Design and Construction of a 6 Bar Kinematic Quick Return Device for use as a Demonstration Tool

A Major Qualifying Project Report:

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By

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Approved:

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Abstract

The purpose of our project was to explore the characteristics of kinematic quick return devices, and characterize the major differences between quick return and other kinematic machines. These differences would allow us to develop a demonstration device which would be used to help future students understand how quick return machines function, and how to produce a specified motion with them. Our machine is a crank-shaper quick return mechanism designed to run at low speed, to highlight the differences between front and back strokes. Additionally, the device allows for easy changing of the time ratio.

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Introduction:

The kinematics course at WPI covers a wide range of machines, and explains the complex mathematics behind them. Further, it teaches design students how to design machines for specific motions, positions, and applications. While the course does an effective job of teaching students, one aspect it lacks is the ability to demonstrate these machines in operation. Several years ago, Professor Norton sponsored students to build a four bar linkage for use as a demonstration in class. The machine was largely successful, and an effective tool to demonstrate the motion of a typical four bar linkage. Another important machine discussed in the kinematics course is the quick return mechanism. Professor Cobb requested that a machine similar to the four bar linkage machine be designed and constructed to demonstrate a quick return mechanism. The machine would show how the forward and return strokes of the quick return mechanism would follow the same path while traveling at greatly different speeds. The demonstration tool would also need to explain where this difference in speed comes from, and show how a different timing ratio can be accomplished by changing the system. Further, the mobility and safety of the machine would need to allow easy use in a class room environment. The goal of the project was to design and construct a quick return mechanism, with the above considerations, that would serve as a demonstration tool for the kinematics course.

In order to accomplish this goal several steps would need to be completed. The initial step was to determine the design parameters that would help appropriately shape the design. Further research into quick return mechanisms, would help determine design parameters while familiarizing our understanding of their specific kinematic function. The initial research would determine the design space for the project. Once determined, design of the mechanism would begin. Firstly, the geometry of the machine would be decided upon, taking into account the spatial constraints, and desired time ratio. Once the geometry was settled upon, a mathematical model describing the system could be built. The model would contain a typical kinematic position, velocity, acceleration, and force analysis. Using the mathematical model, design characteristics like input torque, operating velocity, and time ratio could be determined. Materials and parts would then be possible to select, order and assemble.

The project is broken down into several sections, beginning with general information on kinematic quick return mechanisms. This section will outline the various types of quick return devices and the analysis of said mechanisms. The design process section explains the methodology behind the project and explains how the design went from start to finish. The finalized design and model are then presented followed by a discussion of the project as a whole, including accomplishments, challenges, and suggestions for future work. Lastly, the project is summarized and concluded.

General Kinematic Quick Return Background:

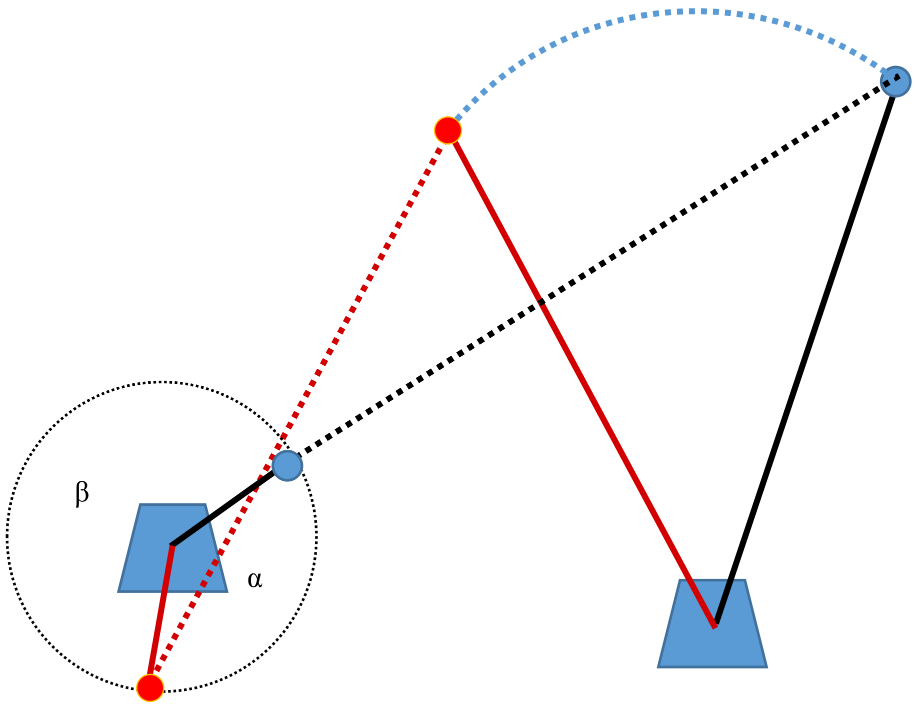


Figure 1: Design of a basic 4 Bar Quick Return Mechanism

The kinematic quick return device is a mechanism typically containing either four or six linkages. Either configuration accomplishes a repeating pattern with a faster return stroke and slower forward stroke. The length of each stroke corresponds to an arc of the crank length, with the boundaries being located at the toggle positions. The ratio of these angles (α and β) corresponds to the difference in time between the two strokes and is called the time ratio. Four bar linkages are effective at producing time ratios smaller than 1:1.5. In order to achieve larger ratios of up to 1:2, a six bar mechanism is required. Both mechanisms are designed to have a single degree of freedom, as determined via Kutzbach equation.

