

**Civil Engineering Design (1)**  
***Analysis and Design of Slabs***

**2006/7**

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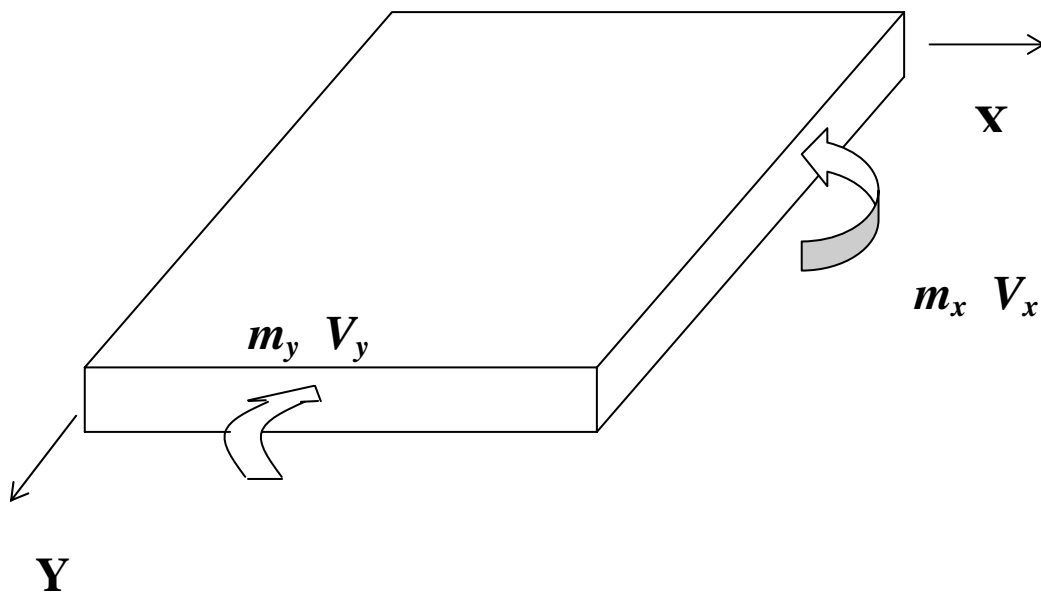
## Contents

<b>1. Elastic Methods.....</b>	<b>3</b>
1.1 Introduction.....	3
1.2 Grillage Analysis .....	4
1.3 Finite Element Analysis.....	6
<b>2. Yield Line Theory.....</b>	<b>9</b>
2.1 Introduction.....	9
2.2 Example 1: Ultimate Behavior of One-Way Spanning Slab.....	11
2.3 Yield Line Basis .....	18
2.4 Example 2 – Yield Lines not Parallel to Axes .....	26
2.5 Example – Simply-supported Rectangular 2-way Slab.....	28
2.6 Example – General Solution of Continuous Rectangular 2-way Slab .....	31
2.7 Example .....	35
2.8 Exam Question 2006 – Q6(b).....	37
<b>3. Lower Bound Methods.....</b>	<b>39</b>

# 1. Elastic Methods

## 1.1 Introduction

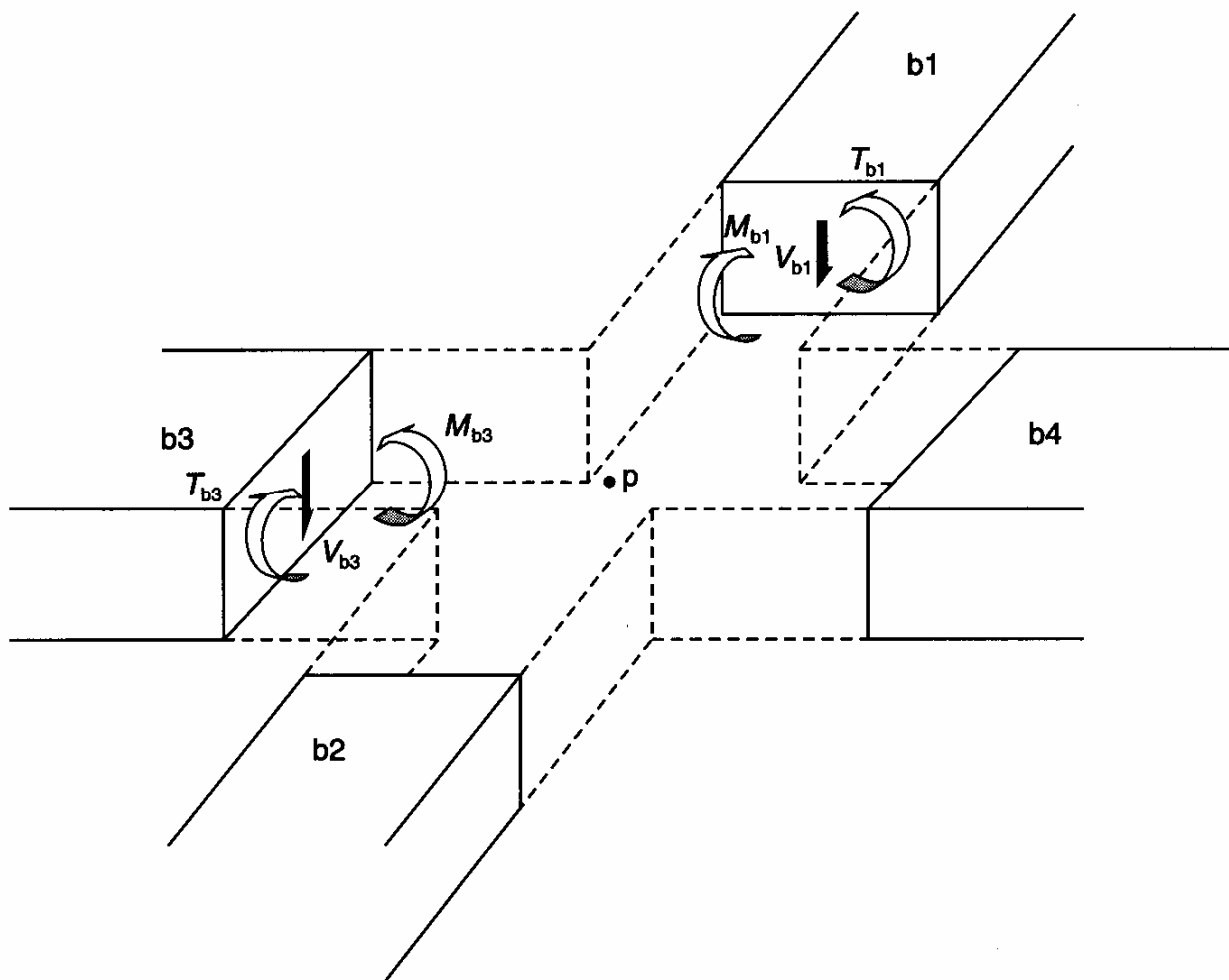
A linear elastic analysis is carried out, generally by computer, to determine the moments and shear forces due to applied loads *in each direction*.

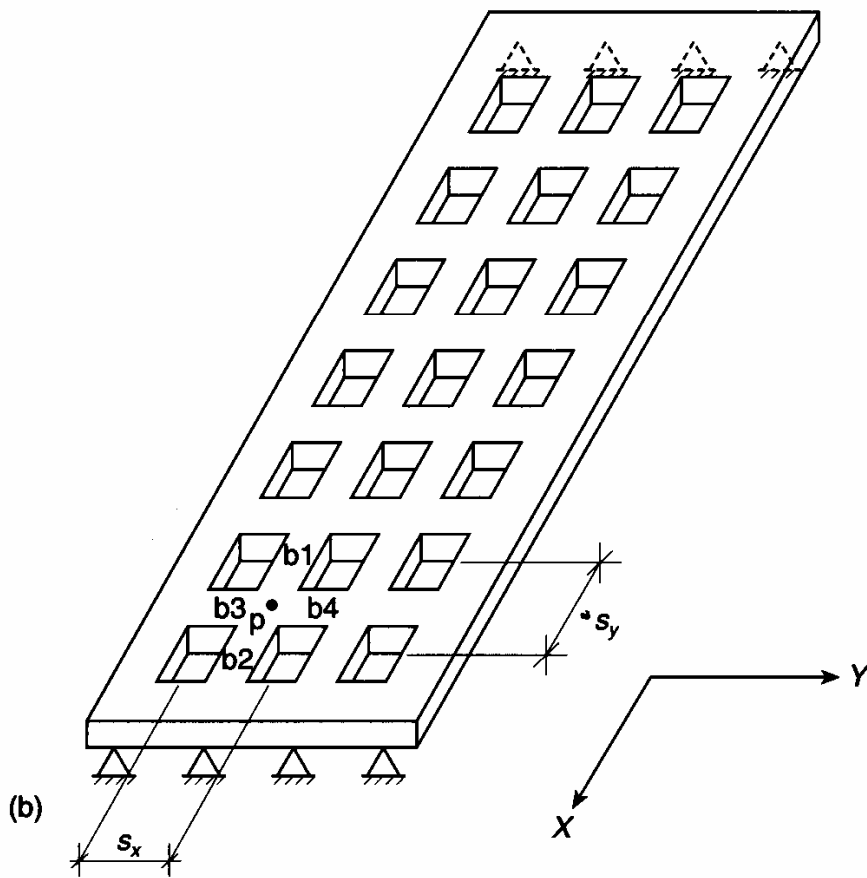
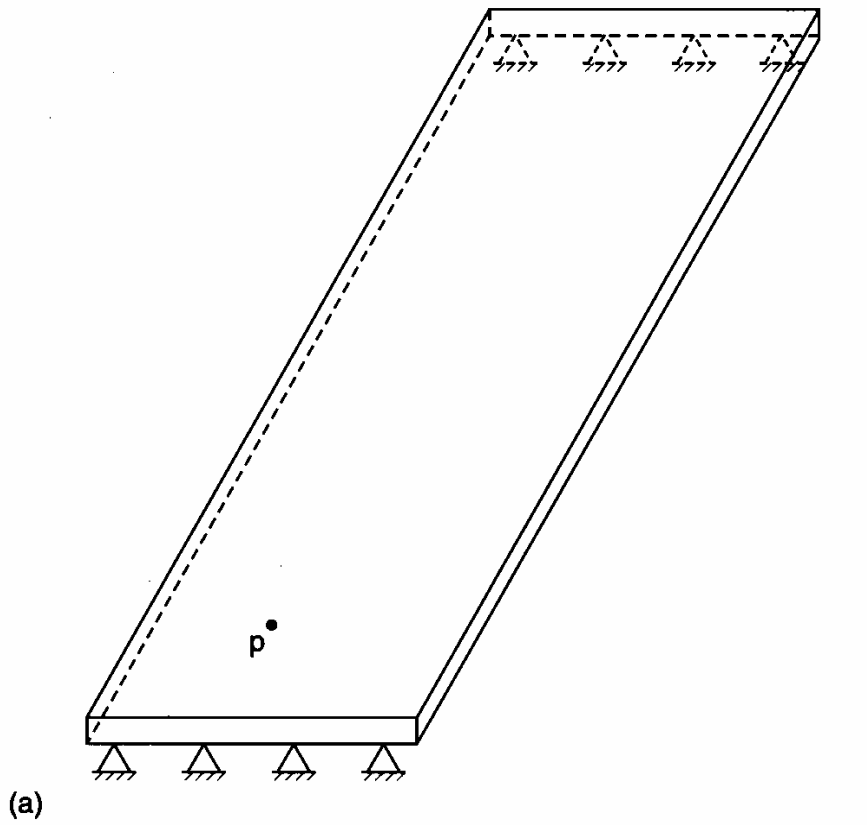


- With slabs,  $m_x$  is *not* moment about the X-axis. It is moment on the X-face, i.e., the face  $\perp$  X-axis
- A unit width of slab is then designed for the moment per unit width found from the computer analysis.
  - You check SLS stresses if the slab is prestressed
  - You compare applied ULS moment to ultimate moment capacity (or, for RC, calc steel needed to give ultimate moment capacity  $\geq$  applied ULS moment)
  - You should check applied ULS shear against the shear capacity
- If 1-way spanning, analyse 1 m strip as a continuous beam
- If 2-way spanning, there are two linear elastic methods of analysis of slabs, Grillage and Finite Element.

