

Course Descriptions

Fall 2020

Department of Mathematics
University of Illinois

Mathematics 500 — Abstract Algebra I

Fall 2020

Instructor: Charles Rezk

Office: 242 Illini Hall

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Webpage: <https://faculty.math.illinois.edu/~rezk/>

Course requirements:

Homework: Weekly homework assignments. (40% of grade.)

Tests: Two midterms (12.5% each) and a final (35%), in class. The final exam will resemble a Comp Exam.

Texts: The primary text will be:

- Dummit & Foote, *Abstract Algebra*, (3rd edition). Wiley, ISBN 978-0-471-43334-7.

Course topics: The course will cover approximately chapters 1–8, and 10–14 of Dummit and Foote. It will be assumed that students are familiar with basic material from an undergraduate algebra class, such as in Math 417. We will cover the following topics.

- Group actions.
- The Sylow theorems.
- Free groups and presentations of groups.
- Basic ring theory.
- Basic module theory.
- Classification of modules over a PID.
- Fields and field extensions.
- Galois theory.

FALL 2020
MATH 502
Introduction to Commutative Algebra

Time: Tu-Th 12:30–1:50

Instructor: Sheldon Katz

Recommended Text: Commutative Ring Theory, H. Matsumura

Prerequisites: Abstract Algebra I (Math 500) and Abstract Algebra II (Math 501)

In addition to being an important research area by itself, commutative algebra is indispensable for other research areas including algebraic geometry and algebraic number theory. This course is designed with all of these areas in mind.

In this course, the topics covered will include:

- Noetherian rings and modules
- Primary decomposition
- Flatness
- Completion
- Hilbert-Samuel polynomial
- Dimension theory
- Integral extensions
- Going-up and going-down theorems
- Noether normalization
- Regular rings
- Depth

Math 514 Complex Algebraic Geometry
Fall 2020 Course Description
Pierre Albin

This is a course on algebraic varieties over the complex numbers. We will spend the bulk of the course studying smooth varieties, but I hope to also spend time discussing singular varieties.

Smooth projective varieties over \mathbb{C} are more than just complex manifolds, they are Kähler. This means that they have compatible complex, Riemannian, and symplectic structures. The interplay of these structures is reflected in a rich interplay of geometric and topological invariants. We will develop techniques from sheaf theory and linear elliptic theory to study the cohomology of Kähler manifolds.

The textbook for the class is “Hodge Theory and Complex Algebraic Geometry I” by Claire Voisin, but we will not follow the text too closely. I expect to provide lecture notes. Grades will be based on the homework, which will be assigned semi-regularly.

Math 518

Professor Fernandes

- **Foundations of Differentiable Manifolds.** Differentiable manifolds and differentiable maps. Tangent space and differential. Immersions and submersions. Embeddings and Whitney's Theorem. Foliations. Quotients.
- **Lie Theory.** Vector fields and flows. Lie derivatives and Lie brackets. Distributions and Frobenius' Theorem. Lie groups and Lie algebras. The Exponential map. Transformation groups.
- **Differential Forms.** Differential forms and Tensor fields. Differential and Cartan Calculus. Integration on manifolds and Stokes Formula.

Textbooks:

You can find my [lecture notes here](#), but the following two textbooks are highly recommended.

Recommended Textbooks:

- John M. Lee, *Introduction to Smooth Manifolds*, Springer-Verlag, GTM vol 218, 2003. See the contents and first chapter [here](#).
- Michael Spivak, *A Comprehensive Introduction to Differential Geometry, Vol. 1*, (3rd edition) Publish or Perish, 1999.

Grading Policy and Exams

There will be weekly homework, 1 midterm and a final exam. All exams/midterms will be closed book.

- **Homework and in class participation (40% of the grade):** Homework problems are to be assigned once a week. They are due the following week, at the beginning of the Friday class. **No late homework will be accepted.** The two worst homework grades will be dropped. On Fridays, one homework problem will be discussed in class, with the participation of the students and this will be taken into account for the grade.
- **Midterm (20% of the grade):** The midterm will take place on Friday, October 11, in the regular classroom (the date is subject to change).
- **Final Exam (40% of the grade):** You have to pass the final to pass the course. According to the non-combined final examination schedule it will take place 7:00-10:00PM, Tuesday, December 17, in the regular classroom.