The mathematics that governs and describes the motion of the quick return follows a fairly straight forward method. Firstly, a position analysis is performed on the mechanism. The position analysis is the first of several analyses which will continue to build off each other. The position analysis involves

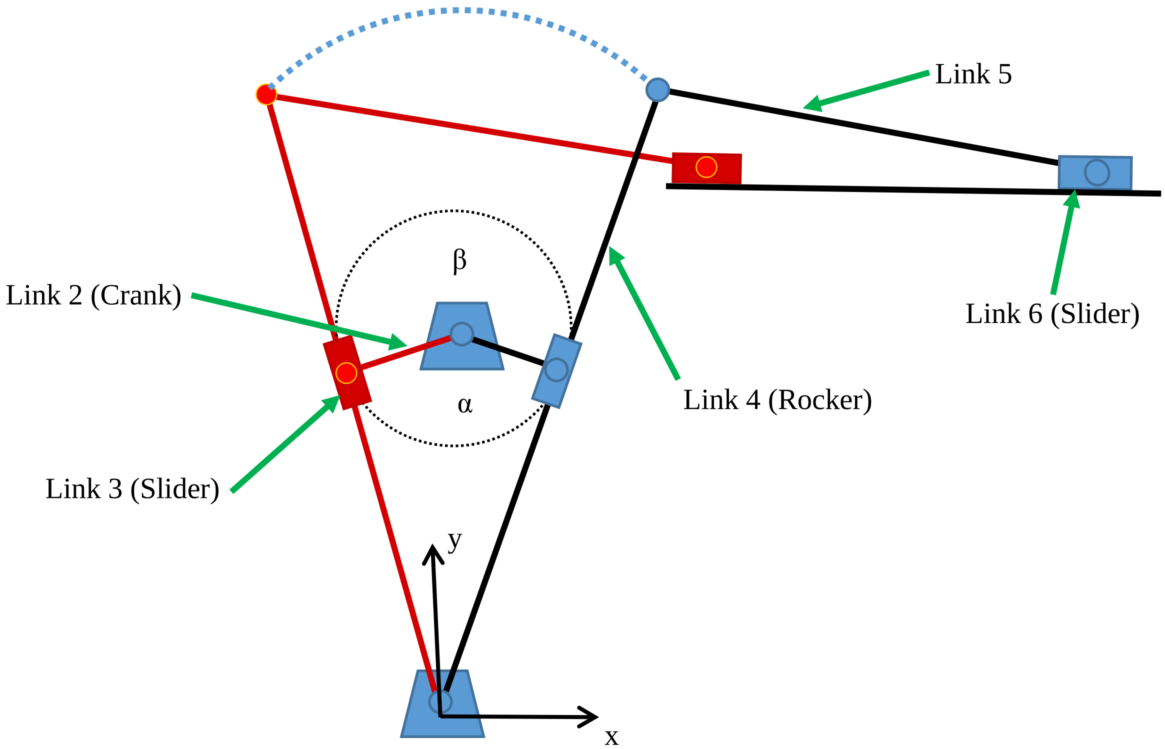


Figure 2: Basic design of a Crank-Shaper 6 Bar Mechanism

determining the location of each link over the course of the entire mechanism's operation. Starting with the crank in an arbitrary pose, each link is represented as a vector, with a magnitude of its length, and a direction based from a non-rotating coordinate system, parallel to the ground coordinate system. By assuming a position for the crank, the slider can be located. Once the slider is located, link 4 is easily defined, followed by link 5 and finally link 6. Once the pose of the mechanism has been fully defined, the velocity analysis can be completed. Once again starting with the crank and assuming an angular velocity, the individual velocities of each subsequent link are determined by vector addition of the various velocity vectors. Using the velocity analysis, accelerations for each member can be derived. Finally free body diagrams are drawn for each link, and summation of forces and moments for each link are generated and simultaneously solved providing the required input torque from the motor as well all the force data determining the operating conditions for the mechanism.

Design Process:

The project began by meeting with Professor Cobb and discussing the best way of constructing the quick return device. Through our research of quick return mechanisms as well as Professor Cobb's requests and suggestions, design conditions were determined. Clear demonstration of the time ratio, and the difference between forward and return strokes were major concerns because these concepts are unique to the quick return mechanism and are central to demonstrating its purpose and function. For this reason it was decided that a larger time ratio would be selected in order to clearly distinguish between forward and return strokes. With four bar quick returns only providing ratios up to 1:1.5, a six bar linkage was decided upon. Another advantaged of the six bar linkage is its configuration. Figure 2, demonstrates the approximate design for our quick return demonstration machine. The machine's vertical nature lends itself to the demonstration application of the class room. Most lecture halls are tiered, as such a sloped panel with the machine mounted to the front makes the most logical sense for demonstration. While the four bar linkage could also be mounted vertically, it wouldn't be as stable and gravity would have more of an effect on the motion of the machine. Another major concern is the ability to change the time ratio in the classroom, so that students can gain an understanding of how to design for a desired time ratio. Determining the best course of action for modifying the time ratio during class took some experimentation. In order for the time ratio to change, a link in the system would need to lengthened or shortened. The first obvious choice is the crank link. It would be relatively simple to make the crank link adjustable, especially because the rocker link can accommodate conditions both smaller or larger. However, through some rough calculations, in order to significantly affect the time ratio, the size of the crank would need to be able to double from its initial size. Further this would cause the end slider to be moving even farther away from the central axis, requiring the machine be overly large. For these reasons, an alternate solution was investigated. Instead of lengthening the crank, the possibility of lengthening the ground link (the distance between the crank and rocker links) was explored to much more promising results. By changing the ground link only a few inches, the time ratio would change from about 1:1.6 to

1:2. This range covers most of the recommended time ratios used in six bar quick return mechanisms which, makes it effective at demonstrating the different possibilities available to design students. Further by making the ground link adjustable, the overall distance that link 6 travels remains manageable.