## 1.2 Grillage Analysis

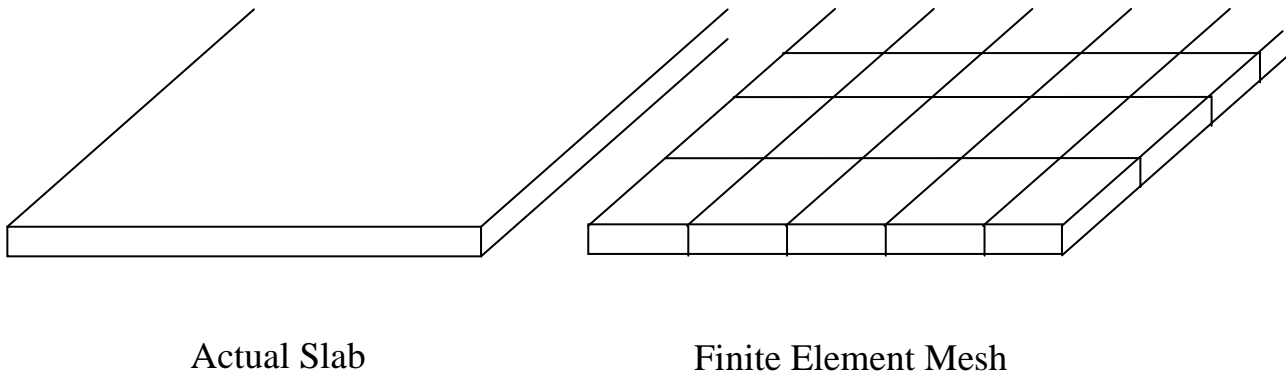
- This is the traditional approach for bridges. Not normally considered worthwhile for a floor slab (grillage takes time).
- Slab is represented in the computer as a mesh of beams. Each beam represents a strip of slab
- see figures below (5.13 & 5.14 from *Bridge Deck Analysis* by OBrien & Keogh)





### 1.3 Finite Element Analysis

The slab is represented by a mesh of slab segments (called elements) joined to each other at nodes. The elements can be triangular, rectangular or quadrilateral.



FE is increasing in popularity:

- Easy to use
- Software becoming more widely available
- Very accurate (*if used properly*) – better than Grillage
- Can improve accuracy by increasing density of mesh and, with increasing computer power, this is now very easy.

But FE can give *very wrong* answers if used improperly:

- Near the supports, FE can give very large shear forces. This results from people specifying point supports (a support of zero width implies an infinite shear force per unit width). There is no simple solution to this - it is currently very difficult to calculate shear forces near point supports in slabs. In a flat slab building, you can do a punching shear calculation based on the reaction but, in a slab bridge, a knowledge of the distribution of shear stresses would be helpful.

- Compatibility between elements can only be guaranteed at the nodes (where it is made to happen). Therefore, avoid 'T-junctions' unless the elements have mid-side nodes.
- Avoid long narrow elements - can lead to round-off errors in some software.

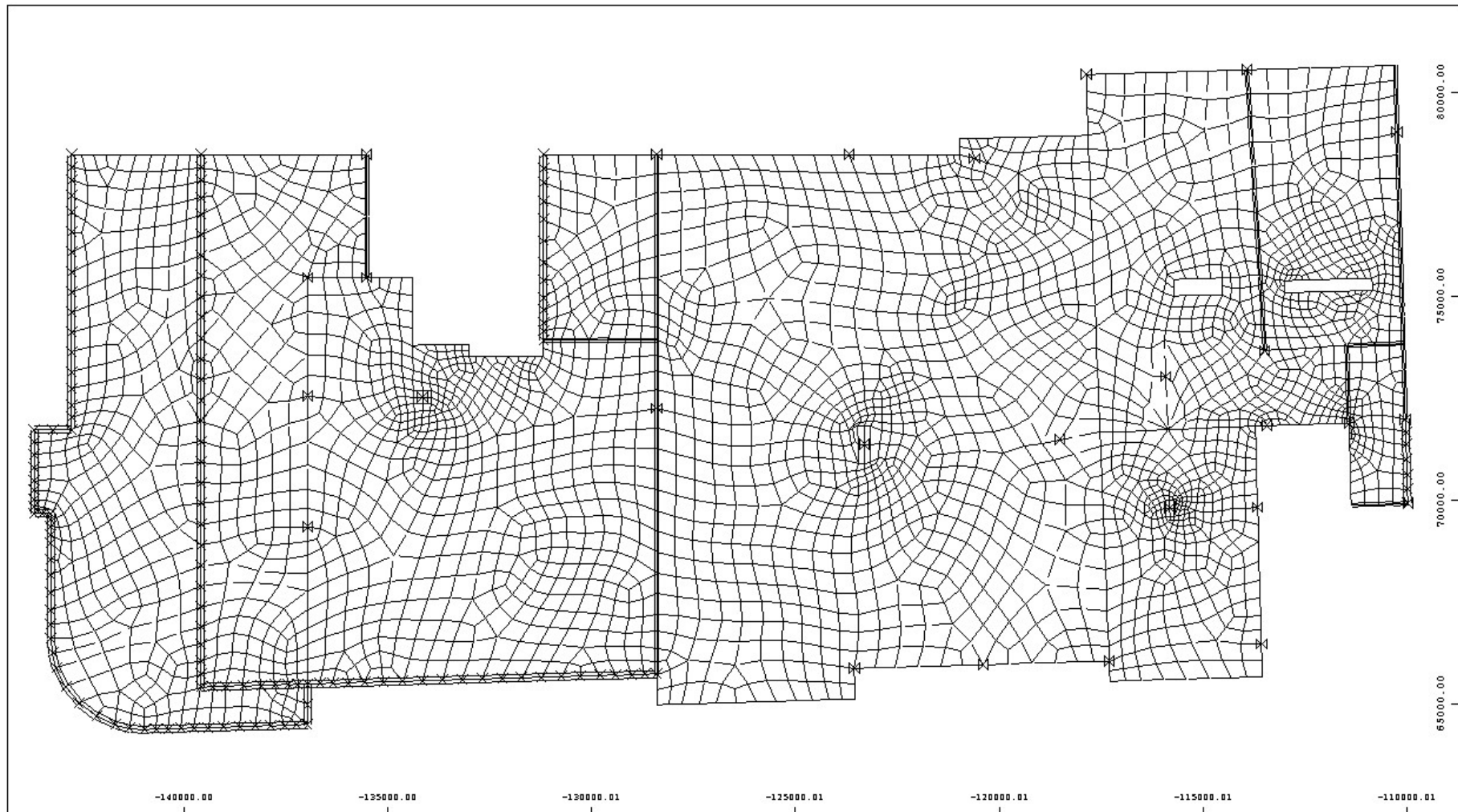
Some benefits of FE analysis of slabs are:

Transfer slabs can be used instead of beam-and-slab construction:

- This saves on formwork over beams;
- Easier to achieve repetition which means a shorter learning curve on site.

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CENTURY BUILDINGS  
SYSTEM

M 1 : 124

Y  
|

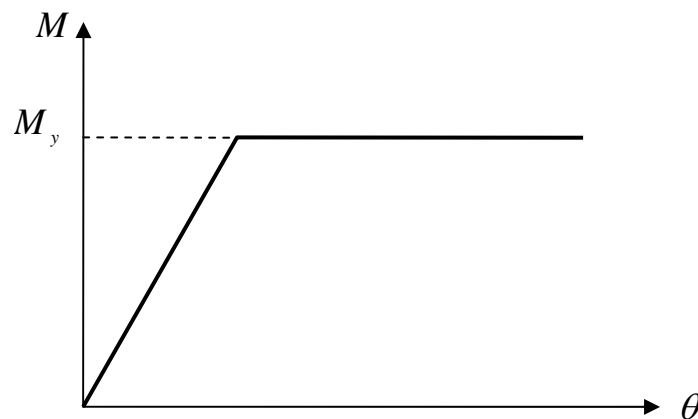


## 2. Yield Line Theory

### 2.1 Introduction

Yield line analysis is an analysis approach for determining the ultimate load capacity of reinforced concrete slabs and was pioneered by K.W. Johansen in the 1940s. It is closely related to the plastic collapse or limit analysis of steel frames, and is an Upper Bound or Mechanism approach.

It is only applicable to *ductile* (under-reinforced) slabs since we assume that the following moment-rotation diagram holds:



Note from the diagram the following:

- A yield line occurs when the moment capacity of the section has been reached.
- No additional moment can be taken at the section.
- The section can undergo any amount of rotation.

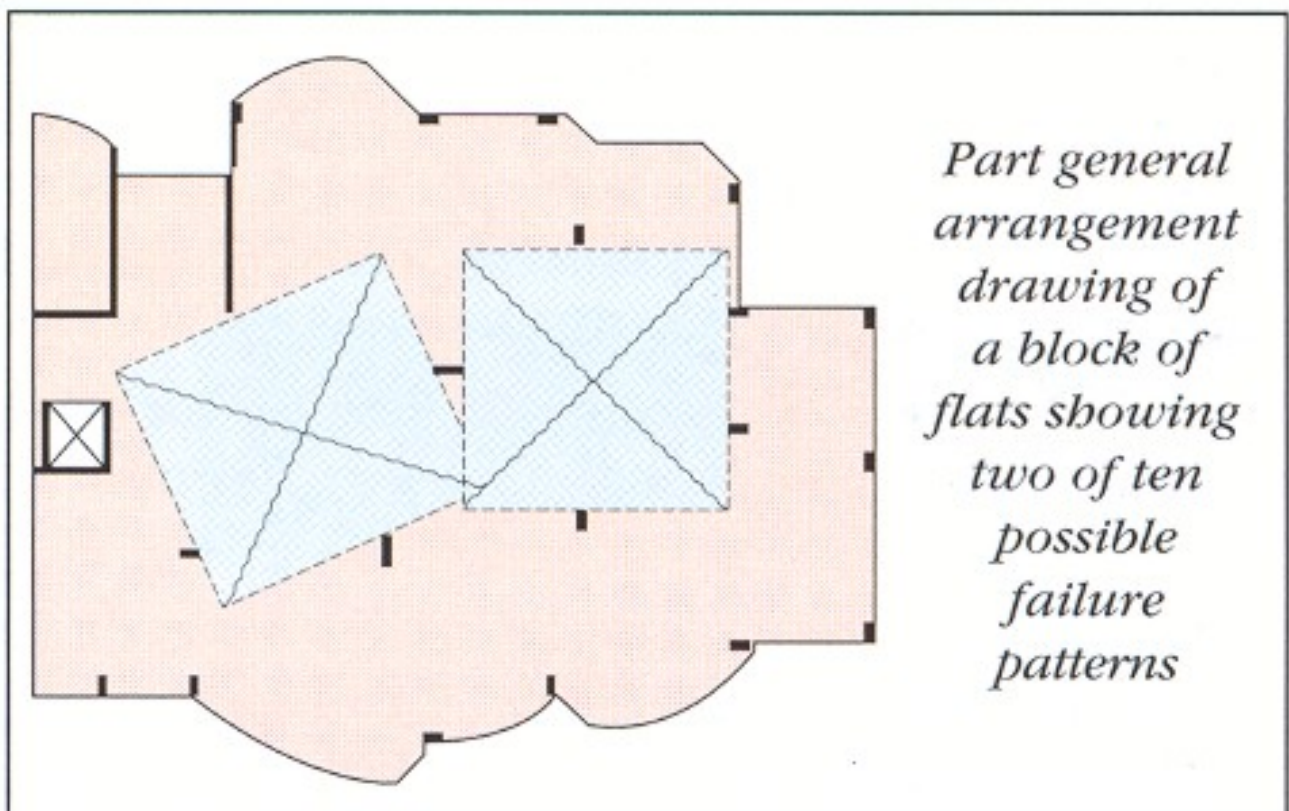
The draft Eurocode EC2 allows yield line analysis provided the ratio,  $x/d$ , is sufficiently low. BS8110 does not allow yield line unless results are ‘similar’ to those from a linear elastic method.

### Advantages of Yield Line over Linear Elastic Analysis

- Simpler to use (computer not necessary);
- Linear elastic only tells you when first yield occurs. Y.L. gives the ultimate capacity of the slab - what it takes to cause collapse;
- Helps understanding of ultimate behaviour;
- Good for awkward shapes.

### Disadvantages are:

- Requires experience to know a likely failure mechanism;
- Dangerous designs are possible without checking or experience;
- Does not give an idea of slab behaviour in service.