MATH 526 Algebraic Topology II

Fall 2020

Instructor: Jeremiah Heller (jbheller@illinois.edu)

Course description: This is the second semester of the algebraic topology sequence. Our focus will be on singular cohomology, its structure and applications. Topics include cup product in cohomology, Poincaré duality, vector bundles, characteristic classes, and cohomology operations. If time permits we'll cover some additional topics such as an introduction to complex K -theory.

Prerequisites: MATH 525 or consent of instructor.

Textbooks: We'll use the following texts:

- *Algebraic Topology*, by Hatcher. (Free pdf made available on author's homepage: <http://www.math.cornell.edu/~hatcher/AT/ATpage.html> .)
- *Geometry and Topology*, by Bredon. (Free pdf available through our library.)
- *Characteristic Classes*, by Milnor and Stasheff. (Free pdf available through our library.)

Other useful references:

- *Algebraic Topology*, by Switzer.
- *A Concise Course in Algebraic Topology*, by May.

Assessment: There will be regular homework and a final project.

FALL 2020, Math 531 : Analytic Theory of Numbers I

Instructor: Alexandru Zaharescu

Office: 449 Altgeld Hall

Phone: 265-5439

E-mail: zaharesc@illinois.edu

Lectures: TR 9:30-10:50am

Course Description. We will follow closely Professor Hildebrand's lecture notes.

Topics will include:

- (1) Arithmetic functions.
- (2) Elementary theorems on the distribution of primes.
- (3) Dirichlet series and Euler products.
- (4) Properties of the Riemann zeta function.
- (5) Analytic proof of the Prime Number Theorem.
- (6) Dirichlet's theorem on primes in arithmetic progressions.

Prerequisite: MATH 448 and either MATH 417 or MATH 453.

Grading Policy: Comprehensive final exam: 35%; Two midterm exams:
 $2 \times 25=50\%$; Homework: 15%.

Recommended Textbooks:

Main reference: A. J. Hildebrand, Introduction to Analytic Number Theory, available on Professor Hildebrand's webpage.

Additional reference: T. M. Apostol, Introduction to Analytic Number Theory, Springer-Verlag, 1st ed. 1976.

Math 535

Professor Lerman

- * Definition and examples of topology, topological spaces and continuous maps, bases, subbases.
- * subspaces, products
- * metrics and pseudometrics
- * quotient topology
- * nets
- * separation axioms: Hausdorff, regular, normal...
- * connectedness, local connectedness, path connectedness
- * compactness, Tychonoff theorem
- * compactness and completeness in metric spaces
- * Urysohn lemma, Tietze extension
- * countability axioms
- * paracompactness and partitions of unity
- * metrizability
- * topology on function spaces
- * categories, functors and natural transformations
- * fundamental groupoid
- * covering spaces

COURSE DESCRIPTION FOR MATH 540

XIAOCHUN LI

This course is mainly about the theory of functions on \mathbb{R}^n , and it is designed for first-year graduate students in mathematics. The following topics are planned to be covered.

- Abstract measure theory, Lebesgue measure, and measurable functions
- Lebesgue theory of integration and convergence theorems
- Differentiation of functions, functions of bounded variation
- L^p spaces
- Hilbert spaces and Fourier series (if time permits)

Lectures: MWF 11:00–11:50am in 441 Altgeld.

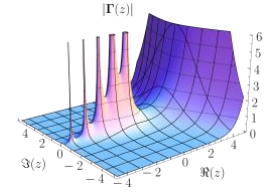
Textbook: G. B. Folland, Real Analysis, John Wiley & Sons

Grading: Homework (15%), 1 or 2 midterms (40%), and a final (45%). No make-up exams.

Exams: The midterm exam will be a 50-minute exam in class. The final will be a 3-hour exam.

Math 542 Complex Variables - Fall 2020

A dry, factual overview might assert that complex analysis investigates analytic and geometric properties of differentiable functions of a complex variable.



Credit: Wikipedia on “Zeros and poles”

Such a statement would do no justice whatsoever to the power or beauty of the subject. Would you believe that if a complex function can be differentiated once, it can be differentiated infinitely often? That adding a point at infinity to the plane and wrapping up to a sphere enables us to map circles and lines to each other simply by rotating? That a huge number of integrals can be evaluated if you know the behavior of the integrand at just a few special points and loop around them in the right way? That the correct generalization of a line to three dimensions is not a plane but rather a saddle? And more...

If you prefer less excitement than this, you may consult instead the list of topics in the official [departmental syllabus](#).

Prerequisites: either Math 446 and 447, or else Math 448.

