DEPARTMENT OF MATHEMATICS
ACADEMIC YEAR – 2008 – 09(ODD SEMESTER)

MA1252-PROBABILITY AND QUEUEING THEORY
UNIT –I
PROBABILITY AND RANDOM VARIABLE

Part - A

1. State the Axioms of probability.
2. If A and B events such that $P(A \cup B) = \frac{3}{4}$, $P(A \cap B) = \frac{1}{4}$ and
3. Let P(AUB)=5/6, $P(A \cap B) = \frac{1}{3} and P\left(\bar{B}\right) = \frac{1}{2.}$. Are the events independent. Explain.
4. Define random variable.
5. From 21 tickets, marked with 20 to 40 numerals, one is drawn at random. Find the chance that it is a multiple of 5.
6. If you twice flip is balanced coins, what is the probability of getting at least one head.
7. In a company ,5% defective components are produced. What is the probability that atleast 5 components are to be examined in order to get 3 defectives?
8. If X and Y are independent random variables with variance 2 and 3. Find the variance of 3X + 4Y.
9. Obtain the probability function or probability distribution from the following distribution function.

x :	0	1	2	3
F(x) :	0.1	0.4	0.9	1.0

10. Let X be a discrete random variable whose cumulative distribution is

$$F(x) = \begin{cases} 0, x < -3 \\ \frac{1}{6}, -3 \le x \le 6 \\ \frac{1}{2}, 6 \le x \le 10 \\ 1, 10 \le x \end{cases}$$

(i) Find $P(X \leq 4)$, $P(-5 < x \leq 4)$ (ii) Find the probability distribution of X.

11. If var(X) = 4, find Var(3X+8), where X is a random variable.

12. The first four moments of a distribution about X = 4 are 1,4,10 and 45 respectively. Show that the mean is 5, variance is 3, $\mu_3 = 0$ and $\mu_4 = 26$.

13. A box contains 5 red and 4 white balls. Two balls are drawn successively from the box without replacement and it is noted that the second one is white. What is the probability that the first is also white ?
14. If a Random variable X takes the values 1, 2, 3, 4 such that $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$. Find the probability distribution of X .
15. Find cumulative distribution function F(x) corresponding to the p.d.f $f(x) = \frac{1}{\pi}\left(\frac{1}{1+x^2}\right), for -\infty < x < \infty.$
16. Define continuous random variable. Given example.
17. A random variable X has p.d.f f(x) given by $f(x) = \{Cxe^{-x}, if x > 0$
= 0, if $x \leq 0$

Find the value of C and C. D. F. of X.

18. The cumulative distribution function of a random variable X is $F(x) = 1-(1+x)e^{-x}, x \geq 0$. Find the probability density function of X

19. Define n th central moment of a random variable.

20. If a RV X has the moment generating function $M_X(t) = \frac{2}{2-t}$ determine the variance of X.

Part – B

1. a) If P (A) = P (B) = P (AB) , prove that P (A$\bar{B}$ + $\bar{A}$B) = 0 [AB = A∩B). **(8)**

   b) $A, B \& C$ in order toss a coins, the first one to throw a head wins. If A starts, find their respective chances of winning; **(8)**

2. a) A manufacture of air plane parts knows that the probability is 0.8 that an order will be ready for shipment on time and it is 0.7 that an order will be ready for shipment and will be delivered on time. delivered on time. What is the probability that such an order will be delivered on time given that it was also ready for shipment on Time. **(8)**

b) Given a binary communication channel, where A is is the input and B is the output, $P(A) = 0.4, P(A/B) = 0.9 and P\left(\frac{\bar{B}}{\bar{A}}\right) = 0.6$.

Find (1) P(A/B) (2) P($A/\bar{B}$). **(8)**

3. a) An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn at random from the 1st urn and placed second urn and then 1 ball is taken at random from the latter. What is the probability that it is a white ball? **(8)**

b) A bolt is manufactured by 3 machines A, B and C . A turns out twice as many items as B , and machines B and C produce equal number of items.2% of bolt produced by A and B are defective and 4% of bolts produced by C are defective. All the bolts are put into 1 stock pile and 1 is chosen from this pile. What is the probability that it is defective. **(8)**

4. a) A random variable X has the following distribution.

x	:	- 2	- 2	0	1	2	3
$P(x)$	:	0.1	k	0.2	$2k$	0.3	$3k$

Find (i) the value of K.
(ii) Evaluate P (X < 2) and P (-2 < X < 2).
(iii) Find the cumulative distribution of X and
(iv) Evaluate the mean of X **(8)**

b) The monthly demand for Allwyn watches is known to have the following probability distribution

Demand	1	2	3	4	5	6	7	8
Probability	0.08	0.12	0.19	0.24	0.16	0.10	0.07	0.04

Determine the expected demand for watches. Also compute the variance. **(8)**

5. a) X is a continuous Random variable with potential distribution function given by f(x) = Kx in $0 \le x \le 2$
= 2K in $2 \le x \le 4$
= 6K – Kx in $4 \le x \le 6$
= 0 elsewhere.