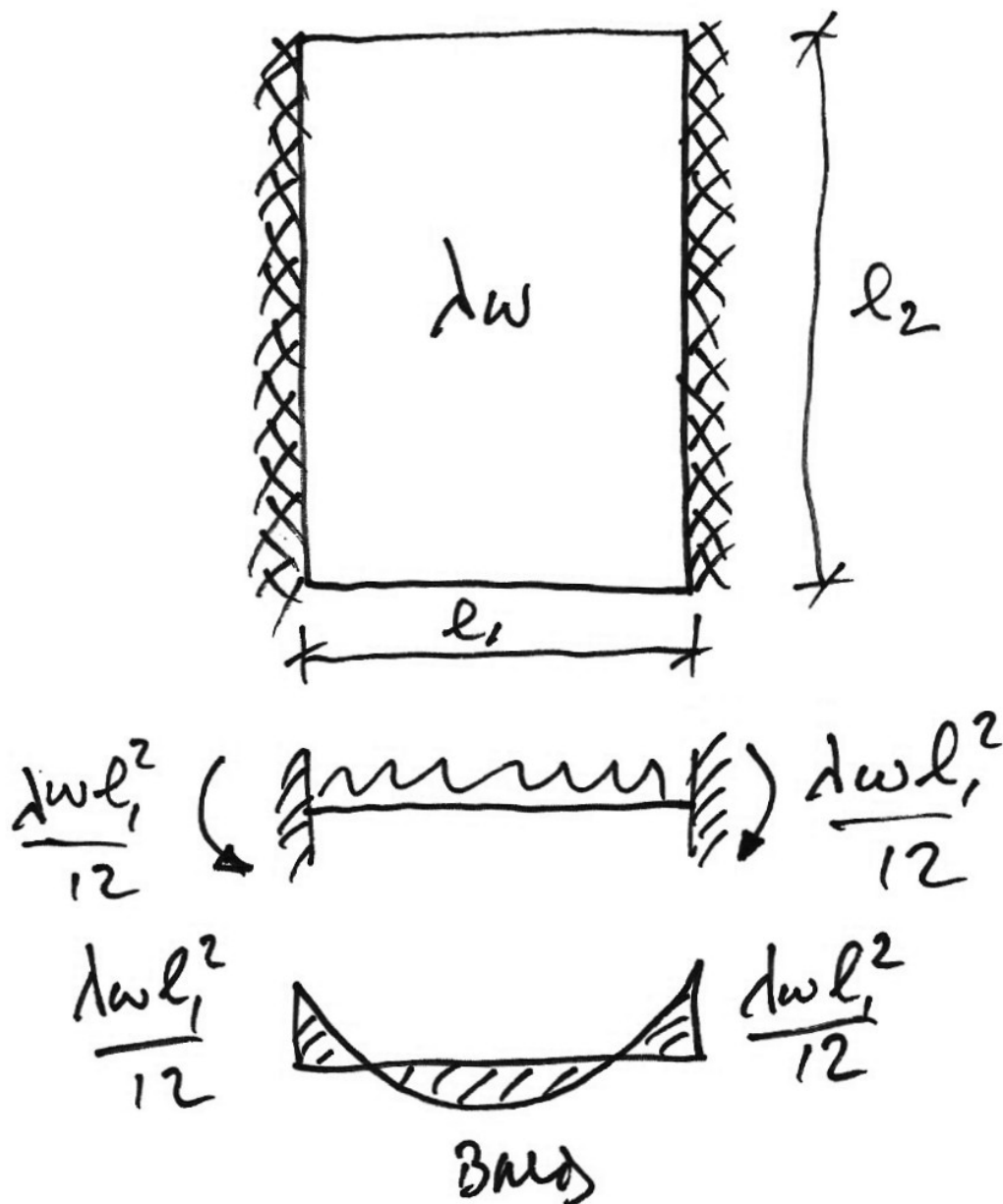


## 2.2 Example 1: Ultimate Behavior of One-Way Spanning Slab

### Linear Elastic Analysis

Methods such as moment distribution are linear elastic methods. To see their result for this slab we will find the load factor,  $\lambda$ , to cause failure. That is, the load factor at which the ULS moment equals the moment capacity.

The following is evident from your knowledge of structures:



If we take some values:

- $w = \text{ULS uniformly distributed load on slab} = 10 \text{ kN/m}^2$ ;
- Moment capacity in sag,  $m = 40 \text{ kNm/m run}$ ;
- Moment capacity in hog,  $m' = 50 \text{ kNm/m run}$ ;
- $l_1 = 8 \text{ m}$  and  $l_2 = 10 \text{ m}$ .

Note that the calculations are carried out for each metre strip of slab.

The first yield in sag occurs when

$$\begin{aligned}\frac{\lambda w l_1^2}{24} &= m \\ \lambda &= \frac{24m}{w l_1^2} \\ &= \frac{24 \times 40}{10 \times 8^2} \\ &= 1.5\end{aligned}$$

The first yield in hog would occur when  $\lambda$

$$\begin{aligned}\frac{\lambda w l_1^2}{12} &= m' \\ \lambda &= \frac{12m'}{w l_1^2} \\ &= \frac{12 \times 50}{10 \times 8^2} \\ &= 0.9375\end{aligned}$$

Clearly, yield occurs in hog at a lower load due to the lower *yield load factor* of 0.9375. Therefore, according to linear elastic analysis, this slab is unsafe for the specified ULS load of  $10 \text{ kN/m}^2$ .

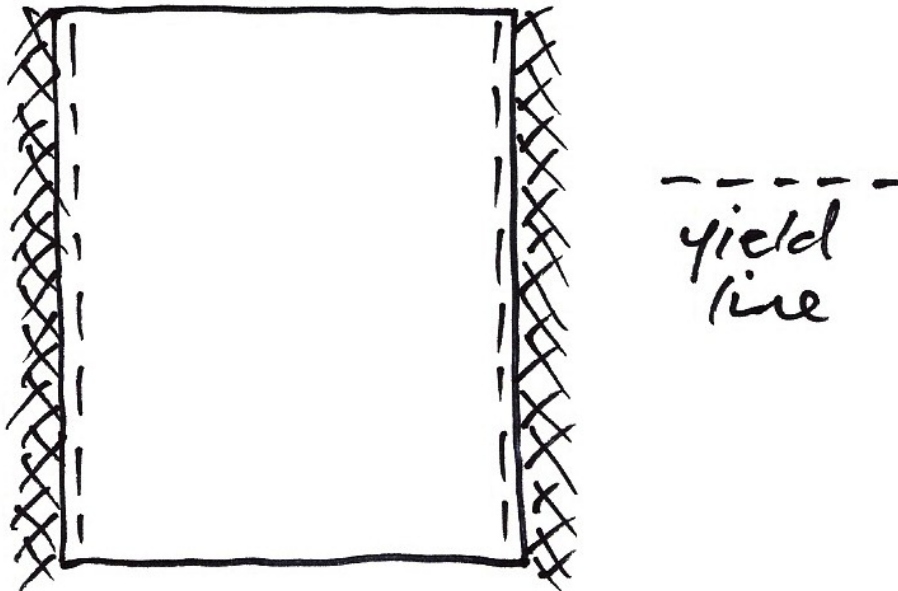
### Elastic Plastic Analysis

Again we are to find the load factor,  $\lambda$ , to cause collapse. The steps are:

#### Step 1

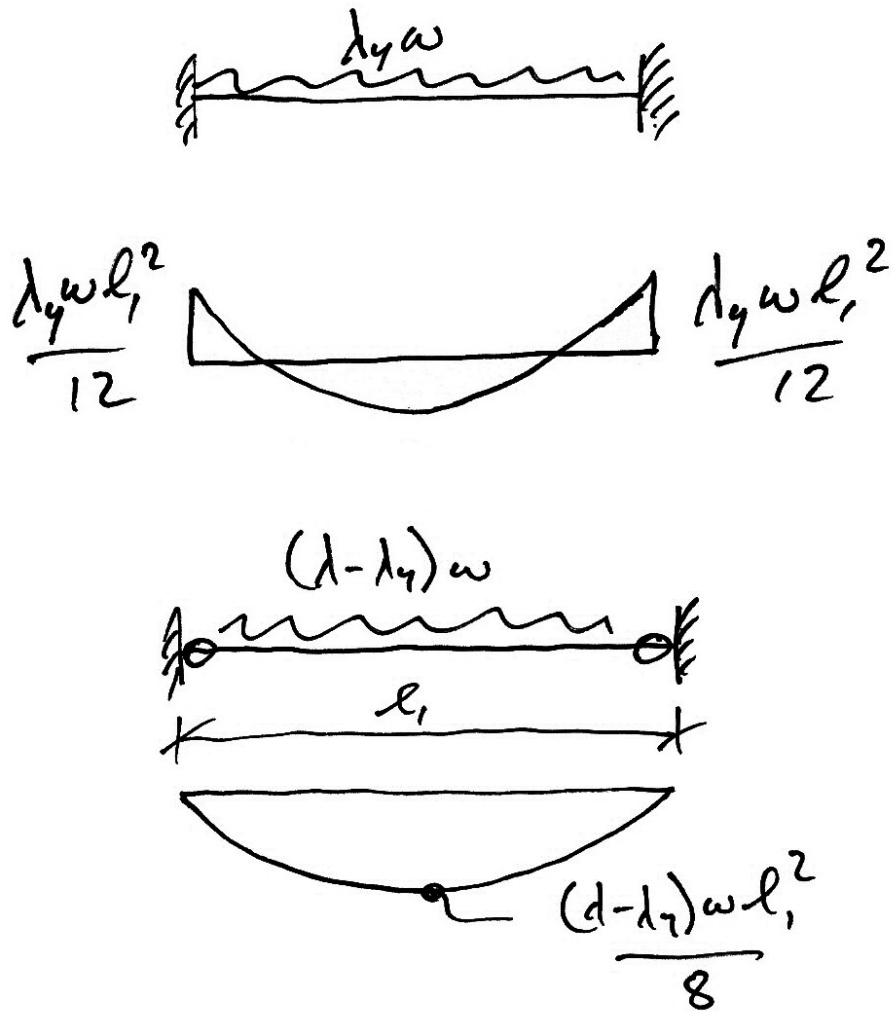
Do a linear elastic analysis to find the point at which yield first begins. From before, we know that the first yield occurs in hog when  $\lambda = \lambda_y = 0.9375$ .

At this load, plastic hinge forms all along the supports. They are called *yield lines*:

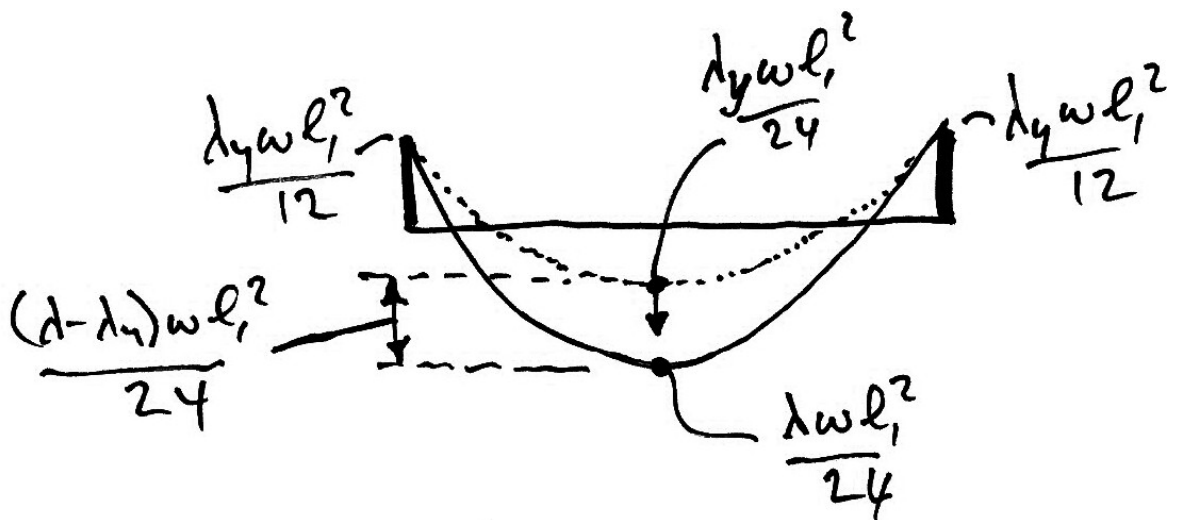


#### Step 2

For  $\lambda$  in excess of 0.9375, there is effectively a hinge all along the length of the support at each side. The BMD is found by adding that from Step 1 plus the extra BMD induced by the increase in load.