In practice this means you must have previously studied undergraduate real analysis, and that some undergraduate complex analysis is highly recommended too. Students who possess plenty of mathematical maturity (e.g. have passed some comps already and know topology) can succeed in Math 542 without having previously taken complex analysis, though they will find the course moves quickly. Otherwise, it would be better to take Math 446 or 448 instead.

Assessment:

Two midterms and a final exam. Weekly homework. Attendance.

Style of the course

Supportive and inclusive. I run weekly collaborative homework sessions, and want you to come to my office hours and work with other students in the course.

Textbook (tentative, could change before the Fall semester)

Bruce Palka, *An Introduction to Complex Function Theory*

Optional reading (for enjoyment and elucidation)

Tristan Needham, *Visual Complex Analysis*, 2nd edition.

His book presents a wonderfully geometric approach to the subject.

- Prof. Richard Laugesen Laugesen@illinois.edu (please email me with questions.)

Beauty is truth, truth beauty,—that is all
Ye know on earth, and all ye need to know.
by John Keats, in his “Ode on ~~Complex Analysis~~ a Grecian Urn”

Math 546 Hilbert Spaces

Instructor: Marius Junge

Hilbert space theory is used in many areas of mathematics, in particular in the interface of analysis, geometry and algebra. The first goal of this course is to show how operations on matrices can be extended to operators, in other words study functional calculus and spectrum of operators. The next aim is to provide the basics for operator algebra theory including C^* -algebras and von Neumann algebras. Last but not least, we will apply these ideas to basics concepts in quantum mechanics and their its foundation, and, if time permits, some applications in quantum information theory.

The following books will be used, with * being the official textbook.

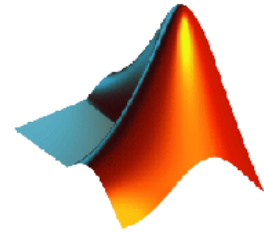
- (1) **John B. Conway:** A course in Functional Analysis, Springer, Graduate Text in Mathematics 1990 (second edition)
- (2) **John B. Conway*:** A Course in Operator Theory, Graduate Studies in Mathematics, Vol. 21, 1999
- (3) **E. Lance:** Hilbert C^* -modules, London Mathematical Society Lecture Note Series, 210., Cambridge University Press, Cambridge, 1995

Math 554. Fall 2020.

Linear Analysis & Partial Differential Equations

Prof. Richard Laugesen <Laugesen@illinois.edu>

Linear algebra on infinite dimensional spaces, such as L^p or Sobolev spaces, generates the tools of functional analysis.



By developing these spaces and tools, Math 554 can present the modern theory of partial differential equations (elliptic, parabolic, and hyperbolic), relying in particular on the concepts of weak convergence and compact imbeddings in Hilbert spaces. The difference from the classical theory in Math 553 is that we no longer have explicit solution formulas - which makes you wonder: how we can say anything at all about the behavior of solutions?!

The answer is that a “hard” core of analytic estimates provides inputs to “soft” structural results from functional analysis. Hence we obtain **qualitative information** on partial differential equations, such as existence, smoothness, maximum principles, energy conservation/decay, and finite speed of propagation, even when the PDEs cannot be solved explicitly.

The course will be valuable to students of **differential equations, numerical analysis, probability, and differential geometry.**

Background/Prerequisites:

Math 442 or 553 Partial Differential Equations, and also Math 540 Real Analysis, are strongly recommended. The partial differential equations background is needed to understand the point of the course. The graduate real analysis is not strictly required provided you have a strong undergraduate background in analysis and know metric space topology, and are willing to work hard and some take facts on faith. Math 541 Functional Analysis is **not** required. Please talk with me if you have any concerns about your background.

Course website <http://www.math.illinois.edu/~laugesen/>

Assessment: Homework 80%, class participation 20%.

I promise an inclusive and supportive experience.

Collaboration is encouraged during the optional, regular homework sessions.

Text: No required textbook. Sources include:

1. G. B. Folland, Real Analysis
2. L. C. Evans, Partial Differential Equations
3. P. D. Lax, Functional Analysis
4. A. Pazy, Semigroups of Linear Operators and Applications to PDEs

MATH 558 — Methods of Applied Mathematics — Fall 2020

Lee DeVille, Mathematics

rdeville@illinois.edu

Course overview. “Applied mathematics” is a rich, broad, and deep field which uses results from many branches of pure mathematics and connects them to problems in science and engineering. The field could not be comprehensively covered in one semester (or perhaps even in one lifetime). What we will do instead is introduce several of the powerful perspectives which make up modern applied mathematics and, for each, present one or more important applications. We will cover both exact mathematical analysis and computational techniques.