Find the value of K and also the cdf f(x). **(8)**

b) If the density function of a continuous random variable X is given by

$$f(x)=\begin{cases} ax, 0 \le x \le 1 \\ a, 1 \le x \le 2 \\ 3a-ax, 2 \le x \le 3 \\ 0, elsewhere \end{cases}$$

Find (i) the value of a

(ii) the cumulative distribution function of X.

(iii) If x_1, x_2, & x_3 are 3 independent observations of X.. What is the probability that exactly one of these 3 is greater than 1.5? **(8)**

6. a) If X is a continuous distribution, the probability density is given by $f(x)=kx(2-x), 0<x<2$. Find k, mean, variance and the distribution function. **(8)**

b) If the probability density of X is given by $f(x)=\begin{cases} 2(1-x)\, for 0<x<1 \\ 0 otherwise \end{cases}$

a. Show that $E\left[X^r\right]=\dfrac{2}{(r+1)(r+2)}$

b. Use this result to evaluate $E\left[(2X+1)^2\right]$ **(8)**

7. a) A man draws 3 balls from an urn containing 5 white and 7 black balls. He get Rs.10 for each white ball and Rs. 5 for each black ball. Find his expectation. **(8)**

b) A random variable X has density function given by $F(x)=\begin{cases} \dfrac{1}{k} for 0<x<k \end{cases}$

Find (1) m.g.f (2) r^{th} Moment (3) mean (4) Variance **(8)**

8. a) The diameter of an electric cable X is a continuous random variable with pdf $f(x)=kx(1-x), 0 \le x \le 1$. Find (A) the value of k (B) the cdf of X (C) $P(X \le 1/2 \,|\, 1/3 < X < 2/3)$ **(8)**

b) The first four moments of a distribution about X4 are 1,4, 10 and 45 Respectively. Show that the mean of variance is 3, $\mu_0=0$ .and $\mu_4=26$ **(8)**

9. a) Find the moment generating function of the random variable with the Probability law $P(X=x)=q^{x-1}p$, x= 1,2,3… Also find the mean and variance. Also find the mean and variance. **(8)**

b) For the triangular distribution $f(x)=\begin{cases} x, 0<x\le 1 \\ 2-x, 1 \le x < 2 \\ 0, otherwise \end{cases}$

Find the mean, variance and the moment generating function. **(8)**

10. a) In each of that following cases $M_x(t)$ the moment generating function of X is given. Determine the distribution of X and its mean.

(1) $M_x(t) = \left(\frac{1}{4}e^t + \frac{3}{4}\right)^4$ (2) $M_x(t) = \left(\frac{\lambda}{\lambda - 1}\right)^n$ **(8)**

b) Let X be a random variable with p.d.f $f(x) = \begin{cases} \frac{1}{2}e^{-\frac{x}{2}}, x > 0 \\ 0, \text{ otherwise.} \end{cases}$

Find (1) P (X>3) (2) M.G.F of X (3) E(X) (4) Var(X). **(8)**

UNIT – II
STANDARD DISTRIBUTIONS

PART – A

1. Define binomial B(n,p) distribution. Obtain its Moment generation function, Mean and variance.
2. The mean of the Binomial distribution is 20 and SD is 4. Find the parameters of the distribution.
3. Define a Poisson distribution and state any two instances where Poisson distribution may be successfully employed.
4. Obtain moment generating function of Poisson Distribution.
5. Find the mean of Poisson distribution.
6. The number of monthly breakdowns of a computer is a RV having a Poisson distribution with mean equal to 1.8.. Find the probability that this computer will function for a month will only one breakdown.
7. If X is a Poisson variate such that P(X=2)=9P(X=4) + 90 P(X=6), find the variance.
8. sharon and ann play a series backganmmon on games until one of the win five games. Suppose that the games are independent Sharon win a game is
   a. (i) Find the probability that the series ends in 7 games.
      (ii) if series ends in series, what is the parobability that Sharon wins.
9. If the probability is 0.40 that a child exposed to a certain contagious will catch it, what is the probability that the tenth child exposed to the disease will be the third to catch it ?
10. One percent of jobs arriving at a computer system need to wait until weekends for scheduling owing to core-size limitations. Find the probability that among a sample of 200 jobs there are no jobs that have to wait until weekends.
11. In a company, 5% defective components are produced. What is the probability that atleast 5 components are to be exmined in order to get 3 defective?

12. Show that for the uniform distribution $f(x) = \frac{1}{2\alpha}$,-a<x<a the moment generating function about origin is $\frac{\sinh at}{at}$

13. If X is uniformally distributed (-1,1) find the pdf of y = sin$\left(\frac{\pi x}{2}\right)$
14. If X is a uniform random variable in [-2,2] find the p.d.f of X and Var(x).
15. If X is uniformly distributed over (0,10) calculate the probability over (0,10) calculate the probability that (a) X>6 (b) 3<X<8
16. The time (in hrs) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{3}$ what is the probability that the repair time exceeds 3 hrs.
17. A fast food chain finds that the average time customers have to wait for service is 45 seconds. If the waiting time can be treated as an exponential random variable, what is the probability that a customer will have to wait more than m5 minutes given that already he waited for 2 minutes?
18. Define exponential r.v with example.
19. The life time of a component measure in hours is Weibull distribution with parameter $\alpha = 0.2$, $\beta = 0.5$. Find the mean life time of the component

The p.d.f of a random variable X is $f(x) = 2x$, 0<x<1, find the p.d.f. of Y Y=3X +1.