Once the overall geometry of the linkage was chosen, the next major step was to determine link lengths. General constraints were selected based on user ability to move and operate the machine. To ease movement of the device, a maximum height of 5’ was selected, with a width of approximately 3’. Assuming the mechanism would only be operating on the top half of the cart our longest link, the rocker, was selected to be 2’ long. This would mean that the linkage’s maximum height would be 2’ with some accommodation for non-negligible link thickness. Our initial ground link was selected to be 12”, making the crank center on the mid-point of the rocker. This made selecting a crank length easier as the maximum travel distance would be four times that of the crank. In order to ensure that mechanism would constrain to the 3’ width, a crank length of 4.5” was selected. This length of crank would mean an overall mechanism width of about 27”. Finally link 5 was selected to be 7” long making the total mechanism width 34” leaving some tolerance.

With the geometry and shape of the mechanism decided, work could begin on creating the equations which would describe the motion of the machine. This began with a position analysis of the kinematic system. While rough estimates had been effective up to this point, determining exact position is vital to the use of the model and machine. The vector addition of the position vectors was a relatively straight forward process.

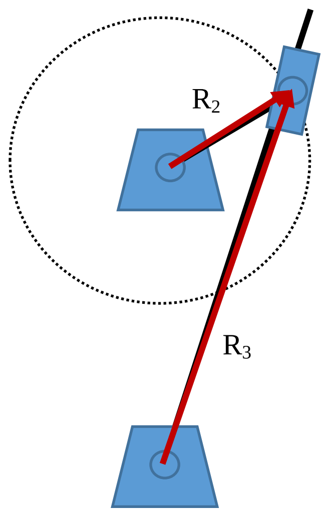


Figure 3: Close up of mechanism showing two position vectors, R_2 and R_3

The complexity of the position analysis came when transferring these vectors into a Mathcad file. One of the major challenges for the position analysis was creating relations that would remain valid no matter which quadrant the crank was currently in. This was especially complicated in determining the angle between crank and rocker, which is constantly changing and would result in different formula depending on which side of the central axis the crank was on. Eventually, the law of cosines was used to establish a relation between the lengths of links 1, 2, and 3 and their interior angles, allowing for the model to correctly represent the system throughout its operation cycle. Without the addition of the law of cosines, the model required separate calculations for each side, and while correct it wasn't as useful a representation of the system.

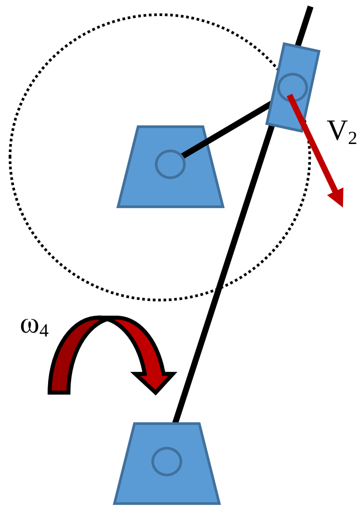


Figure 4: Sketch showing examples of velocity vectors and angular velocities. Shown is the Velocity of joint 2 and 3, and the angular velocity of link 4.

The usefulness of the law of cosines became quickly apparent during the velocity analysis of the mechanism. Similarly to the position analysis, vectors representing the velocities of various links were drawn in order to establish a representative system. Starting with the input of the crank, a velocity was drawn perpendicular to the end of the crank, representing the velocity of the joint between links 2 and 3. This vector is defined by the position vector crossed with the angular velocity of the crank. The angular velocity of the crank is a parameter selected by the designer. In our case, the only constraint we have to the angular velocity is how it will affect the viewing ability of its audience. For this reason, a relatively slow angular velocity was selected. Using the velocity of joint 2 and 3, the velocity of link 3 was calculated. In this configuration, link 3 is fixed to the end of link 2 and moves with the same velocity. In turn, the velocity of link 4 is found by determining how the velocity is transferred from link 3 to link 4. As the crank rotates, link 3 goes through several stages of transmission. At the toggle positions, link 2 and link 4 are perpendicular, this means the velocity is considered to be entirely in slip, and link 4 is considered stopped at its end point. While link 4 is vertical, link 2 is parallel to link 4, meaning the entire velocity of the crank is being transmitted into the rocker. The amount of slip and transmission alternates between these four points over the course of operation. Lastly, using the velocity of link 4, the velocity of link 5 and 6 can be calculated.

The acceleration analysis is an extension of the velocity analysis. Utilizing the position analysis for determination of the center of rotation, and the associated velocity and angular velocity, the acceleration of each link was calculated. Most of the acceleration in the system comes from centripetal forces holding the links in place as they rotate about their joints. During this analysis, the accelerations are broken up into x and y components to be used in the force analysis.

