by general course improvement over time, to which the in-class demonstration made only a partial contribution. Anecdotal assessment data is provided by student response to, and success in, a design project that involved modeling the Neversink Reservoir watershed, part of the New York City water supply system.

The project included model calibration using topographic, soils, and historical rainfall and streamflow data; model verification using an independent data set; and use of the model to predict the impact of future land-use changes. Then, the students were asked to design a storage reservoir, using the HEC1 program, to reduce the post-development peak flow to the pre-development level. This involved designing the reservoir surface area, embankment height, spillway type, and spillway elevation.

Students were required to optimize their design by minimizing total costs (peak discharge should be as close as possible to pre-development levels, reservoir area should be a minimum, and spillway length should be a minimum) subject to constraints in reservoir area, embankment height, and spillway size. Students seemed better prepared to tackle this design after seeing the model reservoir demonstration, and completing a related homework assignment. They had a much better understanding of the relationships among reservoir size, storage, and discharge, and how these combine to attenuate a flood hydrograph. As a result of completing a modeling assignment related to the in-class demonstration, and being able to visualize the effects of reservoir size and discharge capacity on change in storage and hydrograph attenuation, students were more comfortable engaging in the trial and error design process required.

5.0 Lesson Plan

1. Introduce practical engineering problems that require a solution approach involving mass balances; e.g., flood routing, reservoir routing, and detention basin design.
2. Present the mass balance equation and describe the physical meaning of each term: storage changes at a rate equal to the difference between inflow and outflow; inflow rate is governed by watershed hydrology, and may change as land use changes; outflow rate is governed by the hydraulic characteristics of the spillway and the head on the spillway, i.e., water surface elevation.
3. Describe spillway types, and which types may be modeled using the orifice equation (e.g., submerged drop, siphon, and culvert spillways).

4. Describe the physical phenomena of a hydrograph passing through a reservoir: as inflow exceeds outflow, storage increases, and the water surface elevation rises according to the elevation storage relationship; as the water surface rises, the head on the spillway increases, and the outflow (discharge) increases according to the head-discharge relationship.
5. Describe how the unknown, time-dependent terms in the mass balance equation (storage and outflow) can be put in terms of water surface elevation using measured elevation-storage and head-discharge relationships.
6. Demonstrate how elevation-storage curves are developed using topographic data.
7. Measure the elevation-storage curve for the model reservoir.
8. Develop the orifice equation using Bernoulli's equation, and describe how it can be linearized to obtain the value of the discharge coefficient from experimental data.
9. Measure the head-discharge relationship for the model reservoir.
10. Measure the change in water surface elevation as a function of time for a specified inflow hydrograph. Point out that the maximum water surface elevation occurs under steady state conditions.
11. Demonstrate steady-state water surface elevation for a different flow rate.
12. Discuss design implications – trial and error approach to find reservoir size and spillway characteristics that meet design constraints for a design flood hydrograph.

Bibliography

1) Chin, D. *Water Resources Engineering*, Prentice Hall, New Jersey, 2000.
2) Rouse, H. *Elementary Mechanics of Fluids*, Dover Publications, Inc., New York, 1978.

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Biography

JAMES E. KILDUFF is an Assistant Professor of Environmental Engineering in the Department of Civil and Environmental Engineering at Rensselaer. His research and teaching interests include physicochemical processes, applications of adsorptive and membrane separation processes in water and wastewater treatment and the effects of adsorption on pollutant transport. He may be reached via e-mail at kilduff@rpi.edu.