In particular, this course will be very useful for graduate students who are interested in eventually doing research in applied mathematics. Each topic will include an overview of the open questions and “Big Problems” in the field, and a list of resources of where to look next for students interested in the topic.

The topics will include:

1. Nondimensionalization and scaling analysis
 - 8 lectures. Material will include reading from the texts of Barenblatt and Holmes.
2. Regular asymptotics — mean-field laws, mass-action laws, Large Number limits for large systems, van Kampen expansions, Kramers-Moyal expansions, Gronwall’s estimates
 - **Applications:** epidemic sizes in large populations; short-time orbital mechanics; homogeneous steady-states in networks
 - 13 lectures. Material will include reading from the texts of Holmes, Gardiner, and Shwartz/Weiss.
3. Singular asymptotics — multiscale systems, homogenization, WKB expansions, boundary layers, traveling waves in reaction–diffusion systems, large deviations
 - **Applications:** neural models (Hodgkin–Huxley, Fitzhugh–Nagumo) and their dynamics; switching times between equilibria for biochemical and biological systems; epidemics in small populations; semiclassical approximations in quantum mechanics; dynamics of excitable media
 - 12 lectures. Reading will include the books of Holmes, Gardiner, and Keener
4. Complex systems — spectral theory, stability analysis
 - **Applications:** scale-free networks (in social networks, epidemiology, computer networks); synchronization and critical phenomena for biophysical systems
 - 9 lectures. Material will include several recent papers.

Prerequisites. This course will require undergraduate background in ODEs, PDEs, probability theory (MATH 441, 442, 461 or equivalents). However, the more a student brings to the course, the more the student will get out of it, so graduate courses in these areas can only help. Interested students who are not sure if they have sufficient background are encouraged to email me at rdeville@illinois.edu and discuss their readiness.

Math 562 (Probability II)

Instructor: Renming Song
Office: 338 Illini Hall
Phone number: 217 244 6604

Text: Jean-Francois Le Gall : Brownian Motion, Martingales and Stochastic Calculus, 2016, Springer

Course Topics: This is the second half of the basic graduate course in probability theory. This course will concentrate on stochastic calculus and its applications. In particular, we will cover, among other things, the following topics: Brownian motion, stochastic integrals, Ito's formula, martingale representation theorem, Girsanov's theorem, stochastic differential equations, connections to partial differential equations. If time allows, I will also present some applications to mathematical finance.

Math 561 is a prerequisite for this course. However, if you have not taken Math 561, but are willing to invest some extra time to pick up the necessary materials from 561, you may register for this course.

Grading Policy: Your grade will depend on homework assignment and a possible final exam.

MATH 564 — Applied Stochastic Processes — Fall 2020
Lee DeVille, Mathematics
rdeville@illinois.edu

Course overview. This course is designed for both math and non-math graduate students. Measure theory is **not** a prerequisite for this course. However, a student should have a solid knowledge of undergraduate-level probability theory (math 461 or its equivalent).

The goal of this course is to reach fairly rigorous understanding of Markov chains and Markov processes. We are going to cover most of the materials from Norris' book, augmenting the text when necessary. Below is a rough list of some of the topics we will cover:

Strong Markov properties, recurrence and transience, invariant distributions, convergence and ergodicity, time reversal, Q -matrices, holding time, forward and backward equations, martingales and potential theory, queuing networks, Markov decision processes, Markov Chain and Monte Carlo techniques. Depending on the interest of the audience, we may cover some additional materials.

Main text: J.R. Norris, *Markov Chains*.

Will supplement with other materials as well.

Prerequisites. This course will require undergraduate background in probability theory (MATH 461 or equivalent) and some ODEs (MATH 441-ish). However, the more a student brings to the course, the more the student will get out of it, so graduate courses in these areas can only help. Interested students who are not sure if they have sufficient background are encouraged to email me at rdeville@illinois.edu and discuss their readiness.

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
DEPARTMENT OF MATHEMATICS
Course Description — FALL 2020

MATH 581

EXTREMAL GRAPH THEORY

Instructor: A. Kostochka, 234 Illini Hall, 265-8037, kostochk@math.uiuc.edu.

TEXT: D. B. West, "The Art of Combinatorics", Volume I: Extremal Graph Theory. There also could be some handouts. Specialized texts covering the topics of the course will be listed on the course web site and will be on reserve in the library.