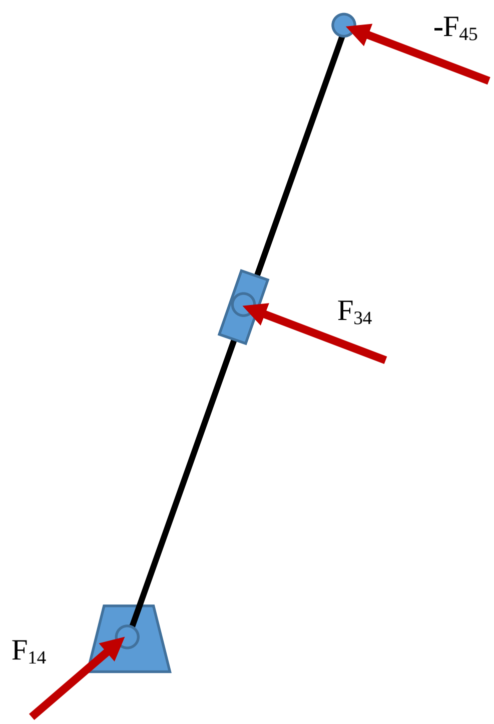


Figure 5: Free body diagram of link 4.

The last major part of our mathematical model is the force analysis. The purpose of this analysis is twofold, first is to provide us information on the forces that each link and joint are affected by. These are vital to ensuring that no joint or link will break during normal operation as well as providing the constraints our joints will need to operate under. The second important piece of information the force analysis provides is the necessary torque required for the system to run. The analysis begins by creating free body diagrams of each linkage, starting with link 6. A force was added to link 6 representing a friction force caused by an induced downward force. This force was added in order to simulate the forces generated by tooling. Coupled with a force from link 5, three equations were written. Two force equations

(F=m*a) for x and y axes respectively, and one moment equation (M=I*α) taken about the center of gravity of each link. Five links, with one ground link leads to five sets of three equations, totaling fifteen equations. Given the accelerations from previous analysis, and masses taken from a CAD model, the various forces and torques were solved for. These four analyses combine to model the mechanical system and allow the designer to provide a robust design. With the analysis complete, changes to the system could be made to alter the output effect as necessary. This is particularly useful to look at how changing the ground link affects the entire system.

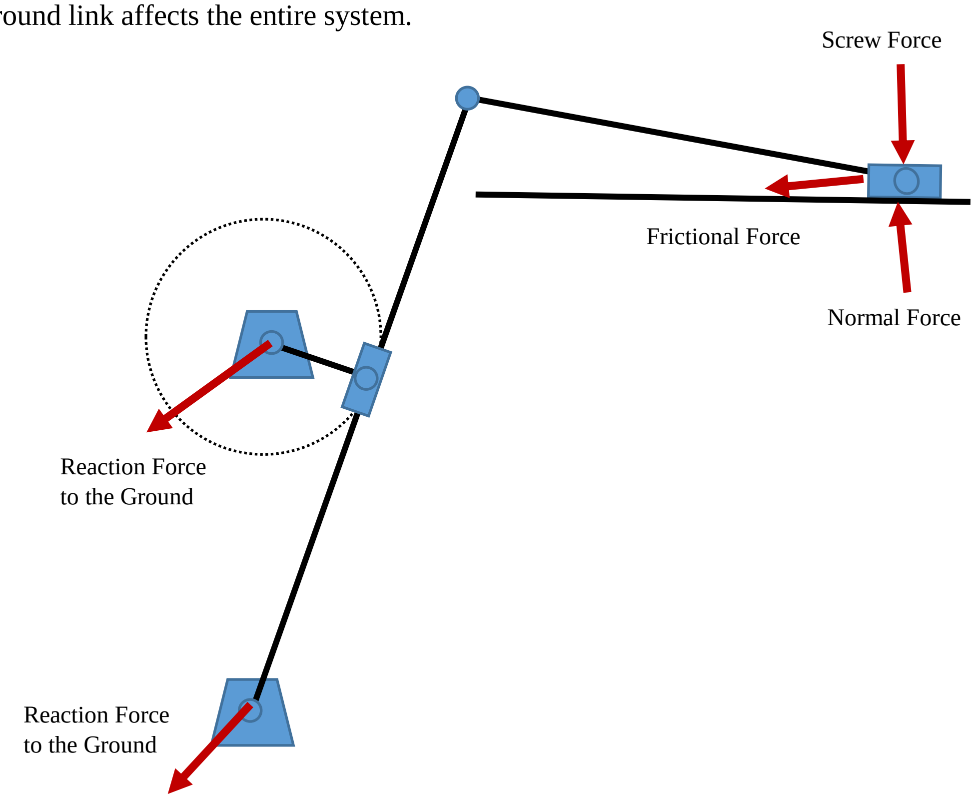


Figure 6: Overall Free Body Diagram of the Mechanism

The final stage of the project was to take the system that we'd modeled and transform it into a physical mechanism. The first part in creating the physical machine was to finalize the platform it is constructed on. The unit would be a rectangular cart where the top half had an angled front. On the front would be mounted the various linkages with the motor mounted on the back side. In order to simulate the tooling force, there would need to be a way to apply a horizontal force to the slider link 6. Moreover, it would be highly desirable to have this force be adjustable to demonstrate the affect it has on the system.

