DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

End-Sem Exam **16/11/15** **PAPER CODE: A**

Note: Write your name & roll no. on answerbook and on summary-answer-sheet provided with the question paper.
You must submit the summary-answer-sheet along with the answerbook.
Closed book, closed notes exam. No formula sheet allowed. No mobile phones allowed in the exam hall.
All questions carry equal marks. Assume suitable data if required and state the same clearly.
Use formulae from provided tables, **if applicable**.

Problem 1

The rectangular tube shown in **Fig. 1** has a uniform wall thickness of **12 mm**. For the given loading, determine:

(a) the **normal stress** acting **on** the **cross section at** points ***A*** and ***B***.

(b) the **distance from *A* to** the **point where** the **neutral axis intersects** line **AB**.

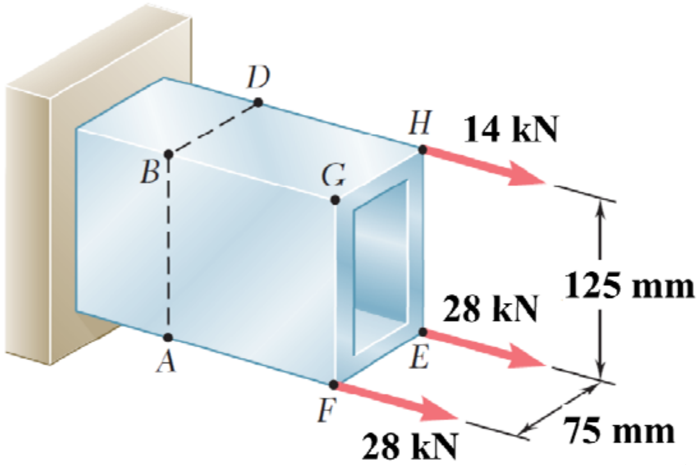

Fig. 1

Problem 2

Two forces are applied to the pipe ***AB*** as shown in **Fig. 2**. The pipe has inner and outer diameters of **35 mm** and **42 mm** respectively. Determine the **maximum shearing stress**

(a) acting **in** the ***xy*** plane at **point a**,

(b) acting **in** the ***zy*** plane at **point b**.

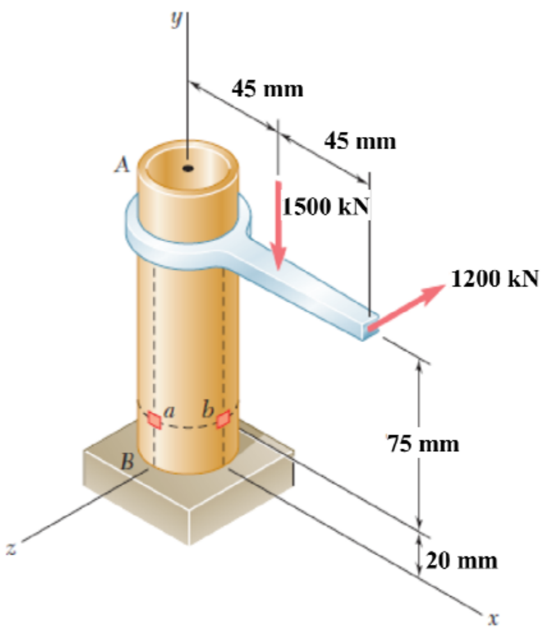

Fig. 2

Problem 3

The compound beam shown in **Fig. 3** has fixed supports at ***A*** and ***D*** and consists of three members that are pinned together at ***B*** and ***C***. Find the vertical deflection of the beam at the point where the load is applied.

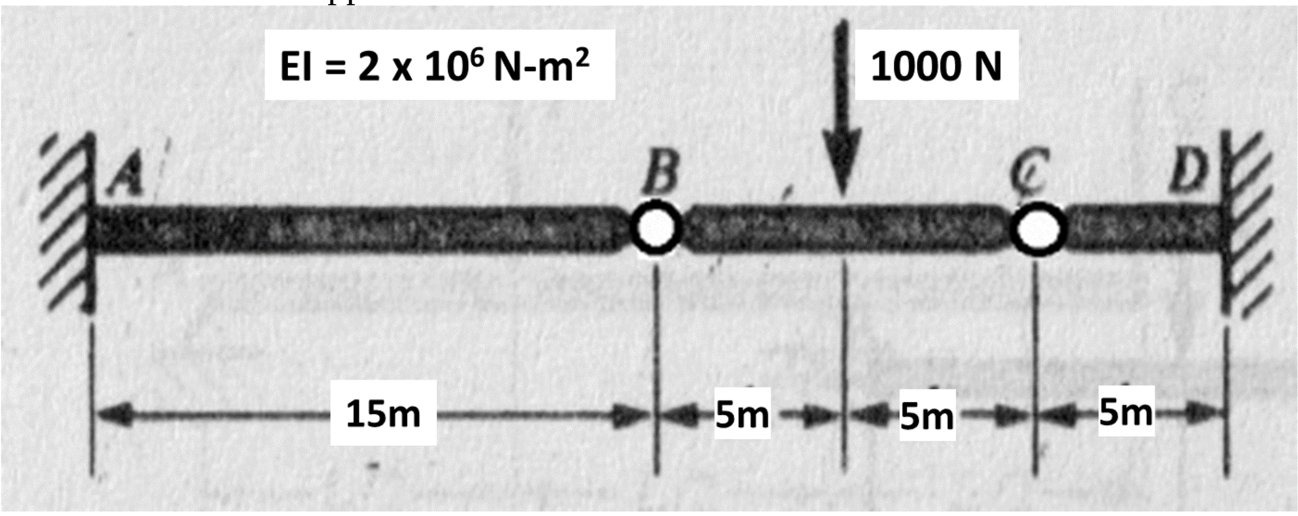

Fig. 3

Problem 4

Three uniform rigid bars of length ***a*** are interconnected by hinges at points **1** and **2** in the rigid bar assemblage shown in **Fig. 4**. The lateral displacements of the assemblage are resisted by linear springs located at each hinge as shown in **Fig. 4**. The assemblage is hinged at point ***O*** and is guided to move vertically by rollers at point **3**. Calculate the critical load ***P*** of the assemblage when $k_1 = k$, $k_2 = 2k$. Your answer should be in terms of ***a*** and ***k*** only.

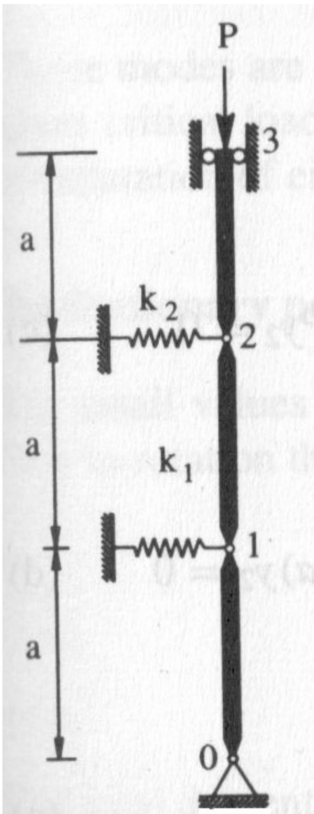

Fig. 4

Problem 5

A cantilever beam carries a downward concentrated load of **10 kN** at its free end. The composite cross-section (see **Fig. 5**) comprises a **300mm x 100mm** rectangular section having Youngs modulus $\boldsymbol{E_1}$**=50 GPa** which is bolted to a symmetric I-section having Youngs modulus $\boldsymbol{E_2}$**=200 GPa**. The cross-sectional area and centroidal moment of inertia of the symmetric I-section are **2.02×10^4 mm^2** and **7.475×10^8 mm^4**, respectively. The bolts are placed two in a row (see **Fig. 5**) and spaced **600mm** apart along the length of the beam. Determine:

(a) The **shear force carried by each bolt**

(b) The **vertical shear stress in** the **rectangular section at** the **interface of the two materials.**

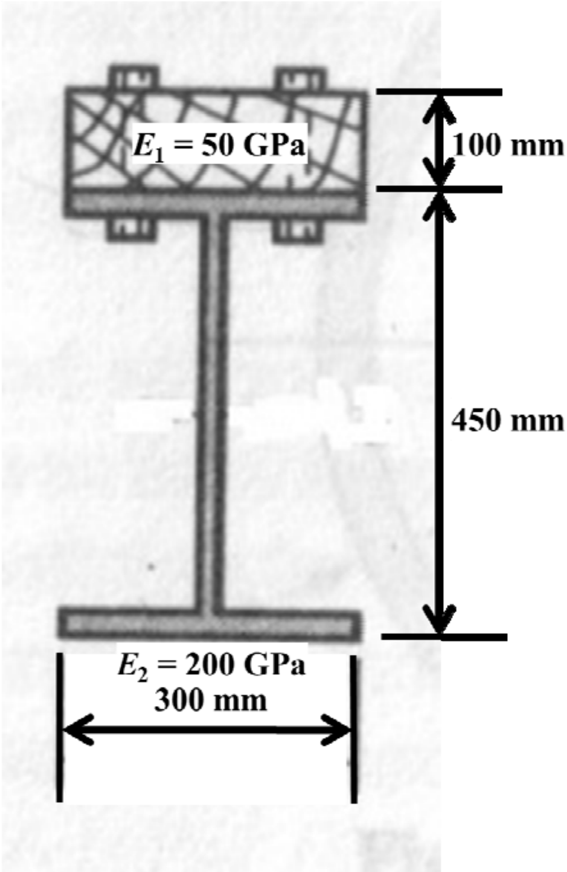

Fig. 5

Problem 6

A simply supported beam carrying a uniform load of **100 kN/m** over its span of **10 m** is made of a solid rectangular section. The allowable normal stress of the beam is **150 MPa**. The maximum deflection of the beam should not exceed **10 mm**. Calculate the width and depth of the cross-section such that the above mentioned capacities are fully utilized. $\boldsymbol{E}$ = **200 GPa.**

SUMMARY ANSWER SHEET

PAPER CODE: A

Name:

Roll no:

Problem 1

(a) σ_A **= 31.52MPa**

σ_B **= -10.39MPa**

(b) **distance from *A* to point where neutral axis intersects** line **AB =** 94.01mm

Problem 2

(a) max shear stress **in *xy*** plane at **a** = 17.58 GPa

(b) max shear stress **in *zy*** plane at **b** = 22.71 GPa

Problem 3

vertical deflection at point where load is applied = 0.15625m

Problem 4

critical load *P* of assemblage = $\left(1-1/\sqrt{3}\right)$=0.4226ka

Problem 5

(a) **shear force carried by each bolt = 3866N**

(b) vertical shear stress in rectangular section at interface = 0.043MPa

Problem 6

width of cross-section = 1.56m

depth of cross-section = 0.02048m

DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

End-Sem Exam **16/11/15** **PAPER CODE: B**

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Use formulae from provided tables, **if applicable**.

Problem 1

The rectangular tube shown in **Fig. 1** has a uniform wall thickness of **14 mm**. For the given loading, determine:

(a) the **normal stress** acting **on** the **cross section at** points ***A*** and ***B***.

(b) the **distance from *A* to** the **point where** the **neutral axis intersects** line **AB**.

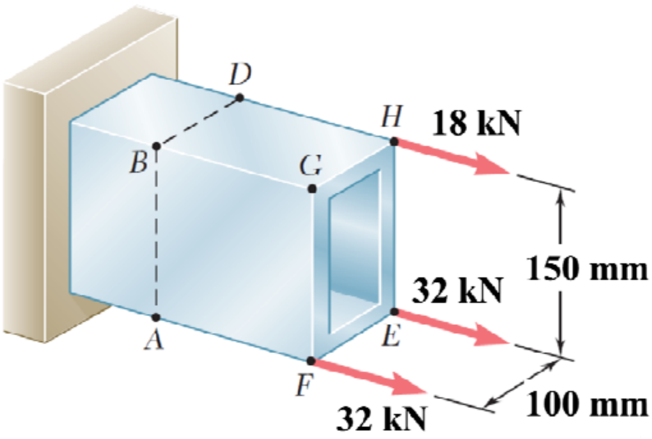

Fig. 1

Problem 2

Two forces are applied to the pipe ***AB*** as shown in **Fig. 2**. The pipe has inner and outer diameters of **32 mm** and **38 mm** respectively. Determine the **maximum shearing stress**

(a) acting **in** the ***xy*** plane at **point a**,

(b) acting **in** the ***zy*** plane at **point b**.

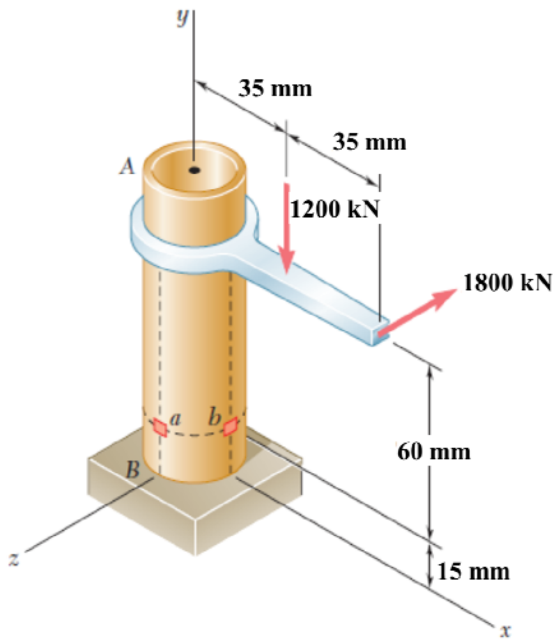

Fig. 2

Problem 3

The compound beam shown in **Fig. 3** has fixed supports at ***A*** and ***D*** and consists of three members that are pinned together at ***B*** and ***C***. Find the vertical deflection of the beam at the point where the load is applied.