TOPICS: This is a companion course to Math 582 (Structure of Graphs). The two courses are independent and discuss advanced material in graph theory. Extremal Graph Theory includes topics drawn from the following.

Trees and Distance: Optimization with trees; diameter and distance; encodings and embeddings.

Matching and Independence: Bipartite matching, min-max relations, van der Waerden conjecture; algorithms and applications (weighted matching, Menger's Theorem, Hopcroft-Karp algorithm, randomized on-line matching, stable matching); factors (Tutte's f -Factor Theorem, Edmonds' Blossom Algorithm); independent sets and covers, dominating sets and hypergraph transversals.

Coloring: Vertex colorings (bounds, generalized colorings); structure of k -chromatic graphs (color-critical graphs, forced subgraphs); edge-colorings (Vizing's Theorem and extensions); variations and generalizations (interval, list, circular colorings). Colorings of hypergraphs.

Perfect Graphs and Intersection Graphs: Perfect and imperfect graphs (Perfect Graph Theorem, partitionable graphs); classes of perfect graphs (chordal, interval, threshold, perfectly orderable, etc.).

Other Extremal Problems: Forbidden subgraphs (Turán's Theorem, Erdős-Stone Theorem); graph decomposition (arboricity, fractional arboricity, paths and cycles); representation parameters (intersection number, boxicity, interval number), etc.

COURSE REQUIREMENTS: There are no exams. There will be 5 problem sets, each requiring 5 out of 6 problems for 50 points total and 4 quizzes each worth 20 points. The problems in homeworks require proofs related to or applying results from class. Quizzes will ask for definitions, statements and applications of the course theorems. Roughly speaking, 85% of 330 points suffices for an A, 66% for a B. Discussions between students about problems can help understanding. Collaborations should be acknowledged, and submitted homework should be written individually. Electronic mail is a good way to ask questions about homework problems or other matters.

PREREQUISITE: Math 580, or Math 412, or CS 473, or consent of instructor.

Instructor: Alexander Yong

Course description: This is a graduate course on methods in algebraic combinatorics. The course will have roughly three components that represent modern preparation in the field. These are enumerative techniques, symmetric functions, and multivariate polynomials.

The intended audience for this course will be PhD students in combinatorics as well as students in representation theory, algebra and geometry where methods from algebraic combinatorics arise. The prerequisite for the course is Math 580 (or instructor approval). This course will be taught from a combinatorial perspective. The only exceptions (representation theory of the symmetric group and GL_n) will only assume modest background in graduate level algebra.

Grading: Based on homework exercises and a final presentation.

Below I give a list of specific topics and approximate class usage:

I. *Generating series*

- Ordinary generating series: e.g., tree enumeration, partition enumeration (including the Jacobi Triple Product identity and Euler's pentagonal number theorem), Lagrange inversion
- Exponential generating series: e.g., permutation enumeration, counting connected graphs, theory of species
- Asymptotics of coefficients of generating series

II. *Additional general enumerative/algebraic methods*

- Möbius inversion (incidence algebra)
- Lindström-Gessel-Viennot lemma
- The Combinatorial Nullstellensatz

III. *Symmetric functions*

- monomial elementary, homogeneous, power sum bases and their transitions
- Representation theory of the symmetric group, characters and the Murnaghan-Nakayama rule
- Schur polynomials
- Schur polynomials as characters of GL_n representations

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IV. Young tableaux:

- The hook-length formula
- RSK correspondence
- The Littlewood-Richardson rule
- Jeu de taquin

V. Polynomials:

- Schubert polynomials
- Symmetric group combinatorics including Bruhat order and reduced words
- Quasisymmetric functions
- Macdonald polynomials and their specializations (Demazure characters, atoms, Hall-Littlewood polynomials)

Textbooks will be

- H. S. Wilf, *Generating Functionology*, available for free download at <https://www.math.upenn.edu/~wilf/DownldGF.html>
- R. P. Stanley, *Enumerative combinatorics*, vol. 1, 2nd edition, Cambridge University Press, 1997.
- R. P. Stanley, *Enumerative combinatorics*, vol. 2, Cambridge University Press, 1999.
- W. Fulton, *Young tableaux*, Cambridge University Press, 1997.