20. Given the random variable with density function $f(x) = \begin{cases} 2x, 0 \le x \le 1 \\ 0, elsewhere \end{cases}$
21. Find the probability density function of Y = 8 X^2
22. If X is a Gaussian r.v with mean zero and variance σ^2 find the probability density function of Y = | x |.

PART - B

1. a) Ten coins are simultaneously find the probability of atleast 7 heads. **(8)**
   b) If X is a binomial distribution R.V with E(X)=2 and Var(X)=4/3, find P(X=5). **(8)**
2. a) For a binomial distribution mean is 6 and standard deviation is $\sqrt{2}$. Find the first two terms of the distribution. **(8)**
   b) Let the random variable X follows binomial distribution with parameter n and p.
   Find (1) probability mass function of X.
   (2) moment generating function X
   (3) mean and variance of X. **(8)**

3. a) VLSI chips, essential to the running of a computer system, fail in accordance with Poisson distribution with the rate of one chip in about weeks. If there two spare chips on hand, and if a new supply will arrive in 8 weeks. What is the probability that during the next 8 weeks the system will be down for a week or more, owing to a lack of chip? **(8)**
   b) The monthly breakdowns of a computer is a random variable having Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month.
   i. without breakdown
   ii. with only one breakdown. **(8)**
4. a) Prove that Poisson distribution is the limiting case of binomial distribution. **(8)**
   b) Find the MGF of a Poisson variable deduce that the sum of two independent Poisson variates is a Poisson variates, while the difference is not a Poisson variate. **(8)**

5.a)Fit a Poisson distribution for the following distribution: **(8)**

x:	0	1	2	3	4	5	Total
f:	142	156	69	27	5	1	400

b)The atoms of radioactive elements are randomly disintegrating. If every gram of this element, on average emits 3.9 alpha particles persecond what is a probability during the next second the number of alpha particles emitted from 1 grams : (i) at most 6 (ii) at least 2 (iii) atleast 3 and atmost 6. **(8)**

6. a)If Poisson variate X is such that P(X=1) = 2P(X=2), find P(X=0) and Var (X). **(8)**
   b) Find the MGF of Poisson variable and hence obtain its mean and variance. **(8)**

7. a) Define geometric distribution. Find the moment generating function of geometric distribution and hence find its mean and variance. Hence or otherwise compute the first four moments. **(8)**
   b) Establish the memoryless property of geometric distribution. **(8)**

8.a). Suppose that the trainee soldiers shots a target in an independent fashion. If the probability that the target shot an any one shot is 0.7 **(8)**

1. what is the probability that the target would be hit on 10th attempt
2. what is the probability that it takes him less than 4 shots
3. what is a probability that take him on even number of shots.
4. what is a average number of shots needed to hit the target.

b) Describe negative binomial distribution X~NB(k,p) where X = number of failures preceding the kth success in a sequence of Bernoulli trails and p= probability success. Obtain the MGF of X, mean and variance of X. **(8)**

9.a) If X is uniformly distributed in [-2,2], find (1) P(X<0) and (2) P$\left(|X-1|\geq\frac{1}{2}\right)$ **(8)**

b) Staring at 5.00 A.M. every half Hour there is a flight from San Francisco airport to los Angles International Airport. Suppose that none of these planes is completely sold out and that they always have room for passengers. A person who wants to fly to L.A. arrives at the airport at a random time between 8.45 a.m. and 9.45 a.m. Find the probability that she waits (1) at most 10 mins (2) at least 15 mins. **(8)**

10. a) If X is uniformly distributed with E(X)=1 and Var(X) = 4/3, find P(X<0). **(8)**
   b) If X_1 given X_1 and X_2 are independent uniform variates (o,1), find the distribution of X_1/X_2 and $X_1 X_2$. **(8)**

11.a)The number of personal computer (PC) sold daily at a computerword is uniformly distributed with minimum of 2000 PC and a maximum of 5000 PC. Find (1) the probability that daily will fall between 2,500 and 3,000 PC.

(2) What is the probability that the compuworld will set atleast 4,000PC's?

(3) What is the probability that the compuworld will exactly sell 2,500 PC's.

