

# DESIGN AND ANALYSIS OF SUSPENSION SYSTEM FOR AN ALL TERRAIN VEHICLE

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**Abstract**—In this paper our work was to study the static and dynamic parameter of the suspension system of an ATV by determining and analyzing the dynamics of the vehicle when driving on an off road racetrack. Though, there are many parameters which affect the performance of the ATV, the scope of this paper work is limited to optimization, determination, design and analysis of suspension systems and to integrate them into whole vehicle systems for best results.

The goals were to identify and optimize the parameters affecting the dynamic performance suspension systems within limitations of time, equipment and data from manufacturer.

In this paper we will also come across the following aspects

- a. Study the static and dynamic parameters of the chassis.
- b. Workout the parameters by analysis, design, and optimization of suspension system.
- c. Study of existing suspension systems and parameters affecting its performance.
- d. Determination of design parameters for suspension system.

**Index terms**—All terrain vehicle, suspension, caster angle, camber angle, toe angle, roll centre

## 1.INTRODUCTION

An All-Terrain Vehicle (ATV) is defined by the American National Standards Institute (ANSI) as a vehicle that travels on low pressure tires, with a seat that is straddled by the operator, along with handlebars for steering control. In some vehicles steering wheel similar to passenger cars is also used. As the name suggests, it is designed to

negotiate a wider variety of terrain than most other vehicles. Although it is a street-legal vehicle in some countries, it is not legal within most states and provinces of Australia, the United States and Canada and definitely not in India. By the current ANSI definition, it is intended for use by a single operator, although a change to include 2-seaters is under consideration.

The All Terrain Vehicle (ATV) was initially developed in the 1960's as a farmtown vehicle in isolated, mountainous areas. During spring thaws and rainy seasons, steep

mountainous roads were often impassable with conventional vehicles. It soon became a recreational vehicle however, providing transportation to areas inaccessible by other motorized transport. Royal Enfield CO built and put on sale a powered Quadra cycle in 1893 that worked in the same way as, and resembles, a modern quad-bike. ATVs were made in the United States a decade before 3- and 4-wheeled vehicles were introduced by Honda and other Japanese companies. During the 1960s, numerous manufacturers offered similar small off-road vehicles that were designed to float and were capable of traversing swamps, ponds and streams, as well as dry land.

The early ATV's were mainly used for agricultural purpose only. But now the definition of ATV is changing. Many countries are allowing ATVs as commercial vehicle, though with the regulations on its use and safety. Now days, ATVs are generally used in defense and sports application redefining the ATV. Now the ATVs are also coming with durable roll cages, added safety of seat and shoulder belts and higher ground clearance making it more rugged vehicle. The rear cargo deck is more useful for hauling camping gear, bales of hay, tools and supplies making it suitable for exploring back country, riding sand dunes, hunting, fishing and camping. ATVs Sport models are built with performance, rather than utility, in mind. To be successful at fast trail riding, an ATV must have light weight, high power, good suspension and a low center of gravity. These machines can be modified for such racing

disciplines as motocross, woods racing, desert racing, hill climbing, ice racing, speedway, tourist trophy, flat track, drag racing and others.

## 1.2. Application of ATV's

Initially the ATVs were solely used for the transportation through the inaccessible areas, but now these vehicles have found their application in different areas as mentioned below:

- a. In Defense Services like army and air force etc to carry and transport guns, ammunition and other supplies to remote areas of rough and varied terrain.
- b. By railways during construction of railway tracks on mountain or on other rough terrain.
- c. By police force.
- d. In sport also like golf for traveling one place to other place.
- e. In Antarctic bases for research things where use of conventional vehicle is impossible.
- f. Now a days ATVs are also used in adventuring like mountaineering, in dirt and in snow.

## 1.3. Objective

The objective of our paper work was to study the static and dynamic parameter of the suspension system of an ATV by determining and analyzing the dynamics of the vehicle when driving on an off road racetrack. Though, there are many parameters which affect the performance of the ATV, the scope of this paper work is limited to optimization, determination, design and analysis

of suspension systems and to integrate them into whole vehicle systems for best results.

The goals were to identify and optimize the parameters affecting the dynamic performance suspension systems within limitations of time, equipment and data from manufacturer.

The objective of the paper includes:

- e. Study the static and dynamic parameters of the chassis.
- f. Workout the parameters by analysis, design, and optimization of suspension system.
- g. Study of existing suspension systems and parameters affecting its performance.
- h. Determination of design parameters for suspension system.

## 2. SUSPENSION SYSTEM

The suspension of vehicles needs to satisfy a number of requirements which depend on different operating conditions of the vehicle (loaded/unloaded, acceleration/braking, level/uneven road, straight running/ cornering). Suspension systems serve a dual purpose contributing to the vehicle's handling and braking for good active safety and driving pleasure, and keeping vehicle occupants comfortable and reasonably well isolated from road noise, bumps, and vibrations. The suspension also protects the vehicle itself and mounted systems from damage and wear.

Suspension is the term given to the system comprise of springs, shock absorbers and linkages that connects a vehicle to its wheels. The design of

front and rear suspension of a vehicle may be different.

### 2.1. Basic Consideration for Suspension System

#### 2.1.1. Vertical loading

When the road wheel comes across the bump or a pit on the road it is subjected to vertical forces (tensile or compressive) depending on the load irregularity which are absorbed by the elastic compression, shear, bending, twisting properties of spring. To reduce the pitching tendency of the vehicle, the front system should be less springing than the rear suspension system.

#### 2.1.2. Rolling

The center of gravity (C.G.) of the vehicle is considerably above the ground. As a result while taking turns the centrifugal force acts outwards on the C.G. of vehicle, while the load resistance acts inwards at the wheels. This give rise to a couple turning the vehicle about the longitudinal axis called rolling.

#### 2.1.3. Brake dip and squat

On applying brakes the nose of the vehicle dips which depends on the position of C.G. relative to the ground, wheel base and other suspension characteristics. This phenomenon is called as dip. In the same way the torque loads

during acceleration tend to lift the front of vehicle. This effect is called as squat.

#### 2.1.4. Side thrust

Centrifugal force during cornering, crosswinds, cambering of the road causes side thrust.

#### 2.1.5. Road holding

The degree to which vehicle maintains the contact with the road surface in various types of directional changes as well as in straight line motion is called as road holding.

#### 2.1.6. Unsprung weight

Unsprung weight is the weight of the vehicle components between suspension and road surface (Rear axle assembly, steering knuckle, front axle, wheels).

### 2.2. Types of Suspension System Used in Automobiles

Suspension systems can be broadly classified into two subgroups – Dependent and Independent.

#### 2.2.1. Dependent suspension system

A dependent suspension normally has a beam or live axle that holds wheels parallel to each other and perpendicular to the axle with the help

of leaf springs to it. In dependent suspension system when the camber of one wheel changes, the camber of the opposite wheel changes in the same way (by convention, on one side this is a positive change in camber and on the other side this a negative change). Depending on the location of system of linkages, the dependent suspension systems have various configurations as:

- a. Satchell link
- b. Panhard rod
- c. Watt's linkage
- d. WOBLink
- e. Mumford linkage
- f. Live axle
- g. Twist beam
- h. Beam axle

Dependent suspension system assures constant camber, it is most commonly used in vehicles that need to carry large loads.