So as the load increase past  $\lambda_y$ , the bending moment diagram moves downwards:



The next yield occurs when the moment at the centre reaches  $m$ :

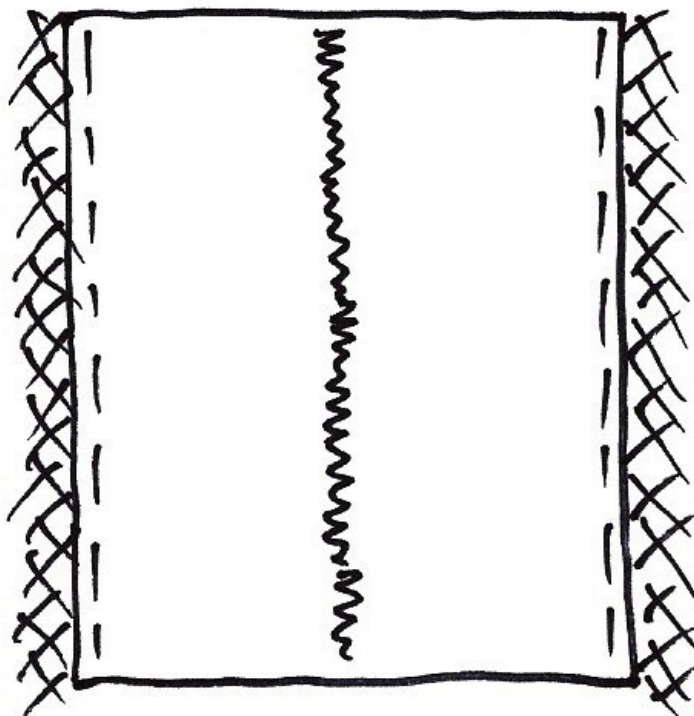
$$\begin{aligned}\frac{\lambda_y w l_1^2}{24} + \frac{(\lambda - \lambda_y) w l_1^2}{8} &= m \\ \lambda &= \frac{8}{w l_1^2} \left( m + \frac{\lambda_y w l_1^2}{12} \right) \\ &= \frac{8}{10 \times 8^2} \left( 40 + \frac{0.9375 \times 10 \times 8^2}{12} \right) \\ &= 1.125\end{aligned}$$

This is the collapse load factor,  $\lambda_c$ .

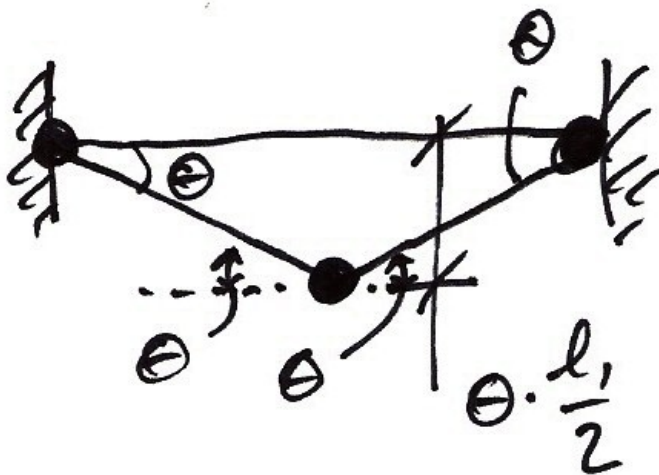
### Plastic Analysis

This time are to go straight to the ultimate load and find the collapse load factor,  $\lambda_c$ . This will give us the same answer as elastic-plastic analysis but it is easier and more direct as the elastic solution is not required.

We need to consider conditions *during* collapse by introducing enough plastic hinges to make the structure a mechanism:



-----  
hogging  
yield line  
~~~~~  
sagging  
yield line





In the diagram we introduce the an amount of rotation,  $\theta$  at the left hand support and relate all other displacements to it. Thus we have:

$$\begin{aligned}\text{External Work} &= \text{Internal Work} \\ (\lambda_c w l_1 l_2)_{\text{Load}} \left( \frac{1}{2} \frac{\theta l_1}{2} \right)_{\text{Ave Disp}} &= (m' l_2 \times \theta)_{\text{LHS}} + (m l_2 \times 2\theta)_{\text{Mid Span}} + (m' l_2 \times \theta)_{\text{RHS}} \\ (\lambda_c \times 10 \times 8 \times 10) \left( \frac{\theta \times 8}{4} \right) &= (50 \times 10\theta) + (40 \times 10 \times 2\theta) + (50 \times 10\theta) \\ 1600 \lambda_c \theta &= 1800\theta \\ \lambda_c &= 1.125\end{aligned}$$

In this development note:

- The external work done is the total load on the slab times the average displacement it moves through;
- Internal work is the moment capacity of the plastic hinge times the rotation it moves through;
- Note that, from the diagram, the rotation of the mid-span hinge is  $2\theta$ ;
- Also note that we work out the total work done on the whole slab and not just per metre: we need to do this for irregular shaped slabs.

## **2.3 Yield Line Basis**

### **Procedure**

Yield line analysis proceeds as:

1. Assume a collapse mechanism by choosing a pattern of yield lines;
2. Calculate the load factor corresponding to that yield line pattern;
3. Repeat for a range of yield line patterns;
4. Actual failure occurs at the lowest collapse load factor.

Note that it requires the slab to have the moment-rotation relationship previously given: it must be ductile.

### **Conventions**

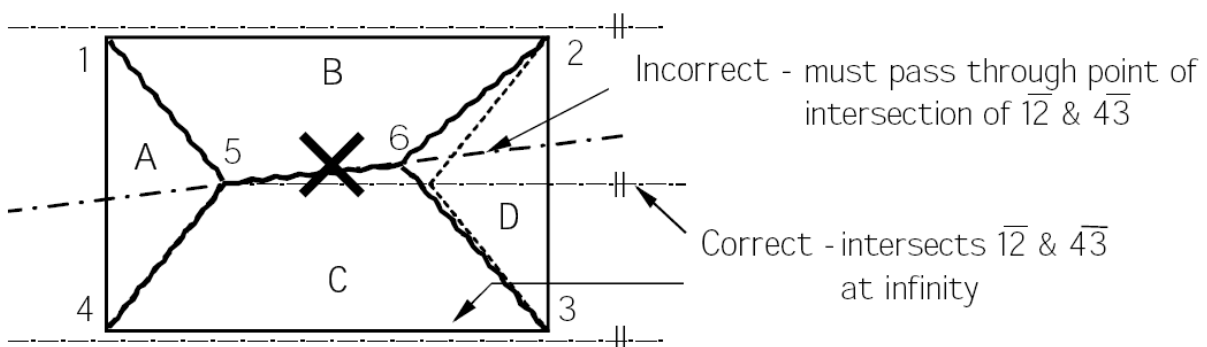
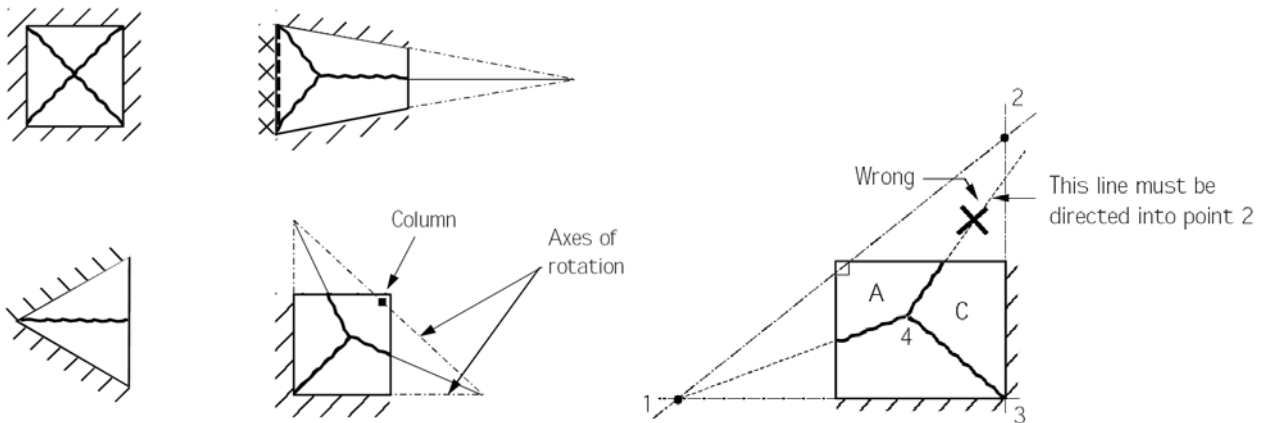
- Sagging yield lines are solid wavy lines;
- Hogging yield lines are dashed straight lines;
- A fixed support is indicated by double hatching;
- A simple support is given by single line hatching;
- A free edge is given by a straight line.

**Rules for Yield Lines:**

There are rules to be observed when postulating a yield line pattern:

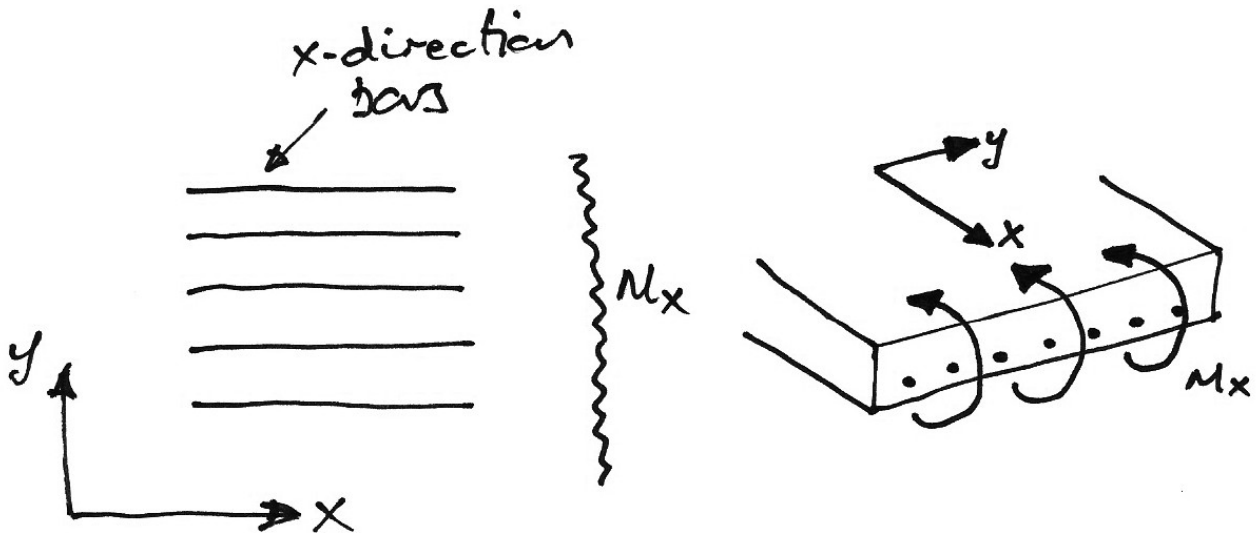
1. Yield lines divide the slab into rigid regions which remain plane through the collapse;
2. Yield lines are straight;
3. Axes of rotation generally lie along lines of support and pass over any columns;
4. Yield lines between adjacent rigid regions must pass through the point of intersection of the axes of rotation of those regions;
5. Yield lines must end at a slab boundary;
6. Continuous supports repel and a simple supports attract yield lines.

Relate the rules to the following patterns:



### Moment Axis Notation

The notation used in YLT can at first be confusing (but does make sense!):

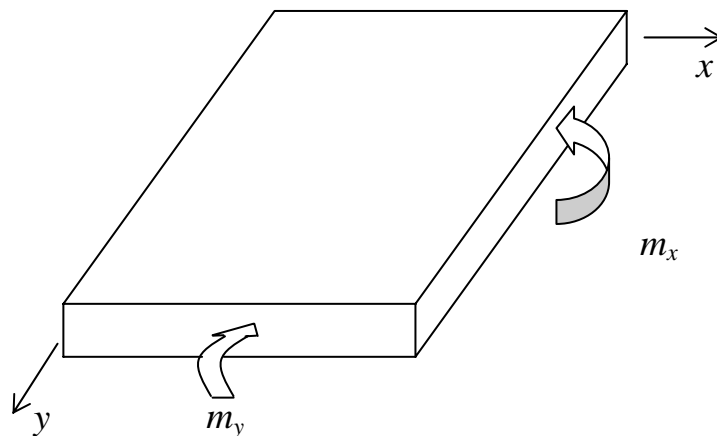


So from the diagram, reinforcement in the  $x$ -direction provides a moment capacity of  $m_x$  parallel to the  $y$ -axis. Thus any yield line parallel to the  $y$ -axis will be opposed by  $x$ -direction reinforcement and hence the  $m_x$  capacity of the slab.

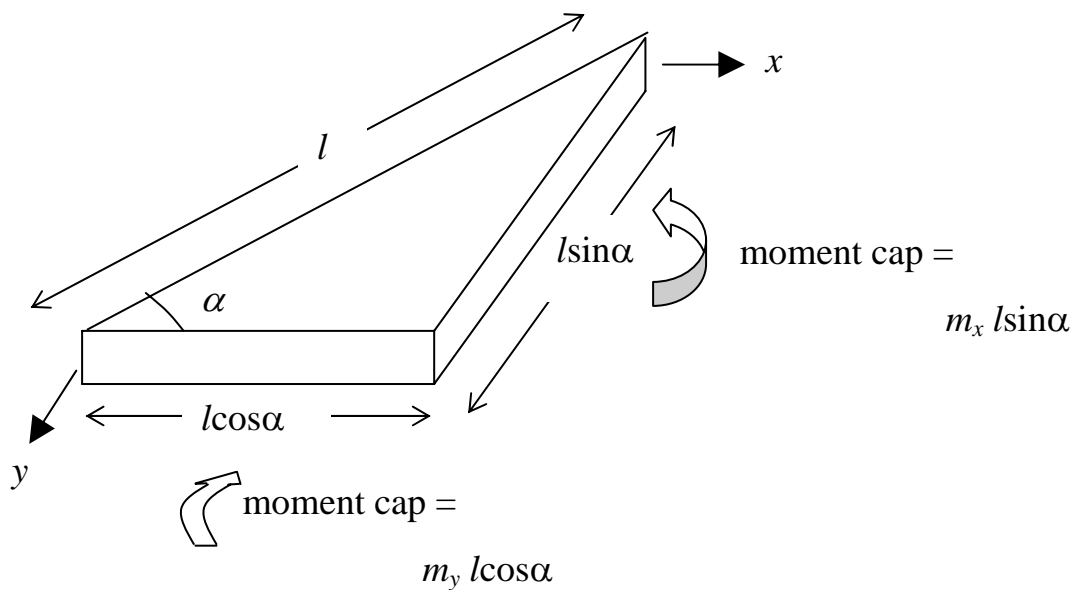
The same goes for yield lines in the  $x$ -direction opposed by the  $m_y$  capacity of the slab.

### Moment Capacity at an Arbitrary Angle

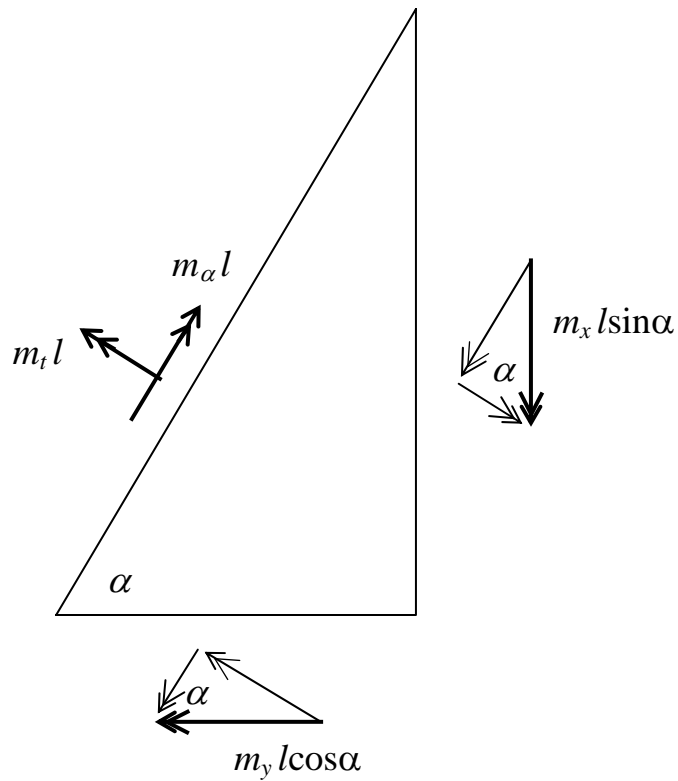
We examine the case of an orthotropic slab (different moment capacities in different orthogonal directions). This is usual for reinforced concrete slabs. Isotropic slabs are a particular case in which  $m_x = m_y$ . Thus, we will have moment capacity per metre,  $m_x$  on the  $x$ -face and moment capacity per metre,  $m_y$  on the  $y$ -face:



The moment capacity in any other direction,  $\alpha$ , is got by looking at the following:



And in plan it looks like:



Resolving vectors:

$$m_\alpha l = m_x l \sin^2 \alpha + m_y l \cos^2 \alpha$$

Thus:

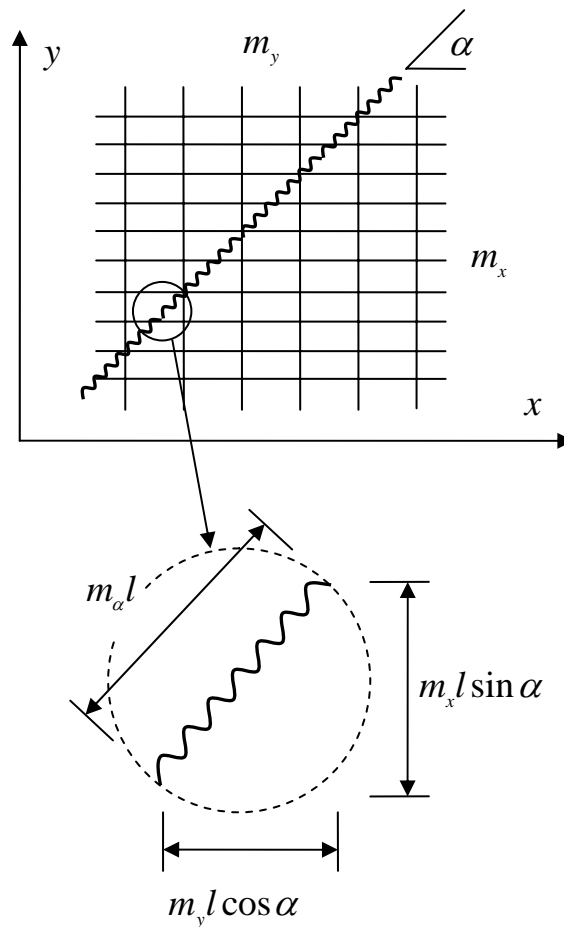
$$m_\alpha = m_x \sin^2 \alpha + m_y \cos^2 \alpha$$

### Johansen's Stepped Yield Line Criterion

This is the basic assumption of yield line theory. It assumes that during the rotation the bars remain in their original direction and do not 'kink'. It is also valid for slabs that are not of reinforced concrete.