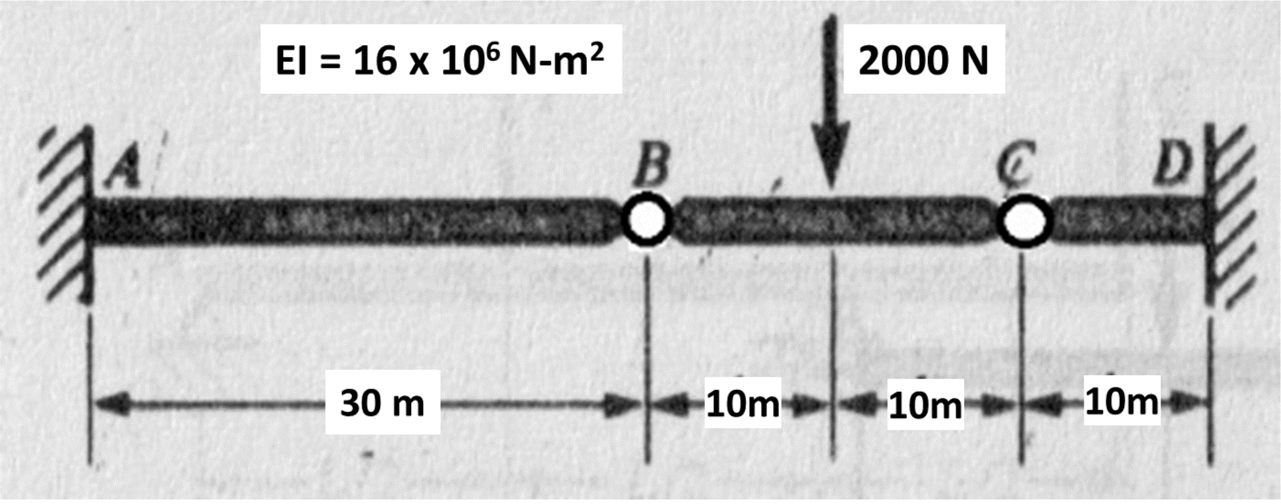

Fig. 3

Problem 4

Three uniform rigid bars of length ***a*** are interconnected by hinges at points **1** and **2** in the rigid bar assemblage shown in **Fig. 4**. The lateral displacements of the assemblage are resisted by linear springs located at each hinge as shown in **Fig. 4**. The assemblage is hinged at point ***O*** and is guided to move vertically by rollers at point **3**. Calculate the critical load ***P*** of the assemblage when $k_1 = 2k$, $k_2 = 3k$. Your answer should be in terms of ***a*** and ***k*** only.

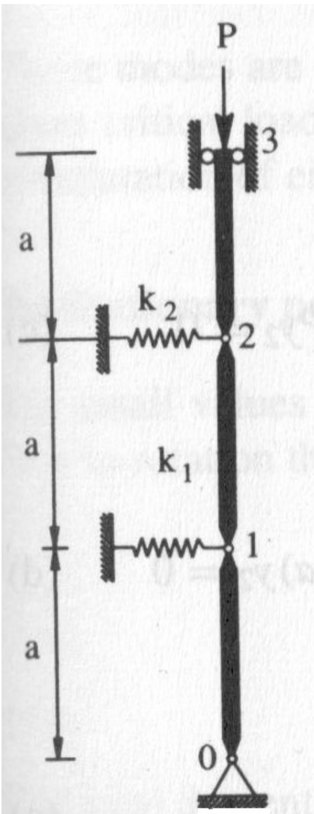

Fig. 4

Problem 5

A cantilever beam carries a downward concentrated load of **12 kN** at its free end. The composite cross-section (see **Fig. 5**) comprises a **300mm x 150mm** rectangular section having Youngs modulus $\boldsymbol{E_1}$**=100 GPa** which is bolted to a symmetric I-section having Youngs modulus $\boldsymbol{E_2}$**=200 GPa**. The cross-sectional area and centroidal moment of inertia of the symmetric I-section are **2.02×10^4 mm^2** and **7.475×10^8 mm^4**, respectively. The bolts are placed two in a row (see **Fig. 5**) and spaced **800mm** apart along the length of the beam. Determine:

(a) The **shear force carried by each bolt**

(b) The **vertical shear stress in** the **rectangular section at** the **interface of the two materials.**

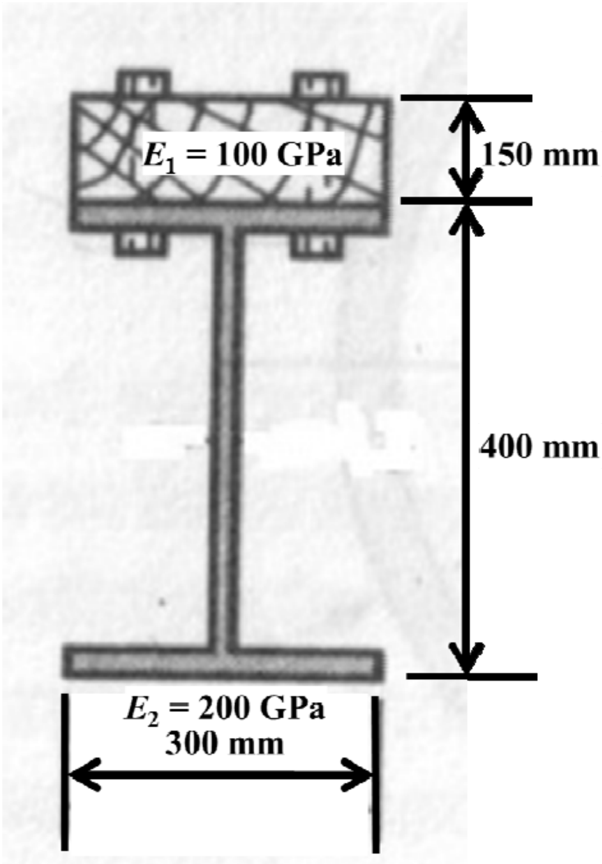

Fig. 5

Problem 6

A simply supported beam carrying a uniform load of **150 kN/m** over its span of **15 m** is made of a solid rectangular section. The allowable normal stress of the beam is **175 MPa**. The maximum deflection of the beam should not exceed **15 mm**. Calculate the width and depth of the cross-section such that the above mentioned capacities are fully utilized. $\boldsymbol{E}$ = **250 GPa.**

SUMMARY ANSWER SHEET

PAPER CODE: B

Name:

Roll no:

Problem 1

(a) σ_A = **23.04MPa**

σ_B = **-6.99MPa**

(b) **distance from *A* to point where neutral axis intersects** line **AB =** 115.07mm

Problem 2

(a) max shear stress **in *xy*** plane at **a** = 29.83 GPa

(b) max shear stress **in *zy*** plane at **b** = 35.77 GPa

Problem 3

vertical deflection at point where load is applied = 0.3125m

Problem 4

critical load *P* of assemblage = $\left(\frac{5-\sqrt{7}}{3}\right)$=0.7848ka

Problem 5

(a) **shear force carried by each bolt = 8810N**

(b) vertical shear stress in rectangular section at interface = 0.073MPa

Problem 6

width of cross-section = 2.1875m

depth of cross-section = 0.03023m

DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

End-Sem Exam **16/11/15** **PAPER CODE: C**

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Use formulae from provided tables, **if applicable**.

Problem 1

The rectangular tube shown in **Fig. 1** has a uniform wall thickness of **10 mm**. For the given loading, determine:

(a) the **normal stress** acting **on** the **cross section at** points ***A*** and ***B***.

(b) the **distance from *A* to** the **point where** the **neutral axis intersects** line **AB**.