The solution was to attach a wing nut to the slider link 6, which when tightened would force the block down and increase the friction force between the slider and the ground. This force would be dependent on how tight the screw was, and could be calculated knowing the threading and size of the bolt, along with how much the nut was turned. Another consideration was the pin joint connecting link 4 to the ground would need to be removable. This would allow a second position to be placed closer to the crank in order to change the time ratio of the system. Several safety concerns would also need to be accounted for including enclosing the mechanism, and providing an emergency stop. Additionally to help demonstrate the time ratio, behind the mechanism would be marked, illustrating the arcs for the two positions. The individual parts would then be collected for final assembly.

Model and Design:

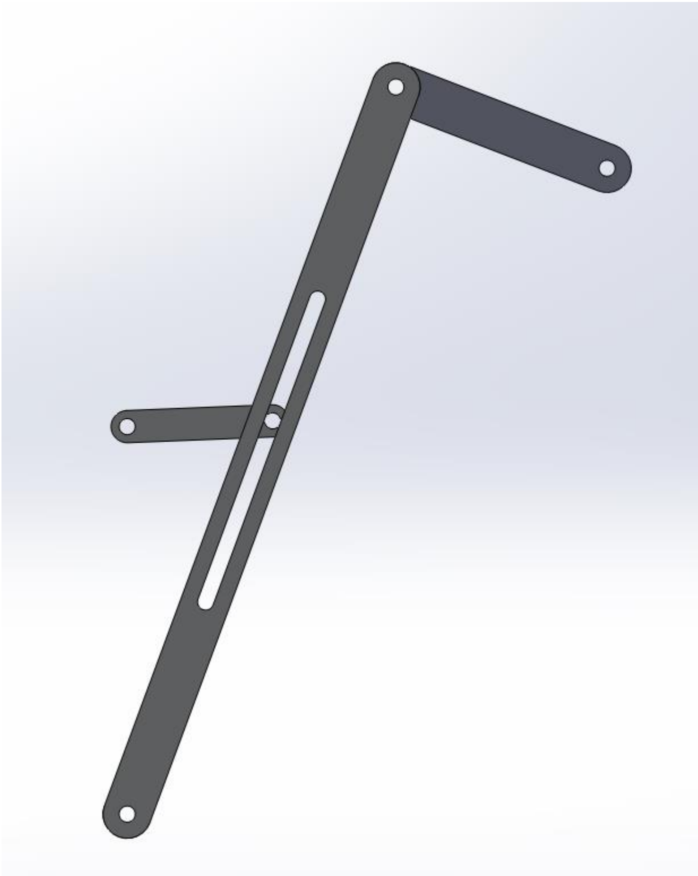


Figure 7: Original Design Model showing crank, rocker, and link 5.

The finalized design of the mechanism is a 6 bar linkage consisting of three typical links, a cam roller, a slider, and a ground link. The crank has an effective length of 4.5”. One end connects to the input motor, while the other is attached to a cam follower. The cam follower acts as link 3, and delivers the input from the crank to the rocker without friction along the axis of slip. The rocker has an effective length of 2’, with a long slot in the middle to allow contact with the roller/crank unit. Link 5 is a typical link with a length of 7”. Link 6 is a simple 2” x 3” slider block that slides along a metal rail attached to the back plate. The motor is mounted to the back of the plate and has its shaft stick through from the back. The links are all made from aluminum stock as well as the back plate which they are mounted to.

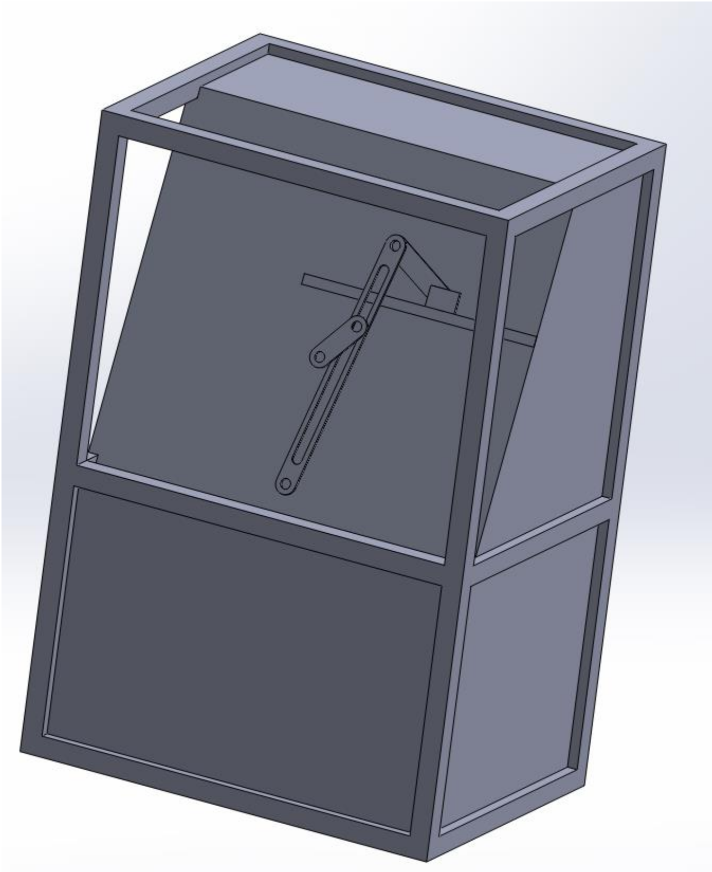


Figure 8: Isometric View of CAD model showing the quick return mechanism.