Johansen's Stepped yield line criterion says that the yield line is actually made up of many small steps in orthogonal directions, as shown. This, in turn, enables us to take the *projections of an inclined yield line upon the axes*, instead of determining the capacity at an angle. Hence we can use:

$$m_{\alpha} l \theta = m_x l \theta_y + m_y l \theta_x$$



### Internal Work Done

For an orthotropic slab, there are two ways of calculating internal work done:

1. Take a section perpendicular to the yield line and find its rotation. Then let internal work done be the moment capacity at that angle times the length of the yield line and the rotation.
2. Use Johansen's approach in which the projections of the yield lines onto the axes of rotations is used.

The latter method is generally easier to apply.

### External Work Done

Consider a general region with loading and deformation as shown. The work done by the elemental load is force by distance:

$$\begin{aligned} W_E &= (w \cdot dA) \delta \\ &= (w \cdot dA) r \theta \end{aligned}$$

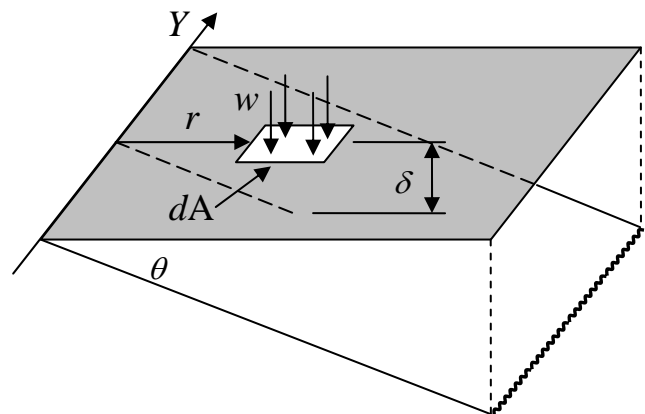
The total work done is:

$$\begin{aligned} W_E &= \int (w \cdot dA) r \theta \\ &= w \theta \int r \cdot dA \end{aligned}$$

But  $\int r \cdot dA$  is the first moment of area, hence the total work done is just:

$$W_E = w \times \text{Area} \times \delta_{\text{centroid}}$$

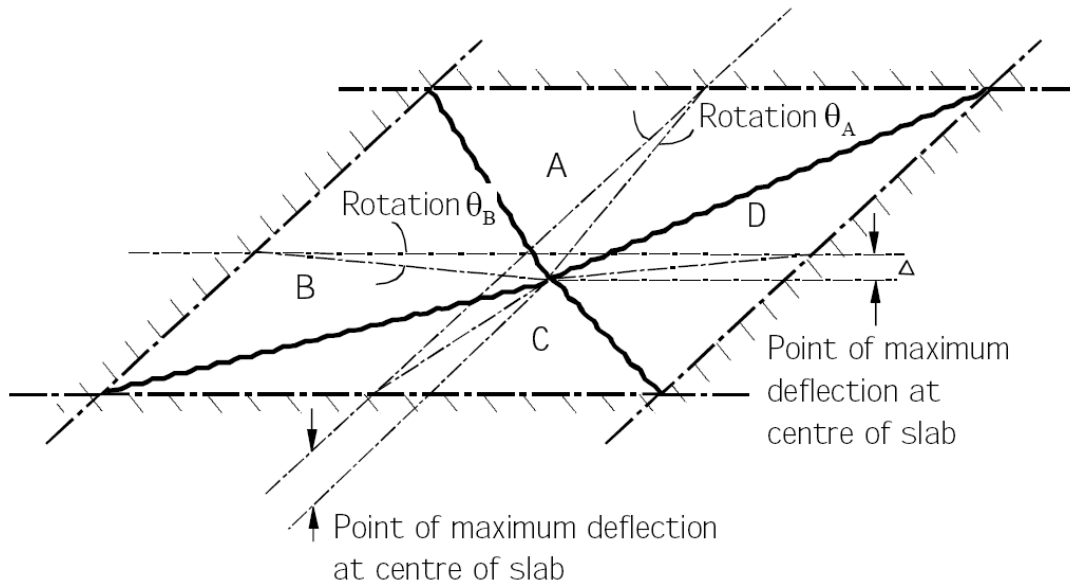
For point loads, it is just the load by deflection under it.





### Setting up the problem

The rotation of a rigid panel is linked to the deflection of the applied load:



Therefore, we assume a value of deflection, say  $\delta$  and relate all rotations to it.

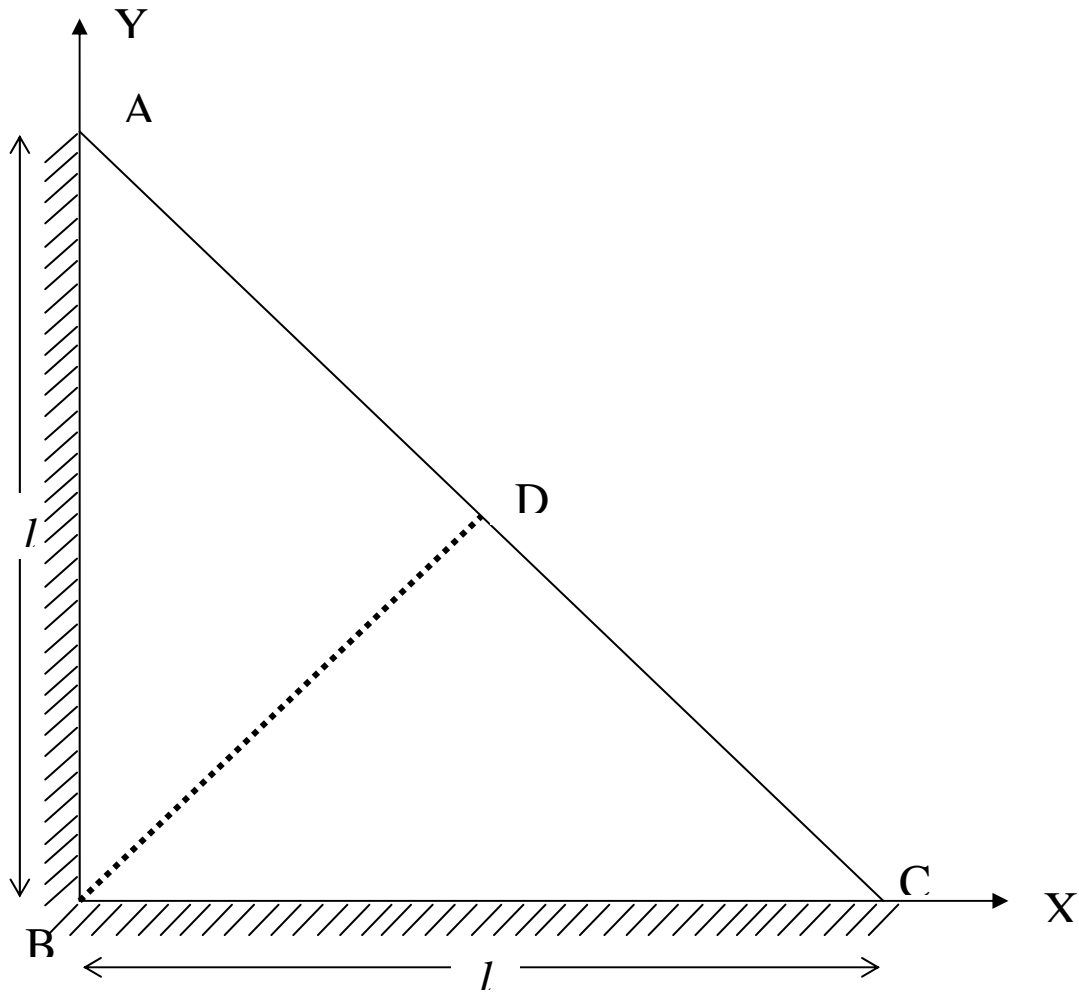
Hence, all loads, moments and deformations in the problem are set up.

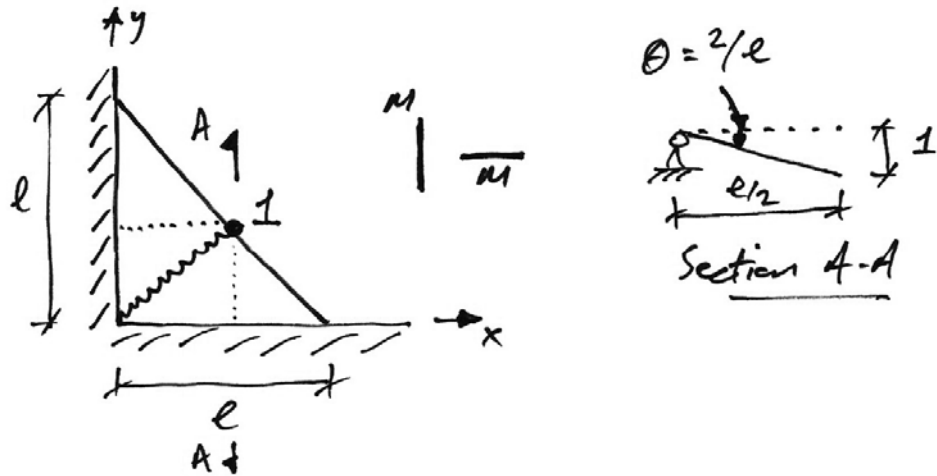
To solve, we equate the work done by the external forces in deflecting, to the work done by the yield lines in rotating:

$$\sum W \cdot \delta = \sum m \cdot \theta$$

### 2.4 Example 2 – Yield Lines not Parallel to Axes

Find required moment capacity to resist uniform (ULS) loading,  $w$ . Assume isotropic, i.e., moment capacity independent of direction.





Ext. Work = Int. Work

Rigid regions

$$2 \left[ \underbrace{\left( \frac{1}{2} \cdot l \cdot \frac{l}{2} \right)}_{\text{Area}} \underbrace{\left( w \left( \frac{1}{3} \right) \right)}_{\text{Load}} \right] = \underbrace{m \left( \frac{l}{2} \right) \left( \frac{2}{l} \right)}_{x\text{-direction}} + \underbrace{m \left( \frac{l}{2} \right) \left( \frac{2}{l} \right)}_{y\text{-direction}}$$

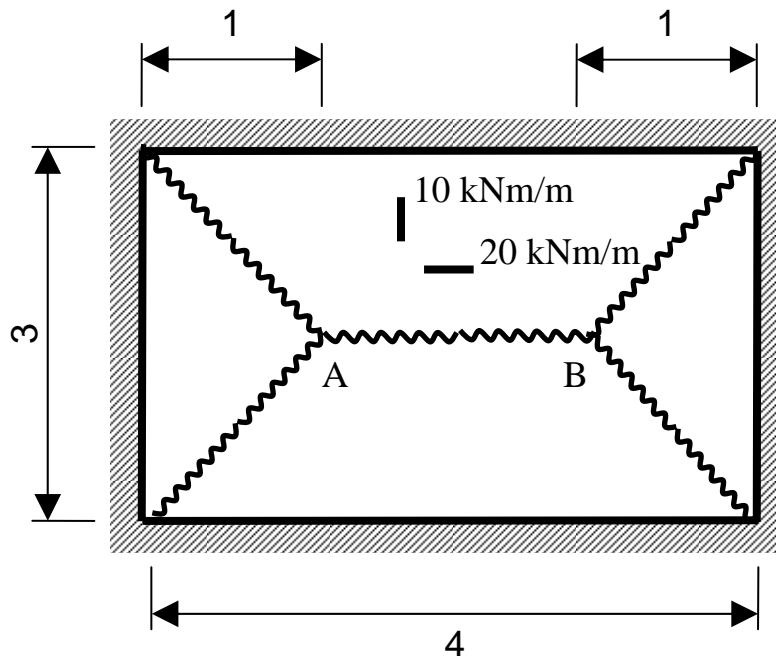
Area    Load    Area    Disp.

$$\frac{wl^2}{3} = 2m$$

$$\therefore m = \frac{wl^2}{6}$$

### 2.5 Example – Simply-supported Rectangular 2-way Slab

This slab is subject to a load of  $10 \text{ kN/m}^2$ . Find the collapse load factor.



#### External Work Done:

Impose a deflection of  $\delta$  along the line AB. For each panel calculate the (load  $\times$  load factor)  $\times$  area  $\times$  deflection at centroid. Add up for each panel. The result will have a number  $\times \delta \times \lambda$  as the load factor and deflection are not known.

#### Internal Work Done:

For each panel, project the yield line onto the axis of rotation and calculate this length. Multiply this by the capacity in that direction. Determine the rotation in terms of  $\delta$  by using the well-known  $S = R\theta$ . Add up for each panel giving a number  $\times \delta$ .

#### Equate:

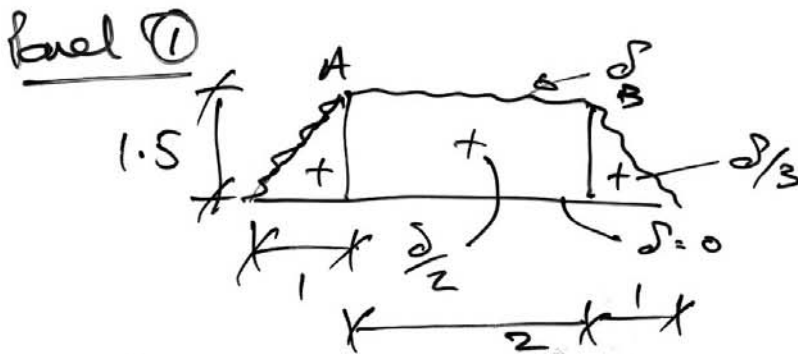
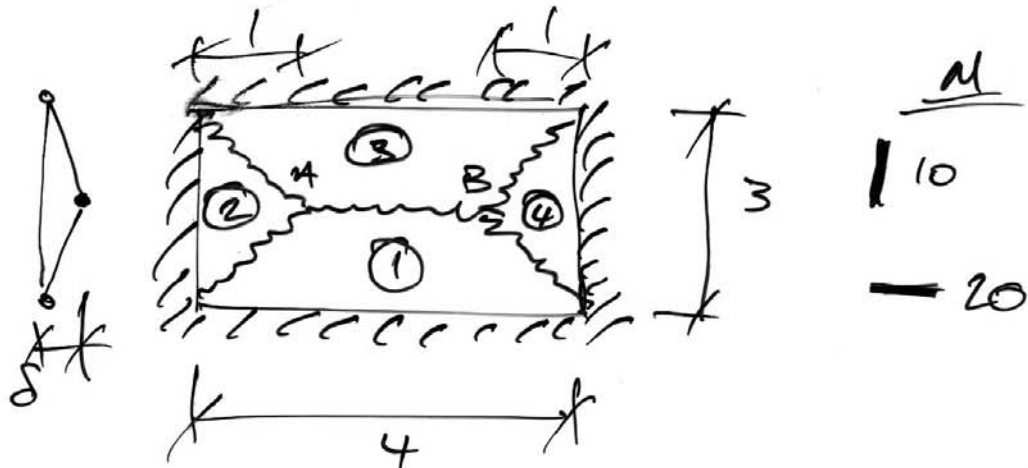
Equate and rearrange for the load factor  $\lambda$ . Note that the deflection term vanishes.

Ans. 1.93

Repeat this example by exchanging the capacities, does  $\lambda$  change?

Repeat this example by changing the 1 m dimension to 1.5 m – does  $\lambda$  change?

Some of the workings:



$$W_E^{①} = 10 \lambda \left[ 2 \times \left( \frac{1}{2} \times 1 \times 1.5 \right) \frac{\delta}{3} + (2 \times 1.5) \frac{\delta}{2} \right]$$

$$= 45 \lambda \delta = W_E^{③}$$

Panel ②

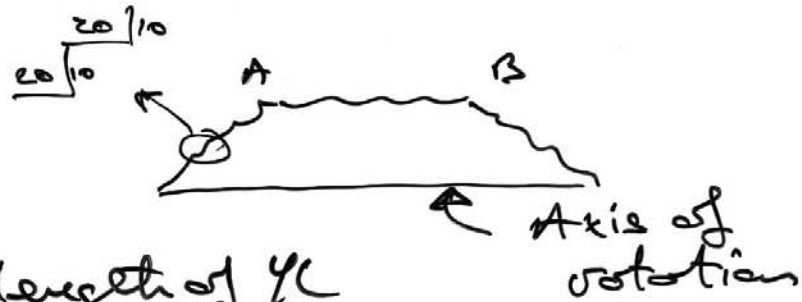
$$W_E^{②} = 10 \lambda \left[ \left( \frac{1}{2} \times 3 \times 1 \right) \left( \frac{\delta}{3} \right) \right]$$

$$= 5 \lambda \delta$$

$$= W_E^{④}$$

$$\therefore W_E = 2 \times 45 \lambda \delta + 2 \times 5 \lambda \delta = \boxed{100 \lambda \delta}$$

t. Work  
 Panel ①:



Projected length of  $YC$   
 onto axis of rotation = 4

$$W_I^{(1)} = \mu L \theta = 20 \times 4 \times \frac{\theta}{1.5}$$

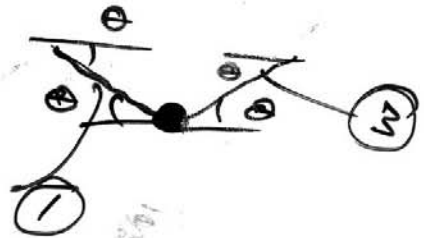
$$= 66.6\theta$$

$$= W_I^{(3)}$$

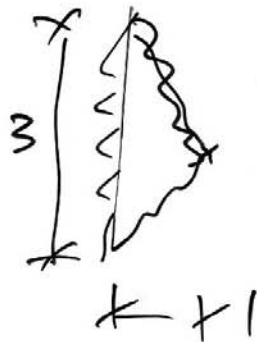
$$S = 20 \theta$$

$$\delta = 1.5 \theta$$

$$\therefore \theta = \frac{S}{1.5}$$



Panel ②:



$$W_I^{(2)} = 10 \times 3 \times \theta$$

$$\begin{cases} S = 20 \theta \\ \delta = 1 \theta \\ \therefore \theta = \delta \end{cases}$$

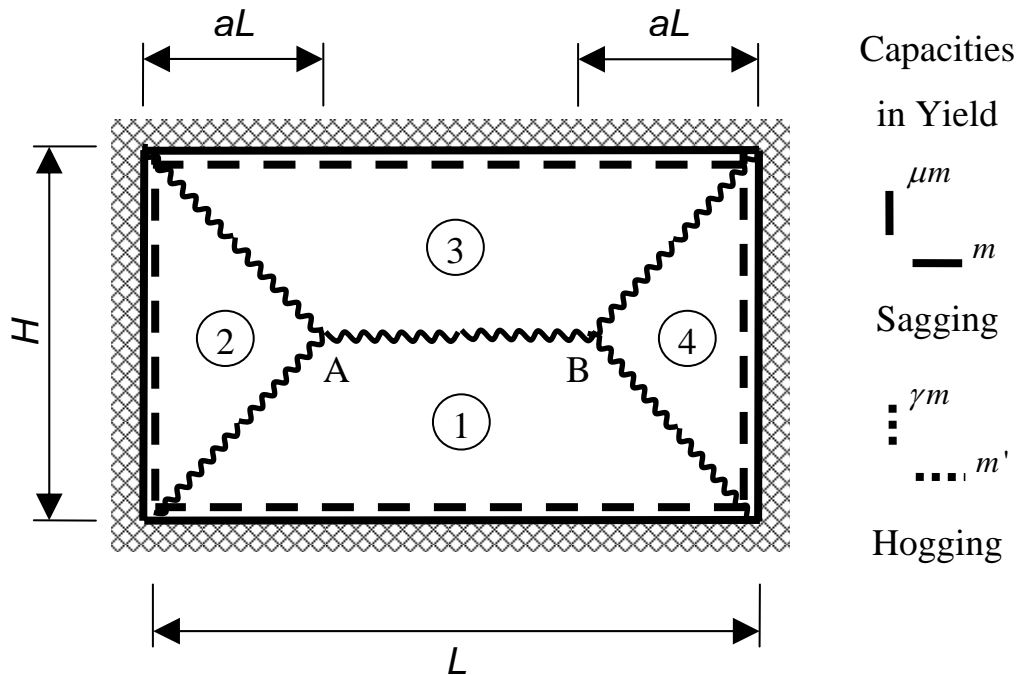
$$= 30\theta$$

$$= W_I^{(4)}$$

$$\therefore W_I = 2 \times 66.6\theta + 2 \times 30\theta$$

$$\boxed{= 193.33\theta}$$

**2.6 Example – General Solution of Continuous Rectangular 2-way Slab**



Note that we have introduced factors to relate the orthogonal strengths to each other.

External Work Done:

Impose a deflection of  $\delta$  along the line AB.

②  $W_E^{(2)} = w \left( \frac{1}{2} \cdot aL \cdot H \right) \cdot \frac{\delta}{3} = wLH\delta \cdot \frac{a}{6}$  and  $W_E^{(4)} = W_E^{(2)}$

① Here we split into two triangles and a rectangle:

$$W_E^{(1)} = 2 \left[ w \left( \frac{1}{2} \cdot aL \cdot \frac{H}{2} \right) \cdot \frac{\delta}{3} \right] + w \left( (L - 2aL) \cdot \frac{H}{2} \right) \cdot \frac{\delta}{2}$$

$$= wLH\delta \left( \frac{3 - 4a}{12} \right)$$

and  $W_E^{(3)} = W_E^{(1)}$ . Hence, in total, we have:

$$\begin{aligned}
 W_E &= W_E^{(1)} + W_E^{(2)} + W_E^{(3)} + W_E^{(4)} \\
 &= wLH\delta \left[ 2 \cdot \frac{a}{6} + 2 \cdot \left( \frac{3-4a}{12} \right) \right] \\
 &= wLH\delta \left( \frac{3-2a}{6} \right)
 \end{aligned}$$

Internal Work Done:

*Sagging Yield Lines:*

- ① The projection of the YL onto the axis of rotation gives us a length of  $L$ . Hence the work done is:

$$W_I^{(1)} = m \cdot L \cdot \frac{\delta}{0.5H} \text{ and } W_I^{(3)} = W_I^{(1)}.$$

- ② This time we have a length of  $H$ . Hence the work done is:

$$W_I^{(2)} = \mu m \cdot H \cdot \frac{\delta}{aL} \text{ and } W_I^{(4)} = W_I^{(2)}.$$

Thus the total sagging work done is:

$$\begin{aligned}
 W_I &= 2 \left[ m \cdot L \cdot \frac{\delta}{0.5H} \right] + 2 \left[ \mu m \cdot H \cdot \frac{\delta}{aL} \right] \\
 &= 2m \left[ 2 \frac{L}{H} + \frac{\mu H}{a L} \right] \delta
 \end{aligned}$$

*Hogging Yield Lines:*

There is no projection needed in these cases and so the problem is simpler:



$$\begin{aligned}
 W_I &= W_I^{(1)} + W_I^{(2)} + W_I^{(3)} + W_I^{(4)} \\
 &= 2 \left[ m' L \frac{\delta}{0.5H} + \gamma m' H \frac{\delta}{aL} \right] \\
 &= 2m' \left[ 2 \frac{L}{H} + \frac{\gamma H}{a L} \right] \delta
 \end{aligned}$$

Total internal work done is thus:

$$W_I = 2m \left[ 2 \frac{L}{H} + \frac{\mu H}{a L} \right] \delta + 2m' \left[ 2 \frac{L}{H} + \frac{\gamma H}{a L} \right] \delta$$

Equate:

$$wLH\delta \left( \frac{3-2a}{6} \right) = 2m \left( 2 \frac{L}{H} + \frac{\mu H}{a L} \right) \delta + 2m' \left( 2 \frac{L}{H} + \frac{\gamma H}{a L} \right) \delta$$

Divide both sides by  $2\delta$ , and multiply by  $\frac{H}{L}$ , and extract  $m$  to get:

$$\frac{wH^2}{12} (3-2a) = m \left[ \left( 2 + \frac{\mu H^2}{a L^2} \right) + \frac{m'}{m} \left( 2 + \frac{\gamma H^2}{a L^2} \right) \right]$$

Introduce  $\bar{m} = \frac{m'}{m}$ ,  $n = \frac{H^2}{L^2}$  and divide across and multiply top and bottom by  $a$ :

$$m = \frac{wH^2}{12} \cdot \frac{(3a - 2a^2)}{(2a + \mu n) + \bar{m}(2a + \gamma n)}$$

The maximum value of  $m$  with respect to a changing  $a$  occurs at:

$$\frac{dm}{da} = [(\mu + \gamma\bar{m})n + 2a(1 + \bar{m})](3 - 4a) - (3a - 2a^2)(2 + 2\bar{m}) = 0$$