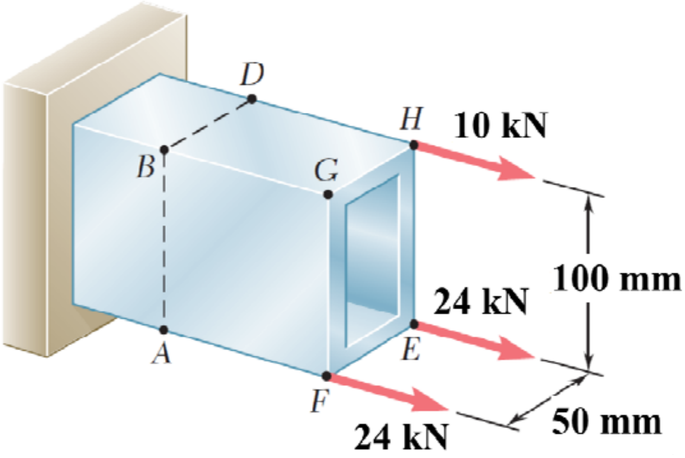

Fig. 1

Problem 2

Two forces are applied to the pipe ***AB*** as shown in **Fig. 2**. The pipe has inner and outer diameters of **42 mm** and **48 mm** respectively. Determine the **maximum shearing stress**

(a) acting **in** the ***xy*** plane at **point a**,

(b) acting **in** the ***zy*** plane at **point b**.

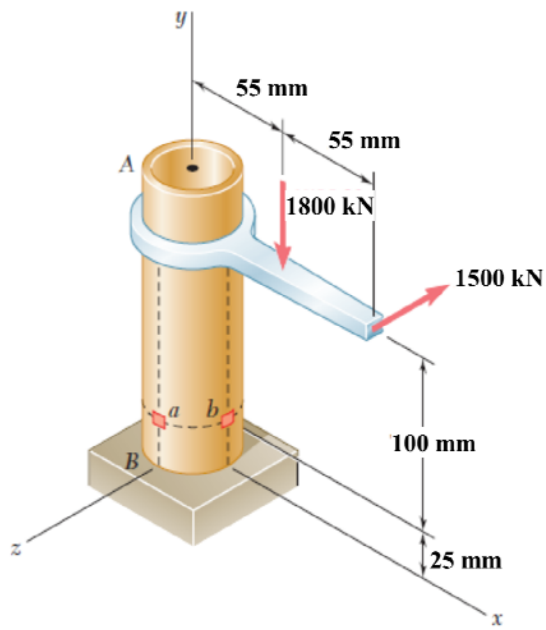

Fig. 2

Problem 3

The compound beam shown in **Fig. 3** has fixed supports at ***A*** and ***D*** and consists of three members that are pinned together at ***B*** and ***C***. Find the vertical deflection of the beam at the point where the load is applied.

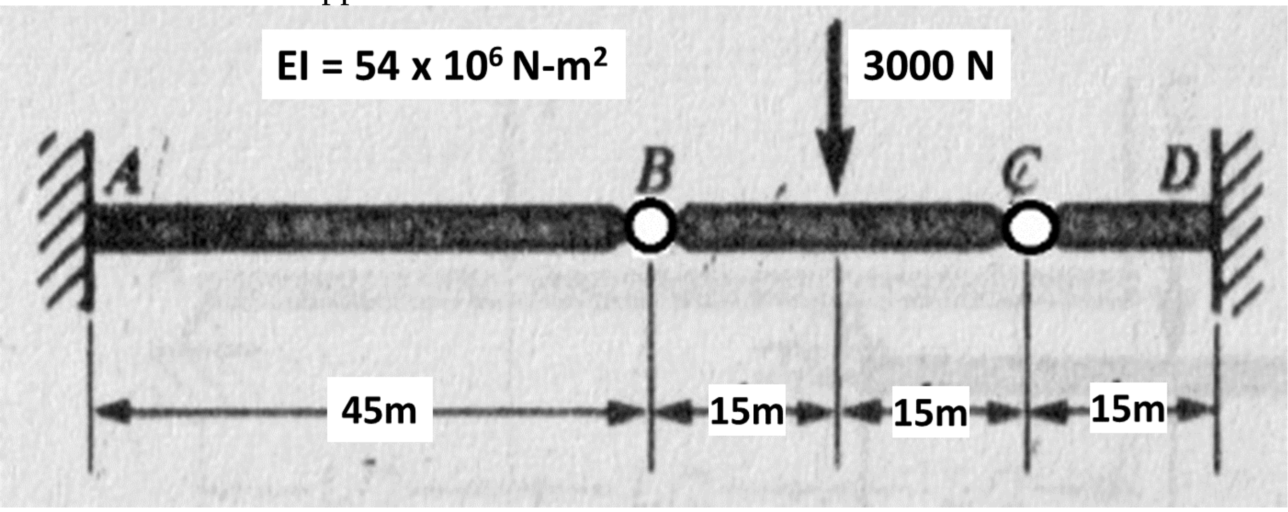

Fig. 3

Problem 4

Three uniform rigid bars of length ***a*** are interconnected by hinges at points **1** and **2** in the rigid bar assemblage shown in **Fig. 4**. The lateral displacements of the assemblage are resisted by linear springs located at each hinge as shown in **Fig. 4**. The assemblage is hinged at point ***O*** and is guided to move vertically by rollers at point **3**. Calculate the critical load ***P*** of the assemblage when $k_1 = 3k$, $k_2 = 4k$. Your answer should be in terms of ***a*** and ***k*** only.

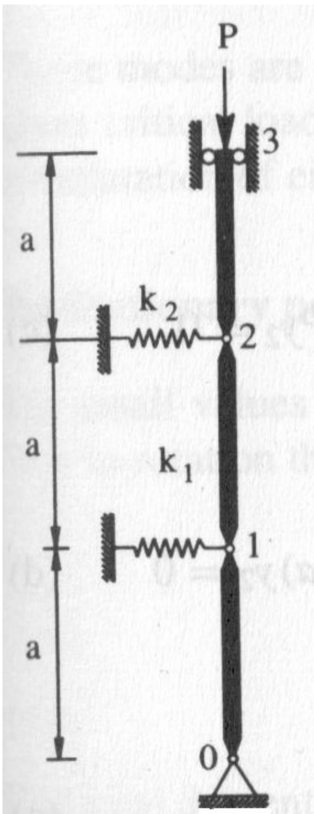

Fig. 4

Problem 5

A cantilever beam carries a downward concentrated load of **18 kN** at its free end. The composite cross-section (see **Fig. 5**) comprises a **300mm x 200mm** rectangular section having Youngs modulus E_1**=75 GPa** which is bolted to a symmetric I-section having Youngs modulus E_2**=200 GPa**. The cross-sectional area and centroidal moment of inertia of the symmetric I-section are **2.02x10^4 mm^2** and **7.475x10^8 mm^4**, respectively. The bolts are placed two in a row (see **Fig. 5**) and spaced **1000mm** apart along the length of the beam. Determine:

(a) The **shear force carried by each bolt**

(b) The **vertical shear stress in** the **rectangular section at** the **interface of the two materials.**