Find the mean and variance of Exponential Distribution **(8)**

b) The daily consumption of milk in excess of 20,000 gallons is approximately distributed with θ = 3000. The city has a daily stock of 35,000 gallons. What is the probability that of two days selected at random, the stock is insufficient for both balls. **(8)**

12.a) Obtain the moment generating function of exponential distribution and hence or otherwise computer the first four moments. Find the mean and variance of Gamma Distribution **(8)**

b) The life time X in hours of a component is modeled by a weibull distribution with shape parameter α =2. Starting with a large number of components, it is observed that 15% of the components that have lasted 90 hours fail before 100 hours. Determine the scale parameter λ. **(8)**

13.a)Define Weibull distribution and its find mean and variance. **(8)**

b)Each of the six tubes of a radio set has a life length (in yrs) with follows a weibull distribution with parameters $\alpha = 25$ and $\beta = 2$. if these tubes function independently of one another, what is the that no tube will have to be the replaced during the first two months of operation. **(8)**

14.a)if the life X (in yrs) of a certain time of car has a weibull distribution with the parameter $\beta = 2$. find the value of the parameter α, given that probability that the life of the car exceeds 5 yrs in $e^{-0.25}$. for this value of α and β,
find the mean and variance of x **(8)**

b)if X ~ N(μ, σ^2). Obtain the probability density function of $U = \frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2$ **(8)**

15.a)The savings bank account of a customer showed an average of balance of Rs. 150 and a standard deviation Rs. 50 assuming that the account balances are normally distributed.

(i) what percentage of account his over Rs. 200?
(ii) What percentage of account is between Rs.120 and Rs.170?
(iii) what percentage of account less than Rs.75? **(8)**

b)The marks obtained a number of students in a certain subject are normally distributed with mean 65 and SD 5.if 3 students are selected at random, from this group what is a probability that atleast one of them would have scored above 75? **(8)**

16.a)X is normally distributed and the mean of X is 12 and S.D. is 4. Find the probability of the following:

(1) $X \geq 20$
(2) $0 \leq X \leq 12$
(3) Find x', when P(X >x')=0.24
(4) Find x_0^1 and x_1^1 , when $P(X_0^1 < x < x_1^1) = 0.50$ and $P(X > x_1^1) = 0.25$. **(8)**

b) Find the moment generating function of a normal distribution. **(8)**

17.a) A man with 'n' keys wants to his door and tries the jays independently and at random. Find the mean and variance of the number of trials required to open the door if unsuccessful keys are not eliminated from further selection. **(8)**

.b) If X ia a continuous r.v having the pdf $f(x) = \begin{cases} 2x, 0 \leq x \leq 1 \\ 0, elsewhere \end{cases}$ and Y = e^{-x}

find the pdf of the r.v Y. **(8)**

18.a) If X is normal r.v with mean 0 and variance σ^2 find the pdf of Y = e^X if X has a exponential distribution with parameter α , find the pdf Y = logX. **(8)**

b) Let the density function of X be $f(x) = \begin{cases} e^{-x}, x > 0 \\ 0, elsewhere \end{cases}$ using the method of transformation find the density function of Y = X$\sqrt{x}$ and z = e^{-x}. **(8)**

19.a) A random variable x has the p.d.f $f_x(x) = \{e^{-2|x|}, -\infty < x < \infty$ If $Y = X^2$,find the p.d.f of Y. **(8)**

b)If X is uniformly distributed in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, find the probability distribution function of y = tan x. **(8)**

20.a) If X is a uniform a random variable in the interval (-2,2) find the p.d.f. of Y = X^2. **(8)**

b) An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800m hours and a standard deviation of 40 hours. Find (1) the probability that a bulb burns more than 834 hours. (23) the probability that bulb burn between 778 and 834 hours. **(8)**

UNIT III
TWO DIMENSIONAL RANDOM VARIABLE

PART-A

1.The joint probability density function of the random variable (X,Y) is given by f(x,y)=Kxy$e^{-(x^2+y^2)}$, x>0, y>0. Find the value of K.

2. If the probability density function of X is $f_X(x) = 2x$, 0<x<1, find the probability density function of Y=3X+1.

3. The joint probability density function of two random variables given by $f_{xy}(x, y) = \frac{1}{8}x(x - y)$; 0<x<2; -x<y<x and find $f_{y/x}(y/x)$

4. If X and Y are random variables having the joint density function f(x,y) = (6-x-y)/8, 0<x<2; 2<y<4,find P(X+Y<3).

5. Find the acute angle between the two lines of regression.

6. State the equations of the two regression lines. What is the angle between them?

7. State central limit theorem in Lindberg-Levy's form.

8. If X and Y are linearly related, find the angle between the two regression lines.

9. Let X and Y be integer valued random variables with P(X = m, Y = n)= $q^2 p^{m+n-2;}$ n,m = 1,2,……….. and p+q = 1. Are X and Y independent?

10. Distinguish between correlation and regression.

11. Can the joint distributions of two random variables X and Y be got of their marginal distributions are known.

12. State the importance of central limit theorem.

13. State central limit theorem in Liapounoff's form.

14. If two random variables X and Y have PDF f(x,y) = k(2x+y) for

$0 \le x \le 2, 0 \le y \le 3$, evaluate k.