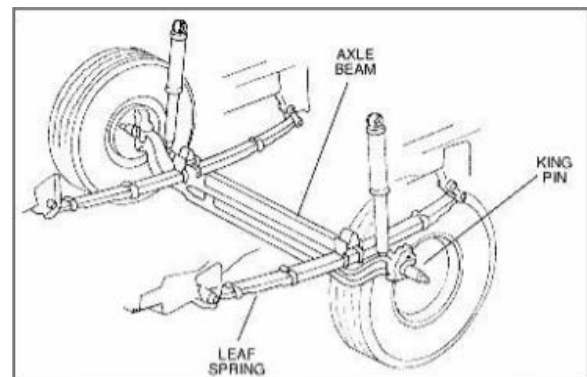


Fig 2. 1: Dependent suspension system using leaf spring

#### 2.2.2. Independent suspension system

In an independent suspension system wheels are allowed to rise and fall on their own without affecting the opposite wheel by using kinematic linkages and coil springs. Suspensions with other devices, such as anti-roll bars that link the wheels are also classified in independent suspension system. The various independent suspension systems are:

- a. Double wishbone suspensions
- b. McPherson struts and strut dampers
- c. Rear axle trailing-arm suspension
- d. Semi-trailing-arm rear axles
- e. Multi-link suspension

In this type of suspension system, the wheels are not constrained to remain perpendicular to a flat road surface in turning, braking and varying load conditions; control of the wheel camber is an important issue.

In double wishbone and multi-link system we can have more control over the geometry of system than swing axle, McPherson strut or swinging arm because of the cost and space requirements.

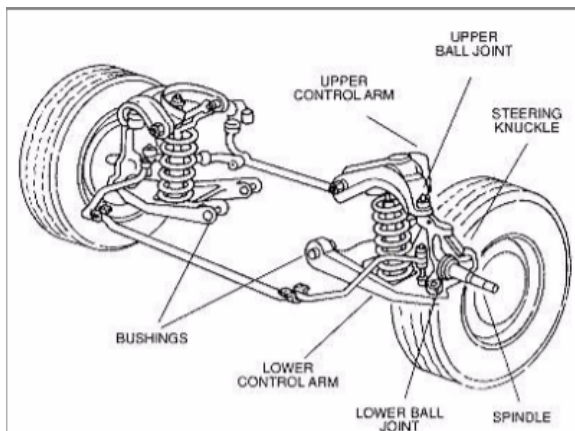


Fig 2.2: Independent suspension system using Double wishbone

### 2.3. Requirements of Suspension Systems

- a. Independent movement of each of the wheels on an axle
- b. Small, unsparing masses of the suspension in order to keep wheel load fluctuation as low as possible
- c. The introduction of wheel forces into the body in a manner favorable to the flow of forces
- d. The necessary room and expenditure for construction purposes, bearing in mind the necessary tolerances with regard to geometry and stability, ease of use
- e. Behavior with regard to the passive safety of passengers and other road users
- f. To preserve stability of the vehicle in pitching and rolling while in motion
- g. Cost

### 2.4. Spring and Dampers

Most suspensions use springs to absorb impacts and dampers (or shock absorbers) to control spring motions. Traditional springs and dampers are referred to as passive suspensions. If the suspension is externally controlled then it is a semi-active or active suspension.

Semi-active suspensions include devices such as air springs and switchable shock absorbers, various self-leveling solutions, as well as systems like Hydro pneumatic,

Hydromantic, and Hydra gas suspensions. Mitsubishi developed the world's first production semi-active electronically controlled suspension system in passenger cars; the system was first incorporated in the 1987 Gallant model.

Fully active suspension systems use electronic monitoring of vehicle conditions, coupled with the means to impact vehicle suspension and behavior in real time to directly control the motion of the car.

With the help of control system, various semi-active/active suspensions could realize an improved design compromise among different vibrations modes of the vehicle, namely bounce, roll, pitch and warp modes. However, the applications of these advanced suspensions are constrained by the cost, packaging, weight, reliability, and/or the other challenges.

Interconnected suspension, unlike semi-active/active suspensions, could easily decouple different vehicle vibration modes in a passive manner. The interconnections can be realized by various means, such as mechanical, hydraulic and pneumatic. Anti-roll bars are one of the typical examples of mechanical interconnections, while it has been stated that fluidic interconnections offer greater potential and flexibility in improving both the stiffness and damping properties.

The leading / trailing swinging arm, fore-aft linked suspension system together with inboard front brakes had a much smaller unsprung weight than existing coil spring or leaf designs. The interconnection transmitted some of the force deflecting a front wheel up over a bump, to push

the rear wheel down on the same side. When the rear wheel met that bump a moment later, it did the same in reverse, keeping the car level front to rear.

The springing balance (which expresses how well the front and rear axles are matched to one another) also needs to be taken into consideration. If a vehicle does not pitch when it goes over bumps in the ground, but instead moves up and down in parallel translation, it has a good springing balance.

#### **2.4.1. Spring rate**

The spring rate (or suspension rate) is a component in setting the vehicle's ride height or its location in the suspension stroke. Vehicles which carry heavy loads will often have heavier springs to compensate for the additional weight that would otherwise collapse a vehicle to the bottom of its travel (stroke). Heavier springs are also used in performance applications when the suspension is constantly forced to the bottom of its stroke causing a reduction in the useful amount of suspension travel which may also lead to harsh bottoming.

Springs that are too hard or too soft will both effectively cause the vehicle to have no suspension at all. Vehicles that commonly experience suspension loads heavier than normal have heavy or hard springs with a spring rate close to the upper limit for that vehicle's weight. This allows the vehicle to perform properly under a heavy load when control is limited by the inertia of

the load. Riding in an empty truck used for carrying loads can be uncomfortable for passengers because of its high spring rate relative to the weight of the vehicle. A race car would also be described as having heavy springs and would also be uncomfortably bumpy. A luxury car, taxi, or passenger bus would be described as having soft springs. Vehicles with worn out or damaged springs ride lower to the ground which reduces the overall amount of compression available to the suspension and increases the amount of body lean. Performance vehicles can sometimes have spring rate requirements other than vehicle weight and load.

#### 2.4.2. Mathematics of the spring rate

Spring rate is a ratio used to measure how resistant a spring is to being compressed or expanded during the spring's deflection. The magnitude of the spring force increases as deflection increases according to Hooke's Law. Briefly, this can be stated as,

$$F = -kx$$

Where,

$F$  is the force the spring exerts  $k$  is the spring rate of the spring.

$x$  is the displacement from equilibrium length i.e.

the length at which the spring is neither compressed or stretched.

Spring rate is confined to a narrow interval by the weight of the vehicle, the load the vehicle will carry, and to a lesser extent by suspension geometry and performance desires.

Spring rates typically have units of N/mm. A non-linear spring rate is one for which the relation between the spring's compression and the force exerted cannot be fitted adequately to a linear model. The spring rate of a coil spring may be calculated by a simple algebraic equation or it may be measured in a spring testing machine. The spring constant  $k$  can be calculated as follows:

$$k = \frac{d^4 G}{8ND^3}$$

Where,  $d$  is the wire diameter,  $G$  is the spring's shear modulus (e.g., about 80 GPa for steel), and  $N$  is the number of wraps and  $D$  is the diameter of the coil.

#### 2.5 Fox Float 3 Air Shock

FOX FLOAT (FOX Load Optimizing Air Technology) 3 air shocks are high-performance shock absorbers that use air as springs, instead of heavy steel coil springs or expensive titanium coil springs. Underneath that air sleeve is a high-performance, velocity-sensitive, shimmed damping system. FLOAT 3 air shock dampers contain high pressure nitrogen gas and FOX high viscosity index shock oil separated by an Internal Floating Piston system. This helps to ensure consistent, fade-free damping in most riding conditions

FLOAT 3 shocks are built using 6061-T6 aluminum for light weight and strength. The chromed damper shaft is super-finished for low friction and long seal life. All of the seals and wipers are engineered specifically for FLOAT 3. The damper shaft and seals are contained within



the air sleeve, protecting them from dirt, water and ice.