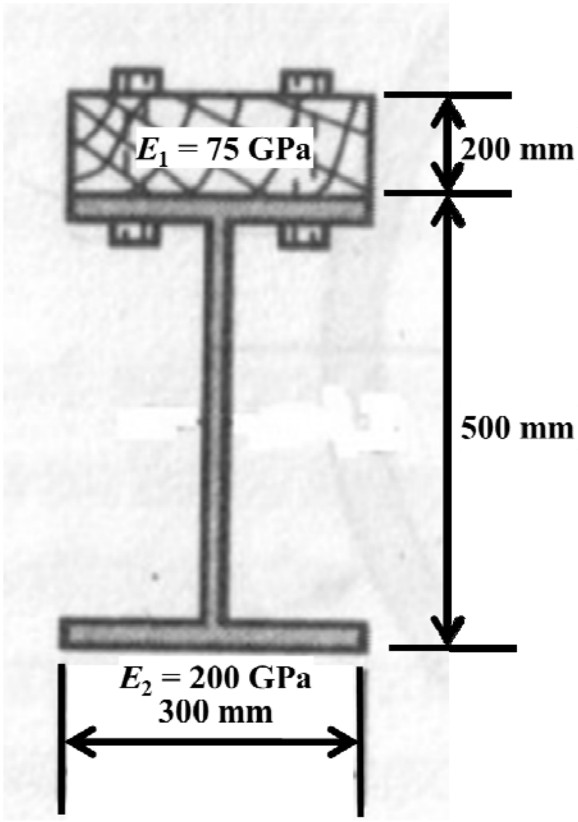

Fig. 5

Problem 6

A simply supported beam carrying a uniform load of **200 kN/m** over its span of **20 m** is made of a solid rectangular section. The allowable normal stress of the beam is **200 MPa**. The maximum deflection of the beam should not exceed **40 mm**. Calculate the width and depth of the cross-section such that the above mentioned capacities are fully utilized. ***E*** **= 150 GPa.**

SUMMARY ANSWER SHEET

PAPER CODE: C

Name:

Roll no:

Problem 1

(a) σ_A = **47.96MPa**

σ_B = **-17.86MPa**

(b) **distance from *A* to point where neutral axis intersects** line **AB = 72.87mm**

Problem 2

(a) max shear stress **in *xy*** plane at **a** = 23.44 GPa

(b) max shear stress **in *zy*** plane at **b** = 28.63 GPa

Problem 3

vertical deflection at point where load is applied = 0.46875m

Problem 4

critical load *P* of assemblage = $\left(\frac{7-\sqrt{13}}{3}\right)$=1.1315ka

Problem 5

(a) **shear force carried by each bolt = 15768N**

(b) vertical shear stress in rectangular section at interface = 0.105MPa

Problem 6

width of cross-section = 2.778m

depth of cross-section = 0.03888m

DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

End-Sem Exam **16/11/15** **PAPER CODE: D**

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Use formulae from provided tables, **if applicable**.

Problem 1

The rectangular tube shown in **Fig. 1** has a uniform wall thickness of **8 mm**. For the given loading, determine:

(a) the **normal stress** acting **on** the **cross section at** points ***A*** and ***B***.

(b) the **distance from *A* to** the **point where** the **neutral axis intersects** line **AB**.

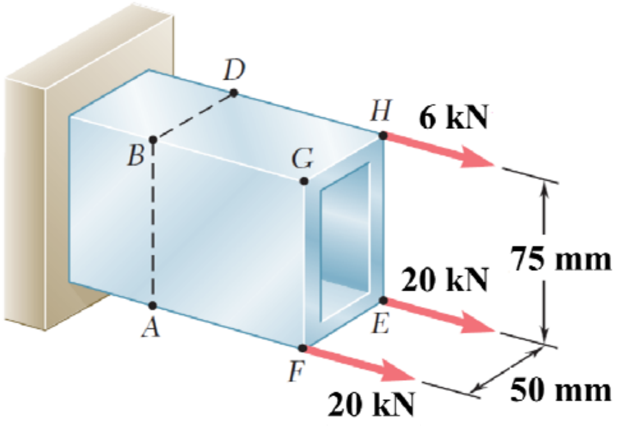


Fig. 1

Problem 2

Two forces are applied to the pipe ***AB*** as shown in **Fig. 2**. The pipe has inner and outer diameters of **28 mm** and **32 mm** respectively. Determine the **maximum shearing stress**

(a) acting **in** the ***xy*** plane at **point a**,

(b) acting **in** the ***zy*** plane at **point b**.

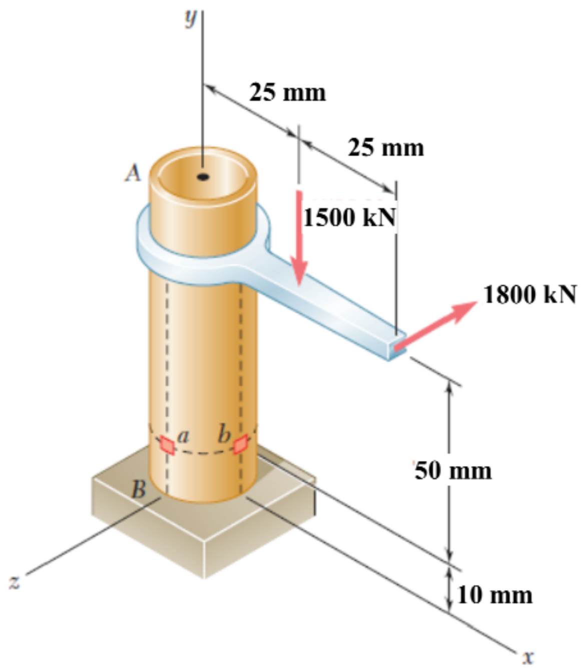


Fig. 2

Problem 3

The compound beam shown in **Fig. 3** has fixed supports at ***A*** and ***D*** and consists of three members that are pinned together at ***B*** and ***C***. Find the vertical deflection of the beam at the point where the load is applied.

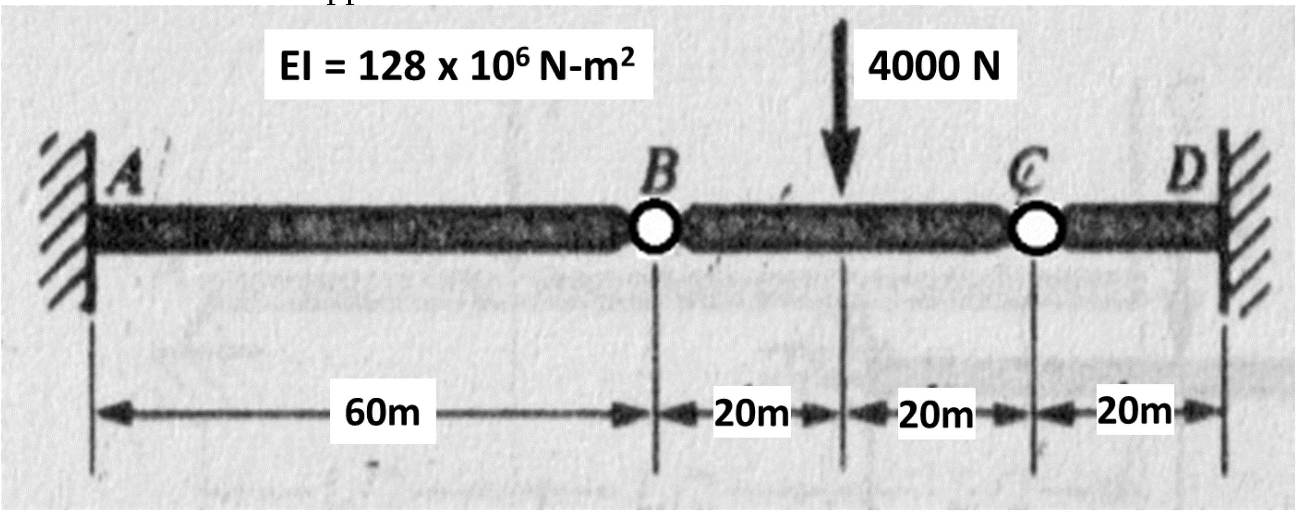


Fig. 3

Problem 4

Three uniform rigid bars of length ***a*** are interconnected by hinges at points **1** and **2** in the rigid bar assemblage shown in **Fig. 4**. The lateral displacements of the assemblage are resisted by linear springs located at each hinge as shown in **Fig. 4**. The assemblage is hinged at point ***O*** and is guided to move vertically by rollers at point **3**. Calculate the critical load ***P*** of the assemblage when $k_1 = 4k$, $k_2 = 5k$. Your answer should be in terms of ***a*** and ***k*** only.