The cart is made from 80/20 tube stock which assembles into the shape required. The entire mechanism is enclosed in Plexiglas to prevent accidents from occurring while the machine is in operation. Another safety consideration is an emergency stop button on the exterior of the device. Casters would be attached to the bottom, to allow easy movement between classroom and storage. Most of the parts are ordered from 80/20 or MSC direct.

The MathCAD model (see appendix for code) was exceptionally useful in defining and describing the design. The model is capable of describing the position, velocity and acceleration of each link at any position in the cycle. Force data is also available although errors in the code make it unreliable. Through discussion and testing of different values, an operation speed of 15 RPM for the motor was decided. This corresponds to 4 seconds per cycle, leading to 1.5 and 2.5 seconds for the return stroke and forward stroke respectively. This cycle should be long enough for students to understand the clear difference between forward and return strokes. Crank 2 becomes the fastest moving link with a top speed around 7in/s.

Discussion:

The process of designing and creating a kinematic quick return mechanism for demonstration in class was a challenging but rewarding undertaking. The project was a classic example of a design problem, and combined many different aspects of engineering, including basic static and dynamic problems, kinematics, stress, and CAD design. Further the project showed a multitude of new aspects to design. Throughout the process numerous design choices had to be made, and while some could be done with clear intent, others simply required making a best guess and refining the choice later on. When encountering this situation there was some apprehension to make a decision for fear of making a poor choice. As the project progressed however, often making a quick decision allowed for work to continue, and any correction to previous best guesses would happen when the analysis would inevitably need to be changed or fixed. The second biggest challenge for the project was taking the design and creating a physical machine. Many different aspects which are either simplified or simulated become complex in the physical world. Even a pin joint, the basis of most kinematic machines, is difficult to manufacture. Further, finding the parts necessary to construct the mechanism was far more complex than initially expected. This ended up causing problems in the amount of time available for the machine to be manufactured. Another setback of the project was in creating the force analysis portion of the model. Throughout the project, as portions of the analysis were done, they were reformatted and added to the MathCAD model. This began to cause problems later in the analysis because there was confusion between individual analyses and how certain values couldn't be used later. Additionally, once the force portion of the analysis was reached, it became apparent that certain pieces were missing from previous steps. In going back and adding sections, while functional, created a mess in the model. Further, there were issues in getting the force analysis to work properly. Setting up the system of equations kept leading to errors and the system was giving answers which didn't make sense. This lead to some assumptions which would need to be corrected later on.

There were several things which in hindsight would've allowed for a smoother project. The process would've benefited from having a full, clear, and clean analysis of the system done by hand before attempting to create a MathCAD model. Problems were often difficult to identify between fundamental analysis errors, and errors in replicating the analysis in MathCAD. Secondly, the selection of parts happened too late. The undertaking of selecting parts and materials was far greater than anticipated, which lead to a lot of critical time lost. Had this process started earlier, and the parts gathered gradually, the project would've come together quite easily.

Additionally, there are some considerations for future work on the project. Primarily, completion of assembly is paramount. Beyond that are numerous ways of improving the projects performance and utility. The second major consideration would be fixing the MathCAD model. To accomplish this would mean taking the existing model, understanding how it is set up, and rebuilding the model. This would reflect a clear consistent model that takes into account the various parts of information that were added as the project progressed. During the selection of parts, several parts were selected based on availability or estimates. These parts should be replaced with more appropriate parts at a future time. Further the utility of the mechanism would be greatly improved with the additions of LED's to help demonstrate the motion, as well as velocity and acceleration sensors to display how these quantities change throughout a cycle, or when the time ratio is adjusted. Lastly, preventing the mechanism from operating while its door is open would be an excellent addition to the mechanisms safety.

Conclusion:

The purpose of the project was to design and construct a kinematic quick return device. The project achieved varying levels of success in its different facets. Beginning with general research into quick return devices, the project followed a methodology of determining the design space, building a mathematical model and then implementing that model. The design process as a whole was experienced from start to finish and incorporated a multitude of different aspects of engineering. Designing this mechanism was an excellent experience in tackling a design project where the majority of constraints were self-imposed. The final design produced is an effective one, however errors in the model do lead to some doubts as well as areas for the project to progress into. Hopefully, with a little work, the mechanism will be operational and seen by future kinematics students for years to come.

Appendix:

MathCAD Code:

$R_1 := 12\text{in}$ Ground Link

$R_2 := 4.5\text{in}$ Crank Length

$R_4 := 24\text{in}$ Rocker Length

$x := 0, .1 .. 2\cdot\pi$

$\theta_1(x) := x\ \text{rad}$ Location of Crank

$\frac{\alpha}{2\pi - \alpha} = 0.607$ Time Ratio

$R_5 := 7\text{in}$ Length of Link 5

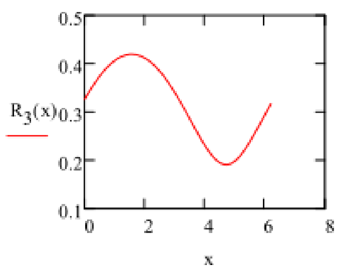
$R_6 := 20\text{in}$ Distance from Ground to Link 6

$\alpha := 2\cdot\text{acos}\left(\frac{R_2}{R_1}\right)$

$b := \frac{0}{2}\pi + \frac{0}{2} = 0$ Test Value

$$R_3(x) := \sqrt{(R_1)^2 + (R_2)^2 - 2\cdot R_1\cdot R_2\cdot\cos\left(\theta_1(x) + \frac{\pi}{2}\right)}$$