This is a quadratic in  $a$  – solve to get:

$$a = \frac{n}{2(1 + \bar{m})} \left[ \sqrt{(\mu + \gamma\bar{m})^2 + 3n^{-1}(\mu + \gamma\bar{m})(1 + \bar{m})} - (\mu + \gamma\bar{m}) \right]$$

It is for this value of  $a$  that the slab will fail, given the dimensional and strength data.

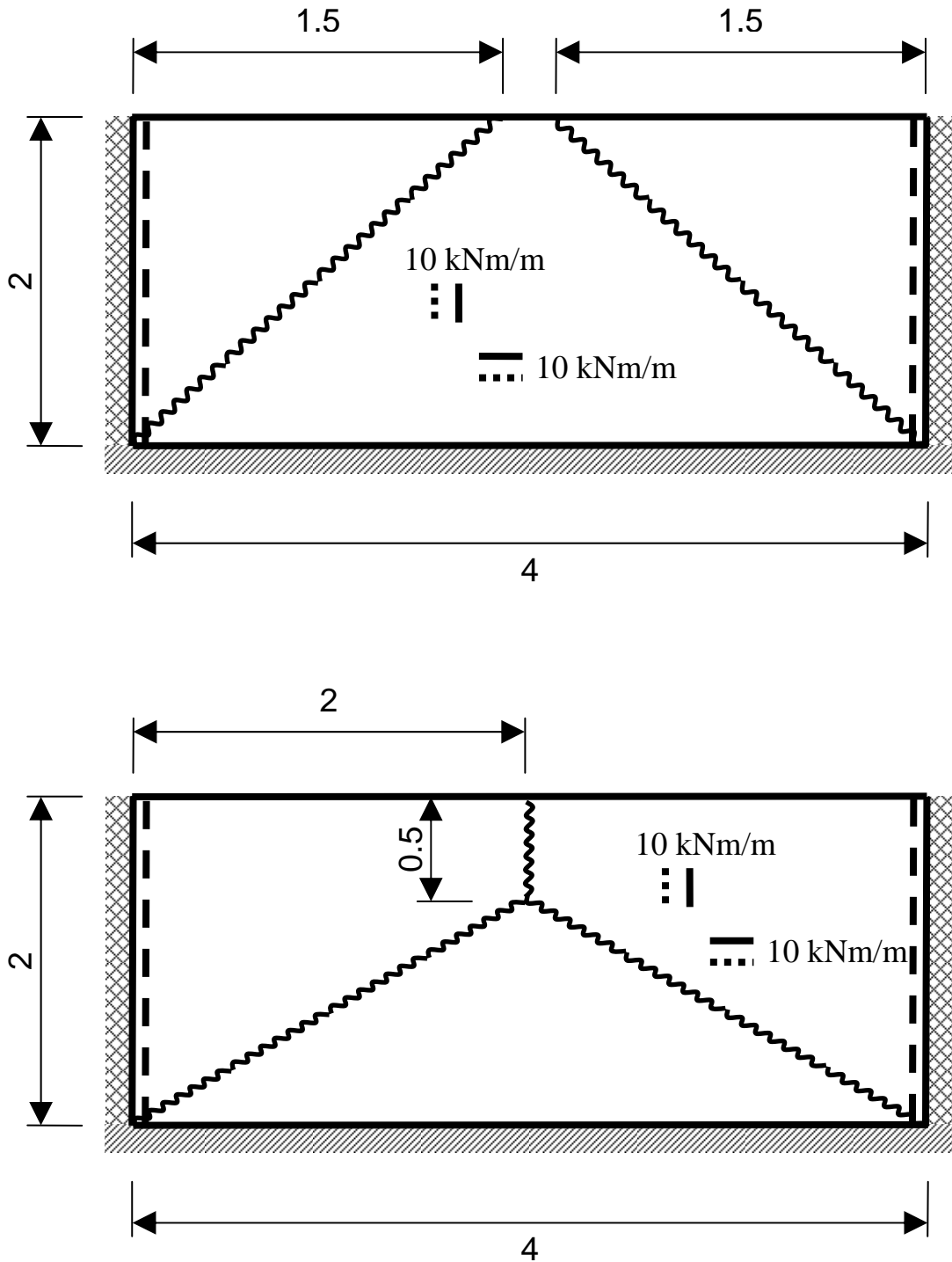
This example shows that although a multitude of yield line patterns are possible, given one particular mode, the critical dimension can be obtained.

**Exercise:**

Derive the general solution for the simply supported case. This is a subset of the calculations presented here. Use this to verify the results of the first example.

### 2.7 Example

Determine which is the critical failure mode for a load  $w$  and capacities as shown.



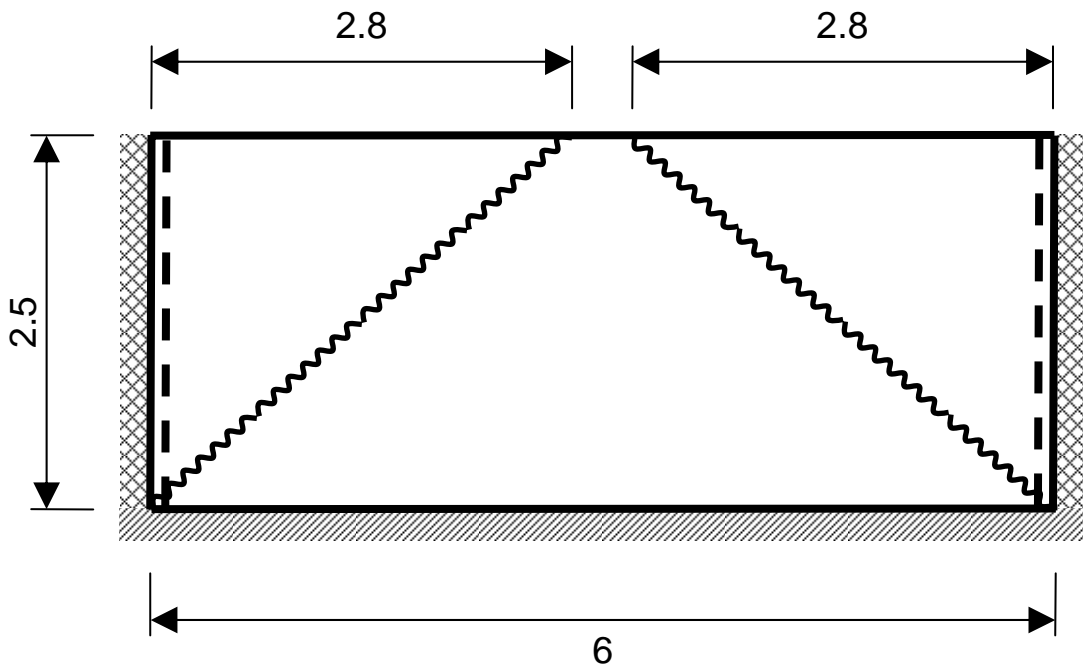
**Long Exercise (on the basis of Tb. 9 of IS 325):**

Using the calculations for this exercise as a basis, derive the general solutions for both failure modes. Reduce to the case when  $m' = m$  and  $\gamma = \mu$ . Derive a table for each mode for various values of  $\mu$  and  $\frac{H}{L}$ . Combine the tables showing the critical mode.

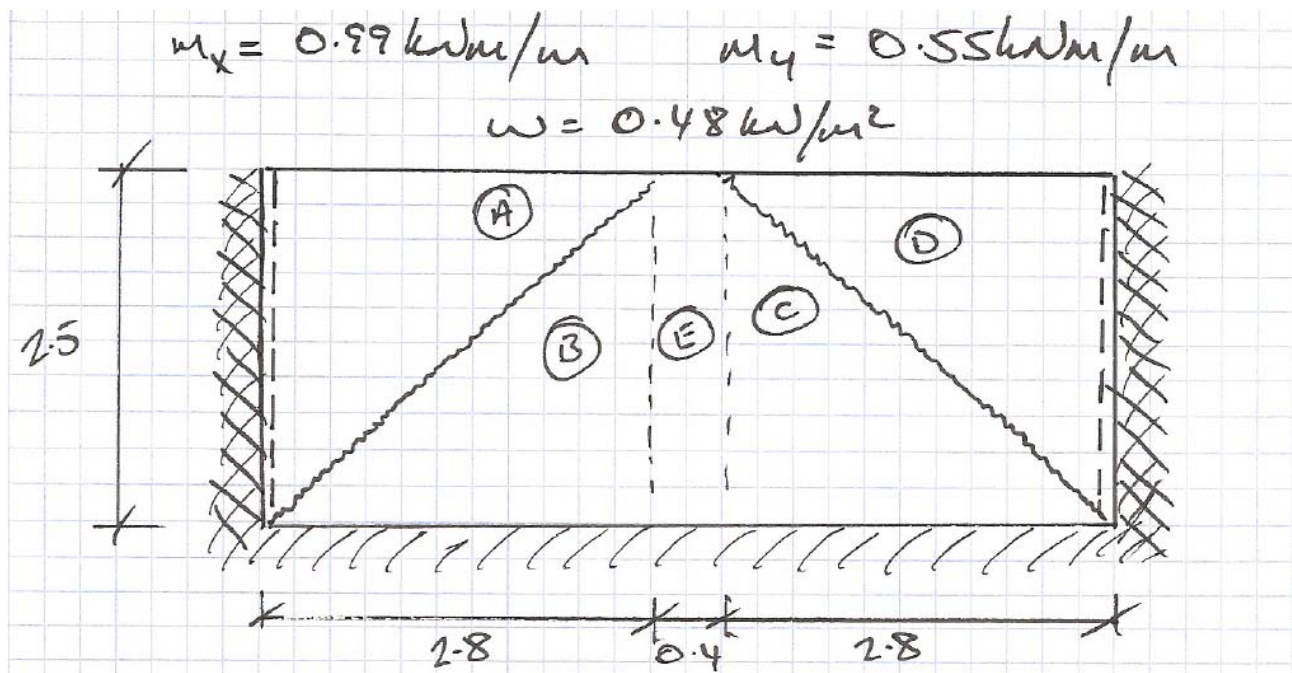
**2.8 Exam Question 2006 – Q6(b)**

Using the capacities of the masonry in the orthogonal directions found from Part (a), determine the collapse load factor for the yield line pattern of Figure Q.6(b)

(60%)



**FIGURE Q.6(b)**



Internal V.W.

$$2 (w_x \times 2.5 \times \frac{1}{2} \times 2.8)^{\text{sag-x}} + 2 (w_x \times 2.5 \times \frac{1}{2} \times 2.8)^{\text{sag-x}} = \frac{10}{2.8} w_x$$

$$+ 2 (w_y \times 2.8 \times \frac{1}{2} \times 2.5)^{\text{sag-y}} \longrightarrow = \frac{5.6}{2.5} w_y$$

$$W_i = 3.57 w_x + 2.33 w_y$$

$$= 4.819$$

$$\Sigma = W_i$$

External V.W.

$$(\frac{1}{2} \times 2.5 \times 2.8) \times \frac{2}{9} \cdot w_d \times 4 \text{ triangles (A), (B), (C), (D)} \longrightarrow \frac{28}{9} w_d$$

$$+ 2.5 \times 0.4 \times \frac{1}{2} \times w_d \longrightarrow \frac{1}{2} w_d$$

$$\therefore W_e = 3.611 w_d$$

$$\Sigma = W_e$$

Equate:  $3.611 w_d = 4.819$

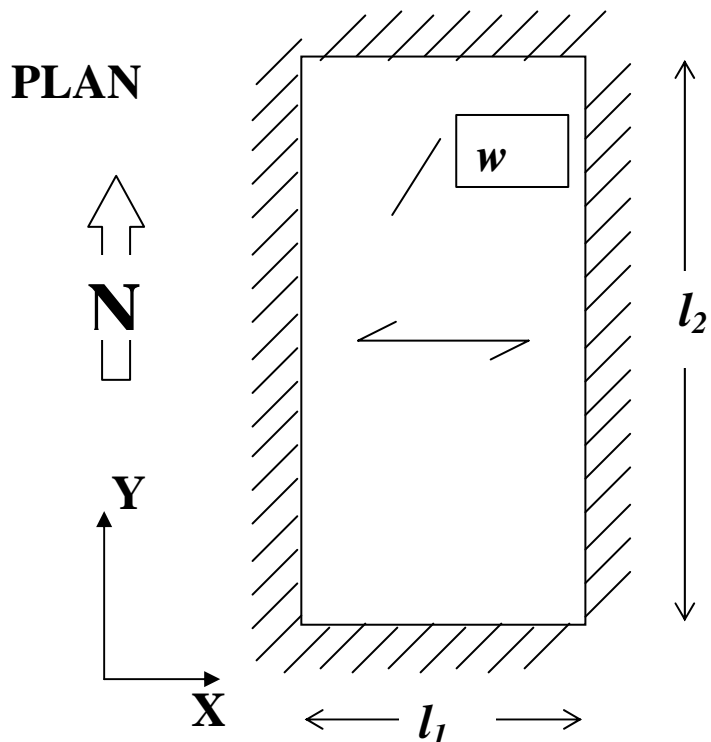
$$\therefore d = \frac{4.819}{3.611 w} = 2.78$$

### 3. Lower Bound Methods

Suppose we assumed that all load on this slab spanned East-West, i.e., across the shorter span,  $l_1$  (as load tends to).

And suppose we provided E-W reinforcement to resist the resulting moment. And provided no reinforcement in N-S direction.

Q: Would this be safe?



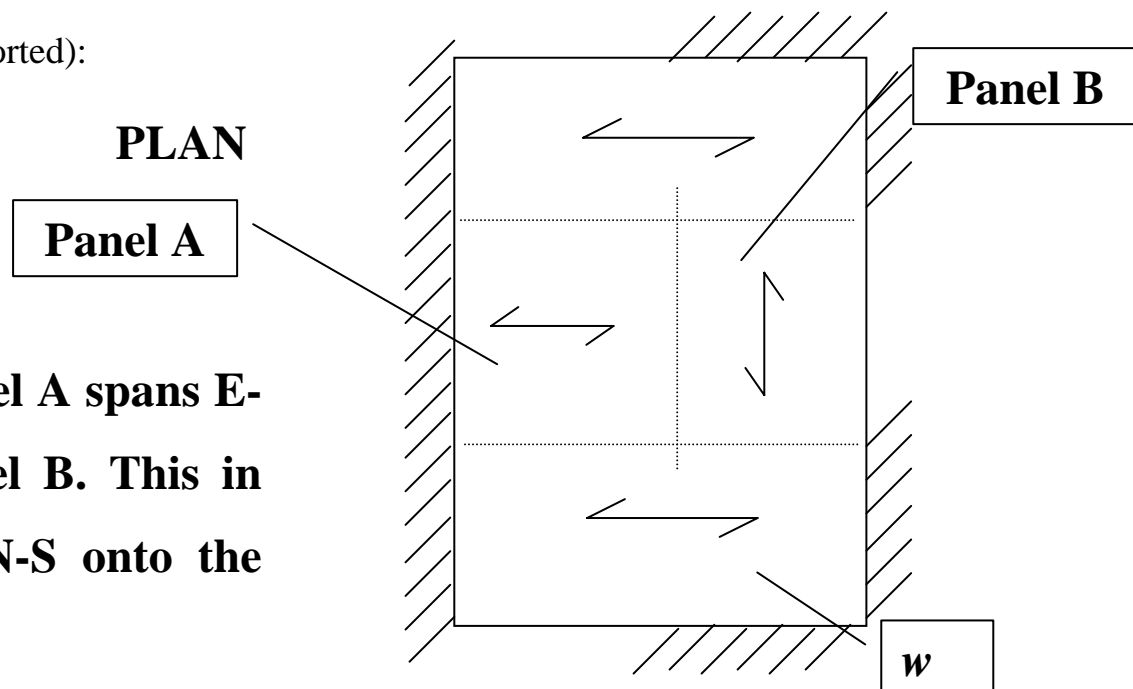
The slab will actually deform in a dish shape and will generate moment on both the X- and Y-faces (suggesting reinforcement parallel to each axis). Therefore, if we have no N-S reinforcement, there will be cracks parallel to the X-axis. However, after this cracking has occurred, the load will find the reinforcement in the E-W direction and will be supported by it - provided no concrete crushing happens first.

A: Yes, provided the slab has sufficient ductility

This kind of approach is known as a 'lower bound' approach - providing enough reinforcement to carry a load by an *assumed load path* (in this case, E-W).

Lower bound approaches are commonly used where there are openings ('opes') or partial support in slabs.

Eg (partially supported):



**Load on Panel A spans E-W onto Panel B. This in turn spans N-S onto the supports.**

**Panel A is designed as a**

Panel B is designed as a 1-way slab spanning NS to the supports. The reaction from Panel A is applied as a load to Panel B. To take this load, we need a strong band of reinforcement in Panel B.

Lower bound approaches are widely used in practice.

- They are safe at ULS if the slab is sufficiently ductile
- They do not guarantee a satisfactory design at SLS - cracking may occur (as in the first eg) if the load path is not reasonable.
- A special case of the Lower Bound approach is the well known 'Hillerborg Strip' method - more of this in the optional Structural Design course.