Math 595

Professor Fernandes

- **Lie groupoids**
 - **Lie algebroids**
 - **Lie functor and integrability**
 - **Differentiable stacks**
 - **Special Topics.** To be chosen from the interests of the students.
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Textbooks:

I will provide to participants some lecture notes as the course progresses, but the following two references should also be very helpful:

- A. Cannas da Silva and A. Weinstein, *Geometric models for noncommutative algebras*, Berkeley Mathematics Lecture Notes, 10. American Mathematical Society, Providence, RI, 1999.
 - M. Crainic and R.L. Fernandes, [Lectures on Integrability of Lie Brackets](#), available as [Geometry & Topology Monographs 17 \(2011\) 1-107](#).
 - D. Metzler, [Topological and Smooth Stacks](#), Preprint arXiv:math/0306176.
 - A. Vistoli, [Grothendieck topologies, fibered categories and descent theory](#), Fundamental algebraic geometry, 1104, Math. Surveys Monogr., 123, AMS Providence, RI (2005).
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Grading Policy

- **Expository Paper:** Students will be encouraged to write (in LaTeX) and present a paper. This is not mandatory. Following the tradition of topics courses, there will be no homework and no written exams.

Math 595 TDGB Fall 2020
Topological Degree and Global Bifurcations

Class time: TR

Lecturer: Eduard Kirr, e-mail: ekirr@illinois.edu

Description: I will first present a modern, axiomatic and rather elementary introduction of the *topological degree theory* together with the fun stuff that comes with it e.g., *the Ham and Sandwich and Hairy Ball Theorems*, and the more profound applications e.g., *the Jordan Curve, Domain Invariance, Brouwer and Schauder Fixed Point Theorems*. No *algebraic topology* will be involved albeit the invariance of the degree under homotopies gives its topological name, and, moreover, the finite dimensional (Brouwer) degree has an equivalent definition via the induced map on the top homological group.

Then I will move to applications to dynamical systems which exhibit important special solutions such as equilibria, periodic solutions, or, more generally, *coherent structures* which include: solitons, solitary waves, kinks, bound states, etc. But they all can be found as zeroes of well chosen maps between certain Banach spaces. Since degree theory is good at counting zeroes of such maps, one of its most important application is *the existence and bifurcations of coherent structures*. We will study Krasnoselski type theorems which give sufficient conditions for new coherent structures to emerge from old ones via *local bifurcation* phenomena. One such theorem has been used to explain *the collapse of Tacoma Narrows Bridge*, see Ref. 5. However, a drawback of local bifurcation theory is that it requires to apriori know a coherent structure in order to find new ones. The trivial, zero solution, is usually the apriori coherent structure since most dynamical system model displacement from a certain equilibrium which is normalized to be zero. Consequently, local bifurcation theory is usually applied in weakly nonlinear regimes and produces branches of small coherent structures.

Global Bifurcation Theory on the other hand attempts to follow branches of coherent structures produced via local bifurcations and determine how far can they be continued. I will present two such results, one using topological degree, the other using real analytic functions. They both show that branches emanating from local bifurcations either form a loop or can be continued until they reach the so called *Fredholm boundary*. While these results generally show the existence of large coherent structures they cannot say where exactly each branch ends (the Fredholm boundary is quite large, usually unbounded) and whether there were other bifurcations along it. However, my very recent results show how a particular dynamical system selects a small set of possible end points on the Fredholm boundary and how local and global bifurcations can be used to trace back the branches from these endpoints, determine the bifurcations along them and, consequently find all branches of coherent structures which do not form loops. I will discuss this result and its possible generalization to a large class of dynamical systems.

References: I will follow my own notes (posted online) based on the following references:

1. Topics in Nonlinear Functional Analysis by L. Nirenberg.
2. Leray-Schauder degree: a half century of extensions and applications by J. Mawhin in *Topol. Methods Nonlinear Anal.* Volume 14, Number 2 (1999), 195-228.
3. The global bifurcation picture for ground states in nonlinear Schrodinger equations by E. Kirr and V. Natarajan available online at <https://arxiv.org/pdf/1811.05716.pdf>
4. Long time dynamics and coherent states in nonlinear wave equations by E. Kirr in *Recent Progress and Modern Challenges in Applied Mathematics, Modeling and Computational Science*, R. Melnik, R. Makarov, J. Belair, Eds. in *Fields Institute Communications*, 2017, pp. 59–88.
5. Large Torsional Oscillations in a Suspension Bridge: Multiple Periodic Solutions to a Nonlinear Wave Equation by K.S. Moore in *SIAM J. Math. Anal.* Vol. 33, No. 6, pp. 1411–1429

Grading Policy: There are no homework assignments or exams for this course. The participants will be asked to make a presentation on the applications of these techniques to a nonlinear differential equation, preferably from their own research area. Grades will be based on class activity, and on the quality of the presentation.