### 2.5.1. Adjustable progressive air spring

Air springs are not just lightweight they are also progressive. What does that mean? As the graph below shows, during the second half of shock travel, the spring force builds rapidly. This virtually eliminates any harsh bottoming of the suspension and provides a “bottomless” feel.

The graph compares the spring forces for three different initial air pressure settings (50, 60 and 70 psi). The progressive air spring pressure is infinitely adjustable (up to a maximum of 150 psi) for different rider weights and terrain conditions using the included FOX High Pressure Pump. The adjustment of the air spring changes both preload and spring rate, making it a much more effective adjustment than preloading a coil spring. This means that air spring pressure adjustments will allow your FLOAT 3 air spring shock to be used in a wide variety of riding conditions without having to buy different rate springs as with a coil-over shock.

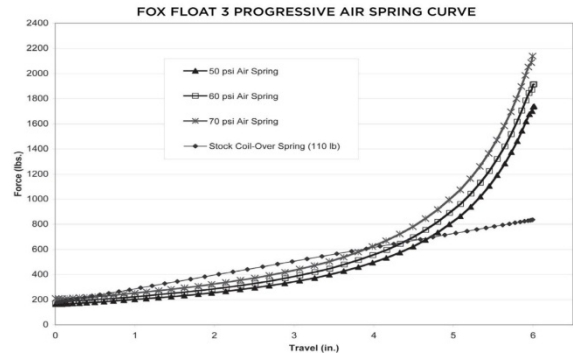


Fig 2.3: Fox Float 3 Progressive air spring curve

The graph also shows a typical stock straight-rate steel coil spring. As you can see, it builds its spring force in a linear straight line. This straight spring rate does not give the progressive bottom-out protection of a FOX FLOAT 3 air shock.

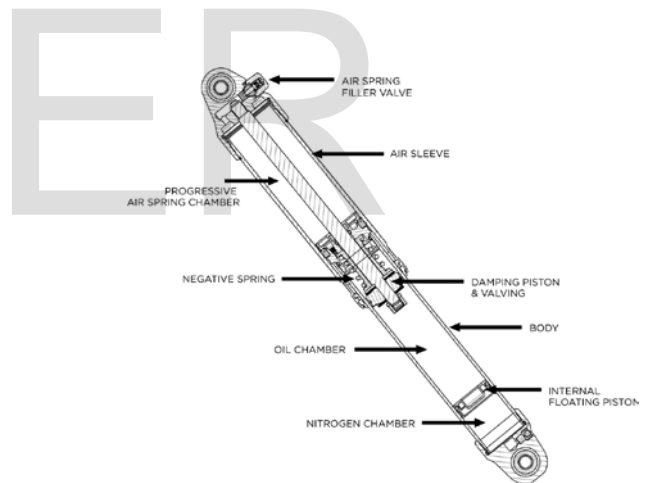


Fig 2.4: Sectional view of Fox float 3

## 2.6. Important Terms in Spring and Dampers

### 2.6.1. Wheel rate



Wheel rate is the effective spring rate when measured at the wheel. Wheel rate is usually equal to or considerably less than the spring rate. Commonly, springs are mounted on control arms, swing arms or some other pivoting suspension member. The wheel rate is calculated by taking the square of the motion ratio times the spring rate. Squaring the ratio is because the ratio has two effects on the wheel rate. The ratio applies to both the force and distance traveled.

Wheel rate on independent suspension is fairly straight-forward. However, special consideration must be taken with some non-independent suspension designs. Yet because the wheels are not independent, when viewed from the side under acceleration or braking the pivot point is at infinity (because both wheels have moved) and the spring is directly in line with the wheel contact patch. The result is often that the effective wheel rate under cornering is different from what it is under acceleration and braking. This variation in wheel rate may be minimized by locating the spring as close to the wheel as possible.

### **2.6.2. Roll couple percentage**

Roll couple percentage is the effective wheel rates, in roll, of each axle of the vehicle as a ratio of the vehicle's total roll rate. Roll Couple Percentage is critical in accurately balancing the handling of a vehicle.

A vehicle with a roll couple percentage of 70% will transfer 70% of its sprung weight at the front of the vehicle during cornering.

### **2.6.3. Weight transfer**

Weight transfer during cornering, acceleration or braking is usually calculated per individual wheel and compared with the static weights for the same wheels. Cornering wheel weights requires knowing the static wheel weights and adding or subtracting the unsprung, sprung and jacking forces at each wheel.

### **2.6.4. Unsprung weight transfer**

Unsprung weight transfer is calculated based on the weight of the vehicle's components that are not supported by the springs. This includes tires, wheels, brakes, spindles, half the control arm's weight and other components. These components are then (for calculation purposes) assumed to be connected to a vehicle with zero sprung weight.

They are then put through the same dynamic loads. The weight transfer for cornering in the front would be equal to the total unsprung front weight times the G-Force times the front unsprung center of gravity height divided by the front track width. The same is true for the rear.

### **2.6.5. Sprung weight transfer**

Sprung Weight Transfer is the weight transferred by only the weight of the vehicle resting on the springs not the total vehicle weight.

Calculating this requires knowing the vehicles sprung weight (total weight less the unsprung weight), the front and rear roll center heights and the sprung center of gravity height (used to calculate the roll moment arm length). Calculating the front and rear sprung weight transfer will also require knowing the roll couple percentage.

The roll axis is the line through the front and rear roll centers that the vehicle rolls around during cornering. The distance from this axis to the sprung center of gravity height is the roll moment arm length. The total sprung weight transfer is equal to the Gforce times the sprung weight times the roll moment arm length divided by the effective track width. The front sprung weight transfer is calculated by multiplying the roll couple percentage times the total sprung weight transfer.

#### **2.6.6. Jacking forces**

Jacking forces can be thought of as the centripetal force pushing diagonally upward from the tire contact patch into the suspension roll center. The front jacking force is calculated by taking the front sprung weight times the G-force times the front roll center height divided by the front track width. The rear is calculated the same way except at the rear.

#### **2.6.7. Travel**

Travel is the measure of distance from the bottom of the suspension stroke to the top of the suspension stroke. Bottoming or lifting a wheel can

cause serious control problems or directly cause damage. "Bottoming" can be the suspension, tires, fenders, etc. running out of space to move the body or other components of the car hitting the road.

The control problems caused by lifting a wheel are less severe if the wheel lifts when the spring reaches its unloaded shape than they are if travel is limited by contact of suspension members.

#### **2.6.8. Damping**

Damping is the control of motion or oscillation, as seen with the use of hydraulic gates and valves in a vehicles shock absorber. This may also vary, intentionally or unintentionally. Like spring rate, the optimal damping for comfort may be less than for control.

Damping controls the travel speed and resistance of the vehicles suspension. An undamped car will oscillate up and down. With proper damping levels, the car will settle back to a normal state in a minimal amount of time. Most damping in modern vehicles can be controlled by increasing or decreasing the resistance to fluid flow in the shock absorber.

#### **2.6.9. Camber control**

A tire wears and brakes best at -1 to -2 degrees of camber from vertical. Depending on the tire, it may hold the road best at a slightly different angle. Small changes in camber, front and rear, are used to tune handling.