15. Define joint distributions of two random variables X and Y and state its properties.
16. The two equations of the variables X and Y are x=19.13-0.87y and y=11.64-0.50x. Find the correlation co-efficient between X and Y.
17. Prove that the correlation coefficient P_{xy} takes value in the range -1 to 1.
18. The Regression equations of X on Y and Y on X are respectively 5x-y=22 and 64x-45y=24. Find the means of X and Y.
19. X and Y are independent random variables with variance 2 and 3 . Find the variance of 3X+4Y
20. Show that $Cov^2(X,Y) \le Var(X).Var(Y)$

PART-B

1. a) The random variable X and Y are satistically independent having a gamma distribution with parameters (m,1/2) and (n,1/2), respectively. Derive the probability density function of a random variable U = X/(X+Y). **(8)**

   b) Find the correlation coefficient for the following data: **(8)**

X	10	14	18	22	26	30
Y	18	12	24	06	30	36

2. a) The joint probability mass function of (X,Y) is given by p(x,y) = K(2x+3y), x = 0,1,2; y =1,2,3. Find all the marginal and conditional probability distributions. **(8)**

   b) Two independent random variables X and Y are defined by, **(8)**

   f(x) = 4ax, $0 \le x \le 1$ f(x) = 4by, $0 \le y \le 1$

   = 0,otherwise   = 0, otherwise

   Show that U=X+Y and V = X-Y are uncorrelated.

3. a) The joint p.d.f of X and Y is given by $f(x,y) = e^{-(x+y),}$, x>0,y>0,find the probability density function of U = (X+Y)/2. **(8)**

   b) Given is the joint distribution X and Y **(8)**

		X		
		0	1	2
Y	0	0.02	0.08	0.10
	1	0.05	0.20	0.25
	2	0.03	0.12	0.15

   Obtain

   (i) Marginal Distributions and

   (ii) The Conditional Distribution of X given Y=0.

4. a) If the joint density of X_1 and X_2 is given by **(8)**

   $f(x_1, x_2) = 6e^{-3x_1 - 2x_2}$ for x_1>0, , x_2>0

   = 0 otherwise.

   Find the probability density of Y= $X_1 + X_2$ and its mean.

   b) Obtain the moment generating function of a gamma variable X. Hence or otherwise calculate the mean and variance of X. **(8)**

5. a) Suppose the joint probability density function is given by **(8)**

$f(x,y) = \frac{6}{5}(x+y^2); 0 \le x \le 1, 0 \le y \le 1$

$= 0$, otherwise.

b) The joint probability mass function of X and Y is given by **(8)**

x/y	-1	1
0	1/8	3/8
1	2/8	2/8

Find the correlation coefficient of (X,Y).

6. a)The joint probability mass function of X and Y is given by **(8)**

P(x,y)		0	1	2
X	0	0.1	0.04	0.02
	1	0.08	0.20	0.06
	2	0.06	0.14	0.30

Compute the marginal PMF of X and Y,P[$x \le 1, y \le 1$] and check if X and Y are independent.

b) Two random variable X and Y have the joint density **(8)**

$f(x,y) = 2-x-y$; $0<x<1$, $0<y<1$

$= 0$, otherwise.

Show that Cov(X,Y) = -1/11.

7. a) Two dimensional random variable (X,Y) has the joint PDF

$f(x,y) = 8xy, 0<x<y<1$; 0 otherwise. Find **(8)**

(i) marginal and conditional distributions
(ii) Check whether X and Y are independent.

b)Two random variables X and Y are defined as Y = 4X+9. Find the coefficient of correlation between X and Y. **(8)**

8. a) If the joint probability density function of a two dimensional random variable (X,Y) is given by $f(x,y) = x^2 + \frac{xy}{3}; 0 < x < 1, 0 < y < 2$ **(8)**

$= 0$, elsewhere

Find

(i) P(X>1/2)
(ii) P(Y<X) and
(iii) P(Y<1/2/ X<1/2).

b) Two dimensional random variable (X,Y) has the joint PDF **(8)**

$f(x,y) = 2, 0<y<x<1$; 0 otherwise. Find

(i) marginal and conditional distributions
(ii) joint distribution function F(x,y)
(iii) Check whether X and Y are independent.

(iv) P(X<1/2/Y<1/4)

9. a) Calculate the correlation coefficient for the following heights(in inches) of fathers X and their sons Y. **(8)**

X: 65 66 67 67 68 69 70 72

Y: 67 68 65 68 72 72 69 71

From the following data, find

I. The two regression equations

II. The coefficient of correlation between the marks in mathematics and statistics

III. The most likely marks in statistics when marks in mathematics are 30.

Marks in Mathematics: 25 28 35 32 31 36 29 38 34 32

Marks in Statistics: 43 46 49 41 36 32 31 30 33 39

b) Given $f_{xy}(x, y) = cx(x - y),\ 0 < x < 2, -x < y < x$

= 0, otherwise

(1) Evaluate C

(2) Find $f_X(x)$

(3) $f_{y/x}(y/x)$ and

(4) $f_y(y)$

10. a) The joint pdf of random variable X and Y is given by f(x,y) = $Kxye^{-(x^2+y^2)}$, x>0,y>0. Find the value of K and prove that X and Y are independent.

b) If y =, 2x – 3 and y = 5x+7 are the two regression lines, find the mean values of x and y . Find the correlation coefficient between x and y. Find an estimate of x when y = 1.