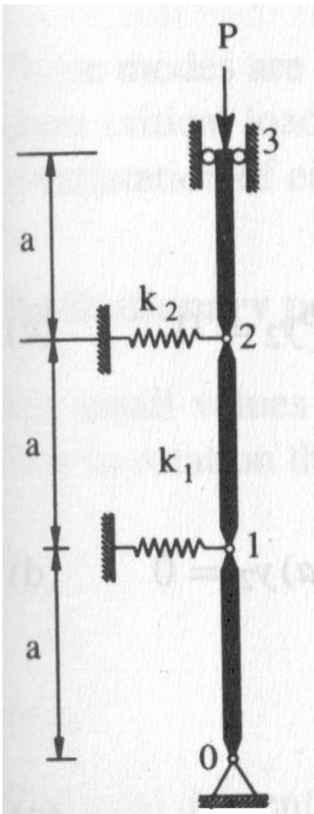


Fig. 4

Problem 5

A cantilever beam carries a downward concentrated load of **30 kN** at its free end. The composite cross-section (see **Fig. 5**) comprises a **300mm x 250mm** rectangular section having Youngs modulus $\boldsymbol{E_1}$**=50 GPa** which is bolted to a symmetric I-section having Youngs modulus $\boldsymbol{E_2}$**=400 GPa**. The cross-sectional area and centroidal moment of inertia of the symmetric I-section are **2.02×10^4 mm^2** and **7.475×10^8 mm^4**, respectively. The bolts are placed two in a row (see **Fig. 5**) and spaced **1200mm** apart along the length of the beam. Determine:

(a) The **shear force carried by each bolt**

(b) The **vertical shear stress in** the **rectangular section at** the **interface of the two materials.**

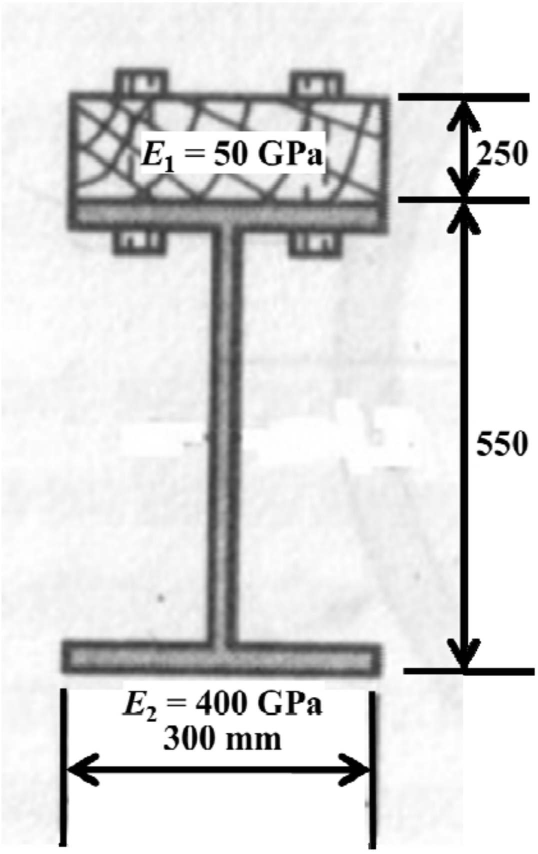


Fig. 5

Problem 6

A simply supported beam carrying a uniform load of **120 kN/m** over its span of **5 m** is made of a solid rectangular section. The allowable normal stress of the beam is **250 MPa**. The maximum deflection of the beam should not exceed **5 mm**. Calculate the width and depth of the cross-section such that the above mentioned capacities are fully utilized. $\boldsymbol{E}$ = **300 GPa.**

SUMMARY ANSWER SHEET

PAPER CODE: D

Name:

Roll no:

Problem 1

(a) σ_A = **60.66MPa**

σ_B = **-20.66MPa**

(b) **distance from *A* to point where neutral axis intersects** line **AB =** 55.94mm

Problem 2

(a) max shear stress **in *xy*** plane at **a** = 45.08 GPa

(b) max shear stress **in *zy*** plane at **b** = 55.90 GPa

Problem 3

vertical deflection at point where load is applied = 0.625m

Problem 4

critical load *P* of assemblage = $\left(\frac{9-\sqrt{21}}{3}\right)$=1.4725ka

Problem 5

(a) **shear force carried by each bolt = 25319N**

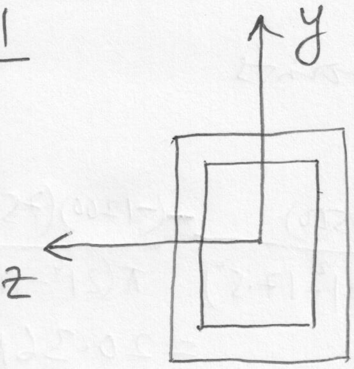
(b) vertical shear stress in rectangular section at interface = 0.141MPa

Problem 6

width of cross-section = 0.8681m

depth of cross-section = 0.01194m

P1



$$P = 14 + 28 + 28 = 70$$

$$M_z = (2)(28)\left(\frac{125}{2}\right) - (14)\left(\frac{125}{2}\right) = 2625 \text{ kN.mm}$$

$$M_y = -(14)\left(\frac{75}{2}\right) = -525$$

$$\sigma_x = \frac{P}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$A = (125)(75) - (125-24)(75-24) = 4224$$

$$I_z = \frac{1}{12}\left[75 * 125^3 - 51 * 101^3\right] = 7828252$$

$$I_y = \frac{1}{12}\left(125 * 75^3 - 101 * 51^3\right] = 3278052 \text{ mm}^4$$

At A: $z = \frac{75}{2}, y = -\frac{125}{2} \Rightarrow (\sigma_x)_A = 31.52$ MPa ◄

At B: $z = \frac{75}{2}, y = \frac{125}{2}, \Rightarrow (\sigma_x)_B = -10.39$ MPa ◄

Put $\sigma_x = 0 \Rightarrow 16.572 - 0.3353y - 0.1602z = 0$

Intercept on AB $\Rightarrow z = \frac{75}{2} \Rightarrow y = 31.51$, ie $(31.51, \frac{75}{2})$

Intercept on BD $\Rightarrow y = \frac{125}{2} \Rightarrow z = -27.37$ ie $(\frac{125}{2}, -27.37)$

Dist from A $= \frac{125}{2} + 31.51 = 94.01$ mm ◄

Code A: $(\sigma_x)_A = 31.52$, $(\sigma_x)_B = -10.39$, Dist from A $= 94.01$ mm

Code B: $= 23.04$, $= -6.99$, $= 115.07$

Code C: $= 47.96$, $= -17.86$, $= 72.87$

Code D: $= 60.66$, $= -20.66$, $= 55.94$

P2 $P = -1500\,N$, $M_y = (1200)(90)$, $M_x = -(1200)(75)$, $M_z = -(1500)(45)$

$V_z = -1200\,N$,

(2)