#### **2.6.10. Roll center height**

This is important to body roll and to front to rear roll moment distribution. However, the roll moment distribution in most cars is set more by the antiroll bars than the RCH. It may affect the tendency to roll over.

### **2.6.11. Instant center**

A tire's force vector points from the contact patch of the tire through a point referred to as the "instant center". This imaginary point is the effective geometric point at which the suspension force vectors are transmitted to the chassis. Another way of looking at this is to imagine each suspension control arm mounted only at the frame. The axis that the arm rotates around creates an imaginary line running through the vehicle. Forces, as far as suspension geometry are concerned, are transmitted either along this axis (usually front to rear) or through this axis at a right angle (usually right to left and intersects the ball joint). When force lines of the upper and lower control arms intersect, where they cross is the Instant Center. The Instant Centers when viewed from the front or side may not seem to have much of a relation to each other until you imagine the points in three dimensions. Sometimes the Instant Center is at ground level or at a distant point due to parallel control arms.

The instant center can also be thought of as having the effect of converting multilink suspension into a single control arm which pivots at the Instant Center. This is only true at a given suspension deflection, because an unequal length,

multi-link system has an instant center that moves as the suspension is deflected.

### **2.6.12. Anti-dive and anti-squat**

Anti-dive and anti-squat are expressed in terms of percentage and refer to the front diving under braking and the rear squatting under acceleration. They can be thought of as the counterparts for braking and acceleration as jacking forces are to cornering. The main reason for the difference is due to the different design goals between front and rear suspension, whereas suspension is usually symmetrical between the left and right of the vehicle.

Anti-dive and anti-squat percentage are always calculated with respect to a vertical plane that intersects the vehicle's center of gravity. The anti-dive is the ratio between the height of where the tire force vector crosses the center of gravity plane expressed as a percentage. An anti-dive ratio of 50% would mean the force vector under braking crosses half way between the ground and the center of gravity. Anti-squat is the counterpart to anti-dive and is for the rear suspension under acceleration. Anti-dive and anti-squat may or may not be desirable depending on the suspension design.

### **2.6.13. Isolation from high frequency shock**

For most purposes, the weight of the suspension components is unimportant, but at

high frequencies, caused by road surface roughness, the parts isolated by rubber bushings act as a multistage filter to suppress noise and vibration better than can be done with only the tires and springs.

#### **2.6.14. Space occupied force distribution**

Designs differ as to how much space they take up and where it is located. It is generally accepted that MacPherson struts are the most compact arrangement for frontengine vehicles, where the wheels is required to place the engine.

#### **2.6.15. Air resistance (drag)**

Certain modern vehicles have height adjustable suspension in order to improve aerodynamics and fuel efficiency. And modern formula cars, that have exposed wheels and suspension, typically use streamlined tubing rather than simple round tubing for their suspension arms to reduce drag. Also typical is the use of rocker arm, push rod, or pull rod type suspensions, that among other things, places the spring/damper unit inboard and out of the air stream to further reduce air resistance.

#### **2.6.16. COST**

Production methods improve, but cost is always a factor. The continued use of the solid rear

axle, with unsprung differential, especially on heavy vehicles, seems to be the most obvious example.

### **2.7. Tires and Wheels**

The tires are crucial functional elements for the transmission of longitudinal, lateral and vertical forces between the vehicle and road. The tire properties should be as constant as possible and hence predictable by the driver. As well as their static and dynamic force transmission properties, the requirements described below – depending on the intended use of the vehicle – are also to be satisfied.

Selecting the right tires for the ATV is not difficult if we know what we are looking for, there are some important things to consider in order to make the best selection, doing a wrong selection can kill the fuel economy, decrease performance and possibly damage the vehicle.

Tread pattern is one of the most important things to consider, there are several patterns like mud tires, trail tires, sand tires and race tires. It is needed to analyze first what type of terrain the vehicle will drive in most, in order to select best performing tires for that particular terrain. Since, the ATV is meant to drive in all kinds of terrains, an aggressive all terrain tires should be the best.

The all terrain tires come in two patterns, flat and round. Flat tires have more treads to the ground, and in the other hand round tires can increase the vehicle speed. But the round tires also

have a tendency to roll under during hard cornering, while the flat tire "puts more rubber to the track".

Then comes the problem with the choice between the tall tire and the short tire, the a tall tire will lift the ATV higher off the ground and give a softer ride, but on the other hand a tall tire has more sidewall flex which will give the ATV a feeling of being loose during hard cornering. Whereas a short tire gives more stability during hard cornering and high speeds, but gives less ground clearance and makes the ride a little bumpier.

Things to remember while selecting ATV tires:

- a. **Ride Comfort:** These tires ride exceptionally smooth on pavement and dirt roads. They also absorb the impact of rocks and other obstacles very well. The driver should feel comfortable and safe while driving the vehicle.
- b. **Steering/Handling:** These tires steer effortlessly and track well over the trail, but they are a little sensitive to uneven surfaces, tending to follow small ruts and grooves etc.
- c. **Puncture Resistance:** Puncture resistance should be very high as this vehicle is going to run through rough terrains, water, mud and many such adverse conditions. Also small bits of gravel caught between the tire bead and rim should be cleaned periodically as it causes to lose all air minimizing life of the tire.
- d. **Mud Traction:** Mud traction is as expected, pretty good for a multi-purpose tire.

e. **Sand/loose dirt track traction:** This is where these tires really shine, especially in very steep terrain. The soft tread cleats that wrap around the tire shoulders and flexible tire construction combine to grab nicely on to most dirt/rocky trail conditions i.e. it should have very high sand tracks in order to deal with the muddy tracks. [2, 6]

## 2.8. ELASTOKINEMATICS

„Elastokinematics“ defines the alterations in the position of the wheels caused by forces and moments between the tires and the road or the longitudinal movement of the wheel, against suspension anchorage required to prevent compliance, kinematics changes.

### 2.8.1. Wheel base

The wheelbase  $l$ , measured from the centre of the front to the centre of the rear axle is an important variable in the vehicle's ride and handling properties.

The short body overhangs to the front and rear, reduce the tendency to pitch oscillations and make it possible to fit soft springing, normally associated with a high level of ride comfort. A short wheelbase, on the other hand, makes cornering easier, i.e. gives a smaller swept turning circle for the same steering input.

### 2.8.2. Track

The track  $b_f$  is measure of centre distance between two front wheels or two rear wheels. When the wheels travel in bump and rebound-travel direction, the track changes on almost all independent wheel suspensions, which may be unavoidable if a higher body roll centre is necessary. However, the track size alteration causes the rolling tire to slip and, on flat cross-sections in particular, causes lateral forces, higher rolling resistance and deterioration in the directional stability of the vehicle, and may even influence the steering.

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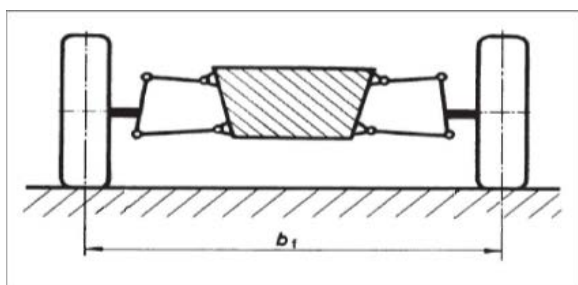


Fig 2.5: Path designations on the front axle

### 2.8.3. Roll center

The roll center of a vehicle is the imaginary point at which the cornering forces in the suspension are reacted to the vehicle body.