UNIT – IV
RANDOM PROCESSES

PART - A

1. Define a Markov process and a Markov chain.
2. Examine whether the Poisson process {X(t)} given by the law

   P[X(t)=r]= $\frac{e^{-\lambda t}(\lambda t)^r}{r!}$, r =0,1,2,….is covariance stationary.
3. Let X be the random variable which gives the inter arrival time (time between successive arrivals), where the arrival process is a Poisson process. What will be the distribution of X? How?
4. Define strict sense stationary process and give an example.
5. A man tosses a fair coin until 3 heads occur in a row. Let Xn denotes the longest string of heads ending at the nth trial. i.e. Xn=k, if at the nth trial, the last occurred at the (n-k) the trial. Find the transition probability matrix.
6. Define wide sense stationary and strict sense stationary random processes.
7. Let X(t) be a poisson process with rate λ. Find the correlation function of X(t)
8. Define a Markov chain and give an example.

9. If the transition probability matrix of a Markov chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$, find the limiting distribution of the chain.
10. Define random process and its classification.
11. Show that the sum of two independent poisson process is a poisson process.
12. The transition probability matrix of a Matrix chain {Xn},n=1,2,3… with three state 0,1 and 2 is P = $\begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ with initial distribution $P^{(0)}$=(1/3,1/3,1/3). Find $P(x_3=1,X_2=2,X_1=1)$.
13. Find the invariant probabilities for the Markov chain {Xn;n≥1} with state space S={0,1} and one step TPM P=$\begin{bmatrix} 1 & 0 \\ 1/2 & 1/2 \end{bmatrix}$
14. What is a Markov chain/ when can you say that a Markov chain is Homogeneous?
15. Define Birth process
16. Consider the random process X(t) = cos ($\omega_0 t+\theta$) where θ uniformly distributed in the interval –π to π. Check whether X(t) is stationary or not?
17. What is continuous random sequence ? Give an example.
18. Define irreducible Markov chain? And state Chapman-Kolmogorov Theorem.
19. What is meant by steady state distribution of Markov chain?
20. State any four properties of Poisson process.

PART - B

1. (a)Show that the random process X(t)= A$\cos\left(\omega_0 t+\theta\right)$ is a wide-sense stationary, if A and ω_0 are constants and θ is a uniformly distributed random variable in (0,2π). **(8)**

   (b)An engineer analyzing a series of digital signals generated by a testing system observes that only 1 out of 15 highly distorted signals follows a highly distorted signal, with no recognizable signal between,whereas 20 out of 23 recognizable signals follow recognizable signals with no highly distorted signal between. Given that only highly distorted signals are not recognizable, find the fraction of signals that are highly distorted. **(8)**
2. (a) Define stationary transition probabilities. **(8)**

   (b)Derive the Chapman-kolmogorov equations for discrete-time Markov chain. **(8)**
3. (a) On a given day, a retired English professor, Dr. Charles Fish, amuses **(8)** himself with only one of the following activities: reading (activity 1), gardening (activity 2) , or working on his book about a river valley (activity 3) . For 1≤i≤3, Let X_n=I if Dr. Fish devotes day 'n' to activity i. Suppose that { X_n ;n=1,2,3…}is a markov chain, and depending on which of these activities on the next day is given by the t.p.m

   P = $\begin{pmatrix} 0.30 & 0.25 & 0.45 \\ 0.40 & 0.10 & 0.50 \\ 0.25 & 0.40 & 0.35 \end{pmatrix}$ Find the proportionl of days Dr. Fish devotes to each activiy

(b) The process {X(t)} whose probability distribution is given by **(8)**

$$P[X(t)=n] = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,\ldots \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$$

Show that it is not stationary.

4. (a) A raining process is considered as a two-state Markov chain. If it rains, it is considered to be in state 0 and if it does not rain, that chain is in state 1.

   The transition probability of the Markov chain is defines as $P = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$.

   Find the probability that it will rain for three days from today assuming that it is raining today. Find also the unconditional probability that it will rain after three days with the initial probabilities of state 0 and state 1 as 0.4 and 0.6 respectively. **(8)**

   (b)A machine goes out of order, whenever a component fails. The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. **(8)**

5. (a) The transition probability matrix of a markov chain $\{X_n\}_{n=1,2,3,\ldots}$ having 3 states 1,2 and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $P^{(0)}=(0.7,0.2,0.1)$. Find $P(X_2=3, X_1=3, X_0=2)$. **(8)**

   (b)Assume that a computer system is in any one of the three states: busy, idle and under repair respectively denoted by 0,1,2. observing its state at 2 pm each day , we get the transition probability matrix as $P=\begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{pmatrix}$.

   Find out the 3^{rd} step transition probability matrix. Determine the limiting probabilities. **(8)**

6. (a)Obtain the steady state or long run probabilities for the population size of a birth death process. **(8)**

   (b)A person owning a scooter has the option to switch over to scooter, bike or a car next time with the probability of (0.3,0.5,0.2) . If the transition probability matrix is

$\begin{pmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.5 & 0.3 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}$, what are the probabilities vehicles related to his fourth purchase? **(8)**

7. (a)A stochastic process is described by X(t)=Asint+Bcost where A and B are independent random variables with zero means and equal standard deviations show that the process is stationary of the second order. **(8)**

   (b)The one-step T.P.M of a markov chain { X_n ;n=0,1,2..}having state space

   S = {1,2,3}is P = $\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is

   π_0 = (0.7,0.2,0.1).