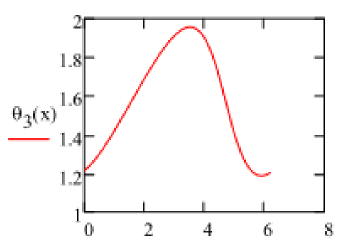
Magnitude of Postion Vector for Link 3



$R_3(b) = 12.816\cdot\text{in}$

$$\theta_3(x) := \frac{\pi}{2} - \text{asin}\left(\frac{R_2}{R_3(x)}\cdot\sin\left(\theta_1(x) + \frac{\pi}{2}\right)\right)$$

Angle between ground and link 4



$\theta_3(b) = 1.212\cdot\text{rad}$

Angle between transmision and crank

$$\theta_5(x) := \begin{cases} \text{acos}\left[\frac{\left[(R_2)^2 + (R_3(x))^2 - R_1{}^2\right]}{2\cdot R_2\cdot R_3(x)}\right] & \text{if } x \le \frac{3}{2}\pi - \frac{\alpha}{2} \vee x \ge \frac{3}{2}\pi + \frac{\alpha}{2} \\ \left[\pi - 1\ \text{acos}\left[\frac{\left[(R_2)^2 + (R_3(x))^2 - R_1{}^2\right]}{2\cdot R_2\cdot R_3(x)}\right]\right] & \text{otherwise} \end{cases}$$

$\theta_5(b) = 1.212 \cdot rad$

$X_4(x) := R_4 \cos(\theta_3(x))$ $\quad$ $X_4(b) = 8.427 \cdot in$ $\quad$ X and Y Components of link 4

$Y_4(x) := R_4 \cdot \sin(\theta_3(x))$ $\quad$ $Y_4(b) = 22.472 \cdot in$

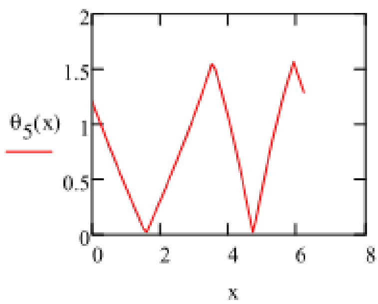
$$\theta_7(x) := \operatorname{asin}\left(\frac{Y_4(x) - R_6}{R_5}\right)$$

Angle of link 5 with relation to ground

$R_7(x) := X_4(x) + R_5 \cdot \cos(\theta_7(x))$

Postion of Slider/Link 6

$R_7(b) = 14.976 \cdot in$



Velocity Analysis

$$\omega_2 := \frac{\pi}{2} \frac{rad}{sec}$$

Crank Speed

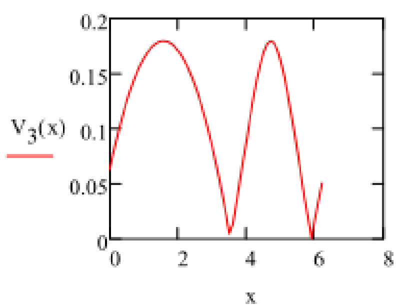
Speed at end of Crank 2

$V_2 := \omega_2 \cdot R_2 = 7.069 \cdot \frac{in}{s}$

Velocity Of Transmission = V_3

$V_3(x) := V_2 \cdot \cos(\theta_5(x))$

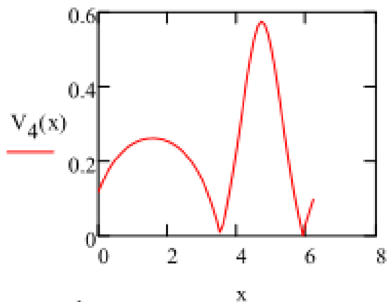
$V_3(b) = 2.482 \cdot \frac{in}{s}$



Velocity of link 4

$$\omega_4(x) := \frac{V_3(x)}{R_3(x)} \qquad \omega_4(b) = 0.194 \cdot \frac{rad}{s}$$

$$V_4(x) := \omega_4(x) \cdot R_4 \qquad V_4(b) = 4.648 \cdot \frac{in}{s}$$



$$V_7(x) := \frac{d}{dx} R_7(x) \qquad V_7(b) = -3.163 \cdot in$$

Velocity of Slide/Link 6

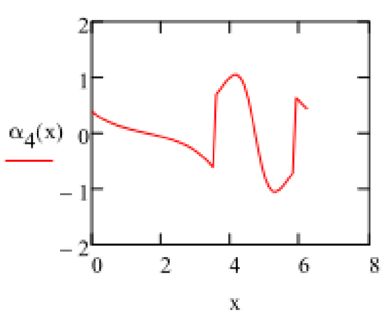
Acceleration Analysis

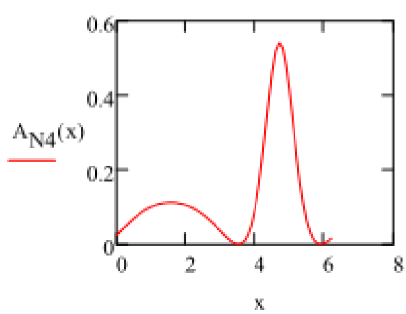
$$A_{N4}(x) := R_4 \cdot \left(\omega_4(x)^2\right)$$

$$A_{N4}(b) = 0.9 \cdot \frac{in}{s^2}$$

$$\alpha_4(x) := \frac{d}{dx} \omega_4(x)$$

$$\alpha_4(b) = 0.389 \frac{1}{s}$$





Force Analysis

$$mass_1 := \frac{.21}{386} \frac{lb \cdot s^2}{in} \qquad mass_4 := \frac{.78}{386} \frac{lb \cdot s^2}{in}$$

$$mass_2 := \frac{.12}{386} \frac{lb \cdot s^2}{in} \qquad mass_{sl} := \frac{.01}{386} \frac{lb \cdot s^2}{in}$$