There are two definitions of roll center. The most commonly used is the geometric (or kinematics) roll center, whereas the Society of Automotive Engineers uses a force based definition.

"The point in the transverse vertical plane through any pair of wheel centers at which lateral forces may be applied to the sprung mass without producing suspension roll".

The roll centers are also defined as the instant center of rotation of the chassis relative to the ground when both suspensions of the same axle are regarded as planar mechanisms.

Load transfer is of critical importance for vehicle stability in vehicle such as ATVs. Ideally in high performance applications load transfer tends to be minimized as a tire's performance is directly affected by the amount of load that it has to transmit. In a steady state turn the final load transfer, summed across all the axles, is only related to the position of the center of mass above the ground, the track width and the lateral acceleration. ATVs must shift their center of mass lower level or decrease their lateral acceleration to avoid tipping. To keep them from tipping the tires used are with lower grip which reduces the vehicles cornering capacity, or another option is altering the roll stiffness balance from front to rear, to encourage under steer or over steer as necessary to limit the maximum lateral acceleration of the vehicle.

The geometric roll center of the vehicle can be found by following basic geometrical procedures when the vehicle is static. However, when the vehicle rolls the roll centers migrate. The rapid movement of roll centers when the system experiences small displacements can lead to stability problems with the vehicle. The roll center height has been shown to affect behavior at the initiation of turns such as nimbleness and initial roll control.

### 2.8.3.1. Method of determining the roll center for double wishbone system

The height of the (instantaneous centre of rotation) P determines the position of the body roll centre Ro

From figure 2.6, the roll center height can be calculated by formula,

$$h_{Ro} = \frac{b_f}{2} \cdot \frac{p}{k \cos \beta + \tan \sigma + r_\sigma}$$

Where,

$$p = k \sin \beta + d$$

$$k = c \cdot \frac{\sin(90^\circ + \sigma - \alpha)}{\sin(\alpha + \beta)}$$

As it can be seen in figure 2.6, for double wishbone suspension only the position of the control arms is important. The lines connecting the inner and outer control arm pivots need to be extended to fix virtual centre of rotation P and, at the same time, its height p.

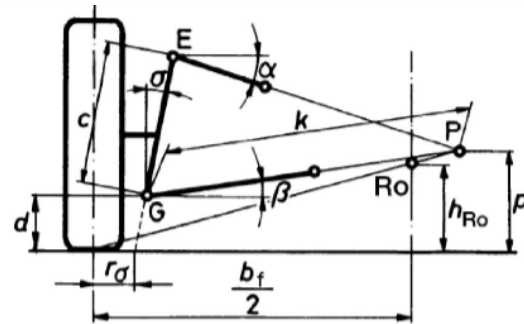


Fig 2. 6: Determination by drawing and calculation of the paths  $h_{Ro}$  and  $p$  on double wishbone suspensions and a multi-link as well as longitudinal transverse axes.

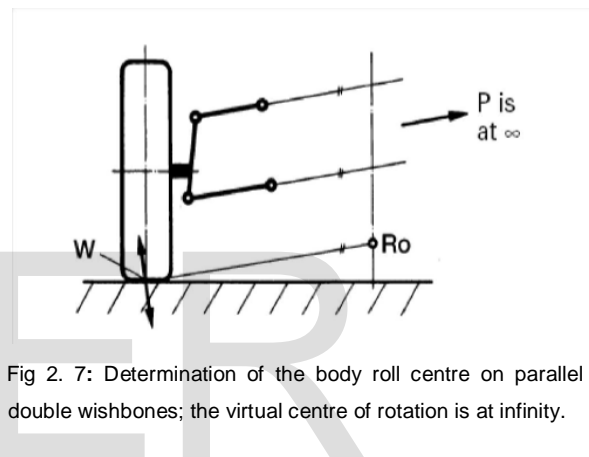


Fig 2. 7: Determination of the body roll centre on parallel double wishbones; the virtual centre of rotation is at infinity.

P linked with the centre of tire contact W gives the body roll centre Ro in the intersection with the vehicle centre plane. In the case of parallel control arms, P is at  $\infty$  and a line parallel to them needs to be drawn through W (Figure 2.7). Where the virtual centre of rotation is a long way from the wheel centre of contact, it is recommended that the distances  $p$  and  $h_{Ro}$  be calculated using the formulae listed above.

Steering control arm axes of rotation, which are sloped when viewed from the side, need E1 and G1 to be moved perpendicularly up or down (Figure 2.8). The points E2 and G2 obtained in this way – linked with E1 and G1 when viewed



from the rear – give the virtual centre of rotation  $P$ , and the line from this axis to the centre of tire contact (as shown in Figure 2.8) gives the body roll centre.

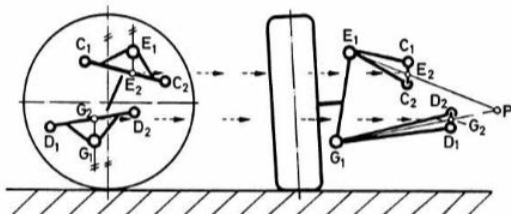


Fig 2. 8: If the suspension control arm axes of rotation are at an angle to one another when viewed from the side, a vertical should first be drawn to the ground through the points  $E_1$  and  $G_1$ ; the intersections with the axes of rotation  $C_1C_2$  and  $D_1D_2$  yield the points  $E_2$  and  $G_2$ , needed for determining the virtual centre of rotation when viewed from the rear.

#### 2.8.4. Roll axis

“Traditionally the vehicle has been assumed to roll about a roll axis which has been defined as an axis joining two imaginary points, the „roll centers“ of the front and rear suspensions.

The roll axis is the line about which the chassis (or car body) rolls when a force (or a pure rolling moment) acts on the car body from the side (which is what happens, for instance, when the car enters a turn). Or the roll axis is the set of the chassis points where a lateral force can be applied without producing any roll movement of the chassis itself.

The roll axis is determined as the line going through the front and rear roll centers of a

car. In the general, the roll axis is determined by introducing the ensuing simplifications:

- a. The front and rear parts of the car are considered separately. Each semi-vehicle is composed of a part (front or rear) of the chassis, together with the suspensions of the corresponding axle.
- b. Any pitch rotation of the chassis of a semi-vehicle is neglected, so that a transverse vertical plane point fixed to the chassis of the semi-vehicle and going through the centers of the wheels at the reference configuration of the vehicle keeps vertical when the chassis moves with respect to the ground.
- c. The spatial kinematic chains of the suspensions connecting the chassis to the two hub carriers of any semi-vehicle are considered as planar (even though they are actually not), the plane of motion being  $pt$ .
- d. The two wheels of any semi-vehicle are supposed as rigid and of infinitesimal thickness.
- e. The toe and steering angles of the wheels are neglected, so that the points of contact between the two wheels of a semi-vehicle and the ground always lie on plane  $pt$ .
- f. The mutual distance of the contact points between the two wheels of a semi-vehicle and the ground is considered as constant.

#### 2.8.5. Camber angle

Camber angle is the angle made by the wheel of an automobile; that is, it is the angle between the vertical axis of the wheel and the vertical axis of the vehicle when viewed from the

front or rear. If the top of the wheel is farther out than the bottom (that is, away from the axle), it is called positive camber; if the bottom of the wheel is farther out than the top, it is called negative camber.

Camber angle alters the handling qualities of a particular suspension design. Negative camber improves grip when cornering. This is because it places the tire at a more optimal angle to the road, transmitting the forces through the vertical plane of the tire, rather than through a shear force across it. Another reason for negative camber is that a rubber tire tends to roll on itself while cornering. If the tire had zero camber, the inside edge of the contact patch would begin to lift off of the ground, thereby reducing the area of the contact patch. By applying negative camber, this effect is reduced, thereby maximizing the contact patch area. Note that this is only true for the outside tire during the turn; the inside tire would benefit most from positive camber.