PLANE STRESS ∵ A, B are boundary points
(xy plane) (zy plane)

At A :
$$\sigma_y = \frac{P}{\pi(r_o^2 - r_i^2)} - \frac{M_x r_o}{\pi(r_o^4 - r_i^4)/4} = \frac{(-1500)}{\pi(21^2 - 17.5^2)} - \frac{(-1200)(75)(21)}{\pi(21^4 - 17.5^4)/4} = 20.36\,MPa$$

$$\tau_{xy} = \text{due to torsion only} = \frac{M_y r_o}{\pi(r_o^4 - r_i^4)/2} = \frac{(1200)(90)(21)}{\pi(21^4 - 17.5^4)/2} = 14.34\,MPa$$

$\sigma_x = 0$

$$\tau_{max} = R = \frac{1}{2}\left(\sqrt{(\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2}\right) = 17.58\,MPa$$ ◀

At B :
$$\sigma_y = \frac{P}{\pi(r_o^2 - r_i^2)} + \frac{M_z r_o}{\pi(r_o^4 - r_i^4)/4} = \frac{(-1500)}{\pi(21^2 - 17.5^2)} + \frac{(-1500)(45)(21)}{\pi(21^4 - 17.5^4)/4} = -21.47\,MPa$$

$$\tau_{yz} = \text{due to torsion + due to bending shear} = \frac{-M_y r_o}{\pi(r_o^4 - r_i^4)/2} + \frac{V_z Q_x}{I_x t_x}$$

$$= \frac{-(1200)(90)(21)}{\pi(21^4 - 17.5^4)/2} + \frac{(-1200)\left(\frac{2\bar{r}}{\pi}\right)(\pi \bar{r})\cancel{(r_o - r_i)}\, t}{2\pi\bar{r}(r_o - r_i)(\bar{r})^2/2\,(r_o - r_i) * 2}, \quad \bar{r} = \text{mean radius.}$$

using thin-walled approx, ie $t \ll \bar{r}$

$$= \; \downarrow \; + \frac{(-1200)}{\pi(19.25)(21 - 17.5)/2} = -25.68\,MPa$$

almost same.

$$= \; \downarrow \; + \frac{(-1200)\left(\frac{\pi r_o^2}{2}\frac{4r_o}{3\pi} - \frac{\pi r_i^2}{2}\frac{4r_i}{3\pi}\right)}{\left(\frac{\pi r_o^2\, r_o^2}{4} - \frac{\pi r_i^2\, r_i^2}{4}\right)(r_o - r_i) * 2} = -25.62\,MPa$$

exact (no thin walled approx used)

$$\tau_{max} = R = \frac{1}{2}\sqrt{21.47^2 + (2 * 25.68)^2} = 27.83\,MPa$$ ◀

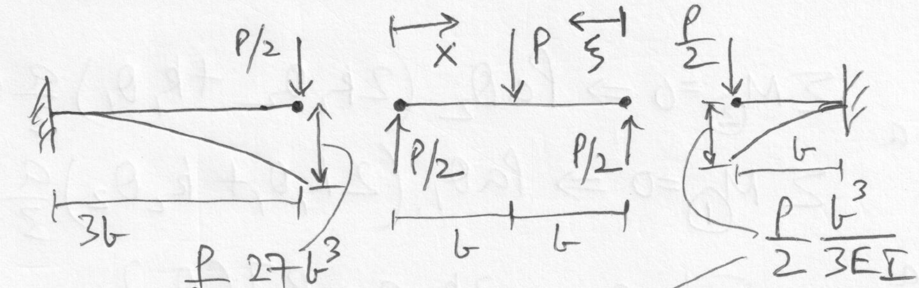
Code A : $(\tau_A)_{max} = 17.58\,MPa$
$(\tau_B)_{max} = 27.83$

Code B : $(\tau_A)_{max} = 29.83$, $(\tau_B)_{max} = 46.37$

Code C : $(\tau_A)_{max} = 23.44\,MPa$
$(\tau_B)_{max} = 35.06$

Code D : $(\tau_A)_{max} = 45.08$
$(\tau_B)_{max} = 74.23$

P3



<u>4th order method</u>

$$EI\,y^{IV} = -W = -P\langle x-b\rangle^{-1}$$
$$EI\,y^{III} = -P\langle x-b\rangle^{0} + C_1$$
$$EI\,y^{II} = -P\langle x-b\rangle^{1} + C_1 x + C_2$$
$$EI\,y^{I} = -\frac{P}{2}\langle x-b\rangle^{2} + \frac{C_1}{2}x^2 + C_2 x + C_3$$
$$EI\,y = -\frac{P}{6}\langle x-b\rangle^{3} + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3 x + C_4$$

$M = EI\,y^{II} = 0$ at $x=0 \Rightarrow C_2 = 0$; $M = EI\,y^{II} = 0$ at $x = 2b \Rightarrow C_1 = \frac{P(2b-b)}{2b} = P/2$

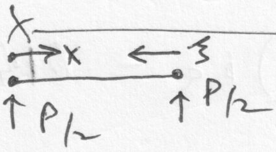
$$y\big|_{x=0} = -\frac{9}{2}\frac{Pb^3}{EI} = \frac{C_4}{EI}$$

$$y\big|_{x=2b} = -\frac{Pb^3}{6EI} = \frac{1}{EI}\left[-\frac{Pb^3}{6} + \frac{P}{2}\frac{8b^3}{6} + 2b\,C_3 - \frac{9}{2}Pb^3\right] \Rightarrow C_3 = \frac{23}{12}Pb^2$$

$$y\big|_{x=b} = \frac{1}{EI}\left[\frac{P}{2}\frac{b^3}{6} + \frac{23}{12}b^2 b - \frac{9}{2}Pb^3\right] = -2.5\frac{Pb^3}{EI}\ (\text{ie.} \downarrow) ◀$$

<u>Code A</u>: $y\big|_{x=b} = 0.15625\,m$; <u>Code B</u>: 0.3125; <u>Code C</u>: 0.46875; Code D: 0.625

<u>2nd order method</u>



$$EI\,y'' = \frac{P}{2}x \qquad (') \equiv \frac{d()}{dx} \qquad\Big|\qquad EI\,y'' = \frac{P}{2}\xi \qquad (') \equiv \frac{d()}{d\xi}$$
$$EI\,y' = \frac{P}{2}\frac{x^2}{2} + C_1 \qquad\Big|\qquad EI\,y' = \frac{P}{2}\frac{\xi^2}{2} + C_3$$
$$EI\,y = \frac{P}{2}\frac{x^3}{6} + C_1 x + C_2 \qquad\Big|\qquad EI\,y = \frac{P}{2}\frac{\xi^3}{6} + C_3\xi + C_4$$

$$y\big|_{x=0} = -\frac{9}{2}\frac{Pb^3}{EI} = \frac{C_2}{EI}; \quad y\big|_{\xi=0} = -\frac{Pb^3}{6EI} = \frac{C_4}{EI}$$

