

DENSITY, PRESSURE, AND BUOYANCY

[DID NOT DISCUSS PASCAL'S PRINCIPLE.]

Test #3 is in two weeks!

- April 12, 7-10pm, Eiesland G24 as usual. Please let me know **BY APRIL 5** if you need a make-up exam.
- **Covers Chapters 7-10.**
 - For gravity section, will only test gravitational force, not escape velocity and altitude-dependent GPE.
- Practice test and equation sheet are on my website.
- **If you're happy with test 1 and 2 scores, you can skip test 3!**

<http://sarahspolaor.faculty.wvu.edu/home/physics-101>

Why does ice float?



And penguins, now that we're talking about it.

This is my first time teaching this class so I don't know the typical background knowledge of attendees. I really just want to know before talking about it if you all already know what density is. I gave you some homeworks on this and you did alright but I want to just query... If 90% of you get this right I'll grade it.

Density clicker trial.

You have a 114.6 kg cube of ice that measures
0.5 x 0.5 x 0.5 meters.
What is the density of ice?

- A. 114.6 kg
- B. 229.2 kg/m²
- C. 458.4 kg/m³
- D. 916.8 kg/m³



Q91

ANSWER: D

Density.

You have a 114.6 kg cube of ice that measures
0.5 x 0.5 x 0.5 meters.
What is the density of ice?

$$\rho = \frac{M}{V} = \frac{\textit{mass}}{\textit{volume}}$$

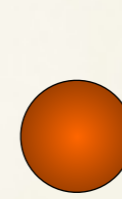
This greek letter
is called "rho"

When writing rho, we usually write it with a little backwards flag at the bottom of it.

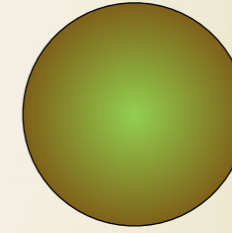
Not necessarily made of same material.

The sphere on the right has twice the mass and twice the radius of the sphere on the left.

Compared to the sphere on the left, the larger sphere on the right has



mass m
radius R



mass $2m$
radius $2R$

$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

$$\rho = \frac{M}{V} = \frac{\text{mass}}{\text{volume}}$$

- A. twice the density.
- B. the same density.
- C. 1/2 the density.
- D. 1/4 the density.
- E. 1/8 the density.



Q92

Answer: D
[See light board notes]

Why does ice float?

- Why is the surface of water always flat?
- Why is sea level the same everywhere?
- Why does oil float on water?
- Why does ANYTHING float?!?



What happens when pressure is uneven?



Weather reporters are always talking about high pressure systems moving in to low pressure systems. Now that we know what pressure is, this should make sense [P=F/A] if pressure is high, [F=PA] the force is high so the force of that high pressure system pushes into that low pressure system. This is SIMPLY NEWTON'S SECOND LAW in action!

Hydrostatic equilibrium

$$P = P_0 + \rho gh$$

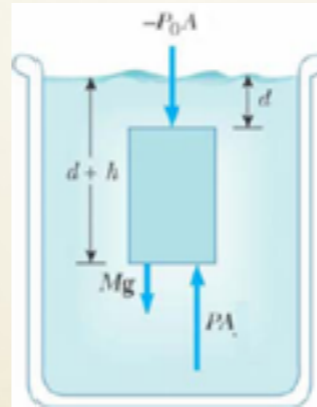
Implications:

Pressure of liquid varies with depth
because of gravity.

Holds things of a given density at a certain
height in a liquid
(keeps them floating)!

Now, we can think about something called “Hydrostatic equilibrium”, which is when a fluid is NOT MOVING. So all of its forces are balanced. We’re going to derive this equation from first principles but I wanted to show you the end product of it first so you can see where we’re going with it. MEANING: Pressure varies with depth because of the gravitational force of the liquid! Specifically, the pressure P at a depth h below the surface of a liquid open to the atmosphere is greater than the atmospheric pressure by the amount ρgh .

Hydrostatic equilibrium: The forces behind the magic.



$$P = P_0 + \rho gh$$

Earth atmospheric pressure
at sea level:
 $P_0 = 1.01 \times 10^5 \text{ Pa}$

Assume:

All the fluid has the same density.
Fluid is not moving around.

[See light board derivation]. I know some of you like derivations and some not, but I wanted to do this because it describes a very basic observable on earth, and I wanted to show you that simply by understanding Newton's laws you can show how this works.

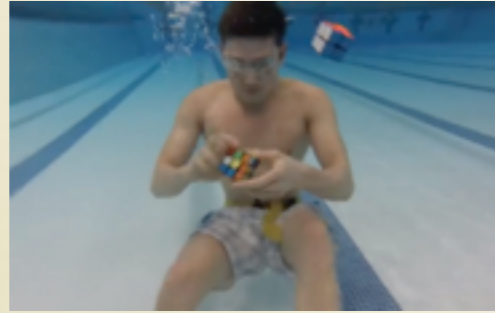
To reiterate:

Pressure varies with depth because of the gravitational force of the liquid! Specifically, the pressure P at a depth h below the surface of a liquid open to the atmosphere is greater than the atmospheric pressure by the amount ρgh . BUT YOU CAN ANALYZE THIS FOR THE PRESSURE DIFFERENCE AT ANY DIFFERENCE IN DEPTH FOR P_0 AND P .

You dive into a lake and start to swim toward the bottom. You feel increasing pressure on your ears as you swim down, and so quickly calculate what depth you can get to before your eardrum will rupture. Eardrums usually rupture at over-pressures above ~50 kPa.

How deep can you swim before this happens?