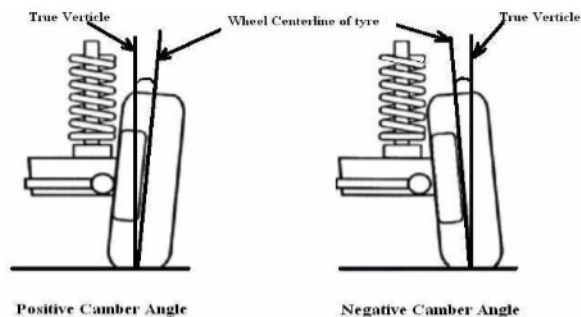


Fig 2.9: Camber angle

On the other hand, for maximum straight-line acceleration, the greatest traction will be attained when the camber angle is zero and the tread is flat on the road. Proper management of camber angle

is a major factor in suspension design, and must incorporate not only idealized geometric models, but also real-life behavior of the components; flex distortion, elasticity, etc.

In cars with double wishbone suspensions, camber angle was usually adjustable, but in newer with McPherson strut suspensions, it is normally fixed. While this may reduce maintenance requirements, if the car is lowered by use of shortened springs, this changes the camber angle (as described in McPherson strut) and can lead to increased tire wear and impaired handling. For this reason, for better handling the car should not only lower the body, but also modify the mounting point of the top of the struts to the body to allow some inward/outward (relative to longitudinal centerline the of vehicle) movement for camber adjustment. Aftermarket plates with slots for strut mounts instead of just holes are available for most of the commonly modified models of cars. Off-Road vehicles such as agricultural tractors, ATVs generally use positive camber. In such vehicles, the positive camber angle helps to achieve a lower steering effort.

### 2.8.6 .Caster angle

Caster angle is the angular displacement from the vertical axis of the suspension of a steered wheel in a vehicle, measured in the longitudinal direction. It is the angle between the pivot line (in a car - an imaginary line that runs through the center of the upper ball joint to the center of the lower ball joint) and vertical.

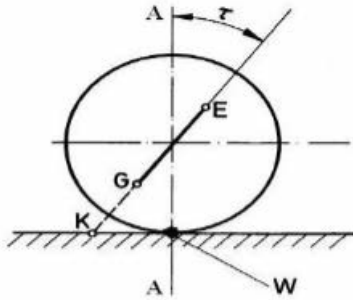


Fig 2.10: caster angle

As shown in figure, the caster angle is the angle between the center plane of the wheel (AA) and the line joining the two pivot points E and G.

The pivot points of the steering are angled such that a line drawn through them intersects the road surface slightly ahead of the contact point of the wheel. The purpose of this is to provide a degree of self-centering for the steering - the wheel casts around so as to trail behind the axis of steering. This makes a car easier to drive and improves its directional stability (reducing its tendency to wander). Excessive caster angle will make the steering heavier and less responsive, although, in racing, large caster angles are used to improve camber gain in cornering. Caster angles over 10 degrees with radial tires are common. Power steering is usually necessary to overcome the jacking effect from the high caster angle.

The steering axis (the dotted line in the diagram above) does not have to pass through the center of the wheel, so the caster can be set independently of the mechanical trail, which is the distance between where the steering axis hits the ground, in side view, and the point directly below the axle. The interaction between caster angle and trail is complex, but roughly speaking they both

aid steering, caster tends to add damping, while trail adds 'feel', and return ability. In the extreme case the system is undamped but stable, as the wheel oscillates around the 'correct' path. Complicating this still further is that the lateral forces at the tire do not act at the center of the contact patch, but at a distance behind the nominal contact patch. This distance is called the pneumatic trail and varies with speed, load, steer angle, surface, tire type, tire pressure and time. A good starting point for this is 30 mm behind the nominal contact patch.

### 2.8.7. Kingpin inclination and kingpin offset at ground

According to ISO 8855, the kingpin inclination is the angle  $\zeta$  which arises between the steering axis EG and a vertical to the road (Figure 2.11). The kingpin offset is the horizontal distance  $r\zeta$  from the steering axis to the intersecting point of line N-N in the wheel center plane with the road.

Larger kingpin inclination angles are necessary to give the vehicle a small or negative kingpin offset. In commercial vehicles, tractors and building-site Lorries, the inclination of the kingpin is often equivalent to the angle  $\zeta$ , whereas the wheels are controlled by ball joints on the front axles of passenger cars. On double wishbone suspensions, the steering axis therefore goes through the centers of the ball sockets E and G indicated; the engineering detail drawing must show the total angle of camber and kingpin inclination.

The McPherson strut and strut damper have a greater effective distance between the lower ball joint G and the upper mounting point E in the wheel house; however, the upper axle parts are next to the wheel, so attention should be paid to creating enough clearance for the rotating tire (possibly for snow chains). As a result, a higher inclination of the steering axis and a higher angle  $\zeta$  has to be accepted. In addition, as can be seen in the illustrations, point G has been shifted to the wheel to obtain a negative kingpin offset. The steering axis then no longer matches the centre line of the suspension strut. Due to the relationship between camber and kingpin inclination shown in Figure 2.11, the angle  $\zeta$  does not need to be tolerance on double wishbone suspensions.

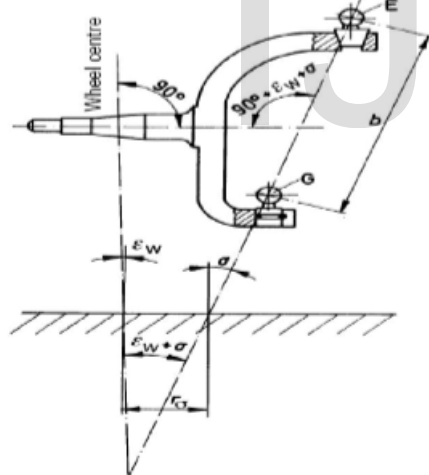


Fig 2. 11: The Kingpin Inclination and Kingpin offset

### 2.8.8. Toe angle

In automotive engineering, toe is the symmetric angle that each wheel makes with the longitudinal axis of the vehicle, as a function of

static geometry, and kinematic and compliant effects.

Positive toe, or toe in, is the front of the wheel pointing in towards the centerline of the vehicle. Negative toe, or toe out, is the front of the wheel pointing away from the centerline of the vehicle. Toe can be measured in linear units, at the front of the tire, or as an angular deflection. In a rear wheel drive car, increased front toe in (i.e. the fronts of the front wheels are closer together than the backs of the front wheels) provides greater straight-line stability at the cost of some sluggishness of turning response, as well as a little more tire wear as they are now driving a bit sideways. On front wheel drive cars, the situation is more complex.

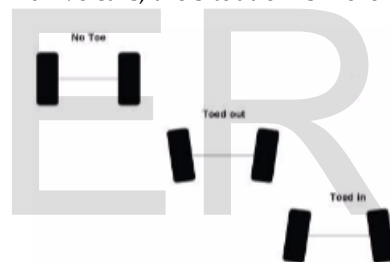


Fig 2.12: Toe Angles

Toe is always adjustable in production automobiles, even though caster angle and camber angle are often not adjustable. Maintenance of front end alignment, which used to involve all three adjustments, currently involves only setting the toe; in most cases, even for a car in which caster or camber are adjustable, only the toe will need adjustment.