   Find (i) $P(X_2=3/X_0=1)$ (ii) $P(X_2=3)$ (iii) $P(x_3=2,X_2=3,X_1=3,X_0=1)$ **(8)**

8. (a)Derive the balance equation of birth and death process. **(8)**

   (b)If the process {N(t);t>0} is a poisson process with parameter λ obtain P[N(t) = n] and E[N(t)]. **(8)**

9. (a)Derive the distribution of Poisson process and find its mean and variance. **(8)**

   (b)Define a Markov chain. Explain how you would clarify the states and identify different classes of a Markov chain. Give an example to each class. **(8)**

10. (a)A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again he is to travel by train. Now suppose that on the first day of the week the man tossed a fair die and drove to work if and only if a "6" appeared. Find (1) the probability that he takes a train on the third day.

    (2) the probability that he drives to work in the long run. **(8)**

    (b)Consider the random process X(t) = cos(t+φ) where φ is a random variable with density function f(φ) = 1/π, - π/2 < φ < π/2, check whether the process is stationary or not. **(8)**

UNIT-V
QUEUEING THEORY

PART-A

1. What are the basic characteristics of a queueing system?
2. Define Kendall's notation.
3. Give the formulas for the waiting time of a customer in the queue and in the system for the (M/M/1):(∞/FIFO) model.
4. In the usual notation of an M/M/1 queueing system, if λ=3/hour and μ=4/hour, find P(X≥5) where X is the number of customers in the system.
5. In the usual notation of an M/M/1 queueing system, if λ=12/hour and μ=24/hour, find the average number of customers in the system.
6. Derive the average number of customers in the system for (M/M/1):(∞/FIFO) model.
7. State Little's formula for an (M/M/1):(∞/FIFO) queueing model.
8. In a given (M/M/1):(∞/FCFS) queue, ρ=0.6, what is the probability that the queue contains 5 or more customers?

9. What is the probability that a customer has to wait more than 15 minutes to get his service completed in a (M/M/1):(∞/FIFO) queue system, if λ=6 per hour and μ=10 per hour?
10. Consider an M/M/1 queueing system. If λ=6 and μ=8, find the probability of atleast 10 customers in the system.
11. Consider an M/M/1 queueing system. Find the probability of finding atleast 'n' customers in the system.
12. What is the probability that an arrival to an infinite capacity 3 server Poisson queue with $\lambda/c\mu = 2/3$ and $P_0=1/9$ enters the service without waiting?
13. Consider an M/M/C queueing system. Find the probability that an arriving customer is forced to join the queue.
14. For (M/M/C):(N/FIFO)model, write down the formula for
    (i) Average number of customers in the queue
    (ii)Average waiting time in the system
15. Give the formulas for the average number of customers in the queue and in the system for the (M/M/s):(∞/FIFO) queueing model.
16. What is the effective arrival rate for (M/M/1):(4/FCFS) queueing model when λ=2 and μ=5.
17. Give the probability that there is no customer in an (M/M/1):(k/FIFO) queueing system.
18. Write the formulas for the average number of customers in the (M/M/1):(k/FIFO) queueing system and also in the queue.
19. Define effective arrival rate with respect to an (M/M/1):(k/FIFO) and (M/M/s):(k/FIFO) queueing models.
20. Write Pollaczek-Khintchine formula and explain the notations.

PART-B

1. Arrivals at a telephone booth are considered to be poisson with an average time of 12 min. between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 min.
    (a) Find the average number of persons waiting in the system.
    (b) What is the probability that a person arriving at the booth will have to wait in the queue?
    (c) What is the probability that it will take him more than 10min. altogether to wait for the phone and complete his call?
    (d) Estimate the fraction of the day when the phone will be in use
    (e) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for at least 3 min. for phone. By how much the flow of arrivals should increase in order to justify a second booth?
    (f) What is the average length of the queue that forms time to time?**(16)**
2. Customers arrive at a one-man barber shop according to a poisson process with a mean interarrival time of 12 min. Customers spend an average of 10min in the barber's chair.
    (a) What is the expected number of customers in the barber shop and in the queue?
    (b) Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait.