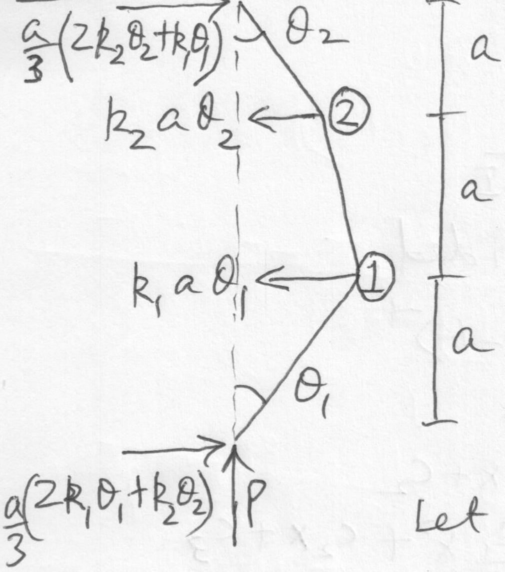
$$y\big|_{x=b} = y\big|_{\xi=b} \Rightarrow \frac{P}{2}\frac{b^3}{6} + C_1 b + C_2 = \frac{P}{2}\frac{b^3}{6} + C_3 b + C_4$$

$$y'\big|_{x=b} = -y'\big|_{\xi=b} \Rightarrow \frac{P}{2}\frac{b^2}{2} + C_1 = -\left(\frac{P}{2}\frac{b^2}{2} + C_3\right)$$

$$\Rightarrow C_1 = C_3 + \frac{C_4 - C_2}{b} \Rightarrow 2C_3 = -\frac{Pb^2}{2} + \frac{C_2 - C_4}{b} \Rightarrow C_3 = -\frac{29}{12}Pb^2;\ C_1 = \frac{23}{12}Pb^2$$

$$\Rightarrow y\big|_{x=b} = \frac{Pb^3}{EI}\left(\frac{1}{12} + \frac{23}{12} - \frac{9}{2}\right) = -2.5\,Pb^3/EI ◀$$

P4



$$\sum M_{②} = 0 \Rightarrow Pa\theta_2 \quad (2k_2\theta_2 + k_1\theta_1)\frac{a}{3}a$$

$$\sum M_{①} = 0 \Rightarrow Pa\theta_1 \quad (2k_1\theta_1 + k_2\theta_2)\frac{a}{3}a$$

$$\begin{bmatrix} k_1\frac{a}{3} & 2k_2\frac{a}{3}-P \\ 2k_1\frac{a}{3}-P & k_2\frac{a}{3} \end{bmatrix}\begin{Bmatrix}\theta_1 \\ \theta_2\end{Bmatrix} = 0$$

det = 0

Let $K_1 = k_1\frac{a}{3}$, $K_2 = k_2\frac{a}{3}$

$$\det = 0 \Rightarrow P^2 - 2(K_1+K_2)P + 3K_1K_2 = 0$$

$$P = (K_1+K_2) \pm \sqrt{K_1^2 + K_2^2 - K_1K_2}$$

−ve sign gives critical (lower load).

Case A : $K_2 = 2K_1 = 2k\frac{a}{3}$, Case B : $K_1 = 2\frac{ka}{3}$; $K_2 = 3k\frac{a}{3}$

Case C : $K_1 = \frac{3ka}{3}$; $K_2 = \frac{4ka}{3}$, Case D : $K_1 = 4\frac{ka}{3}$, $K_2 = 5k\frac{a}{3}$.

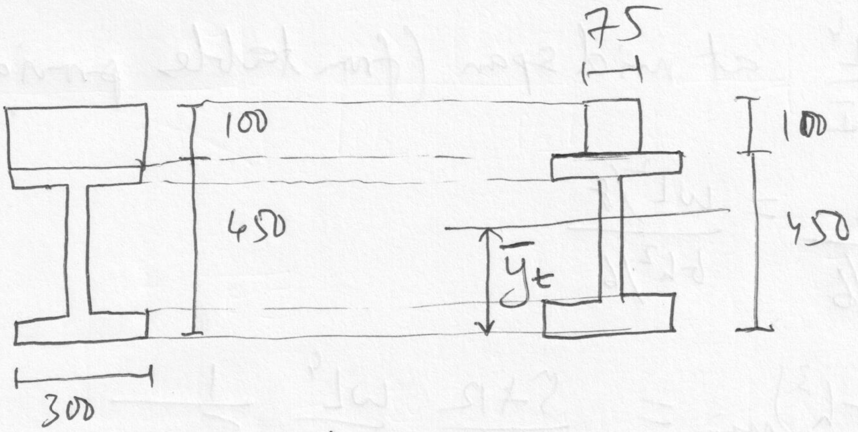
<u>Case A</u> : $P_{cr} = \left(1 - \frac{1}{\sqrt{3}}\right)ka = 0.4226\,ka$

<u>Case B</u> : $P_{cr} = \left(\frac{5-\sqrt{7}}{3}\right)ka = 0.0.7848\,ka$

<u>Case C</u> : $P_{cr} = \left(\frac{7-\sqrt{13}}{3}\right)ka = 1.1315\,ka$

<u>Case D</u> : $P_{cr} = \frac{(9-\sqrt{21})}{3}ka = 1.4725\,ka$

P5



$$\frac{E_1}{E_2} = \frac{1}{4} = \frac{50}{200}$$

for transformed section,

$$\bar{y}_t = \frac{(2.02*10^4)(225) + (100)(300)\left(\frac{1}{4}\right)(500)}{2.02*10^4 + 100*300*\frac{1}{4}} = 299.46\,mm$$

$$I_t = 7.475*10^8 + (2.02*10^4)(225-299.46)^2 + \left(\frac{300}{4}\right)(100)^3\frac{1}{12} + \left(\frac{300}{4}\right)(100)(500-299.46)^2 = 1.167*10^9\,mm^4$$

$$Q_t = \frac{300}{4}*100\,(500-299.46) = 1.504*10^6\,mm^3$$

$$\frac{\Delta H}{\Delta x} = \frac{(10*10^3)(1.504*10^6)}{(1.167*10^9)} = 12.89\,N/mm$$

$$\text{Force in each bolt} = \frac{12.89*600}{2} = 3866\,N$$

$$\tau_{interface} = \frac{12.89}{300} = 0.043\,MPa$$

Code A: Force in each bolt $= 3866\,N$; $\tau_{interface} = 0.043\,MPa$

Code B: $= 8810\,N$; $= 0.073$

Code C: $= 15768\,N$; $= 0.105$

Code D: $= 25319\,N$; $= 0.141$

P6 $\delta_{max} = \frac{5}{384}\frac{wL^4}{EI}$ at mid span (from tables provided)

$$\sigma_{max} = \frac{M_{max}}{bh^2/6} = \frac{wL^2/8}{bh^2/6}$$

From δ_{max}, $(bh^3)_{min} = \frac{5 * 12}{384}\frac{wL^4}{E}\frac{1}{\delta_{max}}$

From σ_{all}, $(bh^2)_{min} = \frac{6}{8}\frac{wL^2}{\sigma_{all}}$

If both limits are reached simultaneously,

$$h = \frac{(bh^3)_{min}}{(bh^2)_{min}} = \left(\frac{5*12}{384}\right)\frac{wL^{4\,2}}{E}\frac{1}{\delta_{max}} \Big/ \left(\frac{6}{8}\frac{wL^2}{\sigma_{all}}\right)$$

$$b = \frac{6}{8}\frac{wL^2}{\sigma_{all}} \Big/ h^2$$

Code A : $h = 1.56\,m$; $b = 0.02048\,m$

Code B : $h = 2.1875\,m$; $b = 0.03023\,m$

Code C : $h = 2.778\,m$; $b = 0.03888\,m$

Code D : $h = 0.8681\,m$; $b = 0.01194\,m$