One related concept is that the proper toe for straight line travel of a vehicle will not be

correct while turning, since the inside wheel must travel around a smaller radius than the outside wheel; to compensate for this, the steering linkage typically conforms more or less to Ackermann steering geometry, modified to suit the characteristics of the individual vehicle. Individuals who decide to adjust their car's static ride height, either by raising or lowering the springs, should have the car properly aligned. The common misconception is that camber angle causes an increased rate of tire wear, when in fact its contribution to tire wear is usually only visible over the entire life of the tire.

### 3.DESIGN

#### 3.1. Tyre Selection for ATV Design

- a. For selection of ATV tyres for loose, muddy and rough track, it should give more grip for better traction. So we used cross groove tires.
- b. For lesser unspurng weight and better heat dissipation from tires, we choose tube less wheel which also give lesser rolling resistance.
- c. Also it gives comfortable ride and slow leakage of air which provides safety to driver and vehicle.
- d. To get rid of all obstacles on rough track high ground clearance required so selecting the rim of larger diameter give large clearance.
- e. Width of tyre is also a criterion for selection, so tyre having maximum width to give more grips in rough track.

- f. Based on our requirements and market constrains we selected tyre of following specification

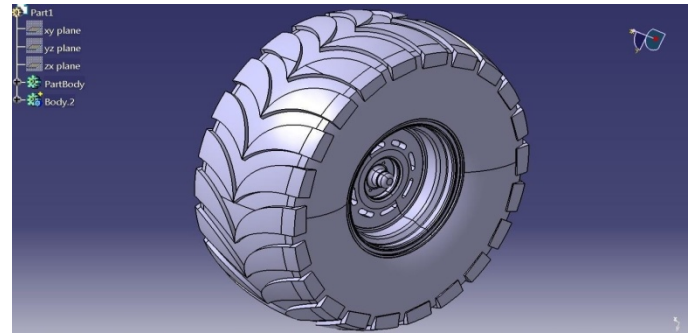


Fig 3.1: Tire Profile

	FRONT	REAR
Width	8"	10"
Outer dia. Of tyre	25"	25"
Outer dia. Of rim	12"	12"

#### 3.2. Suspension System

Depending on the various parameters such as driver comfort, required ground clearance, and rolling tendency of vehicle we selected double wishbone suspension system at front and rear.

##### 3.2.1. Designing of front suspension system

###### 3.2.1.1.Determination of length of wishbones

The overall dimension of the car was decided within constraints by considering

B/L ratio for better performance of differential during cornering, and driver's comfort.

From this we have decided track width as 52" at front and 50" at rear and wheel Base as 57".

According to the required travel for front suspension system of around 8" we have decided to go for Double wishbone system which gives maximum travel amongst all suspension systems. The double wishbone system is more flexible and provides better ride comfort on bumpy terrain; also it is easy to manufacture. Moreover we get more control on parameters of suspension geometry.

We have decided the optimum length of wishbone keeping in mind the required leg space at front, the required ground clearance and the angles at which wishbones were positioned. The values of angles for wishbones were determined by required roll center height at front. To achieve that, we have fixed the feasible range for the height of roll center. Generally for the stability of vehicle it is required that the height of roll center at front is around 10.12" and at rear is 9.44" for a ground clearance of 12" front and 11" rear. This roll center positioning provides better transmission of forces acting on the vehicle along the roll axis which yields good stability of vehicle and increased effect of roll/yaw damping. Then we selected the horizontal distance between roll center and instantaneous center as 53.22". The position of instantaneous center which is more near to infinity is best suitable for a stable suspension design. To get a positive scrub radius of 2.6" we fixed kingpin inclination (steering axis inclination) as 10°.

The parameters which is initially fixed for drawing front suspension geometry for obtaining the optimum length of wishbone are given below.

TABLE 3.1:  
 INPUT VALUES FOR FRONT SUSPENSION  
 GEOMETRY

Track width( $b_f$ )	52"
Wheel base	57"
Scrub radius	2.60"
Toe in	0°
Caster angle	5 °
Camber angle	-2 °
King pin Inclination	10 °
Roll Center Height	10.12"



Fig 3.2: Front view of front suspension geometry

From the above data we have drawn the optimum suspension geometry to fulfill our requirements. Now from these suspension geometry we calculated the exact dimensions of upper wishbone and lower wishbone and also angles and geometry for front suspension.

TABLE 3.2:  
 FINAL VALUES OBTAINED FOR DESIGNING  
 FRONT WISHBONE

Length of upper wish bones	12.07"
Length of lower wish bones	13.59"
Inclination of wishbone with upper horizontal( $\alpha$ )	12 <sup>o</sup>
Inclination of wishbone with lower horizontal( $\beta$ )	17 <sup>o</sup>

TABLE 3.3:  
 SPECIFICATION OF FOX SHOCK USED IN  
 FRONT

Part	Length	Travel	Comp
830-12-301	16.2	4.5	11.5

3.2.1.2. Calculation of spring

Most automotive suspension systems use helical springs. Next step in suspension designing is to get dimensions of helical spring. Depending on wishbone travel the spring and damper travel was determined. Mounting of spring to lower wishbones give better suspension effect.

Spring stiffness for front suspension can be calculated by

$$K_s = 4\pi^2 f_r^2 m_{sm} MR^2$$

$K_s$  = Spring  
 $m_{sm}$  = Sprun  
 $f_r$  = Ride fre  
 MR = Motio

$$\text{Spring stiffness} = 4 \times 3.14^2 \times 1.2^2 \times 205 \times .5625^2 = 20.72 \text{ N/mm}$$

Here we decided to use fox float 3 air shock of following specification which comes in the range of our stiffness value and load of vehicle

3.2.2. Designing of rear suspension system

3.2.2.1 Determination of length of wishbones

Here we initially fix a rollcenter height of 9.44" and distance between the roll center and instantaneous center is taken as 44.58"

TABLE 3.4:  
 INPUT VALUES FOR REAR

Track width( $b_f$ )	50"
Toe in	0o
Roll Center Height	9.44"

suspension geometry

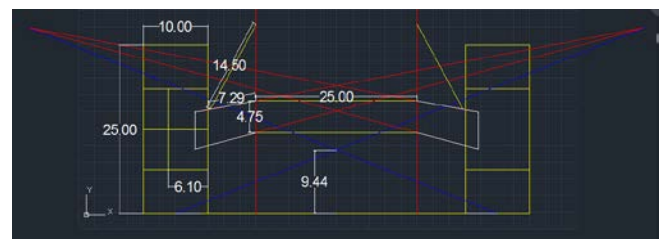


Fig 3.3: Front view of rear suspension geometry



TABLE 3.5:  
 FINAL VALUES OBTAINED FOR DESIGNING  
 FRONT WISHBONE

Length of upper wish bones	<b>11.92"</b>
Length of lower wish bones	<b>12.07"</b>
Inclination of wishbones with upper horizontal ( $\alpha$ )	<b>13.42°</b>
Inclination of wishbones With lower horizontal ( $\beta$ )	<b>16.85°</b>

also it was thoroughly tested as an assembly with the remaining rear suspension parts. A final model is shown below

**3.2.2.2. Calculation of spring**

Similar to front suspension system we followed the same procedure for designing of rear shocker spring.