[area of eardrum ~ 1 cm²]



$$P = P_0 + \rho gh$$

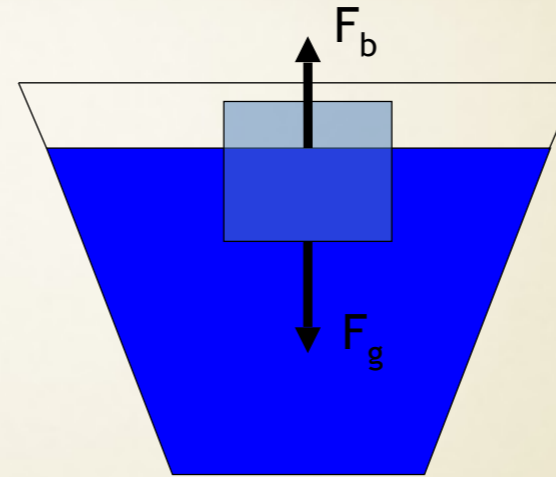
Earth atmospheric pressure
at sea level:

$$P_0 = 1.01 \times 10^5 \text{ Pa}$$

Density of water: 1000 kg/m³

[Light board problem]

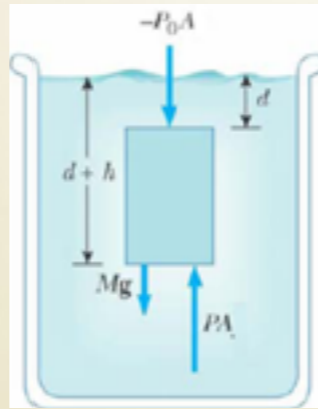
Realize: when you're standing in air, the air inside of your ears is the same pressure as outside of your ears: there's no pressure difference. Go deep underwater or to very high altitude and that changes! You can equalize the pressure by moving extra air into or out of your middle ear from your lungs.



Let's say you throw a weight in a tub. How much water water will it displace?

Will it sink or float?

Famously, Archimedes was asked to determine whether a local tyrant's crown was made of silver or gold without breaking the crown.



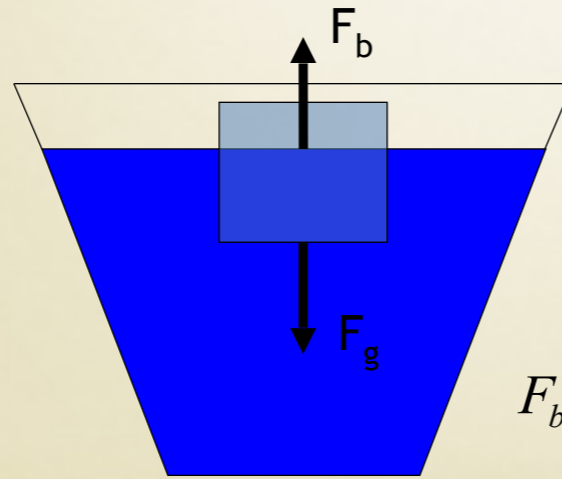
“Buoyant force”

$$F_b = W_{\text{FluidDisplaced}}$$

THINK ABOUT IT if you're looking at the force balance of a small volume of non-mobile water: There are pressures acting upward and downward that equal THE MASS OF THE VOLUME OF FLUID. [LIGHT BOARD DERIV.]

Archimedes' Principle

Any object completely or partially submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced.



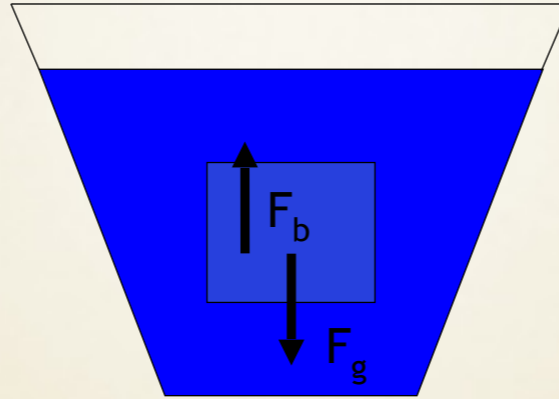
“Buoyant force”

$$F_b = W_{\text{FluidDisplaced}}$$

$$F_b = M_{\text{WaterDisplaced}} g = \rho_{\text{fluid}} g V$$

This leads to Archimedes' principle. [See light board notes]

Totally submerged object



If the density of the fluid is greater than the density of the object, the object floats!

Let's look at the consequences of this, and throw an actual mass in the water.

[Light board notes]



$$\frac{V_{fluid}}{V_{object}} = \frac{\rho_{object}}{\rho_{fluid}}$$

Using this concept allowed Archimedes to determine crown density - eureka!

Two blocks (A and B) have the same size and shape. Block A floats in the water, but Block B sinks in the same water. Which block has the larger buoyant force on it?

$$F_b = W_{\text{FluidDisplaced}}$$

- A. Block A has the larger buoyant force on it.
- B. Block B has the larger buoyant force on it.
- C. Neither; they have the same.
- D. Not enough information



Q93

B, Block B displaces the larger volume of water, so it also has the larger buoyant force on it.



$$\rho_{\text{water}} = 1000 \text{ kg/m}^3$$
$$\rho_{\text{ice}} = 916.8 \text{ kg/m}^3$$

Ice and penguins have a net buoyant force upward.



$$\rho_{\text{penguin}} < \rho_{\text{water}}$$

Why does ice float?

- Why is the surface of water always flat?
[equal pressure at all heights]
- Why is sea level the same everywhere?
[equal pressure at all heights]
- Why does oil float on water?
[it's less dense]
- Why does ANYTHING float?!?
[it's less dense]



How can a steel ship float?



The hull contains mostly air and displaces a lot of water... enough so that $F_b = F_g$ and it floats.

Average density of the ship (including air) is less than water.