CRANK

$$R_{12x}(x) := \frac{R_2}{2} \cdot \cos(\theta_1(x) + \pi)$$

$$R_{12x}(b) = -0.057\ m$$

$$R_{12y}(x) := \frac{R_2}{2} \cdot \sin(\theta_1(x) + \pi)$$

$$R_{12y}(b) = 0\ m$$

$$R_{32x}(x) := \frac{R_2}{2} \cdot \cos(\theta_1(x))$$

$$R_{32x}(b) = 0.057\ m$$

$$R_{32y}(x) := \frac{R_2}{2} \cdot \sin(\theta_1(x))$$

$$R_{32y}(b) = 0\ m$$

$$a_{cg2x}(x) := \frac{R_2}{2} \cdot (\omega_2)^2 \cdot \cos(\theta_1(x) + \pi)$$

$$a_{cg2x}(b) = -0.141\ \frac{m}{s^2}$$

$$a_{cg2y}(x) := \frac{R_2}{2} \cdot (\omega_2)^2 \cdot \sin(\theta_1(x) + \pi)$$

$$a_{cg2x}(b) = -0.141\ \frac{m}{s^2}$$

$$F_{32x}(x) + F_{12x}(x) := mass_2 \cdot a_{cg2x}(x)$$

$$F_{32y}(x) + F_{12y}(x) := mass_2 \cdot a_{cg2y}(x)$$

$$T_{motor}(x) + (R_{12x}(x) \cdot F_{12y}(x) - R_{12y}(x) \cdot F_{12x}(x)) + (R_{32x}(x) \cdot F_{32y}(x) - R_{32y}(x) \cdot F_{32x}(x)) := I_{cg}$$

Slider

$$a_{cg3x}(x) := R_3(x)\cdot\left(\omega_4(x)\right)^2\cdot\cos\left(\theta_3(x)+\pi\right)$$

$$a_{cg3x}(b) = -4.287\times10^{-3}\,\frac{m}{s^2}$$

$$a_{cg3y}(x) := R_3(x)\cdot\left(\omega_4(x)\right)^2\cdot\sin\left(\theta_3(x)+\pi\right)$$

$$a_{cg3x}(b) = -4.287\times10^{-3}\,\frac{m}{s^2}$$

$$F_{43x}(x) - F_{32x}(x) := mass_3\cdot a_{cg3x}(x)$$

$$F_{43y}(x) - F_{32y}(x) := mass_3\cdot a_{cg3y}(x)$$

ARM

$$R_{14x}(x) := \frac{R_4}{2}\cdot\cos\left(\theta_3(x)+\pi\right)$$

$$a_{cg4x}(x) := \frac{R_4}{2}\cdot\left(\omega_4(x)\right)^2\cdot\cos\left(\theta_1(x)+\pi\right)$$

$$R_{14y}(x) := \frac{R_4}{2}\cdot\sin\left(\theta_3(x)+\pi\right)$$

$$R_{54x}(x) := \frac{R_4}{2}\cdot\cos\left(\theta_3(x)\right) \qquad a_{cg4y}(x) := \frac{R_4}{2}\cdot\left(\omega_4(x)\right)^2\cdot\sin\left(\theta_1(x)+\pi\right)$$

$$R_{54y}(x) := \frac{R_4}{2}\cdot\sin\left(\theta_3(x)\right)$$

$$R_{34x}(x) := \begin{cases}\left(R_3(x)-\dfrac{R_4}{2}\right)\cdot\cos\left(\theta_3(x)\right) & \text{if } R_3(x)\ge\dfrac{R_4}{2}\\ \left[\left(\dfrac{R_4}{2}-R_3(x)\right)\cdot\cos\left(\theta_3(x)\right)\right] & \text{otherwise}\end{cases}$$

$$R_{34y}(x) := \begin{cases}\left(R_3(x)-\dfrac{R_4}{2}\right)\cdot\sin\left(\theta_3(x)\right) & \text{if } R_3(x)\ge\dfrac{R_4}{2}\\ \left[\left(\dfrac{R_4}{2}-R_3(x)\right)\cdot\sin\left(\theta_3(x)\right)\right] & \text{otherwise}\end{cases}$$

$$F_{14x}(x) - F_{43x}(x) + F_{54x} := mass_4\cdot a_{cg4x}(x)$$

$$F_{14y}(x) - F_{43y}(x) + F_{54y} := mass_4\cdot a_{cg4y}(x)$$

$$\left(R_{14x}(x)\cdot F_{14y}(x) - R_{14y}(x)\cdot F_{14x}(x)\right) + \left(R_{34y}(x)\cdot F_{43x}(x) - R_{34x}(x)\cdot F_{43y}(x)\right) + \left(R_{54x}(x)\cdot F_{54y}(x) - \right.$$