Fig 3.4: CATIA design of HUB

TABLE 3.6:

SPECIFICATION OF REAR FOX SHOCK

PART	LENGTH	TRAVEL	COMP
Rear 830-12-302	14.5	3.7	10.2

**3.2.3. Designing of hub and upright**

Catia V5 was used extensively to arrive at the final design of the rear upright. After a specific design was developed it was then validated using Catia's FEA package. Not only was the part simulated as an individual piece, but

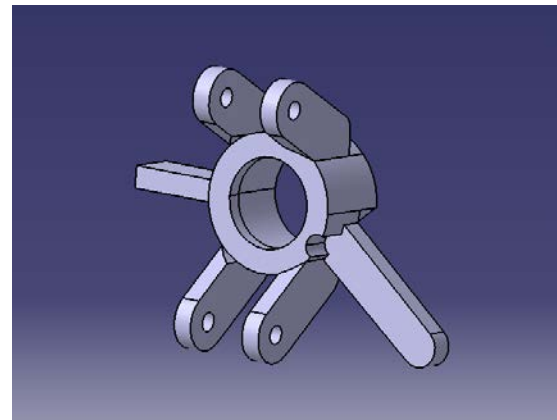


Fig 3.5: Front left upright

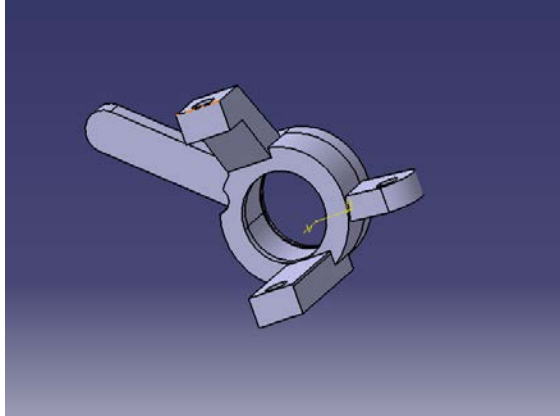


Fig 3.6: Rear left upright

#### 4.ANALYSIS

Testing analysis was done twice during the project time line; prior to fabrication and after fabrication, on the track. The various parts of the vehicle were modeled on the simulation software first in order to get the proper idea of its assembly, fabrication and possible difficulties in fabrication. Another and most important advantage of the modeling was to check for any possibility of the failure of the component. The modeling software provided us with the information of the stress distribution in the component or in the system and its behavior under static and dynamic loading conditions. This has saved lot of redesign work as well as it reduced the overall cost of vehicle. For software modeling and analysis we have utilized CATIA software. The results of these analyses are explained further in this chapter.

#### 4.1 Analysis of Front and Rear Upright

Image 4.1 and 4.2 shows the FEA analysis of front and rear upright. The upright provides the support for the bearings on which hub and ultimately the wheel rotates. This acts large amount of forces on the upright. As it can be seen from the graphical image of the FEA analysis result, with application of load of 500N there is small amount of red zones on the upright. So we decided to harden the upright which made of aluminium to give strength to sustain the forces acting on it.

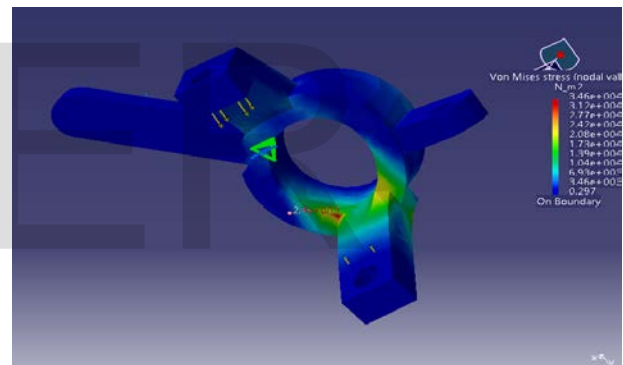


Fig 4.1: FEA analysis of front upright.

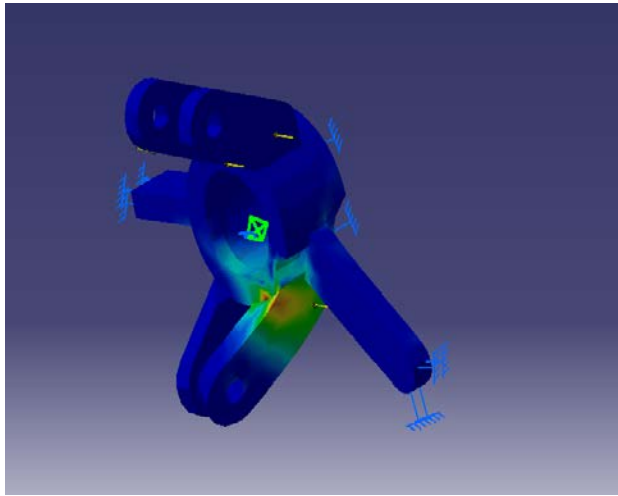


Fig 4.2: FEA analysis of rear s upright

#### 4.2. Analysis of A-arm

Image 4.3 shows the FEA analysis of front upper wishbone. As it can be observed from the image the component shows stress concentration near bearing sleeves, though the force at which the red zone has occurred it is a very critical section where failure can occur hence, in manufacturing extra care is been taken to avoid any possibility for defect occurrence. Image 4.3 shows the possible deflection in the upper wishbone due to same force as applied for the stress distribution analysis.

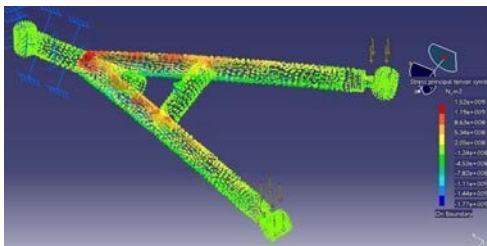


Fig 4.3: FEA analysis of front upper wishbone

## 5.CONCLUSION & RESULT

### 5.1. Result

After the designing and analysis of ATV, some of the following results were obtained:

Position of Center of gravity and roll center obtained from vehicle for better stability and comfortable ride for driver.

Distance from ground level  $h_v = 19''$

Roll center at front = 10.12"

Roll center at rear = 9.44"

To sustain the static and dynamic load on vehicle following parameter are obtain by designing for suspension.

TABLE 5.1:  
 RESULT TABLE

<b>GENERAL SECIFICATIONS</b>	
Wheel Base	57"
Front Track	52"
Rear Track	50"
Target Weight	270 kg
<b>SUSPENSION SPECIFICATIONS</b>	
<b>FRONT</b>	
Type	Double Wishbone, Non parallel
Travel	8 "(5" Comp. and 3" Droop)
Camber	-2
Caster	5
Kingpin Inclination	10
Scrub radius	2.60"
Stiffness Value(N/mm)	20.72
Ride frequency(Hz)	1.2
Roll Centre Height	10.12"
Roll Stiffness (Nm/deg)	147.36

Tyre	25*8*12
<b>REAR</b>	
Type	Double Wishbone, Non Parallel
Travel	6"(4" Comp 2" Droop)
Stiffness Value(N/mm)	37.93
Ride frequency(Hz)	1.7
Roll Centre Height	9.44"
Roll Stiffness (Nm/deg)	281.55
Tyre(in)	25*10*12

## 5.2. Conclusion

The paper describes about designing and analyzing suspension of an All Terrain Vehicle (ATV) and their integration in the whole vehicle. The ATV has been designed and analyzed based on the facts of vehicle dynamics. The primary objective of this paper was to identify the design parameters of a vehicle with a proper study of vehicle dynamics. This paper also helps us to study and analyze the procedure of vehicle suspension designing and to identify the performance affecting parameters. It also helps to understand

and overcome the theoretical difficulties of vehicle design.

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