(c) How much time can a customer expect to spend in the barber's shop?
(d) Management will provide another chair and hire another barber, when a customer's waiting time in the shop exceed 1.25h . how much must the average rate of arrivals increase to warrant a second barber?
(e) What is the average time customer spends in the queue?
(f) What is the probability that the waiting time in the system is greater than 30min?
(g) Calculate the percentage of customers who have to wait prior to getting into the barber's chair.
(h) What is the probability that more than 3 customers are in the system? **(16)**

3.a. Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes, and the service time is an exponential random variable with mean 8 minutes. Find the average number of customers L_S, the average waiting time a customer spends in the shop W_S and the average time a customer spends in the waiting for service W_q. **(8)**

b. A bank has two tellers working on savings account. The first teller handles withdrawals only. The second teller handles deposits only. It has been found that the service time distributions for both deposits and withdrawals are exponential with mean service time of 3 minutes per customer. Depositors are found to arrive in a poisson fashion throughtout the day with mean arrival rate of 16 per hour. Withdrawers also arrive in a Poisson fashion with mean arrival rate 14 per hour. What would be the effect on the average waiting time for the customers if each teller could handle both withdrawals and deposits. **(8)**

4.a. There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour.

(a) what fraction of the time all the typists will be busy?
(b) What is the average number of letters waiting to be typed?
(c) What is the average time a letter has to spend for waiting and for being typed?
(d) What is the probability that a letter will take longer than 20 min. waiting to be typed and being typed? **(8)**

b. A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean of 6 min and cars arrive for service in a poisson process at the rate of 30 cars per hour.

(a) what is the probability that an arrival would have to wait in line?
(b) find the average waiting time, average time spend in the system and the average number of cars in the system.
(c) For what percentage of time would a pump be idle on an average? **(8)**

5.a. A supermarket has 2 girls attending to sales at the counters. If the service times for each customer is exponential with mean 4min and if people arrive in poisson fashion at the rate of 10per hour,

(a) what is the probability that a customer has to wait for service?
(b) What is the expected percentage of idle time for each girl?
(c) If the customer has to wait in the queue, what is the expected length of his waiting time? **(8)**

b. A telephone company is planning to install telephone booths in a new airport. It has established the policy that a person should not have to wait more than 10% of the times he tries to use a phone. The demand for use is estimated to be poisson with an average of 30per hour. The average phone call has an exponential distribution with a mean time of 5 min. how many phone booths should be installed? **(8)**

6.a. In a single server queueing system with poisson input and exponential service times, if the mean arrival rate is 3 calling units per hour, the expected service time is 0.25h and the maximum possible number of calling units in the system is 2. find $P_n (n \geq 0)$, average number of calling units in the system and in the queue and average waiting time in the system and in the queue**(8)**

b. The local one person barber shop can accommodate a maximum of 5 people at a time(4waiting and 1 getting hair-cut) Customers arrive according to a poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour.

(a) what percentage of time is the barber idle?
(b) What fraction of the potential customers are turned away?
(c) What is the expected number of customers waiting for a hair-cut?
(d) How much time can a customer expect to spend in the barber shop?**(8)**

7.a. Patients arrive at clinic according to poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

(a) find the effective arrival rate at the clinic.
(b) What is the probability that an arriving patient will not wait
(c) What is the expected waiting time until a patients is discharged from the clinic? **(8)**

b. Customers arrive at a one window drive-in bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of the window including that for the serviced car can accommodate a maximum of 3 cars. Other can wait outside this space.

(1) What is the probability that an arriving customer can drive directly to the space in front of the window?
(2) What is the probability that an arriving customer will have to wait outside the indicated space?
(3) How long the arriving customer is expected to wait before starting service? **(8)**

8.a. A 2-person barber shop has 5 chairs to accommodate waiting customers. Potential customers, who arrive when all 5 chairs are full, leave without entering barber shop . Customers arrive at the average rate of 4 per hour and spend an average of 12 min in the barber's chair. Compute P_0,P_1,P_7,E[N_q] and E[W]. **(8)**

b. A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitation, only 4 cars are accepted for servicing. The arrival pattern is poisson with 12 cars per day. The service time in both the bays is exponentially distributed with μ=8 cars per day per boy. Find the average number of cars in the service station, the average number of cars waiting for service and the average time a car spends in the system. **(8)**

9.a. A group of engineers has 2 terminals available to aid in their calculations. The average computing job requires 20min of terminal time and each engineer requires some computation about once every half an hour. Assume that these are distributed according to an exponential distribution. If there are 6 engineers in the group, find

(a) The expected number of engineers waiting to use one of the terminals and in the computing centre and

(b) The total time lost per day. **(8)**

b. At a port there are 6 unloading berths and 4 unloading crews. When all the berths are full, arriving ships are diverted to an overflow facility 20kms down the river. Tankers arrive according to a Poisson process with a mean of 1 every 2h. It takes for an unloading crew, on the average, 10h to unload a tanker, the unloading time following an exponential distribution. Find

(a) how many tankers are at the port on the average?

(b) How long does a tanker spend at the port on the average?

(c) What is the average arrival rate at the overflow facility? **(8)**

10.a. Derive Pollaczek-Khinchine formula for the average number of customers in the M/G/1 queueing system. **(8)**

b. Automatic car wash facility operates with only one boy. Cars arrive according to a Poisson process, with mean of 4 cars per hour and may wait in the facility's parking lot if the boy is busy. If the service time for all cars is constant and equal to 10 min, determine

(1) Mean number of customers in the system LS

(2) Mean number of customers in the queue

(3) Mean waiting time in the system

(4) Mean waiting time in the queue. **(8)**